Appendices (On-line Only)

A. Simple Model of Police-Motorist Interaction

We develop a model of police traffic stops and consider the effect of discrimination on the driving behavior of minority motorists. First, we formalize the police decision to stop a motorist for a traffic infraction and demonstrate that the standard presumption of increasing minority share of stops with discrimination (lower stop costs for minority motorists) holds when the speed distribution of minority motorists is fixed. This model is different from traditional models of police search, such as Knowles et al. (1999), because officers observe the violation in most motorist stops, while searches are made of stopped motorists based on the likelihood that an individual is carrying contraband. As Knowles et al. (1999) note, motorists will have an incentive to always break the law if they face no chance of being stopped or punished, but will never break the law if they know that they will be punished with certainty. To create uncertainty in stops when the infraction is observable, we extend the standard police cost function to include a stochastic term that represents factors around the context or circumstance of the infraction. These factors may include whether a police officer observed the infraction, differences in officer stop thresholds, impact of pending stop quotas, rainy or cold weather, the officer's current mood, and whether the officer is busy with other activities.

In the second subsection, we formalize the motorist's problem of selecting infraction severity, e.g. a driving speed or other moving violation. We impose three key assumptions to assure that some motorists choose not to commit infractions. Specifically, we assume that police receive a pay-off and motorists face a cost for any positive infraction level and that the pay-off and cost are both bounded away from zero as the infraction level approaches zero. Then, the stochastic component of stop costs assures that motorists always face a positive expected cost of committing an infraction even for minimal infraction levels.[1] We also assume that benefits from committing an infraction increase smoothly with the infraction level as motorists shift from zero infraction to positive levels. Together, these assumptions assure that motorists with very weak preferences for committing infractions chose not to commit

[1] All results can be derived if instead police establish some threshold such as two miles per hour over the speed limit, never stopping motorists who are going two miles over the speed limit or less, but always having some positive probability of stopping anyone who goes more than two miles over the speed limit.

infractions. We demonstrate that the minority share of stops can fall with discrimination without placing restrictions on the relative preferences of majority and minority motorists.

On the surface, data on infraction severity among stopped motorists, such as the speed travelled over the posted speed limit, would provide a natural approach for examining whether motorists adjust their behavior in response to discrimination. However, any decrease in speeds with discrimination/lower stop costs (intensive margin) can be confounded by the same extensive margin adjustments discussed above. When discrimination increases, motorists who committed the least severe violations exit the population of motorists at risk of being stopped. In the third subsection, we address this problem by demonstrating that, in a sample of stopped motorists, the intensive response of motorists will dominate the extensive response at the upper portions of the speed distribution. Therefore, we can test for behavioral responses of minority motorists by examining the distribution of speeds, rather than average shifts in the level of speeding. The key assumption required to obtain this result is that the right tail of the minority preference distribution is not too heavy. Notably, this result holds for some distributions that have substantially heavier tails than the normal distribution.

As noted above, the results below do not apply to analyses of disparities in policing where the population at risk of treatment is observed, such as models of police search where the population of stopped motorists provides a counterfactual for the population at risk of search. For police search, Knowles et al. (1999) and Anwar and Fang (2006) use theory to provide solutions to concerns that the distribution of unobservable risk variables differs between majority and minority motorists.[2] In our empirical work, we address concerns about differences in the distribution of motorist preferences across race by exploiting the Veil of Darkness (VOD) approach developed by Grogger and Ridgeway (2006), who compare the composition of stops in daylight to the composition in darkness when police cannot observe race. In our theoretical work, we demonstrate that the racial composition of stops or other

[2] Knowles et al. (1999) demonstrate circumstances under which all groups that are observationally equivalent to police and are at risk of being searched must have the same equilibrium likelihood of carrying contraband, and based on these results, defend examining racial differences in unconditional rates of search success. Anwar and Fang (2006) develop rank tests for detecting officer prejudice by demonstrating that the ranking of search success rates should be not vary across officer types under the null that officers do not consider race in search decisions. Also see approaches in Arnold, Dobbie and Yang (2018) on bail decisions.

police treatment may not capture racial disparities, even when using approaches like VOD to account for racial differences in unobserved attributes that contribute to police stops.

A.1. The Police Officer's Problem

Officer's decision to stop a motorist $\gamma(i, d, \phi)$ is made after observing non-negative infraction severity i, motorist type/demography d, and circumstances surrounding the stop ϕ. The resulting officer's problem takes the form

$$\max_{\gamma(i,s_d,\phi)} [u(i) - h(\phi) - s_d]\gamma(i, s_d, \phi) \qquad (1)$$

where s_d as a fixed component of stop costs and $h(\phi)$ represents circumstantial costs. The model appears to assume uniform stop costs, but if motorists receive a random draw of officer when observed committing an infraction then officer specific heterogeneity in stop costs can be included in the circumstances term ϕ and all results will continue to hold. Models of police search must depend upon the equilibrium likelihood of guilt because guilt is unobserved prior to search. In our case, however, the severity of the moving violation is observed by police, and so the individual's behavior is the most relevant information on which to base the police stop decision.[3]

We make the following assumptions about police pay-offs and costs

Assumption 1.1 u is continuous and twice differentiable over positive values of its argument, $\frac{du(i)}{di} > 0$ and $\frac{d^2u(i)}{di^2} > 0 \ \forall \ i > 0$, $\lim_{i\to 0^+} u(i) = u_0 > 0$, and $u(i) = 0 \ \forall \ i \leq 0$;

Assumption 1.2 $\phi \sim Uniform(0,1)$;

Assumption 1.3 h is a continuous, twice differentiable function defined over $[0,1)$, $\frac{dh(\phi)}{d\phi} > 0 \ \forall \ 0 \leq \phi \leq 1$, $lim_{\phi\to 1} h(\phi) = \infty$, and $h(0) = 0$;

