Supplementary material for "MoNK: Mortgages in a New-Keynesian Model"

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Appendix A: Recursive household problem

Capital owner's problem:

$$U(z, S, s^*) = \max_{x_K, n^*, (b^*)', l^*} \left\{ u(c^*, n^*) + \beta E[U(z', S', (s^*)')|z] \right\},$$

where a prime denotes a value next period and the per-period constraints and laws of motion of the sequential problem are to be substituted in the utility and value functions.

$$s^* = [k, b^*, d^*, \gamma^*, R^*]$$

Homeowner's problem:

$$V(z, S, s) = \max_{x_H, n, b'} \left\{ v(c, n, h) + \beta E[V(z', S', s')|z] \right\},$$

where a prime denotes a value next period and the per-period constraints and laws of motion of the sequential problem are to be substituted in the utility and value functions.

$$s = [h, b, d, \gamma, R]$$

Aggregate endogenous state

$$S = [K, H, B, D, \Gamma, \Re],$$

where Γ and $\Re$ are aggregate counterparts to γ and R.

Aggregate exogenous state: z contains the shocks, depending on the specification of the Taylor rule.

Appendix B: Equilibrium conditions

This appendix lists the conditions characterizing the equilibrium. We use these conditions to solve for the steady state (a situation in which all shocks are equal to their unconditional means and all real variables, interest rates, and the inflation rate are constant). The steady state is used to calibrate the model and serves as the point of local approximation of the equilibrium conditions in our solution method. The steady state is the same for both ARM and FRM. This is because, in the steady state, $i_t = i_t^F = R_t$.

Throughout, the notation is that, for instance, u_{ct} denotes the first derivative of the function u with respect to c, evaluated in period t. Alternatively, v_{2t}, for instance, denotes the first derivative of the function v with respect to the second argument, evaluated in period t. To ease the notation, instead of indexing the agents by "1" and "2", we denote the variables with a common notation pertaining to the capital owner by a "*". Variables with a "tilde" are normalized with respect to the price level as defined below.

Capital owner's optimality

First-order conditions:

$$u_{ct}(1-\tau_N)\epsilon_w w_t = -u_{nt}, \tag{1}$$

$$1 = E_t\left\{\beta\frac{u_{c,t+1}}{u_{ct}}\left[1+(1-\tau_K)(r_{t+1}-\delta_K)\right]\right\}, \tag{2}$$

$$1 = E_t\left\{\beta\frac{u_{c,t+1}}{u_{ct}}\left[\frac{(1+i_t)}{1+\pi_{t+1}}\right]\right\}, \tag{3}$$

$$1 = E_t\left\{\beta\frac{\widetilde{U}_{d,t+1}}{u_{ct}} + \beta\frac{U_{\gamma,t+1}}{u_{ct}}\zeta^*_{Dt}\left[\kappa-(\gamma^*_t)^\alpha\right] + \beta\frac{U_{R,t+1}}{u_{ct}}\zeta^*_{Dt}(i^F_t - R^*_t)\right\}. \tag{4}$$

Note: the last equation (for l^*_t) applies only in the FRM case, as explained in the text.

Benveniste-Scheinkman conditions:

$$\widetilde{U}_{dt} = u_{ct}\frac{R^*_t+\gamma^*_t}{1+\pi_t} + \beta\frac{1-\gamma^*_t}{1+\pi_t}E_t\left\{\widetilde{U}_{d,t+1} + \zeta^*_{lt}\left[(\gamma^*_t)^\alpha-\kappa\right]U_{\gamma,t+1} + \zeta^*_{lt}(R^*_t - i^F_t)U_{R,t+1}\right\}, \tag{5}$$

