Appendix A Proofs for Static Model

In this appendix, we provide derivations for the one-period model. We start by expressing the full set of equilibrium conditions as a function of endogenous equilibrium objects and exogenous parameters. We then characterize the response of firm markups, quantities, and sales shares in response to a nominal wage shock. By aggregating across firms, we derive the changes in aggregate variables, which yields a fixed point in changes in output.

A.1 Equilibrium conditions

Note that we can express each firm's output y_θ and price p_θ as a function of the sales density λ_θ and markup μ_θ:

$$y_\theta = \frac{\bar{\mu} L \lambda_\theta A_\theta}{\mu_\theta} \quad \text{and} \quad \frac{p_\theta}{w} = \frac{\mu_\theta}{A_\theta}. \tag{24}$$

This allows us to write all equilibrium conditions in terms of the endogenous equilibrium variables $\lambda_\theta, \mu_\theta, P, \bar{\mu}, L$, and Y, and the exogenous parameters $A_\theta, \delta_\theta, w, \zeta$, and γ. We restate the equilibrium conditions from the main text.

1. *Consumers maximize utility.* Consumer demand for each good is given by the inverse-demand curve in Equation (1), which becomes

$$\frac{\mu_\theta}{A_\theta} = wP\Upsilon'_\theta\left(\frac{\bar{\mu} L \lambda_\theta A_\theta}{\mu_\theta Y}\right), \tag{25}$$

   with the constraint on utility from the consumption bundle, Y, given in Section 2.1, expressed as,

$$1 = \mathbb{E}\left[\Upsilon_\theta\left(\frac{\bar{\mu} L \lambda_\theta A_\theta}{\mu_\theta Y}\right)\right]. \tag{26}$$

   The aggregate markup is

$$\bar{\mu} = \mathbb{E}_\lambda\left[\mu_\theta^{-1}\right]^{-1} = \frac{\Pi}{wL + \Pi}. \tag{27}$$

   Finally, the consumer maximizes overall utility from the consumption bundle and labor supply such that

$$\bar{\mu} L^{1+\frac{1}{\zeta}} = Y^{1-\gamma}. \tag{28}$$

2. *Firms maximize profits.* Flexible-price firms set markups according to the Lerner

formula,

$$\mu_\theta^{\text{flex}} = \mu_\theta(\bar{\mu}\frac{L\lambda_\theta A_\theta}{\mu_\theta Y}), \tag{29}$$

while the markups of sticky-price firms move due to changes in marginal cost that are out of the firm's control,

$$\mu_\theta^{\text{sticky}} = \frac{w_{t-1}}{w_t}\frac{A_{\theta,t}}{A_{\theta,t-1}}\mu_{\theta,t-1}. \tag{30}$$

3. *Markets clear.* The price aggregator satisfies

$$\frac{1}{P} = \frac{Y}{wL}\mathbb{E}\left[\frac{L\lambda_\theta A_\theta}{\mu_\theta Y}\Upsilon'_\theta(\frac{\bar{\mu}L\lambda_\theta A_\theta}{\mu_\theta Y})\right]. \tag{31}$$

A.2 Response to monetary policy shocks

We consider a shock to the nominal wage given by $d\log w$. All other exogenous primitives (A_θ, δ_θ, ζ, and γ) are unchanged by the shock. First, we characterize changes in firm-level markups $d\log\mu_\theta$, quantities $d\log y_\theta$, and sales shares $d\log\lambda_\theta$.

Markups. The markup of a firm of type θ satisfies

$$d\log\mu_\theta = d\log p_\theta - d\log w. \tag{32}$$

The prices of sticky-price firms do not change, hence

$$d\log\mu_\theta^{\text{sticky}} = -d\log w. \tag{33}$$

For flexible-price firms,

$$d\log p_\theta^{\text{flex}} = d\log\mu_\theta(\frac{y_\theta}{Y}) + d\log w \tag{34}$$

$$= \frac{\frac{y_\theta}{Y}\mu'_\theta(\frac{y_i}{Y})}{\mu_\theta}d\log(\frac{y_\theta}{Y}) + d\log w \tag{35}$$

$$= \frac{1-\rho_\theta}{\rho_\theta}\frac{1}{\sigma_\theta}d\log(\frac{y_\theta}{Y}) + d\log w \tag{36}$$

$$= (1-\rho_\theta)d\log P + \rho_\theta d\log w. \tag{37}$$

This yields,

$$d\log\mu_\theta^{\text{flex}} = (1-\rho_\theta)(d\log P - d\log w). \tag{38}$$

Quantities. We have

$$d\log(\frac{y_\theta}{Y}) = -\sigma_\theta\left(d\log p_\theta - d\log P\right) \tag{39}$$

$$= -\sigma_\theta\left(d\log \mu_\theta + d\log w - d\log P\right). \tag{40}$$

This yields,

$$d\log(\frac{y_\theta^{\text{sticky}}}{Y}) = \sigma_\theta\left(d\log P\right), \tag{41}$$

$$d\log(\frac{y_\theta^{\text{flex}}}{Y}) = \sigma_\theta\rho_\theta\left(d\log P - d\log w\right). \tag{42}$$

Sales shares. We have

$$d\log\lambda_\theta = d\log(\frac{y_\theta}{Y}) + d\log\mu_\theta + d\log Y - d\log\bar{\mu} - d\log L. \tag{43}$$

This yields,

$$d\log\lambda_\theta^{\text{flex}} = \left(d\log Y - d\log\bar{\mu} - d\log L\right) + \left(1 + (\sigma_\theta - 1)\rho_\theta\right)\left(d\log P - d\log w\right), \tag{44}$$

$$d\log\lambda_\theta^{\text{sticky}} = \left(d\log Y - d\log\bar{\mu} - d\log L\right) - d\log w + \sigma_\theta d\log P. \tag{45}$$

Now, we aggregate across firms to get the change in aggregate variables, $d\log\bar{\mu}$, $d\log P$, $d\log Y$, and $d\log L$.

Aggregate markup. We start with the equation for the change in the aggregate markup,

$$d\log\bar{\mu} = \mathbb{E}_{\lambda\mu^{-1}}\left[d\log\mu_\theta\right] - \mathbb{E}_{\lambda\mu^{-1}}\left[d\log\lambda_\theta\right]. \tag{46}$$

We use

$$\begin{aligned}\mathbb{E}_{\lambda\mu^{-1}}\left[d\log\lambda_\theta\right] = &\left(d\log Y - d\log\bar{\mu} - d\log L\right)\\ &+ \mathbb{E}_{\lambda\mu^{-1}}\left[\delta_\theta\left[\left(1 + (\sigma_\theta - 1)\rho_\theta\right)\left(d\log P - d\log w\right)\right]\right]\\ &+ \mathbb{E}_{\lambda\mu^{-1}}\left[(1 - \delta_\theta)\left[-d\log w + \sigma_\theta d\log P\right]\right],\end{aligned} \tag{47}$$

which simplifies to

$$\begin{aligned}\mathbb{E}_{\lambda\mu^{-1}}\left[d\log\lambda_\theta\right] = &\left(d\log Y - d\log L - d\log\bar{\mu}\right)\\ &+ \mathbb{E}_{\lambda\mu^{-1}}\left[\delta_\theta\left(1 + (\sigma_\theta - 1)\rho_\theta\right) + (1 - \delta_\theta)\sigma_\theta\right]d\log P\end{aligned}$$

$$+\mathbb{E}_{\lambda\mu^{-1}}\left[(-1-\delta_\theta(\sigma_\theta-1)\rho_\theta)\right]d\log w, \quad (48)$$

and

$$\mathbb{E}_{\lambda\mu^{-1}}\left[d\log\mu_\theta\right] = \mathbb{E}_{\lambda\mu^{-1}}\left[\delta_\theta\left[(1-\rho_\theta)(d\log P - d\log w)\right]\right] + \mathbb{E}_{\lambda\mu^{-1}}\left[(1-\delta_\theta)\left[-d\log w\right]\right] \quad (49)$$

$$= \mathbb{E}_{\lambda\mu^{-1}}\left[\delta_\theta(1-\rho_\theta)\right]d\log P - \mathbb{E}_{\lambda\mu^{-1}}\left[1-\delta_\theta\rho_\theta\right]d\log w. \quad (50)$$

Combining yields,

$$d\log Y - d\log L = -\mathbb{E}_{\lambda\mu^{-1}}\left[\left[\delta_\theta\rho_\theta + (1-\delta_\theta)\right]\sigma_\theta\right]d\log P + \mathbb{E}_{\lambda\mu^{-1}}\left[\delta_\theta\sigma_\theta\rho_\theta\right]d\log w. \quad (51)$$

Price index. We have

$$d\log P = -d\log Y + d\log L + d\log\bar{\mu} + d\log w - \mathbb{E}_\lambda\left[\left(1-\frac{1}{\sigma_\theta}\right)d\log(\frac{y_\theta}{Y})\right]. \quad (52)$$

Using

$$\mathbb{E}_\lambda\left[\left(1-\frac{1}{\sigma_\theta}\right)d\log(\frac{y_\theta}{Y})\right] = \mathbb{E}_\lambda\left[\delta_\theta\left(1-\frac{1}{\sigma_\theta}\right)\sigma_\theta\rho_\theta\left(d\log P - d\log w\right)\right] \quad (53)$$

$$+\mathbb{E}_\lambda\left[(1-\delta_\theta)\left(1-\frac{1}{\sigma_\theta}\right)\sigma_\theta\left(d\log P\right)\right] \quad (54)$$

$$= \mathbb{E}_\lambda\left[\left[1+\delta_\theta\left(\rho_\theta-1\right)\right]\left(\sigma_\theta-1\right)\right]d\log P - \mathbb{E}_\lambda\left[\delta_\theta\rho_\theta\left(\sigma_\theta-1\right)\right]d\log w, \quad (55)$$

we get

$$d\log P = \frac{-\left(d\log Y - d\log L - d\log\bar{\mu}\right) + \mathbb{E}_\lambda\left[1+\delta_\theta\rho_\theta\left(\sigma_\theta-1\right)\right]d\log w}{1+\mathbb{E}_\lambda\left[\left(\sigma_\theta-1\right)\left[(1-\delta_\theta)+\delta_\theta\rho_\theta\right]\right]}. \quad (56)$$

Output. We have

$$d\log Y = d\log L + d\log\bar{\mu} - \mathbb{E}_\lambda\left[d\log\mu_\theta\right] \quad (57)$$

$$= d\log L + d\log\bar{\mu} - \mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]d\log P + \mathbb{E}_\lambda\left[1-\delta_\theta\rho_\theta\right]d\log w. \quad (58)$$

Labor. We have

$$d\log\bar{\mu} + \left(1+\frac{1}{\zeta}\right)d\log L = (1-\gamma)\,d\log Y, \quad (59)$$

which we rearrange to get

$$d\log L = \frac{\zeta(1-\gamma)}{1+\zeta} d\log Y - \frac{\zeta}{1+\zeta} d\log \bar{\mu}. \tag{60}$$

Solving the fixed point. We have a system of four equations in four unknowns, $d\log Y$, $d\log L$, $d\log \bar{\mu}$, and $d\log P$:

$$d\log Y = d\log L + d\log\bar{\mu} - \mathbb{E}_{\lambda}\left[\delta_\theta(1-\rho_\theta)\right] d\log P + \mathbb{E}_{\lambda}\left[1-\delta_\theta\rho_\theta\right] d\log w \tag{61}$$

$$d\log L = \frac{\zeta(1-\gamma)}{1+\zeta} d\log Y - \frac{\zeta}{1+\zeta} d\log \bar{\mu} \tag{62}$$

$$d\log Y - d\log L = -\mathbb{E}_{\lambda\mu^{-1}}\left[\left[\delta_\theta\rho_\theta + (1-\delta_\theta)\right]\sigma_\theta\right] d\log P + \mathbb{E}_{\lambda\mu^{-1}}\left[\delta_\theta\sigma_\theta\rho_\theta\right] d\log w \tag{63}$$

$$d\log P = \frac{-\left(d\log Y - d\log L - d\log\bar{\mu}\right) + \mathbb{E}_{\lambda}\left[1+\delta_\theta\rho_\theta\left(\sigma_\theta-1\right)\right] d\log w}{1+\mathbb{E}_{\lambda}\left[(\sigma_\theta-1)\left[(1-\delta_\theta)+\delta_\theta\rho_\theta\right]\right]}. \tag{64}$$

