

# Online Appendix

## Markups and Inequality

### Not For Publication

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This appendix provides detailed derivations for our optimal regulation results in Section 3, describes the SCF data we used to parameterize the model, and reports the parameter values and moments for the two perturbations considered in the robustness section in the dynamic economy.

## 1 Optimal Regulation

We first analyze the case of complete information and then turn to the economy with private information.

### 1.1 Complete Information

The problem under complete information is described in equations (15)–(18) in the text. The Lagrangean is

$$\begin{aligned} \max_{q(z), c(z), W, Y} & V^w(W) + \alpha \omega \int_0^\infty \frac{c(z)^{1-\theta}}{1-\theta} f(z) dz + \lambda \left[ Y - C^w(W) - \omega \int_0^\infty c(z) f(z) dz \right] \\ & + \kappa \left[ \omega \int_0^\infty \Upsilon(q(z)) f(z) dz - 1 \right] + \nu \left[ L(W) - Y^{\frac{1}{\eta}} \omega \int_0^\infty \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} f(z) dz \right] \end{aligned}$$

The FOC with respect to  $c(z)$  is

$$\alpha c(z)^{-\theta} = \lambda,$$

so all entrepreneurs receive the same level of consumption

$$c(z) = c^e.$$

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The FOC with respect to  $q(z)$  is

$$\kappa \Upsilon'(q(z)) = \nu Y^{\frac{1}{\eta}} \frac{1}{\eta} \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}-1} \frac{1}{z},$$

or arranging terms,

$$\Upsilon'(q(z)) q(z) = \frac{1}{\eta} \Lambda \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}},$$

where  $\Lambda = \frac{\nu Y^{\frac{1}{\eta}}}{\kappa}$ . To find  $\Lambda$ , we note that this expression implicitly determines the relative quantity choice as a function of productivity and  $\Lambda$ ,  $q(z; \Lambda)$ . We can therefore find  $\Lambda$  by requiring that the Kimball aggregator is satisfied. That is,  $\Lambda$  is the implicit solution to

$$\omega \int_0^\infty \Upsilon(q(z; \Lambda)) f(z) dz = 1.$$

Once we have the relative quantity choice, we calculate  $Z$  from equation (7) in text and use the resource constraint to rewrite the regulator's problem as

$$\max_W V^w(W) + \alpha \omega \frac{\left[ \frac{ZL(W)^\eta - C^w(W)}{\omega} \right]^{1-\theta}}{1-\theta}.$$

The FOC with respect to  $W$  is

$$\frac{\partial V^w(W)}{\partial W} = \alpha (c^e)^{-\theta} \left[ \frac{\partial C^w(W)}{\partial W} - \eta ZL(W)^{\eta-1} \frac{\partial L^w(W)}{\partial W} \right].$$

When  $\theta = 1$ , as assumed in the paper,  $L^w$  is constant, so this expression simplifies to

$$(1 - \omega) \frac{1}{W} = \alpha (c^e)^{-\theta} \frac{C^w(W)}{W},$$

which implies that

$$\frac{C^w(W)}{1 - \omega} = \frac{c^e}{\alpha}.$$

## 1.2 Economy with Private Information

We now assume that the regulator does not observe productivity  $z$ . Without loss of generality, we invoke the revelation principle and constrain the regulator to choose functions  $c(z)$  and  $q(z)$  that ensure truth-telling.

Let  $\tau(z)$  denote a subsidy received by a firm that claims to be of type  $z$  and sells  $q(z)$  units of output as prescribed by the regulator. The producer's consumption, if it reports

truth-fully, is

$$c(z) = DY \left[ \Upsilon'(q(z)) q(z) - \frac{WY^{\frac{1}{\eta}-1}}{D} \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} + \tau(z) \right].$$

If the producer claims instead to have productivity  $\hat{z}$ , it receives

$$c(\hat{z}, z) = DY \left[ \Upsilon'(q(\hat{z})) q(\hat{z}) - \frac{WY^{\frac{1}{\eta}-1}}{D} \left( \frac{q(\hat{z})}{z} \right)^{\frac{1}{\eta}} + \tau(\hat{z}) \right]$$

units of consumption. Following the first-order approach, the local incentive constraint is

$$\left. \frac{\partial \hat{c}(\hat{z}, z)}{\partial \hat{z}} \right|_{\hat{z}=z} = 0 = DY \left[ \left( \Upsilon''(q(z)) q(z) + \Upsilon'(q(z)) - \frac{1}{\eta} \frac{WY^{\frac{1}{\eta}-1}}{D} \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{q(z)} \right) q'(z) + \tau'(z) \right].$$