$$\begin{aligned} U_{\gamma t} = {} & u_{ct}\left(\frac{\widetilde{d}^*_t}{1+\pi_t}\right) - \beta\left(\frac{\widetilde{d}^*_t}{1+\pi_t}\right)E_t\widetilde{U}_{d,t+1} \\ & +\beta\left(\frac{\widetilde{d}^*_t}{1+\pi_t}\right)\left\{\zeta^*_{lt}[\kappa-(\gamma^*_t)^\alpha] + \frac{(1-\gamma^*_t)\alpha(\gamma^*_t)^{\alpha-1}}{\frac{1-\gamma^*_t}{1+\pi_t}\widetilde{d}^*_t + \widetilde{l}^*_t}\right\}E_tU_{\gamma,t+1} \\ & +\beta\left(\frac{\widetilde{d}^*_t}{1+\pi_t}\right)\zeta^*_{lt}(i^F_t - R^*_t)E_tU_{R,t+1}, \end{aligned} \tag{6}$$

$$U_{Rt} = u_{ct}\left(\frac{\widetilde{d}^*_t}{1+\pi_t}\right) + \beta\left(\frac{\frac{1-\gamma^*_t}{1+\pi_t}\widetilde{d}^*_t}{\frac{1-\gamma^*_t}{1+\pi_t}\widetilde{d}^*_t + \widetilde{l}^*_t}\right)E_tU_{R,t+1}. \tag{7}$$

Constraints:

$$c^*_t + q_{Kt}x_{Kt} + \widetilde{b}^*_{t+1} + \widetilde{l}^*_t = [(1-\tau_K)r_t + \tau_K\delta_K]k_t + (1+i_{t-1})\frac{\widetilde{b}^*_t}{1+\pi_t} + \widetilde{m}^*_t + (1-\tau_N)\epsilon_w w_t n^*_t + \tau^*_t + \Pi_t, \tag{8}$$

$$\widetilde{m}^*_t = (R^*_t+\gamma^*_t)\frac{\widetilde{d}^*_t}{1+\pi_t}, \tag{9}$$

$$\widetilde{d}^*_{t+1} = \frac{1-\gamma^*_t}{1+\pi_t}\widetilde{d}^*_t + \widetilde{l}^*_t, \tag{10}$$

$$\gamma^*_{t+1} = (1-\phi^*_t)(\gamma^*_t)^\alpha + \phi^*_t\kappa, \tag{11}$$

$$R^*_{t+1} = \begin{cases} (1-\phi^*_t)R^*_t + \phi^*_t i^F_t, & \text{if FRM,} \\ i_t, & \text{if ARM,} \end{cases} \tag{12}$$

$$k_{t+1} = (1-\delta_K)k_t + x_{Kt}. \tag{13}$$

Homeowner's optimality

First-order conditions:

$$v_{ct}(1-\tau_N)w_t = -v_{nt}, \tag{14}$$

$$1 = E_t\left[\beta\frac{v_{c,t+1}}{v_{ct}}\left(\frac{1+i_t+\Upsilon_t}{1+\pi_{t+1}}\right)\right], \tag{15}$$

$$v_{ct}(1-\theta)q_{Ht} = \beta E_t\left\{V_{h,t+1} + q_{Ht}\theta\left[\widetilde{V}_{d,t+1} + \zeta_{Dt}(\kappa-\gamma^\alpha_t)V_{\gamma,t+1} + \zeta_{Dt}(i^M_{t+1}-R_t)V_{R,t+1}\right]\right\}. \tag{16}$$

Note: $i^M_{t+1} = i^F_t$ (FRM case), $i^M_{t+1} = i_t$ (ARM case).

Benveniste-Scheinkman conditions:

$$\widetilde{V}_{dt} = -v_{ct}\frac{R_t+\gamma_t}{1+\pi_t} + \beta\frac{1-\gamma_t}{1+\pi_t}E_t\left[\widetilde{V}_{d,t+1} + \zeta_{lt}\left(\gamma^\alpha_t-\kappa\right)V_{\gamma,t+1} + \zeta_{lt}(R_t-i^M_{t+1})V_{R,t+1}\right], \tag{17}$$

