Inequality in Human Capital and Endogenous Credit Constraints: Web Appendix[*]

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Contents

A   Web Appendix

A.1   Data and Basic Analysis

Table A1: Sample Selection

	Observation Left
Total observations for all individuals	124,099
Keep if male	62,620
Keep if white	30,925
Drop if missing schooling or working information for the entire sample period	25,639

Figure A1: Weeks Worked and Hours Worked Per Week
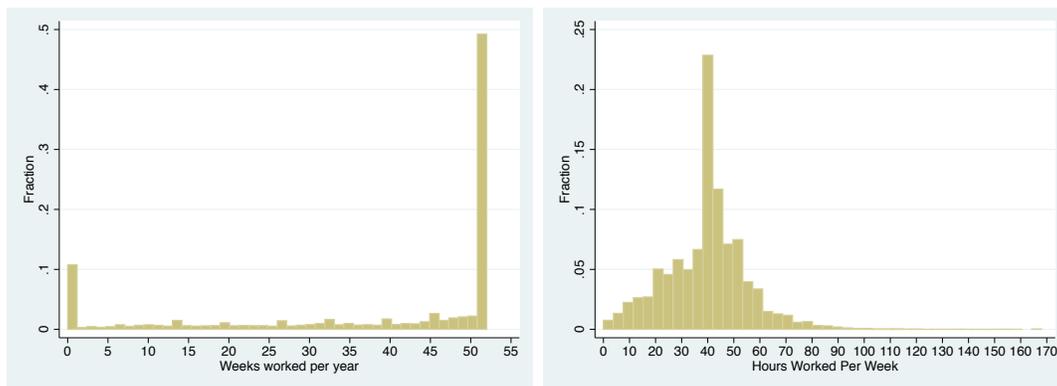
Source: NLSY97 white males.

Figure A2: Total Hours Worked Per Year (Full-time Employed)
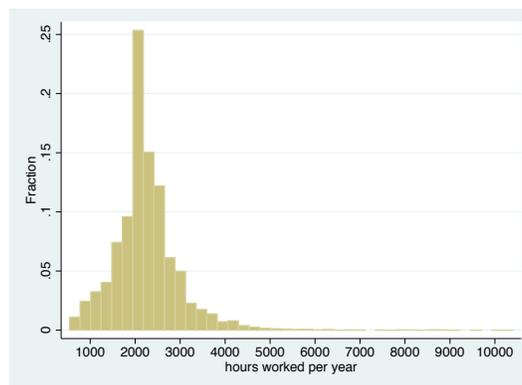
Source: NLSY97 white males.

Table A2: Descriptive Statistics of NLSY97 Sample (Years 1997 to 2011)

	mean	sd	min	max	N
Age	24.04	4.42	17.00	33.00	31,403
Education	12.79	2.38	9.00	20.00	27,828
Years Worked	3.27	3.49	0.00	15.00	26,821
In School	0.24	0.42	0.00	1.00	28,738
Full-Time Working	0.57	0.50	0.00	1.00	28,174
Part-Time Working	0.21	0.41	0.00	1.00	28,174
Full-Time Hourly Wage	13.93	9.44	3.00	123.55	10,874
Part-Time Hourly Wage	10.27	11.82	3.00	200.00	4,609
Net Worth	14917.95	30063.15	-75000.00	100000.00	6,881
Total Parental Transfers	803.20	2800.01	0.00	30000.00	24,882
Parents' Education	13.18	1.95	8.00	16.00	30,393
Parents' Net Worth	162988.31	186831.49	-11769.47	706168.25	24,074

Figure A3: Years Worked
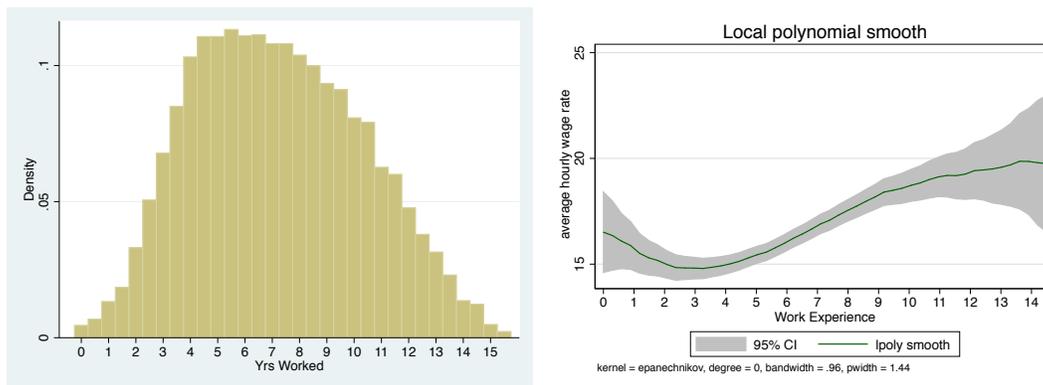
Source: NLSY97 white males who always work upon completing the highest grades.

Table A3: Key Variables by Education (Age 25)

	<12 yrs	12 yrs	13 to 15 yrs	>=16 yrs
Years Worked	5.71	5.30	3.79	1.75
Net Worth	14657.63	24033.25	21432.79	19242.44
Full-Time Hourly Wage	11.06	14.01	15.21	17.26
Part-Time Hourly Wage	15.15	20.78	13.99	14.59

Table A4: OLS regression: Log Hourly Wage

	(1)	(2)	(3)	(4)
Years of Schooling after 4-Year College	0.0151	0.0465**	0.00989	0.0382**
	(0.018)	(0.017)	(0.015)	(0.015)
Years of Schooling $\geq$ 12	0.0926*	0.134**	0.167**	0.212**
	(0.049)	(0.048)	(0.045)	(0.044)
Years of Schooling $\geq$ 14	-0.0107	0.0535*	-0.0125	0.0533**
	(0.027)	(0.028)	(0.026)	(0.026)
Years of Schooling $\geq$ 16	0.141**	0.224**	0.0809**	0.174**
	(0.042)	(0.042)	(0.035)	(0.035)
Years Worked	0.0541**		0.0645**	
	(0.018)		(0.016)	
Years Worked Squared	-0.000960		-0.00134	
	(0.001)		(0.001)	
Yrs Worked (Post School)		0.0816**		0.0807**
		(0.013)		(0.011)
Yrs Worked Squared (Post School)		-0.00272**		-0.00244**
		(0.001)		(0.001)
Cognitive Ability $\times$ High School Dropout	0.0556*	0.0504*	0.0642**	0.0600**
	(0.031)	(0.030)	(0.028)	(0.027)
Noncognitive Ability $\times$ High School Dropout	0.0888**	0.0917**	0.0470	0.0483
	(0.035)	(0.035)	(0.030)	(0.030)
Cognitive Ability $\times$ Years of Schooling 12 to 15	0.0633**	0.0615**	0.0314**	0.0295**
	(0.014)	(0.014)	(0.013)	(0.013)
Noncognitive Ability $\times$ Years of Schooling 12 to 15	0.0600**	0.0609**	0.0648**	0.0667**
	(0.012)	(0.012)	(0.012)	(0.012)
Cognitive Ability $\times$ 4-Year College Graduate	0.120**	0.113**	0.141**	0.123**
	(0.027)	(0.026)	(0.022)	(0.021)
Noncognitive Ability $\times$ 4-Year College Graduate	0.0817**	0.0588**	0.103**	0.0778**
	(0.031)	(0.030)	(0.022)	(0.022)
Work Part-Time			0.0108	0.0226
			(0.054)	(0.053)
Constant	2.169**	2.087**	2.021**	1.981**
	(0.087)	(0.064)	(0.074)	(0.056)
Observations	2600	2693	3534	3649
Adjusted R^2	0.1395	0.1531	0.1346	0.1398
Var of Error Term	0.26	0.25	0.29	0.29
Var of Log Wage	0.31	0.31	0.31	0.31
Mean of Log Wage	2.63	2.63	2.63	2.63

Standard errors in parentheses

Columns (1)-(2): always works full-time after leaving school; Columns (3)-(4): always works after leaving school.

* $p < 0.10$, ** $p < 0.05$

A.2   Additional Parameterizations

We estimate the following Tobit model as a function not only of parents' education and net worth terciles but also of individuals' decisions on schooling and employment:

$$
\begin{aligned}
\log(tr_{p,t}+1) =& \beta_{\text{tr},p,1}d_{e,t}\mathbf{1}(d_{e,t}+e_t \geq 13 \ \& \ d_{e,t}+e_t \leq 14) \\
& + \beta_{\text{tr},p,2}d_{e,t}\mathbf{1}(d_{e,t}+e_t \geq 15 \ \& \ d_{e,t}+e_t \leq 16) + \beta_{\text{tr},p,3}d_{e,t}\mathbf{1}(d_{e,t}+e_t > 16) \\
& + \beta_{\text{tr},p,4}\mathbf{1}(d_{k,t}=0 \ \& \ d_{e,t}=0) \\
& + \beta_{\text{tr},p,5}t + \beta_{\text{tr},p,6}t \cdot d_{e,t}\mathbf{1}(d_{e,t}+e_t \geq 13) + \beta_{\text{tr},p,7}\mathbf{1}(t=17) + \beta_{\text{tr},p,8}\mathbf{1}(t>23) \\
& + \beta_{\text{tr},p,9}\mathbf{1}(s_p=T2) + \beta_{\text{tr},p,10}\mathbf{1}(s_p=T3) \\
& + \beta_{\text{tr},p,11}\mathbf{1}(e_p=12) + \beta_{\text{tr},p,12}\mathbf{1}(e_p \geq 13 \ \& \ \leq 15) \\
& + \beta_{\text{tr},p,13}\mathbf{1}(e_p \geq 16) + \beta_{\text{tr},p,14}\mathbf{1}(e_p \geq 16 \ \& \ s_p=T3) \\
& + \beta_{\text{tr},p,15}\mathbf{1}(e_p=12)d_{e,t}\mathbf{1}(d_{e,t}+e_t \geq 13) \\
& + \beta_{\text{tr},p,16}\mathbf{1}(e_p \geq 13 \ \& \ \leq 15)d_{e,t}\mathbf{1}(d_{e,t}+e_t \geq 13) \\
& + \beta_{\text{tr},p,17}\mathbf{1}(e_p \geq 16)d_{e,t}\mathbf{1}(d_{e,t}+e_t \geq 13) \\
& + \beta_{\text{tr},p,18}\theta_c + \beta_{\text{tr},p,19}\theta_n + \beta_{\text{tr},p,0} + \epsilon_{p,t}.
\end{aligned}
\tag{1}
$$

In our data, we do not observe parental transfers beyond age 30. However, given that we are focusing on parental transfer targeting the youth's college education, it is reasonable to assume that parental transfers are zero for a youth after age 30 due to data limitation, i.e. $tr_{p,t}=0$ for $t \geq 30$. This assumption is appropriate as we focus on the role of parental transfer on young adults' college attainment and the majority of individuals obtain their college degree before age 30. For the same reason, here we only focus on non-negative parental transfers. Negative parental transfers may be important for older adults and their parents for a different purpose, but it is outside the scope of this paper.

