Online Appendix for

“Dispersion in Financing Costs and Development”

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A Data Appendix

This appendix provides a more detailed description of the data (together with summary statistics) and the definition, sources, and values of the moments targeted in our calibration.

A.1 Data Description

Data on bank loans to all formal firms in Brazil are from the Brazilian Public Credit Registry (SCR - *Sistema de Informações de Crédito*). This is a confidential loan level database, managed by the Central Bank of Brazil. It contains information of all formal loans granted from January 2005 until December 2016. For any bank-to-firm loan during the period of analysis, we identify the lender, borrower, size of the loan, the interest rate on the loan, the loan maturity, default rates, the currency denomination of the loan and whether or not it was at an earmarked interest rate. This dataset allows us to construct information on the borrower-lender relationship, such as the length of a firm-bank relationship. We aggregate loan level data to the level of the firm using loan-weighted averages to construct annual data on spreads, maturity, non-performing loans, and other measures. Spreads are the difference between the weighted average of firms' outstanding loans rates and the one-year interbank deposit rate.

The other main dataset that we use in our empirical analysis is RAIS (*Relação Anual de Informações Sociais*), a matched employer-employee administrative dataset covering all formal firms in Brazil. This a mandatory annual survey maintained by the Ministry of Economy. RAIS provides information on firms, such as sector of activity and location, and information on employees, which we use to construct firm-level measures of employment and labor compensation. It is also possible to identify the date of entry and exit of firms. With this dataset, we can capture important firm dynamics for all formal firms in Brazil.

Using the unique firm tax identifier, we merge the SCR and RAIS datasets. Financial firms, public administration, non-governmental organizations and multilateral agencies were excluded in our sample. Our database therefore comprises of annual data of more than 3 million firms from 246 industries between 2006 and 2016, totaling roughly 11.8 million firm-year observations. Our sample period includes two years of solid economic growth (2006 and 2007), the burst and aftermath of the global financial crisis in 2008-2009, a strong economic recovery from 2010 to 2013, and the largest recession of the Brazilian economy in recent history (2014-2016).

To be precise about how the dataset is used, we construct a firm-level dataset in which we weight any loan information at the firm level by the loan size in each year. For instance, a firm might have more than one loan in a year. Then, for instance, the loan interest rate corresponds to the weighted average of all loans a firm had in a particular year. Similar procedure was used to construct loan characteristics at the firm level.

Table A1 provides descriptive statistics for the main variables used in our empirical analysis. As we can see, there are almost 12 million observations and statistics for the following variables are provided: outstanding loan, spread (difference between the loan interest rate and the deposit interest rate) rate, average maturity, non-performing loan (NPL), dummy for a loan denominated in a foreign currency, loan type (working capital, investment, foreign trade), earmarked loan, duration (in months) of the firm-bank relationship, number of banks a firm got loans in a particular year, firm size (number of employees), firm age in years, firm wage bill, firm growth in size and whether or not a firm exit the market. We also have indicator variables for the location (state) of the firm, firm legal nature and economic sector.

Table A1: Summary Statistics

	Obs	Mean	Median	St. Dev.	Min.	P5	P95	Max.
Outst. loan	11,846,248	909,708	60,414	6.92e+07	0.001	3,781	1,093,925	1.12e+11
Spread	11,846,248	44.43	23.47	60.41	-10.23	-3.05	176.74	321.28
Maturity	11,846,248	1.20	0.66	2.05	0	0.08	3.71	7.32
NPL	11,846,248	0.043	0.000	0.122	0	0	0.253	1
Currency dum.	11,846,248	0.037	0	0.190	0	0	0	1
Firm-bank rel.	11,846,248	80.16	45.55	120.67	0	1.49	246.98	1,407
N. of banks	11,846,248	2.12	2	1.415	1	1	5	36
Size	11,846,248	21.03	4	279.10	0	0	49	105,455
Age	11,846,236	14.48	9.24	19.29	0	1.52	38.50	118.62
Wage bill	11,846,248	33,798	3,505	989,213	0	0	61,411	1.10e+09
Got loan	11,846,248	0.659	1	0	0	1	1	-0.67
Firm growth	9,167,466	0.010	0	0.534	-11.20	-0.693	0.754	8.18
Exit	11,284,288	0.082	0	0.274	0	0	1	1

A.2 Moments

Here we provide the definition, source and value of the moments used in Section 4 of the paper to internally calibrate the model parameters.

