Online Appendix

The Value of a Cure: An Asset Pricing Perspective

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A Solution to the Regime-Switching Model with 2 States

We derive below the solution to the regime-switching model with just two states. The solution technique is then applied to solving the regime-switching model with S states.

A.1 Description of States & Regime Switch

State $s \in \{0, \underbrace{1}_{pandemic}\}$ The transition probabilities are $P(s_{t+dt} = 1 | s_t = 0) = \eta dt$ and $P(s_{t+dt} = 0 | s_t = 1) = \lambda dt$. Let $\mathbb{J}^{\text{Off}}$ and $\mathbb{J}^{\text{On}}$ be the two value functions. The HJBs for the two states are:

For the non-Pandemic state

$$0 = \max_{c} \left[f(C, V) + \mathbb{J}_q^{\text{Off}} \left(\bar{l}^{\alpha} \mu q - C \right) + \frac{1}{2} \mathbb{J}_{qq}^{\text{Off}} q^2 \bar{l}^{\alpha} \sigma^2 + \eta \left[\mathbb{J}^{\text{On}} - \mathbb{J}^{\text{Off}} \right] \right] \tag{A.1}$$

and for the Pandemic state

$$0 = \max_{c,l} \left[f(C, V) + \mathbb{J}_q^{\text{On}} \left(l^{\alpha} \mu q - C \right) + \frac{1}{2} \mathbb{J}_{qq}^{\text{On}} q^2 l^{\alpha} \sigma^2 + \zeta [\mathbb{J}^{\text{On}}(q[1 - \chi \Delta]) - \mathbb{J}^{\text{On}}(q)] + \lambda \left[\mathbb{J}^{\text{Off}} - \mathbb{J}^{\text{On}} \right] \right] \tag{A.2}$$

In the Pandemic state, optimal labor supply is

$$\frac{\alpha[\mu - \frac{1}{2}\gamma\sigma^2]}{\zeta \varepsilon \Delta} = l^{1-\alpha} [1 - (\varepsilon l + k + Kl)\Delta]^{-\gamma} \tag{A.3}$$

Define

$$\nu \equiv \left(\frac{\alpha[\mu - \frac{1}{2}\gamma\sigma^2]}{\zeta \varepsilon \Delta} \right)^{-\frac{1}{\gamma}} \tag{A.4}$$

Combine (A.3) and (A.4) to get:

$$\nu l^{* \frac{1-\alpha}{\gamma}} = [1 - (\varepsilon l + k + Kl)\Delta] \tag{A.5}$$

Or,

$$\chi(l) = \varepsilon l + k + Kl = \frac{1}{\Delta} \left[1 - \nu l^{\frac{1-\alpha}{\gamma}} \right] \tag{A.6}$$

Assume $\mathbb{J}^{\text{On}} = H^{\text{On}} \frac{q^{1-\gamma}}{1-\gamma}$ and $\mathbb{J}^{\text{Off}} = H^{\text{Off}} \frac{q^{1-\gamma}}{1-\gamma}$. Then,

$$C^s = \frac{(H^s)^{-\theta\psi} q}{\rho^{-\psi}} \tag{A.7}$$

A.2 Non-Pandemic state

$$0 = \max_c \left[f(C^*, \mathbb{J}^{\text{Off}}) + \mathbb{J}_q^{\text{Off}} \left(\bar{l}^\alpha \mu q - C^* \right) + \frac{1}{2} \mathbb{J}_{qq}^{\text{Off}} q^2 \bar{l}^\alpha \sigma^2 + \eta [\mathbb{J}^{\text{On}} - \mathbb{J}^{\text{Off}}] \right] \tag{A.8}$$