Parental consumption subsidy is given as $tr_{c,t} = \chi \cdot \mathbf{1}(t < 18)$, where χ is the value of direct consumption subsidy provided by the parents such as shared housing and meals when

the youth attends high school.

Government transfers, $tr_{g,t}$, are comprised of two components: an unemployment benefit component $tr_{g,t}^b$ that is offered for individuals who are currently unemployed ($\mathbf{1}(d_{k,t} = 0 \ \& \ d_{e,t} = 0)$), and a means-tested component $tr_{g,t}^c$ that supports a minimum consumption floor c_{min}: i.e., $tr_{g,t} = tr_{g,t}^b \mathbf{1}(d_{k,t} = 0 \ \& \ d_{e,t} = 0) + tr_{g,t}^c$.

Government means-tested transfers $tr_{g,t}^c$ bridge the gap between an individual's available financial resources and the consumption floor c_{min}. Hubbard, Skinner, and Zeldes (1995), Keane and Wolpin (2001), and French and Jones (2011) show that allowing for the effects of means-tested benefits is important in understanding savings behavior of poor households.

A.3 Parameter Estimates

Table A5: Estimation of Parental Transfer Function: Log (Parental Transfers + 1)

	(1)	
In First/Second Year College	6.534**	(0.657)
In Third/Fourth Year College	7.713**	(0.712)
In Graduate School	6.768**	(0.795)
Unemployed	0.258**	(0.076)
Age	-0.215**	(0.014)
Age * in School	-0.286**	(0.030)
Age 17	-0.753**	(0.102)
> Age 23	-0.173*	(0.097)
Parents' Net Worth T2	-0.078	(0.086)
Parents' Net Worth T3	0.096	(0.092)
Parents' Schooling 12 Years	0.042	(0.079)
Parents' Schooling 13 to 15 Years	0.402**	(0.079)
Parents' Schooling $\geq$ 16 Years	0.813**	(0.116)
Parents' Schooling $\geq$ 16 Years * Parents' Net Worth T3	0.363**	(0.123)
Parents' Schooling 12 Years * in School	1.076**	(0.278)
Parents' Schooling 13 to 15 Years * in School	1.319**	(0.260)
Parents' Schooling $\geq$ 16 Years * in School	1.667**	(0.267)
Cognitive Ability	0.134**	(0.026)
Noncognitive Ability	0.149**	(0.042)
Constant	6.309**	(0.307)
Observations	14788	
R^2	0.252	

Standard errors in parentheses

Parental transfers are in 2004 dollars.

* $p < 0.10$, ** $p < 0.05$

Table A6: Parameter Estimates of Joint Initial Distribution of (θ_c, θ_c)*

	θ_c: Cognitive	θ_n: Noncognitive
	Mean	
Parents Wealth 3rd Tercile	0.304	1.507
	(0.050)	(0.093)
Parents Wealth 2nd Tercile	0.000	1.027
	(0.050)	(0.085)
Parents 4-Yr College	0.354	1.378
	(0.069)	(0.141)
Parents Some College	0.314	0.782
	(0.043)	(0.077)
Parents High School	0.241	0.570
	(0.055)	(0.107)
Constant	-0.541	-1.545
	(N.A.)	(N.A.)
	Variance Matrix	
θ_c: Cognitive	1.000	
	(N.A.)	
θ_n: Noncognitive	0.280	1.000
	(0.050)	(N.A.)

Constant terms are normalized such that $\mathbb{E}(\theta_c) = \mathbb{E}(\theta_n) = 0$. *Standard errors in parentheses.

Table A7: Parameter Estimates of Measurement Equations

	ASVAB: Arithmetic Reasoning	ASVAB: Mathematics Knowledge	ASVAB: Paragraph Comprehension	ASVAB: Word Knowledge	Noncognitive: Violent Behavior (1997)	Noncognitive: Had Sex bef. Age 15	Noncognitive: Theft Behavior (1997)
Cognitive Ability	0.743	0.696	0.654	0.569			
	(0.013)	(0.015)	(0.017)	(0.015)			
Noncognitive Ability					-0.661	-1.181	-0.369
					(0.035)	(0.079)	(0.029)
Age in 1997	0.151	0.255	0.152	0.192	0.103	0.104	0.097
	(0.001)	(0.001)	(0.001)	(0.001)	(0.003)	(0.005)	(0.002)
Parental Wealth 3rd Tercile	(0.069)	0.134	0.084	0.017	0.567	1.012	0.448
	0.024	0.026	0.026	0.028	0.088	0.170	0.055
Parental Wealth 2nd Tercile	0.193	0.203	0.242	0.179	0.435	0.814	0.236
	(0.027)	(0.025)	(0.028)	(0.029)	(0.081)	(0.134)	(0.057)
Parents' Yrs of Schooling	0.106	0.110	0.108	0.115	0.030	-0.014	0.018
	(0.001)	(0.001)	(0.001)	(0.001)	(0.004)	(0.006)	(0.002)
Constant	-4.114	-5.600	-4.228	-4.932	-2.218	-2.094	-2.190
	(0.015)	(0.015)	(0.015)	(0.016)	(0.047)	(0.080)	(0.030)
Measurement Error SD	0.426	0.448	0.492	0.518	0.978	1.452	0.463
	(0.012)	(0.011)	(0.011)	(0.010)	(0.059)	(0.096)	(0.026)

Table A8: Parameter Estimates of Flow Utility Function on Schooling and Working (Our Model *vs.* Alternative Model)

		Our Model		Alternative Model	
Description	Parameter	Estimate	S.E.	Estimate	S.E.
Panel A: $d_{e,t} \cdot u_e(\Omega_t)$					
Attending High School	$\phi_{e,0}$	0.4386	0.0113	0.4510	0.0102
Attending College	$\phi_{e,1}$	0.0000	0.0049	-0.0172	0.0029
Attending College After Age 23	$\phi_{e,a}$	-0.1868	0.0075	-0.1605	0.0041
Attending Graduate School	$\phi_{e,2}$	-0.4275	0.0066	-0.4477	0.0033
Schooling × Cognitive Ability	$\alpha_{e,c}$	0.1825	0.0033	0.2537	0.0018
Schooling × Noncognitive Ability	$\alpha_{e,n}$	0.2160	0.0041	0.2508	0.0023
Schooling × Parents 4-Year College	$\phi_{e,p}$	0.0699	0.0061	0.0050	0.0030
Psychic Cost of Returning to School	$\phi_{e,e}$	0.3596	0.0059	0.6515	0.0042
S.D. of Preference Shock to Schooling	σ_e	0.1675	0.0039	0.1000	0.0036
Panel B: $u_k(d_{k,t}, d_{e,t}, \Omega_t)$					
Working Part Time When in School	$\phi_{k,e}$	-0.0773	0.0020	-0.0603	0.0014
Working Part Time When Not in School	$\phi_{k,0}$	-0.0332	0.0010	-0.0338	0.0008
Working Full Time	$\phi_{k,1}$	-0.0822	0.0010	-0.0615	0.0007
Working Full Time × Age	$\phi_{k,2}$	0.0029	0.0001	0.0006	0.0001
Working × Cognitive Ability	$\alpha_{k,c}$	-0.0637	0.0029	-0.0237	0.0084
Working × Noncognitive Ability	$\alpha_{k,n}$	-0.0814	0.0048	-0.0942	0.0048

$$
\begin{aligned}
u_e =& \phi_{e,0}\mathbf{1}(d_{e,t} + e_t \le 12) + (\phi_{e,1} + \phi_{e,a}\mathbf{1}(t > 22)) \cdot \mathbf{1}(d_{e,t} + e_t > 12 \ \& \ d_{e,t} + e_t \le 16) + \phi_{e,2}\mathbf{1}(d_{e,t} + e_t > 16) \\
& + \alpha_{e,c}\theta_c + \alpha_{e,n}\theta_n + \phi_{e,p}(e_p - 12)\mathbf{1}(e_p > 12) - \phi_{e,e}(1 - d_{e,t-1}) + \sigma_e\epsilon_{e,t} \\
u_k =& [\phi_{k,e} \cdot \mathbf{1}(d_{k,t} = 0.5 \ \& \ d_{e,t} = 1) + \phi_{k,0} \cdot \mathbf{1}(d_{k,t} = 0.5 \ \& \ d_{e,t} = 0) + \mathbf{1}(d_{k,t} = 1) \cdot (\phi_{k,1} + \phi_{k,2}(age - 17)) \\
& \cdot (1 + \alpha_{k,c}\theta_c + \alpha_{k,n}\theta_n)
\end{aligned}
$$

Table A9: Subjective Discount Rate: $\rho(\theta_c, \theta_n) = \rho_0(1 - \rho_c\theta_c - \rho_n\theta_n)$ (Our Model *vs.* Alternative Model)