1. Exit rate: 8.2%. Source: Our dataset based on the SCR and RAIS.

2. Average firm size (number of employees): 21. Source: Our dataset based on the SCR and RAIS.

3. Top 10-percentile employment share: 77.54%. Source: Our dataset based on the SCR and RAIS

4. Top 10-percentile earnings share ($\alpha + \theta$): 55.6%. Source: Morgan (2017).

5. Capital share in income: 40%. Source: Capital share in income is calculated from the Penn World Tables (PWT) 9.1. The average value for our period is 0.4, which is similar to the value reported by Abeles, Amarante, and Vega (2015). PWT uses Gollin (2002)'s Method 2 for Brazil, attributing proprietary income to labor and capital according to the split of nonproprietary income.

6. Investment rate: 17%. Source: Penn World Table - Feenstra, Inlaar, and Timmer (2015). Investment divided by GDP. This is the average value from 2005 to 2016.

7. Real risk free interest rate: 2%. For the real interest rate in Brazil, we took the average rate from 2005.1 to 2016.12 of the monthly Over/Selic interest rate (Brazilian Central Bank rate) minus the inflation rate measured by the IGP-DI (General price index from Vargas Foundation). We then annualized the monthly average real interest rate and we get 5.87%. We also deducted country default risks, measured by the sovereign default spreads. Damodaran (2020) shows that default spreads varied from 2 to 4% in the period (see Figure 13 of this paper) and the value in April. Therefore, we set the interest rate in the small open economy to 2%.

8. Net International Investment Position as a share of GDP: -30%. This moment is used for the open economy calibration only. We use the International Financial Statistics of the IMF to calculate the difference between the external financial assets and liabilities divided by GDP of Brazil. The average value from 2007 to 2016 is 30%.

9. Credit to output ratio: we target a value of 48.7%. Following Buera, Kaboski, and Shin (2011), we define this by the sum of the credit to non-financial corporations, private bond market capitalization, and stock market capitalization.

10. The average spreads once some factors are washed out: 38%. We calculate this by using the predicted value of the Regression (4) in Table 1. We set maturity at 1 year (as in our model), non-performing loan at 0 (there is no default in our model), currency denomination at 0, number of banks to 1 and all the other variables at their observed mean. The resulting predicted spread is 37%, which is about 7 percentage points lower than the unconditional average spread. Source: Our dataset based on the SCR and RAIS.

11. Weighted average spreads: credit-weighted spread of 6.01%. Source: Our dataset based on the SCR and RAIS

12. Standard deviation of spreads: 32%. Following the same procedure as above, we calculate the standard deviation of spreads once some loan effects are washed out. The value we target is: 32%, which is about 28 percentage points lower than the unconditional standard deviation of spreads. Source: Our dataset based on the SCR and RAIS.

13. Entrepreneurs with credit: 35%. We use an extensive margin measure of credit. Roughly 35% of entrepreneurs have credit. Source: Our dataset based on the SCR and RAIS.

B Theory Appendix

We provide proofs of Propositions 1 and 2 together with an analytical derivation of Figure 1.

B.1 Proof of Proposition 1

Notice that $x_k = r + \tau(a, z)$ and we have that

$$\tilde{r}(a, z) = r + \tau(a, z) + \frac{\chi}{k^b(a, z) - a} \left(y^b(a, z) + \tau(a, z)a - \kappa - \tilde{w}(a, z) \right).$$

The proof when $\chi = 0$ is trivial.

When $\chi = 1$ and $\tau(a, z) = \tau_0$, then by the implicit function theorem, we have that

$$\frac{\partial \tilde{r}(a, z)}{\partial z} = \frac{\frac{\partial y^b(a,z)}{\partial z} - (\tilde{r}(a, z) - (r + \tau_0))\frac{\partial k^b(a,z)}{\partial z} - \frac{\partial \tilde{w}(a,z)}{\partial z}}{k^b(a, z) - a}.$$

This can be rewritten as:

$$\frac{\partial \tilde{r}(a, z)}{\partial z} = \frac{y^b(a, z) - (\tilde{r}(a, z) - (r + \tau_0))k^b(a, z) - (1 - \alpha - \theta) \left(z a^\alpha \left(\frac{\theta}{w} \right)^\theta \right)^{\frac{1}{1-\theta}} I_{w<\tilde{\pi}}}{z(1 - \alpha - \theta)(k^b(a, z) - a)},$$

where $I_{w<\tilde{\pi}}$ is an indicator function which takes value equal to one when $w < \tilde{\pi}(a, z)$ and zero otherwise. The denominator of this expression is clearly positive. The numerator is positive for any $S^e \geq 0$.