Rearranging yields,

$$d\log P = \frac{\mathbb{E}_{\lambda}\left[\delta_\theta\rho_\theta\sigma_\theta\right]}{\mathbb{E}_{\lambda}\left[\sigma_\theta\left[(1-\delta_\theta)+\delta_\theta\rho_\theta\right]\right]} d\log w. \tag{65}$$

Proof of Proposition 1. Using our equations for $d\log P$ and $d\log Y - d\log L$, we solve for the response of aggregate TFP

$$
\begin{aligned}
d\log A &= d\log Y - d\log L && (66)\\
&= -\mathbb{E}_{\lambda\mu^{-1}}\left[\left[\delta_\theta\rho_\theta + (1-\delta_\theta)\right]\sigma_\theta\right] d\log P + \mathbb{E}_{\lambda\mu^{-1}}\left[\delta_\theta\sigma_\theta\rho_\theta\right] d\log w && (67)\\
&= \left[\mathbb{E}_{\lambda\mu^{-1}}\left[\delta_\theta\sigma_\theta\rho_\theta\right] - \frac{\mathbb{E}_{\lambda\mu^{-1}}\left[\left[\delta_\theta\rho_\theta + (1-\delta_\theta)\right]\sigma_\theta\right]\mathbb{E}_{\lambda}\left[\delta_\theta\rho_\theta\sigma_\theta\right]}{\mathbb{E}_{\lambda}\left[\sigma_\theta\left[(1-\delta_\theta)+\delta_\theta\rho_\theta\right]\right]}\right] d\log w && (68)\\
&= \bar{\mu}\left[\mathbb{E}_{\lambda}\left[\delta_\theta\rho_\theta\left(\sigma_\theta-1\right)\right] - \left[\frac{\mathbb{E}_{\lambda}\left[\left[\delta_\theta\rho_\theta+(1-\delta_\theta)\right](\sigma_\theta-1)\right]\mathbb{E}_{\lambda}\left[\delta_\theta\rho_\theta\sigma_\theta\right]}{\mathbb{E}_{\lambda}\left[\sigma_\theta\left[(1-\delta_\theta)+\delta_\theta\rho_\theta\right]\right]}\right]\right] d\log w && (69)\\
&= \bar{\mu}\frac{\mathbb{E}_{\lambda}\left[1-\delta_\theta\right]\mathbb{E}_{\lambda}\left[\delta_\theta\rho_\theta\sigma_\theta\right] - \mathbb{E}_{\lambda}\left[\sigma_\theta\left(1-\delta_\theta\right)\right]\mathbb{E}_{\lambda}\left[\delta_\theta\rho_\theta\right]}{\mathbb{E}_{\lambda}\left[\sigma_\theta\left[(1-\delta_\theta)+\delta_\theta\rho_\theta\right]\right]} d\log w && (70)\\
&= \bar{\mu}\frac{\mathbb{E}_{\lambda}\left[\delta_\theta\rho_\theta\sigma_\theta\right] - \mathbb{E}_{\lambda}\left[\delta_\theta\right]\mathbb{E}_{\lambda}\left[\delta_\theta\rho_\theta\sigma_\theta\right] - \mathbb{E}_{\lambda}\left[\sigma_\theta\right]\mathbb{E}_{\lambda}\left[\delta_\theta\rho_\theta\right] + \mathbb{E}_{\lambda}\left[\sigma_\theta\delta_\theta\right]\mathbb{E}_{\lambda}\left[\delta_\theta\rho_\theta\right]}{\mathbb{E}_{\lambda}\left[\sigma_\theta\left[(1-\delta_\theta)+\delta_\theta\rho_\theta\right]\right]} d\log w && (71)\\
&= \bar{\mu}\frac{\mathbb{E}_{\lambda}\left[\delta_\theta\right]\left(1-\mathbb{E}_{\lambda}\left[\delta_\theta\right]\right)\mathbb{E}_{\lambda\delta}\left[\rho_\theta\sigma_\theta\right] + \mathbb{E}_{\lambda\delta}\left[\rho_\theta\right]\left(\mathbb{E}_{\lambda}\left[\sigma_\theta\delta_\theta\right]\mathbb{E}_{\lambda}\left[\delta_\theta\right] - \mathbb{E}_{\lambda}\left[\sigma_\theta\right]\mathbb{E}_{\lambda}\left[\delta_\theta\right]\right)}{\mathbb{E}_{\lambda}\left[\sigma_\theta\left[(1-\delta_\theta)+\delta_\theta\rho_\theta\right]\right]} d\log w && (72)\\
&= \bar{\mu}\frac{\mathbb{E}_{\lambda}\left[\delta_\theta\right]\left(1-\mathbb{E}_{\lambda}\left[\delta_\theta\right]\right)Cov_{\lambda\delta}\left[\rho_\theta,\sigma_\theta\right] + \mathbb{E}_{\lambda\delta}\left[\rho_\theta\right]Cov_{\lambda}\left[\sigma_\theta,\delta_\theta\right]}{\mathbb{E}_{\lambda}\left[\sigma_\theta\left[(1-\delta_\theta)+\delta_\theta\rho_\theta\right]\right]} d\log w, && (73)
\end{aligned}
$$

which concludes the proof of Proposition 1.

Proof of Proposition 2. We use the change in firm markups to calculate

$$\mathbb{E}_\lambda\left[d\log\mu_\theta\right] = \mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]d\log P - \mathbb{E}_\lambda\left[1-\delta_\theta\rho_\theta\right]d\log w \tag{74}$$

$$= \left[\frac{\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda\left[\delta_\theta\rho_\theta\sigma_\theta\right]}{\mathbb{E}_\lambda\left[\sigma_\theta\left[(1-\delta_\theta)+\delta_\theta\rho_\theta\right]\right]} - \mathbb{E}_\lambda\left[1-\delta_\theta\rho_\theta\right]\right]d\log w \tag{75}$$

$$= \left[\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\left(\frac{\mathbb{E}_\lambda\left[\delta_\theta\rho_\theta\sigma_\theta\right]}{\mathbb{E}_\lambda\left[\sigma_\theta\left[(1-\delta_\theta)+\delta_\theta\rho_\theta\right]\right]}-1\right) - \mathbb{E}_\lambda\left[1-\delta_\theta\right]\right]d\log w \tag{76}$$

$$= \left[-\frac{\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda\left[\sigma_\theta\left(1-\delta_\theta\right)\right]}{\mathbb{E}_\lambda\left[\sigma_\theta\left[(1-\delta_\theta)+\delta_\theta\rho_\theta\right]\right]} - \mathbb{E}_\lambda\left[1-\delta_\theta\right]\right]d\log w, \tag{77}$$

which yields Equation (9). We have

$$d\log L = \frac{\zeta\left(1-\gamma\right)}{1+\zeta}d\log Y - \frac{\zeta}{1+\zeta}d\log\bar{\mu}, \tag{78}$$

and

$$d\log A = d\log\bar{\mu} - \mathbb{E}_\lambda\left[d\log\mu_\theta\right], \tag{79}$$

which yields,

$$\frac{1+\gamma\zeta}{1+\zeta}d\log Y = \frac{1}{1+\zeta}d\log A - \frac{\zeta}{1+\zeta}\mathbb{E}_\lambda\left[d\log\mu_\theta\right]. \tag{80}$$

Rearranging yields Equation (8), which concludes the proof of Proposition 2.

General form for Phillips curve flattening. Proposition 3 follows immediately from dividing Equation (8) by $d\log w$ and rearranging.

Proposition 6 generalizes Proposition 4 to the case where both price-stickiness and pass-throughs are allowed to be heterogeneous.

Proposition 6. *The flattening of the price Phillips curve due to real rigidities, compared to nominal rigidities alone, is*

$$\frac{\textit{Phillips curve slope w/ nominal rigidities only}}{\textit{Phillips curve slope w/ real rigidities}} = 1 + \frac{1}{\mathbb{E}_\lambda\left[1-\delta_\theta\right]}\frac{\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda\left[\sigma_\theta(1-\delta_\theta)\right]}{\mathbb{E}_\lambda\left[\delta_\theta\right]\mathbb{E}_\lambda\left[\delta_\theta\rho_\theta\sigma_\theta\right]+\mathbb{E}_\lambda\left[\delta_\theta\rho_\theta\right]\mathbb{E}_\lambda\left[\sigma_\theta(1-\delta_\theta)\right]}. \tag{81}$$

The flattening of the price Phillips curve due to the misallocation channel is

$$\frac{\textit{Phillips curve slope w/ real rigidities}}{\textit{Phillips curve slope w/ misallocation}} = 1 + \frac{\bar{\mu}}{\zeta}\frac{\mathbb{E}_\lambda[\delta_\theta]\mathbb{E}_\lambda\left[1-\delta_\theta\right]Cov_{\lambda\delta}[\rho_\theta,\sigma_\theta] + \mathbb{E}_{\lambda\delta}\left[\rho_\theta\right]Cov_\lambda[\sigma_\theta,\delta_\theta]}{\mathbb{E}_\lambda\left[1-\delta_\theta\right]\mathbb{E}_\lambda\left[\delta_\theta\rho_\theta\sigma_\theta\right] + \mathbb{E}_\lambda\left[1-\delta_\theta\rho_\theta\right]\mathbb{E}_\lambda\left[\sigma_\theta(1-\delta_\theta)\right]}. \quad (82)$$

Proof. The flattening due to the misallocation channel is,

$$\text{Flattening due to the misallocation channel} \quad (83)$$

$$= \frac{\frac{d\log A}{d\log w} - \zeta\mathbb{E}_\lambda\left[\frac{d\log\mu_\theta}{d\log w}\right]}{-\zeta\mathbb{E}_\lambda\left[\frac{d\log\mu_\theta}{d\log w}\right]} \quad (84)$$

$$= 1 + \frac{\frac{d\log A}{d\log w}}{-\zeta\mathbb{E}_\lambda\left[\frac{d\log\mu_\theta}{d\log w}\right]} \quad (85)$$

$$= 1 + \frac{1}{\zeta}\frac{\kappa_\rho Cov_{\lambda\delta}[\rho_\theta,\sigma_\theta] + \kappa_\delta Cov_\lambda[\sigma_\theta,\delta_\theta]}{\frac{\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda\left[\sigma_\theta(1-\delta_\theta)\right]}{\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta+(1-\delta_\theta)\right]\sigma_\theta\right]} + \mathbb{E}_\lambda\left[1-\delta_\theta\right]} \quad (86)$$

$$= 1 + \frac{\bar{\mu}}{\zeta}\frac{\mathbb{E}_\lambda[\delta_\theta]\mathbb{E}_\lambda\left[1-\delta_\theta\right]Cov_{\lambda\delta}[\rho_\theta,\sigma_\theta] + \mathbb{E}_{\lambda\delta}\left[\rho_\theta\right]Cov_\lambda[\sigma_\theta,\delta_\theta]}{\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda\left[\sigma_\theta(1-\delta_\theta)\right] + \mathbb{E}_\lambda\left[1-\delta_\theta\right]\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta+(1-\delta_\theta)\right]\sigma_\theta\right]} \quad (87)$$

$$= 1 + \frac{\bar{\mu}}{\zeta}\frac{\mathbb{E}_\lambda[\delta_\theta]\mathbb{E}_\lambda\left[1-\delta_\theta\right]Cov_{\lambda\delta}[\rho_\theta,\sigma_\theta] + \mathbb{E}_{\lambda\delta}\left[\rho_\theta\right]Cov_\lambda[\sigma_\theta,\delta_\theta]}{\mathbb{E}_\lambda\left[1-\delta_\theta\rho_\theta\right]\mathbb{E}_\lambda\left[\sigma_\theta(1-\delta_\theta)\right] + \mathbb{E}_\lambda\left[1-\delta_\theta\right]\mathbb{E}_\lambda\left[\delta_\theta\rho_\theta\sigma_\theta\right]}. \quad (88)$$