		Our Model		Alternative Model	
Description	Parameter	Estimate	S.E.	Estimate	S.E.
Cognitive Ability	ρ_c	0.0874	0.0086	0.0963	0.0127
Noncognitive Ability	ρ_n	0.2910	0.0109	0.1733	0.0128
Level Parameter	ρ_0	0.0245	0.0012	0.0210	0.0008

The associated discount factor is $\exp(-\rho(\theta_c, \theta_n)) = 0.9758$ in our model and $\exp(-\rho(\theta_c, \theta_n)) = 0.9792$ in alternative model.
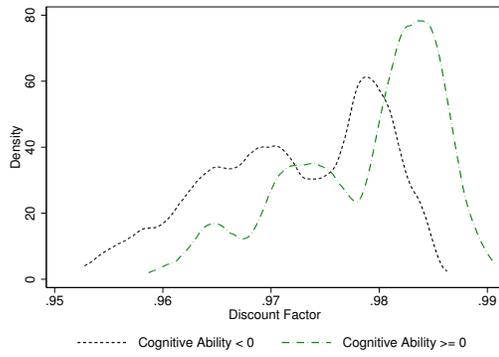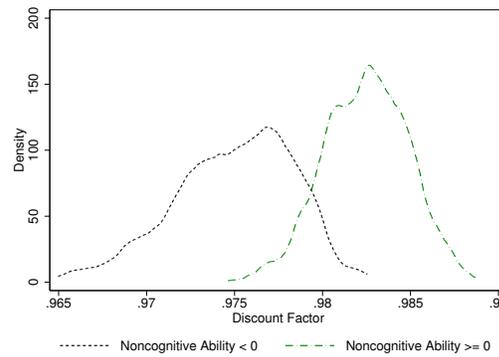
(a) Cognitive Ability (Our Model)
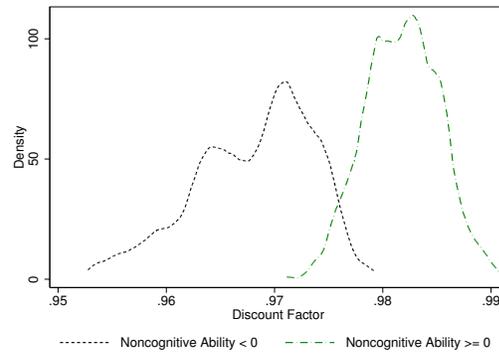
(b) Noncognitive Ability (Our Model)
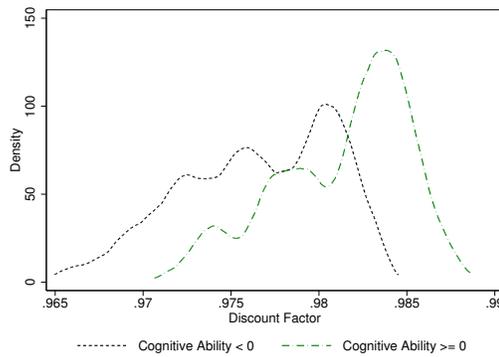
(c) Cognitive Ability (Alternative Model)

(d) Noncognitive Ability (Alternative Model)

Figure A4: Density of Estimated Discount Factors by Abilities: $\exp(-\rho(\theta_c, \theta_n))$ (Our Model *vs.* Alternative Model)

Table A10: Parameter Estimates on Human Capital Production Function and Wage Equation (Our Model *vs.* Alternative Model)
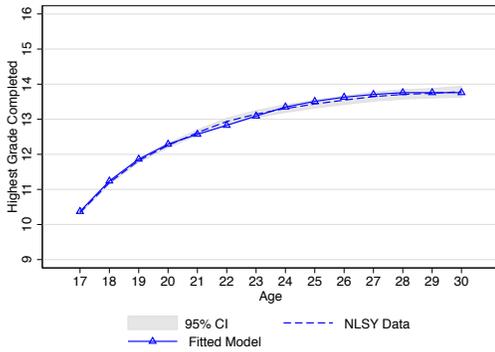
		Our Model		Alternative Model	
Description	Parameter	Estimate	S.E.	Estimate	S.E.
Intercept Parameter	$\beta_{\psi,0}$	1.8884	0.0018	1.9072	0.0042
Experience	$\beta_{\psi,k}$	0.0767	0.0002	0.0533	0.0005
Experience Squared/100	$\beta_{\psi,kk}$	-0.2683	0.0013	-0.2718	0.0019
Years of Schooling -12	$\beta_{\psi,e,0}$	0.0465	0.0005	0.0610	0.0020
Years of Schooling $= 12$	$\beta_{\psi,e,1}$	0.1432	0.0026	0.1320	0.0037
Years of Schooling $> 12, < 16$	$\beta_{\psi,e,2}$	0.1435	0.0015	0.1393	0.0041
Years of Schooling ≥ 16	$\beta_{\psi,e,3}$	0.2806	0.0031	0.2873	0.0044
Cognitive Ability $\times$ (Years of Schooling < 12)	$\alpha_{\psi,c,0}$	0.0529	0.0034	0.0322	0.0070
Cognitive Ability $\times$ (Years of Schooling $\geq 12, < 16$)	$\alpha_{\psi,c,1}$	0.0529	0.0031	0.0322	0.0058
Cognitive Ability $\times$ (Years of Schooling ≥ 16)	$\alpha_{\psi,c,2}$	0.1433	0.0028	0.0922	0.0048
Noncognitive Ability $\times$ (Years of Schooling < 12)	$\alpha_{\psi,n,0}$	0.0275	0.0076	0.0304	0.0035
Noncognitive Ability $\times$ (Years of Schooling $\geq 12, < 16$)	$\alpha_{\psi,n,1}$	0.0512	0.0017	0.0623	0.0029
Noncognitive Ability $\times$ (Years of Schooling ≥ 16)	$\alpha_{\psi,n,2}$	0.0892	0.0017	0.0723	0.0036
Part-Time	$\beta_{w,0}$	-0.0082	0.0263	-0.0099	0.0086
Part-Time $\times$ Enroll	$\beta_{w,1}$	-0.4863	0.0042	-0.5210	0.0058
Scale Parameter of Productivity Shock (Years of Schooling < 16)	b_0	0.1388	0.0051	0.1392	0.0013
Scale Parameter of Productivity Shock (Years of Schooling ≥ 16)	b_1	0.1424	0.0007	0.1384	0.0018
Shape Parameter of Productivity Shock (Years of Schooling < 16)	a_0	15.3558	0.4056	19.8838	0.3523
Shape Parameter of Productivity Shock (Years of Schooling ≥ 16)	a_1	15.8092	0.0615	21.4807	0.4661
Depreciation Rate of Experience	δ_k	0.1135	0.0022	0.1880	0.0035

$$
\begin{aligned}
\log \psi_t =& \beta_{\psi,0} + \beta_{\psi,k} k_t + \beta_{\psi,kk} k_t^2/100 + \beta_{\psi,e,0}(e_t - 12) \\
& + \beta_{w,e,1}\mathbf{1}(e_t = 12) + \beta_{w,e,2}\mathbf{1}(e_t > 12 \ \& \ e_t < 16) + \beta_{w,e,3}\mathbf{1}(e_t \geq 16) \\
& + (\alpha_{\psi,c,0}\theta_c + \alpha_{\psi,n,0}\theta_n) \cdot \mathbf{1}(e_t < 12) \\
& + (\alpha_{\psi,c,1}\theta_c + \alpha_{\psi,n,1}\theta_n) \cdot \mathbf{1}(e_t \geq 12 \ \& \ e_t < 16) \\
& + (\alpha_{\psi,c,2}\theta_c + \alpha_{\psi,n,2}\theta_n) \cdot \mathbf{1}(e_t \geq 16) + \epsilon_{w,t} - \mathbb{E}(\epsilon_{w,t}) \\
\log w_t =& \log \psi_t + \mathbf{1}(d_{k,t} = 0.5)(\beta_{w,0} + \beta_{w,1}d_{e,t}).
\end{aligned}
$$

The density of productivity shock $\epsilon_{w,t}$ is:

$$
p(\epsilon_{w,t}) = \frac{1}{\Gamma(a)b^a}(\epsilon_{w,t})^{a-1}e^{-(\epsilon_{w,t})/b}. \tag{2}
$$