Differentiating the expression for  $c(z)$  with respect to  $z$  and imposing the local incentive constraint gives

$$c'(z) = WY^{\frac{1}{\eta}} \frac{1}{\eta} \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{z}.$$

The Lagrangean is

$$\begin{aligned} & \max_{c(z), q(z), W, Y} V^w(W) + \alpha \omega \int_0^\infty \frac{c(z)^{1-\theta}}{1-\theta} f(z) dz + \\ & \int_0^\infty \hat{\mu}(z) \left[ c'(z) - \frac{1}{\eta} WY^{\frac{1}{\eta}} \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{z} \right] dz + \lambda \left[ Y - C^w(w) - \omega \int_0^\infty c(z) f(z) dz \right] \\ & + \kappa \left[ \omega \int_0^\infty \Upsilon(q(z)) f(z) dz - 1 \right] + \hat{\nu} \left[ L^w(W) - \omega Y^{\frac{1}{\eta}} \int_0^\infty \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} f(z) dz \right], \end{aligned}$$

where  $\hat{\mu}(z)$  is the multiplier on the IC constraint and we now use  $\hat{\nu}$  to denote the multiplier on the labor resource constraint.

Consider the term  $\int_0^\infty \hat{\mu}(z) c'(z) dz$ . Integrating by parts and using the transversality conditions  $\hat{\mu}(0) = \hat{\mu}(\infty) = 0$  gives

$$\int_0^\infty \hat{\mu}(z) c'(z) dz = - \int_0^\infty \hat{\mu}'(z) c(z) dz,$$

which allows us to rewrite the Lagrangean as

$$\begin{aligned} & \max_{c(z), q(z), W, Y} V^w(W) + \alpha\omega \int_0^\infty \frac{c(z)^{1-\theta}}{1-\theta} f(z) dz - \int_0^\infty \hat{\mu}'(z) c(z) dz \\ & - \frac{1}{\eta} W Y^{\frac{1}{\eta}} \int_0^\infty \hat{\mu}(z) \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{z} dz + \lambda \left[ Y - C^w(w) - \omega \int_0^\infty c(z) f(z) dz \right] \\ & + \kappa \left[ \omega \int_0^\infty \Upsilon(q(z)) f(z) dz - 1 \right] + \hat{\nu} \left[ L^w(W) - \omega Y^{\frac{1}{\eta}} \int_0^\infty \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} f(z) dz \right]. \end{aligned}$$

The FOC with respect to  $c(z)$  is

$$\alpha\omega c(z)^{-\theta} f(z) - \hat{\mu}'(z) - \lambda\omega f(z) = 0,$$

or

$$\hat{\mu}'(z) = \omega \left[ \alpha c(z)^{-\theta} - \lambda \right] f(z).$$

The FOC with respect to  $q(z)$  is

$$-\frac{1}{\eta} W Y^{\frac{1}{\eta}} \frac{1}{\eta} \hat{\mu}(z) \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{z} \frac{1}{q(z)} + \kappa\omega \Upsilon'(q(z)) f(z) - \hat{\nu}\omega Y^{\frac{1}{\eta}} \frac{1}{\eta} \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{q(z)} f(z) = 0.$$

The FOC with respect to  $Y$  is

$$-\frac{1}{\eta} \frac{1}{\eta} W Y^{\frac{1}{\eta}-1} \int_0^\infty \hat{\mu}(z) \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{z} dz + \lambda - \hat{\nu}\omega \frac{1}{\eta} Y^{\frac{1}{\eta}-1} \int_0^\infty \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} f(z) dz = 0.$$

The FOC with respect to  $W$  is

$$\frac{\partial V^w(W)}{\partial W} - \lambda \frac{\partial C^w(W)}{\partial W} + \hat{\nu} \frac{\partial L^w(W)}{\partial W} - \frac{1}{\eta} Y^{\frac{1}{\eta}} \int_0^\infty \hat{\mu}(z) \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{z} dz = 0.$$

To derive equation (28) in the text, we note that, since  $\hat{\mu}(\infty) = 0$ ,

$$\hat{\mu}(z) = - \int_z^\infty \hat{\mu}'(x) dx = \omega \int_z^\infty \left[ \lambda - \alpha c(x)^{-\theta} \right] f(x) dx.$$

