On the Distribution of Estates and the Distribution of Wealth:

Evidence from the Dead

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A Estate data coverage

Estate data usually represent only a share of the decedent population, with substantial heterogeneity across countries. Figure 5 shows the share of the decedent population represented in the data in the group of countries analyzed in Figure 1.

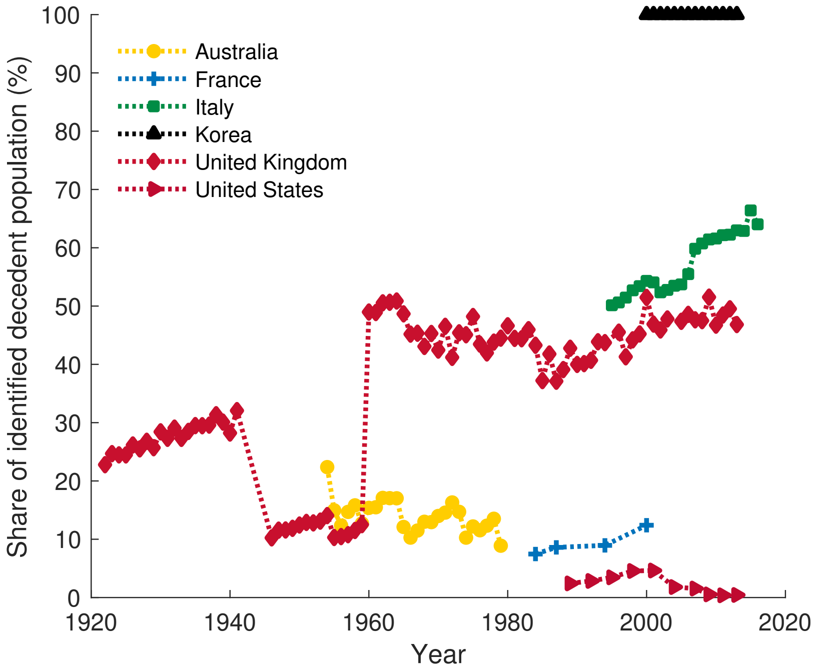

Figure 5: The share of decedents covered in the estate data in Australia, France, Italy, Korea, the United Kingdom, and the United States. The data were taken from Katic and Leigh (2016), Garbinti, Goupille-Lebret and Piketty (2020), Acciari, Alvaredo and Morelli (2020), Kim (2018), Alvaredo, Atkinson and Morelli (2018), and Saez and Zucman (2016), respectively, combined with mortality data from The Human Mortality Database (2018).

B Which homogeneous multiplier?

For implementing the simplified mortality multiplier method one has to choose the homogeneous multiplier to be applied. The choice in the average multiplier of the adult population, $\bar{m}$, is only one possible choice. In the absence of detailed demographic data for the decedents included in the estate tax records, it is possible to use an approximation for their mortality multiplier to estimate the top wealth shares. Even when demographic data are available, homogeneous multipliers can simplify the estimation process. We list below several possible choices of a homogeneous multiplier and present the differences between them, and demonstrate their corresponding resulting top wealth shares in the case of Italy:

- m_1: A simple and natural choice of such a multiplier is the average multiplier $\bar{m}$, which is the ratio between the population size of the living and the dead. Considering such a multiplier makes an implicit assumption that the mortality rate of the observed decedents is similar to that of the unobserved decedents.

- m_2: If detailed demographic data are available, it is possible to take the arithmetic average of the individual multiplier m_i. m_2 is expected to be lower than m_1, since the average multiplier among the observed decedents tends to be lower than the average multiplier; however, this is not always the case.

- m_3: m_2 changes the identified wealth compared to the case in which the individual multiplier m_i is considered, because $\Sigma_i^{N_E} m_i w_{E,i} \neq \frac{\Sigma_i^{N_E} m_i}{N_E} \Sigma_i^{N_E} w_{E,i}$. Another possible choice of homogeneous multiplier would be a multiplier that is consistent with the identified wealth: $m_3 = \frac{\Sigma_i^{N_E} m_i w_{E,i}}{\Sigma_i^{N_E} w_{E,i}}$.