A.4   Model Goodness of Fit

(a) Education by Age (Our Model)
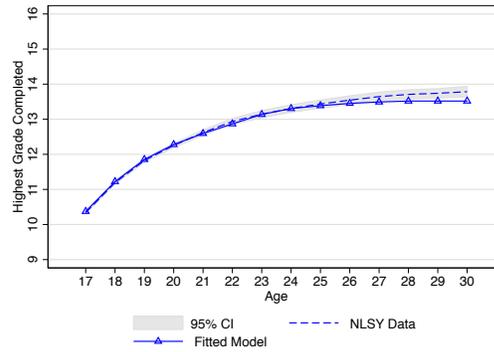
(b) Education by Age (Alternative Model)
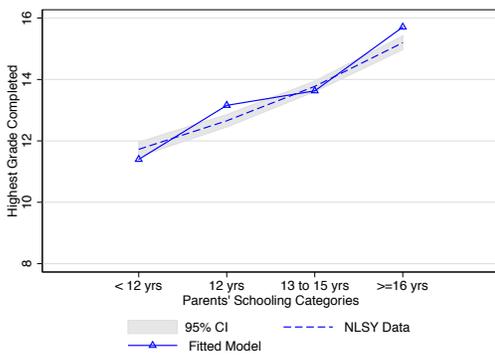
(c) Education by Parental Education (Our Model)
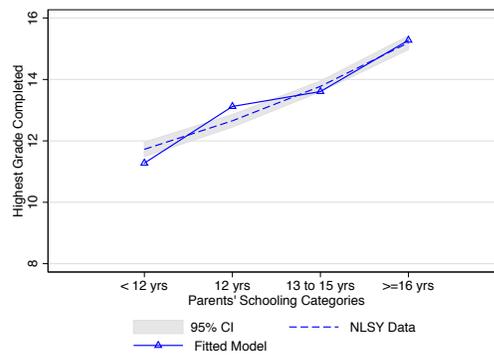
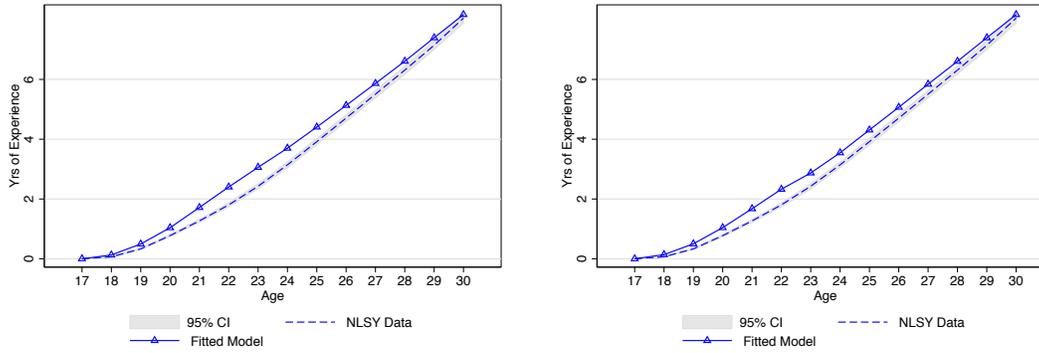(d) Education by Parental Education (Alternative Model)
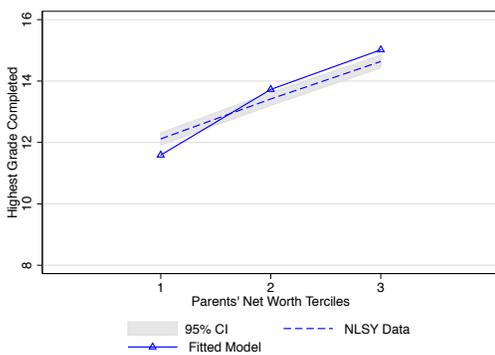
(e) Education by Parental Wealth (Our Model)
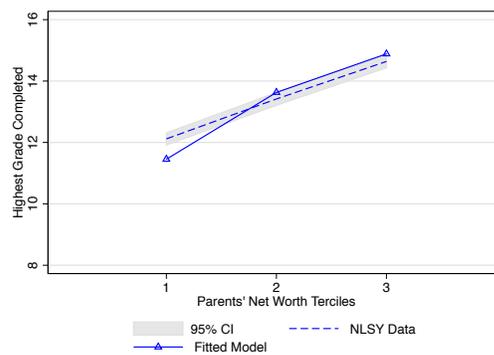
(f) Education by Parental Wealth (Alternative Model)

Figure A5: Model Fit on Years of Schooling

(a) Yrs Worked by Age (Our Model)   (b) Yrs Worked by Age (Alternative Model)

Figure A6: Model Fit on Years Worked

Table A11: Model Fit: Linear Regression on Enrollment

	Our Model	Alternative Model	Data	Data S.E.
Previously in School	0.3265	0.4337	0.474	0.010
Age	-0.0377	-0.0239	-0.033	0.004
Age = 17	0.2347	0.2025	0.186	0.014
Parental Education	0.0847	0.0406	0.082	0.011
Cognitive Ability	0.0837	0.0877	0.051	0.004
Noncognitive Ability	0.1101	0.1073	0.058	0.005

Note: Cognitive ability and noncognitive ability are estimated factor scores from the first stage.
Parameter estimate on the constant term is not reported here.

Table A12: Model Fit: Linear Regression on Full-Time Employment

	Our Model	Alternative Model	Data	Data S.E.
Years of Schooling	0.0235	0.0206	0.013	0.002
Cognitive Ability	0.0006	0.0066	0.005	0.004
Noncognitive Ability	0.0140	0.0092	0.009	0.004

Note: Cognitive ability and noncognitive ability are estimated factor scores from the first stage.
Parameter estimate on the constant term is not reported here.

Table A13: Model Fit: Log Hourly Wage Regression

	Our Model	Alternative Model	Data	Data S.E.
Years Worked	0.0973	0.0873	0.076	0.011
Years Worked Squared	-0.3424	-0.4498	-0.164	0.093
Years of Schooling	0.0130	0.0533	0.043	0.011
Years of Schooling = 12	0.1253	0.0995	0.146	0.041
Years of Schooling > 12, < 16	0.1536	0.1381	0.150	0.054
Years of Schooling ≥ 16	0.3839	0.3477	0.269	0.078
Cognitive Ability × (Years of Schooling < 12)	0.0537	0.0101	0.031	0.024
Noncognitive Ability × (Years of Schooling < 12)	0.0334	-0.0078	0.022	0.021
Cognitive Ability × (Years of Schooling ≥ 12, < 16)	0.0629	0.0524	0.031	0.016
Noncognitive Ability × (Years of Schooling ≥ 12, < 16)	0.0623	0.0667	0.074	0.015
Cognitive Ability × (Years of Schooling ≥ 16)	0.1194	0.0539	0.116	0.019
Noncognitive Ability × (Years of Schooling ≥ 16)	0.0707	0.0274	0.057	0.019
Previously Not Working	-0.0443	-0.0594	-0.089	0.037
Std Dev*	0.5383	0.5489	0.541	0.006

Note: Cognitive ability and noncognitive ability are estimated factor scores from the first stage.
Parameter estimate on the constant term is not reported here.
*Square root of error variance.

Table A14: Model Fit: Linear Regression on Log Net Worth

	Our Model	Alternative Model	Data	Data S.E.
Cognitive Ability	-0.0075	-0.0556	0.003	0.036
Noncognitive Ability	0.0969	-0.0510	0.220	0.033
Log Wage	0.8114	0.4297	0.636	0.061
Age >20	0.5116	1.2872	0.352	0.079
Age >25	0.5747	0.2636	0.544	0.089

Note: Cognitive ability and noncognitive ability are estimated factor scores from the first stage.
Parameter estimate on the constant term is not reported here.

A.5 Additional Results

(a) Multiplier $(\lambda_{s,t})$ vs s_t (Our Model) (b) Multiplier $(\lambda_{s,t})$ vs s_t (Alternative Model)

(c) Multiplier $(\lambda_{s,t})$ vs θ_c (Our Model) (d) Multiplier $(\lambda_{s,t})$ vs θ_c (Alternative Model)

(e) Multiplier $(\lambda_{s,t})$ vs θ_n (Our Model) (f) Multiplier $(\lambda_{s,t})$ vs θ_n (Alternative Model)

