# Appendix A. Proofs

*Proof of Lemma 3.1.* Let us define $V_1^{s_1}\left(D_1, \bar{\lambda}\right)$, $V_1^{s_1}\left(D_1, \underline{\lambda}\right)$, and $\Delta_1^{s_1}(D_1)$ as in section 2. Assumption 1 leads to $B_1^{E,G}\left(\underline{\gamma}\right) > B_1^{E,B}(\bar{\gamma})$. If $q\Delta_1^{\max,G} + (1-q)\Delta_1^{\max,B} > 0$, then

$$D_1^{IC} = D_1^{G,PayIC} - \frac{(1-q)}{q}\left\{\theta\left[B_1^{E,B}(\bar{\gamma}) - B_1^{E,B}\left(\underline{\gamma}\right)\right] + \frac{\varepsilon}{\bar{\lambda} - \underline{\lambda}}\right\} > B_1^{E,G}\left(\underline{\gamma}\right). \qquad (A.1)$$

Otherwise,

$$D_1^{IC} = D_1^{B,PayIC} + \frac{q}{1-q}\left\{(1-\theta)\left[B_1^{E,G}(\bar{\gamma}) - B_1^{E,G}\left(\underline{\gamma}\right)\right] - \frac{\varepsilon}{\bar{\lambda} - \underline{\lambda}}\right\} < B_1^{E,B}(\bar{\gamma}). \qquad (A.2)$$

The solution to $D_1^{IC}$ is unique unless

$$q\left\{(1-\theta)\left[B_1^{E,G}(\bar{\gamma}) - B_1^{E,G}\left(\underline{\gamma}\right)\right] - \frac{\varepsilon}{\bar{\lambda} - \underline{\lambda}}\right\} = (1-q)\left\{\theta\left[B_1^{E,B}(\bar{\gamma}) - B_1^{E,B}\left(\underline{\gamma}\right)\right] + \frac{\varepsilon}{\bar{\lambda} - \underline{\lambda}}\right\},$$
$$\qquad (A.3)$$

in which case we pick the highest solution. At $B_1^{E,G}\left(\underline{\gamma}\right) = C_2$, $D_1^{IC} = D_1^{B,PayIC}$. In this case, $l_0^{IG} = D_1^{B,PayIC} + (1-q)\left(1-\bar{\lambda}\right)B_1^{E,B}\left(\underline{\gamma}\right)$, and $l_0^{CL} = qC_2 + (1-q)\left[\underline{\lambda}B_1^{E,B}(\bar{\gamma}) + \left(1-\underline{\lambda}\right)B_1^{E,B}\left(\underline{\gamma}\right)\right]$. A comparison between them shows that $l_0^{IG} < l_0^{CL}$ if and only if

$$q\left[C_2 - D_1^{B,PayIC}\right] > (1-q)\left[\bar{\lambda}(1-\theta) - \underline{\lambda}\right]\left[B_1^{E,B}(\bar{\gamma}) - B_1^{E,B}\left(\underline{\gamma}\right)\right], \qquad (A.4)$$

which holds if $q$ is sufficiently high. In this case, $\omega^*$ exists. The uniqueness follows from the monotonicity of $l_0^{IG} - l_0^{CL}$ with respect to $\omega_1^{E,G}$ once $B_1^{E,G}\left(\underline{\gamma}\right) < C_2 = B_1^{E,G}(\bar{\gamma})$.

*Proof of Lemma 3.2.* Let us write down the incumbent's IC constraint in choosing high effort:

$$\bar{\lambda}(1-e_1)\left\{q\left[\theta C_2 + (1-\theta)B_1^{E,G}(\bar{\gamma}) - D_1\right] + (1-q)\left[\theta C_2 + (1-\theta)B_1^{E,B}(\bar{\gamma}) - \min\left\{D_1, B_1^{E,B}(\bar{\gamma})\right\}\right]\right\}$$
$$+\bar{\lambda}e_1\left\{q\theta\left[C_2 - B_1^{E,G}(\bar{\gamma})\right] + (1-q)\theta\left[C_2 - B_1^{E,B}(\bar{\gamma})\right]\right\} + \left(1-\bar{\lambda}\right)\left\{q\theta\left[C_2 - B_1^{E,G}(\underline{\gamma})\right] + (1-q)\theta\left[C_2 - B_1^{E,B}(\underline{\gamma})\right]\right\} - \varepsilon$$
$$\geq \underline{\lambda}(1-e_1)\left\{q\left[\theta C_2 + (1-\theta)B_1^{E,G}(\bar{\gamma}) - D_1\right] + (1-q)\left[\theta C_2 + (1-\theta)B_1^{E,B}(\bar{\gamma}) - \min\left\{D_1, B_1^{E,B}(\bar{\gamma})\right\}\right]\right\}$$
$$+\underline{\lambda}e_1\left\{q\theta\left[C_2 - B_1^{E,G}(\bar{\gamma})\right] + (1-q)\theta\left[C_2 - B_1^{E,B}(\bar{\gamma})\right]\right\} + \left(1-\underline{\lambda}\right)\left\{q\theta\left[C_2 - B_1^{E,G}(\underline{\gamma})\right] + (1-q)\theta\left[C_2 - B_1^{E,B}(\underline{\gamma})\right]\right\}.$$
$$(A.5)$$