Moreover, since  $\hat{\mu}(0) = 0$  we have

$$\hat{\mu}(0) = \omega \int_0^\infty \left[ \lambda - \alpha c(z)^{-\theta} \right] f(z) dz = 0,$$

which implies that

$$\lambda = \alpha \int_0^\infty c(z)^{-\theta} f(z) dz.$$

Consider next the  $Y$  FOC:

$$\hat{\nu} \omega \frac{1}{\eta} Y^{\frac{1}{\eta}-1} \int_0^\infty \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} f(z) dz = \lambda - \frac{1}{\eta} \frac{1}{\eta} W Y^{\frac{1}{\eta}-1} \int_0^\infty \hat{\mu}(z) \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{z} dz.$$

Since  $Z^{-\frac{1}{\eta}} = \omega \int_0^\infty \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} f(z) dz$  we can simplify this expression to

$$\hat{\nu} \frac{1}{\eta} Y^{\frac{1}{\eta}-1} Z^{-\frac{1}{\eta}} = \lambda - \frac{1}{\eta} \frac{1}{\eta} W Y^{\frac{1}{\eta}-1} \int_0^\infty \hat{\mu}(z) \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{z} dz,$$

so the multiplier on the labor resource constraint is

$$\hat{\nu} = \eta Y^{1-\frac{1}{\eta}} Z^{\frac{1}{\eta}} \lambda - \frac{1}{\eta} W Z^{\frac{1}{\eta}} \int_0^\infty \hat{\mu}(z) \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{z} dz.$$

Since

$$\eta Y^{1-\frac{1}{\eta}} Z^{\frac{1}{\eta}} = \eta \frac{Y}{L},$$

the marginal rate of substitution between employment and consumption is equal to

$$\nu = \frac{\hat{\nu}}{\lambda} = \eta \frac{Y}{L} - \frac{1}{\lambda} \frac{1}{\eta} W Z^{\frac{1}{\eta}} \int_0^\infty \hat{\mu}(z) \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{z} dz.$$

Let

$$\mu(z) = \frac{\hat{\mu}(z)}{\lambda \omega (1 - F(z))} = 1 - \frac{1}{\lambda} \frac{1}{1 - F(z)} \int_z^\infty \alpha c(x)^{-\theta} f(x) dx$$

and recall that  $c'(z) = W Y^{\frac{1}{\eta}} \frac{1}{\eta} \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{z}$ . We can therefore write

$$\nu = \eta \frac{Y}{L} - \left( \frac{Z}{Y} \right)^{\frac{1}{\eta}} \omega \int_0^\infty \frac{\hat{\mu}(z)}{\omega \lambda (1 - F(z))} W Y^{\frac{1}{\eta}} \frac{1}{\eta} \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{z} (1 - F(z)) dz$$

or

$$\nu = \eta \frac{Y}{L} - \frac{1}{L} \omega \int_0^\infty \mu(z) c'(z) (1 - F(z)) dz.$$

We can finally rewrite the  $q(z)$  FOC as

$$\kappa \Upsilon'(q(z)) q(z) = \hat{\nu} \frac{1}{\eta} Y^{\frac{1}{\eta}} \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} + \frac{1}{\eta} W Y^{\frac{1}{\eta}} \frac{\hat{\mu}(z)}{\omega f(z)} \frac{1}{\eta} \left( \frac{q(z)}{z} \right)^{\frac{1}{\eta}} \frac{1}{z},$$

which implies

$$\Upsilon'(q(z))q(z) = \left(1 + \mu(z) \frac{\frac{1}{\eta} \frac{W}{z} (1 - F(z))}{\nu f(z)}\right) \frac{\hat{\nu} \frac{1}{\kappa} Y^{\frac{1}{\eta}}}{\eta} \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}}.$$

To find  $\kappa$ , we multiply the  $q(z)$  FOC by  $q(z)$  and integrate across all producers:

$$-\frac{1}{\eta} W Y^{\frac{1}{\eta}} \frac{1}{\eta} \int_0^\infty \hat{\mu}(z) \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{z} dz + \kappa \omega \int_0^\infty \Upsilon'(q(z)) q(z) f(z) dz - \hat{\nu} \omega Y^{\frac{1}{\eta}} \frac{1}{\eta} \int_0^\infty \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} f(z) dz = 0.$$

Similarly, multiplying the  $Y$  FOC by  $Y$  gives

$$-\frac{1}{\eta} \frac{1}{\eta} W Y^{\frac{1}{\eta}} \int_0^\infty \hat{\mu}(z) \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} \frac{1}{z} dz + \lambda Y - \hat{\nu} \omega \frac{1}{\eta} Y^{\frac{1}{\eta}} \int_0^\infty \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}} f(z) dz = 0.$$