(g) Multiplier $(\lambda_{s,t})$ vs $F^{\psi}(e_t, k_t, \boldsymbol{\theta}, \epsilon_{w,t})$ (Our Model) (h) Multiplier $(\lambda_{s,t})$ vs $F^{\psi}(e_t, k_t, \boldsymbol{\theta}, \epsilon_{w,t})$ (Alternative Model)
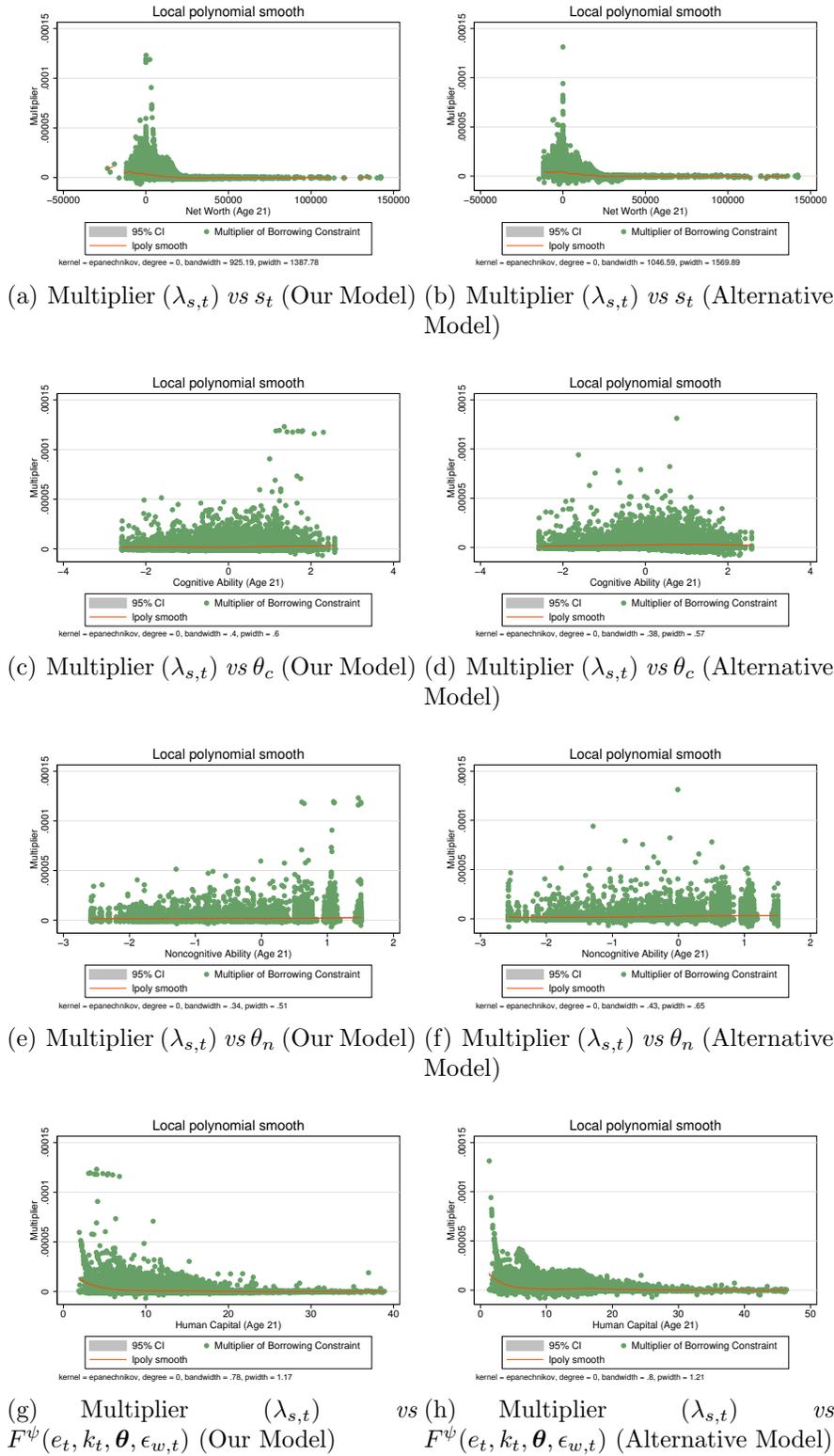
Figure A7: Kuhn-Tucker Multiplier of the Borrowing Constraint at Age 21
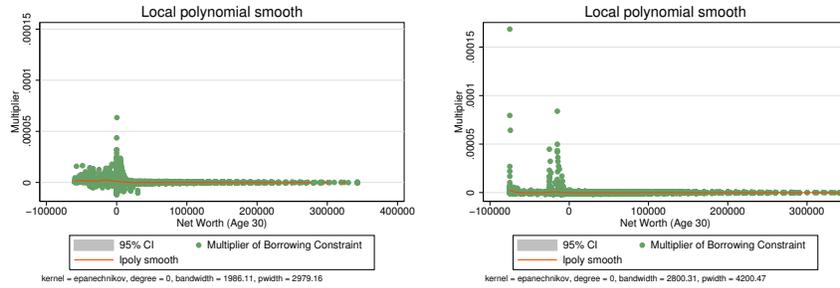
(a) Multiplier $(\lambda_{s,t})$ vs s_t (Our Model) (b) Multiplier $(\lambda_{s,t})$ vs s_t (Alternative Model)
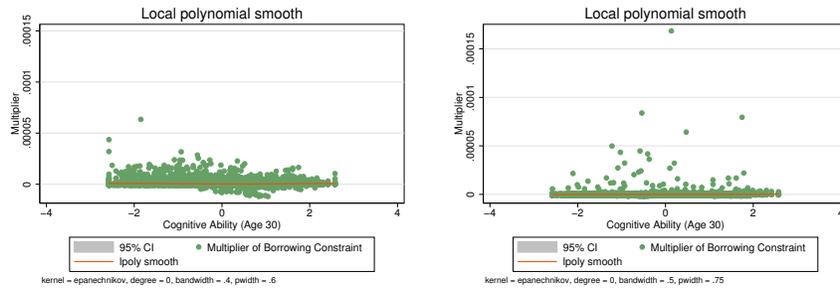
(c) Multiplier $(\lambda_{s,t})$ vs θ_c (Our Model) (d) Multiplier $(\lambda_{s,t})$ vs θ_c (Alternative Model)
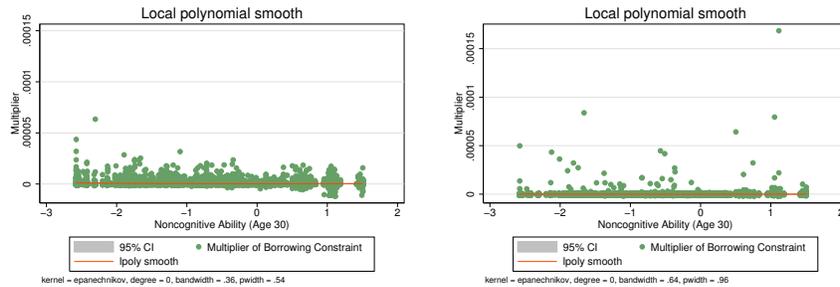
(e) Multiplier $(\lambda_{s,t})$ vs θ_n (Our Model) (f) Multiplier $(\lambda_{s,t})$ vs θ_n (Alternative Model)
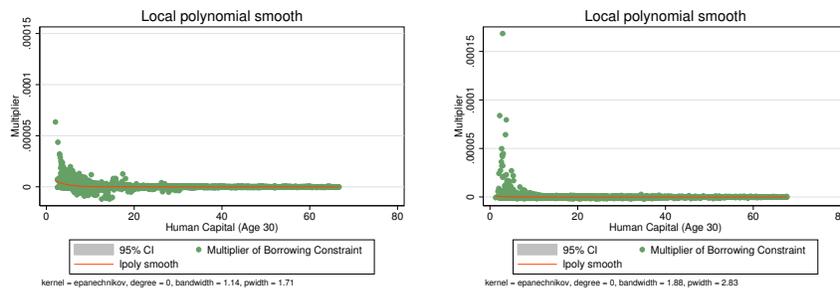
(g) Multiplier $(\lambda_{s,t})$ vs $F^{\psi}(e_t, k_t, \boldsymbol{\theta}, \epsilon_{w,t})$ (Our Model) (h) Multiplier $(\lambda_{s,t})$ vs $F^{\psi}(e_t, k_t, \boldsymbol{\theta}, \epsilon_{w,t})$ (Alternative Model)

Figure A8: Kuhn-Tucker Multiplier of the Borrowing Constraint at Age 30
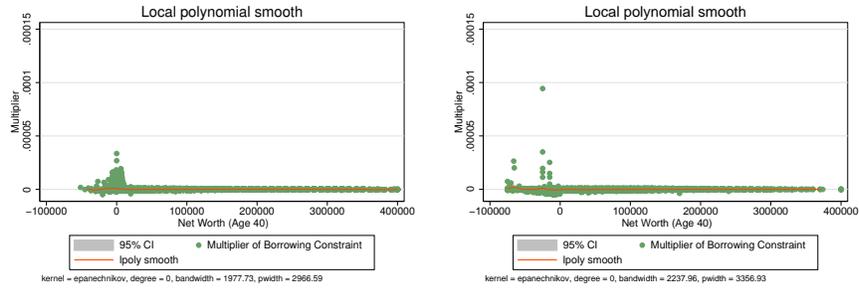
(a) Multiplier $(\lambda_{s,t})$ vs s_t (Our Model) (b) Multiplier $(\lambda_{s,t})$ vs s_t (Alternative Model)
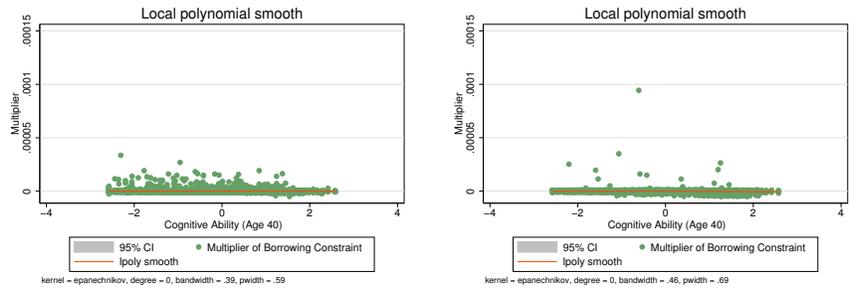
(c) Multiplier $(\lambda_{s,t})$ vs θ_c (Our Model) (d) Multiplier $(\lambda_{s,t})$ vs θ_c (Alternative Model)
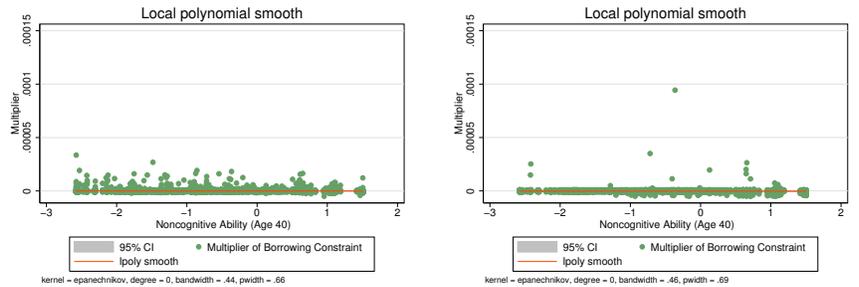
(e) Multiplier $(\lambda_{s,t})$ vs θ_n (Our Model) (f) Multiplier $(\lambda_{s,t})$ vs θ_n (Alternative Model)
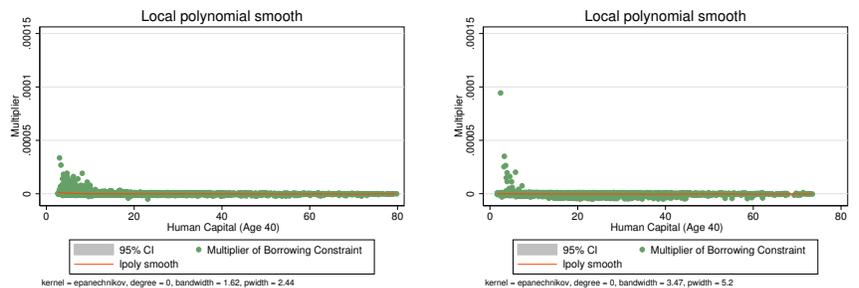
(g)   Multiplier $(\lambda_{s,t})$ vs (h)   Multiplier $(\lambda_{s,t})$ vs
$F^\psi(e_t, k_t, \boldsymbol{\theta}, \epsilon_{w,t})$ (Our Model) $F^\psi(e_t, k_t, \boldsymbol{\theta}, \epsilon_{w,t})$ (Alternative Model)