While seemingly complicated, the inequality is straightforward: with probability $\lambda(1-e_1)$, the realized pledgeability is high, and the covenant is not violated. In this case, the incumbent manager receives $\theta C_2 + (1-\theta)B_1^{E,s_1}(\bar{\gamma})$ and repays $\min\left\{D_1, B_1^{E,s_1}(\bar{\gamma})\right\}$. With probability $\lambda e_1$, however, the type-I error occurs, in which case $D_1$ is augmented to $\tilde{D}_1 = B_1^{E,G}(\bar{\gamma})$. Therefore, the incumbent only receives a payoff $\left[C_2 - B_1^{E,s_1}(\bar{\gamma})\right]$ if she retains her ability. With probability $(1-\lambda)$, the realized pledgeability is low and the covenant is correctly triggered. In this case, the incumbent receives a payoff $\left[C_2 - B_1^{E,s_1}(\underline{\gamma})\right]$ if she retains her ability. Finally, the incumbent incurs the cost of effort $\varepsilon$. This constraint is easily simplified to

$$(1-e_1)\left\{q\left[B_1^{E,G}(\bar{\gamma}) - D_1\right] + (1-q)\max\left\{B_1^{E,B}(\bar{\gamma}) - D_1, 0\right\}\right\} - \frac{\varepsilon}{\bar{\lambda}-\underline{\lambda}}$$
$$\geq \theta\left\{q\left[B_1^{E,G}(\bar{\gamma}) - B_1^{E,G}(\underline{\gamma})\right] + (1-q)\left[B_1^{E,B}(\bar{\gamma}) - B_1^{E,B}(\underline{\gamma})\right]\right\}.$$
$$(A.6)$$

Given that the left-hand side is decreasing in $D_1$, the condition on $e_1$ follows from evaluating the constraint at $D_1 = 0$. In particular, the detailed expression is

$$\bar{e}_1 = 1 - \frac{\theta\left\{q\left[B_1^{E,G}(\bar{\gamma}) - B_1^{E,G}(\underline{\gamma})\right] + (1-q)\left[B_1^{E,B}(\bar{\gamma}) - B_1^{E,B}(\underline{\gamma})\right]\right\} + \frac{\varepsilon}{\bar{\lambda}-\underline{\lambda}}}{qB_1^{E,G}(\bar{\gamma}) + (1-q)B_1^{E,B}(\bar{\gamma})}. \quad (A.7)$$

Evaluating the constraint at $D_1 = B_1^{E,B}(\bar{\gamma})$, we get two cases.

(i) If

$$(1-e_1)q\left[B_1^{E,G}(\bar{\gamma})-B_1^{E,B}(\bar{\gamma})\right]-\frac{\varepsilon}{\bar{\lambda}-\underline{\lambda}}\geq$$
$$\theta\left\{q\left[B_1^{E,G}(\bar{\gamma})-B_1^{E,G}(\underline{\gamma})\right]+(1-q)\left[B_1^{E,B}(\bar{\gamma})-B_1^{E,B}(\underline{\gamma})\right]\right\},$$

(A.8)

then

$$D_1^P=B_1^{E,G}(\bar{\gamma})-\frac{\theta}{q(1-e_1)}\left\{q\left[B_1^{E,G}(\bar{\gamma})-B_1^{E,G}(\underline{\gamma})\right]+(1-q)\left[B_1^{E,B}(\bar{\gamma})-B_1^{E,B}(\underline{\gamma})\right]\right\}$$
$$-\frac{\varepsilon}{q(1-e_1)(\bar{\lambda}-\underline{\lambda})}>B_1^{E,B}(\bar{\gamma}).$$