$$\begin{aligned} V_{\gamma t} = &-v_{ct}\left(\frac{\widetilde{d}_t}{1+\pi_t}\right) - \beta\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)E_t\widetilde{V}_{d,t+1} \\ &+\beta\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\left[\zeta_{lt}(\kappa-\gamma^\alpha_t) + \frac{(1-\gamma_t)\alpha\gamma^{\alpha-1}_t}{\frac{1-\gamma_t}{1+\pi_t}\widetilde{d}_t + \widetilde{l}_t}\right]E_tV_{\gamma,t+1} \\ &+\beta\left(\frac{\widetilde{d}_t}{1+\pi_t}\right)\zeta_{lt}(i^M_{t+1}-R_t)E_tV_{R,t+1}, \end{aligned} \tag{18}$$

$$V_{Rt} = -v_{ct}\left(\frac{\widetilde{d}_t}{1+\pi_t}\right) + \beta\left(\frac{\frac{1-\gamma_t}{1+\pi_t}\widetilde{d}_t}{\frac{1-\gamma_t}{1+\pi_t}\widetilde{d}_t + \widetilde{l}_t}\right)E_tV_{R,t+1}, \tag{19}$$

$$V_{ht} = v_{ht} + \beta(1-\delta_H)E_tV_{h,t+1}. \tag{20}$$

Constraints:

$$\widetilde{l}_t = \theta q_{Ht}x_{Ht}, \tag{21}$$

$$h_{t+1} = (1-\delta_H)h_t + x_{Ht}. \tag{22}$$

Producers' optimality

Profit maximization condition, already aggregated and log-linearized (the New-Keynesian Phillips Curve):

$$\pi_t = \frac{(1-\psi)(1-\beta\psi)}{\psi}\widehat{\chi}_t + \beta E_t \pi_{t+1}. \tag{23}$$

Marginal costs:

$$\chi_t \equiv A^{-1}(r_t/\varsigma)^{\varsigma}[w_t/(1-\varsigma)]^{1-\varsigma}. \tag{24}$$

Cost minimization condition:

$$\frac{w_t}{r_t} = \left(\frac{1-\varsigma}{\varsigma}\right)\frac{K_t}{N_t}. \tag{25}$$

Production function, already aggregate and log-linearized:

$$\widehat{Y}_t = \varsigma \widehat{K}_t + (1-\varsigma)\widehat{N}_t. \tag{26}$$

(A TFP shock $\widehat{A}_t$ is added when required by an experiment.)

PPF curvature

$$q_{Kt} = q^K(X_{Kt}), \tag{27}$$

$$q_{Ht} = q^H(X_{Ht}). \tag{28}$$

Monetary policy

The benchmark Taylor rule (analogously for other experiments)

$$i_t = i + \mu_t - \pi + \nu_\pi(\pi_t - \mu_t) + \eta_t, \tag{29}$$

where $\mu_{t+1} = (1-\rho_\mu)\pi + \rho_\mu \mu_t + \xi_{\mu,t+1}$ and $\eta_{t+1} = \rho_\eta \eta_t + \xi_{\eta,t+1}$.

Market clearing

$$(1-\Psi)\widetilde{l}_t^* = \Psi \widetilde{l}_t, \tag{30}$$

$$(1-\Psi)\widetilde{b}_t^* + \Psi \widetilde{b}_t = 0, \tag{31}$$

$$\epsilon_w(1-\Psi)n_t^* + \Psi n_t = N_t, \tag{32}$$

$$(1-\Psi)k_t = K_t, \tag{33}$$

$$(1-\Psi)c_t^* + \Psi c_t + q_{Kt}(1-\Psi)x_{Kt} + q_{Ht}\Psi x_{Ht} + G = Y_t - \Delta. \tag{34}$$

Government budget constraint

$$G + (1-\Psi)\tau_t^* + \Psi \overline{\tau}_2 = \tau_K(r_t - \delta_K)K_t + \tau_N w_t N_t. \tag{35}$$

Aggregate consistency

$$(1-\Psi)\widetilde{d}_t^* = \Psi \widetilde{d}_t, \tag{36}$$

$$\gamma_t^* = \gamma_t, \tag{37}$$

$$R_t^* = R_t. \tag{38}$$

Equation count: 38 equations (FRM), 37 equations (ARM). Note: the homeowner's budget constraint holds by Walras' law.