- m_4: If no demographic data are available, but mortality data are, it is possible to assume that the representative multiplier of the observed decedents is the multiplier that corresponds to an individual whose age is the average age at death, based on the mortality data. Typically, since this age is higher than the average age of decedents in the tax records, this multiplier will be substantially lower than the other choices of multiplier.

- m_5: The same m_2 but after adding a mortality-wealth gradient to the demographic data to obtain heterogeneous individual multipliers.

The evolution of these multipliers over time and the resulting top 1% wealth shares are presented in Figure 6. The choice of a homogeneous multiplier matters for the estimated top shares. Yet almost all the options considered lead to levels of inequality that closely follow the results when heterogeneous multipliers are used.

The major exception is m_4, the multiplier that corresponds to the average age of decedents in a given year. m_4 is much lower than the other suggested choices, as it effectively ignores the

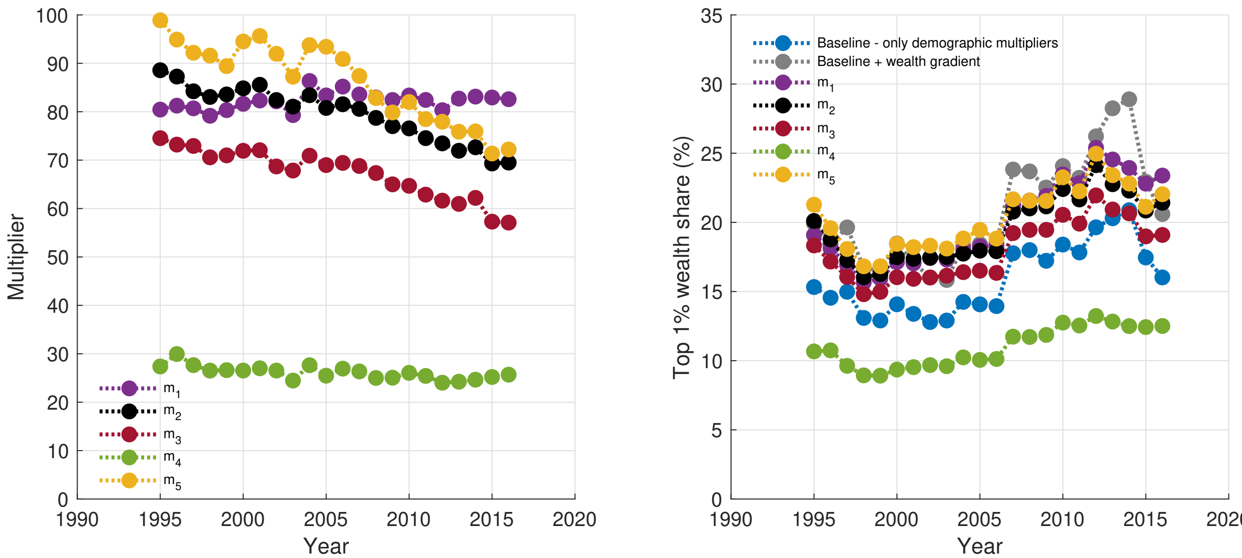

Figure 6: Homogeneous mortality multipliers in Italy 1995–2016. Mortality data were taken from The Human Mortality Database (2018), and the estate tabulations and demographic data were taken from Acciari, Alvaredo and Morelli (2020). The mortality-wealth gradients used were those used for France in Garbinti, Goupille-Lebret and Piketty (2020). The homogeneous multipliers used are: m_1—the average multiplier $\bar{m}$, the ratio between the population size of the living and the dead; m_2—the arithmetic average of the individual multiplier m_i; $m_3 = \frac{\Sigma_i^{N_E} m_i w_{E,i}}{\Sigma_i^{N_E} w_{E,i}}$; m_4—the multiplier that corresponds to an individual whose age is the average age at death, based on the mortality data; m_5 – the same m_2 but after adding a mortality-wealth gradient to the demographic data.

presence of younger decedents among the wealthiest decedents. As seen previously for France in Figure 2, top wealth groups include a significant presence of younger individuals. Since mortality rates are approximately exponential in age, the impact of these younger individuals on the most representative multiplier for decedents is, in fact, substantial.[6]

[6] This is a direct implication of Jensen's inequality for the exponential function:

$$\mathrm{Exp}\left[E\left[a\right]\right] < \mathrm{E}\left[\mathrm{Exp}\left[a\right]\right] \ , \tag{B.1}$$

where a is decedent age. Because the multipliers depend exponentially on age, the multiplier corresponding to the average age at death is much lower than the average multiplier. Had the dependence of the mortality rate on age been linear, for example, the two quantities would have been equal.