Combining these two expressions and using the definition of the demand index,

$$D = \frac{1}{\omega \int_0^\infty \Upsilon'(q(z)) q(z) f(z) dz},$$

implies

$$\kappa = \lambda Y D,$$

which gives equation (28) for  $q(z)$  in the text:

$$\Upsilon'(q(z))q(z) = \left(1 + \mu(z) \frac{\frac{1}{\eta} \frac{W}{z} (1 - F(z))}{\nu f(z)}\right) \frac{1}{\eta} \frac{\nu Y^{\frac{1}{\eta}-1}}{D} \left(\frac{q(z)}{z}\right)^{\frac{1}{\eta}}.$$

Having solved for  $q(z)$ , we use the ICC to find the consumption of each entrepreneur,

$$c(z) = c(0) + \frac{1}{\eta} W Y^{\frac{1}{\eta}} \int_0^z \left(\frac{q(x)}{x}\right)^{\frac{1}{\eta}} \frac{1}{x} dx,$$

where the lump-sum transfer  $c(0)$  adjusts to ensure revenue neutrality

$$C^w(W) + \omega \int c(z) f(z) dz = Y.$$

To find the optimal choice of  $W$ , we note that aggregating workers' optimal consumption and hours choices gives

$$C(W) = (1 - \omega) \left( \int e^{\frac{1+\gamma}{\gamma+\theta}} dH(e) \right) W^{\frac{1+\gamma}{\gamma+\theta}},$$

$$L(W) = (1 - \omega) \left( \int_0^\infty e^{\frac{1+\gamma}{\gamma+\theta}} dH(e) \right) W^{\frac{1-\theta}{\gamma+\theta}},$$

and the overall welfare of workers is

$$V^w(W) = (1 - \omega) \frac{\gamma + \theta}{(1 - \theta)(1 + \gamma)} W^{\frac{(1-\theta)(1+\gamma)}{\gamma+\theta}} \int e^{\frac{(1-\theta)(1+\gamma)}{\gamma+\theta}} dH(e).$$

The FOC for  $W$  can therefore be written

$$\frac{\partial V^w(W)}{\partial W} - \lambda \frac{\partial C^w(W)}{\partial W} + \dot{v} \frac{\partial L^w(W)}{\partial W} - \frac{1}{W} \omega \lambda \int_0^\infty \mu(z) c'(z) (1 - F(z)) dz = 0.$$

When  $\theta = 1$ , we have

$$V^w(W) = (1 - \omega) \left( \log W + \int_0^\infty \log e dH(e) - \frac{1}{1 + \gamma} \right),$$

$$C^w(W) = (1 - \omega) \left( \int_0^\infty e dH(e) \right) W,$$

and

$$L^w(W) = (1 - \omega) \left( \int_0^\infty e dH(e) \right),$$

so the FOC with respect to  $W$  reduces to

$$\frac{1 - \omega}{W} - \lambda \frac{C^w(W)}{W} - \frac{1}{W} \omega \lambda \int_0^\infty \mu(z) c'(z) (1 - F(z)) dz = 0,$$

or

$$\frac{C^w(W)}{1 - \omega} = \frac{1}{\lambda} - \frac{\omega}{1 - \omega} \int_0^\infty \mu(z) c'(z) (1 - F(z)) dz.$$

Since

$$\lambda = \alpha \int_0^\infty c(z)^{-1} f(z) dz,$$

this reduces to equation (29) in text.

## 2 Survey of Consumer Finances

The Survey of Consumer Finances (SCF) is a survey conducted by the National Opinion Research Center at the University of Chicago. This survey is well suited for characterizing the earnings, income, and wealth concentration at the top because it over-samples rich households. The unit of observation we use is the household. Each wave of the survey samples more than 6,000 households and is representative of the US economy.

**Sample Selection.** As is standard in the literature, we exclude households with negative income. In addition, we focus on a sample of households in which the household head is between 22 and 79 years old.

**Wealth.** Our measure of household wealth is the variable constructed by the Federal Reserve for its Bulletin article which accompanies each wave of the SCF. Wealth is defined as total net worth, which equals assets minus debt. Assets includes both financial and non-financial assets. Financial assets include checking and savings accounts, stocks held directly and indirectly, bonds, etc. Non-financial assets, among others, include the value of houses and other real state, the value of farm and private businesses owned by the household.<sup>3</sup> Debt include both housing debt (e.g. mortgages), debt from lines of credit or credit cards, installment loans, etc.

**Income.** Our measure of income includes all sources of income excluding government transfers (e.g. social security and unemployment benefits) and excluding other (non classified) sources of income. Thus, we include wage income, income from businesses, income from interests and dividends, from capital gains, rent income and income from pensions and annuities.

