

Appendix A.

This section briefly considers using the Make table to construct a commodity-level production account using publicly available (from BEA) summary level data.⁸ Summary-level data includes information at the two or three digit level and the common denominator between the input-output data and TFP data at this level of detail is the set of industries included in the BEA-BLS integrated industry level production account (63 total industries). The basic idea is that the Make table includes information on commodities produced by each industry. Therefore, one can use this information to back out commodity level TFP growth given observations on industry level TFP growth. This section lays out the basics and argues that while these estimates are potentially useful and easy to assemble, the underlying assumptions are stronger than those employed in this paper and may yield misleading results.

The basic approach of using the Make table to define a commodity-level production account can be summarized as:

$$\begin{bmatrix} \Delta \ln TFP_{I1} \\ \Delta \ln TFP_{I2} \\ \vdots \\ \Delta \ln TFP_{IN} \end{bmatrix} = \begin{pmatrix} m_{11} & \cdots & m_{1N} \\ m_{21} & \cdots & m_{2N} \\ \vdots & & \\ m_{N1} & \cdots & m_{NN} \end{pmatrix} \begin{bmatrix} \Delta \ln TFP_{C1} \\ \Delta \ln TFP_{C2} \\ \vdots \\ \Delta \ln TFP_{CN} \end{bmatrix}$$

Where the first vector on the left hand side is an Nx1 matrix of measured industry-level TFP growth rates. The right hand side is made up of an NxN matrix of shares of commodities produced by each industry (industry on rows and shares of each commodity in its industry output in each column) multiplied by an Nx1 vector of commodity-level TFP growth rates. This says that observed industry TFP growth is a linear combination of TFP growth rates of the various commodity it produces. Premultiplying both sides by the inverse of the Make share matrix yields an estimate of commodity-level growth TFP rates. To consider how this would work in practice, consider the following stylized example with a Computer hardware sector (Comp) and a Miscellaneous professional, scientific, and technical services sector (Misc). In the BEA-BLS industry production account, the Misc sector holds the establishments primarily engaged in R&D and other establishments that are in the same NAICS group. Suppose that the Make table and industry level TFP growth rates observed were:

$$\begin{bmatrix} 5.8\%_{I_Comp} \\ 0\%_{I_Misc} \end{bmatrix} = \begin{pmatrix} 0.85 & 0.15 \\ 0 & 1 \end{pmatrix} \begin{bmatrix} \Delta \ln TFP_{C_Comp} \\ \Delta \ln TFP_{C_Misc} \end{bmatrix}$$

Applying the matrix inverse to both sides imposes yield the following estimates for commodity-level TFP growth.

⁸ This alternative was considered in John Fernald's discussion of this paper at the March 2022 Conference on Research in Income and Wealth.

$$\begin{bmatrix} \Delta \ln TFP_{C_Comp} \\ \Delta \ln TFP_{C_Misc} \end{bmatrix} = \begin{bmatrix} 6.9\% \\ 0\% \end{bmatrix}$$

This is not unreasonable, but it is worth comparing this to the approach taken in this paper and how it misses underlying data. First, the Make table approach based on the level of detail in this example would impose that the measured TFP growth in the Misc sector corresponds exactly to the TFP growth of the Misc commodity because the Misc sector produces no Computer hardware output. Remember that the Misc sector in the actual data includes industries and commodities other than R&D, including Accounting, Architecture, Management consulting, Advertising, Photography, and Veterinary services. The summary-level Make table approach imposes that measured TFP growth in the Misc commodity produced by the Computer sector (which is R&D) has the same TFP growth rate as all of the industries included in the Misc. sector. That is, this method imposes that TFP growth of the R&D produced by the Comp sector is zero. Furthermore, if one wanted to apply this same approach to decomposing the sources of growth by replacing measured TFP growth with the sources of industry level growth and the sources of commodity level growth we get the equation:

$$\begin{bmatrix} \Delta \ln Y_{I_Comp} - w_{IK} \Delta \ln K_{I_Comp} - w_{IL} \Delta \ln K_{I_Comp} - w_{IX} \Delta \ln X_{I_Comp} \\ \Delta \ln Y_{I_Misc} - w_{IK} \Delta \ln K_{I_Misc} - w_{IL} \Delta \ln K_{I_Misc} - w_{IX} \Delta \ln X_{I_Misc} \end{bmatrix} = \begin{pmatrix} 0.85 & 0.15 \\ 0 & 1 \end{pmatrix} \begin{bmatrix} \Delta \ln Y_{C_Comp} - w_{CK} \Delta \ln K_{C_Comp} - w_{CL} \Delta \ln K_{C_Comp} - w_{CX} \Delta \ln X_{C_Comp} \\ \Delta \ln Y_{C_Misc} - w_{CK} \Delta \ln K_{C_Misc} - w_{CL} \Delta \ln K_{C_Misc} - w_{CX} \Delta \ln X_{C_Misc} \end{bmatrix}$$

If we associate each term in the industry growth equation with the corresponding term in the commodity equation, for example equating the output terms:

$$\begin{bmatrix} \Delta \ln Y_{I_Comp} \\ \Delta \ln Y_{I_Misc} \end{bmatrix} = \begin{pmatrix} 0.85 & 0.15 \\ 0 & 1 \end{pmatrix} \begin{bmatrix} \Delta \ln Y_{C_Comp} \\ \Delta \ln Y_{C_Misc} \end{bmatrix}$$

we see that we get the result that this method would tell us that output growth in the Misc commodity including Computer R&D would be equated to output growth in the Misc industry.

For example, if we observed output growing by 4.6% in the Computer sector and 2.95% in the Misc sector, this Make table method would tell us that Misc commodity output produced by the computer sector also grew by 2.95% per year, and applying this to the factors of production would result in equalized (to the Misc Sector) contributions of capital, labor, and intermediate inputs. While the Make table approach is somewhat intuitive for producing commodity-level TFP estimates, applying this to the sources of growth is less so. The 2.95% real growth of the Misc industry may be of little relevance in measuring the real output growth rate of R&D in the Computer sector. The equivalent decomposition could be done for prices with a similar result; the price growth in the Misc sector may be of little relevance for the price index for the R&D output priced by the Computer hardware sector. The approach taken in this paper is to use the detailed underlying BEA data to move the relevant R&D output and corresponding prices that are included in the measures of output at the industry level out of that sector and into the Misc sector. This gives a relatively clean match between the output that is being moved and its price index. Therefore, the new output in the Misc sector reflects the new nominal R&D output that

gets moved from the Computer sector and the price of this new output reflects the price of R&D that is actually produced by the Computer sector.

A second caveat in using the Make table approach at the summary level is that it assumes that all secondary commodities should be reallocated at summary level; that is, it imposes that the after redefinition Make table is diagonal. In practice, what this means is that in aligning commodities, the summary level Make table approach assumes that all secondary output is relevant for commodity level TFP estimates. While in many cases this may be reasonable, this does not coincide with the approach taken in the official after redefinition input-output accounts. For example, in the 2012 before redefinition Make table, about 12% of the output of the Broadcasting and telecommunications sector is Miscellaneous professional, scientific, and technical services. In the after redefinition table that has moved the commodities to their secondary industries, the Make table indicates that even after redefinitions, 12% of the gross output of the Broadcasting and telecommunications still is Miscellaneous professional, scientific, and technical services. What this means is that using the summary level Make table approach to determine TFP at the commodity level essentially removes this commodity in determining the after redefinition TFP growth rate of the Broadcasting and telecommunications sector even though the official after redefinition Make table indicates that this output should remain in the sector. Because the official After redefinition Make tables move output and intermediates to group items with similar production structures, this implies that the production process for the remaining 12% of gross output more closely aligns with the Broadcasting and telecommunications sector than the Miscellaneous professional, scientific, and technical services sector. Thus, the application of the summary level Make table approach would result in outputs and intermediates being reallocated to sectors that BEA has already ruled out as a target for realignment.