Appendices for Strategic Interactions in U.S. Policies For Online Publication*

Xiaoshan Chen† Eric M. Leeper‡ Campbell Leith§

*July 8, 2020.

†University of Durham; xiaoshan.chen@durham.ac.uk.
‡University of Virginia, and NBER; eleeper@virginia.edu
§University of Glasgow; campbell.leith@glasgow.ac.uk.

Contents

A System of Non-Linear Equations

$$
\begin{aligned}
N_t^{k\varphi}X_t^{k\sigma} &= \frac{W_t}{A_tP_t} \equiv w_t \\
1 &= \beta E_t\left[\left(\frac{X_{t+1}^k\xi_{t+1}}{X_t^k\xi_t}\right)^{-\sigma}\frac{A_t}{A_{t+1}}\frac{P_t}{P_{t+1}}\right]R_t \\
P_t^M &= \beta E_t\left[\left(\frac{X_{t+1}^k\xi_{t+1}}{X_t^k\xi_t}\right)^{-\sigma}\frac{A_t}{A_{t+1}}\frac{P_t}{P_{t+1}}(1+\rho P_{t+1}^M)\right] \\
N_t &= (\frac{Y_t}{A_t})\int_0^1(\frac{P(i)_t}{P_t})^{-\eta}di \\
P_tY_t &= P_tC_t + P_tG_t \\
\frac{P_t^f}{P_t} &= \left(\frac{\eta}{\eta-1}\right)\frac{E_t\sum_{s=0}^{\infty}(\alpha\beta)^s(X_{t+s}\xi_{t+s})^{-\sigma}\mu_{t+s}mc_{t+s}\left(\frac{P_{t+s}\pi^{-s}}{P_t}\right)^{\eta}\frac{Y_{t+s}}{A_{t+s}}}{E_t\sum_{s=0}^{\infty}(\alpha\beta)^s(X_{t+s}\xi_{t+s})^{-\sigma}(1-\tau_{t+s})\left(\frac{P_{t+s}\pi^{-s}}{P_t}\right)^{\eta-1}\frac{Y_{t+s}}{A_{t+s}}} \\
mc_t &= \frac{W_t}{A_tP_t} \\
P_t^b &= P_{t-1}^*\pi_{t-1} \\
\ln P_{t-1}^* &= (1-\zeta)\ln P_{t-1}^f + \zeta P_{t-1}^b \\
(P_t)^{1-\eta} &= \alpha(P_{t-1}\pi)^{1-\eta} + (1-\alpha)(P_t^*)^{1-\eta} \\
b_t^M &= \frac{(1+\rho P_t^M)}{P_{t-1}^M}\frac{Y_{t-1}}{\pi_tY_t}b_{t-1}^M - \tau_t + g_t + z_t + \xi_{tp,t} \\
\ln g_t &= (1-\rho_g)\ln g + \rho_g\ln g_{t-1} + \sigma_g\varepsilon_{g,t} \\
\ln z_t &= (1-\rho_z)\ln z + \rho_z\ln z_{t-1} + \sigma_z\varepsilon_{z,t} \\
\ln A_t &= \ln\gamma + \ln A_{t-1} + \ln q_t \\
\ln q_t &= \rho_q\ln q_{t-1} + \sigma_q\varepsilon_{q,t} \\
\ln\mu_t &= \rho_u\ln\mu_{t-1} + \sigma_\mu\varepsilon_t^\mu \\
\ln\xi_t &= \rho_\xi\ln\xi_{t-1} + \sigma_\xi\varepsilon_{\xi,t}
\end{aligned}
$$

The equation describing the evolution of price dispersion, $\int_0^1(\frac{P(i)_t}{P_t})^{-\eta_t}di$ is not needed to tie down the equilibrium upon log-linearization.

In order to render this model stationary we need to scale certain variables by the non-stationary level of technology, A_t such that $k_t = K_t/A_t$ where $K_t = \{Y_t, C_t, W_t/P_t\}$. Fiscal variables (i.e. $P_t^MB_t^M/P_t$, G_t and Z_t) are normalized with respect to Y_t. All other real variables are naturally stationary. Applying this scaling, the steady-state equilibrium conditions reduce to:

$$\begin{aligned}
N^{\varphi}X^{\sigma} &= w \\
1 &= \beta\left(R\pi^{-1}\right)/\gamma = \beta r/\gamma \\
P^{M} &= \frac{\beta}{\gamma\pi - \beta\rho} \\
y &= N \\
y &= \frac{c}{(1-g)} \\
X &= c(1-\theta) \\
mc &= w \\
\frac{\eta}{\eta-1} &= \frac{1-\tau}{mc} \\
b^{M} &= \left(\frac{\beta}{1-\beta}\right)s
\end{aligned}$$

To determine the steady state value of labor, we substitute for X in terms of y and then, using the aggregate production function, we obtain the following expression,

$$y^{\sigma+\varphi}\left[(1-g)(1-\theta)\right]^{\sigma} = \frac{\eta-1}{\eta}(1-\tau), \tag{A.1}$$

where g is the steady state share of government spending in output. We shall contrast this with the labor allocation/output that would be chosen by a social planner to obtain a measure of the steady-state distortion inherent in this economy which features distortionary taxation, monopolistic competition and the habits externality.

B Derivation of Objective Functional Form

B.1 The Social Planner's Problem

In order to assess the scale of the steady-state inefficiencies caused by the monopolistic competition, tax and habits externalities it is helpful to contrast the decentralized equilibrium with that which would be attained under the social planner's allocation. The social planner ignores the nominal inertia and all other inefficiencies and chooses real allocations that maximize the representative consumer's utility subject to the aggregate resource constraint, the aggregate production function, and the law of motion for habits-adjusted consumption:

$$\max_{\{X_t^*, C_t^*, N_t^*\}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{X_t^{*1-\sigma}\xi_t^{-\sigma}}{1-\sigma} + \chi\frac{(G_t^*/A_t)^{1-\sigma}(\xi_t)^{-\sigma}}{1-\sigma} - \frac{N_t^{*1+\varphi}\xi_t^{-\sigma}}{1+\varphi} \right)$$

$$\begin{aligned}
s.t.\ Y_t^* &= C_t^* + G_t^* \\
Y_t^* &= A_t N_t^* \\
X_t^* &= C_t^*/A_t - \theta C_{t-1}^*/A_{t-1}
\end{aligned}$$

The optimal choice implies the following relationship between the marginal rate of substitution between labor and habit-adjusted consumption and the intertemporal marginal rate of substitution in habit-adjusted consumption

$$\frac{(N_t^*)^{\varphi}}{(X_t^*)^{-\sigma}} = \left[1 - \theta\beta E_t\left(\frac{X_{t+1}^*\xi_{t+1}}{X_t^*\xi_t}\right)^{-\sigma}\right].$$

The steady state equivalent of this expression can be written as,

$$(N^*)^{\varphi+\sigma}\left[(1-\frac{G^*}{Y^*})(1-\theta)\right]^{\sigma} = (1-\theta\beta). \tag{B.1}$$

where the optimal share of government consumption in output is given by,

$$\frac{G_t^*}{Y_t^*} = \chi^{\frac{1}{\sigma}}(\frac{Y_t^*}{A_t})^{-\frac{\sigma+\varphi}{\sigma}}$$

In steady state these can be combined to give the optimal share of government consumption in output,

$$\frac{G^*}{Y^*} = (1+(1-\theta)^{-1}\chi^{-\frac{1}{\sigma}}(1-\theta\beta)^{\frac{1}{\sigma}})^{-1}$$

which can then used to get the steady state level of output under the social planner's allocation. We shall assume that the share of government spending in GDP in the data matches this, such that the data is calibrating the value of χ. Doing so facilitates the construction of a quadratic objective function.

If we contrast this with the allocation achieved in the steady-state of our decentralized equilibrium (A.1), assuming that the steady state share of government consumption to GDP is the same, we can see that the two will be identical whenever the following relationship between the markup, the tax rate and the degree of habits holds,

$$\frac{\eta}{\eta-1} = \frac{1-\tau}{1-\theta\beta}$$

Notice that in the absence of habits this condition could only be supported by a negative tax rate. However, for the data given level of taxation and the estimated degree of habits this condition will define our steady-state markup, enabling us to adopt an efficient steady-state and thereby avoiding a steady-state inflationary bias problem when describing optimal policy.

B.2 Quadratic Representation of Social Welfare

Individual utility in period t is

$$\frac{X_t^{1-\sigma}\xi_t^{-\sigma}}{1-\sigma} + \chi\frac{(G_t/A_t)^{1-\sigma}(\xi_t)^{-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}\xi_t^{-\sigma}}{1+\varphi}$$

where $X_t = C_t - \theta C_{t-1}$ is the habit-adjusted aggregate consumption. Before considering the elements of the utility function, we need to note the following general result relating to second order approximations

$$\frac{Y_t - Y}{Y_t} = \widehat{Y}_t + \frac{1}{2}\widehat{Y}_t^2 + O[2]$$

where $\widehat{Y}_t = \ln\left(\frac{Y_t}{Y}\right)$ and $O[2]$ represents terms that are of order higher than 2 in the bound on the amplitude of the relevant shocks. This will be used in various places in the derivation of welfare. Now consider the second order approximation to the first term,

$$\frac{X_t^{1-\sigma}\xi_t^{-\sigma}}{1-\sigma} = \overline{X}^{1-\sigma}\left(\frac{X_t-\overline{X}}{\overline{X}}\right) - \frac{\sigma}{2}\overline{X}^{1-\sigma}\left(\frac{X_t-\overline{X}}{\overline{X}}\right)^2 - \sigma\overline{X}^{1-\sigma}\left(\frac{X_t-\overline{X}}{\overline{X}}\right)(\xi_t-1) + tip + O[2]$$

where *tip* represents 'terms independent of policy'. Using the results above this can be rewritten in terms of hatted variables

$$\frac{X_t^{1-\sigma}\xi_t^{-\sigma}}{1-\sigma} = \overline{X}^{1-\sigma}\left\{\widehat{X}_t + \frac{1}{2}(1-\sigma)\widehat{X}_t^2 - \sigma\widehat{X}_t\widehat{\xi}_t\right\} + tip + O[2].$$

In pure consumption terms, the value of X_t can be approximated to second order by:

$$\widehat{X}_t = \frac{1}{1-\theta}\left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right) - \frac{\theta}{1-\theta}\left(\widehat{c}_{t-1} + \frac{1}{2}\widehat{c}_{t-1}^2\right) - \frac{1}{2}\widehat{X}_t^2 + O[2]$$

and to a first order,

$$\widehat{X}_t = \frac{1}{1-\theta}\widehat{c}_t - \frac{\theta}{1-\theta}\widehat{c}_{t-1} + O[1]$$

which implies

$$\widehat{X}_t^2 = \frac{1}{(1-\theta)^2}\left(\widehat{c}_t - \theta\widehat{c}_{t-1}\right)^2 + O[2]$$

Therefore,

$$\frac{X_t^{1-\sigma}\xi_t^{-\sigma}}{1-\sigma} = \overline{X}^{1-\sigma}\left\{\frac{1}{1-\theta}\left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right) - \frac{\theta}{1-\theta}\left(\widehat{c}_{t-1} + \frac{1}{2}\widehat{c}_{t-1}^2\right) + \frac{1}{2}(-\sigma)\widehat{X}_t^2 - \sigma\widehat{X}_t\widehat{\xi}_t\right\} + tip + O[2]$$

Summing over the future,

$$\sum_{t=0}^{\infty}\beta^t\frac{X_t^{1-\sigma}\xi_t^{-\sigma}}{1-\sigma} = \overline{X}^{1-\sigma}\sum_{t=0}^{\infty}\beta^t\left\{\frac{1-\theta\beta}{1-\theta}\left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right) - \frac{1}{2}\sigma\widehat{X}_t^2 - \sigma\widehat{X}_t\widehat{\xi}_t\right\} + tip + O[2].$$

Similarly for the term in government spending,

$$\chi\frac{g_t^{1-\sigma}\xi_t^{-\sigma}}{1-\sigma} = \chi\overline{g}^{1-\sigma}\{\widehat{g}_t + \frac{1}{2}(1-\sigma)\widehat{g}_t^2 - \sigma\widehat{g}_t\widehat{\xi}_t\} + tip + O[2]$$

While the term in labour supply can be written as

$$\frac{N_t^{1+\varphi}\xi_t^{-\sigma}}{1+\varphi} = \overline{N}^{1+\varphi}\left\{\widehat{N}_t + \frac{1}{2}(1+\varphi)\widehat{N}_t^2 - \sigma\widehat{N}_t\widehat{\xi}_t\right\} + tip + O[2]$$

Now we need to relate the labour input to output and a measure of price dispersion. Aggregating the individual firms' demand for labour yields,

$$N_t = \left(\frac{Y_t}{A_t}\right)\int_0^1\left(\frac{P(i)_t}{P_t}\right)^{-\eta_t}di$$

It can be shown (see Woodford (2003, Chapter 6)) that

$$\begin{aligned}\widehat{N}_t &= \widehat{y}_t + \ln[\int_0^1 (\frac{P(i)_t}{P_t})^{-\eta_t} di] \\ &= \widehat{y}_t + \frac{\eta}{2} var_i\{p(i)_t\} + O[2]\end{aligned}$$

which implies

$$\widehat{N}_t^2 = \widehat{y}_t^2$$

so we can write

$$\frac{N_t^{1+\varphi}}{1+\varphi} = \overline{N}^{1+\varphi}\left\{\widehat{y}_t + \frac{1}{2}(1+\varphi)\widehat{y}_t^2 - \sigma\widehat{y}_t\widehat{\xi}_t + \frac{\eta}{2} var_i\{p_t(i)\}\right\} + tip + O[2]$$