Endogenous variables

38 variables (FRM), 37 variables (ARM):

Allocations: c_t^*, n_t^*, x_{Kt}, k_t, c_t, n_t, x_{Ht}, h_t, Y_t, N_t, K_t.	11 variables
Prices: χ_t, π_t, i_t, i_t^F (FRM only), r_t, w_t, q_{Kt}, q_{Ht}.	8 vars (FRM), 7 vars (ARM)
Mortgages: m_t^*, $\widetilde{l}_t^*$, $\widetilde{d}_{t+1}^*$, γ_{t+1}^*, R_{t+1}^*, $\widetilde{l}_t$, $\widetilde{d}_{t+1}$, γ_{t+1}, R_{t+1}.	9 variables
Bonds: $\widetilde{b}_{t+1}^*$, $\widetilde{b}_{t+1}$.	2 variables
Transfer: τ_t^*.	1 variable
Value function: $\widetilde{U}_{dt}$, $U_{\gamma t}$, U_{Rt}, $\widetilde{V}_{dt}$, $V_{\gamma t}$, V_{Rt}, V_{ht}.	7 variables

Transformation of variables

To ensure stationarity:

$$\begin{aligned}
\widetilde{U}_{dt} &\equiv p_{t-1} U_{dt}, \\
\widetilde{m}_t^* &\equiv m_t^* / p_t, \\
\widetilde{d}_t^* &\equiv d_t^* / p_{t-1}, \\
\widetilde{l}_t^* &\equiv l_t^* / p_t, \\
\widetilde{b}_t^* &\equiv b_t^* / p_{t-1}, \\
\widetilde{V}_{dt} &\equiv p_{t-1} V_{dt}, \\
\widetilde{d}_t &\equiv d_t / p_{t-1}, \\
\widetilde{l}_t &\equiv l_t / p_t, \\
\widetilde{b}_t &\equiv b_t / p_{t-1}.
\end{aligned}$$

Auxiliary notation:

$$\zeta_{lt}^* \equiv \frac{\widetilde{l}_t^*}{\left(\frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^* + \widetilde{l}_t^*\right)^2} \in (0,1),$$

$$\zeta_{Dt}^{*} \equiv \frac{\frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^*}{\left(\frac{1-\gamma_t^*}{1+\pi_t}\widetilde{d}_t^* + \widetilde{l}_t^*\right)^2} \in (0,1),$$

$$\zeta_{Dt} \equiv \frac{\frac{1-\gamma_t}{1+\pi_t}\widetilde{d}_t}{\left(\frac{1-\gamma_t}{1+\pi_t}\widetilde{d}_t + \widetilde{l}_t\right)^2} \in (0,1),$$

$$\zeta_{lt} \equiv \frac{\widetilde{l}_t}{\left(\frac{1-\gamma_t}{1+\pi_t}\widetilde{d}_t + \widetilde{l}_t\right)^2} \in (0,1),$$

$$\phi_t^* \equiv \widetilde{l}_t^*/\widetilde{d}_{t+1}^* \in (0,1),$$

$$\Xi_t \equiv (1-\Psi)\tau_{Bt}(1-i_{t-1})\widetilde{b}_t^*/(1+\pi_t),$$

$$\Upsilon_t = \Upsilon\left(-\widetilde{b}_{t+1}\right).$$

Appendix C: Real marginal costs and aggregate output

Given that the NKPC in the text is based on a log-linear approximation, our exposition uses log-linear approximations of the other relevant equations as well. First, a log-linear aggregate counterpart to the individual marginal cost is

$$\widehat{\chi}_t = \frac{\varsigma}{1-\varsigma}\frac{1}{1+\overline{Y}/Y}\widehat{Y}_t + \widehat{w}_t - \frac{\varsigma}{1-\varsigma}\widehat{K}_t, \tag{C.1}$$