It can also be shown that

$$\frac{\partial \tilde{r}(a, z)}{\partial a} = \frac{\tilde{r}(a, z) - r - \frac{\partial \tilde{w}(a,z)}{\partial a}}{k^b(a, z) - a}.$$

The denominator for this expression is always positive for entrepreneurs (a, z)

with $a < k^u(a,z)$ and $S^e \geq 0$. If $w > \tilde{\pi}(a,z)$, then $\frac{\partial \tilde{w}(a,z)}{\partial a} = 0$ and

$$\frac{\partial \tilde{r}(a,z)}{\partial a} = \frac{\tilde{r}(a,z) - r}{k^b(a,z) - a} > 0,$$

which is positive, otherwise financial intermediaries surplus would be negative. If $w < \tilde{\pi}(a,z)$, then

$$\frac{\partial \tilde{r}(a,z)}{\partial a} = \frac{\tilde{r}(a,z) - \alpha \left(z a^{\alpha} \left(\frac{\theta}{w}\right)^{\theta}\right)^{\frac{1}{1-\theta}} \frac{1}{a}}{k^b(a,z) - a} < 0.$$

The numerator for this case has to be negative for an agent (a,z) with $a < k^u(a,z)$ and $S^e \geq 0$. In order to see this, notice that $\alpha \left(z a^{\alpha} \left(\frac{\theta}{w}\right)^{\theta}\right)^{\frac{1}{1-\theta}} \frac{1}{a}$ corresponds to the marginal productivity net of labor cost of an additional capital at the collateral of an entrepreneur who is borrowing. This object has to be larger than her cost of capital, $\tilde{r}(a,z)$, otherwise she will not borrow.

If $\tau(a,z) = \tau_0$, $S^e \geq 0$ and $a < k^u(a,z)$ but the incentive compatible constraint is not binding, then clearly $\frac{\partial k^b(a,z)}{\partial a} = 0$. It can also be shown that

$$\frac{\partial k^b(a,z)}{\partial z} = \frac{y^b(a,z) - (\tilde{r}(a,z) - (r+\tau_0))k^b(a,z) - (1-\alpha-\theta)\left(z a^{\alpha} \left(\frac{\theta}{w}\right)^{\theta}\right)^{\frac{1}{1-\theta}} I_{w<\tilde{\pi}}}{(1-\alpha-\theta) z (\tilde{r}(a,z) - (r+\tau_0))}.$$

This is clearly positive for any $S^e \geq 0$, $a < k^u(a,z)$ and when the incentive compatible constraint does not bind.

When $\chi \in (0,1)$, we have that in equilibrium $x_k = r + \tau(a,z)$, which is independent of χ. Therefore $\frac{\partial k^b(a,z)}{\partial \chi} = 0$. In addition,

$$\frac{\partial \tilde{r}(a,z)}{\partial \chi} = \frac{y^b(a,z) + \tau(a,z)a - \kappa - \tilde{w}(a,z)}{k^b(a,z) - a} > 0.$$

B.2 Proof of Proposition 2

When $\chi = 0$ and entrepreneurs' bargaining power is equal to one, then it is optimal to set $S^b = 0$. Then Equation (7) implies

$$\phi x(k^b(a,z)) = (r+\tau)k^b(a,z),$$

and Equation (8) requires:

$$\tilde{r}(a,z) = r + \tau(a,z).$$

Then the interest rate on loan will be negatively related with the asset value of each entrepreneur, a, and with her managerial productivity, z. Using the incentive compatible constraint we can show that $\frac{\partial k^b(a,z)}{\partial a} > 0$ and $\frac{\partial k^b(a,z)}{\partial z} > 0$, as required.

When $\chi = 1$ and $\tau(a,z) = \tau_0$, then it can be shown that

$$\frac{\partial \tilde{r}(a,z)}{\partial z} = \frac{(\phi x_k - \tilde{r})\tilde{w}_z(a,z) + \tilde{r}(1-\phi)x_z}{(1-\phi)x_k(k-a)}.$$

If $w > \tilde{\pi}(a,z)$, then $\tilde{w}_z(a,z) = 0$ and this expression is clearly positive. In addition, for this case $\frac{\partial k^b(a,z)}{\partial z} = -\frac{x_z}{x_k} < 0$. When $w < \tilde{\pi}(a,z)$, then $\tilde{w}_z(a,z) = x_z(a) > 0$ and we cannot determine the sign of $\frac{\partial \tilde{r}(a,z)}{\partial z}$. The first term of the numerator of the above equation is negative ($\phi x_k - \tilde{r} = \frac{\tilde{r}(\alpha+\theta-1)k - \alpha\tilde{r}a}{(1-\theta)k} < 0$.) while the second is positive. Moreover, $\frac{\partial k^b(a,z)}{\partial z} = \frac{x_z(a)-(1-\phi)x_z}{(1-\phi)x_k} < 0$, which can be positive or negative.