Welfare is then given by

$$\begin{aligned}\Gamma_0 \;=\; & \overline{X}^{1-\sigma} E_0 \sum_{t=0}^{\infty} \beta^t \left\{\frac{1-\theta\beta}{1-\theta}\left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right) - \frac{1}{2}\sigma\hat{X}_t^2 - \sigma\widehat{X}_t\widehat{\xi}_t\right\} \\ & + \chi\overline{g}^{1-\sigma} E_0 \sum_{t=0}^{\infty} \beta^t \{\widehat{g}_t + \frac{1}{2}(1-\sigma)\widehat{g}_t^2 - \sigma\widehat{g}_t\widehat{\xi}_t\} \\ & - \overline{N}^{1+\varphi} E_0 \sum_{t=0}^{\infty} \beta^t \left\{\widehat{y}_t + \frac{1}{2}(1+\varphi)\widehat{y}_t^2 - \sigma\widehat{y}_t\widehat{\xi}_t + \frac{\eta}{2} var_i\{p_t(i)\}\right\} \\ & + tip + O[2]\end{aligned}$$

From the steady-state of our model, and its comparison with the social planner's allocation we know that $\overline{X}^{1-\sigma}(1-\theta\beta) = (1-\theta)\frac{c}{y}\overline{N}^{1+\varphi}$. Similarly, assuming the same share of government spending in GDP across the social planner's and decentralized equilibrium, we also know that, $\chi\overline{g}^{1-\sigma} = \frac{g}{y}\overline{N}^{1+\varphi}$. Using the fact that,

$$\frac{c}{y}\widehat{c}_t = \widehat{y}_t - (1-\frac{c}{y})\widehat{g}_t - \frac{1}{2}\frac{c}{y}\widehat{c}_t^2 - \frac{1}{2}(1-\frac{c}{y})\widehat{g}_t^2 + \frac{1}{2}\widehat{y}_t^2 + O[2]$$

we can collect the levels terms and write the sum of discounted utilities as:

$$\Gamma_0 = -\frac{1}{2}\overline{N}^{1+\varphi} E_0 \sum_{t=0}^{\infty} \beta^t \left\{\begin{array}{c} \frac{\sigma(1-\theta)}{1-\theta\beta}\frac{c}{y}\left(\widehat{X}_t + \widehat{\xi}_t\right)^2 + \sigma\frac{g}{y}\left(\widehat{g}_t + \widehat{\xi}_t\right)^2 \\ +\varphi\left(\widehat{y}_t - \frac{\sigma}{\varphi}\widehat{\xi}_t\right)^2 \\ +\eta var_i\{p_t(i)\} \end{array}\right\} + tip + O[2]$$

Using the result from Fabian Eser and Wren-Lewis (2009) that

$$\sum_{t=0}^{\infty} \beta^t var_i[p_t(i)] = \frac{\alpha}{(1-\beta\alpha)(1-\alpha)} \sum_{t=0}^{\infty} \beta^t \left[\widehat{\pi}_t^2 + \frac{\zeta\alpha^{-1}}{(1-\zeta)}(\widehat{\pi}_t - \widehat{\pi}_{t-1})^2\right] + O[2].$$

we can write the discounted sum of utility as,

$$\Gamma_0 = -\frac{1}{2}\overline{N}^{1+\varphi} E_0 \sum_{t=0}^{\infty} \beta^t \left\{\begin{array}{c} \frac{\sigma(1-\theta)}{1-\theta\beta}\frac{c}{y}\left(\widehat{X}_t + \widehat{\xi}_t\right)^2 + (\varphi)\left(\widehat{y}_t - \frac{\sigma}{\varphi}\widehat{\xi}_t\right)^2 \\ +\frac{\alpha\eta}{(1-\beta\alpha)(1-\alpha)}\left[\widehat{\pi}_t^2 + \frac{\zeta\alpha^{-1}}{(1-\zeta)}(\widehat{\pi}_t - \widehat{\pi}_{t-1})^2\right] \end{array}\right\} + tip + O[2]$$

where we have put the terms in public consumption into tip since they are treated as an exogenous process and therefore independent of policy.

After normalising the coefficient on inflation to one, we can write the microfounded objective function as,

$$\Gamma_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \Phi_1 \left(\widehat{X}_t + \widehat{\xi}_t \right)^2 + \Phi_2 \left(\widehat{y}_t - \frac{\sigma}{\varphi} \widehat{\xi}_t \right)^2 \\ + \frac{\zeta \alpha^{-1}}{(1-\zeta)} \left(\widehat{\pi}_t - \widehat{\pi}_{t-1} \right)^2 + \widehat{\pi}_t^2 \end{array} \right\} \tag{B.2}$$

where the weights on the two real terms are functions of model structural parameters, where $\Phi_1 = \frac{\sigma(1-\theta)}{1-\theta\beta} \frac{(1-\beta\alpha)(1-\alpha)}{\alpha\eta} \frac{c}{y}$ and $\Phi_2 = \frac{\varphi(1-\beta\alpha)(1-\alpha)}{\alpha\eta}$.

C Rules-Based Estimation

In this section we undertake an estimation of our model when describing policy using simple rules. This serves to create a set of benchmark results which we can contrast with our estimates which allow for strategic interactions between monetary and fiscal policy. In doing so it is important to note that while we extend the analysis of Bianchi (2012) and Bianchi and Ilut (2017) in some ways, this does not overturn their key results. Bianchi and Ilut argue that restricting the number and transition pattern of regimes is data-preferred largely as a result of the fact that the PM/AF and PM/PF regimes are very similar in terms of their dynamic responses to shocks. This is no longer the case when taxes are assumed to be distortionary where the inflationary impact of variation in taxes becomes a key ingredient in identifying policy regimes. Nevertheless, this results in a similar narrative in terms of the evolution of monetary and fiscal policy to the existing literature - fiscal policy turns active in the late 1960s and monetary policy turns active shortly afterwards, only regaining its activism following the Volcker disinflation in 1982. However, under our Rules-Based estimation the transition to a complementary passive fiscal regime was, unlike Bianchi and Ilut, not decisively achieved in 1982, and really only emerged a decade later in 1992.

When considering policy described by simple rules, we assume fiscal policy follows a simple tax rule,

$$\widetilde{\tau}_t = \rho_{\tau,s_t} \widetilde{\tau}_{t-1} + (1 - \rho_{\tau,s_t}) \left(\delta_{\tau,s_t} \widetilde{b}_{t-1}^M + \delta_y \widehat{y}_t \right) + \sigma_\tau \varepsilon_{\tau,t}$$

where we assume the coefficient on debt, δ_{τ,s_t}, and the persistence of the tax rate, ρ_{τ,s_t} are subject to regime switching with $s_t = 1$ indicating the Passive Fiscal (PF) regime and $s_t = 2$ being the Active Fiscal (AF) regime. The fiscal policy regimes are determined by the value of coefficient on debt with $\delta_{\tau,s_t=1} > \frac{1}{\beta} - 1$ in the PF regime and $\delta_{\tau,s_t=2} = 0$ in the AF regime.

When U.S. monetary policy is described as a generalized Taylor rule, we specify this rule following An and Schorfheide (2007),

$$\widehat{R}_t = \rho_{R,S_t} \widehat{R}_{t-1} + (1 - \rho_{R,S_t})[\psi_{1,S_t} \widehat{\pi}_t + \psi_{2,S_t} (\Delta \widehat{y}_t + \widehat{q}_t)] + \sigma_R \varepsilon_{R,t}$$

where the Fed adjusts interest rates in response to movements in inflation and deviations of output growth from trend. We allow the rule parameters $(\rho_{R,S_t}, \psi_{1,S_t}, \psi_{2,S_t})$ to switch between active and passive policy regimes. The Active Monetary (AM) policy regime corresponds to

$S_t = 1$, while the Passive Monetary (PM) policy regime corresponds to $S_t = 2$. The labeling implies that $\psi_{1,S_t=1} > 1$ and $0 < \psi_{1,S_t=2} < 1$.

By considering both fiscal and monetary policy changes, we can distinguish four policy regimes under Rules-Based policy. They are AM/PF, AM/AF, PM/PF and PM/AF. Leeper (1991) shows that, in the absence of regime switching, the existence of a unique solution to the model depends on the nature of the assumed policy regime. A unique solution can be found under both the AM/PF and PM/AF regimes, what Leeper and Leith (2017) refer to as the M and F-regimes, respectively. In the former monetary policy actively targets inflation and fiscal policy adjusts taxes to stabilize debt, while under the latter combination the fiscal authority does not adjust taxes to stabilize debt and the monetary authority does not actively target inflation in order to facilitate the stabilization of debt. In contrast, no stationary solution and multiple equilibria are obtained under the AM/AF and PM/PF regimes, respectively. However, when regime switching is considered, the existence and uniqueness of a solution also depends on the transition probabilities of the potential regime changes as economic agents anticipate the transition to different policy regimes. Specifically, we allow monetary and fiscal policy rule parameters to switch independently of each other. The transition matrices for monetary policy and fiscal policy are as follows

$$P = \begin{bmatrix} p_{11} & 1-p_{22} \\ 1-p_{11} & p_{22} \end{bmatrix},\ Q = \begin{bmatrix} q_{11} & 1-q_{22} \\ 1-q_{11} & q_{22} \end{bmatrix},$$

where $p_{ii} = \Pr[S_t = i | S_{t-1} = i]$ and $q_{ii} = \Pr[s_t = i | s_{t-1} = i]$. In addition, we also account for a possible shift in fundamental shock volatilities which has been used as a potential explanation of the Great Moderation. Failure to do so could potentially bias the identification of shifts in policy (see Sims and Zha (2006)). Therefore, we allow for independent regime switching in the standard deviations of technology (σ_{q,k_t}), preference (σ_{ξ,k_t}) and cost-push (σ_{μ,k_t}) shocks, with $k_t = 1$ being in the low volatility regime and $k_t = 2$ in the high volatility regime. The transition matrix for the shock volatilities is as follows

$$H = \begin{bmatrix} h_{11} & 1-h_{22} \\ 1-h_{11} & h_{22} \end{bmatrix},$$

where $h_{ii} = \Pr[k_t = i | k_{t-1} = i]$.[1]

We adopt the solution algorithm proposed by Farmer, Waggoner, and Zha (2011) to solve the model with Markov-switching in policy rule parameters. Since this algorithm implies that economic agents anticipate the Markov switching between different policy rules, there will be spillovers across policy regimes which will turn out to be crucial in determining the relative performance of alternative policies.

Table C.1 presents the priors and posterior estimates for the Rules-Based policy. For the interest rate rule parameters, we set symmetric priors for the parameter of the lagged interest rate and the parameter of output growth, whereas asymmetric and truncated priors are used for the parameter of inflation to ensure that $\psi_{1,S_t=1} > 1$ in the AM regime and $0 < \psi_{1,S_t=2} < 1$ in the PM regime. Similarly, for the tax rule, a symmetric prior is used for the parameter of lagged tax rate, while the parameter of debt is restricted to be zero in the AF

[1] The joint transition matrix governing the monetary-fiscal-shock regime is then $P = P \otimes Q \otimes H$. In total, there are eight regimes in the Rules-Based model.

regime and positive in the PF regime. Overall, the priors of the policy rule parameters imply four distinct fiscal and monetary policy regimes: AM/PF, AM/AF, PM/PF and PM/AF. In addition, variances of shocks are chosen to be highly dispersed inverted Gamma distributions to generate realistic volatilities for the endogenous variables.

	Posterior				Prior		
Parameters	Mode	Mean	5%	95%	Type	Mean	Std Dev
			AM/PF				
$\rho_{R,S_t=1}$, lagged interest rate	0.880	0.880	0.853	0.906	B	0.50	0.15
$\psi_{1,S_t=1}$, interest rate resp. to inflation	3.068	2.898	2.098	3.657	G	2.00	0.50
$\psi_{2,S_t=1}$, interest rate resp. to output	0.719	0.670	0.421	0.977	G	0.50	0.25
$\rho_{\tau,s_t=1}$, lagged tax rate	0.955	0.957	0.929	0.985	B	0.70	0.15
$\delta_{\tau,s_t=1}$, tax rate resp. to debt	0.032	0.036	0.014	0.057	G	0.05	0.02
δ_y, tax rate resp. to output	0.000	0.078	0.000	0.177	G	0.10	0.10
			AM/AF				
$\rho_{R,S_t=1}$, lagged interest rate	0.610	0.609	0.525	0.694	B	0.50	0.15
$\psi_{1,S_t=1}$, interest rate resp. to inflation	1.454	1.485	1.289	1.688	G	2.00	0.50
$\psi_{2,St=1}$, interest rate resp. to output	0.686	0.695	0.483	0.926	G	0.50	0.25
$\rho_{\tau,s_t=2}$, lagged tax rate	0.763	0.725	0.610	0.846	B	0.70	0.15
$\delta_{\tau,s_t=2}$, tax rate resp. to debt	0.00	0.00	-	-	Fixed	-	-
δ_y, tax rate resp. to output	0.000	0.078	0.000	0.177	G	0.10	0.10
			PM/PF				
$\rho_{R,S_t=2}$, lagged interest rate	0.869	0.856	0.819	0.896	B	0.50	0.15
$\psi_{1,S_t=2}$, interest rate resp. to inflation	0.982	0.904	0.810	0.990	G	0.80	0.15
$\psi_{2,St=2}$, interest rate resp. to output	0.581	0.583	0.288	0.938	G	0.50	0.25
$\rho_{\tau,s_t=1}$, lagged tax rate	0.437	0.466	0.308	0.623	B	0.70	0.15
$\delta_{\tau,s_t=1}$, tax rate resp. to debt	0.077	0.083	0.055	0.112	G	0.05	0.02
δ_y, tax rate resp. to output	0.000	0.078	0.000	0.177	G	0.10	0.10
			PM/AF				
$\rho_{R,S_t=2}$, lagged interest rate	0.869	0.856	0.819	0.896	B	0.50	0.15
$\psi_{1,S_t=2}$, interest rate resp. to inflation	0.982	0.904	0.810	0.990	G	0.80	0.15
$\psi_{2,St=2}$, interest rate resp. to output	0.581	0.583	0.288	0.938	G	0.50	0.25
$\rho_{\tau,s_t=2}$, lagged tax rate	0.763	0.725	0.610	0.846	B	0.70	0.15
$\delta_{\tau,s_t=2}$, tax rate resp. to debt	0.00	0.00	-	-	Fixed	-	-
δ_y, tax rate resp. to output	0.000	0.078	0.000	0.177	G	0.10	0.10

Table C.1: Rules-Based Policy. Under the Rules-Based policy, we have four alternative policy permutations: AM/PF, AM/AF, PM/PF and PM/AF. For monetary policy switches, $S_t = 1$ is the AM regime and $S_t = 2$ is the PM regime. For fiscal policy switches, $s_t = 1$ is the PF regime and $s_t = 2$ is the AF regime. δ_y is assumed to be time-invariant across regimes.