1. $f(C, \mathbb{J})$:

$$\begin{aligned} f(C^*, \mathbb{J}^{\text{Off}}) =& \frac{\rho}{1-\psi^{-1}} \frac{(C^*)^{1-\psi^{-1}} - \left((1-\gamma)\mathbb{J}^{\text{Off}}\right)^\theta}{\left((1-\gamma)\mathbb{J}^{\text{Off}}\right)^{\theta-1}} \\ =& \frac{\rho}{1-\psi^{-1}} \frac{(C^*)^{1-\psi^{-1}} - \left((1-\gamma)\mathbb{J}^{\text{Off}}\right)^\theta}{\left((1-\gamma)\mathbb{J}^{\text{Off}}\right)^{\theta-1}} \\ =& \frac{\rho^\psi}{1-\psi^{-1}} (H^{\text{Off}})^{1-\theta\psi} q^{1-\gamma} - \frac{\rho}{1-\psi^{-1}} H^{\text{Off}} q^{1-\gamma} \end{aligned} \tag{A.9}$$

2. $\mathbb{J}_q^{\text{Off}} \left(\bar{l}^\alpha \mu q - C^* \right)$:

$$H^{\text{Off}} \left(\bar{l}^\alpha \mu - \frac{(H^{\text{Off}})^{-\theta\psi}}{\rho^{-\psi}} \right) q^{1-\gamma} \tag{A.10}$$

3. $\frac{1}{2} \mathbb{J}_{qq}^{\text{Off}} q^2 \bar{l}^\alpha \sigma^2$:

$$-\frac{1}{2} \gamma H^{\text{Off}} \bar{l}^\alpha \sigma^2 q^{1-\gamma} \tag{A.11}$$

4. $\eta \left[\mathbb{J}^{\text{On}} - \mathbb{J}^{\text{Off}} \right]$:

$$\eta [H^{\text{On}} - H^{\text{Off}}] \frac{q^{1-\gamma}}{1-\gamma} \tag{A.12}$$

HJB simplifies to

$$
\begin{aligned}
0 =& \frac{\rho^{\psi}}{1-\psi^{-1}}\left(H^{\text{Off}}\right)^{1-\theta\psi} q^{1-\gamma} - \frac{\rho}{1-\psi^{-1}} H^{\text{Off}} q^{1-\gamma} + H^{\text{Off}}\left(\bar{l}^{\alpha}\mu - \frac{(H^{\text{Off}})^{-\theta\psi}}{\rho^{-\psi}}\right) q^{1-\gamma} \\
&- \frac{1}{2}\gamma H^{\text{Off}} \bar{l}^{\alpha}\sigma^{2} q^{1-\gamma} + \eta\left[H^{\text{On}} - H^{\text{Off}}\right]\frac{q^{1-\gamma}}{1-\gamma}
\end{aligned} \tag{A.13}
$$

Cancelling out $q^{1-\gamma}$

$$
\begin{aligned}
0 =& \frac{\rho^{\psi}}{1-\psi^{-1}}(H^{\text{Off}})^{1-\theta\psi} - \frac{\rho}{1-\psi^{-1}} H^{\text{Off}} + H^{\text{Off}}\left(\bar{l}^{\alpha}\mu - \frac{(H^{\text{Off}})^{-\theta\psi}}{\rho^{-\psi}}\right) \\
&- \frac{1}{2}\gamma H^{\text{Off}} \bar{l}^{\alpha}\sigma^{2} + \eta\left[H^{\text{On}} - H^{\text{Off}}\right]\frac{1}{1-\gamma}
\end{aligned} \tag{A.14}
$$

Dividing by H^{Off}, we get:

$$
0 = \frac{\rho^{\psi}}{1-\psi^{-1}}(H^{\text{Off}})^{-\theta\psi} - \frac{\rho}{1-\psi^{-1}} + \left(\bar{l}^{\alpha}\mu - \frac{(H^{\text{Off}})^{-\theta\psi}}{\rho^{-\psi}}\right) - \frac{1}{2}\gamma\bar{l}^{\alpha}\sigma^{2} + \eta\left[\frac{H^{\text{On}}}{H^{\text{Off}}} - 1\right]\frac{1}{1-\gamma} \tag{A.15}
$$

