Online Appendix, Not for Publication

A Decentralizing the Labor Supply Decision in the Household Problem

This Section shows that the labor supply decision solving (6) subject to (7) and (5) can be decentralized to individual workers solving equations (5) and (9).
The Lagrangian of problem (6), (7) and (5) is

$$\mathcal{L} = E_0 \left\{ \begin{array}{c} \sum_{t=0}^{\infty} \left[\delta^t \phi^t \left(\overline{L} u \left(c_\ell^t \right) - \widetilde{\lambda}^t \left(\overline{L} P^{F,t} c_\ell^t + B^{t+1} - \Pi^t - R^t B^t \right) \right) \right] + \\ \int_0^{\overline{L}} \left[\sum_{t=0}^{\infty} \left[\delta^t \phi^t \left(\begin{array}{c} \left(1 - e_\ell^t\right) \left(-C_{k_\ell^t, k_\ell^{t+1}} + \omega_{\ell, k_\ell^{t+1}}^t + b_{k_\ell^{t+1}} \right) + \\ e_\ell^t \eta_{k_\ell^t} + \widetilde{\lambda}^t \left(\sum_{k=1}^{K} \mathcal{I} \left(k_\ell^{t+1} = k \right) e_\ell^t w_k^t \left(x_\ell^t \right) \right) \end{array} \right) \right] \right] d\ell \end{array} \right\} \tag{A.1}$$

Because each worker is infinitesimal, and the allocation of one worker does not interfere with the allocation/utility of other individual workers (conditional on aggregates), maximizing

$$\int_0^{\overline{L}} \left[\sum_{t=0}^{\infty} \delta^t \phi^t \left(\begin{array}{c} \left(1 - e_\ell^t\right) \left(-C_{k_\ell^t, k_\ell^{t+1}} + \omega_{\ell, k_\ell^{t+1}}^t + b_{k_\ell^{t+1}} \right) + \\ e_\ell^t \eta_{k_\ell^t} + \widetilde{\lambda}^t \left(\sum_{k=1}^{K} \mathcal{I} \left(k_\ell^{t+1} = k \right) e_\ell^t w_k^t \left(x_\ell^t \right) \right) \end{array} \right) \right] d\ell \tag{A.2}$$

means maximizing each individual term. Therefore, the planner solves, for each individual, the recursive problem:

$$\mathcal{L}_W^t \left(k_\ell^t, e_\ell^t, x_\ell^t, \omega_\ell^t \right) = \max_{k_\ell^{t+1}, \widetilde{e}_k^{t+1}(.)} \left\{ \begin{array}{c} \left(1 - e_\ell^t\right) \left(-C_{k_\ell^t, k_\ell^{t+1}} + \omega_{\ell, k_\ell^{t+1}}^t + b_{k_\ell^{t+1}} \right) + e_\ell^t \eta_{k_\ell^t} \\ + \widetilde{\lambda}_t \sum_{k=1}^{K} \mathcal{I} \left(k_\ell^t = k \right) e_\ell^t w_k^t \left(x_\ell^t \right) + \\ \delta \widehat{\phi}^{t+1} E_t \mathcal{L}_W^{t+1} \left(k_\ell^{t+1}, e_\ell^{t+1}, x_\ell^{t+1}, \omega_\ell^{t+1} \right) \end{array} \right\}, \tag{A.3}$$

where $\widehat{\phi}^{t+1} \equiv \frac{\phi^{t+1}}{\phi^t}$.

Denote by $\mathcal{F}^t$ the set of information at t. So, $E_t(.) = E\left(.|\mathcal{F}^t\right)$. For an unemployed worker in sector k at time t, $k_\ell^t = k$, $e_\ell^t = 0$:

$$\mathcal{L}_W^t \left(k_\ell^t = k, e_\ell^t = 0, x_\ell^t, \omega_\ell^t \right) = \max_{k', \left\{ \widetilde{e}_k^{t+1}(.) \right\}} - C_{kk'} + \omega_{\ell,k'}^t + b_{k'} + \delta \widehat{\phi}^{t+1} E_t \mathcal{L}_W^{t+1} \left(k', e_\ell^{t+1}, x_\ell^{t+1}, \omega_\ell^{t+1} \right). \tag{A.4}$$

Using the law of iterated expectations we obtain:

$$
\begin{aligned}
&\mathcal{L}_W^t\left(k_\ell^t=k, e_\ell^t=0, x_\ell^t, \omega_\ell^t\right)=\max_{k', \left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'}+\omega_{\ell,k'}^t+b_{k'} \\
&+\delta\widehat{\phi}^{t+1} E\left\{E\left[\mathcal{L}_W^{t+1}\left(k', 1, x_\ell^{t+1}, \omega_\ell^{t+1}\right) | x_\ell^{t+1}, \mathcal{F}^t\right] \times \Pr\left(k_\ell^{t+1}=k', e_\ell^{t+1}=1 | x_\ell^{t+1}, \mathcal{F}^t\right) | \mathcal{F}^t\right\} \\
&+\delta\widehat{\phi}^{t+1} E\left\{E\left[\mathcal{L}_W^{t+1}\left(k', 0, x_\ell^{t+1}, \omega_\ell^{t+1}\right) | x_\ell^{t+1}, \mathcal{F}^t\right] \times \Pr\left(k_\ell^{t+1}=k', e_\ell^{t+1}=0 | x_\ell^{t+1}, \mathcal{F}^t\right) | \mathcal{F}^t\right\} \\
&=\max_{k', \left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'}+\omega_{\ell,k'}^t+b_{k'} \\
&+\delta\widehat{\phi}^{t+1}\theta_{k'}^t q\left(\theta_{k'}^t\right) E\left\{\mathcal{L}_W^{t+1}\left(k', 1, x_{\ell'}^{t+1}, \omega_\ell^{t+1}\right) \widetilde{e}_{k'}^{t+1}\left(x_\ell^{t+1}\right) | \mathcal{F}^t\right\} \\
&+\delta\widehat{\phi}^{t+1} E\left\{\left(1-\theta_k^t q\left(\theta_{k'}^t\right) \widetilde{e}_{k'}^{t+1}\left(x_\ell^{t+1}\right)\right) \mathcal{L}_W^{t+1}\left(s', 0, x_\ell^{t+1}, \omega_\ell^{t+1}\right) | \mathcal{F}^t\right\} \qquad \text{(A.5)}
\end{aligned}
$$

For an employed worker in sector k, $k_\ell^t = k$, $e_\ell^t = 1$:

$$
\begin{aligned}
\mathcal{L}_W^t\left(k_\ell^t=k, e_\ell^t=1, x_\ell^t, \omega_\ell^t\right) &= \max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t\left(x_\ell^t\right)+\eta_k+\delta\widehat{\phi}^{t+1} E_t \mathcal{L}_W^{t+1}\left(k, e_\ell^{t+1}, x_\ell^t, \omega_\ell^{t+1}\right) \\
&= \max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t\left(x_\ell^t\right)+\eta_k \\
&+\delta\widehat{\phi}^{t+1} E\left\{\begin{array}{c} E\left[\mathcal{L}_W^{t+1}\left(k, 1, x_\ell^t, \omega_\ell^{t+1}\right) | k_\ell^{t+1}=k, , e_\ell^{t+1}=1, x_\ell^{t+1}, \mathcal{F}^t\right] \times \\ \Pr\left(k_\ell^{t+1}=k, e_\ell^{t+1}=1 | x_\ell^{t+1}, \mathcal{F}^t\right) | \mathcal{F}^t \end{array}\right\} \\
&+\delta\widehat{\phi}^{t+1} E\left\{\begin{array}{c} E\left[\mathcal{L}_W^{t+1}\left(k, 0, x_\ell^t, \omega_\ell^{t+1}\right) | k_\ell^{t+1}=k, e_\ell^{t+1}=0, x_\ell^{t+1}, \mathcal{F}^t\right] \times \\ \Pr\left(k_\ell^{t+1}=k, e_\ell^{t+1}=0 | x_\ell^{t+1}, \mathcal{F}^t\right) | \mathcal{F}^t \end{array}\right\} \\
&= \max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t\left(x_\ell^t\right)+\eta_k \\
&+\delta\widehat{\phi}^{t+1}\left(1-\chi_k\right) E\left[\begin{array}{c} \widetilde{e}_k^{t+1}\left(x_\ell^t\right) \mathcal{L}_W^{t+1}\left(k, 1, x_\ell^t, \omega_\ell^{t+1}\right) \\ +\left(1-\widetilde{e}_k^{t+1}\left(x_\ell^t\right)\right) \mathcal{L}_W^{t+1}\left(k, 0, x_\ell^t, \omega_\ell^{t+1}\right) | \mathcal{F}^t \end{array}\right] \\
&+\delta\widehat{\phi}^{t+1}\chi_k E\left[\mathcal{L}_W^{t+1}\left(k, 0, x_\ell^t, \omega_\ell^{t+1}\right) | \mathcal{F}^t\right] \qquad \text{(A.6)}
\end{aligned}
$$

Make the following definitions

$$
\begin{aligned}
\widetilde{U}_k^t\left(\omega_\ell^t\right) &\equiv \mathcal{L}_W^t\left(k_\ell^t=k, e_\ell^t=0, x_\ell^t, \omega_\ell^t\right), \text{ and} \\
W_k^t(x) &\equiv \mathcal{L}_W^t\left(k_\ell^t=k, e_\ell^t=1, x, \omega_\ell^t\right). \qquad \text{(A.7)}
\end{aligned}
$$

$\widetilde{U}_k^t\left(\omega_\ell^t\right)$ is the value of unemployment in sector k, conditional on the preference shocks ω_ℓ^t, and $W_k^t(x)$ is the value of a job with match productivity x. Note that $\mathcal{L}_W^t\left(k_\ell^t=k, e_\ell^t=0, x_\ell^t, \omega_\ell^t\right)$ does not depend on x_ℓ^t and $\mathcal{L}_W^t\left(k_\ell^t=k, e_\ell^t=1, x, \omega_\ell^t\right)$ does not depend on ω_ℓ^t. Rewrite $\widetilde{U}_k^t\left(\omega_\ell^t\right)$ as

$$
\begin{aligned}
\widetilde{U}_k^t\left(\omega_\ell^t\right) &= \max_{k', \left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'}+\omega_{\ell,k'}^t+b_{k'} \\
&+\delta\widehat{\phi}^{t+1}\theta_{k'}^t q\left(\theta_{k'}^t\right) \int W_{k'}^{t+1}(x)\, \widetilde{e}_{k'}^{t+1}(x)\, dG_{k'}(x) \\
&+\delta\widehat{\phi}^{t+1}\left(1-\theta_{k'}^t q\left(\theta_{k'}^t\right) \Pr\left(\widetilde{e}_{k'}^{t+1}\left(x_\ell^{t+1}\right)=1\right)\right) E_\omega\left(\widetilde{U}_{k'}^{t+1}\left(\omega_\ell^{t+1}\right)\right), \qquad \text{(A.8)}
\end{aligned}
$$

and so:

$$\widetilde{U}_k^t\left(\omega_\ell^t\right) = \max_{k',\left\{\widetilde{e}_k^{t+1}(.)\right\}} - C_{kk'} + \omega_{\ell,k'}^t + b_{k'}$$
$$+ \delta\widehat{\phi}^{t+1}\theta_{k'}^t q\left(\theta_{k'}^t\right)\int\left(\begin{array}{c} W_{k'}^{t+1}(x)\,\widetilde{e}_{k'}^{t+1}(x) + \\ E_\omega\left(\widetilde{U}_{k'}^{t+1}\left(\omega_\ell^{t+1}\right)\right)\left(1-\widetilde{e}_{k'}^{t+1}(x)\right)\end{array}\right)dG_{k'}(x)$$
$$+ \delta\widehat{\phi}^{t+1}\left(1-\theta_{k'}^t q\left(\theta_{k'}^t\right)\right)E_\omega\left(\widetilde{U}_{k'}^{t+1}\left(\omega_\ell^{t+1}\right)\right). \tag{A.9}$$

Now, wewrite $W_k^t(x)$ as:

$$W_k^t(x) = \max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t(x) + \eta_k$$
$$+ \delta\widehat{\phi}^{t+1}\left(1-\chi_k\right)\widetilde{e}_k^{t+1}(x)\,W_k^{t+1}(x)$$
$$+ \delta\widehat{\phi}^{t+1}\left(1-\left(1-\chi_k\right)\widetilde{e}_k^{t+1}(x)\right)E\left(\widetilde{U}_k^{t+1}\left(\omega_\ell^{t+1}\right)\right), \tag{A.10}$$

and so

$$W_k^t(x) = \max_{\left\{\widetilde{e}_k^{t+1}(.)\right\}} \widetilde{\lambda}^t w_k^t\left(x_\ell^t\right) + \eta_k$$
$$+ \delta\widehat{\phi}^{t+1}\left(1-\chi_k\right)\left(\widetilde{e}_k^{t+1}(x)\,W_k^{t+1}(x) + \left(1-\widetilde{e}_k^{t+1}(x)\right)E_\omega\left(\widetilde{U}_k^{t+1}\left(\omega_\ell^{t+1}\right)\right)\right)$$
$$+ \delta\widehat{\phi}^{t+1}\chi_k E_\omega\left(\widetilde{U}_k^{t+1}\left(\omega_\ell^{t+1}\right)\right). \tag{A.11}$$

It is now clear that the optimal policy $\widetilde{e}_k^{t+1}(.)$ is:

$$\widetilde{e}_k^{t+1}(x) = \left\{\begin{array}{c} 1 \text{ if } W_k^{t+1}(x) > E_\omega\left(\widetilde{U}_k^{t+1}\left(\omega_\ell^{t+1}\right)\right) \\ 0 \text{ otherwise}\end{array}\right\}. \tag{A.12}$$

Define $U_k^t \equiv E_\omega\left(\widetilde{U}_k^t\left(\omega_\ell^t\right)\right)$. We therefore have the following Bellman equations:

$$U_k^t = E_\omega\left(\begin{array}{c} \max_{k'} - C_{kk'} + b_{k'} + \omega_{\ell,k'}^t \\ +\delta\widehat{\phi}^{t+1}\theta_{k'}^t q\left(\theta_{k'}^t\right)\int \max\left\{W_{k'}^{t+1}(x), U_{k'}^{t+1}\right\}dG_{k'}(x) \\ +\delta\widehat{\phi}^{t+1}\left(1-\theta_{k'}^t q\left(\theta_{k'}^t\right)\right)U_{k'}^{t+1}\end{array}\right) \tag{A.13}$$

$$W_k^t(x) = \widetilde{\lambda}^t w_k^t(x) + \eta_k + \delta\widehat{\phi}^{t+1}\left(1-\chi_k\right)\left(\max\left\{W_k^{t+1}(x), U_k^{t+1}\right\}\right) + \delta\widehat{\phi}^{t+1}\chi_k U_k^{t+1} \tag{A.14}$$

B Steady State Equilibrium

In this section we derive the equations characterizing the steady state equilibrium. The key conditions that we impose is that variables are constant over time, inflows of workers into each sector equal outflows, and job destruction rates equal job creation rates. We also impose that the preference shifters $\left\{\phi_i^t\right\}$ are constant and equal to 1 in the long run.