The flattening due to real rigidities is,

$$\text{Flattening due to real rigidities} \quad (89)$$

$$= \left(\underbrace{\frac{1-\mathbb{E}_\lambda\left[1-\delta_\theta\right] - \frac{\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda\left[\sigma_\theta(1-\delta_\theta)\right]}{\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta+(1-\delta_\theta)\right]\sigma_\theta\right]}}{-\zeta\left[-\mathbb{E}_\lambda\left[1-\delta_\theta\right] - \frac{\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda\left[\sigma_\theta(1-\delta_\theta)\right]}{\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta+(1-\delta_\theta)\right]\sigma_\theta\right]}\right]}}_{\text{Slope with real rigidities}}\right)^{-1}\underbrace{\frac{1-\mathbb{E}_\lambda\left[1-\delta_\theta\right]}{\zeta\mathbb{E}_\lambda\left[1-\delta_\theta\right]}}_{\text{Slope in CES model}} \quad (90)$$

$$= \frac{\mathbb{E}_\lambda\left[\delta_\theta\right]\mathbb{E}_\lambda\left[1-\delta_\theta\right] + \frac{\mathbb{E}_\lambda[\delta_\theta]\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda\left[\sigma_\theta(1-\delta_\theta)\right]}{\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta+(1-\delta_\theta)\right]\sigma_\theta\right]}}{\mathbb{E}_\lambda\left[\delta_\theta\right]\mathbb{E}_\lambda\left[1-\delta_\theta\right] - \frac{\mathbb{E}_\lambda[1-\delta_\theta]\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda\left[\sigma_\theta(1-\delta_\theta)\right]}{\mathbb{E}_\lambda\left[\left[\delta_\theta\rho_\theta+(1-\delta_\theta)\right]\sigma_\theta\right]}} \quad (91)$$

$$= 1 + \frac{1}{\mathbb{E}_\lambda\left[1-\delta_\theta\right]}\frac{\mathbb{E}_\lambda\left[\delta_\theta(1-\rho_\theta)\right]\mathbb{E}_\lambda\left[\sigma_\theta(1-\delta_\theta)\right]}{\mathbb{E}_\lambda\left[\delta_\theta\right]\mathbb{E}_\lambda\left[\delta_\theta\rho_\theta\sigma_\theta\right] + \mathbb{E}_\lambda\left[\delta_\theta\rho_\theta\right]\mathbb{E}_\lambda\left[\sigma_\theta(1-\delta_\theta)\right]}. \quad (92)$$

Setting $\delta_\theta = \delta$ in both equations yields Proposition 4.

Appendix B Proofs for Dynamic Model

Firms choose reset prices to maximize future discounted profits,

$$\max_{p_{i,t}} \mathbb{E}\left[\sum_{k=0}^{\infty} \frac{1}{\prod_{j=0}^{k-1}(1+r_{t+j})}(1-\delta_i)^k y_{i,t+k}(p_{i,t} - \frac{w_{t+k}}{A_i})\right]. \tag{93}$$

The first order condition yields,

$$\mathbb{E}\left[\sum_{k=0}^{\infty} \frac{1}{\prod_{j=0}^{k-1}(1+r_{t+j})}(1-\delta_i)^k y_{i,t+k}\left[\frac{dy_{i,t+k}}{dp_{i,t}}\frac{p^*_{i,t}}{y_{i,t+k}}\frac{p^*_{i,t}-\frac{w_{t+k}}{A_i}}{p^*_{i,t}}+1\right]\right] = 0. \tag{94}$$

Using $\sigma_{i,t} = -\frac{p_i}{y_{i,t}}\frac{dy_{i,t}}{dp_i}$,

$$\mathbb{E}\left[\sum_{k=0}^{\infty} \frac{1}{\prod_{j=0}^{k-1}(1+r_{t+j})}(1-\delta_i)^k y_{i,t+k}\left(-\sigma_{i,t+k}\left(1-\frac{w_{t+k}}{p^*_{i,t}A_i}\right)+1\right)\right] = 0. \tag{95}$$

Rearranging gives us

$$\frac{p^*_{i,t}A_i}{w_t} = \frac{\mathbb{E}\left[\sum_{k=0}^{\infty} \frac{1}{\prod_{j=0}^{k-1}(1+r_{t+j})}(1-\delta_i)^k y_{i,t+k}\left(-\sigma_{i,t+k}\frac{w_{t+k}}{w_t}\right)\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty} \frac{1}{\prod_{j=0}^{k-1}(1+r_{t+j})}(1-\delta_i)^k y_{i,t+k}\left(1-\sigma_{i,t+k}\right)\right]}. \tag{96}$$

We now log-linearize around a perfect foresight, no-inflation steady state. This steady state is characterized by a constant discount factor such that $\left[\prod_{j=0}^{k-1}(1+r_{t+j})\right]^{-1} = \beta^k$. After removing all second-order terms, we get:

$$\frac{p^*_{i,t}A_i}{w_t} = \frac{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k y_{i,t+k}\sigma_{i,t+k}\left(d\log\left(\frac{w_{t+k}}{w_t}\right)+1\right)\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k y_{i,t+k}\left(\sigma_{i,t+k}-1\right)\right]} \tag{97}$$

$$= \frac{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k y_{i,t}\sigma_{i,t}(1+d\log(y_{i,t+k}\sigma_{i,t+k}))\left(d\log\left(\frac{w_{t+k}}{w_t}\right)+1\right)\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k y_{i,t}\left(\sigma_{i,t}-1\right)\left(1+d\log(y_{i,t+k}(\sigma_{i,t+k}-1))\right)\right]} \tag{98}$$

$$= \mu_{i,t}\frac{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k\right]+\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k d\log\left(\frac{w_{t+k}}{w_t}\right)\right]+\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k d\log(y_{i,t+k}\sigma_{i,t+k})\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k\right]+\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k d\log(y_{i,t+k}(\sigma_{i,t+k}-1))\right]}. \tag{99}$$

Rearranging, we get

$$d\log\mu^*_{i,t} = \frac{\frac{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k d\log\left(\frac{w_{t+k}}{w_t}\right)\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k\right]} + \frac{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k d\log(y_{i,t+k}\sigma_{i,t+k})\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k\right]} - \frac{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k d\log(y_{i,t+k}(\sigma_{i,t+k}-1))\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k\right]}}{1 + \frac{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k d\log(y_{i,t+k}(\sigma_{i,t+k}-1))\right]}{\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k\right]}} \tag{100}$$

$$= \frac{\Gamma^{-1}\left[\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k d\log\left(\frac{w_{t+k}}{w_t}\right)\right] + \mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k d\log(\mu_{i,t+k})\right]\right]}{1+\Gamma^{-1}\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k d\log(y_{i,t+k}(\sigma_{i,t+k}-1))\right]}, \tag{101}$$

where $\Gamma = \mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k\right] = \left[1-\beta(1-\delta_i)\right]^{-1}$. We multiply both sides by the denominator and remove second order terms to get

$$d\log\mu^*_{i,t} = \Gamma^{-1}\left[\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k d\log\left(\frac{w_{t+k}}{w_t}\right)\right] + \mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k d\log(\mu_{i,t+k})\right]\right]. \tag{102}$$

At the time of a price reset, we know that

$$\mu_{i,t} = \mu_i(\frac{y_{i,t}}{Y_t}). \tag{103}$$

Then,

$$d\log(\mu(\frac{y_{i,t+k}}{Y_{t+k}})) = \frac{\frac{y_{i,t}}{Y_t}\mu_i'(\frac{y_{i,t}}{Y_t})}{\mu_{i,t}} d\log(\frac{y_{i,t+k}}{Y_{t+k}}) \tag{104}$$

$$= \frac{1-\rho_{i,t}}{\rho_{i,t}}\frac{1}{\sigma_{i,t}} d\log(\frac{y_{i,t+k}}{Y_{t+k}}) \tag{105}$$

$$= \frac{1-\rho_{i,t}}{\rho_{i,t}}\left(d\log P_{t+k} - d\log w_{t+k} - d\log\mu_{i,t+k}\right) \tag{106}$$

$$= \frac{1-\rho_{i,t}}{\rho_{i,t}}\left(d\log P_{t+k} - d\log w_{t+k} - d\log\mu^*_{i,t} + d\log\frac{w_{t+k}}{w_t}\right), \tag{107}$$

where in the last line, we use the fact that the change in the markup $d\log\mu_{i,t+k}$ includes changes that occur at the time of the price change ($d\log\mu^*_{i,t}$) and subsequent changes due to the shifts in the nominal wage.

Plugging this in yields,

$$\frac{1}{\rho_{i,t}} d\log\mu^*_{i,t} = \Gamma^{-1}\left[\frac{1}{\rho_{i,t}}\mathbb{E}\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k d\log\left(\frac{w_{t+k}}{w_t}\right)\right] + E\left[\sum_{k=0}^{\infty}\beta^k(1-\delta_i)^k\frac{1-\rho_{i,t}}{\rho_{i,t}}\left(d\log P_{t+k} - d\log w_{t+k}\right)\right]\right]. \tag{108}$$

Finally, since $d\log\mu^*_{i,t} = d\log p^*_{i,t} - d\log w_t$, we get

$$d\log p^*_{i,t} = [1-\beta(1-\delta_i)] \sum_{k=0}^{\infty} \beta^k (1-\delta_i)^k \left[\rho_i d\log w_{t+k} + (1-\rho_i) d\log P_{t+k}\right]. \tag{109}$$

We can also write this equation recursively as

$$d\log p^*_{i,t} = (1-\beta(1-\delta_i)) \left[\rho_i d\log w_t + (1-\rho_i) d\log P_t\right] + \beta(1-\delta_i) p^*_{i,t+1}, \tag{110}$$

or in terms of firm types as,

$$d\log p^*_{\theta,t} = (1-\beta(1-\delta_\theta)) \left[\rho_\theta d\log w_t + (1-\rho_\theta) d\log P_t\right] + \beta(1-\delta_\theta) p^*_{\theta,t+1}. \tag{111}$$

Now that we have a recursive formulation for the optimal reset price, we can solve for the movement in the expected price for firms of type θ. Here, we use $\mathbb{E}$ to indicate the expectation over a continuum of identical firms of type θ, some of which will have the opportunity to change their prices and the remainder of which will not. The expected price for a firm of type θ follows,

$$\mathbb{E}\left[d\log p_{\theta,t+1}\right] = \delta_\theta d\log p^*_{\theta,t+1} + (1-\delta_\theta) d\log p_{\theta,t}, \tag{112}$$

since with probability δ_θ the firm is able to change its price to the optimal reset price at time $t+1$. Combining this with the recursive formula for optimal reset prices above, we get

$$\begin{aligned} &\mathbb{E}[d\log p_{\theta,t} - d\log p_{\theta,t-1}] - \beta\mathbb{E}[d\log p_{\theta,t+1} - d\log p_{\theta,t}] \\ &\qquad = \frac{\delta_\theta}{1-\delta_\theta}(1-\beta(1-\delta_\theta)) \left[-\mathbb{E}[d\log p_\theta] + \rho_\theta d\log w_t + (1-\rho_\theta) d\log P_t\right]. \end{aligned} \tag{113}$$

We can then aggregate this equation over firm types to get the modified New Keynesian Phillips curve and to get the Endogenous TFP equation.

