

Interest Rates and the Design of Financial Contracts

Internet Appendix

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This appendix provides supplementary analysis for “Interest Rates and the Design of Financial Contracts.” First, we provide supplementary details on our sample and robustness tables for the main analysis in the paper. Second, we outline the procedure used to construct the CLO simulation discussed in Section 3.3 of the paper.

Contents

A1	Supplementary Empirical Analysis	2
A2	CLO Simulation	11

A1 Supplementary Empirical Analysis

Figure A1: Principal Components of the Treasury Yield Curve

This figure presents the yield curve characteristics used in our regression analysis. Panels A through C plot the time-series of the first three principal components of the Treasury yield curve. Following the literature, we refer to these principal components as Level, Slope, and Curvature. Panel D plots the MOVE Index, a weighted average of implied volatilities from one-month options on the 2-year, 5-year, 10-year, and 30-year Treasury yields. Data on zero-coupon Treasury yields are from Gurkaynak, Sack, and Wright (2007), while the MOVE Index is collected from Bloomberg.

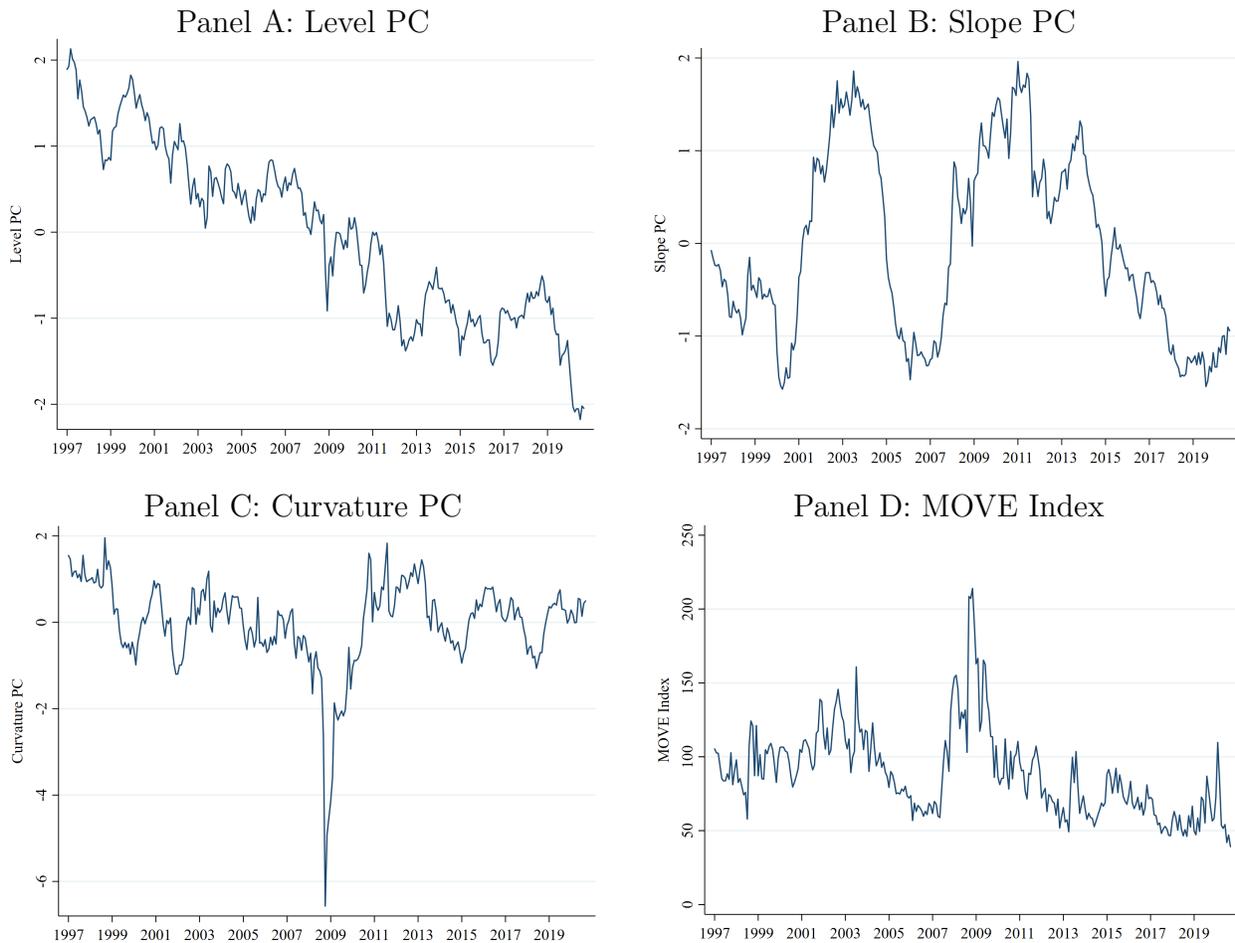


Figure A2: Distribution of LIBOR Floor Levels

This figure presents box-and-whisker plots of the cross-sectional distribution of LIBOR floor levels over time. The horizontal lines of the box denote the quartiles of the distribution. Boxes with fewer than three lines correspond to a concentration of observations. The whiskers extend 1.5 quartiles away from the median of the distribution. Observations beyond 1.5 quartiles from the median are displayed individually as dots.