Figure A9: Kuhn-Tucker Multiplier of the Borrowing Constraint at Age 40
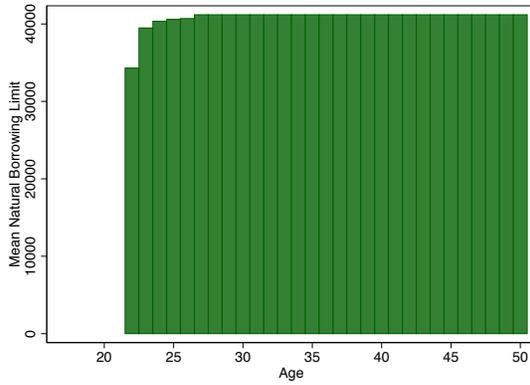
(a) Fixed Borrowing Limit over Ages 17 to 50   (b) Fixed Borrowing Limit *vs* Human Capital
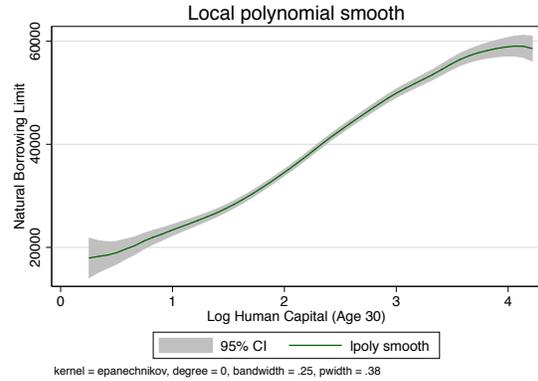
Figure A10: Mean of Borrowing Limit $\bar{L}_t^s(e_{t+1}, k_{t+1}, \boldsymbol{\theta})$ for Alternative Model
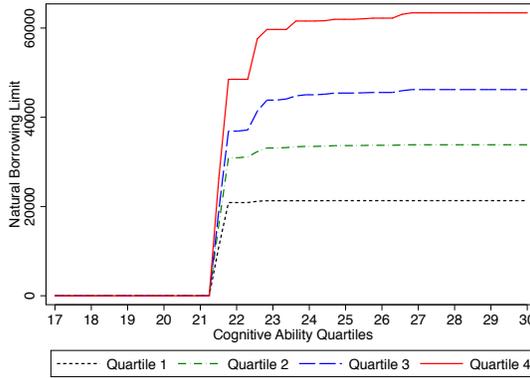
(a) Fixed Borrowing Limit *vs* Cognitive Ability   (b) Fixed Borrowing Limit *vs* Noncog. Ability
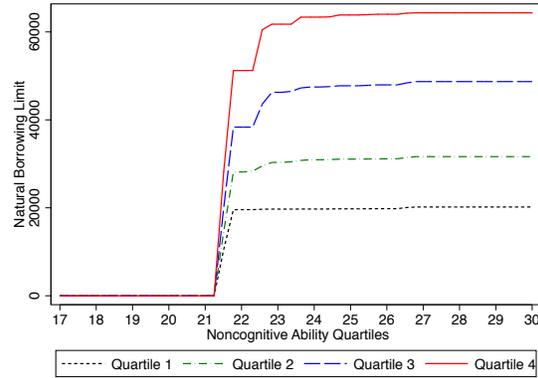
Figure A11: Evolution of Average Borrowing Limit by Ability Endowments for Alternative Model

Local polynomial smooth

(a) Fraction Constrained *vs* s_t

Local polynomial smooth

(b) Fraction Constrained *vs* θ_c

Local polynomial smooth

(c) Fraction Constrained *vs* θ_n

Local polynomial smooth

(d) Fraction Constrained *vs* $\log F^\psi(e_t, k_t, \boldsymbol{\theta}, \epsilon_{w,t})$

Figure A12: Borrowing Constrained Youths $(s_{t+1} \leq -\bar{L}_t^s(e_{t+1}, k_{t+1}, \boldsymbol{\theta})\ \&\ \lambda_{s,t} > 0)$ at Age 21 for Alternative Model
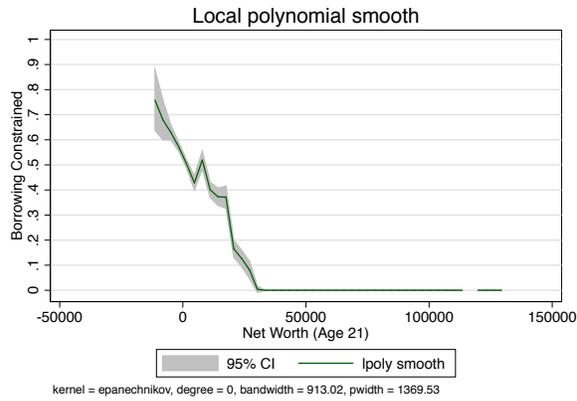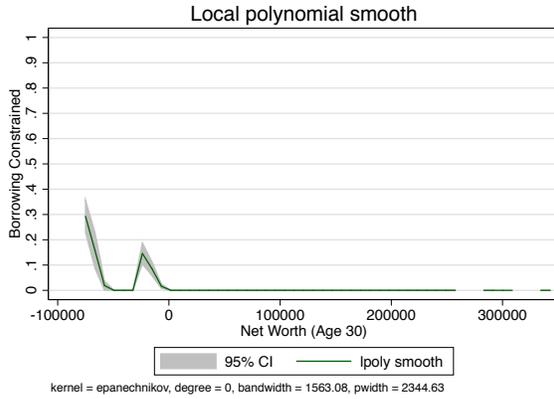
(a) Fraction Constrained $vs\ s_t$
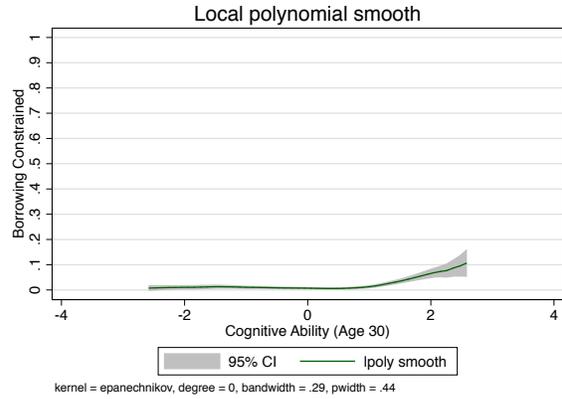
(b) Fraction Constrained $vs\ \theta_c$
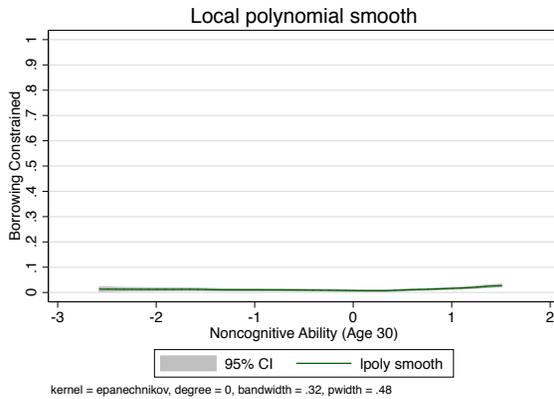
(c) Fraction Constrained $vs\ \theta_n$
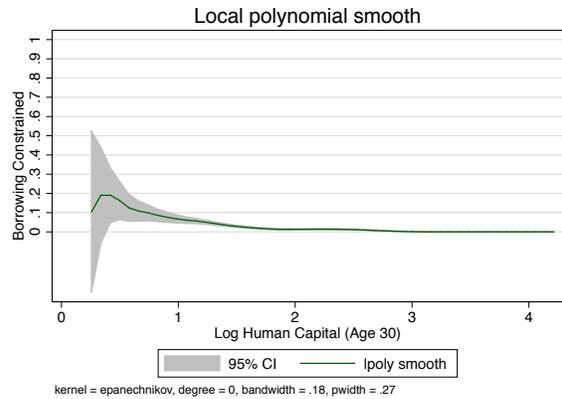
(d) Fraction Constrained $vs\ \log F^{\psi}(e_t, k_t, \boldsymbol{\theta}, \epsilon_{w,t})$

Figure A13: Borrowing Constrained Youths $(s_{t+1} \leq -\bar{L}_t^s(e_{t+1}, k_{t+1}, \boldsymbol{\theta})\ \&\ \lambda_{s,t} > 0)$ at Age 30 for Alternative Model

(a) Fraction Constrained vs s_t

(b) Fraction Constrained vs θ_c

(c) Fraction Constrained vs θ_n
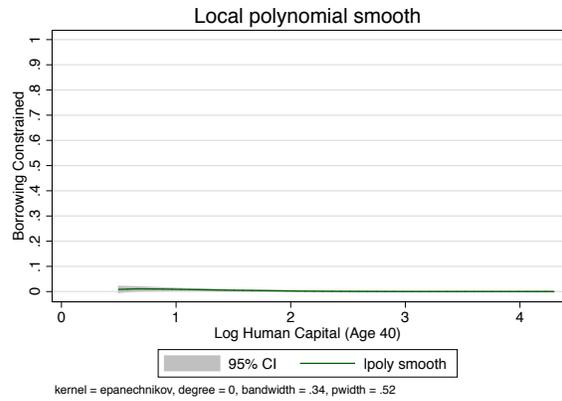
(d) Fraction Constrained vs $\log F^\psi(e_t, k_t, \boldsymbol{\theta}, \epsilon_{w,t})$

Figure A14: Borrowing Constrained Youths $(s_{t+1} \leq -\bar{L}_t^s(e_{t+1}, k_{t+1}, \boldsymbol{\theta})$ & $\lambda_{s,t} > 0)$ at Age 40 for Alternative Model