C The concentration of estates at the top

As shown by Alvaredo, Atkinson and Morelli (2018), relying on unadjusted tax data on estate value can also be informative about the concentration of wealth at the top. To show that, we first need to define the top estate share of quantile q

$$(1-L_q)^E = \frac{\sum_{i=1}^{qN_E} w_{E,i}}{W_E}\,. \tag{C.1}$$

This requires summing the estates of the richest qN_E decedents and estimating the total value of estates of the full decedent population, W_E. However, the estimation of the latter is not a trivial exercise. It requires the estimation of the total value of unobserved estates of the deceased excluded from the tax records, W_E^{exc}. This creates uncertainty in the top estate share estimates.

In practice, estimating W_E^{exc} can be done using the total wealth of the living population not represented by the re-weighted tax records (excluded population), $N^{exc} = N - \sum_{i=1}^{N_E} m_i$. The latter can be directly estimated from external sources of data, such as surveys or other administrative records, if the general identity of the excluded population could be inferred.

The total identified wealth is known through the multipliers and observed estate values:

$$W^{iden} = \sum_{i=1}^{N_E^{tax}} m_i w_{E,i}\,. \tag{C.2}$$

In the absence of heterogeneous multipliers this becomes

$$W^{iden} = \sum_{i=1}^{N_E^{tax}} \bar{m} w_{E,i} = \bar{m} W_E^{iden}\,. \tag{C.3}$$

The total excluded wealth is then

$$W^{exc} = W - W^{iden}\,. \tag{C.4}$$

At the same time

$$W^{exc} = \bar{m}^{exc} W_E^{exc}\,, \tag{C.5}$$

where $\bar{m}^{exc}$ is the average multiplier of the excluded decedents. $\bar{m}^{exc}$ can be estimated depending on how refined the demographic data and mortality data available are. If mortality by age and gender is available, it is possible to define a different multiplier for the excluded decedents in each age and gender:

$$m_{a,g}^{exc} = \frac{N_{a,g}^{exc}}{N_{E,a,g}^{exc}}\,, \tag{C.6}$$

where $N_{a,g}^{exc}$ is the number of living with age a and gender g not observed by the tax records, and $N_{E,a,g}^{exc}$ is the number of decedents with age a and gender g not observed by the tax records. In this case $\bar{m}^{exc}$ would be the average of all multipliers $m^{exc}_{a,g}$. Alternatively, in the absence of such data, $\bar{m}^{exc}$ can be defined as the ratio between the excluded living population and the excluded decedent population:

$$\bar{m}^{exc} = \frac{N^{exc}}{N_E - N_E^{tax}} \,. \tag{C.7}$$

It is clear that different sets of multipliers would lead to different estimates of W_E^{exc}. This leads to different total values of estates, which, in turn, leads to different top estate share estimates. In Sec. 3 we use this calculation to provide different estimates of top estate shares and compare them to top wealth shares reported in the literature.

D Mortality rates by age

Age is the most important statistical determinant of mortality. Figure 7 shows the mortality rates in France, Italy, the United Kingdom, and the United States in 1950, 1970, 1990, and 2010, based on The Human Mortality Database (2018) data. It illustrates that mortality rates increase exponentially with age above the age of 40.