Wage Equation

$$w_{k,i}(x) = \beta_{k,i}\widetilde{w}_{k,i}x + \frac{(1-\beta_{k,i})(1-\delta)U_{k,i} - (1-\beta_{k,i})\eta_{k,i}}{\widetilde{\lambda}_i} \tag{A.15}$$

Firms' value function

$$J_{k,i}(x) = \frac{1-\beta_{k,i}}{1-(1-\chi_{k,i})\delta}\widetilde{\lambda}_i\widetilde{w}_{k,i}\left(x - \underline{x}_{k,i}\right) \tag{A.16}$$

Probability of filling a vacancy

$$q_i(\theta_{k,i}) = \frac{\kappa_{k,i}P_i^F}{\widetilde{w}_{k,i}} \times \frac{1-\delta(1-\chi_{k,i})}{\delta(1-\beta_{k,i})I_{k,i}\left(\underline{x}_{k,i}\right)} \tag{A.17}$$

where

$$I_{k,i}\left(\underline{x}_{k,i}\right) \equiv \int_{\underline{x}_{k,i}}^{x_{\max}} \left(s - \underline{x}_{k,i}\right) dG_{k,i}(s) \tag{A.18}$$

Unemployed workers' Bellman equation

$$U_{k,i} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}\frac{\kappa_{k',i}P_i^F}{\widetilde{w}_{k',i}} \times \widetilde{\lambda}_i\widetilde{w}_{k',i}\frac{\beta_{k',i}}{\left(1-\beta_{k',i}\right)} + \delta U_{k',i}}{\zeta_i}\right\}\right) \tag{A.19}$$

Transition rates

$$s_{k\ell,i} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}\frac{\kappa_{\ell,i}P_i^F}{\widetilde{w}_{\ell,i}}\times\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{\left(1-\beta_{\ell,i}\right)}+\delta U_{\ell,i}}{\zeta_i}\right\}}{\sum_{\overline{k}}\exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta_{\overline{k},i}\frac{\kappa_{\overline{k},i}P_i^F}{\widetilde{w}_{\overline{k},i}}\times\widetilde{\lambda}_i\widetilde{w}_{\overline{k},i}\frac{\beta_{\overline{k},i}}{\left(1-\beta_{\overline{k},i}\right)}+\delta U_{\overline{k},i}}{\zeta_i}\right\}} \tag{A.20}$$

Steady-state unemployment rates

$$u_{k,i} = \frac{\chi_{k,i}}{\theta_{k,i}q_i(\theta_{k,i})\left(1-G_{k,i}\left(\underline{x}_{k,i}\right)\right)+\chi_{k,i}} \tag{A.21}$$

Trade + Price System
Input Bundle Price

$$P^M_{k,i} = \prod_{\ell=1}^{K} \left(\frac{P^I_{\ell,i}}{\nu_{k\ell,i}} \right)^{\nu_{k\ell,i}} \tag{A.22}$$

Domestic Sectoral Output Price

$$c_{k,i} = \left(\frac{\widetilde{w}_{k,i}}{\gamma_{k,i}} \right)^{\gamma_{k,i}} \left(\frac{P^M_{k,i}}{1-\gamma_{k,i}} \right)^{1-\gamma_{k,i}} \tag{A.23}$$

Price of Composite Sector-Specific Intermediate Good

$$P^I_{k,i} = \Gamma_{k,i} \left[\sum_{j=1}^{N} \frac{A_{k,j}}{(c_{k,j} d_{k,ji})^{\lambda}} \right]^{-1/\lambda} \tag{A.24}$$

where $\Gamma_{k,i}$ is a sector and country specific constant.

Price of Final Consumption Good

$$P^F_i = \prod_{k=1}^{K} \left(\frac{P^I_{k,i}}{\mu_{k,i}} \right)^{\mu_{ki}} \tag{A.25}$$

Trade Shares

$$\pi_{k,oi} = \frac{A_{k,o} \left(c_{k,o} d_{k,oi}\right)^{-\lambda}}{\Phi_{k,i}}, \tag{A.26}$$

where

$$\Phi_{k,i} = \sum_{o=1}^{N} A_{k,o} \left(c_{k,o} d_{k,oi}\right)^{-\lambda}. \tag{A.27}$$

Zero net flows condition

$$\left(L_i.u_i\right) = \left(\sum_{\ell=1}^{K} s_{\ell k,i} L_{\ell,i} u_{\ell,i} \right)_{k=1}^{K} = s'_i \left(L_i.u_i\right) \tag{A.28}$$

Product market clearing
Gross Output

$$\gamma_{k,o} Y_{k,o} = \widetilde{w}_{k,o} L_{k,o} \left(1-u_{k,o}\right) \int_{\underline{x}_{k,o}}^{x_{\max}} \frac{s}{1-G_{k,i}\left(\underline{x}_{k,o}\right)} dG_{k,o}\left(s\right) \tag{A.29}$$

$$= \widetilde{w}_{k,o} \widetilde{L}_{k,o} \tag{A.30}$$

Expenditure with Vacancies

$$E^V_{k,o} = \kappa_{k,o} P^F_o \theta_{k,o} u_{k,o} L_{k,o} \tag{A.31}$$

Market Clearing System

$$Y_{k,o} = \sum_{i=1}^{N} \pi_{k,oi} E_{k,i} \tag{A.32}$$

$$E_{k,i} = \mu_{k,i} \left(\sum_{\ell=1}^{K} \gamma_{\ell,i} Y_{\ell,i} \right) + \sum_{\ell=1}^{K} (1 - \gamma_{\ell,i}) \, \nu_{\ell k,i} Y_{\ell,i} - \mu_{k,i} NX_i \tag{A.33}$$

Normalization: World total revenue is the numeraire

$$\sum_{i=1}^{N} \sum_{k=1}^{K} Y_{k,i} = 1 \tag{A.34}$$

Final Good Consumption Expenditure

$$E_i^C = \sum_{k=1}^{K} \gamma_{k,i} Y_{k,i} - \sum_{k=1}^{K} E_{k,i}^V - NX_i \tag{A.35}$$

Lagrange multipliers

$$\widetilde{\lambda}_i = \frac{\overline{L}_i}{E_i^C} \tag{A.36}$$

C Solution Methods

This Section presents the different algorithms we developped to estimate the model and to perform counterfactual simulations. Section C.1 details the estimation algorithm and Section C.2 obtains expressions for simulated moments. Section C.3 outlines an exact hat alebra algorithm to compute changes in the steady state equilibrium in response to shocks in trade costs, productivities or net exports. Section C.4 develops the algorithm solving for the transition path of our complete model with trade imbalances. Section C.5 adapts this algorithm to the case where we have exogenous deficits. Finally, Section C.6 outlines the procedure we use in Section 5.1 to extract the shocks in trade costs, productivities and inter-temporal shocks.

C.1 Estimation Algorithm

Define $I_{k,i}(x) \equiv \int_x^{x_{\max}} (s-x)\, dG_{k,i}(s)$. Imposing $G_{k,i} \sim \log \mathcal{N}\left(0, \sigma_{k,i}^2\right)$ and a bit of algebra leads to:

- $G_{k,i}(x) = \Phi\left(\frac{\ln x}{\sigma_{k,i}}\right)$
- $I_{k,i}(x) = \exp\left(\frac{\sigma_{k,i}^2}{2}\right)\Phi\left(\sigma_{k,i} - \frac{\ln x}{\sigma_{k,i}}\right) - x\Phi\left(-\frac{\ln x}{\sigma_{k,i}}\right)$
- $I_{k,i}(0) = \exp\left(\frac{\sigma_{k,i}^2}{2}\right)$
- $\int_{\underline{x}_{k,i}}^{x_{\max}} \frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}\right)} dG_{k,i}(s) = \exp\left(\frac{\sigma_{k,i}^2}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}$

Note: The estimation procedure we describe takes trade shares $\pi_{k,oi}^{Data}$ and net exports NX_i^{Data} as given.

Step 1: Solve for $\{Y_{k,i}\}$ using:

$$Y_{k,o} = \sum_{i=1}^{N}\sum_{\ell=1}^{K} \pi_{k,oi}^{Data}\left(\mu_{k,i}\gamma_{\ell,i} + (1-\gamma_{\ell,i})\,\nu_{\ell k,i}\right) Y_{\ell,i} - \sum_{i=1}^{N} \pi_{k,oi}^{Data}\mu_{k,i} NX_i$$

$$\sum_{o=1}^{N}\sum_{k=1}^{K} Y_{k,o} = 1$$

The rest of the procedure conditions on these values of $\{Y_{k,i}\}$.

Step 2: Guess model parameters Ω. We treat $\widetilde{\kappa}_{k,i} \equiv \frac{\kappa_{k,i} P_i^F}{\widetilde{w}_{k,i}}$ as parameters to be estimated.

Step 3: Define

$$\varpi_{k,i} \equiv \frac{\left(1-\left(1-\chi_{k,i}\right)\delta\right)\widetilde{\kappa}_{k,i}}{\delta\left(1-\beta_{k,i}\right)}$$

If $\frac{\left(1-\left(1-\chi_{k,i}\right)\delta\right)\widetilde{\kappa}_{k,i}}{\delta\left(1-\beta_{k,i}\right)I_{k,i}(0)} = \frac{\varpi_{k,i}}{I_{k,i}(0)} \geq 1$, the free entry condition cannot be satisfied—$I_{k,i}$ is decreasing. Abort the procedure and highly penalize the objective function.

Step 4: Find $\underline{x}^{ub}_{k,i}$ such that $\frac{\left(1-\left(1-\chi_{k,i}\right)\delta\right)\widetilde{\kappa}_{k,i}}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^{ub}_{k,i}\right)} = 1 \iff I_{k,i}\left(\underline{x}^{ub}_{k,i}\right) = \varpi_{k,i}$. If along the algorithm $\underline{x}_{k,i}$ goes above $\underline{x}^{ub}_{k,i}$, we update it to be equal to $\underline{x}^{ub}_{k,i}$ (minus a small number).

Step 5: Guess $\{L_{k,i}\}$, and $\left\{\underline{x}_{k,i}\right\}$

Step 6: Compute $I_{k,i}\left(\underline{x}_{k,i}\right)$, $G_{k,i}\left(\underline{x}_{k,i}\right)$, $\theta_{k,i}$ and $u_{k,i}$.

- $\theta_{k,i} = q_i^{-1}\left(\frac{\varpi_{k,i}}{I_{k,i}\left(\underline{x}_{k,i}\right)}\right)$ where $q_i^{-1}\left(y\right) = \left(\frac{1-y^{\xi_i}}{y^{\xi_i}}\right)^{1/\xi_i}$
- $u_{k,i} = \frac{\chi_{k,i}}{\theta_{k,i}q_i\left(\theta_{k,i}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}\right)\right)+\chi_{k,i}}$

Step 7: Compute $\left\{\widetilde{L}_{k,i}\right\}$

$$\begin{aligned}
\widetilde{L}_{k,i} &\equiv L_{k,i}\left(1-u_{k,i}\right)\int_{\underline{x}_{k,i}}^{x_{\max}} \frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}\right)} dG_{k,i}\left(s\right) \\
&= L_{k,i}\left(1-u_{k,i}\right)\exp\left(\frac{\sigma_{k,i}^2}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}
\end{aligned}$$

Step 8: Compute $\{\widetilde{w}_{k,i}\}$

$$\widetilde{w}_{k,i} = \frac{\gamma_{k,i}Y_{k,i}}{\widetilde{L}_{k,i}}$$

Step 9: Compute $\left\{E^V_{k,i}\right\}$

$$E^V_{k,i} = \widetilde{\kappa}_{k,i}\widetilde{w}_{k,i}\theta_{k,i}u_{k,i}L_{k,i}$$

Step 10: Compute $\left\{E^C_i\right\}$

$$E^C_i = \sum_{k=1}^{K}\gamma_{k,i}Y_{k,i} - \sum_{k=1}^{K}E^V_{k,i} - NX_i$$

Step 11: Compute $\left\{\widetilde{\lambda}_i\right\}$

$$\widetilde{\lambda}_i = \frac{\overline{L}_i}{E^C_i}$$

Step 12: Obtain $\{U_{k,i}\}$.

- Step 12a: Guess $\left\{U_{ki}^0\right\}$
- Step 12b: Compute until convergence

$$U_{k,i}^{g+1} = \zeta_i \log\left(\sum_{\ell=1}^{K} \exp\left\{\frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}\widetilde{\kappa}_{\ell,i}\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{(1-\beta_{\ell,i})} + \delta U_{\ell,i} - \delta U_{k,i}^g}{\zeta_i}\right\}\right) + \delta U_{k,i}^g$$

Step 13: Update $\{L_{k,i}\}$.

- Step 13a: Given knowledge of $\{U_{k,i}\}$, compute transition rates $s_{k\ell,i}$.

$$s_{k\ell,i} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}\widetilde{\kappa}_{\ell,i}\widetilde{\lambda}_i\widetilde{w}_{\ell,i}\frac{\beta_{\ell,i}}{1-\beta_{\ell,i}}+\delta U_{\ell,i}}{\zeta_i}\right\}}{\sum_{\overline{k}}\exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta_{\overline{k},i}\widetilde{\kappa}_{\overline{k},i}\widetilde{\lambda}_i\widetilde{w}_{\overline{k},i}\frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}}+\delta U_{\overline{k},i}}{\zeta_i}\right\}}$$

- Step 13b: Find y_i such that

$$\left(I - s_i'\right) y_i = 0$$

- Step 13c: Find allocations $L_{k,i}$

$$L_{k,i}u_{k,i} = \varphi y_{k,i}$$

$$\begin{aligned}
&\Rightarrow L_{k,i} = \varphi \underbrace{y_{k,i}/u_{k,i}}_{\widetilde{y}_{k,i}} \\
&\Rightarrow L_i' 1_{K\times 1} = \varphi \widetilde{y}_{k,i}' 1_{K\times 1} = \overline{L}_i \\
&\Rightarrow \varphi = \frac{\overline{L}_i}{\widetilde{y}_{k,i}' 1_{K\times 1}}
\end{aligned}$$

$$\begin{aligned}
(L_{k,i})' &= \varphi \widetilde{y}_{k,i} \\
L_{k,i}^{new} &= (1-\lambda_L) L_{k,i} + \lambda_L (L_{k,i})'
\end{aligned}$$

Step 14: Update $\left\{\underline{x}_{k,i}\right\}$.

Note that in equilibrium:

$$\widetilde{\lambda}_i \widetilde{w}_{k,i} \underline{x}_{k,i} = (1-\delta) U_{k,i} - \eta_{k,i} \tag{A.37}$$

So, we update $\underline{x}_{k,i}$ according to:

$$\left(\underline{x}_{k,i}\right)' = \frac{(1-\delta) U_{k,i} - \eta_{k,i}}{\widetilde{\lambda}_i \widetilde{w}_{k,i}}$$

$$\underline{x}_{k,i}^{new} = \min\left\{(1-\lambda_x)\,\underline{x}_{k,i} + \lambda_x \left(\underline{x}_{k,i}\right)', \underline{x}_{k,i}^{ub}\right\}$$

Step 15: Armed with $L_{k,i}^{new}$ and $\underline{x}_{k,i}^{new}$ go to Step 6 until $\left\|\left\{L_{k,i}^{new}-L_{k,i}\right\}\right\| \to 0$ and $\left\|\left\{\underline{x}_{k,i}^{new}-\underline{x}_{k,i}\right\}\right\| \to 0$.

Note that $\left\|\left\{\underline{x}_{k,i}^{new}-\underline{x}_{k,i}\right\}\right\| \to 0$ does not imply that (A.37) is satisfied. Therefore, we penalize deviations from (A.37) in the objective function.