	Posterior				Prior		
Parameters	Mode	Mean	5%	95%	Type	Mean	Std Dev
Deep parameters							
σ, Inv. of intertemp. elas. of subst.	2.500	2.509	2.134	2.898	N	2.50	0.25
α, Calvo parameter	0.798	0.800	0.772	0.827	B	0.75	0.02
ζ, inflation inertia	0.387	0.339	0.206	0.458	B	0.50	0.10
θ, habit persistence	0.464	0.524	0.359	0.658	B	0.50	0.10
φ, Inverse of Frisch elasticity	2.00	2.00	-	-	Fixed	-	-
Serial correlation of exogenous processes							
ρ_ξ, AR coeff., taste shock	0.893	0.886	0.844	0.927	B	0.50	0.15
ρ_μ, AR coeff., cost-push shock	0.153	0.209	0.078	0.346	B	0.50	0.15
ρ_q, AR coeff., productivity shock	0.427	0.406	0.293	0.519	B	0.50	0.15
ρ_z, AR coeff., transfers	0.977	0.976	0.966	0.987	B	0.50	0.15
ρ_g, AR coeff., government spending	0.981	0.980	0.970	0.99	B	0.50	0.15
Standard deviations of exogenous processes							
$\sigma_{\xi,k_t=1}$, taste shock	0.555	0.532	0.375	0.684	IG	0.50	2.00
$\sigma_{\xi,k_t=2}$ taste shock	1.235	1.215	0.883	1.532	IG	2.00	2.00
$\sigma_{\mu,k_t=1,}$ cost-push shock	4.845	4.051	3.080	5.000	IG	0.50	2.00
$\sigma_{\mu,k_t=2,}$ cost-push shock	12.734	11.772	7.851	15.792	IG	2.00	2.00
$\sigma_{q,k_t=1,}$ productivity shock	0.510	0.572	0.479	0.660	IG	0.50	2.00
$\sigma_{q,k_t=2,}$ productivity shock	1.111	1.275	1.059	1.462	IG	2.00	2.00
σ_{tp}, term premium shock	3.258	3.293	2.996	3.570	IG	2.00	2.00
σ_g, government spending shock	0.246	0.249	0.229	0.269	IG	0.10	2.00
σ_z, transfers shock	0.300	0.303	0.279	0.327	IG	0.50	2.00
σ_τ, tax rate shock	0.359	0.361	0.330	0.393	IG	0.50	2.00
σ_R, interest rate shock	0.205	0.211	0.189	0.232	IG	0.50	2.00
Transition probabilities							
p_{11}, monetary policy: remaining active	0.972	0.971	0.955	0.989	B	0.95	0.02
p_{22}, monetary policy: remaining passive	0.933	0.915	0.877	0.956	B	0.95	0.02
q_{11}, fiscal policy: remaining passive	0.955	0.952	0.929	0.978	B	0.95	0.02
q_{22}, fiscal policy: remaining active	0.935	0.918	0.882	0.954	B	0.95	0.02
h_{11}, volatility: remaining with low volatility	0.958	0.951	0.926	0.977	B	0.95	0.02
h_{22}, volatility: remaining with high volatility	0.910	0.905	0.875	0.935	B	0.95	0.02

Table C.1: Rules-Based Policy (continued). For volatility, $k_t = 1$ is the low volatility regime and $k_t = 2$ is the high volatility regime.

C.1 Posterior Estimates: Rules-Based Policy

The posterior parameter estimates of the Rules-Based policy are reported in Table C.1. Our estimates of the structural parameters are broadly in line with other studies: an intertemporal elasticity of substitution, $\sigma = 2.5$; a measure of price stickiness, $\alpha = 0.8$, implying that price contracts typically last for just over one year; a degree of price indexation, $\zeta = 0.34$, and a significant estimate of the degree of habits, $\theta = 0.52$.

Under the Rules-Based policy, we have four alternative policy permutations: AM/PF, AM/AF, PM/PF and PM/AF. In order to allow for maximum flexibility in describing the policy regimes, we initially allowed for variations in rule parameters across the four policy regimes. Therefore, for example, the active monetary policy rule parameters in the AM/PF regime can differ from those in the AM/AF regime. Indeed, we find significant variations in the AM and PF regimes depending on which policy they are combined with. However, the PM and AF regimes appeared to be similar regardless of which policy they were paired with. Therefore, we restrict the PM and AF to be the same across their respective paired regimes. The resultant policy regimes imply that the passive monetary policy is inertial, $\rho_{R,S_t=2} = 0.86$, and only falling slightly short of the Taylor principle, $\psi_{1,S_t=2} = 0.9$, with a significant coefficient on output, $\psi_{2,S_t=2} = 0.58$. While an active monetary policy paired with a passive fiscal policy (AM/PF) is both inertial, $\rho_{R,S_t=1} = 0.88$, and very aggressive in targeting inflation, $\psi_{1,S_t=1} = 2.9$, with a relatively strong response to output, $\psi_{2,S_t=1} = 0.67$. When fiscal policy is active, then an associated active monetary policy is far less aggressive as interest rate inertia falls, $\rho_{R,S_t=1} = 0.61$, along with the response to inflation, $\psi_{1,S_t=1} = 1.48$, while the response to output increases, $\psi_{2,S_t=1} = 0.70$. Since the AM/AF regime is inherently unstable, it would appear that the conflict between the monetary and fiscal authority results in a moderation in the conservatism of monetary policy even while that policy remains active. Similarly, the passive fiscal policy is far more inertial, $\rho_{\tau,s_t=1} = 0.96$, and less responsive to debt, $\delta_{\tau,s_t=1} = 0.04$, when it is paired with an active monetary policy (AM/PF) than when the passive fiscal policy is paired with a passive monetary policy (PM/PF) where tax rate inertia falls, $\rho_{\tau,s_t=1} = 0.46$, and the response to debt rises, $\delta_{\tau,s_t=1} = 0.08$. These kinds of differences in estimation across regimes could reflect the nature of the interaction between monetary and fiscal policy. In the case of the AM/AF regime the policy is unstable and only rendered determinate because of spillovers from other policy permutations, so that the moderation in monetary policy would serve to mitigate the unstable debt dynamics caused by rising debt service costs under the active policy policy. Similarly, a passive fiscal policy which raises distortionary taxes to stabilize debt is likely to fuel inflation and lead to rising debt service costs when monetary policy is active. This is less of a danger when monetary policy is passive, so that fiscal policy can be relatively more aggressive in responding to debt in the latter case. These results suggest that the stance of one (or both) policy maker(s) is dependent on the policies of the other. This can be analyzed more formally by considering optimal policy where one policy maker takes into account the actions of the other.

C.2 Regime Switching Rules-Based Policy

Figure C.1 details the movements across fiscal and monetary policy regimes when the policy is described by Rules-Based policy. The first panel describes the probability of being in the passive fiscal policy regime, the second the active fiscal policy regime, and the third panel gives the probability of being in the passive monetary policy regime (with its complement being the active monetary regime). Taking these together, we observe that the conventional policy assignment (i.e. AM/PF) prevails right up until the late 1960s in contrast to the findings in Bianchi (2012) or Bianchi and Ilut (2017) who suggest that policy had already deviated from the textbook assignment by then. Fiscal policy then turns active in 1969, and monetary policy turns passive shortly afterwards. There is a brief attempt at disinflation in 1973, but we essentially stay in the PM regime until Volcker. Afterwards monetary policy stays active, and there are brief flirtations with passive fiscal policy around 1975 and 1981-1982, although none stick until 1992. Therefore, the AM/PF regime did not re-emerge until 1992. This result is consistent with Bianchi (2012) but different with Bianchi and Ilut (2017). Finally, we find a brief relaxation of monetary policy in the aftermath of the bursting of the dot com bubble around 2001, while fiscal policy remains passive.

Our estimates suggest that regimes that are determinate because of the expectations of returning to either the AM/PF or PM/AF regime actually describe observed policy configurations for much of our sample period. The AM/PF and PM/AF regimes are estimated to be in place for 60% and 12% of the sample period, respectively, while the PM/PF regime appears to be the least frequently observed regime which is only present for 2% of the time. This is consistent with Bianchi and Ilut (2017) in that the PM/PF regime does not appear to be a significant regime. The remaining 26% of the sample period is described by the AM/AF regime, which is inherently unstable in the absence of expectations that we would return to either the AM/PF or PM/AF regimes.

In short, the Rules-Based estimation is consistent with a narrative where fiscal policy ceases to act to stabilize debt in the 1970s, with monetary policy turning passive shortly afterwards. Monetary policy then actively targets inflation following the appointment of Paul Volcker, but fiscal policy does not decisively turn passive in support of that policy until the early 1990s. That the Rules-Based estimation would identify this pattern of regime change can easily be seen in the broad trends in inflation, interest rates and debt contained in Figure 1 in the paper. The PM/AF regime of the 1970s is associated with high inflation, the AM/AF regime of the 1980s with the tightening of monetary policy, falling inflation and rising debt, the AM/PF regime of the 1990s with the ongoing stabilization in inflation and the debt to GDP ratio. We shall see in the main text that the estimation based on optimal strategic policy allows for a more nuanced description of the evolution of policy regimes

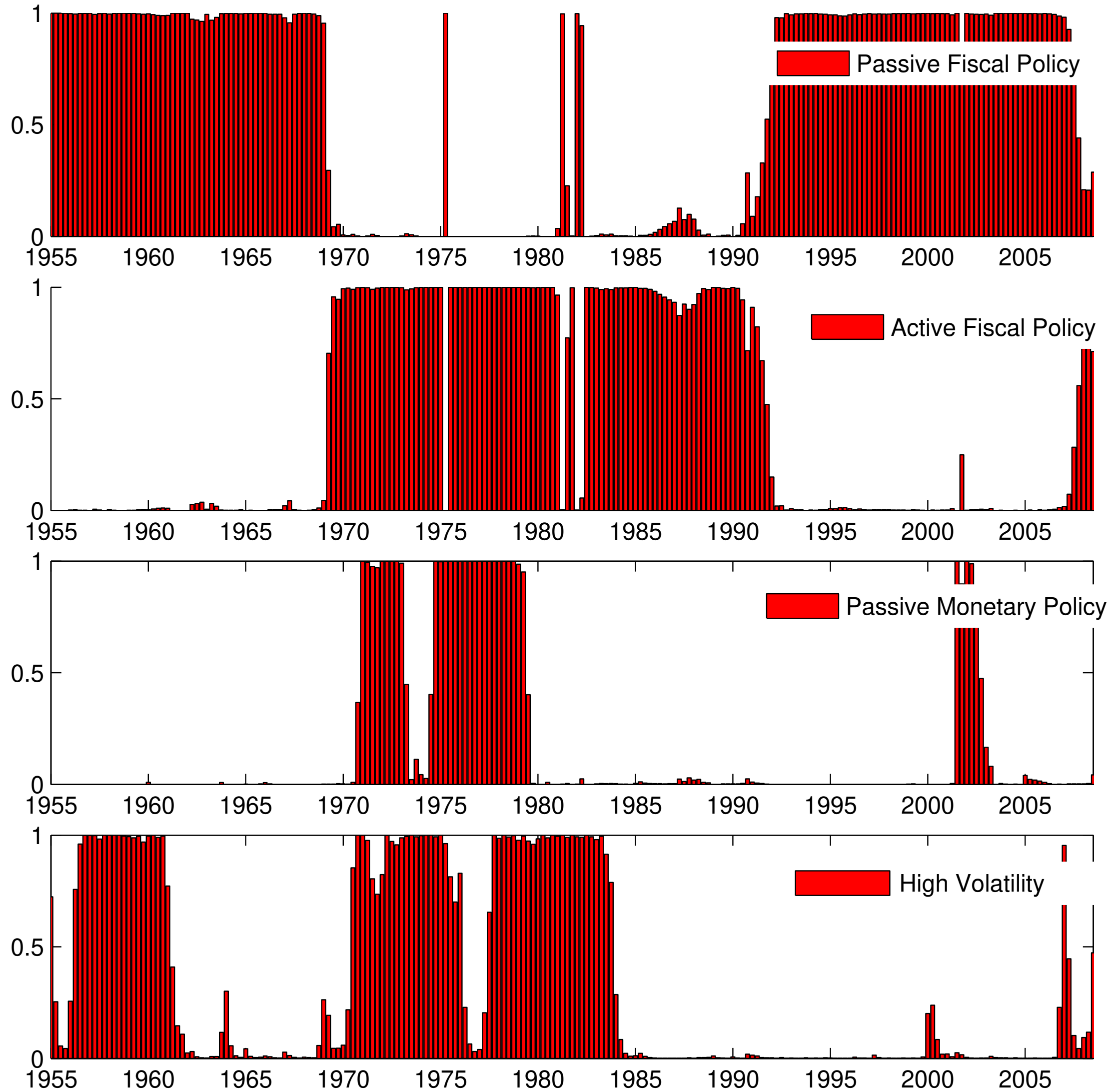

Figure C.1: Markov Switching Probabilities: Policy and Volatility Switches under Rules-Based Policy