New Keynesian Phillips curve with misallocation. We list a few identities that will be helpful in the subsequent derivations. The first four are derived in the main text, and the latter two can be formed be rearranging the above.

$$d\log P_t - d\log P^Y_t = \bar{\mu}^{-1}\, d\log A_t \tag{114}$$

$$d\log P^Y_t - d\log w_t = \mathbb{E}_\lambda\left[d\log\mu_\theta\right] \tag{115}$$

$$d \log A_t = d \log \bar{\mu}_t - \mathbb{E}_\lambda \left[d \log \mu_{\theta,t} \right] \tag{116}$$

$$d \log Y_t = \frac{1}{1 + \gamma\zeta} \left(d \log A_t - \zeta \mathbb{E}_\lambda \left[d \log \mu_{\theta,t} \right] \right) \tag{117}$$

$$-\mathbb{E}_\lambda \left[d \log \mu_{\theta,t} \right] = \left(\frac{1 + \gamma\zeta}{\zeta} \right) d \log Y_t - \frac{1}{\zeta} d \log A_t \tag{118}$$

$$d \log w_t - d \log P_t = \frac{1 + \gamma\zeta}{\zeta} d \log Y_t - \left(\frac{1}{\zeta} + \frac{1}{\bar{\mu}} \right) d \log A_t. \tag{119}$$

We now take the sales-weighted expectation of Equation (113) to get:

$$\begin{aligned} d \log \pi_t - \beta d \log \pi_{t+1} &= \varphi \left[-d \log P_t^Y + \mathbb{E}_\lambda \left[\rho_\theta \right] d \log w_t + \left(1 - \mathbb{E}_\lambda \left[\rho_\theta \right] \right) d \log P_t \right] && (120) \\ &= \varphi \left[\left(d \log P_t - d \log P_t^Y \right) + \mathbb{E}_\lambda \left[\rho_\theta \right] \left(d \log w_t - d \log P_t \right) \right] && (121) \\ &= \varphi \left[\left(\bar{\mu}^{-1} d \log A_t \right) + \mathbb{E}_\lambda \left[\rho_\theta \right] \left(\frac{1 + \gamma\zeta}{\zeta} d \log Y_t - \left(\frac{1}{\zeta} + \frac{1}{\bar{\mu}} \right) d \log A_t \right) \right] && (122) \\ &= \varphi \mathbb{E}_\lambda \left[\rho_\theta \right] \frac{1 + \gamma\zeta}{\zeta} d \log Y_t + \varphi \left[\frac{1}{\bar{\mu}} - \mathbb{E}_\lambda \left[\rho_\theta \right] \left(\frac{1}{\zeta} + \frac{1}{\bar{\mu}} \right) \right] d \log A_t, && (123) \end{aligned}$$

which concludes the proof.

Endogenous TFP equation. Start by subtracting $\mathbb{E} \left[d \log w_t - d \log w_{t-1} \right] - \beta \mathbb{E} \left[d \log w_{t+1} - d \log w_t \right]$ from both sides of Equation (113). This yields,

$$\begin{aligned} &\mathbb{E} \left[d \log \mu_{\theta,t} - d \log \mu_{\theta,t-1} \right] - \beta \mathbb{E} \left[d \log \mu_{\theta,t+1} - d \log \mu_{\theta,t} \right] \\ &\qquad = - \left[\mathbb{E} \left[d \log w_t - d \log w_{t-1} \right] - \beta \mathbb{E} \left[d \log w_{t+1} - d \log w_t \right] \right] \\ &\qquad\qquad + \varphi \left[-\mathbb{E} \left[d \log \mu_{\theta,t} \right] + \left(\rho_\theta - 1 \right) d \log w_t + \left(1 - \rho_\theta \right) d \log P_t \right]. \end{aligned} \tag{124}$$

We can write

$$d \log A_t = d \log \bar{\mu} - \mathbb{E}_\lambda \left[d \log \mu_\theta \right] = \bar{\mu} \left(\frac{\mathbb{E}_\lambda \left[\sigma_\theta d \log \mu_{\theta,t} \right]}{\mathbb{E}_\lambda \left[\sigma_\theta \right]} - \mathbb{E}_\lambda \left[d \log \mu_\theta \right] \right). \tag{125}$$

Now, we take Equation (125) and (1) multiply all terms by σ_θ, take the sales-weighted expectation, and divide by $\mathbb{E}_\lambda[\sigma_\theta]$; (2) take the sales-weighted expectation of (125); and multiply (1) − (2) by $\bar{\mu}$. This yields,

$$\left(d \log A_t - d \log A_{t-1} \right) - \beta \left(d \log A_{t+1} - d \log A_t \right) \tag{126}$$

$$= \varphi \left[-d\log A_t + \bar{\mu} \left(1 - \frac{\mathbb{E}_\lambda [\sigma_\theta \rho_\theta]}{\mathbb{E}_\lambda [\sigma_\theta]} - (1 - \mathbb{E}_\lambda [\rho_\theta]) \right) (d\log P_t - d\log w_t) \right] \tag{127}$$

$$= \varphi \left[-d\log A_t + \bar{\mu} \left(\frac{Cov_\lambda [\rho_\theta, \sigma_\theta]}{\mathbb{E}_\lambda [\sigma_\theta]} \right) (d\log w_t - d\log P_t) \right] \tag{128}$$

$$= \varphi \left[-d\log A_t + \bar{\mu} \left(\frac{Cov_\lambda [\rho_\theta, \sigma_\theta]}{\mathbb{E}_\lambda [\sigma_\theta]} \right) \left(\frac{1+\gamma\zeta}{\zeta} d\log Y_t - \left(\frac{1}{\zeta} + \frac{1}{\bar{\mu}} \right) d\log A_t \right) \right] \tag{129}$$

$$= \varphi \left[- \left(1 + \bar{\mu} \left(\frac{Cov_\lambda [\rho_\theta, \sigma_\theta]}{\mathbb{E}_\lambda [\sigma_\theta]} \right) \left(\frac{1}{\zeta} + \frac{1}{\bar{\mu}} \right) \right) d\log A_t + \bar{\mu} \frac{Cov_\lambda [\rho_\theta, \sigma_\theta]}{\mathbb{E}_\lambda [\sigma_\theta]} \frac{1+\gamma\zeta}{\zeta} d\log Y_t \right], \tag{130}$$

which concludes the proof.

Appendix C Empirical Calibration: Additional Details

In this appendix, we provide additional results from our calibration exercise. C.1 provides additional comparative statics from the calibration of the static model as we change the average markup and the degree of price-stickiness. C.2 shows additional impulse responses for the dynamic calibration of a 25bp interest rate shock.

Our procedure for extracting pass-throughs over the firm distribution from estimates provided by Amiti et al. (2019) is described in Appendix A of Baqaee and Farhi (2020a). We refer interested readers to that appendix.

C.1 Static model: Additional results

We vary the average markup $\bar{\mu}$ from just over one to 1.60 in Figure C.1. We do so by re-calculating markups of all firms according to the differential equation in Equation (22) according to the boundary condition implied by $\bar{\mu}$. As expected, the average markup does not affect the CES or real rigidities models, but the strength of the misallocation channel increases in $\bar{\mu}$. This reflects the dependence of the productivity response on $\bar{\mu}$.

In Figure C.2, we vary the degree of price-stickiness between zero (complete rigidity) and one (complete flexibility). We find that the flattening of the price Phillips curve due to real rigidities increases as the price becomes more rigid, and the flattening of the price Phillips curve due to the misallocation channel decreases as the price becomes more rigid. These comparative statics match the intuitions provided in the main text (see the discussion of Proposition 4).

Figure C.1: Decomposition of Phillips curve slope, varying the average markup $\bar{\mu}$.

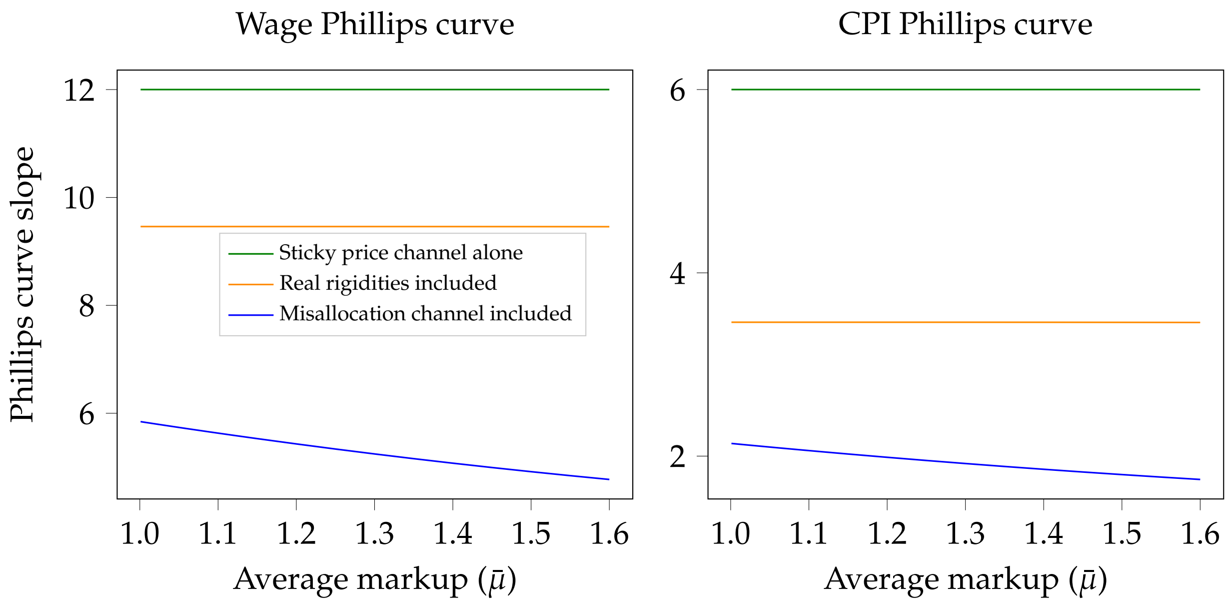
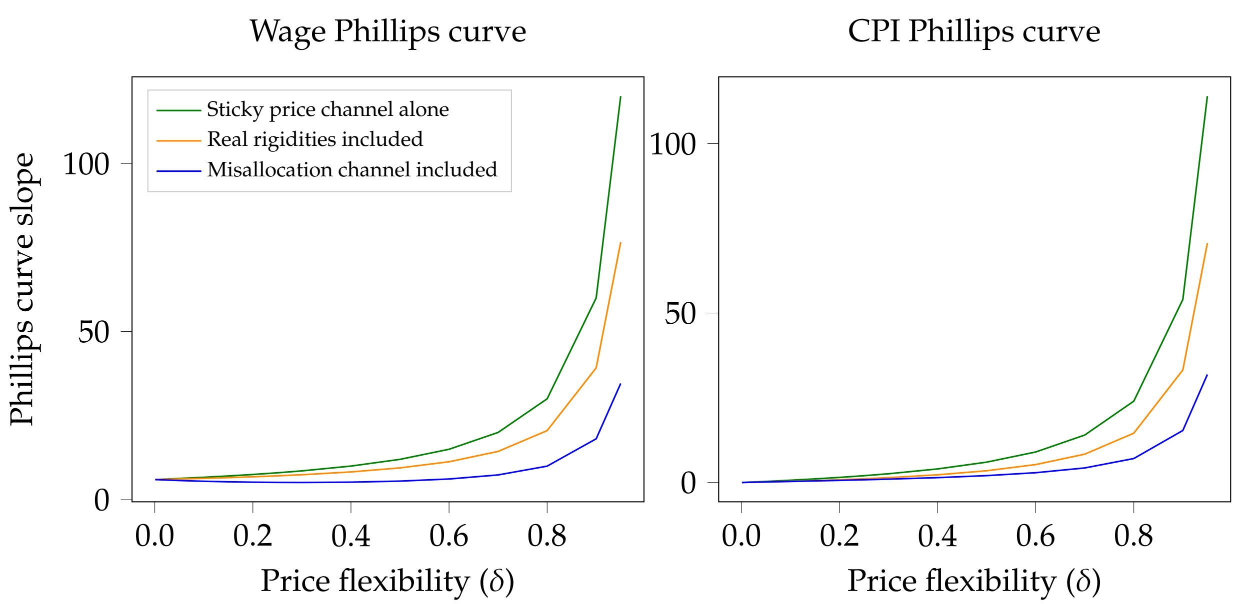


Figure C.2: Decomposition of Phillips curve slope, varying the degree of price-stickiness δ.