(A.9)

(ii) Otherwise,

$$D_1^P=\left[qB_1^{E,G}(\bar{\gamma})+(1-q)B_1^{E,B}(\bar{\gamma})\right]-\frac{\theta}{1-e_1}\left\{q\left[B_1^{E,G}(\bar{\gamma})-B_1^{E,G}(\underline{\gamma})\right]+(1-q)\left[B_1^{E,B}(\bar{\gamma})-B_1^{E,B}(\underline{\gamma})\right]\right\}$$
$$-\frac{\varepsilon}{(1-e_1)(\bar{\lambda}-\underline{\lambda})}\leq B_1^{E,B}(\bar{\gamma}).$$

(A.10)

<u>Lemma A.1</u>: *in performance-pricing debt, it is without loss of generality to assume*
$\tilde{D}_1=B_1^{E,G}(\bar{\gamma})$.

*Proof:* Let us write down the IC constraint under $\left\{D_1,\tilde{D}_1\right\}$ :

$$\bar{\lambda}(1-e_1)\left\{q\left[\theta C_2+(1-\theta)B_1^{E,G}(\bar{\gamma})-D_1\right]+(1-q)\left[\theta C_2+(1-\theta)B_1^{E,B}(\bar{\gamma})-\min\left\{D_1,B_1^{E,B}(\bar{\gamma})\right\}\right]\right\}$$
$$+\bar{\lambda}(1-e_1)\left\{q\left[\theta C_2+(1-\theta)B_1^{E,G}(\bar{\gamma})-\tilde{D}_1\right]+(1-q)\left[\theta C_2+(1-\theta)B_1^{E,B}(\bar{\gamma})-\min\left\{\tilde{D}_1,B_1^{E,B}(\bar{\gamma})\right\}\right]\right\}$$
$$+\left(1-\bar{\lambda}\right)\left\{q\left[\theta C_2+(1-\theta)B_1^{E,G}(\underline{\gamma})-\tilde{D}_1\right]+(1-q)\left[\theta C_2+(1-\theta)B_1^{E,B}(\underline{\gamma})-\min\left\{\tilde{D}_1,B_1^{E,B}(\underline{\gamma})\right\}\right]\right\}-\varepsilon$$
$$\geq\underline{\lambda}(1-e_1)\left\{q\left[\theta C_2+(1-\theta)B_1^{E,G}(\bar{\gamma})-D_1\right]+(1-q)\left[\theta C_2+(1-\theta)B_1^{E,B}(\bar{\gamma})-\min\left\{D_1,B_1^{E,B}(\bar{\gamma})\right\}\right]\right\}$$
$$+\underline{\lambda}(1-e_1)\left\{q\left[\theta C_2+(1-\theta)B_1^{E,G}(\bar{\gamma})-\tilde{D}_1\right]+(1-q)\left[\theta C_2+(1-\theta)B_1^{E,B}(\bar{\gamma})-\min\left\{\tilde{D}_1,B_1^{E,B}(\bar{\gamma})\right\}\right]\right\}$$
$$+\left(1-\underline{\lambda}\right)\left\{q\left[\theta C_2+(1-\theta)B_1^{E,G}(\underline{\gamma})-\tilde{D}_1\right]+(1-q)\left[\theta C_2+(1-\theta)B_1^{E,B}(\underline{\gamma})-\min\left\{\tilde{D}_1,B_1^{E,B}(\underline{\gamma})\right\}\right]\right\}.$$

(A.11)

Meanwhile, the goal is to maximize

$$l_0^P=\bar{\lambda}(1-e_1)\left\{qD_1+(1-q)\min\left\{D_1,B_1^{E,B}(\bar{\gamma})\right\}\right\}+\bar{\lambda}(1-e_1)\left\{q\tilde{D}_1+(1-q)\min\left\{\tilde{D}_1,B_1^{E,B}(\bar{\gamma})\right\}\right\}$$
$$+\left(1-\bar{\lambda}\right)\left\{q\tilde{D}_1+(1-q)B_1^{E,B}(\underline{\gamma})\right\}-\varepsilon.$$

(A.12)

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Let $\left\{D_1^*, \tilde{D}_1^*\right\}$ be the solution. If we increase $\tilde{D}_1^*$ and decrease $D_1^*$ such that $l_0^P$ is unchanged, it is straightforward to see that the IC constraint gets more slack because $\bar{\lambda} > \underline{\lambda}$.