Step 16: Generate moments, compute Loss Function, guess new parameter set Ω and go to Step 3, until objective function is minimized.

Note: Given that we condition on the trade shares $\pi_{k,oi}^{Data}$, we can estimate the model country by country, separately. However, in practice, we will first estimate the model for the US and obtain all of the US specific parameters. Next, armed with US-specific mobility costs $C_{kk'}$ and sector-specific exogenous exit components χ_k (we will impose $\chi_{k,i} = \chi_i + \chi_k$), we estimate the remaining countries' parameters separately, in parallel.

C.2 Expressions for Simulated Moments

C.2.1 Employment Shares

$$emp_{k,i} = \frac{L_{k,i}\left(1-u_{k,i}\right)}{\sum\limits_{k=1}^{K} L_{k,i}\left(1-u_{k,i}\right)}$$

C.2.2 National Unemployment Rate

$$unemp_i = \frac{\sum\limits_{k=1}^{K} L_{k,i}u_{k,i}}{\sum\limits_{k=1}^{K} L_{k,i}}$$

C.2.3 Sector-Specific Average Wages

$$w_{k,i}\left(x\right) = \left(1-\beta_{k,i}\right)\widetilde{w}_{k,i}\underline{x}_{k,i} + \beta_{k,i}\widetilde{w}_{k,i}x$$

$$\begin{aligned}
\overline{w}_{k,i} &= \frac{\int_{\underline{x}_{k,i}}^{x_{\max}} w_{k,i}\left(s\right)dG_{k,i}\left(s\right)}{1-G_{k,i}\left(\underline{x}_{k,i}\right)} \\
&= \left(1-\beta_{k,i}\right)\widetilde{w}_{k,i}\underline{x}_{k,i} + \beta_{k,i}\widetilde{w}_{k,i}\int_{\underline{x}_{k,i}}^{x_{\max}} \frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}\right)}dG_{k,i}\left(s\right) \\
&= \left(1-\beta_{k,i}\right)\widetilde{w}_{k,i}\underline{x}_{k,i} + \beta_{k,i}\widetilde{w}_{k,i}\exp\left(\frac{\sigma_{k,i}^2}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}}{\sigma_{k,i}}\right)}
\end{aligned}$$

C.2.4 Sector-Specific Variance of Wages

$$\sigma_{w,k,i}^{2} = \frac{\int_{\underline{x}_{k,i}}^{\infty}\left(w_{k,i}\left(s\right)-\overline{w}_{k,i}\right)^{2}dG_{k,i}\left(s\right)}{1-G_{k,i}\left(\underline{x}_{k,i}\right)}$$
$$= \left(\beta_{k,i}\widetilde{w}_{k,i}\right)^{2}\times\frac{\int_{\underline{x}_{k,i}}^{\infty}\left(s-\exp\left(\frac{\sigma_{k,i}^{2}}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln\underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln\underline{x}_{k,i}}{\sigma_{k,i}}\right)}\right)^{2}dG_{k,i}\left(s\right)}{1-G_{k,i}\left(\underline{x}_{k,i}\right)}$$
$$= \left(\beta_{k,i}\widetilde{w}_{k,i}\right)^{2}\times\left(\exp\left(2\sigma_{k,i}^{2}\right)\frac{\Phi\left(2\sigma_{k,i}-\frac{\ln\underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln\underline{x}_{k,i}}{\sigma_{k,i}}\right)}-\exp\left(\sigma_{k,i}^{2}\right)\left(\frac{\Phi\left(\sigma_{k,i}-\frac{\ln\underline{x}_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln\underline{x}_{k,i}}{\sigma_{k,i}}\right)}\right)^{2}\right)$$

C.2.5 Transition Rates

Note that the transition rates in equation (10) are transitions from unemployment in sector k to search in sector k' within period t. There are no data counterfactuals for this variable. However, we can construct a matrix with transition rates between all possible (model) states between time t and time $t+N$ (where N is even)—where variables are measured at the t_a stage (which is the production stage). From this matrix, we can obtain N-period transition rates between all states observed in the data (employment in each of the sectors and unconditional unemployment). First, we obtain the one-year transition matrix $\widetilde{s}^{t,t+1}$ between states $\{\widetilde{u}_1,...,\widetilde{u}_K,1,...,K\}$. Here, we abuse notation to mean $\widetilde{u}_k$ as sector-k unemployment at the very beginning of a period.

The one-year transition rate between sector-ℓ unemployment and sector-k unemployment is given by:

$$\widetilde{s}_{\widetilde{u}_\ell\widetilde{u}_k,i}^{t,t+1} = s_{\ell k,i}^{t,t+1}\left(1-\theta_{k,i}^{t}q_i\left(\theta_{k,i}^{t}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)\right), \tag{A.38}$$

that is, a share $s_{\ell k,i}^{t}$ of individuals starting period t unemployed in sector ℓ choose to search in sector k. A fraction $\left(1-\theta_{k,i}^{t}q_i\left(\theta_{k,i}^{t}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)\right)$ of those do not find a match that survives until $t+1$. Similarly, the one-year transition rate between sector-ℓ unemployment and sector-k employment is given by:

$$\begin{aligned}\widetilde{s}_{\widetilde{u}_\ell k,i}^{t,t+1} &= s_{\ell k,i}^{t,t+1}\theta_{k,i}^{t}q_i\left(\theta_{k,i}^{t}\right)\left(1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)\\ &= s_{\ell k,i}^{t,t+1}-\widetilde{s}_{\widetilde{u}_\ell\widetilde{u}_k,i}^{t,t+1}.\end{aligned} \tag{A.39}$$

According to the timing assumptions of the model, the one-year transition rate between employment in sector k and employment in sector k' is zero if $k\neq k'$. However, the persistence rate of employment in sector k is given by the probability that a match does not receive a death shock times the probability that the match is not dissolved because the threshold for production increases in the following period:

$$\widetilde{s}_{kk',i}^{t,t+1} = \begin{cases} 0 \text{ if } k\neq k' \\ \left(1-\chi_{k,i}\right)\Pr\left(x\geq\underline{x}_{k,i}^{t+1}|x\geq\underline{x}_{k,i}^{t}\right) \text{ if } k=k' \end{cases}. \tag{A.40}$$

Finally, the one-year transition rate between sector-k employment and unemployment in sector ℓ

is given by:

$$\widetilde{s}^{t,t+1}_{k\widetilde{u}_\ell,i} = \begin{cases} 0 \text{ if } k \neq \ell \\ \chi_{k,i} + (1-\chi_{k,i}) \Pr\left(x < \underline{x}^{t+1}_{k,i} | x \geq \underline{x}^{t}_{k,i}\right) \text{ if } k = \ell \end{cases} . \tag{A.41}$$

That is, if a worker is employed in sector k at t, she cannot start next period unemployed in sector ℓ if $k \neq \ell$. Otherwise, workers transition between sector k employment to sector k unemployment if their match is hit with a death shock or if their employer's productivity goes below the threshold for production at $t+1$.

We can now write the N-period transition matrix as:

$$\widetilde{s}^{t,t+N} = \widetilde{s}^{t+k-1,t+k} \times ... \times \widetilde{s}^{t+1,t+2} \times \widetilde{s}^{t,t+1}, \tag{A.42}$$

and we can write transition rates between unemployment $\widetilde{u}$ and sector-k employment between t and $t+N$ as:

$$\widetilde{s}^{t,t+N}_{\widetilde{u},k,i} = \frac{\sum_{\ell=1}^{K} L^{t-1}_{\ell,i} \widetilde{u}^{t-1}_{\ell,i} \widetilde{s}^{t,t+N}_{\widetilde{u}_\ell,k}}{\sum_{\ell=1}^{K} L^{t-1}_{\ell,i} \widetilde{u}^{t-1}_{\ell,i}}. \tag{A.43}$$

Finally, we can write transition rates between sector-k employment and unemployment $\widetilde{u}$ as:

$$\widetilde{s}^{t,t+N}_{k,\widetilde{u},i} = 1 - \sum_{k'=1}^{K} \widetilde{s}^{t,t+N}_{k,k',i}. \tag{A.44}$$

1-period transition rates

$$\widetilde{s}_{\widetilde{u}_\ell \widetilde{u}_k,i} = s_{\ell k,i} \left(1 - \theta_{k,i} q_i\left(\theta_{k,i}\right)\left(1 - G_{k,i}\left(\underline{x}_{k,i}\right)\right)\right)$$

$$\widetilde{s}_{\widetilde{u}_\ell k,i} = s_{\ell k,i} \theta_{k,i} q_i\left(\theta_{k,i}\right)\left(1 - G_{k,i}\left(\underline{x}_{k,i}\right)\right)$$

$$\widetilde{s}_{\ell k,i} = \begin{cases} 0 \text{ if } \ell \neq k \\ (1-\chi_{\ell,i}) \text{ if } \ell = k \end{cases}$$

$$\widetilde{s}_{\ell \widetilde{u}_k,i} = \begin{cases} 0 \text{ if } \ell \neq k \\ \chi_{k,i} \text{ if } \ell = k \end{cases}$$

N-period transition rates from and to unconditional unemployment: $\widetilde{s}^N$

$$\widetilde{s}^{N}_{\widetilde{u},k,i} = \frac{\sum_{\ell=1}^{K} L_{\ell,i} u_{\ell,i} \widetilde{s}^{N}_{\widetilde{u}_\ell,k,i}}{\sum_{\ell=1}^{K} L_{\ell,i} u_{\ell,i}}$$

$$\widetilde{s}^{N}_{k,\widetilde{u},i} = 1 - \sum_{\ell=1}^{K} \widetilde{s}^{N}_{k,\ell,i}.$$

C.3 Algorithm: Steady-State Equilibrium Following Shock

Consider shocks $\left\{A^0_{k,i}\right\} \to \left\{A^1_{k,i}\right\}$, $\left\{d^0_{k,oi}\right\} \to \left\{d^1_{k,oi}\right\}$, $\{NX^0\} \to \{NX^1_i\}$

We will be using 0 superscripts to denote the initial steady state, and 1 superscripts to denote the final steady state.

Start from estimated Steady State: $\left\{L^0_{k,i}\right\}$, $\left\{\underline{x}^0_{k,i}\right\}$, $\left\{\widetilde{w}^0_{k,i}\right\}$, $\left\{\pi^0_{k,oi}\right\}$

Note that $\pi^0_{k,oi} = \pi^{Data}_{k,oi}$

We also have $\widetilde{\kappa}^0_{k,i} = \frac{\kappa_{k,i} P^{F,0}_i}{\widetilde{w}^0_{k,i}}$, but we do not know $\left\{P^{F,0}_i\right\}$

Denote relative changes in variable a by $\widehat{a} = \frac{a^1}{a^0}$

Step 1: Guess $\left\{L^1_{k,i}\right\}$ and $\left\{\underline{x}^1_{k,i}\right\}$

Step 2: Guess $\left\{\widetilde{w}^1_{k,i}\right\}$

- Step 2a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}^1_{k,i}}{\widetilde{w}^0_{k,i}}$ and iteratively solve for $\widehat{P}^I_{k,i}$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}^I_{\ell,i}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}^I_{k,i} = \left(\sum_{o=1}^{N} \pi^0_{k,oi} \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 2b: Compute $\widehat{P}^F_{k,i}$:

$$\widehat{P}^F_i = \prod_{k=1}^{K} \left(\widehat{P}^I_{k,i}\right)^{\mu_{ki}}$$

- Step 2c: Compute $\widehat{\pi}_{k,oi}$:

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}^I_{k,i}}\right)^{-\lambda}$$

- Step 2d: Compute

– $\pi^1_{k,oi} = \pi^0_{k,oi}\widehat{\pi}_{k,oi}$

– $\widetilde{\kappa}^1_{k,i} \equiv \frac{\kappa_{k,i}P_i^{F,1}}{\widetilde{w}^1_{k,i}} = \frac{\kappa_{k,i}P_i^{F,0}}{\widetilde{w}^0_{k,i}}\frac{P_i^{F,1}}{P_i^{F,0}}\frac{\widetilde{w}^0_{k,i}}{\widetilde{w}^1_{k,i}} = \widetilde{\kappa}^0_{k,i}\frac{\widehat{P}_i^F}{\widehat{\widetilde{w}}_{k,i}}$

Step 3: If $\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)} \geq 1$ abort, set $\underline{x}^1_{k,i}$ such that $\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)} = 1-\varepsilon$ and go back to Step 1 with this new guess. If $\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)} < 1$, proceed to Step 4.

Step 4: Compute

$$q_i\left(\theta^1_{k,i}\right) = \widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)}$$

$$\theta^1_{k,i} = q_i^{-1}\left(\widetilde{\kappa}^1_{k,i} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}^1_{k,i}\right)}\right)$$

$$u^1_{k,i} = \frac{\chi_{k,i}}{\theta^1_{k,i}q_i\left(\theta^1_{k,i}\right)\left(1-G_{k,i}\left(\underline{x}^1_{k,i}\right)\right)+\chi_{k,i}}$$

Step 5: Solve system in $\left\{Y^1_{k,o}\right\}$

$$Y^1_{k,o} = \sum_{i=1}^{N}\pi^1_{k,oi}\left(\mu_{k,i}\left(\sum_{\ell=1}^{K}\gamma_{\ell,i}Y^1_{\ell,i}\right)+\sum_{\ell=1}^{K}\left(1-\gamma_{\ell,i}\right)\nu_{\ell k,i}Y^1_{\ell,i}\right)-\sum_{i=1}^{N}\pi^1_{k,oi}\mu_{k,i}NX^1_i$$

$$\sum_{i=1}^{N}\sum_{k=1}^{K}Y^1_{k,i} = 1$$

Step 6: Compute $\left\{\widetilde{L}^1_{k,i}\right\}$

$$\widetilde{L}^1_{k,i} \equiv L^1_{k,i}\left(1-u^1_{k,i}\right)\int_{\underline{x}^1_{k,i}}^{x_{\max}}\frac{s}{1-G_{k,i}\left(\underline{x}^1_{k,i}\right)}dG_{k,i}\left(s\right)$$

$$= L^1_{k,i}\left(1-u^1_{k,i}\right)\exp\left(\frac{\sigma^2_{k,i}}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln \underline{x}^1_{k,i}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}^1_{k,i}}{\sigma_{k,i}}\right)}$$

Step 7: Update $\left\{\widetilde{w}^1_{k,i}\right\}$

$$\left(\widetilde{w}^1_{k,i}\right)^{new} = \frac{\gamma_{k,i}Y^1_{k,i}}{\widetilde{L}^1_{k,i}}$$

Go back to Step 2a and repeat until converegence of $\left\{\widetilde{w}^1_{k,i}\right\}$.

Step 8: Compute

$$E^{V,1}_{k,i} = \widetilde{\kappa}^1_{k,i}\widetilde{w}^1_{k,i}\theta^1_{k,i}u^1_{k,i}L^1_{k,i}$$

$$E^{C,1}_i = \sum_{k=1}^{K}\gamma_{k,i}Y^1_{k,i} - \sum_{k=1}^{K}E^{V,1}_{k,i} - NX^1_i$$

Step 9: Obtain Lagrange Multipliers

$$\widetilde{\lambda}^1_i = \frac{\overline{L}_i}{E^{C,1}_i}$$

Step 10: Compute Bellman Equations

$$U^1_{k,i} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta^1_{k',i}\widetilde{\kappa}^1_{k',i}\widetilde{\lambda}^1_i\widetilde{w}^1_{k',i}\frac{\beta_{k',i}}{\left(1-\beta_{k',i}\right)} + \delta U^1_{k',i}}{\zeta_i}\right\}\right)$$

Step 11: Update $\left\{L^1_{k,i}\right\}$.