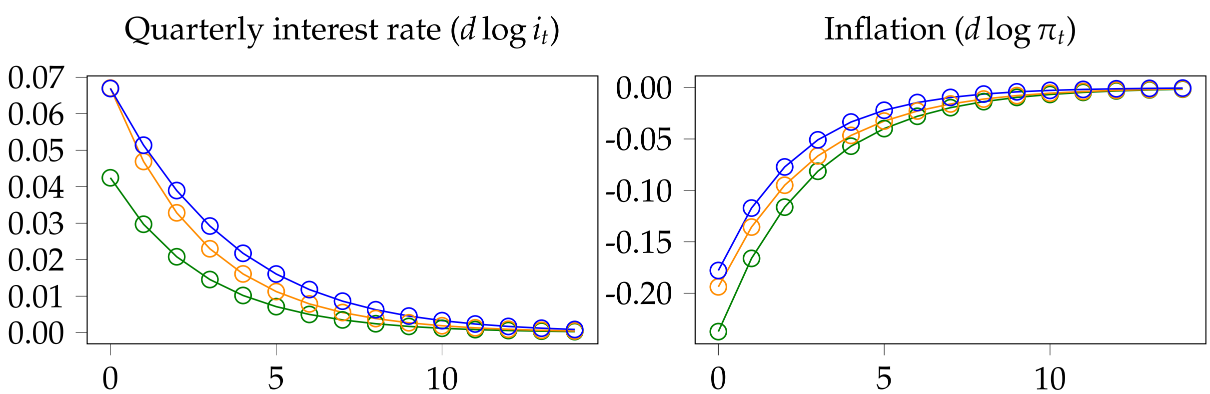


C.2 Dynamic model: Additional results

Figure C.3 shows the impulse response of the nominal interest rate and inflation following the 25bp contractionary monetary policy shock calibrated in the main text. The nominal interest rate differs across models since the monetary authority responds to the contemporaneous output and inflation gap. Compared to the CES and homogeneous firm models, the full model predicts less deflation following the shock.

Figure C.4 shows the change in sales shares of different firm types following the 25bp contractionary monetary policy shock calibrated in the main text. The contractionary shock leads to an expansion in the sales of smaller firms and a contraction in the sales of larger firms.

Figure C.3: Impulse response functions (IRFs) following a 25bp monetary shock. Green, orange, and blue IRFs indicate the CES, homogeneous firms, and heterogeneous firms models respectively.



Appendix D Dynamic Calibration: Money Supply Shock

Suppose the monetary shock takes the form of an exogenous shock to the money supply, rather than the interest rate rule. We calibrate the impulse response functions for the dynamic model, as in Section 6.4, for such a shock.

Money supply is linked to real variables via a cash-in-advance constraint, so that

$$d \log M = d \log P^Y + d \log Y. \tag{131}$$

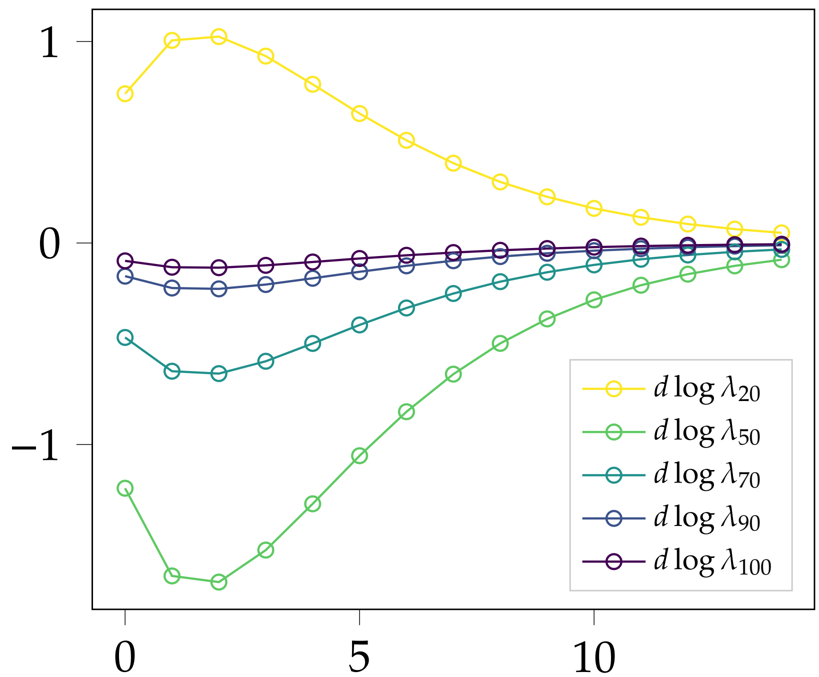
As in Galí (2015), we assume that the money supply follows an exogenous AR(1) process,

$$\Delta d \log M_t = \rho_m \Delta d \log M_{t-1} + \epsilon_t^m. \tag{132}$$

where $\Delta d \log M_t = d \log M_t - d \log M_{t-1}$ and ϵ_t^m is white noise. We choose $\rho_m = 0.5$ and calibrate impulse response functions for an expansionary money supply shock where $\epsilon_t^m = 0.25$ for $t = 0$ and zero in all subsequent periods.

Figure D.1 shows the response of output to the money supply shock, and Figure D.2 shows the response of other variables. Like an interest rate shock, the money supply

Figure C.4: Change in sales shares following a 25bp contractionary monetary policy shock by firm type. In the legend, $d\log\lambda_j$ refers to the change in the sales share of a firm at the j'th percentile of cumulative sales.



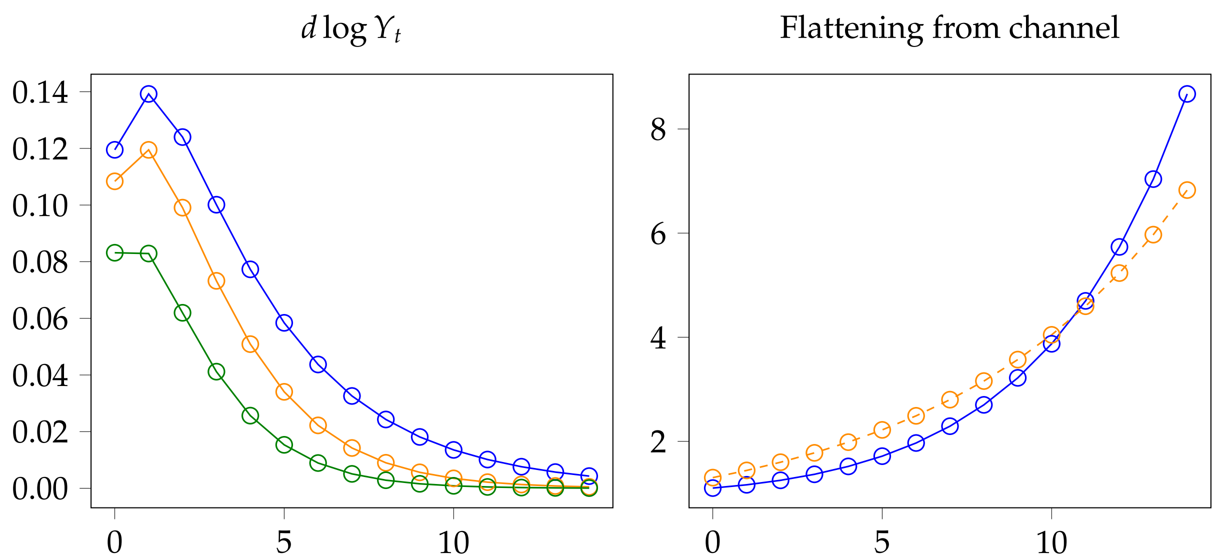
shock generates procyclical aggregate TFP and countercyclical dispersion in firm-level TFPR. Real rigidities and the misallocation channel both increase the responsiveness of output to the monetary shock.

The effects on output are summarized in Table D.1. The misallocation channel increases the half-life of the shock by 28% and increases the total output impact by 45% compared to the model with real rigidities alone.

Table D.1: Effect of exogenous money supply shock on output. The cumulative output impact is calculated as in Alvarez et al. (2016).

Model	Output effect at $t = 0$	Half life	Cumulative output impact
CES	0.083	2.98	0.330
With real rigidities	0.108	3.85	0.545
Full model	0.119	4.93	0.792

Figure D.1: Impulse response function of output following an expansionary money supply shock.



Appendix E Multiple Sectors, Multiple Factors, and Sticky Wages

In this appendix, we provide an extension of the model to multiple sectors and multiple factors, following the general network production structure provided by Baqaee and Farhi (2018). We use Ω to refer to the revenue-based input-output matrix,

$$\Omega_{ij} = \frac{p_j x_{ij}}{p_i y_i}, \tag{133}$$

where Ω_{ij} is share of producer i's costs spent on good j as a fraction of producer i's total revenue. Similarly, the cost-based input-output matrix,

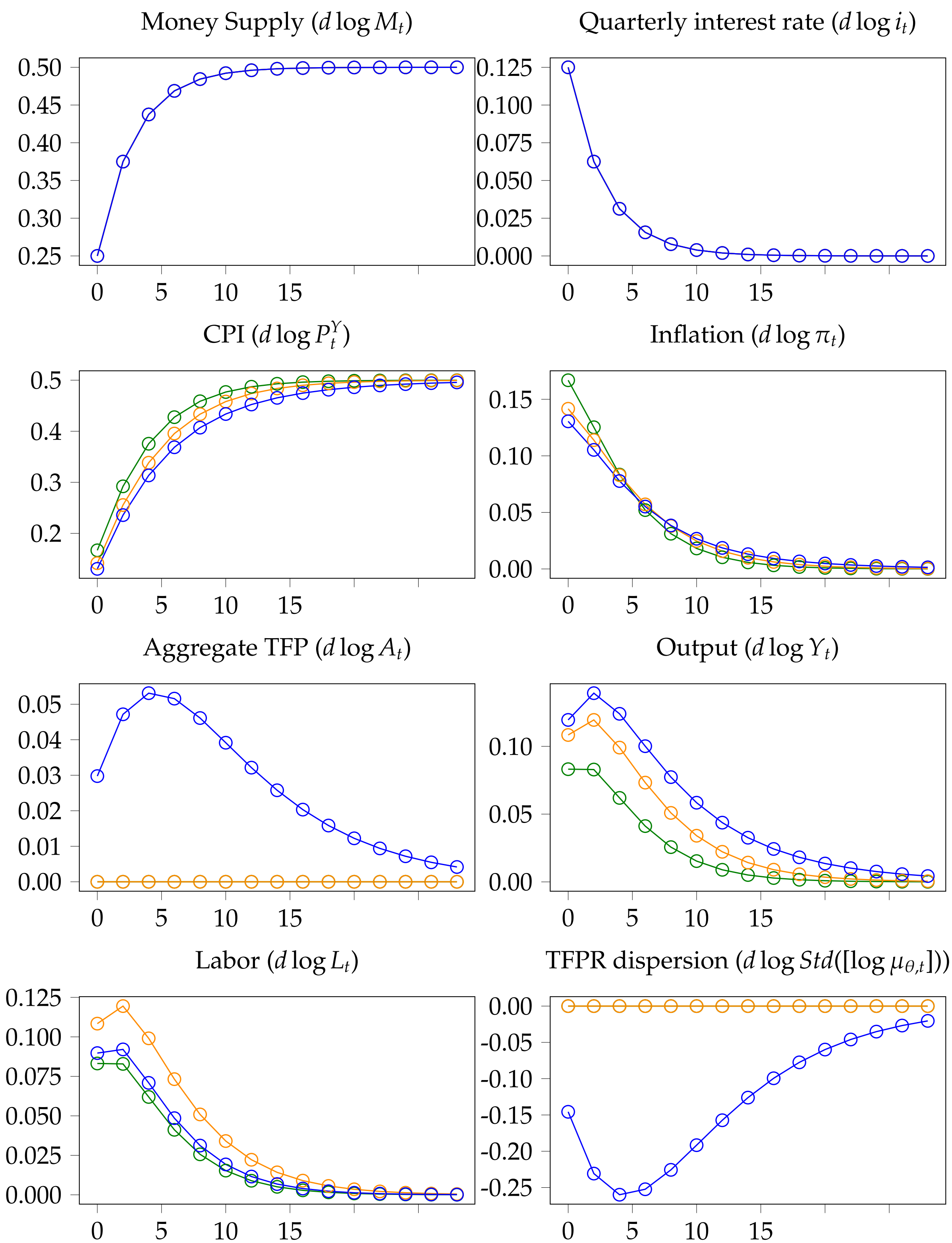
$$\tilde{\Omega}_{ij} = \frac{p_j x_{ij}}{\sum_l p_l x_{il}}, \tag{134}$$