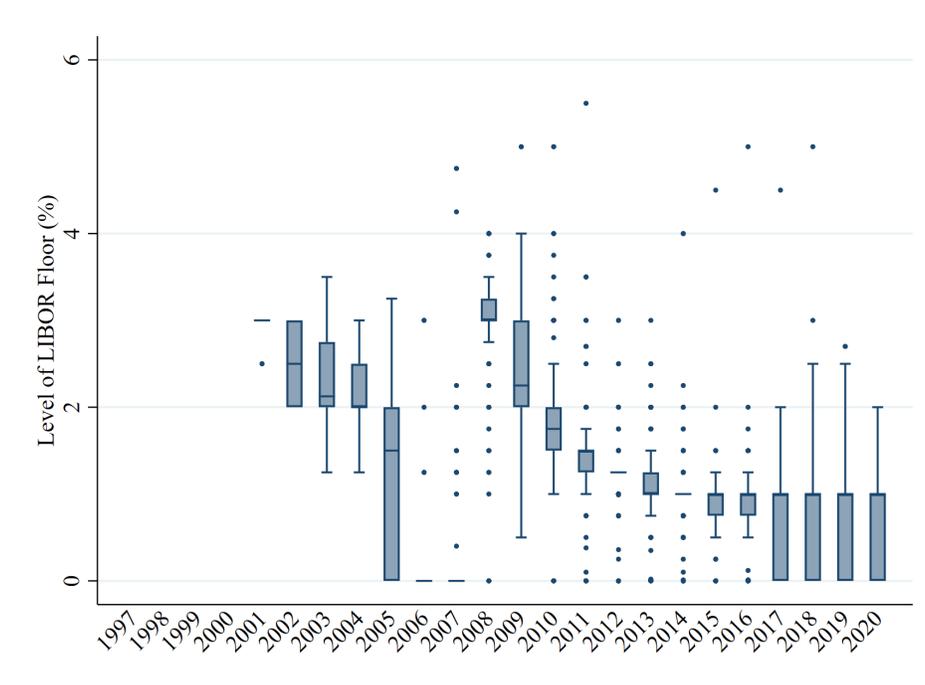


Figure A3: Interest Rates and Contract Terms – Institutional and Pro Rata Tranches

This figure presents time-series plots of contract terms against short-term interest rates. Each panel presents two quarterly time series. The dotted green line presents the average three-month LIBOR rate for the quarter. The solid blue and dashed red lines correspond to a component of the contract pricing mechanism, expressed as an average weighted by loan size, for pro rata and institutional tranches, respectively. For ease of presentation, each quarterly series is presented as a moving average with one-half weight on the current quarter and one-quarter weight on each of the neighboring quarters.

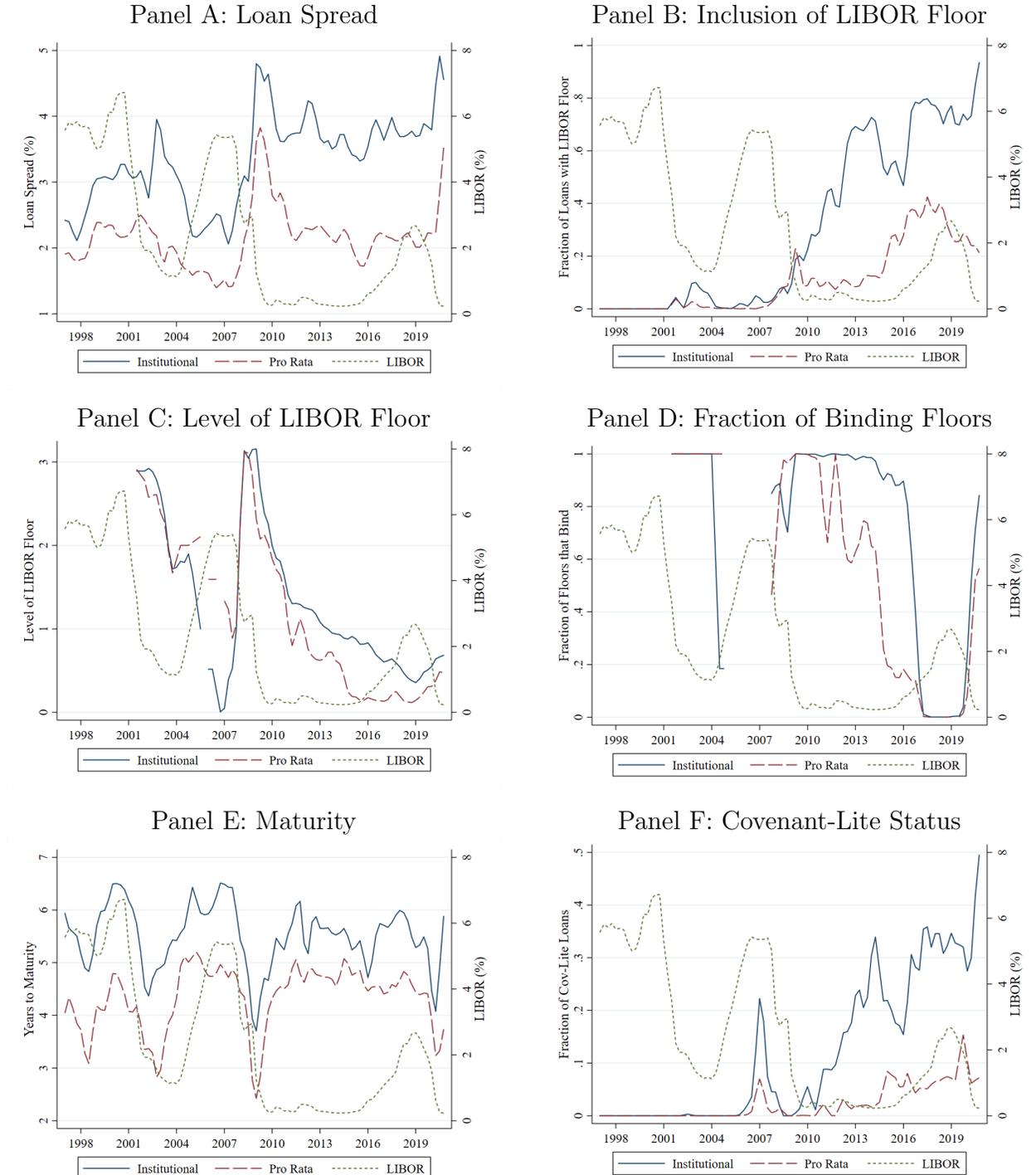


Table A1: Aggregate Determinants of Contract Terms – Institutional and Pro Rata Tranches

This table reports panel regressions of loan contract terms on macroeconomic variables. Panels A and B split the sample into institutional and pro rata tranches, respectively. Each column has a different contract term as the dependent variable. Loan spreads and floors are in percentage terms and maturity is in years. Floor Val. is the spread adjustment implied by the Black (1976) value of the floor. Adj. Spread is the spread of a floating-rate loan with the same value as the loan with a floor. LIBOR is the three-month LIBOR rate on the loan's start date, in percentage terms. Equity CAPE is the cyclically adjusted price-earnings ratio from Robert Shiller's website. Real GDP Growth is lagged by one quarter. 1(Recession) is an indicator for NBER U.S. recession dates, lagged by one month. Both panels control for borrower and lead arranger fixed effects, loan credit rating and borrower industry indicators, log issue size, and indicators for revolving lines of credit and Term Loan A facilities. *t*-statistics based on standard errors clustered by firm and year-quarter are reported in parentheses. *, **, and *** denote *p*-values less than 0.10, 0.05, and 0.01, respectively.