D Leadership Equilibria Under Discretion

This section demonstrates how to solve non-cooperative dynamic games in the Markov jump-linear quadratic systems. Consider an economy with two policy makers: a leader (L) and a follower (F).

$$X_{t+1} = A_{11k_{t+1}}X_t + A_{12k_{t+1}}x_t + B_{11k_{t+1}}u_t^L + B_{12k_{t+1}}u_t^F + C_{k_{t+1}}\varepsilon_{t+1}, \quad \text{(D.1)}$$
$$E_t H_{k_{t+1}}x_{t+1} = A_{21j_t}X_t + A_{22j_t}x_t + B_{21j_t}u_t^L + B_{22j_t}u_t^F. \quad \text{(D.2)}$$

where X_t is a n_1 vector of predetermined variables; x_t is a n_2 vector of forward-looking variables; u_t^L and u_t^F are the control variables, and ε_t contains a vector of zero mean $i.i.d.$ shocks. Without loss of generality, the shocks are normalized so that the covariance matrix of ε_t is an identity matrix, and the covariance matrix of the shocks to X_{t+1} is $C'_{k_{t+1}}C_{k_{t+1}}$.

The matrices $A_{11k_{t+1}}$, $A_{12k_{t+1}}$, $H_{k_{t+1}}$, $B_{11k_{t+1}}$, $B_{12k_{t+1}}$, A_{21j_t}, A_{22j_t}, B_{21j_t}, and B_{22j_t} can each take n different values, corresponding to the n modes $k_{t+1} = 1, 2, ..., n$ in period $t+1$, and $j_t = 1, 2, ..., n$ in period t. The modes follow a Markov process with constant transition probabilities:

$$P_{jk} = Pr\left\{k_{t+1} = k | j_t = j\right\}, \;\; j, k = 1, 2, ..., n$$

Let P denote the $n \times n$ transition matrix $[P_{jk}]$ and the $1 \times n$ vector $p \equiv (p_{1_t}, ..., p_{n_t})$ denote the probability distribution of the modes in period t,

$$p_{t+1} = p_t P.$$

Finally, the $1 \times n$ vector $\overline{p}$ denotes the unique stationary distribution of the modes,

$$\overline{p} = \overline{p}P.$$

We assume that the intertemporal loss functions of the two policy makers are defined by the quadratic loss function

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} \frac{1}{2}\beta^\tau L^u_{j_{t+\tau}},$$

where $L^u_{j_t}$ is the period loss with $u = F$ for the follower and $u = L$ for the leader, respectively. The period loss, $L^u_{j_t}$, can take different value corresponding to the n modes in period t. The period loss satisfies

$$L^u_{j_t} = Y_t^{u\prime}\Lambda^u_{j_t}Y_t^u,$$

where $\Lambda^u_{j_t}$ is a symmetric and positive semi-definite weight matrix. Y_t^u are n_Y vectors of target variables for the follower and leader.

$$Y_t^u = D^u \begin{bmatrix} X_t \\ x_t \\ u_t^L \\ u_t^F \end{bmatrix}.$$

It follows that the period loss function can be rewritten as

$$L_{j_t}^u = \begin{bmatrix} X_t \\ x_t \\ u_t^L \\ u_t^F \end{bmatrix}' W_{j_t}^u \begin{bmatrix} X_t \\ x_t \\ u_t^L \\ u_t^F \end{bmatrix}, \tag{D.3}$$

where $W_{j_t}^u = D^{u\prime}\Lambda_{j_t}^u D^u$ is symmetric and positive semidefinite, and

$$W_{j_t}^u = \begin{bmatrix} Q_{11j_t}^u & Q_{12j_t}^u & P_{11j_t}^u & P_{12j_t}^u \\ Q_{21j_t}^u & Q_{22j_t}^u & P_{21j_t}^u & P_{22j_t}^u \\ P_{11j_t}^{u\prime} & P_{21j_t}^{u\prime} & R_{11j_t}^u & R_{12j_t}^u \\ P_{12j_t}^{u\prime} & P_{22j_t}^{u\prime} & R_{12j_t}^{u\prime} & R_{22j_t}^u \end{bmatrix}$$

is partitioned with X_t, x_t, $u_t^L$and u_t^F.

The follower and leader decide their policy u_t^F and u_t^L in period t to minimize their intertemporal loss functions defined in (D.3) under discretion subject to (D.1), (D.2), X_t and j_t given. The follower also observes the current decision u_t^L of the leader. Furthermore, two policy makers anticipate that they will reoptimize in period $t+1$. Reoptimization will result in the two instruments and the forward-looking variables in period $t+1$ being functions of the predetermined variables and the mode in period $t+1$ according to

$$u_{t+1}^L = -F_{k_{t+1}}^L X_{t+1}, \tag{D.4}$$
$$u_{t+1}^F = -G_{k_{t+1}}^F X_{t+1} - D_{k_{t+1}}^F u_{t+1}^L, \tag{D.5}$$
$$x_{t+1} = -N_{k_{t+1}} X_{t+1}, \tag{D.6}$$

where $k_{t+1} = 1, ..., n$ are the n modes at period $t+1$. The dynamics of the predetermined variables will follow

$$X_{t+1} = M_{j_t k_{t+1}} X_t + C_{k_{t+1}} \varepsilon_{t+1},$$

where

$$M_{j_t k_{t+1}} = A_{11k_{t+1}} - A_{12k_{t+1}} N_{j_t} - B_{11k_{t+1}} F_{j_t}^L - B_{12k_{t+1}} G_{j_t}^F + B_{12k_{t+1}} D_{j_t}^F F_{j_t}^L,$$

First, by (D.6) and (D.1) we have,

$$\begin{aligned} E_t H_{k_{t+1}} x_{t+1} &= -E_t H_{k_{t+1}} N_{k_{t+1}} X_{t+1} \\ &= -E_t H_{k_{t+1}} N_{k_{t+1}} \left(A_{11k_{t+1}} X_t + A_{12k_{t+1}} x_t + B_{11k_{t+1}} u_t^L + B_{12k_{t+1}} u_t^F \right) \end{aligned}$$

where $E_t H_{k_{t+1}} N_{k_{t+1}} = \sum_{k=1}^n P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}}$, conditional on $j_t = 1, 2, ...n$ at the period. Combining this with (D.2) gives

$$\begin{aligned} & -E_t H_{k_{t+1}} N_{k_{t+1}} \left(A_{11k_{t+1}} X_t + A_{12k_{t+1}} x_t + B_{11k_{t+1}} u_t^L + B_{12k_{t+1}} u_t^F \right) \\ = \; & A_{21j_t} X_t + A_{22j_t} x_t + B_{21j_t} u_t^L + B_{22j_t} u_t^F. \end{aligned}$$

Solving for x_t we obtain

$$x_t = -J_{j_t} X_t - K^L_{j_t} u^L_t - K^F_{j_t} u^F_t, \tag{D.7}$$

where

$$\begin{aligned}
J_{j_t} &= \left(A_{22j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}} \right)^{-1} \left(A_{21j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{11k_{t+1}} \right), \\
K^L_{j_t} &= \left(A_{22j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}} \right)^{-1} \left(B_{21j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} B_{11k_{t+1}} \right), \\
K^F_{j_t} &= \left(A_{22j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}} \right)^{-1} \left(B_{22j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} B_{12k_{t+1}} \right).
\end{aligned}$$

We assume that $A_{22j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}}$ is invertible.

Second, substituting x_t from (D.1) using (D.7) gives

$$X_{t+1} = \widetilde{A}_{j_t k_{t+1}} X_t + \widetilde{B}^L_{j_t k_{t+1}} u^L_t + \widetilde{B}^F_{j_t k_{t+1}} u^F_t + C_{k_{t+1}} \varepsilon_{t+1}, \tag{D.8}$$

where

$$\begin{aligned}
\widetilde{A}_{j_t k_{t+1}} &= A_{11k_{t+1}} - A_{12k_{t+1}} J_{j_t}, \\
\widetilde{B}^L_{j_t k_{t+1}} &= B_{11k_{t+1}} - A_{12k_{t+1}} K^L_{j_t}, \\
\widetilde{B}^F_{j_t k_{t+1}} &= B_{12k_{t+1}} - A_{12k_{t+1}} K^F_{j_t}.
\end{aligned}$$

D.1 Policy of the Follower

Using (D.7) in the follower's loss function (D.3) gives

$$\begin{aligned}
L^F_{j_t} &= \begin{bmatrix} X_t \\ x_t \\ u^L_t \\ u^F_t \end{bmatrix}' \begin{bmatrix} Q^F_{11j_t} & Q^F_{12j_t} & P^F_{11j_t} & P^F_{12j_t} \\ Q^F_{21j_t} & Q^F_{22j_t} & P^F_{21j_t} & P^F_{22j_t} \\ P^{F\prime}_{11j_t} & P^{F\prime}_{21j_t} & R^F_{11j_t} & R^F_{12j_t} \\ P^{F\prime}_{12j_t} & P^{F\prime}_{22j_t} & R^{F\prime}_{12j_t} & R^F_{22j_t} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \\ u^L_t \\ u^F_t \end{bmatrix} \\
&= \begin{bmatrix} X_t \\ u^L_t \\ u^F_t \end{bmatrix}' \begin{bmatrix} \widetilde{Q}^F_{j_t} & \widetilde{P}^F_{1j_t} & \widetilde{P}^F_{2j_t} \\ \widetilde{P}^{F\prime}_{1j_t} & \widetilde{R}^F_{11j_t} & \widetilde{R}^F_{12j_t} \\ \widetilde{P}^{F\prime}_{2j_t} & \widetilde{R}^{F\prime}_{12j_t} & \widetilde{R}^F_{22j_t} \end{bmatrix} \begin{bmatrix} X_t \\ u^L_t \\ u^F_t \end{bmatrix}
\end{aligned} \tag{D.9}$$

where

$$\begin{aligned}
\widetilde{Q}^F_{j_t} &= Q^F_{11j_t} - Q^F_{12j_t}J_{j_t} - J'_{j_t}Q^F_{21j_t} + J'_{j_t}Q^F_{22j_t}J_{j_t},\\
\widetilde{P}^F_{1j_t} &= P^F_{11j_t} - Q^F_{12j_t}K^L_{j_t} + J'_{j_t}Q^F_{22j_t}K^L_{j_t} - J'_{j_t}P^F_{21j_t},\\
\widetilde{P}^F_{2j_t} &= P^F_{12j_t} - Q^F_{12j_t}K^F_{j_t} + J'_{j_t}Q^F_{22j_t}K^F_{j_t} - J'_{j_t}P^F_{22j_t},\\
\widetilde{R}^F_{11j_t} &= K^{L\prime}_{j_t}Q^F_{22j_t}K^L_{j_t} - K^{L\prime}_{j_t}P^F_{21j_t} - P^{F\prime}_{21j_t}K^L_{j_t} + R^F_{11j_t},\\
\widetilde{R}^F_{12j_t} &= K^{L\prime}_{j_t}Q^F_{22j_t}K^F_{j_t} - K^{L\prime}_{j_t}P^F_{22j_t} - P^{F\prime}_{21j_t}K^F_{j_t} + R^F_{12j_t},\\
\widetilde{R}^F_{22j_t} &= K^{F\prime}_{j_t}Q^F_{22j_t}K^F_{j_t} - K^{F\prime}_{j_t}P^F_{22j_t} - P^{F\prime}_{22j_t}K^F_{j_t} + R^F_{22j_t}.
\end{aligned}$$

The optimal value of the problem in period t is associated with the symmetric positive semidefinite matrix $V^F_{k_{t+1}}$ and it satisfies the Bellman equation:

$$X_t V^F_{j_t} X_t = \min_{u^F_{j_t}} \left\{ L^F_{j_t} + \beta E_t \left[X'_{t+1} V^F_{k_{t+1}} X_{t+1} \right] \right\} \tag{D.10}$$

subject to (D.8) and (D.9). The first-order condition with respect to u^F_t is

$$\begin{aligned}
0 &= X'_t\widetilde{P}^F_{2j_t} + u^{L\prime}_t\widetilde{R}^F_{12j_t} + u^{F\prime}_t\widetilde{R}^F_{22j_t} + \beta E_t X'_t\widetilde{A}'_{j_tk_{t+1}}V^F_{k_{t+1}}\widetilde{B}^F_{j_tk_{t+1}}\\
&\quad + \beta E_t u^{L\prime}_t\widetilde{B}^{L\prime}_{j_tk_{t+1}}V^F_{k_{t+1}}\widetilde{B}^F_{j_tk_{t+1}} + \beta E_t u^{F\prime}_t\widetilde{B}^{F\prime}_{j_tk_{t+1}}V^F_{k_{t+1}}\widetilde{B}^F_{j_tk_{t+1}}.
\end{aligned}$$