On average, the average natural borrowing limits increase with cognitive ability, noncognitive ability, and education for both our model and alternative model. However, there is substantial heterogeneity in the amount of natural borrowing limit within each education category.
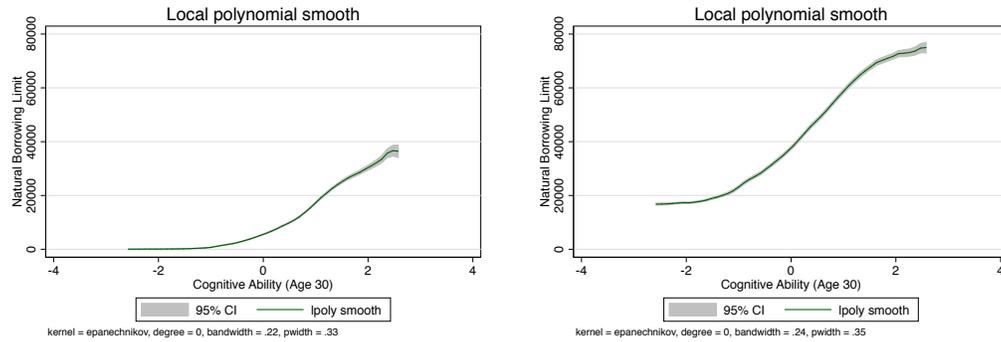
(a) Natural Borrowing Limit & Cognitive Ability at Age 30 (Our Model)

(b) Fixed Borrowing Limit & Cognitive Ability at Age 30 (Alternative Model)
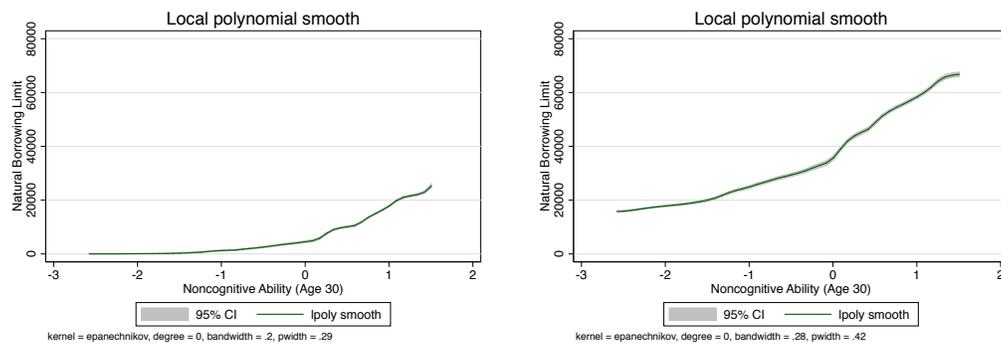
(c) Fixed Borrowing Limit & Noncognitive Ability at Age 30 (Our Model)

(d) Fixed Borrowing Limit & Noncognitive Ability at Age 30 (Alternative Model)
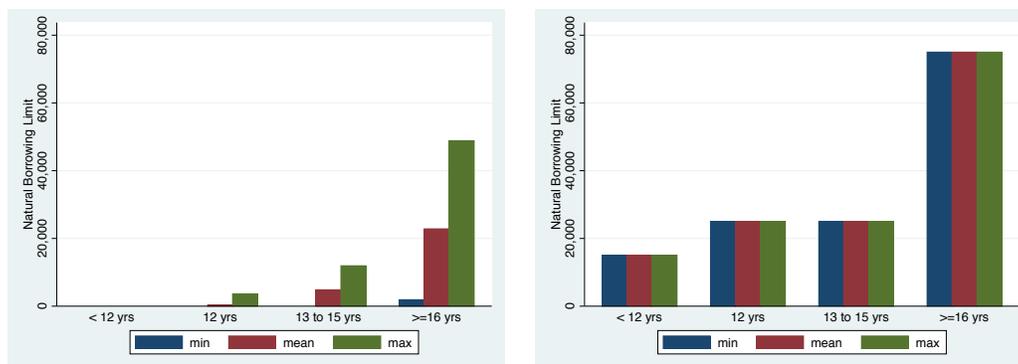
(e) Fixed Borrowing Limit & Education at Age 30 (Our Model)

(f) Fixed Borrowing Limit & Education at Age 30 (Alternative Model)

Figure A15: Fixed Borrowing Limit & Education (Age 30)

Table A15: Inequality in Education, Wages, and Consumption under Different Experiments at Age 30 (Our Model)

	Inequality (Var of log)			Changes in Inequality (%)		
	Educ	Wage	C	Educ	Wage	C
Benchmark	0.0395	0.3313	0.1002	N.A.	N.A.	N.A.
	Counterfactual Experiments					
Subsidizing College Tuition	0.0389	0.3319	0.1010	-1.43	0.17	0.79
Increasing Student Loan Limits	0.0397	0.3346	0.1116	0.57	0.98	11.39

Note: Inequality in Education (Educ), wages, and consumption (C) are measured using variance of log years of schooling, log hourly wage rates, and log consumption at age 30, respectively. Changes in inequality is calculated as the percentage changes in inequality compared to the benchmark model. When equalizing cognitive ability, we set every individual's cognitive ability equal to the population mean, i.e., zero. Similarly, we set every individual's noncognitive ability equal to zero when equalizing noncognitive ability. When equalizing parental factors, we set parents' education equal to 12 years of schooling and parent's net worth to be in the second tercile.

A.5.1 Experiment: Subsidizing College Tuition (Our Model v.s. Alternative Model)
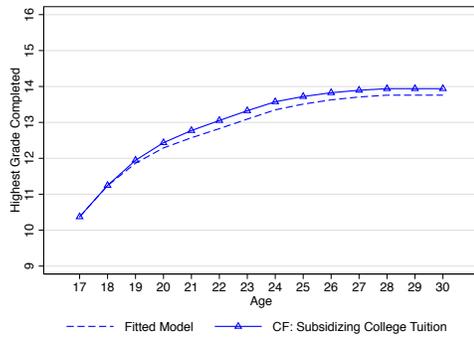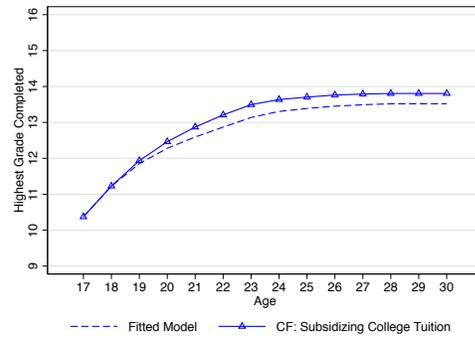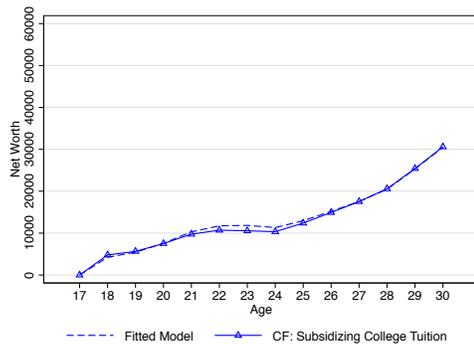
(a) Education (Our Model)

(b) Education (Alternative Model)

(c) Net Worth (Our Model)
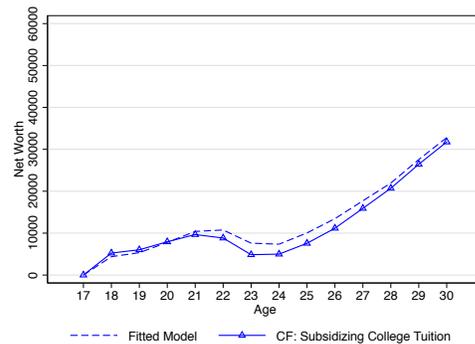
(d) Net Worth (Alternative Model)
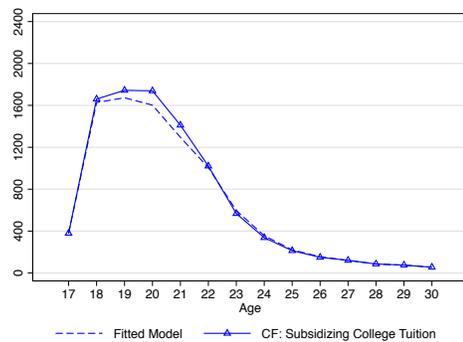
(e) Parental Transfers (Our Model)
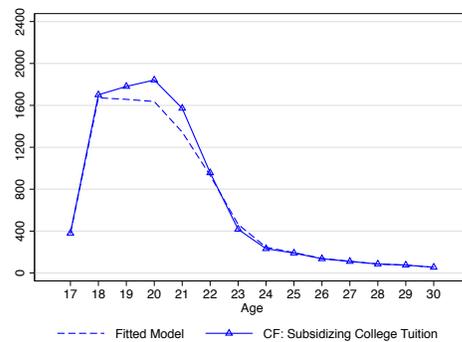
(f) Parental Transfers (Alternative Model)