Notice also that:

$$\frac{\partial \tilde{r}(a,z)}{\partial a} = \frac{(\phi x_k - \tilde{r})(r + \tilde{w}_a(a,z)) + \tilde{r}(1-\phi)x_k}{(1-\phi)x_k(k-a)},$$

and

$$\frac{\partial k^b(a,z)}{\partial a} = \frac{r + \tilde{w}_a(a,z)}{(1-\phi)x_k}.$$

If $w > \tilde{\pi}(a,z)$, then $\tilde{w}_a(a,z) = 0$ and $\frac{\partial k^b(a,z)}{\partial a} = \frac{r}{(1-\phi)x_k} > 0$ and $\frac{\partial \tilde{r}(a,z)}{\partial a} = \frac{(\phi x_k - \tilde{r})r + \tilde{r}(1-\phi)x_k}{(1-\phi)x_k(k-a)}$. It can be shown that the first term of the numerator is negative while the second is positive. Therefore, it is not clear how the loan interest rate varies with assets a.

If $w < \tilde{\pi}(a,z)$, then $\tilde{w}_a(a,z) = x_k(a) - r$ and $\frac{\partial k^b(a,z)}{\partial a} = \frac{x_k(a)}{(1-\phi)x_k(k)} = \frac{1}{1-\phi}\left(\frac{k}{a}\right)^{\frac{1-\alpha-\theta}{1-\theta}} > 1$. In addition, $\frac{\partial \tilde{r}(a,z)}{\partial a} = \frac{(\phi x_k - \tilde{r})x_k(a) + \tilde{r}(1-\phi)x_k}{(1-\phi)x_k(k-a)}$. Once more it is not possible to sign the numerator of the above expression and therefore it is not clear how the loan interest rate varies with assets a.

For the case in which $\chi \in (0,1)$ and $\tau(a,z) = \tau_0$, we can easily show that

$$\frac{\partial k^b(a,z)}{\partial \chi} < 0,$$

while

$$\frac{\partial \tilde{r}(a,z)}{\partial \chi} = -\frac{((1-\theta-\alpha)\tilde{r}k + \alpha\tilde{r}a)\frac{\partial k^b(a,z)}{\partial \chi}}{k^b(a,z) - a} > 0.$$

B.3 Analytical Derivation of Figure 1

Consider the case in which $\chi = 0$. Let $d \leq a$ be the amount of assets entrepreneurs use in their business, and let l be loans, such that $k = d + l$. Clearly, since $\tilde{r} = r + \tau(a,z) \geq r$ for all finite a, then if $l > 0$, then $d = a$. The problem of the entrepreneur can be rewritten as

$$\pi(a,z) = \max_{n,d,l \geq 0} z(d+l)^\alpha n^\theta - wn - \tilde{r}l - rd - \kappa, \tag{16}$$

subject to

$$\begin{aligned} l &\leq \frac{\phi(z(d+l)^\alpha n^\theta - wn)}{\tilde{r}}, \text{ with } \tilde{r} = r + \tau_0 + \frac{\tau_a}{1+a} + \frac{\tau_z}{1+z}, & (17)\\ d &\leq a. & (18) \end{aligned}$$

The Lagrangean associated to this problem is:

$$L = z(d+l)^{\alpha}n^{\theta} - wn - \tilde{r}l - rd - \chi + \lambda\left[\frac{\phi(z(d+l)^{\alpha}n^{\theta} - wn)}{\tilde{r}} - l\right] + \mu[a-d]$$

The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial n} = \left(\theta\frac{y}{n} - w\right)\left(1 + \lambda\frac{\phi}{\tilde{r}}\right) \leq 0,\ n \geq 0,\ \frac{\partial L}{\partial n}n = 0, \tag{19}$$

$$\frac{\partial L}{\partial d} = \alpha\frac{y}{k}\left(1 + \lambda\frac{\phi}{\tilde{r}}\right) - r - \mu \leq 0,\ d \geq 0,\ \frac{\partial L}{\partial d}d = 0, \tag{20}$$

$$\frac{\partial L}{\partial l} = \alpha\frac{y}{k}\left(1 + \lambda\frac{\phi}{\tilde{r}}\right) - \tilde{r} - \lambda \leq 0,\ l \geq 0,\ \frac{\partial L}{\partial l}l = 0, \tag{21}$$

$$\mu[a-d] = 0, \tag{22}$$

$$\lambda\left[\frac{\phi(z(d+l)^{\alpha}n^{\theta} - wn)}{\tilde{r}} - l\right] = 0. \tag{23}$$

Case 1: If $0 < d < a$, then $\mu = 0$ and $\lambda = 0$. Therefore:

$$\theta\frac{y}{n} = w,\ \alpha\frac{y}{k} = r,\ \text{ and }\ \alpha\frac{y}{k} < \tilde{r}.$$