where all variables are in percentage deviations from steady state and $\overline{Y}$ is output adjusted for the fixed cost.[1] For a given $\widehat{w}_t$ and the state variable $\widehat{K}_t$, this equation provides a positive relationship between aggregate output and the marginal cost. The wage rate can be eliminated from equation (C.1) by utilizing first-order conditions of the two agents for labor supply: $u_{ct}(1-\tau_N)\epsilon_w w_t + u_{nt} = 0$ and $v_{ct}(1-\tau_N)w_t + v_{nt} = 0$. At the aggregate level, in a log-linearized form, these conditions become

$$\Phi_{1n}\widehat{N}_{1t} = \Phi_{1w}\widehat{w}_t + \Phi_{1c}\widehat{C}_{1t} \quad \text{and} \quad \Phi_{2n}\widehat{N}_{2t} = \Phi_{2w}\widehat{w}_t + \Phi_{2c}\widehat{C}_{2t}, \tag{C.2}$$

where for standard utility functions (e.g., log additive) $\Phi_{1c} < 0$, $\Phi_{1w} > 0$, and $\Phi_{1n} > 0$, and similarly for the second agent. Further, from the production function of intermediate goods producers follows

$$(1+\overline{Y}/Y)^{-1}\widehat{Y}_t = (1-\varsigma)[\upsilon\widehat{N}_{1t} + (1-\upsilon)\widehat{N}_{2t}] + \varsigma\widehat{K}_t, \tag{C.3}$$

where $\upsilon \equiv \epsilon_w N_1/N$.[2] For tractability, focus on immediate responses from steady state. This allows us to set $\widehat{K}_t = 0$. Furthermore, to simplify the exposition, assume that the agents can

[1] Equation (C.1) contains the well-known result that an aggregation bias—occurring due to price dispersion across the intermediate good producers—disappears under log-linearization around a zero steady-state inflation rate.

[2] Equation (C.3) also contains the result that the Calvo aggregation bias due to price dispersion disappears under a log-linearization around zero steady-state inflation rate.

smooth out consumption reasonably well so that changes in consumption are small, $\widehat{C}_{1t} \approx 0$ and $\widehat{C}_{2t} \approx 0$. The above equations, once $\widehat{\chi}_t$, $\widehat{w}_t$, $\widehat{N}_{1t}$ and $\widehat{N}_{2t}$ are substituted out, then provide a negative relationship between $E_t\pi_{t+1} - \pi_t$ and $\widehat{Y}_t$ in the NKPC as argued in the text.

Appendix D: Alternative formulations of the policy shocks

To facilitate understanding of the inner workings of the model, we have modeled the two policy shocks as independent AR(1) processes. This formulation also had the benefit that it allowed a straightforward mapping from the model to the standard, orthogonal, level and slope factors. Here we provide two examples of alternative formulations of the shocks. In particular, we show that a persistent shock in the Taylor rule can take the form of a statement shock, in the sense of Gürkaynak, Sack and Swanson (2005a), or of an information shock about the future state of the economy, in the sense of Nakamura and Steinsson (2018). In both cases, the shock, like in the benchmark specification, persistently affects expectations of future short rates (Cieslak and Schrimpf, 2018, document that different types of central bank statements have different content, in terms of pure statement shocks vs. information shocks).

Persistent policy shock as a pure statement shock

A statement shock is a policy shock whereby the central bank surprises the private sector by changing the expected path of future policy rates (e.g., by a press release) while leaving the current policy rate unchanged. To provide such formulation of the persistent shock in the model, consider an orthogonal rotation of the two basic policy shocks in the Taylor rule in the main text. First, rewrite the Taylor rule (16) as

$$i_t = i + \nu_\pi(\pi_t - \pi) + v^\top z_t$$

where $v^\top \equiv [1 - \nu_\pi, 1]$ and $z_t^\top \equiv [z_{1t}, z_{2t}]$, with $z_{1t} \equiv \mu_t - \pi$ being the persistent shock and $z_{2t} \equiv \eta_t$ being the temporary shock. Next, consider an invertible two-by-two matrix M and define a new vector of shocks $z_t^* = Mz_t$. The Taylor rule can then be written in terms of the new shocks as