$$
= \frac{\rho^{\psi}\psi^{-1}}{1-\psi^{-1}}(H^{\text{Off}})^{-\theta\psi} - \frac{\rho}{1-\psi^{-1}} + \bar{l}^{\alpha}\mu - \frac{1}{2}\gamma\bar{l}^{\alpha}\sigma^{2} + \eta\left[\frac{H^{\text{On}}}{H^{\text{Off}}} - 1\right]\frac{1}{1-\gamma} \tag{A.16}
$$

$$
= \frac{\rho^{\psi}\psi^{-1}}{1-\psi^{-1}}(H^{\text{Off}})^{-\theta\psi} - \frac{\rho}{1-\psi^{-1}} + \bar{l}^{\alpha}\mu - \frac{1}{2}\gamma\bar{l}^{\alpha}\sigma^{2} + \eta\left[\frac{H^{\text{On}}}{H^{\text{Off}}} - 1\right]\frac{1}{1-\gamma} \tag{A.17}
$$

$$
= \frac{\rho^{\psi}}{\psi-1}(H^{\text{Off}})^{-\theta\psi}(1-\gamma) - \frac{\rho(1-\gamma)}{1-\psi^{-1}} + (1-\gamma)\bar{l}^{\alpha}\left(\mu - \frac{1}{2}\gamma\sigma^{2}\right) + \eta\left[\frac{H^{\text{On}}}{H^{\text{Off}}} - 1\right] \tag{A.18}
$$

Define

$$
g(\bar{l},0) \equiv \frac{\rho(1-\gamma)}{(1-\psi^{-1})} - \bar{l}^{\alpha}(1-\gamma)(\mu - \frac{1}{2}\gamma\sigma^{2}) \tag{A.19}
$$

Then, (A.18) can be written as:

$$
0 = \frac{\rho^{\psi}}{\psi-1}(H^{\text{Off}})^{-\theta\psi}(1-\gamma) - g(\bar{l},0) + \eta\left[\frac{H^{\text{On}}}{H^{\text{Off}}} - 1\right] \tag{A.20}
$$

Rearranging

$$
H^{\text{Off}} = \left(\frac{g(\bar{l},0) - \eta\left[\frac{H^{\text{On}}}{H^{\text{Off}}} - 1\right]}{(1-\gamma)\frac{\rho^{\psi}}{\psi-1}}\right)^{-\frac{1}{\theta\psi}} \tag{A.21}
$$

Define

$$1+\delta \equiv \frac{H^{\text{On}}}{H^{\text{Off}}} \tag{A.22}$$

Then, we get:

$$H^{\text{Off}} = \left(\frac{g(\bar{l},0)-\eta\delta}{(1-\gamma)\frac{\rho^{\psi}}{\psi-1}}\right)^{-\frac{1}{\theta\psi}} \tag{A.23}$$

A.3 Pandemic state

$$0 = \max_{C,l}\left[f(C^*,\mathbb{J}^{\text{On}}) + \mathbb{J}^{\text{On}}_q\left(l^{\alpha}\mu q - C^*\right) + \frac{1}{2}\mathbb{J}^{\text{On}}_{qq} q^2 l^{\alpha}\sigma^2 + \zeta\left[\mathbb{J}^{\text{On}}(q[1-\chi\Delta]) - \mathbb{J}^{\text{On}}(q)\right] + \lambda\left[\mathbb{J}^{\text{Off}} - \mathbb{J}^{\text{On}}\right]\right] \tag{A.24}$$

1. $f(C,\mathbb{J})$:

$$\begin{aligned} f(C^*,\mathbb{J}^{\text{On}}) =& \frac{\rho}{1-\psi^{-1}}\frac{(C^*)^{1-\psi^{-1}} - \left((1-\gamma)\mathbb{J}^{\text{On}}\right)^{\theta}}{\left((1-\gamma)\mathbb{J}^{\text{On}}\right)^{\theta-1}} \\ =& \frac{\rho}{1-\psi^{-1}}\frac{(C^*)^{1-\psi^{-1}} - \left((1-\gamma)\mathbb{J}^{\text{On}}\right)^{\theta}}{\left((1-\gamma)\mathbb{J}^{\text{On}}\right)^{\theta-1}} \\ =& \frac{\rho^{\psi}}{1-\psi^{-1}}(H^{\text{On}})^{1-\theta\psi}q^{1-\gamma} - \frac{\rho}{1-\psi^{-1}}H^{\text{On}}q^{1-\gamma} \end{aligned} \tag{A.25}$$