describes producer i's spending on good j as a fraction of producer i's total costs. The revenue-based Leontief inverse matrix and cost-based Leontief inverse matrix are defined as,

$$\Psi = (1 - \Omega)^{-1}, \tag{135}$$

$$\tilde{\Psi} = (1 - \tilde{\Omega})^{-1}. \tag{136}$$

Figure D.2: Impulse response functions (IRFs) following an expansionary money supply shock. Green, orange, and blue IRFs indicate the CES, homogeneous firms, and heterogeneous firms models respectively.



Some additional notation: We use $\tilde{\Lambda}_f$ and Λ_f to refer to the share of factor f as a fraction of nominal GDP and as a fraction of total factor costs, respectively, and use $\lambda_{\mathcal{I}}$ to refer to the sales share of sector $\mathcal{I}$. The parameter ζ_f is the elasticity of factor f to its real price (or wage, in the case of labor), and $\gamma_f \zeta_f$ is the elasticity of factor f to income. The parameter $\theta_{\mathcal{I}}$ is the elasticity of substitution between inputs for sector $\mathcal{I}$. We use the notation of the covariance operator $Cov_{\Omega^{(j)}}$ as defined in Baqaee and Farhi (2018).

We can now derive the aggregate productivity and markup of any sector $\mathcal{I}$ just as in the one-sector model:

$$d\log A_{\mathcal{I}} = \mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}}\left[\mu_\theta^{-1}\right] \frac{\mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}}[\delta_\theta]\left(1-\mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}}[\delta_\theta]\right)Cov_{\frac{\lambda}{\lambda_{\mathcal{I}}}\delta}[\rho_\theta,\sigma_\theta] + \mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}\delta}[\rho_\theta]\,Cov_{\frac{\lambda}{\lambda_{\mathcal{I}}}}[\sigma_\theta,\delta_\theta]}{\mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}}\left[\left[\delta_\theta\rho_\theta + (1-\delta_\theta)\right]\sigma_\theta\right]} \cdot \left[\sum_{\mathcal{J}} \tilde{\Omega}_{\mathcal{I}\mathcal{J}}\, d\log\frac{p_{\mathcal{J}}}{P} + d\log P\right]. \quad (137)$$

$$d\log\mu_{\mathcal{I}} = -\left[\frac{\mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}}[\delta_\theta(1-\rho_\theta)]\,\mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}}[\sigma_\theta(1-\delta_\theta)]}{\mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}}\left[\left[\delta_\theta\rho_\theta+(1-\delta_\theta)\right]\sigma_\theta\right]} + \mathbb{E}_{\frac{\lambda}{\lambda_{\mathcal{I}}}}[1-\delta_\theta]\right]\left[\sum_{\mathcal{J}}\tilde{\Omega}_{\mathcal{I}\mathcal{J}}\, d\log\frac{p_{\mathcal{J}}}{P} + d\log P\right] + d\log A_{\mathcal{I}}. \quad (138)$$

The remaining aggregation equations follow directly from Baqaee and Farhi (2018). The change in output is:

$$d\log Y = \frac{1}{\sum_f \tilde{\Lambda}_f \frac{1+\gamma_f\zeta_f}{1+\zeta_f}}\left[\sum_{\mathcal{I}}\tilde{\lambda}_{\mathcal{I}}\left(d\log A_{\mathcal{I}} - d\log\mu_{\mathcal{I}}\right) - \frac{1}{1+\zeta_f}\sum_f \tilde{\Lambda}_f d\log\Lambda_f\right]. \quad (139)$$

The change in the sales share of sector $\mathcal{K}$ is:

$$\begin{aligned} d\log\lambda_{\mathcal{K}} = &\sum_{\mathcal{I}}\left(\delta_{\mathcal{K}\mathcal{I}} - \lambda_{\mathcal{I}}\frac{\Psi_{\mathcal{I}\mathcal{K}}}{\lambda_{\mathcal{K}}}\right)d\log\mu_{\mathcal{I}} \\ &+ \sum_{\mathcal{J}}(\theta_{\mathcal{J}}-1)\lambda_{\mathcal{J}}\mu_{\mathcal{J}}^{-1}Cov_{\tilde{\Omega}^{(\mathcal{J})}}\left(\sum_{\mathcal{I}}\tilde{\Psi}_{(\mathcal{I})}\left(d\log A_{\mathcal{I}} - d\log\mu_{\mathcal{I}}\right), \frac{\Psi_{(\mathcal{K})}}{\lambda_{\mathcal{K}}}\right) \\ &- \sum_{\mathcal{J}}(\theta_{\mathcal{J}}-1)\lambda_{\mathcal{J}}\mu_{\mathcal{J}}^{-1}Cov_{\tilde{\Omega}^{(\mathcal{J})}}\left(\sum_g \frac{\tilde{\Psi}_{(g)}}{1+\zeta_g}\left(d\log\Lambda_g + (\gamma_g\zeta_g - \zeta_g)\log Y\right), \frac{\Psi_{(\mathcal{K})}}{\lambda_{\mathcal{K}}}\right). \quad (140)\end{aligned}$$

The change in the share of income going to factor f is:

$$d\log\Lambda_f = -\sum_{\mathcal{I}}\lambda_{\mathcal{I}}\frac{\Psi_{\mathcal{I}f}}{\Lambda_f}d\log\mu_{\mathcal{I}}+\sum_{\mathcal{J}}(\theta_{\mathcal{J}}-1)\lambda_{\mathcal{J}}\mu_{\mathcal{J}}^{-1}Cov_{\tilde{\Omega}(\mathcal{J})}\left(\sum_{\mathcal{I}}\tilde{\Psi}_{(\mathcal{I})}\left(d\log A_{\mathcal{I}}-d\log\mu_{\mathcal{I}}\right),\frac{\Psi_{(f)}}{\Lambda_f}\right)$$
$$-\sum_{\mathcal{J}}(\theta_{\mathcal{J}}-1)\lambda_{\mathcal{J}}\mu_{\mathcal{J}}^{-1}Cov_{\tilde{\Omega}(\mathcal{J})}\left(\sum_{g}\frac{\tilde{\Psi}_{(g)}}{1+\zeta_g}\left(d\log\Lambda_g+(\gamma_g\zeta_g-\zeta_g)\log Y\right),\frac{\Psi_{(f)}}{\Lambda_f}\right). \quad (141)$$

Factor and sector prices follow:

$$d\log\frac{w_f}{P} = \frac{1}{1+\zeta_f}d\log\Lambda_f + \frac{1+\gamma_f\zeta_f}{1+\zeta_f}d\log Y, \quad (142)$$

$$d\log\frac{p_{\mathcal{I}}}{P} = -\sum_{\mathcal{K}}\tilde{\Psi}_{\mathcal{I}\mathcal{K}}\left(d\log A_{\mathcal{K}}-d\log\mu_{\mathcal{K}}\right)+\sum_{f}\tilde{\Psi}_{\mathcal{I}f}d\log\frac{w_f}{P}. \quad (143)$$

To illustrate the results, we consider a simple example with two factors (capital and labor) and sticky wages.

E.1 Example: Two factors and sticky wages

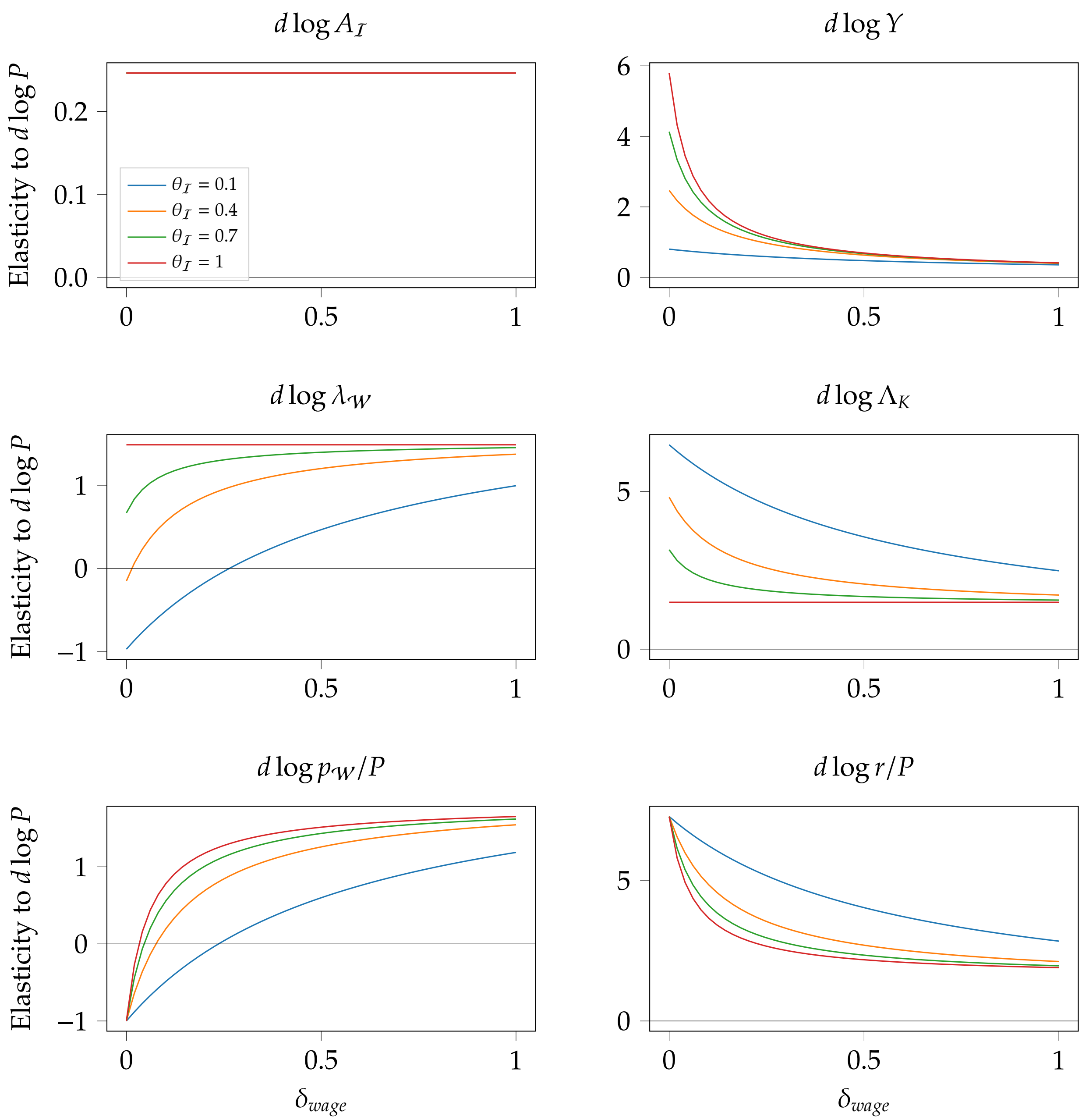
We apply the multiple factor and multiple sector model above. Consider an economy with two factors, labor and capital. Labor is elastic, with a Frisch elasticity of 0.2, as in the model considered in the main text, while capital is inelastic. We allow for sticky wages by introducing a "labor union sector": this sector buys all labor, and then supplies labor to firms in the industry sector at a price which is subject to nominal rigidities.