- Step 11a: Given knowledge of $\left\{U^1_{k,i}\right\}$, compute transition rates $s^1_{k\ell,i}$.

$$s^1_{k\ell,i} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta^1_{\ell,i}\widetilde{\kappa}^1_{\ell,i}\widetilde{\lambda}^1_i\widetilde{w}^1_{\ell,i}\frac{\beta_{\ell,i}}{\left(1-\beta_{\ell,i}\right)}+\delta U^1_{\ell,i}}{\zeta_i}\right\}}{\sum_{\overline{k}}\exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta^1_{\overline{k},i}\widetilde{\kappa}^1_{\overline{k},i}\widetilde{\lambda}^1_i\widetilde{w}^1_{\overline{k},i}\frac{\beta_{\overline{k},i}}{\left(1-\beta_{\overline{k},i}\right)}+\delta U^1_{\overline{k},i}}{\zeta_i}\right\}}$$

- Step 11b: Find y_i such that

$$\left(I - \left(s^1_i\right)^T\right)y_i = 0$$

- Step 11c: Find allocations $L_{k,i}$

$$L^1_{k,i}u^1_{k,i} = \varphi y_{k,i}$$

$$\begin{aligned}
&\Rightarrow L^1_{k,i} = \varphi\underbrace{y_{k,i}/u^1_{k,i}}_{\widetilde{y}_{k,i}} \\
&\Rightarrow \left(L^1_i\right)^T 1_{K\times 1} = \varphi\widetilde{y}^T_{k,i}1_{K\times 1} = \overline{L}_i \\
&\Rightarrow \varphi = \frac{\overline{L}_i}{\widetilde{y}^T_{k,i}1_{K\times 1}}
\end{aligned}$$

$$\left(L_{k,i}^1\right)' = \varphi \widetilde{y}_{k,i}$$
$$\left(L_{k,i}^1\right)^{new} = \left(1 - \lambda_L\right) L_{k,i}^1 + \lambda_L \left(L_{k,i}^1\right)'$$

Step 12: Update $\left\{\underline{x}_{k,i}^1\right\}$.

Note that in equilibrium:

$$\widetilde{\lambda}_i^1 \widetilde{w}_{k,i}^1 \underline{x}_{k,i}^1 = \left(1 - \delta\right) U_{k,i}^1 - \eta_{k,i}$$

So, we update $\underline{x}_{k,i}^1$ according to:

$$\left(\underline{x}_{k,i}^1\right)' = \frac{\left(1 - \delta\right) U_{k,i}^1 - \eta_{k,i}}{\widetilde{\lambda}_i^1 \widetilde{w}_{k,i}^1}$$

$$\left(\underline{x}_{k,i}^1\right)^{new} = \min\left\{\left(1 - \lambda_x\right) \underline{x}_{k,i}^1 + \lambda_x \left(\underline{x}_{k,i}^1\right)', \underline{x}_{k,i}^{ub}\right\}$$

Step 13: Armed with $\left(L_{k,i}^1\right)^{new}$ and $\left(\underline{x}_{k,i}^1\right)^{new}$ go to Step 2 until $\left\|\left\{\left(L_{k,i}^1\right)^{new} - L_{k,i}^1\right\}\right\| \to 0$ and $\left\|\left\{\left(\underline{x}_{k,i}^1\right)^{new} - \underline{x}_{k,i}^1\right\}\right\| \to 0$.

C.4 Algorithm: Out-of-Steady-State Transition

Inner Loop: conditional on paths for expenditures $\left\{E_i^{C,t}\right\}$—determined in the Outer Loop below.

Consider paths $\left\{A_{k,i}^t\right\}_{t=0}^{T_{SS}}$ and $\left\{d_{o,i,k}^t\right\}_{t=0}^{T_{SS}}$ with $A_{k,i}^0 = 1$ and $d_{o,i,k}^0 = 1$. Also, consider paths $\left\{\phi_i^t\right\}_{t=0}^{T_{SS}}$ with $\phi_i^0 = 1$ and $\widehat{\phi}_i^t = 1$ for $T \leq t \leq T_{SS}$, for some $T << T_{SS}$.

Step 0: Given paths $\left\{E_i^{C,t}\right\}$, compute paths $\left\{\widetilde{\lambda}_i^t\right\}$: $\widetilde{\lambda}_i^t = \frac{\overline{L}_i}{E_i^{C,t}}$

Step 1: Guess paths $\left\{\widetilde{w}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ for each sector k and country i.

Step 2: Compute $\underline{x}_{k,i}^{T_{SS}}$ consistent with $\widetilde{w}_{k,i}^{T_{SS}}$ and $\widetilde{\lambda}_i^{T_{SS}}$. Obtain $\theta_{k,i}^{T_{SS}}$, $U_{k,i}^{T_{SS}}$, $s_{k\ell,i}^{T_{SS},T_{SS}+1}$ and $\pi_{k,oi}^{T_{SS}}$.

- Step 2a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}_{k,i}^{T_{SS}}}{\widetilde{w}_{k,i}^0}$, $\widehat{A}_{k,i} = \frac{A_{k,i}^{T_{SS}}}{A_{k,i}^0}$ and $\widehat{d}_{k,i} = \frac{d_{o,i,k}^{T_{SS}}}{d_{o,i,k}^0}$. Iteratively solve for $\widehat{P}_{k,i}^I$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^I\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^I = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 2b: Compute $\widehat{P}_{k,i}^F$:

$$\widehat{P}_i^F = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^I\right)^{\mu_{ki}}$$

- Step 2c: Compute

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}_{k,i}^I}\right)^{-\lambda}$$

  And obtain $\pi_{k,oi}^{T_{SS}} = \pi_{k,oi}^0 \widehat{\pi}_{k,oi}$

- Step 2d: Compute
  - $\widetilde{\kappa}_{k,i}^{T_{SS}} = \widetilde{\kappa}_{k,i}^0 \frac{\widehat{P}_i^F}{\widehat{\widetilde{w}}_{k,i}}$
- Step 2e: Guess $\left\{\underline{x}_{k,i}^{T_{SS}}\right\}$
- Step 2f: Compute

$$\theta_{k,i}^{T_{SS}} = q_i^{-1} \left(\widetilde{\kappa}_{k,i}^{T_{SS}} \times \frac{1 - \delta\left(1 - \chi_{k,i}\right)}{\delta\left(1 - \beta_{k,i}\right) I_{k,i}\left(\underline{x}_{k,i}^{T_{SS}}\right)}\right)$$

- Step 2g: Compute Bellman Equations

$$U_{k,i}^{T_{SS}} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}^{T_{SS}} \widetilde{\kappa}_{k',i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k',i}^{T_{SS}} \frac{\beta_{k',i}}{\left(1-\beta_{k',i}\right)} + \delta U_{k',i}^{T_{SS}}}{\zeta_i}\right\}\right)$$

- Step 2h: Compute

$$\left(\underline{x}_{k,i}^{T_{SS}}\right)' = \frac{(1-\delta)\, U_{k,i}^{T_{SS}} - \eta_{k,i}}{\widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k,i}^{T_{SS}}}$$

- Step 2i: Update $\underline{x}_{k,i}^{T_{SS}} = (1-\lambda_x)\, \underline{x}_{k,i}^{T_{SS}} + \lambda_x \left(\underline{x}_{k,i}^{T_{SS}}\right)'$, for a small step size λ_x, and go back to Step 2d until convergence.
- Step 2j: Compute $s_{kk'}^{T_{SS},T_{SS}+1}$

$$s_{k\ell,i}^{T_{SS},T_{SS}+1} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}^{T_{SS}}\widetilde{\kappa}_{\ell,i}^{T_{SS}}\widetilde{\lambda}_i^{T_{SS}}\widetilde{w}_{\ell,i}^{T_{SS}}\frac{\beta_{\ell,i}}{1-\beta_{\ell,i}}+\delta U_{\ell,i}^{T_{SS}}}{\zeta_i}\right\}}{\sum_{\overline{k}} \exp\left\{\frac{-C_{k\overline{k},i}+b_{\overline{k},i}+\theta_{\overline{k},i}^{T_{SS}}\widetilde{\kappa}_{\overline{k},i}^{T_{SS}}\widetilde{\lambda}_i^{T_{SS}}\widetilde{w}_{\overline{k},i}^{T_{SS}}\frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}}+\delta U_{\overline{k},i}^{T_{SS}}}{\zeta_i}\right\}}$$

Step 3: Obtain series $\left\{\pi_{k,oi}^t\right\}_{t=0}^{T_{SS}}$, $\left\{\widetilde{\kappa}_{k,i}^t\right\}_{t=0}^{T_{SS}}$. Define $\widehat{x}^t \equiv \frac{x^t}{x^0}$.

- Step 3a: For $t = 1, ..., T_{SS}-1$ compute $\widehat{\widetilde{w}}_{k,i}^t = \frac{\widetilde{w}_{k,i}^t}{\widetilde{w}_{k,i}^0}$ and iteratively solve for $\widehat{P}_{k,i}^{I,t}$ and $\widehat{c}_{k,i}^t$ using the system

$$\widehat{c}_{k,i}^t = \left(\widehat{\widetilde{w}}_{k,i}^t\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^{I,t}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^{I,t} = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o}^t \left(\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 3b: Compute $\widehat{P}_{k,i}^{F,t}$ for $t = 1, ..., T_{SS}-1$:

$$\widehat{P}_i^{F,t} = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^{I,t}\right)^{\mu_{ki}}$$

- Step 3c: Compute $\widehat{\pi}_{k,oi}^t$ for $t = 1, ..., T_{SS}-1$:

$$\widehat{\pi}_{k,oi}^t = \widehat{A}_{k,o}^t \left(\frac{\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t}{\widehat{P}_{k,i}^{I,t}}\right)^{-\lambda}$$

- Step 3d: Compute or $t = 1, ..., T_{SS}-1$:

- $\pi^t_{k,oi} = \pi^0_{k,oi}\widehat{\pi}^t_{k,oi}$
- $\widetilde{\kappa}^t_{k,i} \equiv \frac{\kappa_{k,i}P^{F,t}_i}{\widetilde{w}^t_{k,i}} = \frac{\kappa_{k,i}P^{F,0}_i}{\widetilde{w}^0_{k,i}}\frac{P^{F,t}_i}{P^{F,0}_i}\frac{\widetilde{w}^0_{k,i}}{\widetilde{w}^t_{k,i}} = \widetilde{\kappa}^0_{k,i}\frac{\widehat{P}^{F,t}_i}{\widehat{\widetilde{w}}^t_{k,i}}$

Step 4: Given knowledge of $\widetilde{w}^{T_{SS}}_{k,i}$, $\widetilde{\lambda}^{T_{SS}}_i$ and $\underline{x}^{T_{SS}}_{k,i}$ (and therefore $J^{T_{SS}}_{k,i}(s)$), start at $t = T_{SS} - 1$ and sequentially compute (backwards) for each $t = T_{SS} - 1, ..., 1$

- Step 4a: Given $\widetilde{w}^t_{k,i}$, $\underline{x}^{t+1}_{k,i}$, $\widetilde{\kappa}^t_{k,i}$, $\widetilde{\lambda}^t_i$ and $J^{t+1}_{k,i}(s)$ compute $\theta^t_{k,i}$.

If $\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int^{x_{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)dG_{k,i}(s)} \leq 1$ then

$$\theta^t_{k,i} = q^{-1}_i\left(\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int^{x_{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)\,dG_{k,i}(s)}\right)$$

If $\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int^{x_{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)dG_{k,i}(s)} > 1$, it is not possible to satisfy $V^t_{k,i} = 0$, so that $V^t_{k,i} < 0$ and

$$\theta^t_{k,i} = 0.$$

- Step 4b: Given $\underline{x}^{t+1}_{k,i}$, $W^{t+1}_{k,i}(x) = \frac{\beta_{k,i}}{1-\beta_{k,i}}J^{t+1}_{k,i}(x) + U^{t+1}_{k,i}$ (for $x \geq \underline{x}^{t+1}_{k,i}$), $\theta^t_{k,i}$, $U^{t+1}_{k,i}$ compute $U^t_{k,i}$. Notice that $\int^{x_{\max}}_{\underline{x}^{t+1}_{k,i}}W^{t+1}_{k,i}(s)\,dG_{k,i}(s) = \frac{\beta_{k,i}}{1-\beta_{k,i}}\int^{x_{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)\,dG_{k,i}(s) + \left(1 - G_{k,i}\left(\underline{x}^{t+1}_{k,i}\right)\right)U^{t+1}_{k,i}$ so that:

$$U^t_{k,i} = \zeta_i\log\left(\sum_{k'}\exp\left\{\frac{\begin{array}{c}-C_{kk',i} + b_{k',i}\\ +\delta\widehat{\phi}^{t+1}_i\theta^t_{k',i}q_i\left(\theta^t_{k',i}\right)\frac{\beta_{k,i}}{1-\beta_{k,i}}\int^{x_{\max}}_{\underline{x}^{t+1}_{k,i}}J^{t+1}_{k,i}(s)\,dG_{k,i}(s) + \delta\widehat{\phi}^{t+1}_iU^{t+1}_{k',i}\end{array}}{\zeta_i}\right\}\right)$$

- Step 4c: Given $J^{t+1}_{k,i}(x)$, $\widetilde{w}^t_{k,i}$, $U^t_{k,i}$, $U^{t+1}_{k,i}$ and $\underline{x}^{t+1}_{k,i}$ compute $J^t_{k,i}(x)$

$$\begin{aligned}J^t_{k,i}(x) &= (1-\beta_{k,i})\,\widetilde{\lambda}^t_i\widetilde{w}^t_{k,i}x + (1-\beta_{k,i})\,\eta_{k,i}\\ &\quad - (1-\beta_{k,i})\left(U^t_{k,i} - \delta\widehat{\phi}^{t+1}_iU^{t+1}_{k,i}\right) + (1-\chi_{k,i})\,\delta\widehat{\phi}^{t+1}_i\max\left\{J^{t+1}_{k,i}(x), 0\right\}\end{aligned}$$

- Step 4d: Solve for $\underline{x}^t_{k,i}$: $J^t_{k,i}\left(\underline{x}^t_{k,i}\right) = 0$

Step 5: Compute transition rates $\left\{s^{t,t+1}_{kk',i}\right\}^{T_{SS}-1}_{t=1}$ for all countries i according to:

$$s^{t,t+1}_{kk',i} = \frac{\exp\left\{\begin{array}{c}-C_{kk',i} + b_{k',i}+\\ \delta\widehat{\phi}^{t+1}_i\theta^t_{k',i}q(\theta^t_{k',i})\frac{\beta_{k',i}}{1-\beta_{k',i}}\int^{x_{\max}}_{\underline{x}^{t+1}_{k',i}}J^{t+1}_{k',i}(x)\,dG_{k',i}(x) + \delta\widehat{\phi}^{t+1}_iU^{t+1}_{k',i}\end{array}\right\}}{\sum_{k''}\exp\left\{\begin{array}{c}-C_{kk'',i} + b_{k'',i}+\\ \delta\widehat{\phi}^{t+1}_i\theta^t_{k'',i}q(\theta^t_{k'',i})\frac{\beta_{k'',i}}{1-\beta_{k'',i}}\int^{x_{\max}}_{\underline{x}^{t+1}_{k'',i}}J^{t+1}_{k'',i}(x)\,dG_{k'',i}(x) + \delta\widehat{\phi}^{t+1}_iU^{t+1}_{k'',i}\end{array}\right\}}.$$

Step 6: Start loop over t going forward ($t = 0$ to $t = T_{SS} - 1$)

Initial conditions: we know $\widetilde{u}_{k,i}^{t=-1} = u_{k,i}^{t=0}$, $L_{k,i}^{t=-1} = L_{k,i}^{t=0}$, and $\theta_{k,i}^{t=0}$ from the initial steady state computation. Obtain $\widetilde{u}_{k,i}^{t}$ and $L_{k,i}^{t}$ using flow conditions and sequences $\left\{\theta_{k,i}^{t}\right\}$, $\left\{\underline{x}_{k,i}^{t}\right\}$.