2. $\mathbb{J}^{\text{On}}_q\left(l^{\alpha}\mu q - C^*\right)$:

$$H^{\text{On}}\left(l^{\alpha}\mu - \frac{(H^{\text{On}})^{-\theta\psi}}{\rho^{-\psi}}\right)q^{1-\gamma} \tag{A.26}$$

3. $\frac{1}{2}\mathbb{J}^{\text{On}}_{qq}q^2\bar{l}^{\alpha}\sigma^2$:

$$-\frac{1}{2}\gamma H^{\text{On}} l^{\alpha}\sigma^2 q^{1-\gamma} \tag{A.27}$$

4. $\zeta[\mathbb{J}^{\text{On}}(q[1-\chi\Delta]) - \mathbb{J}^{\text{On}}(q)]$

$$\zeta\left[H^{\text{On}}(q[1-\chi\Delta]) - H^{\text{On}}(q)\right]q^{1-\gamma}\frac{1}{1-\gamma} \tag{A.28}$$

5. $\lambda \left[\mathbb{J}^{\text{Off}} - \mathbb{J}^{\text{On}} \right]$:

$$\lambda \left[H^{\text{Off}} - H^{\text{On}} \right] \frac{q^{1-\gamma}}{1-\gamma} \tag{A.29}$$

HJB simplifies to

$$\begin{aligned} 0 = & \frac{\rho^{\psi}}{1-\psi^{-1}} (H^{\text{On}})^{1-\theta\psi} q^{1-\gamma} - \frac{\rho}{1-\psi^{-1}} H^{\text{On}} q^{1-\gamma} + H^{\text{On}} \left(\bar{l}^{\alpha} \mu - \frac{(H^{\text{On}})^{-\theta\psi}}{\rho^{-\psi}} \right) q^{1-\gamma} \\ & - \frac{1}{2} \gamma H^{\text{On}} l^{\alpha} \sigma^2 q^{1-\gamma} + \zeta [H^{\text{On}}([1-\chi\Delta])^{1-\gamma} - H^{\text{On}}] q^{1-\gamma} \frac{1}{1-\gamma} + \lambda \left[H^{\text{Off}} - H^{\text{On}} \right] \frac{q^{1-\gamma}}{1-\gamma} \end{aligned} \tag{A.30}$$

Cancelling out $q^{1-\gamma}$

$$\begin{aligned} 0 = & \frac{\rho^{\psi}}{1-\psi^{-1}} (H^{\text{On}})^{1-\theta\psi} - \frac{\rho}{1-\psi^{-1}} H^{\text{On}} + H^{\text{On}} \left(\bar{l}^{\alpha} \mu - \frac{(H^{\text{On}})^{-\theta\psi}}{\rho^{-\psi}} \right) \\ & - \frac{1}{2} \gamma H^{\text{On}} l^{\alpha} \sigma^2 + \zeta [H^{\text{On}}([1-\chi\Delta]^{1-\gamma} - 1)] \frac{1}{1-\gamma} + \lambda \left[H^{\text{Off}} - H^{\text{On}} \right] \frac{1}{1-\gamma} \end{aligned} \tag{A.31}$$