Q.E.D.

### Appendix A.1. Type-II error

For simplicity, we have assumed that the signal $\phi$ only involves a type-I error. A type-II error, where low pledgeability may not trigger a covenant violation, will only change the results qualitatively, as we show in this subsection – it makes it harder to incentivize effort. Specifically, let us assume that if $\gamma_2 = \underline{\gamma}$, the signal is $\phi = \phi^H$ with probability $e_2$ and $\phi = \phi^L$ with probability $1 - e_2$. In this case, we need to redefine $\bar{\pi} = \frac{\bar{\lambda} e_1}{\bar{\lambda} e_1 + (1-\bar{\lambda})(1-e_2)}$. Again, we are going to focus on the case without the aggregate risk.

In the case of performance-pricing debt, the incumbent's payoff stays unchanged (and remains no better than straight debt.) The incumbent could retain control by paying $B_1^E(\underline{\gamma})$.

The presence of the type-II error reduces the incumbent's incentive to choose high effort under monitoring. To see this, suppose that the bank monitors following a covenant violation and liquidates the project if it learns the pledgeability is low. In this case, the incumbent's IC constraint in choosing high effort becomes

$$
\begin{aligned}
&\bar{\lambda}\left(1 - e_1\right)\left[\theta C_2 + (1 - \theta) B_1^E(\bar{\gamma}) - D_1\right] + \bar{\lambda} e_1 \theta\left[C_2 - B_1^E(\bar{\gamma})\right] + \left(1 - \bar{\lambda}\right) e_2 \theta\left[C_2 - B_1^E(\underline{\gamma})\right] - \varepsilon \\
&\geq \underline{\lambda}\left(1 - e_1\right)\left[\theta C_2 + (1 - \theta) B_1^E(\bar{\gamma}) - D_1\right] + \underline{\lambda} e_1 \theta\left[C_2 - B_1^E(\bar{\gamma})\right] + \left(1 - \underline{\lambda}\right) e_2 \theta\left[C_2 - B_1^E(\underline{\gamma})\right].
\end{aligned}
\tag{A.13}
$$

If $e_2 = 0$, this constraint is always slack whenever $D_1 \leq B_1^E(\bar{\gamma})$. Under a general $e_2$, the constraint holds if and only if $D_1 \leq \frac{\theta(1-e_2)C_2 + (1-e_1-\theta)B_1^E(\bar{\gamma}) + e_2 \theta B_1^E(\underline{\gamma})}{(1-e_1)} - \frac{\varepsilon}{(\bar{\lambda}-\underline{\lambda})(1-e_1)}$. The higher is $e_2$, the lower the right hand side, and lower the incentive compatible level of debt.

By a similar argument, in the case of aggregate uncertainty an increase in the probability of a high type-II error will reduce the amount that can be raised with performance

pricing, whenever performance pricing raises more than straight debt.

*Appendix A.2. Current and Future Pledgeability*

Let us elaborate on how our results vary when the incumbent's effort affects both contemporaneous and future pledgeability. We keep assuming $\gamma_2 \in \{\underline{\gamma}, \bar{\gamma}\}$ and now allow $\gamma_1 \in \{\underline{\gamma}_1, \bar{\gamma}_1\}$. The model presented in the paper corresponds to the case where $\underline{\gamma}_1 = \bar{\gamma}_1 = 0$. In this model, whenever the incumbent chooses effort $\lambda \in \{\bar{\lambda}, \underline{\lambda}\}$, with probability $\lambda$, $\gamma_2 = \bar{\gamma}$ and $\gamma_1 = \bar{\gamma}_1$. With probability $1 - \lambda$, $\gamma_2 = \underline{\gamma}$ and $\gamma_1 = \underline{\gamma}_1$. Throughout, we focus on the case without aggregate uncertainty so that the results can be directly comparable with those in section 2. Let $\tilde{V}_1\left(D_1, \lambda\right)$ be the incumbent's payoff when she chooses $\lambda$ :