It can be shown that

$$k^u(z) = \left(z\left(\frac{\alpha}{r}\right)^{(1-\theta)}\left(\frac{\theta}{w}\right)^{\theta}\right)^{\frac{1}{1-\alpha-\theta}},\ n^u(z) = \left(z\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\theta}{w}\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha-\theta}}.$$

And

$$y^u(z) = \left(z\left(\frac{\alpha}{r}\right)^{\alpha}\left(\frac{\theta}{w}\right)^{\theta}\right)^{\frac{1}{1-\alpha-\theta}},$$

with

$$\pi^u(z) = (1-\alpha-\theta)y^u(z) - \kappa.$$

Therefore, $\pi^u(z) \geq w$ defines a threshold ability level z^u given by

$$z^u = \left(\frac{w+\kappa}{1-\alpha-\theta}\right)^{1-\alpha-\theta}\left(\frac{r}{\alpha}\right)^{\alpha}\left(\frac{w}{\theta}\right)^{\theta},$$

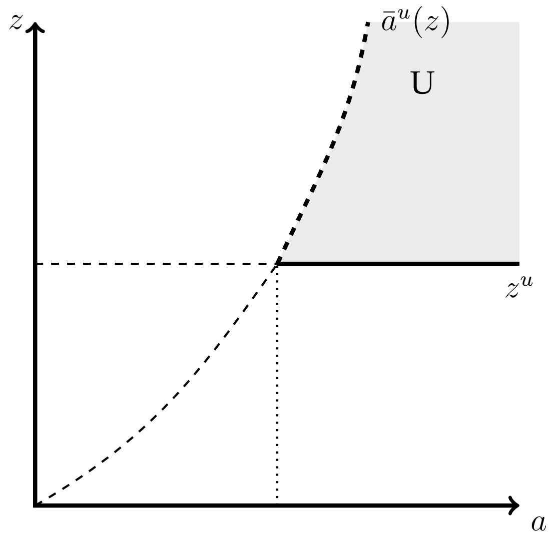

Figure B1: Case 1: $k^u(z) \leq a$. Light gray shaded area shows the measure of agents who are unconstrained entrepreneurs.

such that for all (a, z) with $a > k^u(z)$, and $z \geq z^u$ agents are entrepreneurs. Notice that z^u is independent of a. Since $k^u(z)$ is increasing with z, we can define a threshold of assets $\bar{a}^u(z)$, such that all agents with $z > z^u$ and $a > \bar{a}^u(z)$ are unconstrained entrepreneurs. Figure B1 shows the region in which entrepreneurs are unconstrained and do not borrow.

Case 2: If $d = a > 0$ and $l = 0$, then $\mu > 0$ and $\lambda = 0$. Consequently:

$$\theta \frac{y}{n} = w, \; \alpha \frac{y}{k} = r + \mu, \; \text{ and } \; \alpha \frac{y}{k} < \tilde{r}.$$

It can be shown that

$$k^{nb}(a, z) = a,$$

$$n^{nb}(a, z) = \left(z a^{\alpha} \left(\frac{\theta}{w} \right) \right)^{\frac{1}{1-\theta}},$$

$$y^{nb}(a, z) = \left(z a^{\alpha} \left(\frac{\theta}{w} \right)^{\theta} \right)^{\frac{1}{1-\theta}},$$

and

$$\pi^{nb}(a,z) = (1-\theta)y^{nb}(a,z) - ra - \kappa.$$

Condition $\pi^{nb}(a,z) \geq w$ defines a treshold ability level $z^{nb}(a,z)$ given by

$$z^{nb}(a,z) = \left(\frac{w+\kappa+ra}{1-\theta}\right)^{1-\theta}\left(\frac{w}{\theta}\right)^{\theta}\frac{1}{a^{\alpha}},$$

such that for all agents with $k^{nb}(a,z) = a$, and $z \geq z^{nb}(a)$ agents are entrepreneurs. Observe that since $1-\alpha-\theta > 0$, then $\lim_{a\to 0} z^{nb}(a) = \infty$. It can be shown that

$$sign\left(\frac{\partial z^{nb}(a)}{\partial a}\right) = sign\left((1-\alpha-\theta)ra - \alpha(w+\kappa)\right).$$

Notice that since $\alpha\frac{y}{a} = r+\mu$ at $z^{nb}(a)$, such that $(1-\theta)y^{nb}(a,z) - ra - \kappa = w$, we have

$$sign\left(\frac{\partial z^{nb}(a)}{\partial a}\right) = sign\left(-\mu a\right).$$

This is clearly negative, as long as $\mu > 0$. Therefore as $a \to \bar{a}^u$, then $z^{nb}(a) \to z^u$.