Dividing by H^{On}, we get:

$$0 = \frac{\rho^{\psi}}{1-\psi^{-1}}(H^{\text{On}})^{-\theta\psi} - \frac{\rho}{1-\psi^{-1}} + \left(\bar{l}^{\alpha}\mu - \frac{(H^{\text{On}})^{-\theta\psi}}{\rho^{-\psi}}\right) - \frac{1}{2}\gamma l^{\alpha}\sigma^2 + \zeta[([1-\chi\Delta]^{1-\gamma}) - 1]\frac{1}{1-\gamma} + \lambda\left[\frac{H^{\text{Off}}}{H^{\text{On}}} - 1\right]\frac{1}{1-\gamma} \tag{A.32}$$

$$= \frac{\rho^{\psi}}{1-\psi^{-1}}(H^{\text{On}})^{-\theta\psi} - \frac{\rho}{1-\psi^{-1}} + \left(\bar{l}^{\alpha}\mu - \frac{H^{\text{On}-\theta\psi}}{\rho^{-\psi}}\right) - \frac{1}{2}\gamma l^{\alpha}\sigma^2 + \zeta[([1-\chi\Delta]^{1-\gamma}) - 1]\frac{1}{1-\gamma} + \lambda\left[\frac{H^{\text{Off}}}{H^{\text{On}}} - 1\right]\frac{1}{1-\gamma} \tag{A.33}$$

$$= \frac{\rho^{\psi}\psi^{-1}}{1-\psi^{-1}}(H^{\text{On}})^{-\theta\psi} - \frac{\rho}{1-\psi^{-1}} + \bar{l}^{\alpha}\mu - \frac{1}{2}\gamma l^{\alpha}\sigma^2 + \zeta[([1-\chi\Delta]^{1-\gamma}) - 1]\frac{1}{1-\gamma} + \lambda\left[\frac{H^{\text{Off}}}{H^{\text{On}}} - 1\right]\frac{1}{1-\gamma} \tag{A.34}$$

$$= \frac{\rho^{\psi}\psi^{-1}}{1-\psi^{-1}}(H^{\text{On}})^{-\theta\psi}(1-\gamma) - \frac{\rho(1-\gamma)}{1-\psi^{-1}} + \left[\bar{l}^{\alpha}\mu - \frac{1}{2}\gamma l^{\alpha}\sigma^2 + \zeta[([1-\chi\Delta]^{1-\gamma}) - 1]\frac{1}{1-\gamma}\right](1-\gamma) + \lambda\left[\frac{H^{\text{Off}}}{H^{\text{On}}} - 1\right] \tag{A.35}$$

$$= \frac{\rho^{\psi}}{\psi-1}(H^{\text{On}})^{-\theta\psi}(1-\gamma) - \frac{\rho(1-\gamma)}{1-\psi^{-1}} + (1-\gamma)\bar{l}^{\alpha}\left(\mu - \frac{1}{2}\gamma\sigma^2\right) + \zeta[([1-\chi\Delta]^{1-\gamma}) - 1] + \lambda\left[\frac{H^{\text{Off}}}{H^{\text{On}}} - 1\right] \tag{A.36}$$

Define

$$g(l,\zeta) \equiv \frac{\rho(1-\gamma)}{(1-\psi^{-1})} - l^{\alpha}(1-\gamma)(\mu - \frac{1}{2}\gamma\sigma^2) - \zeta[([1-\chi(l)\Delta]^{1-\gamma}) - 1] \tag{A.37}$$

Then, (A.24) can be written as:

$$0 = \frac{\rho^{\psi}}{\psi-1}(H^{\text{On}})^{-\theta\psi}(1-\gamma) - g(l,\zeta) + \lambda\left[\frac{H^{\text{Off}}}{H^{\text{On}}} - 1\right] \tag{A.38}$$

Rearranging

$$H^{\text{On}} = \left(\frac{g(l,\zeta) - \lambda\left[\frac{H^{\text{Off}}}{H^{\text{On}}} - 1\right]}{(1-\gamma)\frac{\rho^{\psi}}{\psi-1}}\right)^{-\frac{1}{\theta\psi}} \tag{A.39}$$

Using the definition of δ

$$H^{\text{On}} = \left(\frac{g(l,\zeta) + \lambda \frac{\delta}{1+\delta}}{(1-\gamma)\frac{\rho^{\psi}}{\psi-1}} \right)^{-\frac{1}{\theta\psi}} \tag{A.40}$$

We can solve for δ from (A.23) and (A.40).