- Step 6a: Compute

$$JC_{k,i}^{t} = L_{k,i}^{t} u_{k,i}^{t} \theta_{k,i}^{t} q_i\left(\theta_{k,i}^{t}\right)\left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)$$

$$JD_{k,i}^{t} = \left(\chi_{k,i} + (1-\chi_{k,i}) \max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}, 0\right\}\right) L_{k,i}^{t-1}\left(1 - \widetilde{u}_{k,i}^{t-1}\right)$$

$$\widetilde{u}_{k,i}^{t} = \frac{L_{k,i}^{t} u_{k,i}^{t} - JC_{k,i}^{t} + JD_{k,i}^{t}}{L_{k,i}^{t}}$$

- Step 6b: Compute

$$L_{k,i}^{t+1} = L_{k,i}^{t} + IF_{k,i}^{t+1} - OF_{k,i}^{t+1},$$

where

$$IF_{k,i}^{t+1} = \sum_{\ell \neq k} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2},$$

and

$$OF_{k,i}^{t+1} = L_{k,i}^{t} \widetilde{u}_{k,i}^{t}\left(1 - s_{kk,i}^{t+1,t+2}\right).$$

- Step 6c: Compute

$$u_{k,i}^{t+1} = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2}}{L_{k,i}^{t+1}}$$

- Step 6d: Compute

$$\begin{aligned}
\widetilde{L}_{k,i}^{t+1} &= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \int_{\underline{x}_{k,i}^{t+1}}^{\infty} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)} dG_{k,i}(s) \\
&= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \exp\left(\frac{\sigma_{k,i}^{2}}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}
\end{aligned}$$

- Step 6e: Compute expenditure with vacancies

$$E_{k,i}^{V,t+1} = \widetilde{\kappa}_{k,i}^{t+1} \widetilde{w}_{k,i}^{t+1} \theta_{k,i}^{t+1} u_{k,i}^{t+1} L_{k,i}^{t+1}$$

- Step 6f: Solve for $\left\{Y_{k,i}^{t+1}\right\}$ in the system

$$E_{k,i}^{t+1} = \mu_{k,i} E_i^{C,t+1} + \sum_{\ell=1}^{K} \left(\mu_{k,i} E_{\ell,i}^{V,t+1} + (1 - \gamma_{\ell,i}) \, \nu_{\ell k,i} Y_{\ell,i}^{t+1} \right).$$

$$Y_{k,o}^{t+1} = \sum_{i=1}^{N} \pi_{k,oi}^{t+1} E_{k,i}^{t+1}.$$

- Step 6g: Compute $\left(\widetilde{w}_{k,i}^{t+1}\right)' = \frac{\gamma_{k,i} Y_{k,i}^{t+1}}{\widetilde{L}_{k,i}^{t+1}}$

Step 7: Compute distance $dist\left(\left\{\widetilde{w}_{k,i}^{t}\right\}, \left\{\left(\widetilde{w}_{k,i}^{t}\right)'\right\}\right)$

- Step 7b: Update $\widetilde{w}_{k,i}^{t} = (1 - \lambda_w) \, \widetilde{w}_{k,i}^{t} + \lambda_w \left(\widetilde{w}_{k,i}^{t}\right)'$ $t = 1, ..., T_{SS}$, for a small step size λ_w.
- Step 7c: At this point, we have a new series for $\left\{\widetilde{w}_{k,i}^{t}\right\}$ – go back to Step 2 until convergence of $\left\{\widetilde{w}_{k,i}^{t}\right\}$.

Step 8: Compute disposable income $\left\{I_i^t\right\}_{t=1}^{T_{SS}}$

$$I_i^t = \sum_{\ell=1}^{K} \left(\gamma_{\ell,i} Y_{\ell,i}^{t} - E_{\ell,i}^{V,t} \right)$$

Outer Loop: iteration on $\left\{NX_i^t\right\}$

Step 0: Impose a change in a subset of parameters that happens at $t = 0$, but between t_c and t_d. That is, the shock occurs **after** production, workers' decisions of where to search and after firms post vacancies at $t = 0$. Impose a large value for T_{SS}. Assume that for $t \geq T_{SS}$ the system will have converged to a new steady state. World expenditure with final goods $\sum_{i=1}^{I} E_i^{C,t}$ is normalized to 1 for every t.

Step 1: Start with estimated state equilibrium at $t = 0$. Remember that we used the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ during the estimation procedure. **Change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ to $\sum_{i=1}^{I} E_i^C = 1$. Nominal variables to be renormalized: $\left\{Y_{k,i}^0\right\}$, $\left\{\widetilde{w}_{k,i}^0\right\}$, $\left\{E_i^{C,0}\right\}$, $\left\{NX_i^0\right\}$.

Step 2: Obtain B_i^0 with respect to the normalization $\sum_{i=1}^{I} E_i^C = 1$. Equation (37) gives us:

$$B_i^0 = \frac{NX_i^0}{\left(1 - \frac{1}{\delta}\right)}$$

Step 3: Make initial guess for $NX_i^{T_{SS}}$ (with respect to the normalization $\sum_{i=1}^{I} E_i^C = 1$).

Step 4: Compute steady state equilibrium at T_{SS}, conditional on $NX_i^{T_{SS}}$, and the change in parameter values.

- Step 4a: Notice that the steady-state algorithm uses the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. Normalize $NX_i^{T_{SS}}$ with respect to normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. To perform such normalization, use revenue $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in the initial steady state if this is the first outer loop iteration, otherwise use revenue $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in Step 6 below.

- Step 4b: After computing the final steady state, **change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ to $\sum_{i=1}^{I} E_i^C = 1$ using $\left\{E_i^C\right\}$ obtained in Step 3a. Nominal variables to be renormalized: $\left\{Y_{k,i}^{T_{SS}}\right\}$, $\left\{\widetilde{w}_{k,i}^{T_{SS}}\right\}$, $\left\{E_i^{C,T_{SS}}\right\}$, $\left\{NX_i^{T_{SS}}\right\}$.

Step 5: Start at $t = T_{SS} - 1$ and go backward until $t = 1$ and sequentially compute:

$$R^{t+1} = \frac{1}{\delta} \frac{\sum_{i=1}^{N} \frac{E_i^{C,t+1}}{\widehat{\phi}_i^{t+1}}}{\sum_{i=1}^{N} E_i^{C,t}} = \frac{1}{\delta} \sum_{i=1}^{N} \frac{E_i^{C,t+1}}{\widehat{\phi}_i^{t+1}},$$

$$E_i^{C,t} = \frac{E_i^{C,t+1}}{\delta \widehat{\phi}_i^{t+1} R^{t+1}}$$

to obtain paths for $\left\{R^t\right\}$ and $\left\{E_i^{C,t}\right\}$. Note that, because B_i^1 is decided at $t = 0$, before the shock, $R^1 = R^0 = \frac{1}{\delta}$.

Step 6: Solve for the out-of-steady-state dynamics conditional on aggregate expenditures $\left\{E_i^{C,t}\right\}$.

Step 7: Using the path for disposable income $\left\{I_i^t\right\}_{t=1}^{T_{SS}}$ obtained in Step 6 and equation (7) compute:

$$\left(NX_i^t\right)' = I_i^t - E^{C,t} \text{ for } 1 \le t < T_{SS}$$

$$\left(NX_i^{T_{SS}}\right)' = -\frac{1-\delta}{\delta} \frac{1}{\left(\prod_{\tau=1}^{T_{SS}-1} \left(R^\tau\right)^{-1}\right)} \left(B_i^0 + \sum_{t=1}^{T_{SS}-1} \left(\prod_{\tau=1}^{t} \left(R^\tau\right)^{-1}\right) \left(NX_i^t\right)'\right)$$

Step 8: Compute

$$dist\left(\left\{NX_i^{T_{SS}}\right\}, \left\{\left(NX_i^{T_{SS}}\right)'\right\}\right)$$

Step 9: Update $NX_i^{T_{SS}}$

$$NX_i^{T_{SS}} = (1 - \lambda_o)\, NX_i^{T_{SS}} + \lambda_o \left(NX_i^{T_{SS}} \right)',$$

for a small step size λ_o Go back to Step 4 until convergence of $\left\{ NX_i^{T_{SS}} \right\}$.

C.5 Algorithm: Out-of-Steady-State Transition, Exogenous Deficits (No Bonds)

Consider paths $\left\{A_{k,i}^{t}\right\}_{t=0}^{T_{SS}}$ and $\left\{d_{o,i,k}^{t}\right\}_{t=0}^{T_{SS}}$ with $A_{k,i}^{0}=1$ and $d_{o,i,k}^{0}=1$. Also, consider paths $\left\{\phi_i^t\right\}_{t=0}^{T_{SS}}$ with $\phi_i^0=1$ and $\widehat{\phi}_i^t=1$ for $T\leq t\leq T_{SS}$, for some $T<<T_{SS}$.

We condition on an exogenous path for $\left\{NX_i^t\right\}_{t=1}^{T_{SS}}$.

Step 1: Guess paths $\left\{\widetilde{\lambda}_i^t\right\}_{t=1}^{T_{SS}}$ for each country i.

Step 2: Guess paths $\left\{\widetilde{w}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ for each sector k and country i.

Step 3: Compute $\underline{x}_{k,i}^{T_{SS}}$ consistent with $\widetilde{w}_{k,i}^{T_{SS}}$ and $\widetilde{\lambda}_i^{T_{SS}}$. Obtain $\theta_{k,i}^{T_{SS}}$, $U_{k,i}^{T_{SS}}$, $s_{k\ell,i}^{T_{SS},T_{SS}+1}$ and $\pi_{k,oi}^{T_{SS}}$.

- Step 3a: Compute $\widehat{\widetilde{w}}_{k,i}=\frac{\widetilde{w}_{k,i}^{T_{SS}}}{\widetilde{w}_{k,i}^{0}}$, $\widehat{A}_{k,i}=\frac{A_{k,i}^{T_{SS}}}{A_{k,i}^{0}}$ and $\widehat{d}_{k,i}=\frac{d_{o,i,k}^{T_{SS}}}{d_{o,i,k}^{0}}$. Iteratively solve for $\widehat{P}_{k,i}^{I}$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i}=\left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}}\prod_{\ell=1}^{K}\left(\widehat{P}_{\ell,i}^{I}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^{I}=\left(\sum_{o=1}^{N}\pi_{k,oi}^{0}\widehat{A}_{k,o}\left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 3b: Compute $\widehat{P}_{k,i}^{F}$:

$$\widehat{P}_{i}^{F}=\prod_{k=1}^{K}\left(\widehat{P}_{k,i}^{I}\right)^{\mu_{ki}}$$

- Step 3c: Compute

$$\widehat{\pi}_{k,oi}=\widehat{A}_{k,o}\left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}_{k,i}^{I}}\right)^{-\lambda},$$

  and obtain $\pi_{k,oi}^{T_{SS}}=\pi_{k,oi}^{0}\widehat{\pi}_{k,oi}$

- Step 3d: Compute

  - $\widetilde{\kappa}_{k,i}^{T_{SS}}=\widetilde{\kappa}_{k,i}^{0}\frac{\widehat{P}_i^F}{\widehat{\widetilde{w}}_{k,i}}$

- Step 3e: Guess $\left\{\underline{x}_{k,i}^{T_{SS}}\right\}$

- Step 3f: Compute

$$\theta_{k,i}^{T_{SS}}=q_i^{-1}\left(\widetilde{\kappa}_{k,i}^{T_{SS}}\times\frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right)I_{k,i}\left(\underline{x}_{k,i}^{T_{SS}}\right)}\right)$$

- Step 3g: Compute Bellman Equations

$$U_{k,i}^{T_{SS}} = \zeta_i \log \left(\sum_{k'} \exp \left\{ \frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}^{T_{SS}} \widetilde{\kappa}_{k',i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k',i}^{T_{SS}} \frac{\beta_{k',i}}{(1-\beta_{k',i})} + \delta U_{k',i}^{T_{SS}}}{\zeta_i} \right\} \right)$$

- Step 3h: Compute

$$\left(\underline{x}_{k,i}^{T_{SS}} \right)' = \frac{(1-\delta)\, U_{k,i}^{T_{SS}} - \eta_{k,i}}{\widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k,i}^{T_{SS}}}$$

- Step 3i: Update $\underline{x}_{k,i}^{T_{SS}} = (1-\lambda_x)\, \underline{x}_{k,i}^{T_{SS}} + \lambda_x \left(\underline{x}_{k,i}^{T_{SS}} \right)'$, for a small step size λ_x, and go back to Step 2d until convergence.

- Step 3j: Compute $s_{kk'}^{T_{SS},T_{SS}+1}$

$$s_{k\ell,i}^{T_{SS},T_{SS}+1} = \frac{\exp \left\{ \frac{-C_{k\ell,i} + b_{\ell,i} + \theta_{\ell,i}^{T_{SS}} \widetilde{\kappa}_{\ell,i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\ell,i}^{T_{SS}} \frac{\beta_{\ell,i}}{1-\beta_{\ell,i}} + \delta U_{\ell,i}^{T_{SS}}}{\zeta_i} \right\}}{\sum_{\overline{k}} \exp \left\{ \frac{-C_{k\overline{k},i} + b_{\overline{k},i} + \theta_{\overline{k},i}^{T_{SS}} \widetilde{\kappa}_{\overline{k},i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\overline{k},i}^{T_{SS}} \frac{\beta_{\overline{k},i}}{1-\beta_{\overline{k},i}} + \delta U_{\overline{k},i}^{T_{SS}}}{\zeta_i} \right\}}$$

Step 4: Obtain series $\left\{ \pi_{k,oi}^t \right\}_{t=0}^{T_{SS}}$, $\left\{ \widetilde{\kappa}_{k,i}^t \right\}_{t=0}^{T_{SS}}$. Define $\widehat{x}^t \equiv \frac{x^t}{x^0}$.