Case 3: If $d = a > 0$ and $0 < l < \frac{\phi(z(d+l)^{\alpha}n^{\theta} - wn)}{\tilde{r}}$, then $\mu > 0$ and $\lambda = 0$. Consequently:

$$\theta\frac{y}{n} = w,\ \alpha\frac{y}{k} = r+\mu,\ \text{ and }\ \alpha\frac{y}{k} = \tilde{r}.$$

It can be shown that

$$k^b(a,z) = \left(z\left(\frac{\alpha}{\tilde{r}}\right)^{(1-\theta)}\left(\frac{\theta}{w}\right)^{\theta}\right)^{\frac{1}{1-\alpha-\theta}},$$

$$n^b(a,z) = \left(z\left(\frac{\alpha}{\tilde{r}}\right)^{\alpha}\left(\frac{\theta}{w}\right)^{1-\alpha}\right)^{\frac{1}{1-\alpha-\theta}},$$

$$y^b(a,z) = \left(z\left(\frac{\alpha}{\tilde{r}}\right)^{\alpha}\left(\frac{\theta}{w}\right)^{\theta}\right)^{\frac{1}{1-\alpha-\theta}},$$

and

$$\pi^b(a,z) = (1-\alpha-\theta)y^b(a,z) + (\tilde{r}-r)a - \kappa.$$

Therefore, given that $\tilde{r} = r + \tau_0 + \frac{\tau_a}{1+a} + \frac{\tau_z}{1+z}$, the inequality $\pi^b(a,z) \geq w$ defines an ability level $z^b(a,z)$ given by

$$z^b(a,z) = \left(\frac{w+\kappa-(\tilde{r}-r)a}{1-\alpha-\theta}\right)^{1-\alpha-\theta} \left(\frac{r}{\alpha}\right)^{\alpha} \left(\frac{w}{\theta}\right)^{\theta},$$

such that for all agents with $a < k^b(a,z) < \frac{\phi(z(d+l)^\alpha n^\theta - wn)}{\tilde{r}}$, and $z \geq z^b(a,z)$, then agents are entrepreneurs. Observe that $\frac{\partial z^b(a,z)}{\partial a} < 0$.

Cases 2 and 3 imply that for all $a \in [0, \bar{a}^u]$, there will be a productivity level $z^e(a,z) = \max\{z^u, \min(z^{nb}(a,z), z^b(a,z))\}$ such that $z^e(\tilde{r}, w, \bar{a}^u(r,w)) = z^u(r,w)$, $\frac{\partial z^e(\tilde{r},w,a)}{\partial a} < 0$, and $\lim_{a\to 0} z^{nb}(a,z) = \infty$. In addition, whenever $z \geq z^e(\bar{a}^u, z)$, then the agent is an entrepreneur. See Figure B3

Case 4: If $d = a > 0$ and $l = \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$, then $\mu > 0$ and $\lambda > 0$. Consequently:

$$\theta\frac{y}{n} = w,\ \alpha\frac{y}{k}\left(1+\lambda\frac{\phi}{\tilde{r}}\right) = r+\mu,\ \text{ and }\ \alpha\frac{y}{k}\left(1+\lambda\frac{\phi}{\tilde{r}}\right) = \tilde{r}+\lambda.$$

Given that the amount of capital is constrained, it must be the case that $\alpha\frac{y}{k} > \tilde{r}$. The labor first-order condition yields:

$$n(w; k^c, z) = \left(z\,(k^c)^\alpha \left(\frac{\theta}{w}\right)\right)^{\frac{1}{1-\theta}},$$

where k^c solves

$$k^c = a + \frac{\phi(z(k^c)^\alpha n(w;k^c,z)^\theta - wn(w;k^c,z))}{\tilde{r}}.$$

This equation defines

$$k^c = k^c(\tilde{r}, w; z, a),\ \text{ with }\ \frac{\partial k^c}{\partial a} > 0,\ \frac{\partial k^c}{\partial z} > 0.$$

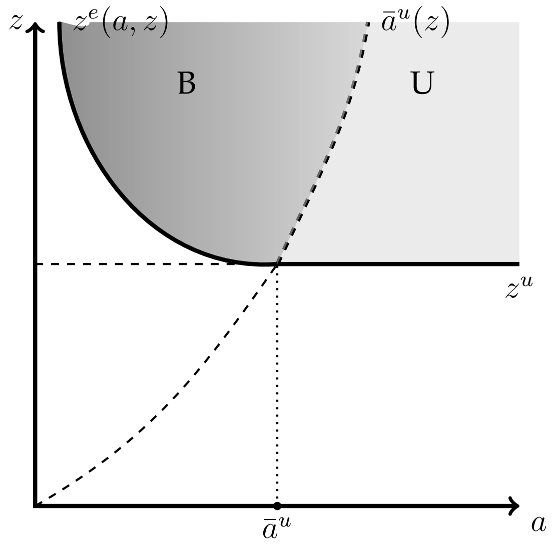

Figure B2: Cases 2 and 3: $k^u(z) < a$ and $0 \leq k - a < \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$. Light gray shaded area shows the measure of agents who are unconstrained entrepreneurs. Dark gray shaded area shows the measure of agents who are not constrained borrowers.