The industry sector consists of firms in monopolistic competition who use capital and labor provided by the labor union to produce varieties. Just as in the main text, firms in the industry sector have heterogeneous productivities and endogenous markups and pass-throughs; we use the same parameters and objects from the firm distribution given in the main text for this calibration. Additionally, we set the share of labor to $\tilde{\Lambda}_L = 2/3$ and the share of capital to $\tilde{\Lambda}_K = 1/3$. We allow both the elasticity of substitution between labor and capital used by firms in the industry sector, denoted $\theta_{\mathcal{I}}$, and the degree of wage-stickiness, denoted δ_w, to vary across calibrations.

We show the results of this model in Figure E.1. The plot shows the change in aggregate productivity in the firm sector, ($d\log A_{\mathcal{I}}$), the change in output ($d\log Y$), the change in the shares of income to labor and capital ($d\log\lambda_{\mathcal{W}}$ and $d\log\Lambda_K$), and the real price of labor and capital ($d\log p_{\mathcal{W}/P}$ and $d\log r/P$) following a shock to the price level ($d\log P$).[41]

[41]We focus on the labor share and the real wage of the labor union sector, since these are the labor share

Figure E.1: Response to shock to price level ($d \log P$) in one period model with capital, labor, and sticky wages. The degree of wage-stickiness varies along the x-axis, from complete rigidity (zero) to complete flexibility (one). Lines indicate calibrations with different elasticities of substitution between capital and labor.



One immediate implication of this exercise is that the productivity response in the firm sector is independent of frictions upstream, such as sticky wages or complementarity in inputs. As a result, the importance of the misallocation channel in transmitting mone-

and real wage that would be observed.

tary shocks is robust to the addition of wage rigidities or deviating from Cobb-Douglas production. Furthermore, note that the cyclicality of labor's share of income is, in general, ambiguous. With sufficiently rigid wages, it is possible to make the labor share countercyclical (and the share of income accruing to profits and capital procyclical).

Appendix F Klenow-Willis Calibration

Under Klenow and Willis (2016) preferences, the markup and pass-through functions are

$$\mu_\theta = \mu(\frac{y_\theta}{Y}) = \frac{1}{1 - \frac{1}{\sigma}(\frac{y_\theta}{Y})^{\frac{\epsilon}{\sigma}}}, \tag{144}$$

$$\rho_\theta = \rho(\frac{y_\theta}{Y}) = \frac{1}{1 + \frac{\epsilon}{\sigma - (\frac{y_\theta}{Y})^{\frac{\epsilon}{\sigma}}}} = \frac{1}{1 + \frac{\epsilon}{\sigma}\mu_\theta}. \tag{145}$$

where the parameters σ and ϵ are the elasticity and superelasticity (i.e., the rate of change in the elasticity) that firms would face in a symmetric equilibrium. This functional form imposes a maximum output of $(y_\theta/Y)^{\max} = \sigma^{\frac{\sigma}{\epsilon}}$, at which markups approach infinity.

Unfortunately, these preferences are unable to match the empirical distribution of firm pass-throughs without counterfactually large markups. To see why, note that the pass-through function $\rho(\cdot)$ is strictly decreasing, and that the maximum pass-through admissible (for a firm with $y_\theta/Y = 0$) is

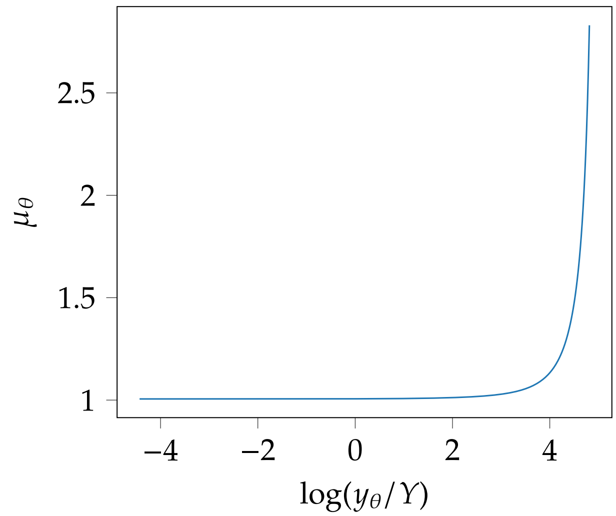
$$\rho^{\max} = \frac{1}{1 + \epsilon/\sigma}.$$

Amiti et al. (2019) estimate the average pass-through for the smallest 75% of firms in ProdCom is 0.97. In order to match the nearly complete pass-through for small firms, we must choose ϵ/σ to be around $0.01 - 0.03$.

This makes it difficult, however, to match the incomplete pass-throughs estimated for the largest firms. To match a pass-through of $\rho_\theta = 0.3$ with $\epsilon/\sigma \in [0.01, 0.03]$, for example, we need a markup of $\mu_\theta \in [78, 233]$ for the largest firms. In contrast, our non-parametric procedure matches the pass-through distribution with moderate markups for the largest firms, shown in Figure F.1. Importantly, since markups and pass-throughs depend on the elasticity of $\Upsilon'(\cdot)$, incorporating additional modeling elements (such as demand shifters correlated with firm productivity) does not avoid the counterfactual properties shown here.

Rather than attempting to match the empirical pass-through distribution, suppose we used a set of parameters from the literature. We adopt the calibration from Appendix

Figure F.1: Firm markups μ_θ estimated using nonparametric approach with $\bar{\mu} = 1.15$.



D of Amiti et al. (2019): $\sigma = 5$, $\epsilon = 1.6$, and firm productivities are drawn from a Pareto distribution with shape parameter equal to 8.[42] The simulated distributions of firm pass-throughs and sales shares are shown in Figure F.2. Over the range of drawn productivities, we see little variation in pass-through. Figure F.3 shows the response of output to an interest rate shock, calibrated with the same parameters as in Section 6.4. We find that the parametric specification dramatically understates the misallocation channel, compared to the nonparametric approach adopted in the main text.

Appendix G Oligopoly Calibration

An alternative to using the monopolistic competition framework is analyzing monetary policy through the lens of oligopoly. We adopt the nested CES model of Atkeson and Burstein (2008) and calibrate it according to typical parameters, as given in the appendix of Amiti et al. (2019). We set the elasticity of substitution across sectors to one, and the elasticity within sectors to 10. We draw firm productivities from a Pareto distribution with shape parameter equal to 8.[43]

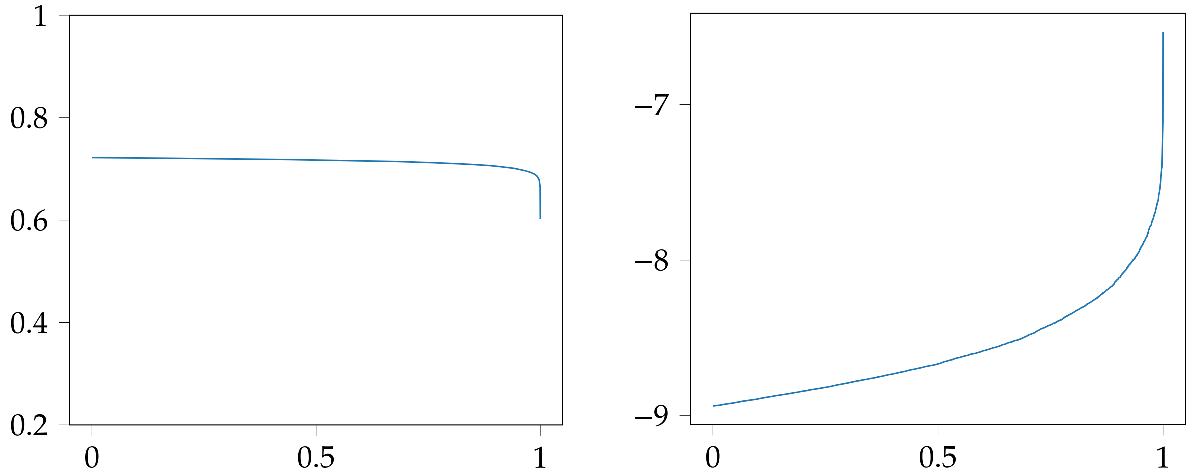
We order firms by market share within sector, and plot the markups and pass-throughs of firms in Figure G.1.[44] The markups and pass-throughs generated by the nested CES

[42]We calibrate the model by drawing 1000 firms and finding a fixed point in output. Since the Pareto distribution is unbounded, we could theoretically draw firms with zero pass-throughs and infinite sales shares; the simulated distributions are bounded away from these extremes.

[43]These parameters are chosen by Amiti et al. (2019) to match moments of the empirical distribution. We refer readers to Appendix D of their paper for more detail.

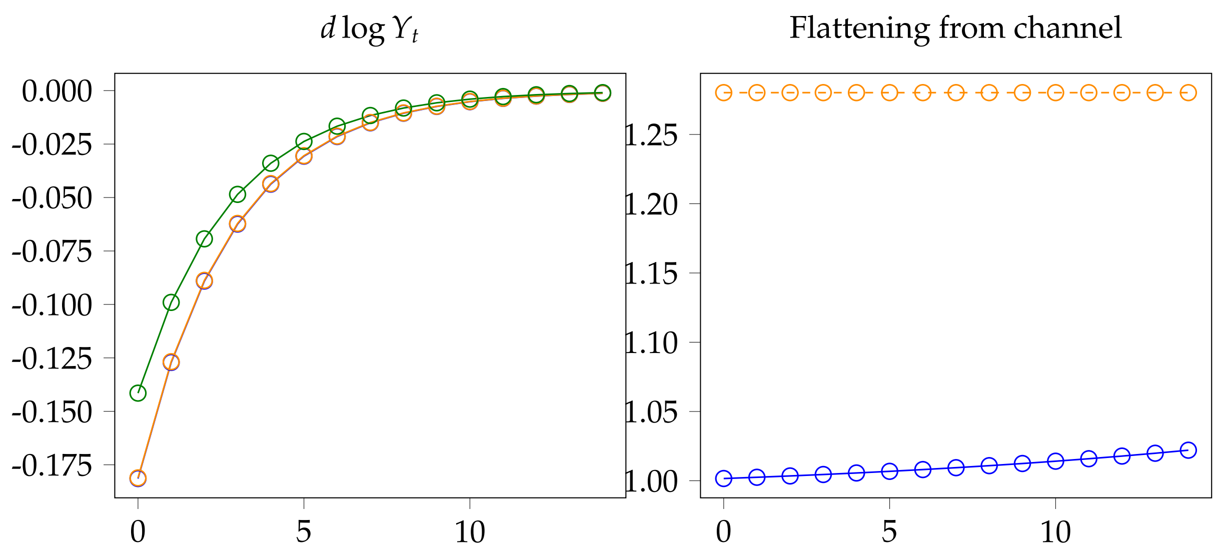
[44]If we instead plot markups and pass-throughs against firm market shares, we exactly replicate Figure

Figure F.2: Pass-through ρ_θ and sales share density $\log\lambda_\theta$ for Klenow and Willis (2016) calibration.