$$\tilde{V}_1\left(D_1, \lambda\right) = \lambda\left\{\theta\left[C_2 - Min\left[B_1^E(\bar{\gamma}), D_1\right]\right] + (1 - \theta)\left[B_1^E(\bar{\gamma}) - Min\left[B_1^E(\bar{\gamma}), D_1\right]\right] + (1 - \bar{\gamma}_1)C_1\right\}$$
$$+ (1 - \lambda)\left\{\theta\left[C_2 - Min\left[B_1^E(\underline{\gamma}), D_1\right]\right] + (1 - \theta)\left[B_1^E(\underline{\gamma}) - Min\left[B_1^E(\underline{\gamma}), D_1\right]\right] + \left(1 - \underline{\gamma}_1\right)C_1\right\} - \varepsilon 1_{\{\lambda = \bar{\lambda}\}},$$
$$= V_1\left(D_1, \lambda\right) + \lambda(1 - \bar{\gamma}_1)C_1 + (1 - \lambda)\left(1 - \underline{\gamma}_1\right)C_1,$$

(A.14)

where $V_1\left(D_1, \lambda\right)$ is the payoff shown in subsection 2.A under $\gamma_1 \equiv 0$. Similarly, let us define $\tilde{\Delta}_1\left(D_1\right) = \tilde{V}_1\left(D_1, \bar{\lambda}\right) - \tilde{V}_1\left(D_1, \underline{\lambda}\right)$, which simplifies into $\tilde{\Delta}_1\left(D_1\right) = \Delta_1\left(D_1\right) - \left(\bar{\lambda} - \underline{\lambda}\right)\left(\bar{\gamma}_1 - \underline{\gamma}_1\right)C_1$, where $\Delta_1\left(D_1\right)$ is the same as in Lemma 2.1. Define $\tilde{D}_1^{PayIC} = \theta B_1^E(\underline{\gamma}) + (1 - \theta)B_1^E(\bar{\gamma}) - \left(\bar{\gamma}_1 - \underline{\gamma}_1\right)C_1 - \frac{\varepsilon}{(\bar{\lambda} - \underline{\lambda})}$, which is shown also as $\tilde{D}_1^{PayIC} = D_1^{PayIC} - \left(\bar{\gamma}_1 - \underline{\gamma}_1\right)C_1$. There are two cases.

(i) If $\tilde{D}_1^{PayIC} > B_1^E(\underline{\gamma})$, then under straight debt, $\tilde{D}_1^{PayIC}$ is the maximum face value that still induces high effort. In this case, the initial expert can borrow

$$\tilde{l}_0^{IG} \equiv \bar{\lambda}\left(\bar{\gamma}_1 C_1 + \tilde{D}_1^{PayIC}\right) + \left(1 - \bar{\lambda}\right)\left(\underline{\gamma}_1 C_1 + B_1^E(\underline{\gamma})\right). \qquad (A.15)$$

Note that in the benchmark model where $\gamma_1 \equiv \underline{\gamma}_1$,

$$l_0^{IG} \equiv \bar{\lambda}\left(\underline{\gamma}_1 C_1 + D_1^{PayIC}\right) + \left(1 - \bar{\lambda}\right)\left(\underline{\gamma}_1 C_1 + B_1^E(\underline{\gamma})\right) = \tilde{l}_0^{IG}. \qquad (A.16)$$

In other words, the ability to increase current pledgeability $\gamma_1$ does not increase the overall amount that the experts can borrow under internal-governance debt. Obviously, the amount that can be raised under covenant-lite debt will stays unchanged. Given so, the amount under performance-pricing debt also stays unchanged. Bank loan with monitoring is able to raise more

$$\tilde{l}_0^M = \bar{\lambda}\left(\bar{\gamma}_1 C_1 + B_1^E(\bar{\gamma})\right) + \left(1 - \bar{\lambda}\right) L - \left[\bar{\lambda} e_1 + \left(1 - \bar{\lambda}\right)\right] \psi, \qquad (A.17)$$

which exceeds the amount

$$l_0^M = \bar{\lambda}\left(\underline{\gamma}_1 C_1 + B_1^E(\bar{\gamma})\right) + \left(1 - \bar{\lambda}\right) L - \left[\bar{\lambda} e_1 + \left(1 - \bar{\lambda}\right)\right] \psi \qquad (A.18)$$

if $\gamma_1 \equiv \underline{\gamma}_1$. Therefore, bank loans can be more attractive.

(ii) If $\tilde{D}_1^{PayIC} \leq B_1^E(\underline{\gamma})$, then under straight debt, high effort in choosing pledgeability can never been induced. Performance-pricing debt cannot do better either, because the elevated face value later will require an even lower face value to begin with. Bank monitoring in this case will be more helpful, because the threat of liquidation is still sufficient to induce high effort.