This leads to the optimal policy function u^F_t of the follower

$$u^F_t = -G^F_{k_{t+1}}X_{t+1} - D^F_{k_{t+1}}u^L_{t+1}, \tag{D.11}$$

where

$$\begin{aligned}
G^F_{k_{t+1}} &= \left(\widetilde{R}^F_{22j_t} + \beta\sum_{k=1}^n P_{j_tk_{t+1}}\widetilde{B}^{F\prime}_{j_tk_{t+1}}V^F_{k_{t+1}}\widetilde{B}^F_{j_tk_{t+1}}\right)^{-1}\left(\widetilde{P}^{F\prime}_{2j_t} + \beta\sum_{k=1}^n P_{j_tk_{t+1}}\widetilde{B}^{F\prime}_{j_tk_{t+1}}V^F_{k_{t+1}}\widetilde{A}_{j_tk_{t+1}}\right),\\
D^F_{k_{t+1}} &= \left(\widetilde{R}^F_{22j_t} + \beta\sum_{k=1}^n P_{j_tk_{t+1}}\widetilde{B}^{F\prime}_{j_tk_{t+1}}V^F_{k_{t+1}}\widetilde{B}^F_{j_tk_{t+1}}\right)^{-1}\left(\widetilde{R}^{F\prime}_{12j_t} + \beta\sum_{k=1}^n P_{j_tk_{t+1}}\widetilde{B}^{F\prime}_{j_tk_{t+1}}V^F_{k+1}\widetilde{B}^L_{j_tk_{t+1}}\right).
\end{aligned}$$

Furthermore, using (D.4) and (D.11) in (D.7) gives

$$x_t = -N_{j_t}X_t, \tag{D.12}$$

where

$$N_{j_t} = J_{j_t} - K^L_{j_t}F^L_{j_t} - K^F_{j_t}G^F_{j_t} + K^F_{j_t}D^F_{j_t}F^L_{j_t},$$

and using (D.4) and (D.11) and (D.12) in (D.1) gives

$$X_{t+1} = M_{j_tk_{t+1}}X_t + C_{k_{t+1}}\varepsilon_{t+1},$$

where

$$M_{j_tk_{t+1}} = A_{11k_{t+1}} - A_{12k_{t+1}}N_{j_t} - B_{11k_{t+1}}F^L_{j_t} - B_{12k_{t+1}}G^F_{j_t} + B_{12k_{t+1}}D^F_{j_t}F^L_{j_t}.$$

Finally, using (D.4), (D.8), (D.9) and (D.11) in (D.10) results in

$$
\begin{aligned}
V^F_{j_t} \equiv\ & \widetilde{Q}^F_{j_t} - \widetilde{P}^F_{1j_t} F^L_{j_t} - F^{L\prime}_{j_t} \widetilde{P}^{F\prime}_{1j_t} + F^{L\prime}_{j_t} \widetilde{R}^F_{11j_t} F^L_{j_t} \\
& + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right)' V^F_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right) \\
& - \left[\widetilde{P}^F_{2j_t} - \widetilde{R}^F_{12j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} + \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right)' V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right] \\
& \left(\widetilde{R}^{F\prime}_{22j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right)^{-1} \\
& \left[\widetilde{P}^{F\prime}_{2j_t} - \widetilde{R}^{F\prime}_{12j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} + \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right) \right],
\end{aligned}
$$

D.2 Policy of the Leader

Using (D.7) and (D.11) in the leader's loss function (D.3) gives

$$
\begin{aligned}
L^L_{j_t} &= \begin{bmatrix} X_t \\ x_t \\ u^L_t \\ u^F_t \end{bmatrix}' \begin{bmatrix} Q^L_{11j_t} & Q^L_{12j_t} & P^L_{11j_t} & P^L_{12j_t} \\ Q^L_{21j_t} & Q^L_{22j_t} & P^L_{21j_t} & P^L_{22j_t} \\ P^{L\prime}_{11j_t} & P^{L\prime}_{21j_t} & R^L_{11j_t} & R^L_{12j_t} \\ P^{L\prime}_{12j_t} & P^{L\prime}_{22j_t} & R^{L\prime}_{12j_t} & R^L_{22j_t} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \\ u^L_t \\ u^F_t \end{bmatrix} \\
&= \begin{bmatrix} X_t \\ u^L_t \end{bmatrix}' \begin{bmatrix} \widetilde{Q}^L_{j_t} & \widetilde{P}^L_{j_t} \\ \widetilde{P}^{L\prime}_{j_t} & \widetilde{R}^L_{j_t} \end{bmatrix} \begin{bmatrix} X_t \\ u^L_t \end{bmatrix}, \qquad \text{(D.13)}
\end{aligned}
$$

where

$$
\begin{aligned}
\widetilde{Q}^L_{j_t} &= Q^L_{11j_t} - P^L_{12j_t} G^F_{j_t} - G^{F\prime}_{j_t} P^{L\prime}_{12j_t} + G^{F\prime}_{j_t} R^L_{22j_t} G^F_{j_t} - Q^L_{12j_t} \widetilde{J}_{j_t} \\
&\quad - \widetilde{J}'_{j_t} Q^L_{21j_t} + \widetilde{J}'_{j_t} Q^L_{22j_t} \widetilde{J}_{j_t} + \widetilde{J}'_{j_t} P^L_{22j_t} G^F_{j_t} + G^{F\prime}_{j_t} P^{L\prime}_{22j_t} \widetilde{J}_{j_t}, \\
\widetilde{P}^L_{j_t} &= P^L_{11j_t} - Q^L_{12j_t} \widetilde{K}_{j_t} - P^L_{12j_t} D^F_{j_t} + \widetilde{J}'_{j_t} Q^L_{22j_t} \widetilde{K}_{j_t} - \widetilde{J}'_{j_t} P^L_{21j_t} \\
&\quad + \widetilde{J}'_{j_t} P^L_{22j_t} D^F_{j_t} + G^{F\prime}_{j_t} P^{L\prime}_{22j_t} \widetilde{K}_{j_t} - G^{F\prime}_{j_t} R^{L\prime}_{12j_t} + G^{F\prime}_{j_t} R^L_{22j_t} D^F_{j_t}, \\
\widetilde{R}^L_{j_t} &= R^L_{11j_t} + \widetilde{K}'_{j_t} Q^L_{22j_t} \widetilde{K}_{j_t} - R^L_{12j_t} D^F_{j_t} - D^{F\prime}_{j_t} R^{L\prime}_{12j_t} + D^{F\prime}_{j_t} R^L_{22j_t} D^F_{j_t} \\
&\quad - \widetilde{K}'_{j_t} P^L_{21j_t} + \widetilde{K}'_{j_t} P^L_{22j_t} D^F_{j_t} - P^{L\prime}_{21j_t} \widetilde{K}_{j_t} + D^{F\prime}_{j_t} P^{L\prime}_{22j_t} \widetilde{K}_{j_t}.
\end{aligned}
$$

and $\widetilde{J}_{j_t} = \left(J_{j_t} - K^F_{j_t} G^F_{j_t}\right)$ and $\widetilde{K}_{j_t} = \left(K^L_{j_t} - K^F_{j_t} D^F_{j_t}\right)$

The value of the problem in period t is associated with the symmetric positive semidefinite matrix $V^L_{k_{t+1}}$ and it satisfies the Bellman equation

$$
X_t V^L_{j_t} X_t = \min_{u^L_{j_t}} \left\{ L^L_{j_t} + \beta E_t \left[X'_{t+1} V^L_{k_{t+1}} X_{t+1} \right] \right\}, \qquad \text{(D.14)}
$$

subject to (D.8), (D.11) and (D.13) . The first-order condition with respect to u_t^L is

$$\begin{aligned} 0 \;=\; & X_t'\widetilde{P}_{j_t}^L + u_t^{L\prime}\widetilde{R}_{j_t}^L + \beta E_t X_t' \left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}_{j_tk_{t+1}}^F G_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right) \\ & + \beta E_t u_t^{L\prime} \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right). \end{aligned}$$

This leads to the optimal policy function of the leader

$$u_t^L = -F_j^L X_t, \tag{D.15}$$

where

$$\begin{aligned} F_j^L \;=\; & \left[\widetilde{R}_{j_t}^L + \beta \sum_{k=1}^{n} P_{j_tk_{t+1}} \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)\right]^{-1} \\ & \left[\widetilde{P}_{j_t}^{L\prime} + \beta \sum_{k=1}^{n} P_{j_tk_{t+1}} \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}_{j_tk_{t+1}}^F G_{j_t}^F\right)\right]. \end{aligned}$$

Furthermore, using (D.11) and (D.15) in (D.7) gives

$$x_t = -N_{j_t} X_t, \tag{D.16}$$

where

$$N_{j_t} = J_{j_t} - K_{j_t}^L F_{j_t}^L - K_{j_t}^F G_{j_t}^F + K_{j_t}^F D_{j_t}^F F_{j_t}^L,$$

and using (D.11), (D.15) and (D.16) in (D.1) gives

$$X_{t+1} = M_{j_tk_{t+1}} X_t + C_{k_{t+1}} \varepsilon_{t+1},$$

where

$$M_{j_tk_{t+1}} = A_{11k_{t+1}} - A_{12k_{t+1}} N_{j_t} - B_{11k_{t+1}} F_{j_t}^L - B_{12k_{t+1}} G_{j_t}^F + B_{12k_{t+1}} D_{j_t}^F F_{j_t}^L$$

Finally, using (D.8), (D.11), (D.13) and (D.15) in (D.14) results in

$$\begin{aligned} V_{j_t}^L = & \widetilde{Q}_{j_t}^L + \beta \sum_{k=1}^{n} P_{j_tk_{t+1}} \left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}_{j_tk_{t+1}}^F G_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}_{j_tk_{t+1}}^F G_{j_t}^F\right) \\ & - \left[\widetilde{P}_{j_t}^L + \beta \sum_{k=1}^{n} P_{j_tk_{t+1}} \left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}_{j_tk_{t+1}}^F G_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)\right] \\ & \left[\widetilde{R}_{j_t}^L + \beta \sum_{k=1}^{n} P_{j_tk_{t+1}} \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)\right]^{-1} \\ & \left[\widetilde{P}_{j_t}^{L\prime} + \beta \sum_{k=1}^{n} P_{j_tk_{t+1}} \left(\widetilde{B}_{j_tk_{t+1}}^L - \widetilde{B}_{j_tk_{t+1}}^F D_{j_t}^F\right)' V_{k_{t+1}}^L \left(\widetilde{A}_{j_tk_{t+1}} - \widetilde{B}_{j_tk_{t+1}}^F G_{j_t}^F\right)\right] \end{aligned}$$

To sum up, the first order conditions to the optimization problem (D.1), (D.2) and (D.3) can be written in the following form:

$$N_{j_t} = J_{j_t} - K^L_{j_t} F^L_{j_t} - K^F_{j_t} G^F_{j_t} + K^F_{j_t} D^F_{j_t} F^L_{j_t},$$

$$\begin{aligned} V^F_{j_t} \equiv\; & \widetilde{Q}^F_{j_t} - \widetilde{P}^F_{1j_t} F^L_{j_t} - F^{L\prime}_{j_t} \widetilde{P}^{F\prime}_{1j_t} + F^{L\prime}_{j_t} \widetilde{R}^F_{11j_t} F^L_{j_t} \\ & + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right)' V^F_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right) \\ & - \left[\widetilde{P}^F_{2j_t} - \widetilde{R}^F_{12j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} + \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right)' V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right] \\ & \left(\widetilde{R}^{F\prime}_{22j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right)^{-1} \\ & \left[\widetilde{P}^{F\prime}_{2j_t} - \widetilde{R}^{F\prime}_{12j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} + \widetilde{B}^L_{j_t k_{t+1}} F^L_{j_t} \right) \right], \end{aligned}$$

$$\begin{aligned} V^L_{j_t} = {} & \widetilde{Q}^L_{j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} G^F_{j_t} \right)' V^L_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} G^F_{j_t} \right) \\ & - \left[\widetilde{P}^L_{j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} G^F_{j_t} \right)' V^L_{k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right) \right] \\ & \left[\widetilde{R}^L_{j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right)' V^L_{k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right) \right]^{-1} \\ & \left[\widetilde{P}^{L\prime}_{j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right)' V^L_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} G^F_{j_t} \right) \right] \end{aligned}$$

$$\begin{aligned} F^L_{j} = {} & \left[\widetilde{R}^{L\prime}_{j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right)' V^L_{k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right) \right]^{-1} \\ & \left[\widetilde{P}^{L\prime}_{j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{B}^L_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} D^F_{j_t} \right)' V^L_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^F_{j_t k_{t+1}} G^F_{j_t} \right) \right], \end{aligned}$$

$$\begin{aligned} G^F_{k_{t+1}} &= \left(\widetilde{R}^{F\prime}_{22j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right)^{-1} \left(\widetilde{P}^{F\prime}_{2j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{A}_{j_t k_{t+1}} \right), \\ D^F_{k_{t+1}} &= \left(\widetilde{R}^{F\prime}_{22j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k_{t+1}} \widetilde{B}^F_{j_t k_{t+1}} \right)^{-1} \left(\widetilde{R}^{F\prime}_{12j_t} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{F\prime}_{j_t k_{t+1}} V^F_{k+1} \widetilde{B}^L_{j_t k_{t+1}} \right). \end{aligned}$$

The discretion equilibrium is a fixed point $\left(N, V^L, V^F\right) \equiv \left\{N_{j_t}, V^L_{j_t}, V^F_{j_t}\right\}^n_{j_t=1}$ of the mapping and a corresponding $\left(F^L, G^F, D^F\right) \equiv \left\{F^L_{j_t}, G^F_{j_t}, D^F_{j_t}\right\}^n_{j_t=1}$. The fixed point can be obtained as the limit of $\left(N_t, V^L_t, V^F_t\right)$ when $t \to -\infty$.