Figure A16: Counterfactual Simulation of Subsidizing College Tuition on Education, Net Worth, and Transfers
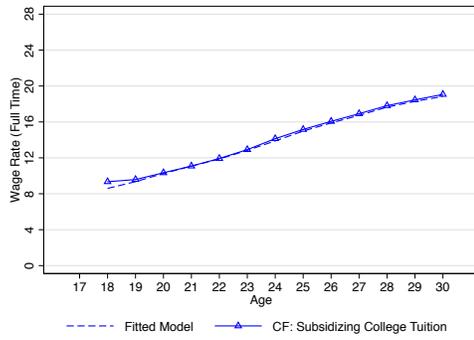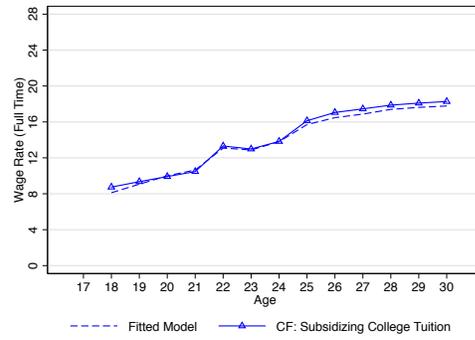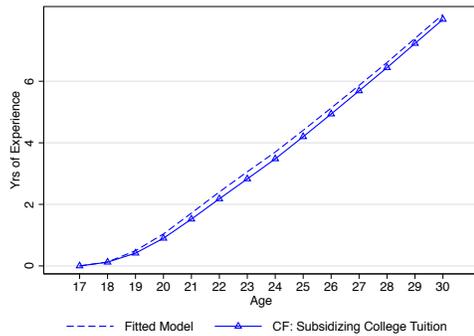
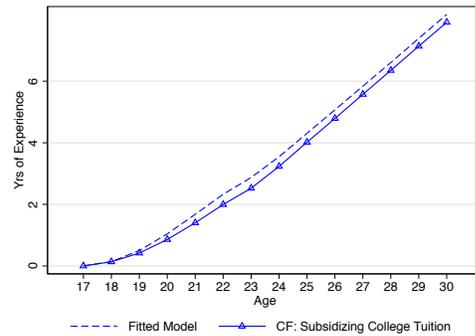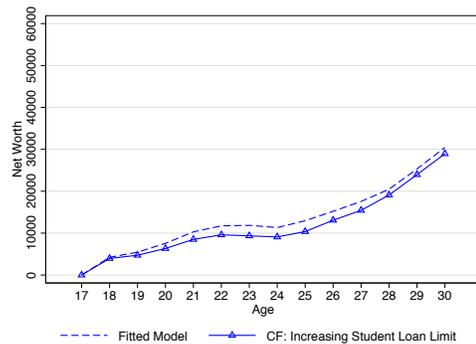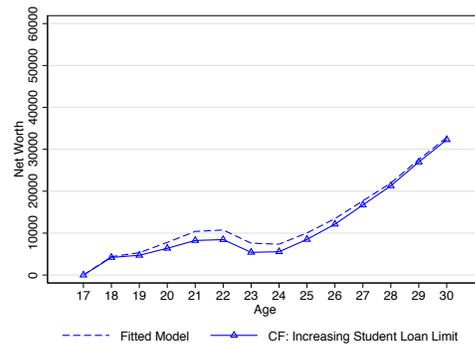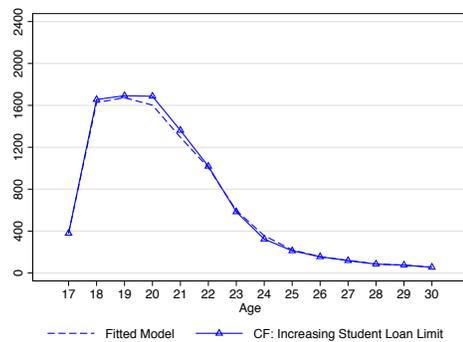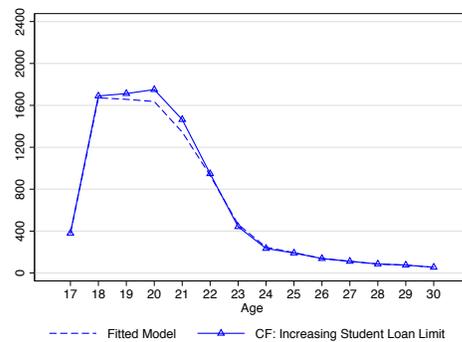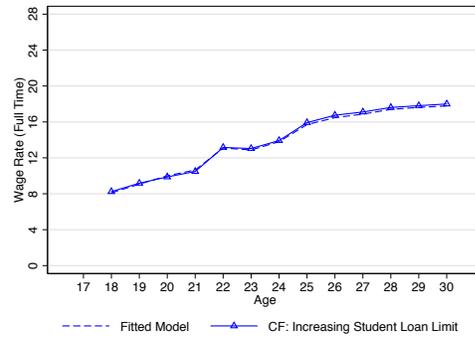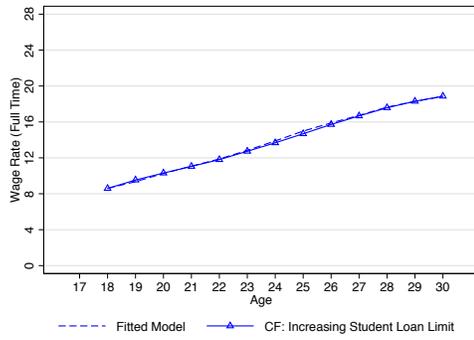(a) Accepted Full-Time Wage (Our Model)   (b) Accepted Full-Time Wage (Alternative Model)
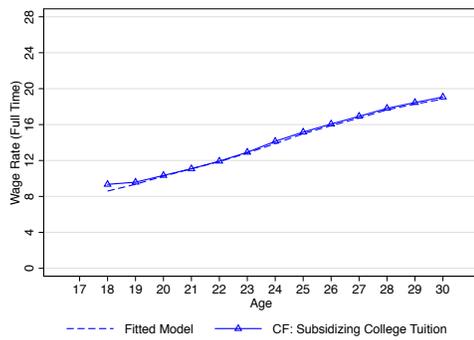
(c) Accepted Part-Time Wage (Our Model)   (d) Accepted Part-Time Wage (Alternative Model)
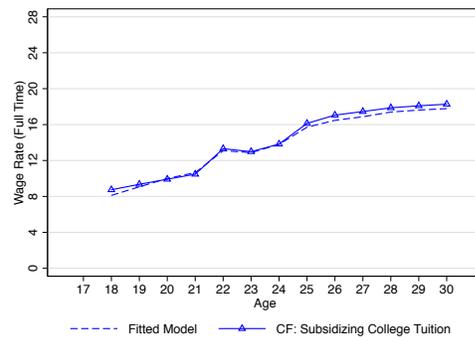
(e) Yrs of Working (Our Model)   (f) Yrs of Working (Alternative Model)

Figure A17: Counterfactual Simulation of Subsidizing College Tuition on Wage and Years of Working

A.5.2 Experiment: Relaxing Student Loan Limit (Our Model v.s. Alternative Model)
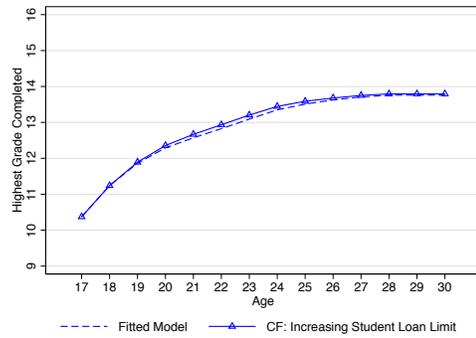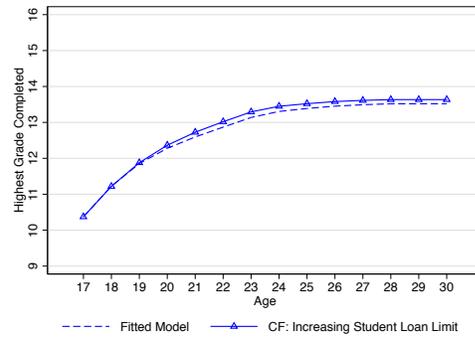
(a) Education (Our Model)

(b) Education (Alternative Model)

(c) Net Worth (Our Model)

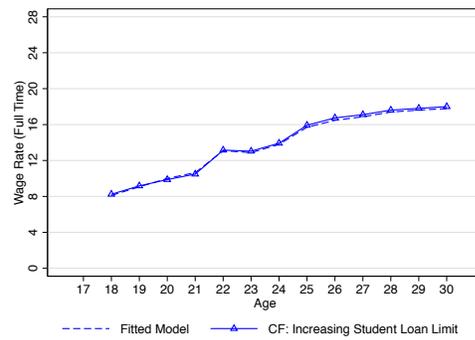(d) Net Worth (Alternative Model)
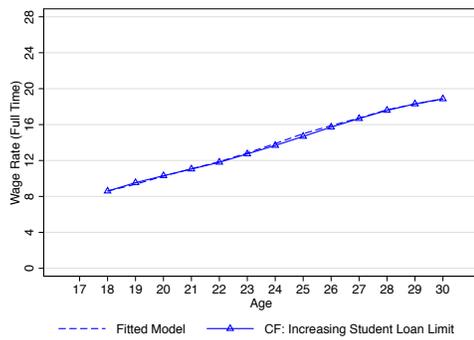
(e) Parental Transfers (Our Model)

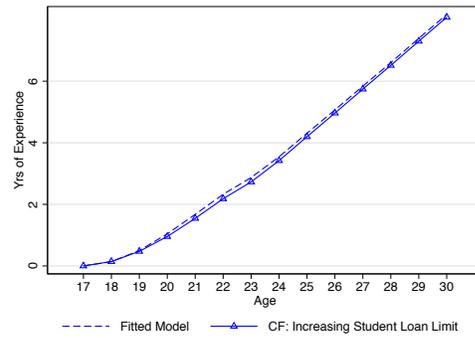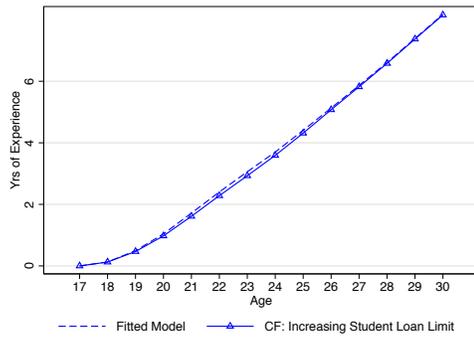(f) Parental Transfers (Alternative Model)

Figure A18: Counterfactual Simulation of Increasing Student Loan Limits on Education, Net Worth, and Transfers

(a) Accepted Full-Time Wage (Our Model) (b) Accepted Full-Time Wage (Alternative Model)

(c) Accepted Part-Time Wage (Our Model) (d) Accepted Part-Time Wage (Alternative Model)

(e) Yrs of Working (Our Model)   (f) Yrs of Working (Alternative Model)

Figure A19: Counterfactual Simulation of Increasing Student Loan Limits on Wage and Years of Working

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Hubbard, R. Glenn, Jonathan Skinner, and Stephen P. Zeldes, 1995, Precautionary Saving and Social Insurance, *Journal of Political Economy* 103, 360–399.

Keane, Michael P., and Kenneth I. Wolpin, 2001, The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment, *International Economic Review* 42, 1051–1103.