**Definition of entrepreneurs.** In contrast to ?, we consider a broader measure of entrepreneurship that includes all households in which the household head owns a business, excluding those who own C-corporations.<sup>4</sup>

### 3 Robustness Economies

Here we report the moments we targeted and the parameter values in the two robustness economies we study. Table 1 shows the calibrated parameters values in the economy with a super-star state. Table 2 shows that the model matches well the moments we target, including the top 1% wealth and income shares, for all households, as well as separately for entrepreneurs and workers.

Table 3 shows the calibrated parameters in the economy calibrated to match the ? estimates of the labor and business income process. Notice that the persistence of both labor and entrepreneurial ability is substantially lower than in our baseline model. Table 4 shows the model’s predictions for the untargeted moments of wealth and income inequality. This

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<sup>3</sup>The value of houses, real state and businesses is self-reported. E.g. with respect to housing the survey asks: “What is the current value of this (home and land/apartment/property)?”. For businesses the survey asks: “What is the net worth of (your share of) this business?”

<sup>4</sup>The exact question in the survey is: (does the household head) “own privately-held businesses?” The SCF reports the legal status of up to two businesses own by the household. We identify households as owners of C-corporations if at least one of their businesses is reported to be a C-corporation.

parameterization fails to match the large degree of wealth and income inequality in the SCF data.

Table 1: Parameter Values in Economy with Super-Star State

$\beta$	0.967	discount factor
$\rho_e$	0.986	AR(1) $e$
$\sigma_e$	0.144	std. dev. $e$ shocks
$p_e$	2.2e-6	prob. enter super-star $e$ state
$q_e$	0.986	prob. stay super-star $e$ state
$\bar{e}$	6.008	log ability super-star $e$ state, rel. mean
$\rho_z$	0.972	AR(1) $z$
$\sigma_z$	0.141	std. dev. $z$ shocks
$p_z$	2.9e-6	prob. enter super-star $z$ state
$q_z$	0.980	prob. stay super-star $z$ state
$\bar{z}$	2.930	log ability super-star $z$ state, rel. mean
$\sigma$	12.30	demand elasticity at $q = 1$
$\mu_c$	1.340	mean productivity corporations
$\bar{K}$	0.068	fixed entry cost / GDP

Table 2: Moments Used to Calibrate Economy with Super-Star State

	Data	Model		Data	Model
Wealth to income ratio	6.57	6.61	Wealth share top 1%	0.35	0.37
Wealth share of entrepr.	0.46	0.46	Income share top 1%	0.22	0.23
Income share of entrepr.	0.31	0.31	Wealth share top 1% entrepr.	0.24	0.23
Gini wealth, all hhs	0.85	0.86	Income share top 1% entrepr.	0.23	0.23
Gini income, all hhs	0.64	0.63	Wealth share top 1% workers	0.31	0.30
Gini wealth, entrepr.	0.78	0.78	Income share top 1% workers	0.16	0.16
Gini income, entrepr.	0.68	0.66	Fraction of corporate firms	0.05	0.05
Gini wealth, workers	0.83	0.84	Sales share corporate firms	0.63	0.63
Gini income, workers	0.59	0.57			

Table 3: Parameter Values in Economy Calibrated to IRS Data

$\beta$	0.971	discount factor
$\rho_e$	0.908	AR(1) $e$
$\sigma_e$	0.217	std. dev. $e$ shocks
$\rho_z$	0.960	AR(1) $z$
$\sigma_z$	0.091	std. dev. $z$ shocks
$\sigma$	14.20	demand elasticity at $q = 1$
$\mu_c$	1.054	mean productivity corporations
$\bar{K}$	0.059	fixed entry cost / GDP

Table 4: Untargeted Moments in Economy Calibrated to IRS Data

	Data	Model		Data	Model
Wealth to income ratio	6.57	6.63	Wealth share top 1%	0.35	0.08
Wealth share of entrepr.	0.46	0.35	Income share top 1%	0.22	0.05
Income share of entrepr.	0.31	0.35	Wealth share top 1% entrepr.	0.24	0.08
Gini wealth, all hhs	0.85	0.57	Income share top 1% entrepr.	0.23	0.05
Gini income, all hhs	0.64	0.36	Wealth share top 1% workers	0.31	0.05
Gini wealth, entrepr.	0.78	0.60	Income share top 1% workers	0.16	0.04
Gini income, entrepr.	0.68	0.36	Fraction of corporate firms	0.05	0.05
Gini wealth, workers	0.83	0.54	Sales share corporate firms	0.63	0.65
Gini income, workers	0.59	0.33			