(a) Pass-through ρ_θ (b) Log sales share $\log\lambda_\theta$

Figure F.3: Impulse response function of output following a monetary policy shock, calibrated using Klenow and Willis (2016) preferences. The real rigidities model IRF and full model IRF coincide in the left panel.

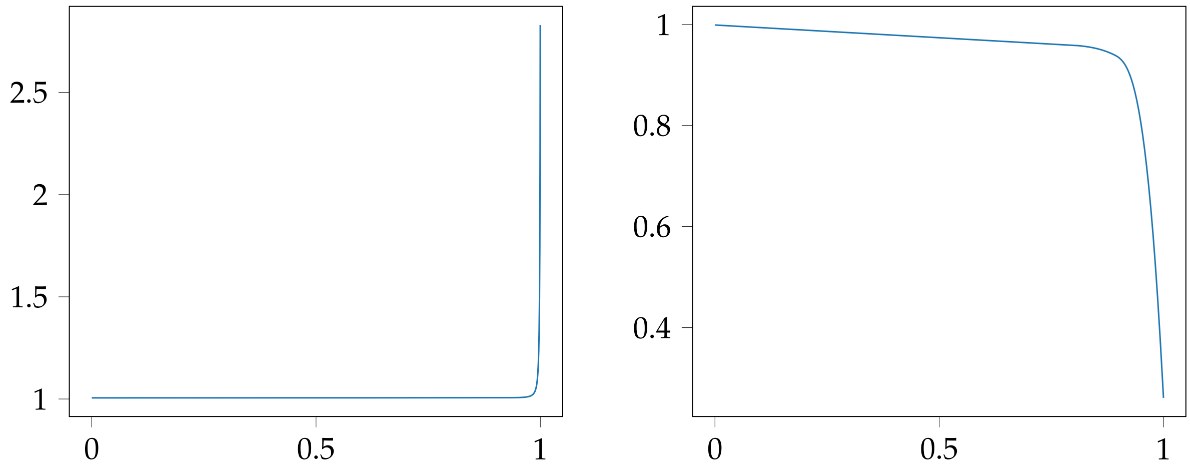


model satisfy Marshall's strong second law of demand: markups are increasing in firm productivity, and pass-throughs are decreasing in productivity.

We calculate the slope of the wage and price Phillips curves in a one-period setting, mirroring the timing of the one-period model presented in the main text. The flattening

A3 from Amiti et al. (2019).

Figure G.1: Markups μ_i and pass-throughs ρ_i for firms in the oligopoly calibration, ordered by market share.



(a) Markup μ_i (b) Pass-through ρ_i

of the Phillips curves due to real rigidities and the misallocation channel are presented in Table G.1. In this setting, as in the setting with monopolistic competition, we find that the misallocation channel is quantitatively important: the misallocation channel flattens both the wage and price Phillips curves by 31%, compared to real rigidities, which flatten the wage Phillips curve by 17% and the price Phillips curve by 42%.

Table G.1: Estimates of Phillips curve flattening due to real rigidities and the misallocation channel in oligopoly calibration.

Flattening	Wage Phillips curve	Price Phillips curve
Real rigidities	1.17	1.42
Misallocation channel	1.31	1.31

Appendix H Variation in markups and pass-throughs

The calibration in the main text assumes that firm markups and pass-throughs are related one-for-one with firm size. Other factors unrelated to firm size may influence markups and pass-throughs, however. We allow the demand elasticity and desired pass-throughs of a firm i to vary due to factors unrelated to firm size,

$$\sigma_i = \underbrace{\mathbb{E}\left[\sigma_i|\lambda_i\right]}_{\sigma_\lambda} + \epsilon_i,$$

$$\rho_{\theta,i} = \underbrace{\mathbb{E}\left[\rho_i|\lambda_i\right]}_{\rho_\lambda} + \nu_i,$$

where ϵ_i and ν_i are orthogonal to λ_i (and hence to σ_λ and ρ_λ), but may be correlated with each other ($\mathbb{E}[\epsilon_i \nu_i] \neq 0$). This nests the most general case in which flexible perturbations to the Kimball aggregator by firm cause linearly independent variation in elasticities and pass-throughs faced by firms. We consider how this flexibility changes the sales-weighted elasticity, sales-weighted pass-through, and covariance of elasticities and pass-throughs, which are sufficient to determine the model's results.

Introducing variation unrelated to firm size does not change the sales-weighted average elasticity and pass-through, due to the law of iterated expectations,

$$\begin{aligned}
\mathbb{E}_\lambda\left[\sigma_i\right] &= \mathbb{E}\left[\mathbb{E}\left[\lambda_i \sigma_i|\lambda_i\right]\right] / \mathbb{E}\left[\lambda_i\right] \\
&= \mathbb{E}\left[\lambda_i \sigma_\lambda\right] / \mathbb{E}\left[\lambda_i\right] \\
&= \mathbb{E}_\lambda\left[\sigma_\lambda\right].
\end{aligned}$$

The covariance of elasticities and pass-throughs may change, however:

$$\begin{aligned}
Cov_\lambda\left[\sigma_i, \rho_i\right] &= Cov_\lambda\left(\sigma_\lambda + \epsilon_i, \rho_\lambda + \nu_i\right) \\
&= Cov_\lambda\left(\sigma_\lambda, \rho_\lambda\right) + Cov_\lambda\left(\epsilon_i, \nu_i\right) \\
&= Cov_\lambda\left(\sigma_\lambda, \rho_\lambda\right) \underbrace{+ \sqrt{Var_\lambda\left(\epsilon_i\right) Var_\lambda\left(\nu_i\right)}\, Corr_\lambda\left(\epsilon_i, \nu_i\right)}_{\text{Bias}}.
\end{aligned}$$

Whether the bias attenuates or magnifies the supply-side effects in the model depends on the correlation between ϵ_i and ν_i, and the magnitude of the bias is bounded by the sales-weighted variance of both errors.

For example, consider the case where the consumer bundle aggregator includes demand shifters B_i (i.e., $\Upsilon_i(\cdot) = B_i \Upsilon(\cdot)$):

$$\int_0^1 B_i \Upsilon(\frac{y_i}{Y}) di = 1.$$

Suppose we perturb B_i for some firm i away from one, and hold $B_j = 1$ for all $j \neq i$. To a first order, the changes in the elasticity and pass-through of firm i are,

$$d\log\sigma_i = d\log B_i$$
$$d\log\rho_i = \frac{\mu_i}{\sigma_i\rho_i}\left[1 - \frac{\frac{y_i}{Y}\Upsilon'''(\frac{y_i}{Y})}{-\Upsilon''(\frac{y_i}{Y})} + \frac{-\frac{y_i}{Y}\Upsilon''(\frac{y_i}{Y})}{\Upsilon'(\frac{y_i}{Y})}\right] d\log\sigma_i$$

If $\Upsilon'''(\cdot)$ is positive or sufficiently close to zero, then $Corr(\epsilon_i, \nu_i) > 0$, and the supply-side effects are magnified, rather than attenuated.

More generally, we can bound the bias in the supply-side effects using the result from from Proposition 1 (assuming $\delta_i = \delta$ across firms):

$$d\log A = \bar{\mu}\left(\frac{\delta(1-\delta)\,Cov_\lambda[\sigma_i, \rho_i]}{(1-\delta)\,\mathbb{E}_\lambda[\sigma_i] + \delta\left(Cov_\lambda[\sigma_i, \rho_i] + \mathbb{E}_\lambda[\sigma_i]\,\mathbb{E}_\lambda[\rho_i]\right)}\right) d\log w.$$

The true supply-side effect, $d\log A^{true}$ (calculated using $Cov_\lambda[\sigma_i, \rho_i]$) is related to the supply-side effect calculated using variation due to sales share alone, $d\log A$ (calculated using $Cov_\lambda[\sigma_i, \rho_i]$), by

$$\frac{d\log A^{true}}{d\log A} = 1 + \frac{1 - d\log A}{d\log A + \frac{Cov_\lambda(\sigma_\lambda, \rho_\lambda)}{\sqrt{Var_\lambda(\epsilon_i)Var_\lambda(\nu_i)}Corr_\lambda(\epsilon_i, \nu_i)}}.$$

To illustrate, suppose 90% of variation in elasticities and pass-throughs comes from sales share, and 10% from other factors. For the calibration exercise given in the main paper, we find $\frac{d\log A^{true}}{d\log A} \in (0.69, 1.27)$; i.e., if variation not due to sales share in elasticities and pass-throughs is perfectly negatively correlated, the supply-side effect is attenuated by 31%, and if this variation is perfectly positively correlated, the supply-side effect is magnified by 27%.

Appendix I Gini coefficient in US data

We use Business Dynamic Statistics (BDS) data from the US Census to calculate the Gini coefficient in firm employment. Figure I.1 shows the Lorenz curve in employment for the firm distribution in 2018. We calculate the ratio of the shaded area (approximated using trapezoids) to the area under the 45-degree line to measure the Gini coefficient.

Figure I.2 plots the estimated Gini coefficients from 1978-2018 for all firms, as well as within sectors provided by the BDS. The trends by sector are consistent with the trends described in Figure A.1 of Autor et al. (2020), who measure HHI across sectors: we find increasing concentration in retail, wholesale trade, utilities, and finance, and flat or decreasing concentration in manufacturing. We use the beginning and end of the time series for all firms and for the retail sector for calibrations in the main text.

Figure I.1: Lorenz curve of cumulative firm employment by share of firms in 2018.

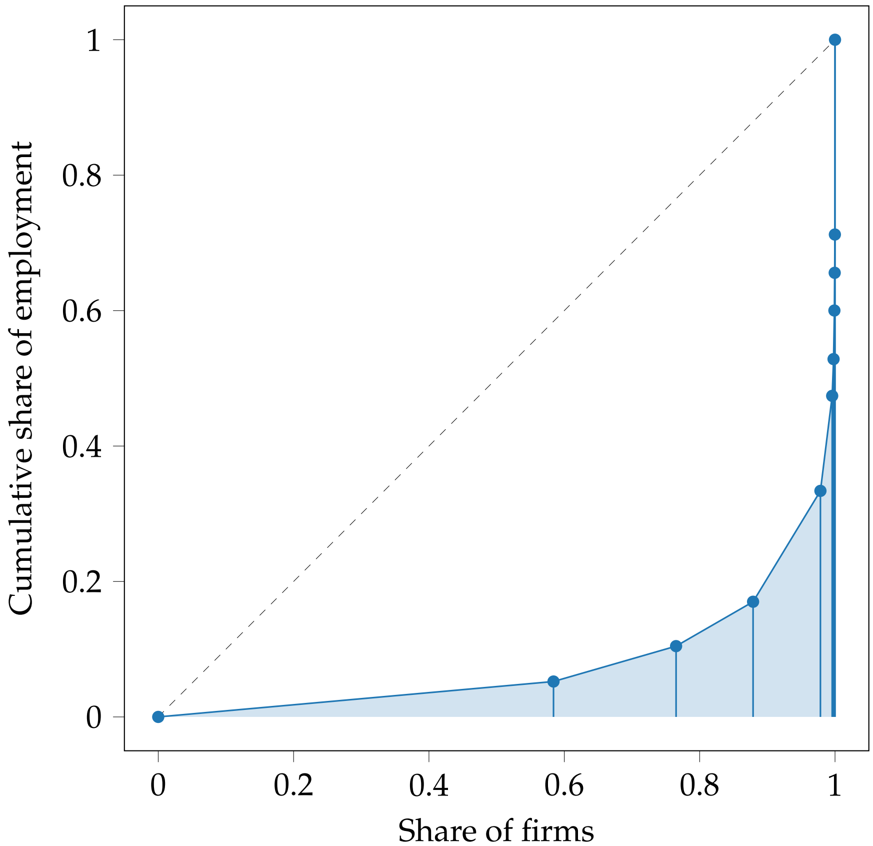


Figure I.2: Estimated Gini coefficients in Census BDS data from 1978-2018.