$$i_t = i + \nu_\pi(\pi_t - \pi) + v^\top M^{-1} z_t^*. \tag{D.1}$$

The stochastic process for z_t^* is derived from the stochastic process for z_t and the matrix M. Consolidating the two independent AR(1) processes for z_{1t} and z_{2t} into a VAR(1), z_t evolves as $z_{t+1} = \rho z_t + \xi_{t+1}$, where ρ is a diagonal matrix with ρ_μ and ρ_η on the diagonal. The process for z_t^* is then

$$z_{t+1}^* = \lambda z_t^* + M\xi_{t+1},$$

where $\lambda = M\rho M^{-1}$. Observe that since z_t^* is just a linear combination of z_t, λ has the same eigenvalues as ρ and, thus, the new process has the same persistence as the original process (recall that the highest eigenvalue of the the original process is 0.99).

The four elements of M are identified from four restrictions. First, like the original shocks, we require the new shocks to be orthogonal to each other: $E(z_{1t}^* z_{2t}^*) = 0$. Then, we normalize the variance of the shocks to be unity: $E(z_{1t}^*) = 1$ and $E(z_{2t}^*) = 1$. We are free to do so as we are not interested in data decomposition. Finally, we impose a restriction that

allows z_{1t}^* to be interpreted as a statement shock. Specifically, combining the Taylor rule (D.1) with the Fisher equation in the main text, an equivalent expression to equation (18) in the main text (for the equilibrium short rate) can be derived for the new shocks

$$i_t \approx i - v^\top M^{-1} \frac{1}{\nu_\pi} \lambda \left(I - \frac{1}{\nu_\pi} \lambda \right)^{-1} z_t^* + \sum_{s=0}^{\infty} \left(\frac{1}{\nu_\pi} \right)^s E_t r_{t+s}^*. \tag{D.2}$$

To derive this expression, we have used the stochastic process for z_t^* to evaluate the expectations of future values of the vector z_t^*. Further, while r_{t+s}^* potentially depends on the values of all state variables in period $t+s$, suppose that a reasonable approximation is $r_{t+s}^* \approx g^\top z_{t+s}^*$, where the vector $g^\top$ is determined in equilibrium, as defined in the main text. In this case, equation (D.2) can be written as

$$i_t \approx i - v^\top M^{-1} \frac{1}{\nu_\pi} \lambda \left(I - \frac{1}{\nu_\pi} \lambda \right)^{-1} z_t^* + g^\top \left(I - \frac{1}{\nu_\pi} \lambda \right)^{-1} z_t^* \tag{D.3}$$

or $i_t \approx i - a_1 z_{1t}^* - a_2 z_{2t}^*$, where a_1 and a_2 are the corresponding loadings from equation (D.3) on z_{1t}^* and z_{2t}^*, respectively. That is, the equilibrium short rate again maps into the general framework whereby it is an affine function of two orthogonal factors. The final restriction on M, which makes z_{1t}^* a statement shock, is $a_1 = 0$. That is, z_{1t}^* has no effect on the equilibrium short rate but, through the transition matrix λ, it forecasts future values of z_{2t}^* and therefore future values of the short rate. Thus, z_{1t}^* is a statement shock about the future course of monetary policy, while z_{2t}^* is an action shock (it affects the current interest rate i_t).

Persistent policy shock as an information shock

Nakamura and Steinsson (2018) argue that surprises in central bank statements about the future path of policy rates reflect new information about the likely future state of the economy, rather than being genuine policy surprises.[3] In this case, policy statements contain information about the likely future path of some state variable affecting inflation (or output gap in a broader formulation of the Taylor rule).