Panel A: Institutional Tranches (TL B)

Dependent Variable	Spread	1(Floor)	Floor Level	Floor Val.	Adj. Spread	Maturity	1(Cov-Lite)
LIBOR (%)	-0.279*** (-14.12)	-0.107*** (-13.73)	0.045 (1.26)	-0.048*** (-10.80)	-0.328*** (-14.61)	0.124*** (7.00)	-0.019*** (-3.56)
Equity CAPE	0.002 (0.14)	0.016*** (6.87)	-0.089*** (-10.34)	-0.005** (-2.29)	-0.003 (-0.21)	0.030*** (3.68)	0.013*** (4.89)
Real GDP Growth	-0.007* (-1.68)	-0.004*** (-3.20)	0.003 (0.85)	0.000 (0.00)	-0.008 (-1.34)	0.011** (2.49)	-0.002*** (-2.66)
1(Recession)	0.642*** (6.72)	-0.048* (-1.95)	0.429*** (3.06)	0.147*** (2.97)	0.775*** (7.40)	-0.617*** (-6.07)	-0.010 (-0.61)
Borrower FE	X	X	X	X	X	X	X
Lead Arranger FE	X	X	X	X	X	X	X
Controls	X	X	X	X	X	X	X
Outcome Mean	3.879	0.491	0.953	0.180	4.048	5.773	0.161
Outcome SD	1.802	0.500	0.569	0.275	1.935	1.601	0.367
Adj. R ²	0.631	0.674	0.659	0.461	0.640	0.471	0.501
Observations	16,763	16,763	7,713	16,139	16,139	16,172	16,763

Panel B: Pro Rata Tranches (RC and TL A)

Dependent Variable	Spread	1(Floor)	Floor Level	Floor Val.	Adj. Spread	Maturity	1(Cov-Lite)
LIBOR (%)	-0.208*** (-12.79)	-0.049*** (-10.02)	0.268*** (6.32)	-0.017*** (-7.32)	-0.225*** (-12.57)	0.110*** (7.73)	-0.007*** (-3.28)
Equity CAPE	0.003 (0.42)	0.009*** (5.04)	-0.137*** (-11.54)	-0.001* (-1.77)	0.002 (0.28)	0.003 (0.53)	0.004*** (3.95)
Real GDP Growth	-0.007 (-1.34)	-0.005*** (-3.20)	0.012*** (3.09)	-0.001 (-0.77)	-0.010* (-1.73)	0.018* (1.94)	-0.001 (-1.41)
1(Recession)	0.539*** (4.80)	0.017 (0.73)	0.545*** (4.35)	0.049*** (2.84)	0.578*** (4.76)	-0.699*** (-5.40)	-0.007 (-0.96)
Borrower FE	X	X	X	X	X	X	X
Lead Arranger FE	X	X	X	X	X	X	X
Controls	X	X	X	X	X	X	X
Outcome Mean	2.971	0.187	0.798	0.061	3.020	4.875	0.036
Outcome SD	1.430	0.390	0.836	0.180	1.498	1.394	0.186
Adj. R ²	0.659	0.451	0.815	0.284	0.658	0.370	0.341
Observations	17,096	17,096	2,440	16,851	16,851	15,809	17,096

Table A2: Alternative Estimates of Merton (1974) Model Interest Rate Sensitivities

This table reports the sensitivity of loan spreads to risk-free rates in the Merton (1974) model calibrated to the borrowers and loan terms in our sample. For each loan, we compute the model-implied change in the credit spread over the risk-free rate with respect to a one percentage point change in the risk-free rate. We summarize the distribution of these sensitivities for the full sample and by loan credit rating, grouping the investment-grade (IG) loans together. We calibrate asset volatility in the model to match the observed loan spread, setting the risk-free rate to maturity-matched zero-coupon LIBOR rate interpolated from the swap curve, the expected life of the debt to the loan's contractual maturity times the average ratio of realized life to contractual maturity by credit rating (to account for prepayment), and the face value of debt equal to the borrower's loan debt outstanding at the prior quarter-end plus the face value of the present loan. Panel A uses the contractual maturity of the loan instead of its expected life. Panel B uses the three-month LIBOR rate instead of the maturity-matched zero-coupon LIBOR rate. Both panels are based on the full sample, interpolating the capital structure data for loans that could not be matched to Capital IQ by using the average loan-to-assets ratio by credit rating.

Panel A: Contractual Maturity instead of Expected Life

Loan Rating	Any	IG	BB+	BB	BB-	B+	B	B-	NR
Mean	-0.382	-0.270	-0.351	-0.368	-0.397	-0.425	-0.484	-0.635	-0.357
St. Dev.	0.120	0.096	0.108	0.117	0.108	0.102	0.096	0.123	0.108
p25	-0.461	-0.322	-0.411	-0.440	-0.469	-0.505	-0.557	-0.757	-0.439
p50	-0.381	-0.273	-0.348	-0.359	-0.384	-0.408	-0.499	-0.655	-0.356
p75	-0.304	-0.218	-0.285	-0.292	-0.323	-0.351	-0.410	-0.523	-0.290
Expected Life	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Observations	47,142	984	1,158	1,937	3,256	4,151	3,094	728	26,169

Panel B: LIBOR instead of Bootstrapped Zero-Coupon Rate

Loan Rating	Any	IG	BB+	BB	BB-	B+	B	B-	NR
Mean	-0.336	-0.218	-0.262	-0.287	-0.316	-0.351	-0.383	-0.559	-0.333
St. Dev.	0.112	0.086	0.095	0.106	0.099	0.094	0.088	0.117	0.108
p25	-0.416	-0.263	-0.308	-0.343	-0.371	-0.413	-0.444	-0.672	-0.414
p50	-0.335	-0.218	-0.255	-0.275	-0.305	-0.339	-0.386	-0.578	-0.333
p75	-0.262	-0.165	-0.205	-0.216	-0.251	-0.287	-0.317	-0.461	-0.264
Expected Life	0.655	0.597	0.470	0.485	0.486	0.505	0.451	0.477	0.751
Observations	47,084	983	1,154	1,932	3,249	4,144	3,085	728	26,144