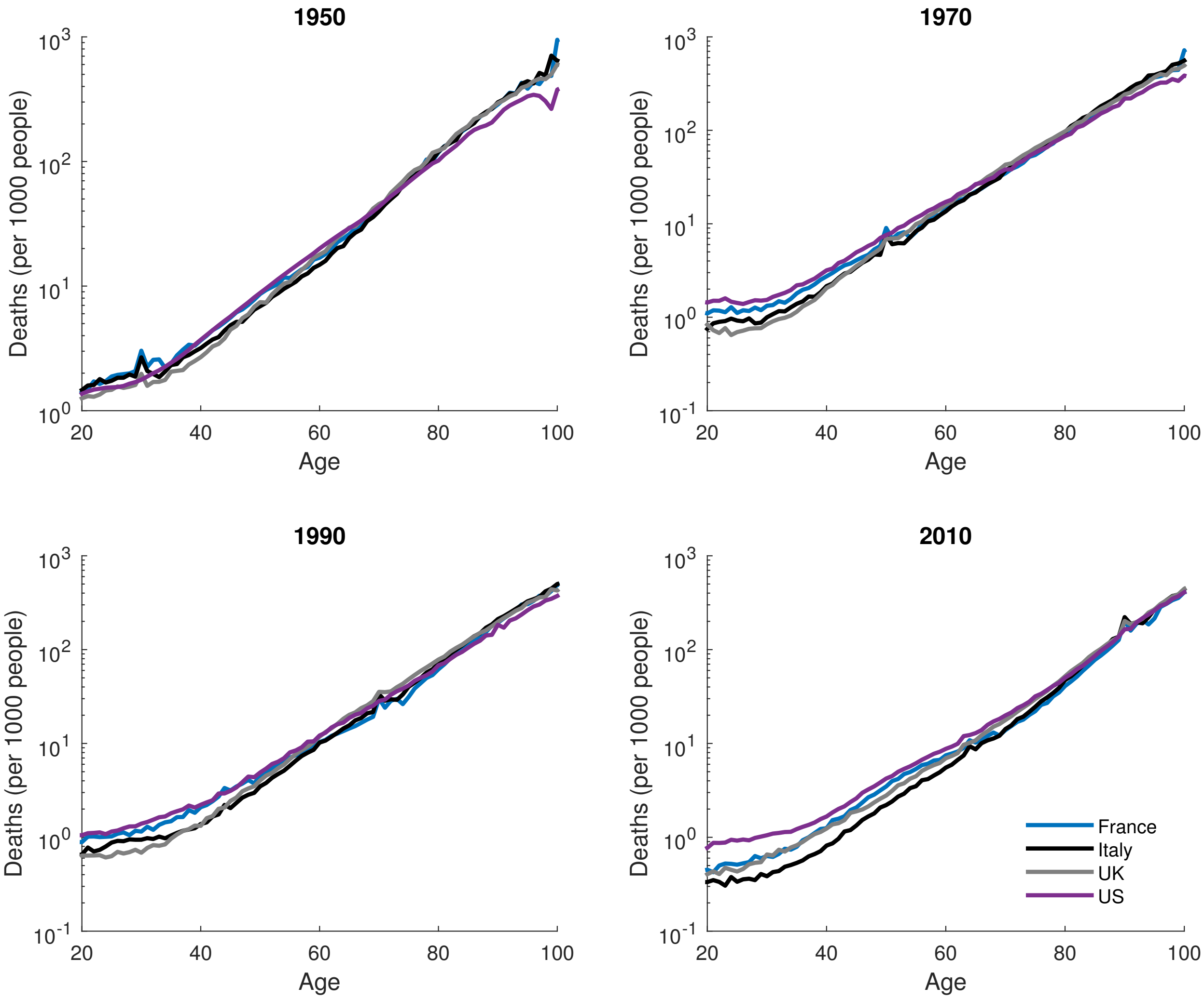

Figure 7: Mortality rates in France, Italy, the United Kingdom, and the United States in 1950, 1970, 1990, and 2010. Source: The Human Mortality Database (2018).

E The coefficient of variation of estates and of wealths

To illustrate the similarity between the concentration of wealth and of estates it is possible to compare the coefficient of variation (CV) of the wealth distribution with and without multipliers. The derivation is inspired by a derivation presented in Atkinson and Harrison (1978), comparing the CV between capital income and wealth for the capitalization method. It clarifies the intuition for the result obtained for top shares discussed above. Yet it is conceptually simpler, since the index I_q does not play a role in the CV. It is also not limited to a specific quantile q, but involves the entire distribution.

The coefficient of variation of estates, denoted Y_E, follows

$$Y_E^2 = \frac{\sigma_E^2}{\bar{w}_E^2} \,. \tag{E.1}$$

The coefficient of variation of wealths, denoted Y_E, follows

$$Y_W^2 = \frac{\sigma_W^2}{\bar{w}_W^2} \,, \tag{E.2}$$

where σ_E^2 is the variance of estates, σ_W^2 is the variance of wealths, $\bar{w}_E$ is the average estate, and $\bar{w}_W$ is the average wealth.