The derivatives can be checked using the Implicit Function Theorem.

We have that

$$y^c(\tilde{r}, w; z, a) = \left(zk^c(\tilde{r}, w; z, a)^\alpha \left(\frac{\theta}{w} \right)^\theta \right)^{\frac{1}{1-\theta}}$$

and

$$\pi^c(\tilde{r}, w; z, a) = (1-\theta)(1-\phi)y^c(\tilde{r}, w; z, a) - ra - \kappa.$$

Condition $\pi^c(\tilde{r}, w; z, a) \geq w$ defines a threshold ability level $\bar{z}^c(\tilde{r}, w; a)$, which is decreasing in a as long as $\lambda > 0$. We can show that $\lim_{a \to 0} \bar{z}^c(\tilde{r}, w; a) = \infty$. Observe that when $\lambda = 0$ and $l = \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$, then for agents who are indifferent to be entrepreneurs or workers, we have that $\bar{z}^c(\tilde{r}, w; a) = \bar{z}^b(\tilde{r}, w; a)$. This defines a value $\bar{a}^c(w, \tilde{r})$, such that whenever $a < \bar{a}^c(w, \tilde{r})$ and $\bar{z}^b(\tilde{r}, w; a) \leq z \leq \bar{z}^c(\tilde{r}, w; a)$, the leverage constraint is binding. For such agents, then $l = \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$ and $\lambda > 0$, and $\bar{z}^c(\tilde{r}, w; a) > \bar{z}^b(\tilde{r}, w; a)$, in order to compensate for the low capital used. Therefore for any $\bar{z}^b(\tilde{r}, w; a) \leq z \leq \bar{z}^c(\tilde{r}, w; a)$

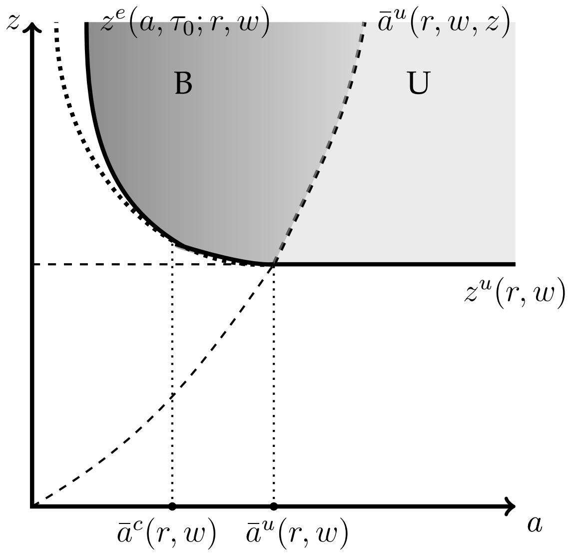

Figure B3: Cases 4: $k^u(r, w) < a$ and $k - a = \frac{\phi(z(a+l)^\alpha n^\theta - wn)}{\tilde{r}}$. Light gray shaded area shows the measure of agents who are unconstrained entrepreneurs. Dark gray shaded area shows the measure of agents who are not constrained borrowers.

and $a < \bar{a}^c(w, \tilde{r})$, the occupational choice is restricted by the leverage ratio. This is shown in the figure below.

C Quantitative Results for the Small Open Economy

For the small open economy (SOE) we fix the interest rate at 2 percent and instead calibrate to the international asset position of Brazil, which averages 34% of GDP over the sample. The tables below reproduce the analogs of Tables 3-5 for the SOE below. As is well known, (e.g., Midrigan and Xu, 2014) the aggregate impacts are smaller in a SOE, since the desire to self-finance does not drive the interest rate down, which would hamper the ability to self-finance. Nevertheless, the results show that the overall findings of the relative importance of spreads vs. collateral constraints, and the impact of the various spread-causing frictions, are robust to a SOE.