Table A3: Summary Statistics from Dealscan and the IHS Markit-Dealscan Merge

This table reports summary statistics on the Dealscan sample (Panel A) and the intersection of the IHS Markit and Dealscan samples (Panel B). The Dealscan sample includes U.S. dollar denominated revolving credit facilities and term loans that have nonmissing data on the spread and issue size. The IHS Markit and Dealscan samples are merged by the following algorithm. For each loan in IHS Markit, we identify all loans in Dealscan of the same broad type (credit line or term loan), start date within 30 days, end date within 366 days, and issue size within 25%. We then clean the issuer names to improve the match quality by standardizing common words (e.g., Company to Co, Limited Partnership to LP, etc.). We retain all loans that are an exact match on the cleaned issuer name. Finally, if there are multiple matches, we narrow down to the nearest matches on specific loan type (e.g., Term Loan C), then maturity date, then issue size. Panel B reports statistics from both sources in the merged sample.

Panel A: Dealscan Sample

	Mean	Std.Dev.	Min.	p10	p50	p90	Max.	Obs.
Spread (%)	2.60	1.70	0.01	0.75	2.25	4.75	22.5	117,535
Maturity	4.30	2.18	0.01	1.00	5.00	7.00	36.0	117,535
Loan Amount (\$MM)	291.7	643.8	0.00	12.5	100.0	700.0	30,000	117,535

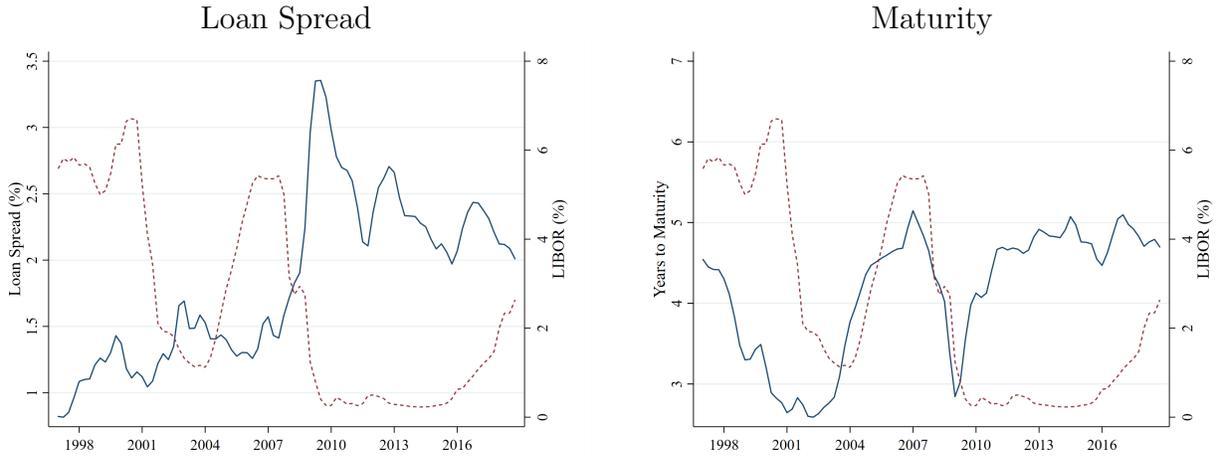
Panel B: IHS Markit-Dealscan Merge

	Mean	Std.Dev.	Min.	p10	p50	p90	Max.	Obs.
<u>Spread (%)</u>								
IHS Markit	3.08	1.47	0.09	1.50	2.75	5.00	15.0	10,273
Dealscan	3.04	1.46	0.03	1.50	2.75	5.00	17.5	10,273
<u>Maturity</u>								
IHS Markit	5.32	1.31	0.09	3.83	5.00	7.00	15.0	10,273
Dealscan	5.31	1.30	0.09	3.91	5.00	7.00	15.0	10,273
<u>Loan Amount (\$MM)</u>								
IHS Markit	440.1	711.5	1.00	30.0	200.0	1,050	10,700	10,273
Dealscan	438.2	707.3	1.00	30.0	200.0	1,050	10,700	10,273

Figure A4: Interest Rates and Contract Terms – Dealscan Sample

This figure presents time-series plots of contract terms against short-term interest rates. Each panel presents two quarterly time series. The dashed red line presents the average three-month LIBOR rate for the quarter. The solid blue line corresponds to a component of the contract pricing mechanism, expressed as an average weighted by loan size. For ease of presentation, each quarterly series is presented as a moving average with one-half weight on the current quarter and one-quarter weight on each of the neighboring quarters. Panel A is based on loans in Dealscan meeting the sample criteria and Panel B is based on the intersection of the IHS Markit and Dealscan samples.