We begin by writing down expressions for the variance estates and wealths:

$$\sigma_E^2 \quad = \tfrac{1}{N_E} \textstyle\sum_{i=1}^{N_E} w_{E,i}^2 - \tfrac{1}{N_E^2} \left(\textstyle\sum_{i=1}^{N_E} w_{E,i} \right)^2 ; \tag{E.3}$$

$$\sigma_W^2 \quad = \tfrac{1}{N} \textstyle\sum_{i=1}^{N_E} m_i w_{E,i}^2 - \tfrac{1}{N^2} \left(\textstyle\sum_{i=1}^{N_E} m_i w_{E,i} \right)^2 . \tag{E.4}$$

Therefore we get

$$Y_E^2 = \frac{\tfrac{1}{N_E} \sum_{i=1}^{N_E} w_{E,i}^2 - \tfrac{1}{N_E^2} \left(\sum_{i=1}^{N_E} w_{E,i} \right)^2}{\tfrac{1}{N_E^2} \left(\sum_{i=1}^{N_E} w_{E,i} \right)^2} \,, \tag{E.5}$$

and

$$Y_W^2 = \frac{\tfrac{1}{N} \sum_{i=1}^{N_E} m_i w_{E,i}^2 - \tfrac{1}{N^2} \left(\sum_{i=1}^{N_E} m_i w_{E,i} \right)^2}{\tfrac{1}{N^2} \left(\sum_{i=1}^{N_E} m_i w_{E,i} \right)^2} \,. \tag{E.6}$$

μ is the ratio between the average estate and the average wealth

$$\mu = \frac{\tfrac{1}{N_E} \sum_{i=1}^{N_E} w_{E,i}}{\tfrac{1}{N} \sum_{i=1}^{N_E} m_i w_{E,i}} = \bar{m} \frac{\sum_{i=1}^{N_E} w_{E,i}}{\sum_{i=1}^{N_E} m_i w_{E,i}} \,, \tag{E.7}$$

so

$$\frac{1}{N^2}\left(\sum_{i=1}^{N_E} m_i w_{E,i}\right)^2 = \frac{1}{\mu^2}\cdot\frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2 , \tag{E.8}$$

and therefore

$$Y_W^2 = \frac{\frac{1}{N}\sum_{i=1}^{N_E}\mu^2 m_i w_{E,i}^2 - \frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2}{\frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2} . \tag{E.9}$$

We can then rearrange Y_W^2 and get

$$Y_W^2 = Y_E^2 - \frac{\frac{1}{N_E}\sum_{i=1}^{N_E} w_{E,i}^2}{\frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2} + \frac{\frac{1}{N}\sum_{i=1}^{N_E}\mu^2 m_i w_{E,i}^2}{\frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2} . \tag{E.10}$$

Taking $N = \bar{m} N_E$ we get

$$Y_W^2 = Y_E^2\left(1 + \frac{\frac{1}{N_E}\sum_{i=1}^{N_E}\left(\frac{\mu^2 m_i}{\bar{m}} - 1\right) w_{E,i}^2}{\sigma_E^2}\right) . \tag{E.11}$$

This result leads to several important observations that clarify the similarity between inequality of estates and of wealth. First, the difference between the CV of wealth and estates is mainly driven by the multipliers at the top of the distribution. This is because the difference $\left(\frac{\mu^2 m_i}{\bar{m}} - 1\right)$ is weighted by the level of estates. Thus, the similarity between Y_W and Y_E, like the top shares, mainly depends on the interaction between estates and multipliers among the richest decedents.

Second, there is a dampening effect that limits the extent to which Y_W and Y_E are distant from one another. If the multipliers at the top are high in comparison to the average multiplier then $m_i/\bar{m} > 1$. μ is then likely to be lower than 1. The inverse is true if $m_i/\bar{m} < 1$. This creates a dampening effect that makes the expressions $\left(\frac{\mu^2 m_i}{\bar{m}} - 1\right)$ in Eq. (E.11) generally close to 0.

Third, comparing the coefficients of variation further demonstrates that the similarity in inequality measures between estates and wealth may not be limited to top shares, but also when full distributions are taken into account.