- Step 4a: For $t = 1, ..., T_{SS}-1$ compute $\widehat{\widetilde{w}}_{k,i}^t = \frac{\widetilde{w}_{k,i}^t}{\widetilde{w}_{k,i}^0}$ and iteratively solve for $\widehat{P}_{k,i}^{I,t}$ and $\widehat{c}_{k,i}^t$ using the system

$$\widehat{c}_{k,i}^t = \left(\widehat{\widetilde{w}}_{k,i}^t \right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^{I,t} \right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^{I,t} = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o}^t \left(\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t \right)^{-\lambda} \right)^{-1/\lambda}$$

- Step 4b: Compute $\widehat{P}_{k,i}^{F,t}$ for $t = 1, ..., T_{SS}-1$:

$$\widehat{P}_i^{F,t} = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^{I,t} \right)^{\mu_{ki}}$$

- Step 4c: Compute $\widehat{\pi}_{k,oi}^t$ for $t = 1, ..., T_{SS}-1$:

$$\widehat{\pi}_{k,oi}^t = \widehat{A}_{k,o}^t \left(\frac{\widehat{c}_{k,o}^t \widehat{d}_{k,oi}^t}{\widehat{P}_{k,i}^{I,t}} \right)^{-\lambda}$$

- Step 4d: Compute or $t = 1, ..., T_{SS}-1$:

– $\pi^t_{k,oi} = \pi^0_{k,oi}\widehat{\pi}^t_{k,oi}$

– $\widetilde{\kappa}^t_{k,i} \equiv \frac{\kappa_{k,i}P^{F,t}_i}{\widetilde{w}^t_{k,i}} = \frac{\kappa_{k,i}P^{F,0}_i}{\widetilde{w}^0_{k,i}}\frac{P^{F,t}_i}{P^{F,0}_i}\frac{\widetilde{w}^0_{k,i}}{\widetilde{w}^t_{k,i}} = \widetilde{\kappa}^0_{k,i}\frac{\widehat{P}^{F,t}_i}{\widehat{\widetilde{w}}^t_{k,i}}$

Step 5: Given knowledge of $\widetilde{w}^{T_{SS}}_{k,i}$, $\widetilde{\lambda}^{T_{SS}}_i$ and $\underline{x}^{T_{SS}}_{k,i}$ (and therefore $J^{T_{SS}}_{k,i}(s)$), start at $t = T_{SS} - 1$ and sequentially compute (backwards) for each $t = T_{SS} - 1, ..., 1$

- Step 5a: Given $\widetilde{w}^t_{k,i}$, $\widetilde{\lambda}^t_i$, $\underline{x}^{t+1}_{k,i}$, $\widetilde{\kappa}^t_{k,i}$ and $J^{t+1}_{k,i}(s)$ compute $\theta^t_{k,i}$.

If $\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s)dG_{k,i}(s)} \leq 1$ then

$$\theta^t_{k,i} = q_i^{-1}\left(\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s)\,dG_{k,i}(s)}\right)$$

If $\frac{\widetilde{\lambda}^t_i\widetilde{\kappa}^t_{k,i}\widetilde{w}^t_{k,i}}{\delta\widehat{\phi}^{t+1}_i\int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s)dG_{k,i}(s)} > 1$, it is not possible to satisfy $V^t_{k,i} = 0$, so that $V^t_{k,i} < 0$ and

$$\theta^t_{k,i} = 0.$$

- Step 5b: Given $\underline{x}^{t+1}_{k,i}$, $W^{t+1}_{k,i}(x) = \frac{\beta_{k,i}}{1-\beta_{k,i}}J^{t+1}_{k,i}(x) + U^{t+1}_{k,i}$ (for $x \geq \underline{x}^{t+1}_{k,i}$), $\theta^t_{k,i}$, $U^{t+1}_{k,i}$ compute $U^t_{k,i}$. Notice that $\int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} W^{t+1}_{k,i}(s)\,dG_{k,i}(s) = \frac{\beta_{k,i}}{1-\beta_{k,i}}\int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s)\,dG_{k,i}(s) + \left(1 - G_{k,i}\left(\underline{x}^{t+1}_{k,i}\right)\right)U^{t+1}_{k,i}$ so that:

$$U^t_{k,i} = \zeta_i \log\left(\sum_{k'}\exp\left\{\frac{\begin{array}{c}-C_{kk',i} + b_{k',i}\\ +\delta\widehat{\phi}^{t+1}_i\theta^t_{k',i}q_i\left(\theta^t_{k',i}\right)\frac{\beta_{k,i}}{1-\beta_{k,i}}\int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s)\,dG_{k,i}(s) + \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k',i}\end{array}}{\zeta_i}\right\}\right)$$

- Step 5c: Given $\widetilde{\lambda}^t_i$, $J^{t+1}_{k,i}(x)$, $\widetilde{w}^t_{k,i}$, $\theta^t_{k,i}$, δ, $U^t_{k,i}$, $U^{t+1}_{k,i}$ and $\underline{x}^{t+1}_{k,i}$ compute $J^t_{k,i}(x)$

$$\begin{aligned}J^t_{k,i}(x) &= (1-\beta_{k,i})\,\widetilde{\lambda}^t_i\widetilde{w}^t_{k,i}x + (1-\beta_{k,i})\,\eta_{k,i}\\ &\quad - (1-\beta_{k,i})\left(U^t_{k,i} - \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k,i}\right) + (1-\chi_{k,i})\,\delta\widehat{\phi}^{t+1}_i\max\left\{J^{t+1}_{k,i}(x), 0\right\}\end{aligned}$$

- Step 5d: Solve for $\underline{x}^t_{k,i}$: $J^t_{k,i}\left(\underline{x}^t_{k,i}\right) = 0$

Step 6: Compute transition rates $\left\{s^{t,t+1}_{kk',i}\right\}_{t=1}^{T_{SS}-1}$ for all countries i according to:

$$s^{t,t+1}_{kk',i} = \frac{\exp\left\{\begin{array}{c}-C_{kk',i} + b_{k',i} +\\ \delta\widehat{\phi}^{t+1}_i\theta^t_{k',i}q(\theta^t_{k',i})\frac{\beta_{k',i}}{1-\beta_{k',i}}\int_{\underline{x}^{t+1}_{k',i}}^{x_{\max}} J^{t+1}_{k',i}(x)\,dG_{k',i}(x) + \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k',i}\end{array}\right\}}{\sum_{k''}\exp\left\{\begin{array}{c}-C_{kk'',i} + b_{k'',i} +\\ \delta\widehat{\phi}^{t+1}_i\theta^t_{k'',i}q(\theta^t_{k'',i})\frac{\beta_{k'',i}}{1-\beta_{k'',i}}\int_{\underline{x}^{t+1}_{k'',i}}^{x_{\max}} J^{t+1}_{k'',i}(x)\,dG_{k'',i}(x) + \delta\widehat{\phi}^{t+1}_i U^{t+1}_{k'',i}\end{array}\right\}}.$$

Step 7: Start loop over t going forward ($t = 0$ to $t = T_{SS} - 1$)

Initial conditions: we know $\widetilde{u}_{k,i}^{t=-1} = u_{k,i}^{t=0}$, $L_{k,i}^{t=-1} = L_{k,i}^{t=0}$, and $\theta_{k,i}^{t=0}$ from the initial steady state computation. Obtain $\widetilde{u}_{k,i}^{t}$ and $L_{k,i}^{t}$ using flow conditions and sequences $\left\{\theta_{k,i}^{t}\right\}$, $\left\{\underline{x}_{k,i}^{t}\right\}$.

- Step 7a: Compute

$$JC_{k,i}^{t} = L_{k,i}^{t} u_{k,i}^{t} \theta_{k,i}^{t} q_i\left(\theta_{k,i}^{t}\right)\left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)$$

$$JD_{k,i}^{t} = \left(\chi_{k,i} + (1-\chi_{k,i}) \max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}, 0\right\}\right) L_{k,i}^{t-1}\left(1 - \widetilde{u}_{k,i}^{t-1}\right)$$

$$\widetilde{u}_{k,i}^{t} = \frac{L_{k,i}^{t} u_{k,i}^{t} - JC_{k,i}^{t} + JD_{k,i}^{t}}{L_{k,i}^{t}}$$

- Step 7b: Compute

$$L_{k,i}^{t+1} = L_{k,i}^{t} + IF_{k,i}^{t+1} - OF_{k,i}^{t+1},$$

where

$$IF_{k,i}^{t+1} = \sum_{\ell \neq k} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2},$$

and

$$OF_{k,i}^{t+1} = L_{k,i}^{t} \widetilde{u}_{k,i}^{t}\left(1 - s_{kk,i}^{t+1,t+2}\right).$$

- Step 7c: Compute

$$u_{k,i}^{t+1} = \frac{\sum_{\ell=1}^{K} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2}}{L_{k,i}^{t+1}}$$

- Step 7d: Compute

$$\begin{aligned}\widetilde{L}_{k,i}^{t+1} &= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \int_{\underline{x}_{k,i}^{t+1}}^{\infty} \frac{s}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)} dG_{k,i}(s) \\ &= L_{k,i}^{t}\left(1 - \widetilde{u}_{k,i}^{t}\right) \exp\left(\frac{\sigma_{k,i}^{2}}{2}\right) \frac{\Phi\left(\sigma_{k,i} - \frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln \underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}\end{aligned}$$

$$\text{and } Y_{k,i}^{t+1} = \widetilde{w}_{k,i}^{t+1} \widetilde{L}_{k,i}^{t+1}$$

- Step 7e: Compute expenditure with vacancies

$$E_{k,i}^{V,t+1} = \widetilde{\kappa}_{k,i}^{t+1} \widetilde{w}_{k,i}^{t+1} \theta_{k,i}^{t+1} u_{k,i}^{t+1} L_{k,i}^{t+1}$$

- Step 7f: Compute $E_i^{C,t+1} = \frac{\overline{L}_i}{\widetilde{\lambda}_i^{t+1}}$

- Step 7g: Solve for $\left\{ Y_{k,i}^{t+1} \right\}$ in the system

$$E_{k,i}^{t+1} = \mu_{k,i} E_i^{C,t+1} + \sum_{\ell=1}^{K} \left(\mu_{k,i} E_{\ell,i}^{V,t+1} + (1-\gamma_{\ell,i}) \nu_{\ell k,i} Y_{\ell,i}^{t+1} \right).$$

$$Y_{k,o}^{t+1} = \sum_{i=1}^{N} \pi_{k,oi}^{t+1} E_{k,i}^{t+1}.$$

- Step 7h: Normalize $\left\{ Y_{k,i}^{t+1} \right\}$ to make sure it sums to 1 across sectors and countries.

- Step 7i: Compute $\left(\widetilde{w}_{k,i}^{t+1} \right)' = \frac{\gamma_{k,i} Y_{k,i}^{t+1}}{\widetilde{L}_{k,i}^{t+1}}$

Step 8: Compute $dist \left(\left\{ \widetilde{w}_{k,i}^{t} \right\}_{t=1}^{T_{SS}}, \left\{ \left(\widetilde{w}_{k,i}^{t} \right)' \right\}_{t=1}^{T_{SS}} \right)$

Step 9: Update $\widetilde{w}_{k,i}^{t} = (1-\alpha_w) \widetilde{w}_{k,i}^{t} + \alpha_w \left(\widetilde{w}_{k,i}^{t} \right)'$ for $t = 1, ..., T_{SS}$, for a small step size α_w, and go back to Step 3 until convergence of $\left\{ \widetilde{w}_{k,i}^{t} \right\}$

Step 10: Compute disposable income $\left\{ I_i^t \right\}_{t=1}^{T_{SS}}$

$$I_i^t = \sum_{\ell=1}^{K} \left(\gamma_{\ell,i} Y_{\ell,i}^{t} - E_{\ell,i}^{V,t} \right)$$

Step 11: Update $\left\{ E_i^{C,t} \right\}_{t=1}^{T_{SS}}$ using

$$E_i^{C,t} = I_i^t - NX_i^t$$

Step 12a: Compute $\left(\widetilde{\lambda}_i^t \right)' = \frac{\overline{L}_i}{E_i^{C,t}}$ for all $t = 1, ..., T_{SS}$

- Step 12b: Compute $dist \left(\left\{ \widetilde{\lambda}_i^t \right\}_{t=1}^{T_{SS}}, \left\{ \left(\widetilde{\lambda}_i^t \right)' \right\}_{t=1}^{T_{SS}} \right)$

- Step 12c: Update $\widetilde{\lambda}_i^t = (1-\alpha_\lambda) \widetilde{\lambda}_i^t + \alpha_\lambda \left(\widetilde{\lambda}_i^t \right)'$ for $t = 1, ..., T_{SS}$, for a small step size α_λ, and go back to Step 2 until convergence of $\left\{ \widetilde{\lambda}_i^t \right\}$.

C.6 Algorithm: Recovering Shocks

Inner Loop: conditional on paths for expenditures $\left\{E_i^{C,t}\right\}_{t=1}^{T_{SS}}$ and shocks $\left\{\widehat{\phi}_i^t\right\}_{t=2}^{T_{SS}}$, $\left\{\widehat{d}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ and $\widehat{A}_{k,i} \equiv \frac{A_{k,i}^{T_{SS}}}{A_{k,i}^0}$ – determined in the Outer Loop below.

As before, we denote changes relative to $t = 0$ by $\widehat{x}^t = \frac{x^t}{x^0}$. This loop **conditions** on data on $\left\{\widehat{\pi}_{k,oi}^t\right\}_{t=1}^T$ and $\left\{\widehat{P}_{k,i}^{I,t}\right\}_{t=1}^T$, where T is the last period we have data on these variables. We assume $t = 0$ is the estimated steady state. Define $\widehat{d}_{k,i} \equiv \frac{d_{o,i,k}^{T_{SS}}}{d_{o,i,k}^0}$.

Step 1: Given paths $\left\{E_i^{C,t}\right\}$, compute paths $\left\{\widetilde{\lambda}_i^t\right\}$: $\widetilde{\lambda}_i^t = \frac{\overline{L}_i}{E_i^{C,t}}$.

Step 2: Guess paths $\left\{\widetilde{w}_{k,i}^t\right\}_{t=1}^{T_{SS}}$ for each sector k and country i.

Step 3: Compute $\underline{x}_{k,i}^{T_{SS}}$ consistent with $\widetilde{w}_{k,i}^{T_{SS}}$ and $\widetilde{\lambda}_i^{T_{SS}}$. Obtain $\theta_{k,i}^{T_{SS}}$, $U_{k,i}^{T_{SS}}$, $s_{k\ell,i}^{T_{SS},T_{SS}+1}$ and $\pi_{k,oi}^{T_{SS}}$.