Panel A: Dealscan Sample



Panel B: IHS Markit-Dealscan Merge

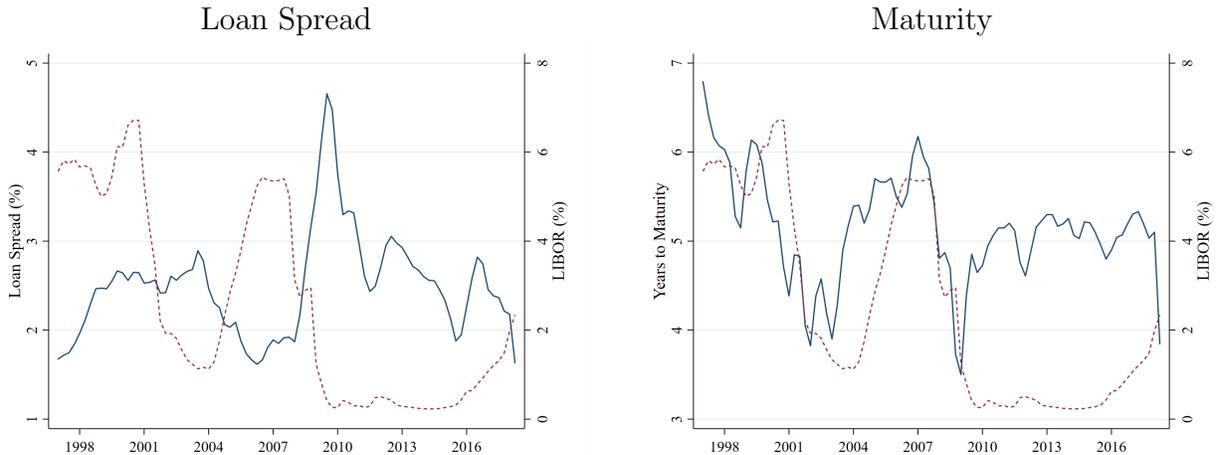


Table A4: Aggregate Determinants of Contract Terms – Dealscan Sample

This table reports panel regressions of loan contract terms on macroeconomic variables. The left two columns are based on loans in Dealscan meeting the sample criteria and the right two columns are based on the intersection of the IHS Markit and Dealscan samples. Each column has a different contract term as the dependent variable. Loan spreads are in percentage terms and maturity is in years. LIBOR is the three-month LIBOR rate on the loan’s start date, in percentage terms. Equity CAPE is the cyclically adjusted price-earnings ratio from Robert Shiller’s website. Real GDP Growth is lagged by one quarter. 1(Recession) is an indicator for NBER U.S. recession dates, lagged by one month. The regression controls include borrower fixed effects, loan credit rating and borrower industry indicators, log issue size, and indicators for revolving lines of credit and Term Loan A facilities. t -statistics based on standard errors clustered by firm and year-quarter are reported in parentheses. *, **, and *** denote p -values less than 0.10, 0.05, and 0.01, respectively.

Dependent Variable	Dealscan Sample		Markit-Dealscan Merge	
	Spread	Maturity	Spread	Maturity
LIBOR (%)	-0.159*** (-12.66)	0.105*** (4.75)	-0.217*** (-12.43)	0.111*** (9.05)
Equity CAPE	0.012* (1.93)	-0.044*** (-4.65)	-0.009 (-0.77)	0.018** (2.51)
Real GDP Growth	-0.018 (-1.43)	-0.004 (-0.19)	-0.008 (-0.44)	0.000 (0.02)
1(Recession)	0.241** (2.16)	-0.687*** (-5.37)	0.842*** (4.10)	-0.552*** (-4.16)
Borrower FE	X	X	X	X
Controls	X	X	X	X
Outcome Mean	1.927	4.207	3.093	5.353
Outcome SD	1.392	1.972	1.427	1.276
Adj. R ²	0.702	0.486	0.668	0.495
Observations	21,995	21,995	9,168	9,168

A2 CLO Simulation

This section explains our simulation of the effects of LIBOR floors on collateralized loan obligation (CLO) payoffs. The simulation uses annual data on the terms of institutional term loans from the Markit sample and the structure of CLOs from CLO-i, a database provided by Creditflux. Based on these inputs, we compute the initial equity yield of the CLO, equal to the difference between the coupons on the loan pool and the coupons on the CLO tranches divided by the principal amount of the equity tranche, under three scenarios. The first uses the observed level of loan spreads and LIBOR floor terms (inclusion, level). The second uses the same loan spreads and assumes no loans have floors. The third also assumes no loans have floors, but adjusts the loan spread upwards by an amount that sets the market value of the loan without a floor equal to the market value of a loan with a floor.

Our sample of CLOs includes U.S. dollar denominated deals issued between 2003 and 2018. To ensure data quality and uniformity, we include only deals with an AAA-rated senior tranche, non-missing data on the equity tranche, and debt referencing 3-month U.S. dollar LIBOR as the base rate. We begin the sample in 2003 because our data have fewer than five deals in each prior year. We also exclude the period 2008 to 2011 because there are no deals satisfying our sample criteria in 2009 and 2010, and only two deals in each of 2008 and 2011, due to collapse of private-label securitization after the financial crisis.

Figure A5 plots the time-series of CLO variables used in the simulation. The top two panels report the number and dollar volume of CLO issues in each year of the sample. Compared with figures from SIFMA, we have near-comprehensive coverage from 2013 to 2018 but imperfect coverage of the pre-crisis and immediate post-crisis period. The bottom two panels describe the capital structure and pricing of CLOs. The equity share is the ratio of the initial equity balance to the original deal balance. The CLO debt spread is the principal value-weighted spread over LIBOR for the debt tranches. We present two series of averages, equal-weighted and value-weighted by deal balance. Due to limited variability in

CLO sizes, there are not large differences between the two series.¹

One of the counterfactual scenarios described above uses the fair value of the LIBOR floor.² We apply the Black (1976) model of interest rate derivatives to estimate this value. An interest rate floor is a portfolio of options to lend at a fixed rate K on a series of dates, which in this context correspond to the quarterly coupon dates of a loan. For example, a LIBOR floor of 1% with a maturity of five years is an option to lend at 1%, which equates to receiving the difference between LIBOR and 1%, at each of 19 coupon dates starting in one quarter and ending at the maturity date.³ Each of the individual options is a floorlet, which offers the payoff

$$\frac{m \times \max\{L - K, 0\}}{1 + mL}, \quad (1)$$

where K is the level of the floor, L is the LIBOR rate fixed at the start of the coupon period, and m is the length of the coupon period according to the ACT/360 daycount convention used in U.S. dollar money markets. Settlement occurs at the start of the coupon period, with discounting. For simplicity, we assume that each quarterly coupon period is 90 days, so $m = 0.25$.

In this context, conventional application of the Black (1976) model assumes that the forward rate is distributed log-normally with constant volatility σ . Based on this distributional assumption, the value of each floorlet is

$$p_t = d_t \times m \times [K\Phi(-z_2) - F_t\Phi(-z_1)] \quad (2)$$

$$z_1 = \frac{\log F_t/K + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{t}}, \quad z_2 = z_1 - \sigma\sqrt{t},$$

where d_t is the discount factor from the trade date to the end of coupon period t and F_t is

¹Unreported in this figure, the typical CLO has a management fee of 50 bps, which we incorporate uniformly across years in each specification of the simulation.

²This discussion borrows from Section 3.5 of Duffie (2019).

³By market convention, the first coupon period is excluded because LIBOR for that period is already known on the trade date.

the forward rate for a LIBOR loan starting at the beginning of period t . Finally, the value of a five-year floor is

$$P_T = \sum_{t=2}^{t=20} p_t. \quad (3)$$

To apply the model, we bootstrap the LIBOR discount and forward rates using the spot LIBOR curve out to 12 months and the LIBOR interest rate swap curve beyond the one-year maturity, interpolating the annual rates to quarterly intervals. We obtain the implied volatility of over-the-counter interest rate floor contracts from Bloomberg. From May 10, 2005 to the end of the sample, we use the five-year contract with a strike of 1%. Prior to that, we use the 1.5% strike because the 1% strike was not available.