To make this notion concrete, we follow Nakamura and Steinsson (2018) and consider shocks affecting the real interest rate that the policy rule of the central bank needs to accommodate, in order to stabilize inflation. In contrast to their model, where the real rate is shocked directly, the real interest rate here is endogenous and the driving exogenous force is shocks to total factor productivity. The Taylor rule (16) is modified by removing the μ_t shock and instead making the intercept time-varying. That is,

$$i_t = r_t^* + \pi + \nu_\pi(\pi_t - \pi) + \eta_t, \tag{D.4}$$

where η_t is the standard temporary shock. In contrast to the Taylor rule in the main text and the above Taylor rule (D.1), there are no persistent policy shocks per se, but the central bank allows the policy rate to move one-for-one with the ex-ante one-period real rate r_t^*, which was defined in the Fisher equation (17) in the main text. Such a Taylor rule is common in the literature (see, e.g., Galí, 2015). Combining the Taylor rule (D.4) with the Fisher

[3] See also Jarocinski and Karadi (2018).

equation in the main text implies that, absent η_t shocks, this policy fully stabilizes inflation; i.e., $\pi_t = \pi \ \forall t$. As a result, movements in the policy rate reflect one-for-one movements in the real rate as in Nakamura and Steinsson (2018).

A news about the future state of the economy is an exogenous random variable S_t, which is informative about the future path of total factor productivity A_t (which we now allow to be time-varying). We capture this property of S_t by a VAR(1) process

$$\begin{bmatrix} A_{t+1} \\ S_{t+1} \end{bmatrix} = \begin{bmatrix} \rho_A & 1 \\ 0 & \rho_S \end{bmatrix} \begin{bmatrix} A_t \\ S_t \end{bmatrix} + \begin{bmatrix} \xi_{A,t+1} \\ \xi_{S,t+1} \end{bmatrix},$$

where $\rho_A, \rho_S \in (0,1)$ and ξ_{At} and ξ_{St} are orthogonal innovations. Suppose that ρ_A is such that A_t has autocorrelation equal to the typical estimates for the Solow residual, the spillover effect from S_t to A_{t+1} is equal to one, and ρ_S is such that the equilibrium persistence of i_t in response to the S_t shock is the same as in response to the μ_t shock in the Taylor rule (16). This requires $\rho_S = 0.99$.

To see how this VAR(1) process impacts on the real, and thus the nominal short rate, observe that it implies

$$\Delta A_{t+1} = (\rho_A - 1)A_t + S_t + \xi_{A,t+1},$$

where S_t is close to random walk. The specification of the VAR(1) process in productivity level is thus equivalent to a process for productivity growth that contains a random slow-moving component, S_t, as in Bansal and Yaron (2004). From this perspective, a realization of S_t is a news about a very persistent change in the productivity growth rate. As persistently higher productivity growth leads to persistently higher output growth and consumption growth, a realization of a higher S_t leads to a persistent increase in the ex-ante real rate (through an Euler equation) and, through the Taylor rule (D.4), in the nominal policy rate. We can thus write

$$i_t = i + r_A A_t + r_S S_t + \Xi_t + \pi + \nu_\pi(\pi_t - \pi) + \eta_t, \tag{D.5}$$

for some equilibrium coefficients r_A and r_S, with Ξ_t capturing the equilibrium effects on the real rate of the endogenous state variables. Suppose that the effect of Ξ_t is relatively small, compared with the direct effects of the shocks. Then the Taylor rule (D.5) has approximately the same form as the Taylor rule (16) in the main text, with two orthogonal "policy shocks": S_t being the persistent shock and η_t being the standard temporary shock.[4] Of course, in this case the orthogonal two-by-two matrix M^* that relates these new policy shocks to the benchmark shocks in the Taylor rule in the main text is a diagonal matrix, with M^*_{11} a scaling factor and $M^*_{22} = 1$. The shocks S_t and η_t can be further rotated, using another orthogonal two-by-two matrix M, so that one is a statement shock (it does not impact on i_t in period t) and the other is an action shock.

[4]The only caveat is that the Taylor rule is now also subject to the A_t shock.

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