- Step 3a: Compute $\widehat{\widetilde{w}}_{k,i} = \frac{\widetilde{w}_{k,i}^{T_{SS}}}{\widetilde{w}_{k,i}^0}$. Iteratively solve for $\widehat{P}_{k,i}^I$ and $\widehat{c}_{k,i}$ using the system

$$\widehat{c}_{k,i} = \left(\widehat{\widetilde{w}}_{k,i}\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^I\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}}$$

$$\widehat{P}_{k,i}^I = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}\widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}$$

- Step 3b: Compute $\widehat{P}_{k,i}^F$:

$$\widehat{P}_i^F = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^I\right)^{\mu_{ki}}$$

- Step 3c: Compute

$$\widehat{\pi}_{k,oi} = \widehat{A}_{k,o} \left(\frac{\widehat{c}_{k,o}\widehat{d}_{k,oi}}{\widehat{P}_{k,i}^I}\right)^{-\lambda}$$

  And obtain $\pi_{k,oi}^{T_{SS}} = \pi_{k,oi}^0 \widehat{\pi}_{k,oi}$

- Step 3d: Compute

  - $\widetilde{\kappa}_{k,i}^{T_{SS}} = \widetilde{\kappa}_{k,i}^0 \frac{\widehat{P}_i^F}{\widehat{\widetilde{w}}_{k,i}}$

- Step 3e: Guess $\left\{\underline{x}_{k,i}^{T_{SS}}\right\}$

- Step 3f: Compute

$$\theta_{k,i}^{T_{SS}} = q_i^{-1}\left(\widetilde{\kappa}_{k,i}^{T_{SS}} \times \frac{1-\delta\left(1-\chi_{k,i}\right)}{\delta\left(1-\beta_{k,i}\right) I_{k,i}\left(\underline{x}_{k,i}^{T_{SS}}\right)}\right)$$

- Step 3g: Compute Bellman Equations

$$U_{k,i}^{T_{SS}} = \zeta_i \log\left(\sum_{k'} \exp\left\{\frac{-C_{kk',i} + b_{k',i} + \theta_{k',i}^{T_{SS}} \widetilde{\kappa}_{k',i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k',i}^{T_{SS}} \frac{\beta_{k',i}}{(1-\beta_{k',i})} + \delta U_{k',i}^{T_{SS}}}{\zeta_i}\right\}\right)$$

- Step 3h: Compute

$$\left(\underline{x}_{k,i}^{T_{SS}}\right)' = \frac{(1-\delta)\, U_{k,i}^{T_{SS}} - \eta_{k,i}}{\widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{k,i}^{T_{SS}}}$$

- Step 3i: Update $\underline{x}_{k,i}^{T_{SS}} = (1-\lambda_x)\, \underline{x}_{k,i}^{T_{SS}} + \lambda_x \left(\underline{x}_{k,i}^{T_{SS}}\right)'$, for a small step size λ_x and go back to Step 2d until convergence.
- Step 3j: Compute $s_{kk'}^{T_{SS},T_{SS}+1}$

$$s_{k\ell,i}^{T_{SS},T_{SS}+1} = \frac{\exp\left\{\frac{-C_{k\ell,i}+b_{\ell,i}+\theta_{\ell,i}^{T_{SS}} \widetilde{\kappa}_{\ell,i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\ell,i}^{T_{SS}} \frac{\beta_{\ell,i}}{1-\beta_{\ell,i}}+\delta U_{\ell,i}^{T_{SS}}}{\zeta_i}\right\}}{\sum_{\bar{k}} \exp\left\{\frac{-C_{k\bar{k},i}+b_{\bar{k},i}+\theta_{\bar{k},i}^{T_{SS}} \widetilde{\kappa}_{\bar{k},i}^{T_{SS}} \widetilde{\lambda}_i^{T_{SS}} \widetilde{w}_{\bar{k},i}^{T_{SS}} \frac{\beta_{\bar{k},i}}{1-\beta_{\bar{k},i}}+\delta U_{\bar{k},i}^{T_{SS}}}{\zeta_i}\right\}}$$

Step 4: Obtain series $\left\{\pi_{k,oi}^t\right\}_{t=T+1}^{T_{SS}}$ and $\left\{\widetilde{\kappa}_{k,i}^t\right\}_{t=1}^{T_{SS}}$.

- Step 4a: For $t = T+1, ..., T_{SS}$ do:

Compute $\widehat{\widetilde{w}}_{k,i}^t = \frac{\widetilde{w}_{k,i}^t}{\widetilde{w}_{k,i}^0}$ and iteratively solve for $\widehat{P}_{k,i}^{I,t}$ and $\widehat{c}_{k,i}^t$ using the system

$$\widehat{c}_{k,i}^t = \left(\widehat{\widetilde{w}}_{k,i}^t\right)^{\gamma_{k,i}} \prod_{\ell=1}^{K} \left(\widehat{P}_{\ell,i}^{I,t}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}},$$

$$\widehat{P}_{k,i}^{I,t} = \left(\sum_{o=1}^{N} \pi_{k,oi}^0 \widehat{A}_{k,o} \left(\widehat{c}_{k,o}^t \widehat{d}_{k,oi}\right)^{-\lambda}\right)^{-1/\lambda}.$$

- Step 4b: Compute $\widehat{P}_{k,i}^{F,t}$ for $t = 1, ..., T_{SS}-1$ (remember $\widehat{P}_{k,i}^{I,t}$ is data for $t = 1, ..., T$):

$$\widehat{P}_i^{F,t} = \prod_{k=1}^{K} \left(\widehat{P}_{k,i}^{I,t}\right)^{\mu_{ki}}$$

- Step 4c: Compute $\widehat{\pi}^{t}_{k,oi}$ and $\pi^{t}_{k,oi}$ for $t = T+1, ..., T_{SS}-1$.

For $t = 1, ..., T_{SS}-1$ do:
First Case: If $t \leq T$ then $\widehat{\pi}^{t}_{k,oi}$ is data, so do:

$$\pi^{t}_{k,oi} = \pi^{0}_{k,oi}\widehat{\pi}^{t}_{k,oi}$$

End of First Case
Second Case if $t \geq T+1$ do:

$$\widehat{\pi}^{t}_{k,oi} = \left(\widehat{A}^{t}_{k,o}\right)' \left(\frac{\widehat{c}^{t}_{k,o}\widehat{d}^{t}_{k,oi}}{\widehat{P}^{I,t}_{k,i}}\right)^{-\lambda}$$

$$\pi^{t}_{k,oi} = \pi^{0}_{k,oi}\widehat{\pi}^{t}_{k,oi}$$

End of Second Case

- Step 4d: Compute for $t = 1, ..., T_{SS}-1$
  - $\widetilde{\kappa}^{t}_{k,i} \equiv \frac{\kappa_{k,i}P^{F,t}_{i}}{\widetilde{w}^{t}_{k,i}} = \frac{\kappa_{k,i}P^{F,0}_{i}}{\widetilde{w}^{0}_{k,i}}\frac{P^{F,t}_{i}}{P^{F,0}_{i}}\frac{\widetilde{w}^{0}_{k,i}}{\widetilde{w}^{t}_{k,i}} = \widetilde{\kappa}^{0}_{k,i}\frac{\widehat{P}^{F,t}_{i}}{\widehat{\widetilde{w}}^{t}_{k,i}}$

Step 5: Given knowledge of $\widetilde{w}^{T_{SS}}_{k,i}$, $\widetilde{\lambda}^{T_{SS}}_{i}$ and $\underline{x}^{T_{SS}}_{k,i}$ (and therefore $J^{T_{SS}}_{k,i}(s)$), start at $t = T_{SS}-1$ and sequentially compute (backwards) for each $t = T_{SS}-1, ..., 1$

- Step 5a: Given $\widetilde{w}^{t}_{k,i}$, $\underline{x}^{t+1}_{k,i}$, $\widetilde{\kappa}^{t}_{k,i}$, $\widetilde{\lambda}^{t}_{i}$ and $J^{t+1}_{k,i}(s)$ compute $\theta^{t}_{k,i}$.

If $\frac{\widetilde{\lambda}^{t}_{i}\widetilde{\kappa}^{t}_{k,i}\widetilde{w}^{t}_{k,i}}{\delta\widehat{\phi}^{t+1}_{i}\int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s)dG_{k,i}(s)} \leq 1$ then

$$\theta^{t}_{k,i} = q^{-1}_{i}\left(\frac{\widetilde{\lambda}^{t}_{i}\widetilde{\kappa}^{t}_{k,i}\widetilde{w}^{t}_{k,i}}{\delta\widehat{\phi}^{t+1}_{i}\int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s)\,dG_{k,i}(s)}\right)$$

If $\frac{\widetilde{\lambda}^{t}_{i}\widetilde{\kappa}^{t}_{k,i}\widetilde{w}^{t}_{k,i}}{\delta\widehat{\phi}^{t+1}_{i}\int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s)dG_{k,i}(s)} > 1$, it is not possible to satisfy $V^{t}_{k,i} = 0$, so that $V^{t}_{k,i} < 0$ and

$$\theta^{t}_{k,i} = 0.$$

- Step 5b: Given $\underline{x}^{t+1}_{k,i}$, $W^{t+1}_{k,i}(x) = \frac{\beta_{k,i}}{1-\beta_{k,i}}J^{t+1}_{k,i}(x) + U^{t+1}_{k,i}$ (for $x \geq \underline{x}^{t+1}_{k,i}$), $\theta^{t}_{k,i}$, $U^{t+1}_{k,i}$ compute $U^{t}_{k,i}$. Notice that $\int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} W^{t+1}_{k,i}(s)\,dG_{k,i}(s) = \frac{\beta_{k,i}}{1-\beta_{k,i}}\int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s)\,dG_{k,i}(s) + \left(1 - G_{k,i}\left(\underline{x}^{t+1}_{k,i}\right)\right)U^{t+1}_{k,i}$ so that:

$$U^{t}_{k,i} = \zeta_{i}\log\left(\sum_{k'}\exp\left\{\frac{\begin{array}{c}-C_{kk',i} + b_{k',i}\\ +\delta\widehat{\phi}^{t+1}_{i}\theta^{t}_{k',i}q_{i}\left(\theta^{t}_{k',i}\right)\frac{\beta_{k,i}}{1-\beta_{k,i}}\int_{\underline{x}^{t+1}_{k,i}}^{x_{\max}} J^{t+1}_{k,i}(s)\,dG_{k,i}(s) + \delta\widehat{\phi}^{t+1}_{i}U^{t+1}_{k',i}\end{array}}{\zeta_{i}}\right\}\right)$$

- Step 5c: Given $J_{k,i}^{t+1}(x)$, $\widetilde{w}_{k,i}^{t}$, $U_{k,i}^{t}$, $U_{k,i}^{t+1}$ and $\underline{x}_{k,i}^{t+1}$ compute $J_{k,i}^{t}(x)$

$$\begin{aligned} J_{k,i}^{t}(x) &= (1-\beta_{k,i})\,\widetilde{\lambda}_i^t \widetilde{w}_{k,i}^{t} x + (1-\beta_{k,i})\,\eta_{k,i} \\ &\quad - (1-\beta_{k,i})\left(U_{k,i}^{t} - \delta\widehat{\phi}_i^{t+1} U_{k,i}^{t+1}\right) + (1-\chi_{k,i})\,\delta\widehat{\phi}_i^{t+1} \max\left\{J_{k,i}^{t+1}(x), 0\right\} \end{aligned}$$

- Step 5d: Solve for $\underline{x}_{k,i}^{t}$: $J_{k,i}^{t}\left(\underline{x}_{k,i}^{t}\right) = 0$

Step 6: Compute transition rates $\left\{s_{kk',i}^{t,t+1}\right\}_{t=1}^{T_{SS}-1}$ for all countries i according to:

$$s_{kk',i}^{t,t+1} = \frac{\exp\left\{\begin{array}{c} -C_{kk',i} + b_{k',i} + \\ \delta\widehat{\phi}_i^{t+1}\theta_{k',i}^{t} q(\theta_{k',i}^{t})\frac{\beta_{k',i}}{1-\beta_{k',i}}\int_{\underline{x}_{k',i}^{t+1}}^{x_{\max}} J_{k',i}^{t+1}(x)\,dG_{k',i}(x) + \delta\widehat{\phi}_i^{t+1} U_{k',i}^{t+1} \end{array}\right\}}{\sum_{k''}\exp\left\{\begin{array}{c} -C_{kk'',i} + b_{k'',i} + \\ \delta\widehat{\phi}_i^{t+1}\theta_{k'',i}^{t} q(\theta_{k'',i}^{t})\frac{\beta_{k'',i}}{1-\beta_{k'',i}}\int_{\underline{x}_{k'',i}^{t+1}}^{x_{\max}} J_{k'',i}^{t+1}(x)\,dG_{k'',i}(x) + \delta\widehat{\phi}_i^{t+1} U_{k'',i}^{t+1} \end{array}\right\}}.$$

Step 7: Start loop over t going forward ($t = 0$ to $t = T_{SS} - 1$)

Initial conditions: we know $\widetilde{u}_{k,i}^{t=-1} = u_{k,i}^{t=0}$, $L_{k,i}^{t=-1} = L_{k,i}^{t=0}$, and $\theta_{k,i}^{t=0}$ from the initial steady state computation. Obtain $\widetilde{u}_{k,i}^{t}$ and $L_{k,i}^{t}$ using flow conditions and sequences $\left\{\theta_{k,i}^{t}\right\}$, $\left\{\underline{x}_{k,i}^{t}\right\}$.

- Step 7a: Compute

$$JC_{k,i}^{t} = L_{k,i}^{t} u_{k,i}^{t} \theta_{k,i}^{t} q_i\left(\theta_{k,i}^{t}\right)\left(1 - G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)\right)$$

$$JD_{k,i}^{t} = \left(\chi_{k,i} + (1-\chi_{k,i})\max\left\{\frac{G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right) - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}{1 - G_{k,i}\left(\underline{x}_{k,i}^{t}\right)}, 0\right\}\right) L_{k,i}^{t-1}\left(1 - \widetilde{u}_{k,i}^{t-1}\right)$$

$$\widetilde{u}_{k,i}^{t} = \frac{L_{k,i}^{t} u_{k,i}^{t} - JC_{k,i}^{t} + JD_{k,i}^{t}}{L_{k,i}^{t}}$$

- Step 7b: Compute

$$L_{k,i}^{t+1} = L_{k,i}^{t} + IF_{k,i}^{t+1} - OF_{k,i}^{t+1},$$

where

$$IF_{k,i}^{t+1} = \sum_{\ell \neq k} L_{\ell,i}^{t} \widetilde{u}_{\ell,i}^{t} s_{\ell k,i}^{t+1,t+2},$$

and

$$OF_{k,i}^{t+1} = L_{k,i}^{t} \widetilde{u}_{k,i}^{t}\left(1 - s_{kk,i}^{t+1,t+2}\right).$$

- Step 7c: Compute

$$u_{k,i}^{t+1}=\frac{\sum_{\ell=1}^{K}L_{\ell,i}^{t}\widetilde{u}_{\ell,i}^{t}s_{\ell k,i}^{t+1,t+2}}{L_{k,i}^{t+1}}$$

- Step 7d: Compute

$$\begin{aligned}\widetilde{L}_{k,i}^{t+1}&=L_{k,i}^{t}\left(1-\widetilde{u}_{k,i}^{t}\right)\int_{\underline{x}_{k,i}^{t+1}}^{\infty}\frac{s}{1-G_{k,i}\left(\underline{x}_{k,i}^{t+1}\right)}dG_{k,i}\left(s\right)\\&=L_{k,i}^{t}\left(1-\widetilde{u}_{k,i}^{t}\right)\exp\left(\frac{\sigma_{k,i}^{2}}{2}\right)\frac{\Phi\left(\sigma_{k,i}-\frac{\ln\underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}{\Phi\left(-\frac{\ln\underline{x}_{k,i}^{t+1}}{\sigma_{k,i}}\right)}\end{aligned}$$

- Step 7e: Compute expenditure with vacancies

$$E_{k,i}^{V,t+1}=\widetilde{\kappa}_{k,i}^{t+1}\widetilde{w}_{k,i}^{t+1}\theta_{k,i}^{t+1}u_{k,i}^{t+1}L_{k,i}^{t+1}$$

- Step 7f: Solve for $\left\{Y_{k,i}^{t+1}\right\}$ in the system

$$E_{k,i}^{t+1}=\mu_{k,i}E_{i}^{C,t+1}+\sum_{\ell=1}^{K}\left(\mu_{k,i}E_{\ell,i}^{V,t+1}+\left(1-\gamma_{\ell,i}\right)\nu_{\ell k,i}Y_{\ell,i}^{t+1}\right).$$

$$Y_{k,o}^{t+1}=\sum_{i=1}^{N}\pi_{k,oi}^{t+1}E_{k,i}^{t+1}.$$

- Step 7g: Compute $\left(\widetilde{w}_{k,i}^{t+1}\right)'=\frac{\gamma_{k,i}Y_{k,i}^{t+1}}{\widetilde{L}_{k,i}^{t+1}}$

Step 8: Compute distance $dist\left(\left\{\widetilde{w}_{k,i}^{t}\right\},\left\{\left(\widetilde{w}_{k,i}^{t}\right)'\right\}\right)$

- Step 8b: Update $\widetilde{w}_{k,i}^{t}=\left(1-\lambda_{w}\right)\widetilde{w}_{k,i}^{t}+\lambda_{w}\left(\widetilde{w}_{k,i}^{t}\right)'$ $t=1,...,T_{SS}$, for a small step size λ_{w}.
- Step 8c: At this point, we have a new series for $\left\{\widetilde{w}_{k,i}^{t}\right\}$ – go back to Step 3 until convergence of $\left\{\widetilde{w}_{k,i}^{t}\right\}$.