We adjust the loan spread to account for the value of the floor as follows. First, we compute the yield-to-maturity of a loan issued at par with the coupon rate set to the average spread plus the five-year swap rate, without a floor. Next, we compute the yield-to-maturity of a loan with the same coupon rate, issued at par minus the theoretical value of the average LIBOR floor. The difference between the two yields is the “spread equivalent” value of the floor, or the increase in spread necessary to compensate lenders for the theoretical value of the floor.⁴

Figure A6 presents the result of this exercise. The top two panels report the rate of LIBOR floor inclusion and the average level of LIBOR floors. Panel C contains the series of implied volatility, which increases before the crisis and declines afterwards. Panel D reports the spread equivalent value of the average floor. We compute two sets of floor values, using equal- and value-weighted floor levels from the Markit sample. The higher level of floors for smaller loans leads to significantly higher floor equivalent spreads using the equal-weighted inputs. The equal-weighted estimates are consistent with the floor spreads estimated by Arscott (2018).

Our simulation uses the average terms of CLOs and institutional term loans to compute

⁴Although most loans in our sample are callable at par shortly after issuance, we assume that the interest rate floor survives through the contractual maturity because prepayment risk adds substantial complexity to the theoretical valuation.

initial equity yields, which equal the difference between coupons received on loans and the coupons paid on debt tranches, divided by the principal amount of equity. We adjust the loan terms to remove the LIBOR floor and increase the spread by the spread equivalent floor value to assess the impact of LIBOR floors on CLO equity payoffs. Figure A7 reports the results of this exercise, which are discussed in Section 3.3 of the paper. LIBOR floors have little effect prior to the financial crisis, when they were rarely included, and at the end of the sample period, when they were out of the money, but add a significant boost to CLO equity yields in the low-rate period after the crisis.

As a check on our estimates, we report the realized payoffs of CLO equity tranches in Figure A8. This figure is based on the same sample of CLOs used to construct the simulation inputs. It reports the mean (equal- and value-weighted) and median initial equity yield, measured as the annualized distribution over the year after each CLO's first payment. The level of equity payoffs is a bit lower than our estimated equity yields, likely due to loan delinquency or the diversion of cash flows to senior tranches after coverage test failures. Overall, the time-series pattern of payoffs is consistent with Figure A7 and provides support for our simulation framework.

Figure A5: CLO Characteristics for Simulation

This figure summarizes the sample of CLOs used in the simulation exercise. The sample includes U.S. dollar denominated CLOs issued between 2003 and 2018. Panels A and B report the number and dollar volume of CLOs issued each year. Panel C reports the average share of CLOs that is funded by equity. Panel D reports the average spread over LIBOR for the debt tranches of CLOs. The averages in Panels C and D are computed by equal-weighting observations and by value-weighting by original deal balance.