E Nash Equilibrium under Discretion

Consider an economy with two policy makers, A and B, who decide their policy simultaneously.

$$X_{t+1} = A_{11k_{t+1}} X_t + A_{12k_{t+1}} x_t + B_{11k_{t+1}} u^A_t + B_{12k_{t+1}} u^B_t + C_{k_{t+1}} \varepsilon_{t+1}, \tag{E.1}$$
$$E_t H_{k_{t+1}} x_{t+1} = A_{21j_t} X_t + A_{22j_t} x_t + B_{21j_t} u^A_t + B_{22j_t} u^B_t, \tag{E.2}$$

where X_t is a n_1 vector of predetermined variables; x_t is a n_2 vector of forward-looking variables; u^A_t and u^B_t are the two policy makers' instruments, and ε_t contains a vector of zero mean *i.i.d.* shocks. Without loss of generality, the shocks are normalized so that the covariance matrix of ε_t is an identity matrix, and the covariance matrix of the shocks to X_{t+1} is $C'_{j_t} C_{j_t}$.

The period loss function of policy makers, A and B, is defined as in equation (D.3) with $u = A$ and $u = B$, respectively. Policy makers A and B simultaneously decide their policy u^A_t and u^B_t in period t to minimize their intertemporal loss functions defined in (D.3) under discretion subject to (E.1), (D.2), X_t and j_t given. Reoptimization in period $t+1$ result in the two instruments and the forward-looking variables being functions of the predetermined variables and the mode as follows

$$u^A_{t+1} = -F^A_{k_{t+1}} X_{t+1}, \tag{E.3}$$
$$u^B_{t+1} = -F^B_{k_{t+1}} X_{t+1}, \tag{E.4}$$
$$x_{t+1} = -N_{k_{t+1}} X_{t+1}. \tag{E.5}$$

Combining equations (E.1), (E.2) and (E.5), we solve for x_t

$$x_t = -J_t X_t - K^A_{j_t} u^A_t - K^B_{j_t} u^B_t, \tag{E.6}$$

where

$$J_{j_t} = \left(A_{22j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}}\right)^{-1} \left(A_{21j} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{11k_{t+1}}\right),$$
$$K^A_{j_t} = \left(A_{22j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}}\right)^{-1} \left(B_{21j} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} B_{11k_{t+1}}\right),$$
$$K^B_{j_t} = \left(A_{22j_t} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}}\right)^{-1} \left(B_{22j} + \sum_{k=1}^{n} P_{j_t k_{t+1}} H_{k_{t+1}} N_{k_{t+1}} B_{12k_{t+1}}\right).$$

By substituting x_t from (E.1) using (E.6) gives

$$X_{t+1} = \widetilde{A}_{j_t k_{t+1}} X_t + \widetilde{B}^A_{j_t k_{t+1}} u^A_t + \widetilde{B}^B_{j_t k_{t+1}} u^B_t + C_{k_{t+1}} \varepsilon_{t+1}, \tag{E.7}$$

where

$$\begin{aligned}
\widetilde{A}_{j_t k_{t+1}} &= A_{11k_{t+1}} - A_{12k_{t+1}} J_{j_t}, \\
\widetilde{B}^A_{j_t k_{t+1}} &= B_{11k_{t+1}} - A_{12k_{t+1}} K^A_{j_t}, \\
\widetilde{B}^B_{j_t k_{t+1}} &= B_{12k_{t+1}} - A_{12k_{t+1}} K^B_{j_t}.
\end{aligned}$$

E.1 Policy Maker A

Substitute (E.4) and (E.6) in the policy maker A's period loss function gives

$$\begin{aligned}
L^A_{j_t} &= \begin{bmatrix} X_t \\ x_t \\ u^A_t \\ u^B_t \end{bmatrix}' \begin{bmatrix} Q^A_{11j_t} & Q^A_{12j_t} & P^A_{11j_t} & P^A_{12j_t} \\ Q^A_{21j_t} & Q^A_{22j_t} & P^A_{21j_t} & P^A_{22j_t} \\ P^{A\prime}_{11j_t} & P^{A\prime}_{21j_t} & R^A_{11j_t} & R^A_{12j_t} \\ P^{A\prime}_{12j_t} & P^{A\prime}_{22j_t} & R^{A\prime}_{12j_t} & R^A_{22j_t} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \\ u^A_t \\ u^B_t \end{bmatrix} \\
&= \begin{bmatrix} X_t \\ u^A_t \end{bmatrix}' \begin{bmatrix} \widetilde{Q}^A_{j_t} & \widetilde{P}^A_{j_t} \\ \widetilde{P}^{A\prime}_{j_t} & \widetilde{R}^A_{j_t} \end{bmatrix} \begin{bmatrix} X_t \\ u^A_t \end{bmatrix}
\end{aligned} \tag{E.8}$$

where

$$\begin{aligned}
\widetilde{Q}^A_{j_t} &= Q^A_{11j_t} - Q^A_{12j_t} \widetilde{J}^B_{j_t} - \widetilde{J}^{B\prime}_{j_t} Q^A_{21j_t} + \widetilde{J}^{B\prime}_{j_t} Q_{22} \widetilde{J}^B_{j_t} \\
&\quad + F^{B\prime}_{j_t} R^A_{22j_t} F^B_{j_t} + F^{B\prime}_{j_t} P^{A\prime}_{22j_t} \widetilde{J}^B_{j_t} + \widetilde{J}^{B\prime}_{j_t} P^A_{22j_t} F^B_{j_t} \\
&\quad - P^A_{12j_t} F^B_{j_t} - F^{B\prime}_{j_t} P^{A\prime}_{12j_t} \\
\widetilde{P}^A_{j_t} &= -Q^A_{12j_t} K^A_{j_t} + \widetilde{J}^{B\prime}_{j_t} Q^A_{22j_t} K^A_{j_t} + P^A_{11j_t} - \widetilde{J}^{B\prime}_{j_t} P^A_{21j_t} + F^{B\prime}_{j_t} P^{A\prime}_{22j_t} K^A_{j_t} - F^{B\prime}_{j_t} R^{A\prime}_{12j_t}, \\
\widetilde{R}^A_{j_t} &= K^{A\prime}_{j_t} Q^A_{22j_t} K^A_{j_t} - K^{A\prime}_{j_t} P^A_{21j_t} - P^{A\prime}_{21j_t} K^A_{j_t} + R^A_{11j_t}
\end{aligned}$$

and $\widetilde{J}^B_{j_t} = J_{j_t} - K^B_{j_t} F^B_{j_t}$.

The optimal value of the problem in period t is associated with the symmetric positive semidefinite matrix $V^A_{k_{t+1}}$ and it satisfies the Bellman equation:

$$X_t V^A_{j_t} X_t = \min_{u^A_{j_t}} \left\{ L^A_{j_t} + \beta E_t \left[X'_{t+1} V^A_{k_{t+1}} X_{t+1} \right] \right\} \tag{E.9}$$

subject to (E.4), (E.6) and (E.8). The first-order condition with respect to u^A_t is

$$0 = X'_t \widetilde{P}^A_{j_t} + u^{A\prime}_t \widetilde{R}^A_{j_t} + \beta E_t X'_t \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^B_{j_t k_{t+1}} F^B_{j_t} \right)' V^A_{k_{t+1}} \widetilde{B}^A_{j_t k_{t+1}} + \beta E_t u^{A\prime}_t \widetilde{B}^{A\prime}_{j_t k_{t+1}} V^A_{k_{t+1}} \widetilde{B}^A_{j_t k_{t+1}}.$$

The optimal policy function of the leader is given by

$$u_t^A = -F_{j_t}^A X_t, \tag{E.10}$$

where

$$\begin{aligned} F_{j_t}^A &= \left(\widetilde{R}_{j_t}^A + \beta \sum_{k=1}^n P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^A \widetilde{B}_{j_t k_{t+1}}^A \right)^{-1} \\ &\left[\widetilde{P}_{j_t}^{A\prime} + \beta \sum_{k=1}^n P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^A \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^B F_{j_t}^B \right) \right] \end{aligned}$$

Furthermore, using (E.4) and (E.10) in (E.6) gives

$$x_t = -N_{j_t} X_t, \tag{E.11}$$

where

$$N_{j_t} = J_t - K_{j_t}^A F_{j_t}^A - K_{j_t}^B F_{j_t}^B$$

and using (E.4), and (E.10) and (E.11) in (20) gives

$$X_{t+1} = M_{j_t k_{t+1}} X_t + C_{k_{t+1}} \varepsilon_{t+1},$$

where

$$M_{j_t k_{t+1}} = A_{11k_{t+1}} - A_{12k_{t+1}} N_{j_t} - B_{11k_{t+1}} F_{j_t}^A - B_{12k_{t+1}} F_{j_t}^B$$

Finally, using (E.4), (E.7), (E.8) and (E.10) in (E.9) results in

$$\begin{aligned} V_{j_t}^A &= \widetilde{Q}_{j_t}^A + \beta \sum_{k=1}^n P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^B F_{j_t}^B \right)' V_{k_{t+1}}^A \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^B F_{j_t}^B \right) \\ &- \left(\widetilde{P}_{j_t}^A + \beta \sum_{k=1}^n P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^B F_{j_t}^B \right)' V_{k_{t+1}}^A \widetilde{B}_{j_t k_{t+1}}^A \right) \\ &\left(\widetilde{R}_{j_t}^A + \beta \sum_{k=1}^n P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^A \widetilde{B}_{j_t k_{t+1}}^A \right)^{-1} \\ &\left[\widetilde{P}_{j_t}^{A\prime} + \beta \sum_{k=1}^n P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^A \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^B F_{j_t}^B \right) \right] \end{aligned}$$

E.2 Policy Maker B

Using (E.10) and (E.6) in policy maker B's period loss function gives

$$\begin{aligned} L_{j_t}^B &= \begin{bmatrix} X_t \\ x_t \\ u_t^A \\ u_t^B \end{bmatrix}' \begin{bmatrix} Q_{11j_t}^B & Q_{12j_t}^B & P_{11j_t}^B & P_{12j_t}^B \\ Q_{21j_t}^B & Q_{22j_t}^B & P_{21j_t}^B & P_{22j_t}^B \\ P_{11j_t}^{B\prime} & P_{21j_t}^{B\prime} & R_{11j_t}^B & R_{12j_t}^B \\ P_{12j_t}^{B\prime} & P_{22j_t}^{B\prime} & R_{12j_t}^{B\prime} & R_{22j_t}^B \end{bmatrix} \begin{bmatrix} X_t \\ x_t \\ u_t^A \\ u_t^B \end{bmatrix} \\ &= \begin{bmatrix} X_t \\ u_t^B \end{bmatrix}' \begin{bmatrix} \widetilde{Q}_{j_t}^B & \widetilde{P}_{j_t}^B \\ \widetilde{P}_{j_t}^{B\prime} & \widetilde{R}_{j_t}^B \end{bmatrix} \begin{bmatrix} X_t \\ u_t^B \end{bmatrix} \end{aligned} \tag{E.12}$$

where

$$\begin{aligned}
\widetilde{Q}^B_{j_t} &= Q^B_{11j_t} - Q^B_{12j_t}\widetilde{J}^A_{j_t} - \widetilde{J}^{A\prime}_{j_t}Q^B_{21j_t} + \widetilde{J}^{A\prime}_{j_t}Q^B_{22j_t}\widetilde{J}^A_{j_t} + F^{A\prime}_{j_t}R^B_{11j_t}F^A_{j_t} \\
&\quad + F^{A\prime}_{j_t}P^{B\prime}_{21j_t}\widetilde{J}^A_{j_t} + \widetilde{J}^{A\prime}_{j_t}P^B_{21j_t}F^A_{j_t} - P^B_{11j_t}F^A_{j_t} - F^{A\prime}_{j_t}P^{B\prime}_{11j_t}, \\
\widetilde{P}^B_{j_t} &= -Q^B_{12j_t}K^B_{j_t} + \widetilde{J}^{A\prime}_{j_t}Q^B_{22j_t}K^B_{j_t} - F^{A\prime}_{j_t}R^B_{12j_t} + P^B_{12j_t} - \widetilde{J}^{A\prime}_{j_t}P^B_{22j_t} + F^{A\prime}_{j_t}P^{B\prime}_{21j_t}K^B_{j_t}, \\
\widetilde{R}^B_{j_t} &= K^B_{j_t}Q^B_{22j_t}K^B_{j_t} - K^B_{j_t}P^B_{22j_t} - P^{B\prime}_{22j_t}K^B_{j_t} + R^B_{22j_t},
\end{aligned}$$

and $\widetilde{J}^A_{j_t} = \left(J_{j_t} - K^A_{j_t}F^A_{j_t}\right)$.