Table C2: Small Open Economy: Calibration and Model Fit

Parameter Values		
Parameter	**Description**	**Value**
2 assigned parameters		
σ	Coefficient of relative risk aversion	1.50
δ	Depreciation rate	0.06
11 calibrated parameters		
ρ	Subjective discount rate	0.17
α	Elast. of y with respect to k	0.27
θ	Elast. of y with respect to n	0.31
κ	Fixed cost of production	0.68
η	Curvature of the Pareto distr.	2.15
γ	New productivity arrival rate	0.08
ϕ	Enforcement parameter	0.22
τ_0	Interm. costs - independent factor	0.02
τ_a	Interm. costs - elast. of assets	0.64
τ_z	Interm. costs - elast. of productivity	0.88
χ	Bank barg. power in a loan	0.01

Model Fit		
12 Targeted Moments	**Data**	**Model**
Net foreign assets to GDP ratio	0.30	0.31
Capital's share of income	0.40	0.32
Investment rate	0.17	0.11
Top 10% earners' income share	0.56	0.64
Average firm size	21.0	18.6
Top 10% firms' employment share	0.78	0.71
Firm exit rate	0.08	0.07
External finance to GDP ratio	0.49	0.52
Fraction firms with credit	0.35	0.36
Average spread (unweighted)	0.38	0.39
Average spread (credit-weighted)	0.06	0.06
Standard deviation of spread	0.32	0.31

Table C3: Small Open Economy: Impacts of Credit Frictions on Development

Value Relative to Perfect Credit:	Perfect Credit (1)	Benchmark (2)	No Quant. Constr. (3)	No Spread Frictions (4)	No Spread Disp. (5)	All Quant. Constr. (6)
Aggregate values relative to perfect credit world:						
GDP	1.00	0.72	0.73	0.94	0.74	0.76
TFP	1.00	0.92	0.92	1.01	0.95	0.97
Wage	1.00	0.69	0.71	0.93	0.76	0.76
Capital	1.00	0.41	0.42	0.77	0.40	0.42
Assets	1.00	1.25	1.24	1.08	1.19	1.24
Credit/GDP	1.00	**0.20**	0.24	0.72	**0.20**	**0.21**
Firm growth	0.02	0.07	0.05	0.04	0.03	0.04
Exit rate	0.07	0.07	0.07	0.07	0.07	0.07
Avg. firm size	8	19	17	6	9	9

Column (2) parameter values are those calibrated in Table C2. Relative to these values, Column (1): $\tau_i = \chi = 0$ and $\phi = 1$, Column (3): $\phi = 1$, Column (4): $\chi = \tau_i = 0$, Column (5): $\tau_a = \tau_z = \chi = 0$ and $\tau_0 = 0.08$, and Column (6): $\tau_i = \chi = 0$ and $\phi = 0.06$, calibrated to match credit/GDP in the benchmark and data.

Table C4: Small Open Economy: Isolated Impacts of Spread-Causing Frictions

		Eliminating Frictions				Single Friction Calibrations			
	Benchmk	No Market Power	No Uniform Cost	No $a-$ depend. Cost	No $z-$ depend. Cost	All Market Power	All Uniform Cost	All $a-$ depend. Cost	All $z-$ depend. Cost
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Aggregate values relative to perfect credit world:									
GDP	0.72	0.72	0.74	0.74	0.80	0.54	0.75	0.57	0.74
TFP	0.92	0.92	0.92	0.94	0.93	0.71	0.95	0.79	0.94
Wage	0.69	0.69	0.71	0.74	0.77	0.50	0.78	0.54	0.71
Capital	0.41	0.41	0.45	0.41	0.58	0.37	0.42	0.31	0.41
Assets	1.25	1.25	1.24	1.23	1.19	0.78	1.17	0.99	1.26
Credit/GDP	0.20	0.20	0.28	0.21	0.49	0.44	0.25	0.17	0.19
Firm credit spread moments:									
Avg. (weighted)	0.06	0.06	0.04	0.07	0.03	**0.05***	**0.06**	**0.06**	**0.06**
Avg. (unweighted)	0.39	0.39	0.35	0.20	0.13	0.06	0.06	35	0.26
Std. deviation	0.31	0.31	0.30	0.05	0.20	0.03	0.00	57	0.08
Frac. with credit	0.36	0.36	0.43	0.59	0.89	0.87	0.93	0.05	0.37
Firm growth	0.07	0.07	0.07	0.04	0.08	0.04	0.03	0.08	0.04
Exit rate	0.07	0.07	0.07	0.07	0.07	0.06	0.07	0.06	0.07
Avg. firm size	19	19	19	11	18	26	9	29	14

* indicates this is the closest match obtained. Column (1) parameter values are those calibrated in Table C2. Relative to these values, Column (2): $\chi = 0$; Column (3): $\tau_0 = 0$, Column (4): $\tau_a = 0$; Column (5): $\tau_z = 0$; Column (6): $\tau_i = 0$ and $\chi = 0.1$; Column (7): $\tau_a = \tau_z = \chi = 0$ and $\tau_0 = 0.06$; Column (8): $\tau_0 = \tau_z = \chi = 0$ and $\tau_a = 126.6$; and Column (9): $\tau_0 = \tau_a = \chi = 0$ and $\tau_z = 1.45$. For Columns (6)-(9), the calibrated value is chosen to match the weighted spread in the benchmark.