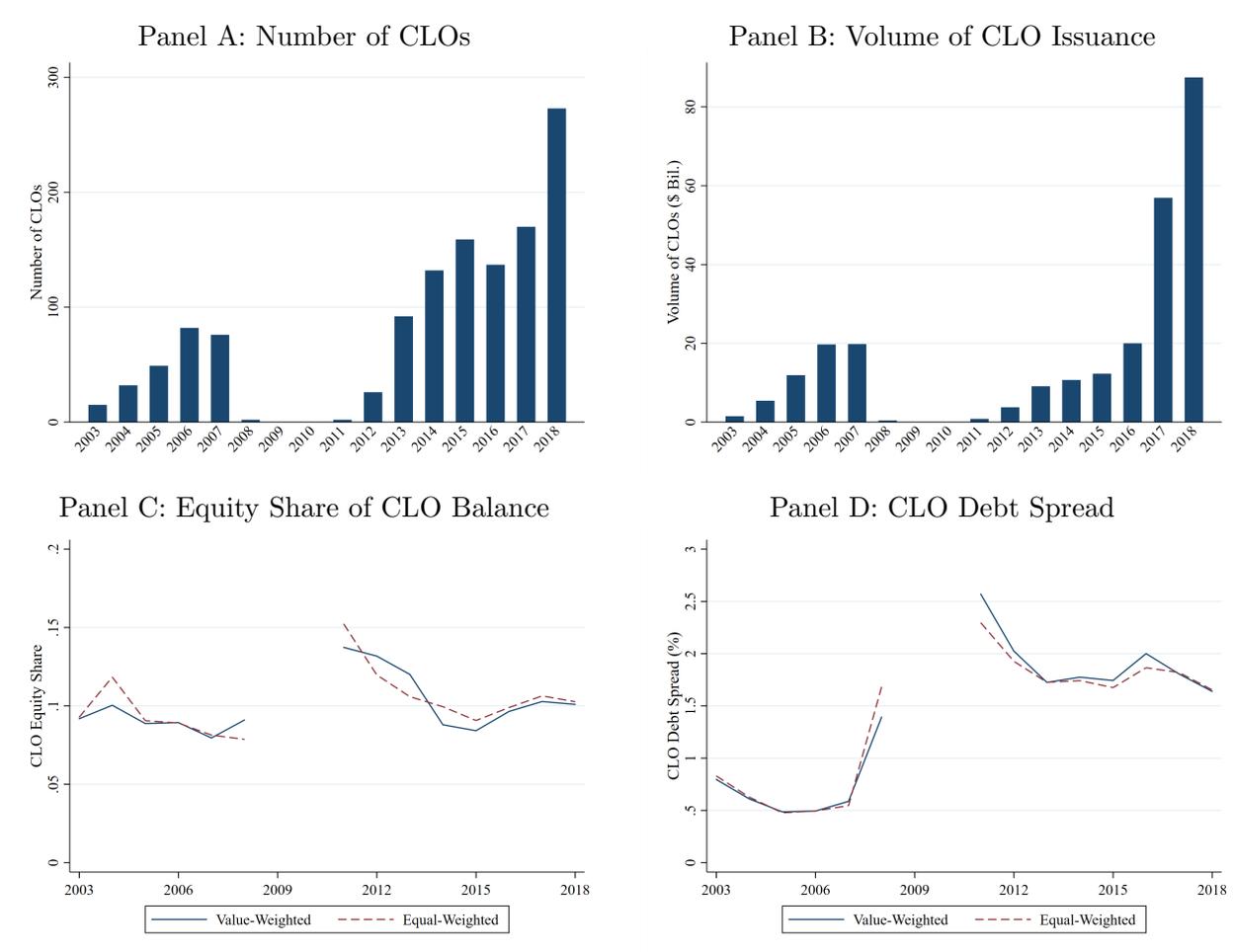


Figure A6: Valuation of LIBOR Floors

This figure presents the model inputs and fair value estimates for LIBOR floors in loan contracts. The sample is restricted to institutional tranches from the Markit data. Panels A and B report the fraction of loans with a floor and the average level of the floor. Panel C reports the implied volatility from the over-the-counter market for interest rate floors using data from Bloomberg. From May 10, 2005 to the end of the sample, we use the five-year contract with a strike of 1%. Prior to that, we use the 1.5% strike because the 1% strike was not available. Panel D reports the spread equivalent value of a floor from the Black (1976) model, using the strike from Panel B, the implied volatility from Panel C, and a maturity of five years.

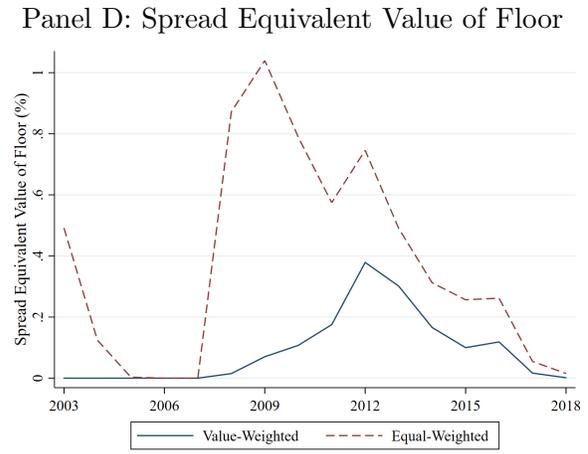
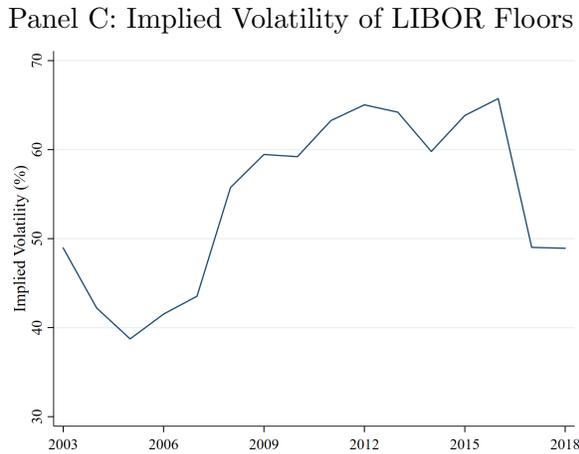
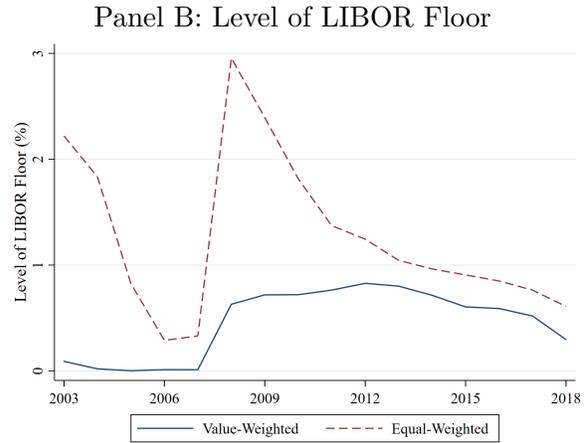
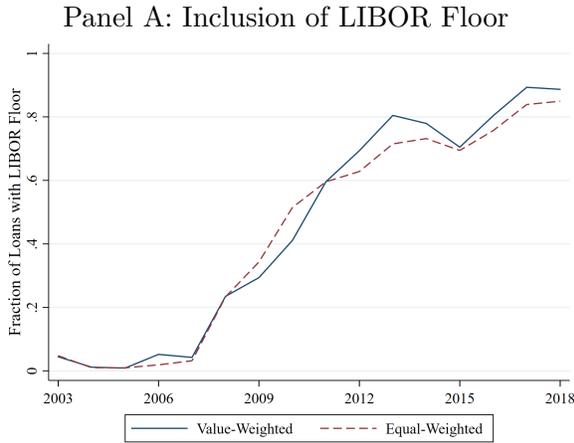


Figure A7: Effect of LIBOR Floors on CLO Equity Distributions

This figure illustrates the effect of LIBOR floors on the equity yields of collateralized loan obligations (CLOs). The calculation uses the principal value-weighted terms of loans and CLOs in each year from 2003 to 2018, including loan spreads, the inclusion of floors and their levels, the capital structure of CLOs, and the coupon rates paid on CLO tranche securities. The equity yield paid on the typical CLO in each year is calculated as the difference between coupons paid to the collateral pool and coupons paid on CLO tranche securities divided by the principal amount of the CLO equity tranche. The solid blue line reports the equity yield using the average loan spread, the fraction of loans with a LIBOR floor, and the average level of the LIBOR floor. The dashed red line is based on the alternative assumption that no loans have a floor. The dashed green line also assumes no loans have a floor but adjusts the loan spread upwards in an amount that offsets the theoretical value of the average LIBOR floor and fraction of loans with a floor each year. The period 2008 to 2011 is excluded due to a dearth of new CLO issuance during and after the financial crisis.