The optimal value of the problem in period t is associated with the symmetric positive semidefinite matrix $V^B_{k_{t+1}}$ and it satisfies the Bellman equation:

$$X_t V^B_{j_t} X_t = \min_{u^B_{j_t}} \left\{ L^B_{j_t} + \beta E_t \left[X'_{t+1} V^B_{k_{t+1}} X_{t+1} \right] \right\} \tag{E.13}$$

subject to (E.10), (E.6) and (E.12). The first-order condition with respect to u^B_t is

$$0 = X'_t\widetilde{P}^B_{j_t} + u^{B\prime}_t\widetilde{R}^B_{j_t} + \beta E_t X'_t \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^A_{j_t k_{t+1}}F^A_{j_t}\right)' V^B_{k_{t+1}}\widetilde{B}^B_{j_t k_{t+1}} + \beta E_t u^{B\prime}_t \widetilde{B}^{B\prime}_{j_t k_{t+1}} V^B_{k_{t+1}} \widetilde{B}^B_{j_t k_{t+1}},$$

The optimal policy function of the follower is given by

$$u^B_t = -F^B_{j_t}X_t, \tag{E.14}$$

where

$$\begin{aligned}
F^B_{j_t} &= \left(\widetilde{R}^{B\prime}_{j_t} + \beta\sum_{k=1}^{n} P_{j_t k_{t+1}}\widetilde{B}^{B\prime}_{j_t k_{t+1}}V^B_{k_{t+1}}\widetilde{B}^B_{j_t k_{t+1}}\right)^{-1} \\
&\quad \left[\widetilde{P}^{B\prime}_{j_t} + \beta\sum_{k=1}^{n} P_{j_t k_{t+1}}\widetilde{B}^{B\prime}_{j_t k_{t+1}}V^B_{k_{t+1}}\left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^A_{j_t k_{t+1}}F^A_{j_t}\right)\right].
\end{aligned}$$

Furthermore, using (E.10) and (E.14) in (E.6) gives

$$x_t = -N_{j_t}X_t, \tag{E.15}$$

where

$$N_{j_t} = J_t - K^A_{j_t}F^A_{j_t} - K^B_{j_t}F^B_{j_t},$$

and using (E.10) and (E.14) and (E.15) in (20) gives

$$X_{t+1} = M_{j_t k_{t+1}}X_t + C_{k_{t+1}}\varepsilon_{t+1},$$

where

$$M_{j_t k_{t+1}} = A_{11k_{t+1}} - A_{12k_{t+1}}N_{j_t} - B_{11k_{t+1}}F^A_{k_{t+1}} - B_{12k_{t+1}}F^B_{j_t}$$

Finally, using (E.7), (E.10), (E.12) and (E.14) in (E.13) results in

$$
\begin{aligned}
V_{j_t}^{B} &= \widetilde{Q}_{j_t}^{B} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{k_{t+1}}^{A} \right)' V_{k_{t+1}}^{B} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{k_{t+1}}^{A} \right) \\
&\quad - \left[\widetilde{P}_{j_t}^{B} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{j_t}^{A} \right)' V_{k_{t+1}}^{B} \widetilde{B}_{j_t k_{t+1}}^{B} \right] \\
&\quad \left(\widetilde{R}_{j_t}^{B} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{B\prime} V_{k_{t+1}}^{B} \widetilde{B}_{j_t k_{t+1}}^{B} \right)^{-1} \\
&\quad \left[\widetilde{P}_{j_t}^{B\prime} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{B\prime} V_{k_{t+1}}^{B} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{j_t}^{A} \right) \right]
\end{aligned}
$$

To sum up, the first order conditions to the optimization problem can be written in the following form:

$$
N_{j_t} = J_{j_t} - K_{j_t}^{A} F_{j_t}^{A} - K_{j_t}^{B} F_{j_t}^{B},
$$

$$
\begin{aligned}
V_{j_t}^{A} &= \widetilde{Q}_{j_t}^{A} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{B} F_{j_t}^{B} \right)' V_{k_{t+1}}^{A} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{B} F_{j_t}^{B} \right) \\
&\quad - \left(\widetilde{P}_{j_t}^{A} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{B} F_{j_t}^{B} \right)' V_{k_{t+1}}^{A} \widetilde{B}_{j_t k_{t+1}}^{A} \right) \\
&\quad \left(\widetilde{R}_{j_t}^{A} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^{A} \widetilde{B}_{j_t k_{t+1}}^{A} \right)^{-1} \\
&\quad \left[\widetilde{P}_{j_t}^{A\prime} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{A\prime} V_{k_{t+1}}^{A} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{B} F_{j_t}^{B} \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
V_{j_t}^{B} &= \widetilde{Q}_{j_t}^{B} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{k_{t+1}}^{A} \right)' V_{k_{t+1}}^{B} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{k_{t+1}}^{A} \right) \\
&\quad - \left[\widetilde{P}_{j_t}^{B} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{j_t}^{A} \right)' V_{k_{t+1}}^{B} \widetilde{B}_{j_t k_{t+1}}^{B} \right] \\
&\quad \left(\widetilde{R}_{j_t}^{B} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{B\prime} V_{k_{t+1}}^{B} \widetilde{B}_{j_t k_{t+1}}^{B} \right)^{-1} \\
&\quad \left[\widetilde{P}_{j_t}^{B\prime} + \beta \sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}_{j_t k_{t+1}}^{B\prime} V_{k_{t+1}}^{B} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}_{j_t k_{t+1}}^{A} F_{j_t}^{A} \right) \right]
\end{aligned}
$$

$$F^A_{j_t} = \left(\widetilde{R}^{A\prime}_{j_t} + \beta\sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{A\prime}_{j_t k_{t+1}} V^A_{k_{t+1}} \widetilde{B}^A_{j_t k_{t+1}}\right)^{-1} \left[\widetilde{P}^{A\prime}_{j_t} + \beta\sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{A\prime}_{j_t k_{t+1}} V^A_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^B_{j_t k_{t+1}} F^B_{j_t}\right)\right]$$

$$F^B_{j_t} = \left(\widetilde{R}^{B\prime}_{j_t} + \beta\sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{B\prime}_{j_t k_{t+1}} V^B_{k_{t+1}} \widetilde{B}^B_{j_t k_{t+1}}\right)^{-1} \left[\widetilde{P}^{B\prime}_{j_t} + \beta\sum_{k=1}^{n} P_{j_t k_{t+1}} \widetilde{B}^{B\prime}_{j_t k_{t+1}} V^B_{k_{t+1}} \left(\widetilde{A}_{j_t k_{t+1}} - \widetilde{B}^A_{j_t k_{t+1}} F^A_{j_t}\right)\right]$$

The discretion equilibrium is a fixed point $\left(N, V^A, V^B\right) \equiv \left\{N_{j_t}, V^A_{j_t}, V^B_{j_t}\right\}^n_{j_t=1}$ of the mapping and a corresponding $\left(F^A, F^B\right) \equiv \left\{F^A_{j_t}, F^B_{j_t}\right\}^n_{j_t=1}$. The fixed point can be obtained as the limit of $\left(N_t, V^A_t, V^B_t\right)$ when $t \longrightarrow -\infty$.

F Data Appendix

We follow Bianchi and Ilut (2017) in constructing our fiscal variables. The data for government spending, tax revenues and transfers, are taken from National Income and Product Accounts (NIPA) Table 3.2 (Federal Government Current Receipts and Expenditures) released by the Bureau of Economics Analysis. These data series are nominal and in levels.

Government Spending. Government spending is defined as the sum of consumption expenditure (line 21), gross government investment (line 42), net purchases of nonproduced assets (line 44), minus consumption of fixed capital (line 45), minus wage accruals less disbursements (line 33).

Total tax revenues. Total tax revenues are constructed as the difference between current receipts (line 38) and current transfer receipts (line 16).

Transfers. Transfers is defined as current transfer payments (line 22) minus current transfer receipts (line 16) plus capital transfers payments (line 43) minus capital transfer receipts (line 39) plus subsidies (line 32).

Federal government debt. Federal government debt is the market value of privately held gross Federal debt, which is downloaded from Dallas Fed web-site

The above three fiscal variables are normalized with respect to Nominal GDP. **Nominal GDP** is taken from NIPA Table 1.1.5 (Gross Domestic Product).

Real GDP. Real GDP is take download from NIPA Table 1.1.6 (Real Gross Domestic Product, Chained Dollars)

The GDP deflator. The GDP deflator is obtained from NIPA Table 1.1.5 (Gross Domestic Product).

Effective Federal Funds Rate. Effective Federal Funds Rate is taken from the St. Louis Fed website.

G Convergence

A random walk Metropolis-Hastings algorithm is then used to generate four chains consisting of 540,000 draws each (with the first 240,000 draws being discarded and 1 in every 100 draws being saved). Brooks-Gelman-Rubin potential reduction scale factors (PRSF) are all below the 1.1 benchmark value used as an upper bound for convergence. FPSR values for Rules-Based Policy and Optimal Policy are presented in Table G.1.

Parameters	PSRF	Parameters	PSRF	Parameters	PSRF	Parameters	PSRF	Parameters	PSRF
				Rules-based policy					
AM/PF		PM/PF		σ	1.00	$\sigma_{\xi(k=1)}$	1.00	p_{11}	1.00
$\rho_{R,S_t=1}$	1.00	$\rho_{R,S_t=2}$	1.06	α	1.00	$\sigma_{\xi(k=2)}$	1.00	p_{22}	1.00
$\psi_{1,S_t=1}$	1.00	$\psi_{1,S_t=2}$	1.01	ζ	1.00	$\sigma_{\mu(k=1)}$	1.00	q_{11}	1.00
$\psi_{2,S_t=1}$	1.00	$\psi_{2,St=2}$	1.01	θ	1.00	$\sigma_{\mu(k=2)}$	1.00	q_{22}	1.01
$\rho_{\tau,s_t=1}$	1.00	$\rho_{\tau,s_t=1}$	1.00	φ	fixed	$\sigma_{q(k=1)}$	1.00	h_{11}	1.00
$\delta_{\tau,s_t=1}$	1.00	$\delta_{\tau,s_t=1}$	1.00	ρ_ξ	1.01	$\sigma_{q(k=2)}$	1.00	h_{22}	1.00
δ_y	1.02	δ_y	1.02	ρ_μ	1.00	σ_{tp}	1.00		
AM/AF		PM/AF		ρ_q	1.00	σ_g	1.00		
$\rho_{R,S_t=1}$	1.00	$\rho_{R,S_t=2}$	1.06	ρ_z	1.00	σ_z	1.00		
$\psi_{1,S_t=1}$	1.00	$\psi_{1,S_t=2}$	1.01	ρ_g	1.00	σ_τ	1.00		
$\psi_{2,St=1}$	1.00	$\psi_{2,St=2}$	1.01			σ_R	1.00		
$\rho_{\tau,s_t=2}$	1.02	$\rho_{\tau,s_t=2}$	1.02						
$\delta_{\tau,s_t=2}$	fixed	$\delta_{\tau,s_t=2}$	fixed						
δ_y	1.02	δ_y	1.02						
				Optimal policy					
ω_1	1.00	σ	1.00	$\sigma_{\mu(k_t=1)}$	1.01	ϕ_{11}	1.00		
ω_2	1.00	α	1.01	$\sigma_{\mu(k_t=2)}$	1.00	ϕ_{22}	1.01		
ω_3	1.00	ζ	1.00	$\sigma_{q(k_t=1)}$	1.00	ψ_{11}	1.01		
$\omega_{\pi,s_t=1}$	fixed	θ	1.02	$\sigma_{q(k_t=2)}$	1.01	ψ_{12}	1.01		
$\omega_{\pi,s_t=2}$	1.00	φ	fixed	$\sigma_{\xi(k_t=1)}$	1.01	ψ_{22}	1.00		
ω_R	1.00	ρ_ξ	1.02	$\sigma_{\xi(k_t=2)}$	1.00	ψ_{23}	1.02		
ω_π^f	1.00	ρ_μ	1.01	σ_{tp}	1.01	ψ_{33}	1.00		
ω_τ	1.01	ρ_q	1.00	σ_g	1.00	ψ_{31}	1.00		
$\rho_{\tau,S_t=2}$	1.03	ρ_z	1.02	σ_z	1.00	h_{11}	1.00		
$\rho_{\tau,S_t=3}$	1.01	ρ_g	1.01	σ_τ	1.01	h_{22}	1.00		
$\delta_{\tau,S_t=2}$	1.02								
$\delta_{\tau,S_t=3}$	fixed								
δ_y	1.00								

Table G.1: Brooks-Gelman-Rubin potential reduction scale factors.

H Model Identification

We apply the Komunjer and Ng (2011) identification test to analyze our optimal policy model. Komunjer and Ng (2011) study the local identification of a DSGE model from its linearized solution. Their test uses the restrictions implied by equivalent spectral densities to obtain rank and order conditions for identification. Minimality and left-invertibility are necessary and sufficient conditions for identification. It is important to note that the Komunjer and Ng (2011) identification test only applies to covariance stationary processes. Therefore, the parameters associated with Markov-switching shock variances cannot be incorporated into the test.

Nevertheless, it is possible to test the identification of structural parameters and the transition probabilities associated with policy changes. We can solve our model assuming that policy stays in one regime, while the private agents in the economy are aware that there are probabilities of policy switching to a different regime. In total, we have six policy regimes: MC/OF, LC/OF, MC/PF, LC/PF, MC/AF and LC/AF.