Step 9: Compute disposable income $\left\{I_{i}^{t}\right\}_{t=1}^{T_{SS}}$

$$I_{i}^{t}=\sum_{\ell=1}^{K}\left(\gamma_{\ell,i}Y_{\ell,i}^{t}-E_{\ell,i}^{V,t}\right)$$

Step 10: Compute $\left\{\left(\widehat{A}^t_{k,i}\right)'\right\}$.

For $t=1,...,T$ compute $\widehat{\widetilde{w}}^t_{k,i}=\frac{\widetilde{w}^t_{k,i}}{\widetilde{w}^0_{k,i}}$, obtain $\widehat{c}^t_{k,i}$:

$$\widehat{c}^t_{k,i}=\left(\widehat{\widetilde{w}}^t_{k,i}\right)^{\gamma_{k,i}}\prod_{\ell=1}^{K}\left(\widehat{P}^{I,t}_{\ell,i}\right)^{(1-\gamma_{k,i})\nu_{k\ell,i}},$$

and compute $\left(\widehat{A}^t_{k,i}\right)'$:

$$\left(\widehat{A}^t_{k,i}\right)'=\frac{\widehat{\pi}^t_{k,ii}}{\left(\widehat{P}^{I,t}_{k,i}\right)^{\lambda}}\left(\widehat{c}^t_{k,i}\right)^{\lambda}.$$

For $t\geq T+1$ set:

$$\left(\widehat{A}^t_{k,i}\right)'=\left(\widehat{A}^T_{k,i}\right)'$$

Feed outer loop with $\left\{\left(\widehat{A}^{T_{SS}}_{k,i}\right)'\right\}$.

Outer Loop: iteration on $\{NX^t_i\}$

Step 0: Compute changes in trade costs $\left\{\widehat{d}^t_{k,oi}\right\}_{t=1}^{T}$:

$$\widehat{d}^t_{k,oi}=\left(\frac{\widehat{\pi}^t_{k,oo}}{\widehat{\pi}^t_{k,oi}}\right)^{1/\lambda}\frac{\widehat{P}^{I,t}_{k,i}}{\widehat{P}^{I,t}_{k,o}}$$

Set $\widehat{d}^t_{k,oi}=\widehat{d}^T_{k,oi}$ for $t>T$.

Step 1: Start with estimated state equilibrium at $t=0$. Remember that we used the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K}Y_{k,i}=1$ during the estimation procedure. **Change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K}Y_{k,i}=1$ to $\sum_{i=1}^{I}E^C_i=1$. Nominal variables to be renormalized: $\left\{Y^0_{k,i}\right\}$, $\left\{\widetilde{w}^0_{k,i}\right\}$, $\left\{E^{C,0}_i\right\}$, $\{NX^0_i\}$.

Step 2: Compute $E^{C,t}_i=\frac{E^{C,0}_i\left(\widehat{E}^{C,t}_i\right)_{Data}}{\sum_{i=1}^{N}E^{C,0}_i\left(\widehat{E}^{C,t}_i\right)_{Data}}$ for $t=1,...,T$ where $E^{C,0}_i$ is aggregate consumption expenditure in the estimated steady state, and $\left(\widehat{E}^{C,t}_i\right)_{Data}$ comes from the data.

Step 3: Normalize $\widehat{\phi}^t_{US}=1$ for $t=2,...,T-1$. This yields

$$R^{t+1}=\frac{E^{C,t+1}_{US}}{\delta E^{C,t}_{US}}\text{ for }t=1,...,T-1$$

Obtain remaining shocks $\left\{\widehat{\phi}^t_i\right\}_{t=2}^{T}$ using

$$\widehat{\phi}^{t+1}_i=\frac{E^{C,t+1}_i}{\delta E^{C,t}_iR^{t+1}}\text{ for }t=1,...,T-1$$

Set $\widehat{\phi}_i^t = 1$ for $t \geq T+2$. The value of $\widehat{\phi}_i^{T+1}$ will depend on $E_i^{C,T_{SS}}$ and will be recovered in Step 7. Note: the value of $\widehat{\phi}_i^{t=1}$ does not matter. Individuals made decisions at $t=0$ assuming $\widehat{\phi}_i^{t=1} = 1$, as the economy was assumed to be in steady state at $t=0$.

Step 4: Obtain B_i^0 with respect to the normalization $\sum_{i=1}^{I} E_i^C = 1$. Equation (37) gives us:

$$B_i^0 = \frac{NX_i^0}{(1 - \frac{1}{\delta})}$$

Step 5: Make initial guess for $NX_i^{T_{SS}}$ (with respect to the normalization $\sum_{i=1}^{I} E_i^C = 1$) and for $\widehat{A}_{k,i}^{T_{SS}} = \widehat{A}_{k,i}^T \equiv \frac{A_{k,i}^T}{A_{k,i}^0}$.

Step 6: Compute steady state equilibrium at T_{SS}, conditional on $NX_i^{T_{SS}}$, $\widehat{A}_{k,i}^{T_{SS}}$ and $\widehat{d}_{k,oi}^{T_{SS}}$.

- Step 6a: Notice that the steady-state algorithm uses the normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. Normalize $NX_i^{T_{SS}}$ with respect to normalization $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$. To perform such normalization, use revenue $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in the initial steady state if this is the first outer loop iteration, otherwise use revenue $\left\{Y_{k,i}^{T_{SS}}\right\}$ obtained in Step 9 below.

- Step 6b: After computing the final steady state, **change the normalization** from $\sum_{i=1}^{I}\sum_{k=1}^{K} Y_{k,i} = 1$ to $\sum_{i=1}^{I} E_i^C = 1$ using $\left\{E_i^C\right\}$ obtained in Step 3a. Nominal variables to be renormalized: $\left\{Y_{k,i}^{T_{SS}}\right\}$, $\left\{\widetilde{w}_{k,i}^{T_{SS}}\right\}$, $\left\{E_i^{C,T_{SS}}\right\}$, $\left\{NX_i^{T_{SS}}\right\}$.

Step 7: Set $E_i^{C,t} = \begin{cases} E_i^{C,0}\left(\frac{E_i^{C,2000+t}}{E_i^{C,2000}}\right)_{Data} & \text{for } t = 0, ..., T, \text{ as in Step 0} \\ E_i^{C,T_{SS}} & \text{for } t = T+1, ..., T_{SS} \end{cases}$

Note: $\widehat{\phi}_i^t = 1$ for $t \geq T+2 \Rightarrow E_i^{C,t} = E_i^{C,T_{SS}}$ for at $t \geq T+1$ (Euler Equation)

Step 8: Compute $\widehat{\phi}_i^{T+1}$, imposing $\widehat{\phi}_{US}^{T+1} = 1$:

$$\widehat{\phi}_i^{T+1} = \frac{E_i^{C,T_{SS}}}{\delta R^{T+1} E_i^{C,T}},$$

$$R^{T+1} = \frac{E_{US}^{C,T_{SS}}}{\delta E_{US}^{C,T}}.$$

We now have a full path for $\left\{\widehat{\phi}_i^t\right\}_{t=2}^{T_{SS}}$.

Step 9: Solve for the out-of-steady-state dynamics conditional on aggregate expenditures $\left\{E_i^{C,t}\right\}_{t=0}^{T_{SS}}$, on preference shifters $\left\{\widehat{\phi}_i^t\right\}_{t=2}^{T_{SS}}$ and steady-state productivity shocks $\left\{\widehat{A}_{k,i}^{T_{SS}}\right\}$.

Step 10: Using the path for disposable income $\left\{I_i^t\right\}_{t=1}^{T_{SS}}$ obtained in Step 9 and equation (7) compute:

$$\left(NX_i^t\right)' = I_i^t - E^{C,t} \text{ for } 1 \leq t < T_{SS}$$

$$\left(NX_i^{T_{SS}}\right)' = -\frac{1-\delta}{\delta}\frac{1}{\left(\prod_{\tau=1}^{T_{SS}-1}\left(R^\tau\right)^{-1}\right)}\left(B_i^0 + \sum_{t=1}^{T_{SS}-1}\left(\prod_{\tau=1}^{t}\left(R^\tau\right)^{-1}\right)\left(NX_i^t\right)'\right)$$

Step 11: Compute

$$dist\left(\left\{NX_i^{T_{SS}}\right\},\left\{\left(NX_i^{T_{SS}}\right)'\right\}\right) \text{ and}$$

$$dist\left(\left\{\widehat{A}_{k,i}^{T_{SS}}\right\},\left\{\left(\widehat{A}_{k,i}^{T_{SS}}\right)'\right\}\right),$$

using the values of $\left\{\left(\widehat{A}_{k,i}^{T_{SS}}\right)'\right\}$ obtained in Step 9.

Step 12: Update $NX_i^{T_{SS}}$

$$NX_i^{T_{SS}} = \left(1-\lambda_o\right)NX_i^{T_{SS}} + \lambda_o\left(NX_i^{T_{SS}}\right)'$$

and $\widehat{A}_{k,i}^{T_{SS}}$

$$\widehat{A}_{k,i}^{T_{SS}} = \left(1-\lambda_A\right)\widehat{A}_{k,i}^{T_{SS}} + \lambda_A\left(\widehat{A}_{k,i}^{T_{SS}}\right)'$$

for small step sizes λ_o and λ_A.
Go back to Step 6 until convergence of $\left\{NX_i^{T_{SS}}\right\}$ and $\left\{\widehat{A}_{k,i}^{T_{SS}}\right\}$.

C.7 Parameter Estimates

In this section, we display the complete set of parameter estimates.

Table A.1: Final Expenditure Shares $\mu_{k,i}$

Sector ↓ Country →	USA	China	Europe	Asia/Oceania	Americas	ROW
Agr.	0.01	0.12	0.02	0.01	0.02	0.09
LT Manuf.	0.03	0.02	0.04	0.03	0.04	0.03
MT Manuf.	0.05	0.11	0.08	0.07	0.1	0.11
HT Manuf.	0.1	0.15	0.11	0.1	0.11	0.1
LT Serv.	0.3	0.35	0.34	0.38	0.33	0.39
HT Serv.	0.51	0.25	0.41	0.41	0.4	0.29

Table A.2: Labor Shares in Production $\gamma_{k,i}$

Sector ↓ Country →	USA	China	Europe	Asia/Oceania	Americas	ROW
Agr.	0.45	0.58	0.56	0.54	0.62	0.67
LT Manuf.	0.37	0.25	0.32	0.35	0.28	0.27
MT Manuf.	0.33	0.28	0.31	0.37	0.32	0.28
HT Manuf.	0.39	0.24	0.33	0.32	0.31	0.25
LT Serv.	0.61	0.37	0.49	0.54	0.56	0.48
HT Serv.	0.62	0.55	0.63	0.67	0.67	0.68

Table A.3: Input-Output Table – Average Across Countries $\frac{1}{N}\sum_i \nu_{k\ell,i}$

User ↓ Supplier →	Agr.	LT Manuf.	MT Manuf.	HT Manuf.	LT Serv.	HT Serv.
Agr.	0.267	0.079	0.118	0.138	0.26	0.139
	(0.056)	(0.019)	(0.036)	(0.029)	(0.06)	(0.07)
LT Manuf.	0.195	0.376	0.043	0.081	0.223	0.082
	(0.047)	(0.064)	(0.009)	(0.015)	(0.041)	(0.04)
MT Manuf.	0.222	0.066	0.287	0.106	0.225	0.095
	(0.034)	(0.017)	(0.04)	(0.022)	(0.052)	(0.044)
HT Manuf.	0.022	0.157	0.067	0.463	0.184	0.106
	(0.018)	(0.022)	(0.015)	(0.052)	(0.044)	(0.05)
LT Serv.	0.057	0.136	0.105	0.101	0.343	0.259
	(0.042)	(0.03)	(0.027)	(0.034)	(0.073)	(0.11)
HT Serv.	0.007	0.076	0.033	0.11	0.267	0.507
	(0.005)	(0.023)	(0.019)	(0.071)	(0.069)	(0.171)

Note: Standard Deviation of $\nu_{k\ell}$ across countries.

Table A.4: Mobility Costs Estimates $C_{k\ell}$

From ↓ / To →	Agr.	LT Manuf.	MT Manuf.	HT Manuf.	LT Serv.	HT Serv.
Agriculture	0	0.825	1.560	0.454	0.189	1.676
LT Manufacturing	0.414	0	0.005	0.000	0.799	2.034
MT Manufacturing	2.033	0.000	0	0.002	0.866	2.646
HT Manufacturing	0.015	0.001	0.003	0	0.276	0.917
LT Services	0.268	0.972	1.221	0.466	0	0.002
HT Services	0.790	1.826	2.150	1.201	0.004	0

Table A.5: Sector-Specific Utility and Variance Estimates $\eta_{k,i}, \sigma^2_{k,i}$

Sector ↓ / Country →	USA	China	Europe	Asia/Oceania	Americas	ROW
Agriculture	0	0	0	0	0	0
LT Manuf.	-0.383	-0.943	-0.428	-0.521	-0.588	-0.316
MT Manuf.	0.026	-0.566	-0.245	-0.229	-0.195	-0.072
HT Manuf.	-0.551	-1.557	-0.743	-0.607	-1.292	-0.934
LT Services	0.085	-0.454	-0.382	-0.236	-0.405	-0.588
HT Services	-0.052	-0.467	-0.603	-0.518	-0.894	-1.011
σ^2_{US}				0.727		

Table A.6: Exogenous Death Rates Estimates $\chi_{k,} = \chi_i + \chi_k$

Country Component χ_i	USA	0.026
	China	0.022
	Europe	0.054
	Asia/Oceania	0.021
	Americas	0.027
	ROW	0.020
Sector Component χ_k	Agriculture	0
	Low Tech Man	0.005
	Med Tech Man	0.014
	High Tech Man	0.002
	Other Services	0.008
	Hi Tech Services	-0.001

Table A.7: Unemployment Utility $b_{k,i} = b_i$

USA	-14.4
China	-12.7
Europe	-7.1
Asia/Oceania	-8.5
Americas	-8.6
ROW	-14.9

Table A.8: Vacancy Posting Costs Estimates $\widetilde{\kappa}_{k,i}$

Sector ↓ / Country →	USA	China	Europe	Asia/Oceania	Americas	ROW
Agriculture	0.539	0.450	0.451	0.664	0.451	0.452
LT Manuf.	0.632	0.810	0.681	0.865	0.809	0.583
MT Manuf.	0.472	0.615	0.549	0.660	0.540	0.447
HT Manuf.	0.698	0.984	0.821	0.922	1.076	0.830
LT Services	0.449	0.617	0.592	0.660	0.649	0.639
HT Services	0.511	0.674	0.672	0.840	0.872	0.813