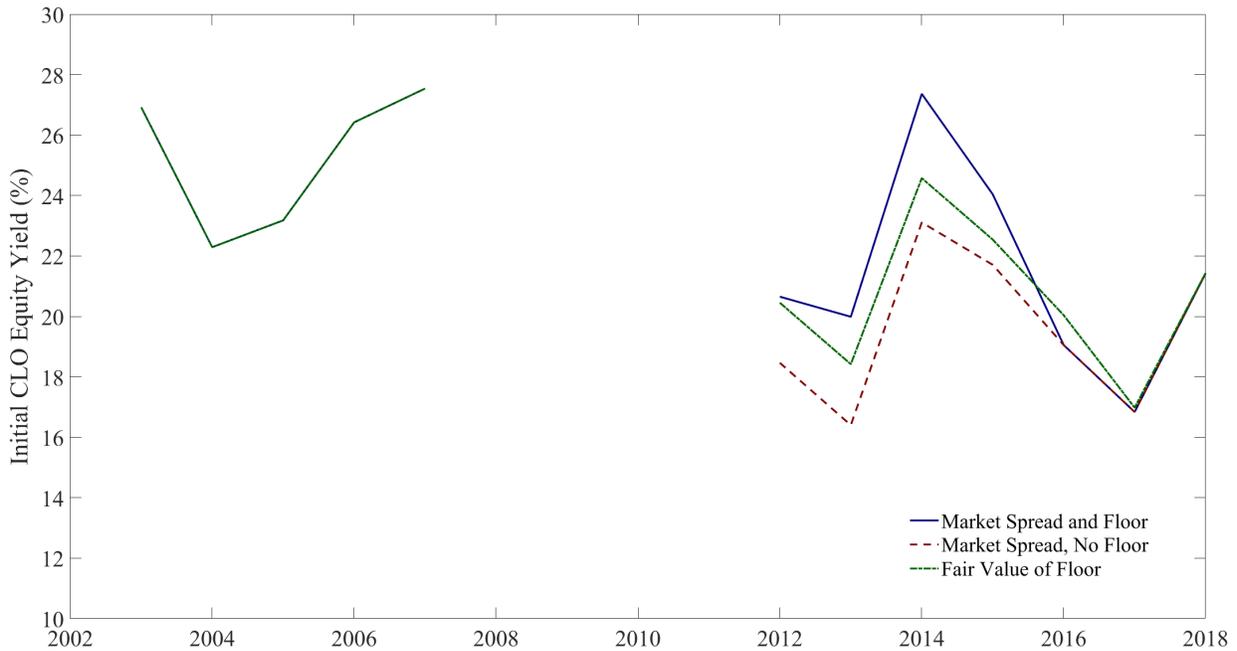
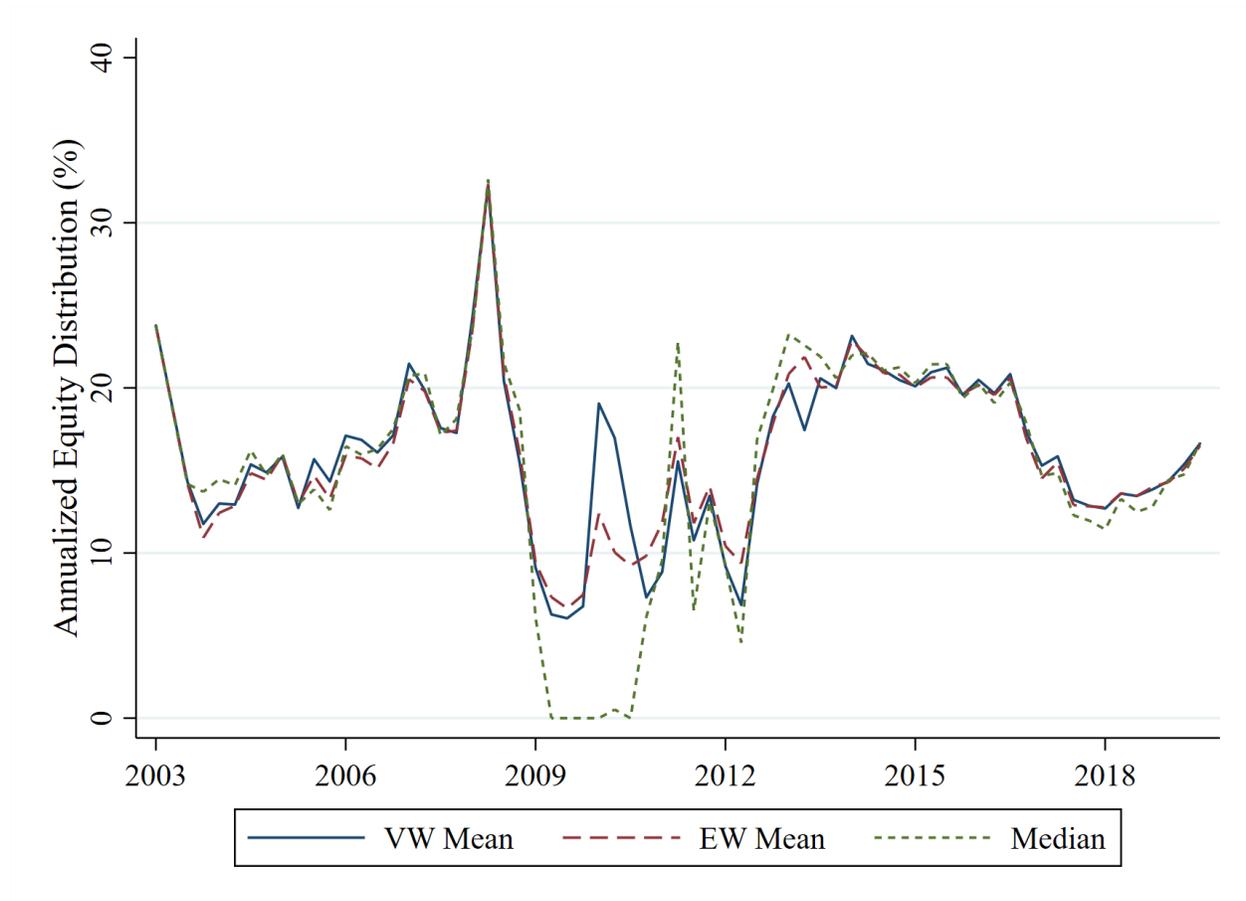


Figure A8: Realized CLO Equity Distributions

This figure reports the time-series of realized CLO equity yields. The sample from CLO-i contains U.S. dollar denominated CLOs closing between 2003 and 2018. Equity yield is measured as the annualized distribution to the equity tranche. For comparability between the realized distributions and the initial equity yields in the simulation, attention is restricted to distributions made within one year of the first payment by each CLO. The three quarterly series are the average equity yield weighted by the original deal balance (solid blue), the equal-weighted equity yield (dashed red), and the median equity yield (dashed green).



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