Our optimal policy model has an estimated parameter vector of dimension $n_\theta = 35$, seven observables and seven exogenous shocks (i.e. $n_Y = n_\varepsilon = 7$). The model is square. Thus, Proposition 2-S in Komunjer and Ng (2011) is employed to assess identification. Overall, the test does not indicate that any parameters are unidentified. In Table H.1 the required rank for identification of each regime is presented, along with the Tol at which the model passes the rank requirement.[2]

In addition, we plot draws from the the prior and posterior distributions of parameters for the optimal policy model in Figure H.1. Visual inspection reveals that the priors are widely dispersed around the respective means, whereas posteriors are more concentrated. In other words, the data are informative with respect to these parameters.

	Tolerance	Δ^s_Λ	Δ^s_T	Δ^s_U	Δ^s	Pass
MC/OF	$1.0e-03$	35	144	49	228	YES
LC/OF	$1.0e-03$	35	144	49	228	YES
MC/PF	$1.0e-04$	35	100	49	184	YES
LC/PF	$1.0e-04$	35	100	49	184	YES
MC/AF	$1.0e-04$	35	100	49	184	YES
LC/AF	$1.0e-04$	35	100	49	184	YES

Table H.1: Komunjer and Ng (2011) Identification Test.

[2] Using the same notation as in Komunjer and Ng (2011), the required rank for identification is $rank(\Delta^s) = rank(\Delta^s_\Lambda + \Delta^s_T + \Delta^s_U) = n_\theta + n_X^2 + n_\varepsilon^2$, where n_θ is the number of estimated parameters, n_X is the number of minimal state variables, and n_ε is the number of exogenous shocks.

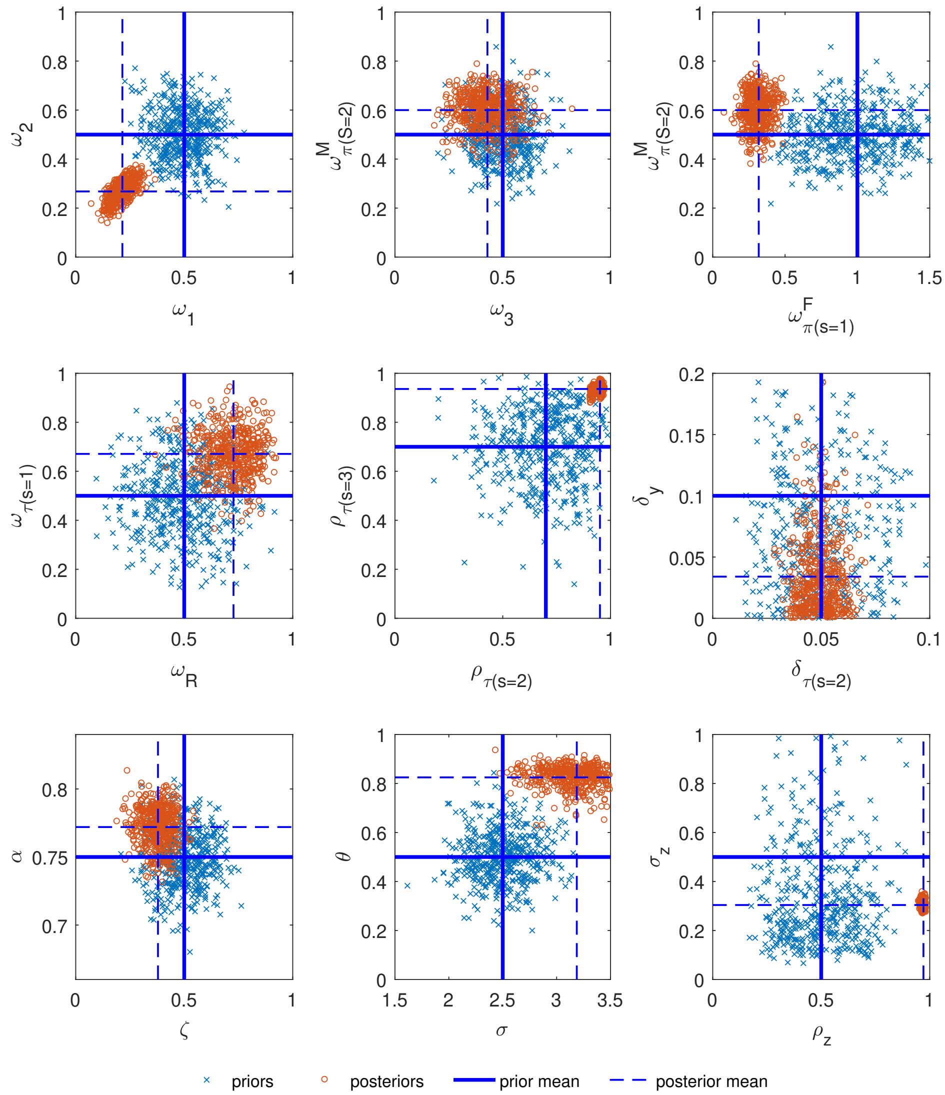

Figure H.1: Prior and Posterior Distributions of Parameters. The panels depict 500 draws from prior and posterior distributions from the estimates of our optimal policy model. The draws are plotted for pairs of estimated parameters and the intersections of lines signify prior (solid) and posterior (dashed) means, respectively.

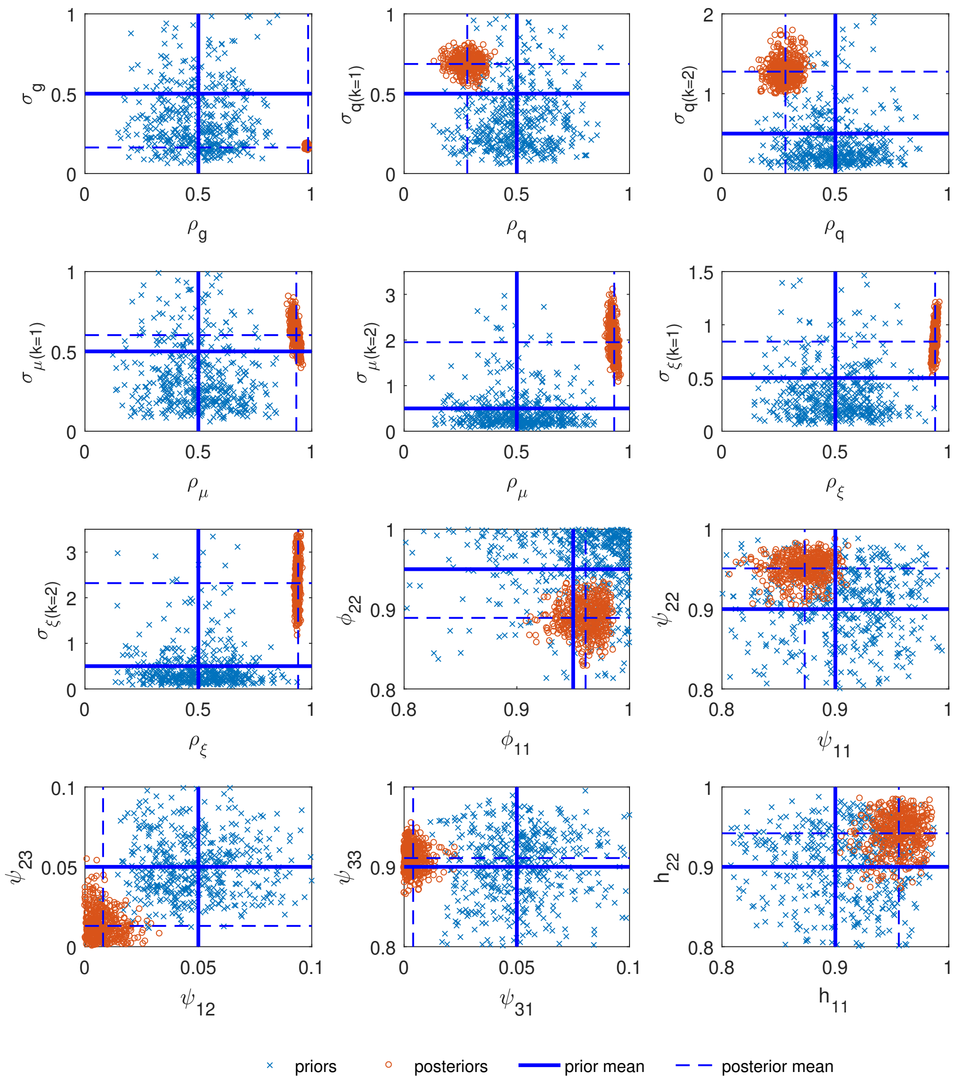

Figure H.1: Prior and Posterior Distributions of Parameters (continued). The panels depict 500 draws from prior and posterior distributions from the estimates of our optimal policy model. The draws are plotted for pairs of estimated parameters and the intersections of lines signify prior (solid) and posterior (dashed) means, respectively.

I Alternative Social Planner's Allocation

In this section we outline the social planner's allocation associated with our estimated model. Normally such an allocation would be obtained by maximising utility subject to resource and technology constraints as in Appendix B above. However, in order to generate insight into our policy maker's decisions we need to consider the estimated objective function. Therefore we maximise the following objective function,

$$L = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \omega_1 \left(\widehat{X}_t^* + \widehat{\xi}_t \right)^2 + \omega_2 \left(\widehat{y}_t^* - \frac{\sigma}{\varphi} \widehat{\xi}_t \right)^2 \right\},$$

subject to the definition of habits adjusted consumption,

$$\widehat{X}_t^* = (1-\theta)^{-1}(\widehat{y}_t^* - \frac{1}{1-g}\widetilde{g}_t - \theta \widehat{y}_{t-1}^* + \theta \frac{1}{1-g}\widetilde{g}_{t-1})$$

where the star superscripts denote the fact that we are considering the social planner's allocation. The first order condition this implies is given by,

$$\begin{aligned} & \omega_1 \left((1-\theta)^{-1}(\widehat{y}_t^* - \frac{1}{1-g}\widetilde{g}_t - \theta \widehat{y}_{t-1}^* + \theta \frac{1}{1-g}\widetilde{g}_{t-1}) + \widehat{\xi}_t \right) (1-\theta)^{-1} + \omega_2 \left(\widehat{y}_t^* - \frac{\sigma}{\varphi}\widehat{\xi}_t \right) \\ = \; & \theta \beta \omega_1 \left((1-\theta)^{-1}(E_t \widehat{y}_{t+1}^* - \frac{1}{1-g}\rho_g \widetilde{g}_t - \theta \widehat{y}_t^* + \theta \frac{1}{1-g}\widetilde{g}_t) + \rho^{\xi} \widehat{\xi}_t \right) (1-\theta)^{-1}. \end{aligned}$$

This describes the desired path for output that would be chosen by the social planner conditional on the exogenous path for government spending. This can be used to construct a welfare relevant output gap $\widehat{y}_t - \widehat{y}_t^*$ which captures the extent to which the policy maker is unable to achieve this desired level of output due to nominal inertia, the habits externality, fiscal constraints and time-consistency problems. Effectively, it reflects the welfare trade-offs between inflation and the real economy implied by the estimated objective function, but reduces those to a single measure.

In order to identify why the estimations adopts particular regimes at particular points of time it is also helpful to get a measure of various fiscal gaps, specifically the tax and debt gaps. The tax gap is the difference between $\widetilde{\tau}_t$ and the tax rate that the social planner would choose to eliminate cost-push shocks, $\widetilde{\tau}_t^* = -(1-\tau)\widehat{\mu}_t$, so that we have a tax gap, $\widetilde{\tau}_t - \widetilde{\tau}_t^*$.

References

An, S., and F. Schorfheide (2007): “Bayesian Analysis of DSGE Models,” *Econometric Reviews*, 26(2–4), 113–172.

Bianchi, F. (2012): “Evolving Monetary/Fiscal Policy Mix in the United States,” *American Economic Review Papers & Proceedings*, 101(3), 167–172.

Bianchi, F., and C. Ilut (2017): “Monetary/Fiscal Policy Mix and Agents’ Beliefs,” *Review of Economic Dynamics*, 26(October), 113–139.

Fabian Eser, C. L., and S. Wren-Lewis (2009): “When is Monetary Policy All We Need?,” University of Oxford Working Paper No. 430, May.

Farmer, R. E. A., D. F. Waggoner, and T. Zha (2011): “Minimal State Variable Solutions to Markov-Switching Rational Expectations Models,” *Journal of Economic Dynamics and Control*, 35(12), 2150–2166.

Komunjer, I., and S. Ng (2011): “Dynamic Identification of Dynamic Stochastic General Equilibrium Models,” *Econometrica*, 79(6), 1995–2032.

Leeper, E. M. (1991): “Equilibria Under ‘Active’ and ‘Passive’ Monetary and Fiscal Policies,” *Journal of Monetary Economics*, 27(1), 129–147.

Leeper, E. M., and C. Leith (2017): “Understanding Inflation as a Joint Monetary-Fiscal Phenomenon,” in *Handbook of Macroeconomics*, ed. by J. B. Taylor, and H. Uhlig, vol. 2B, pp. 2305–2416. Elsevier Press, Amsterdam.

Sims, C. A., and T. Zha (2006): “Were There Regime Switches in US Monetary Policy?,” *American Economic Review*, 96(1), 54–81.

Woodford, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton, NJ.