Assumption 1.4 $u_0 - s_d > 0,\ u_0 > 0,\ \ s_d > 0 \ \forall \ d$

In Assumption 1.1, we assume u is discontinuous at zero so that the officer receives no pay-off for stopping a non-infracting motorist, but has a pay-off bounded away from zero

[3] In principle, police may also care about aggregate stop patterns and adjust to aggregate changes in motorist driving behavior. However, our results on the ambiguity of stop-rate based tests would still hold since our model is a special case of this possible generalization.

for any positive infraction level. We also assume that u has increasing total and marginal pay-off with respect to infraction severity. These assumptions are consistent with the penalty structures in many states. In Assumption 1.2, we assume circumstances are drawn from a uniform $(0,1)$ distribution and allow the monotonically increasing function $h(\phi)$ to capture possible non-linearities in the mapping between circumstances and costs. Therefore, Assumption 1.3 does not directly impose sign restrictions on the second derivative of h to allow for generality over circumstance costs. However, the assumption $lim_{\phi \to 1} h(\phi) = \infty$ implies that the second derivative of h must be positive as ϕ approaches one. Finally, Assumption 1.4 requires a positive net pay-off of stop under favorable circumstances, sufficiently low ϕ, even for small positive infraction levels. Therefore, the probability of stop is bounded away from zero for any non-zero infraction level creating a situation where motorists might choose to not commit infractions (modeling requirement 2 above).

The solution to the officer's problem implies an optimal infraction threshold above which the officer makes a stop with certainty and below which the officer does not make a stop.[4] Given the officer's net utility of $u(i) - h(\phi) - s_d \ \forall \ i$, The solution to her problem is

$$\gamma(i, s_d, \phi) = \begin{cases} 1, & \text{if } u(i) > h(\phi) + s_d \\ 0, & \text{otherwise.} \end{cases}$$

Officer will stop all motorists at any infraction level above some threshold severity level

$$i^*(\phi, s_d) = u^{-1}(h(\phi) + s_d) \tag{2}$$

where u^{-1} maps from stop costs (u_0, ∞) to stop thresholds within $(0, \infty)$.[5]

Conditional on infraction severity and stop costs, we can solve Equation (2) for the circumstances $\phi^*(i, s_d)$ when net pay-off is zero by exploiting the monotonicity of $h(\phi)$.

$$\phi^*(i, s_d) = h^{-1}(u(i) - s_d) \tag{3}$$

[4] In principle, γ could be a probability between zero and one if the net return were zero, but since ϕ follows a continuous distribution and h is a monotonic, continuous function zero return to stop only arises on a set of measure zero. Unlike Knowles, Persico, and Todd (1999) and Persico and Todd (2006), circumstantial costs imply that motorists' adjustment no longer yields police indifference between stopping and not stopping motorists.

[5] We also note that $h(\phi) + s_d$ is always greater than u_0 for all combinations i and ϕ where $u(i) = h(\phi) + s_d$.

Based on Assumption 1.3, h^{-1} maps from stop costs $(0, \infty)$ to stop circumstances $(0,1)$.[6] ϕ is distributed uniform, and so Equation (3) represents the probability that an officer stops a motorist with infraction level i.

Lemma 1. *(i) The infraction level representing the optimal stop-threshold,* $i^*(\phi, s_d) = u^{-1}(h(\phi) + s_d)$*, is increasing in officer circumstances and demographic stop cost, and these derivatives are finite for a finite* ϕ*. (ii) The probability of an officer making a stop,* $\phi^*(i, s_d) = h^{-1}(u(i) - s_d)$*, is decreasing in stop cost and increasing in the level of infraction, and these derivatives are finite for finite* i*. (iii) The* $\lim_{i \to 0} \phi^*(i, s_d) > 0$ *for all* s_d.

Proof of Lemma 1. (i) Assumption 1.1 and the Implicit Function Theorem imply that the derivative $u^{-1\prime}(\cdot) > 0$ and finite over its domain (u_0, ∞). Then by Assumption 1.3 and inspection it is clear the derivative of Equation (2) implies $\frac{\partial i^*}{\partial \phi} = u^{-1\prime} \frac{dh}{d\phi} > 0$, and $\frac{\partial i^*}{\partial s_d} = u^{-1\prime} > 0$.

(ii) Assumption 1.3 and the Implicit Function Theorem imply that the derivative $h^{-1\prime}(\cdot) > 0$ and finite over its domain, and by Assumption 1.1 and inspection it is clear the derivative of Equation (3) implies $\frac{\partial \phi*}{\partial s_d} = -h^{-1\prime} < 0$, and $\frac{\partial \phi^*}{\partial i} = h^{-1\prime} \frac{du}{di} > 0$.

(iii) Based on Equation (3) and the continuity of h, we can rewrite $\lim_{i \to 0} \phi^*(i, s_d)$ as $\lim_{i \to 0} h^{-1}[u(i) - s_d] = h^{-1}\left[\lim_{i \to 0} u(i) - s_d\right] = h^{-1}(u_0 - s_d) > 0$, which is greater than zero based on Assumption 1.4 and the definition of h (Assumption 1.3). **QED**

In this model, discrimination arises if police officers have lower demographic cost of stopping a minority *(m)* relative to the majority *(w)*, $s_m < s_w$. A standard statistic for evaluating racial discrimination in stops is the relative share of stops involving minority motorists, or

Definition 1. $K_f \equiv \frac{p[m|stopped, s_m, f(i,m)]}{p[w|stopped, s_w, f(i,w)]} = \frac{\int_0^\infty f(i,m)\phi^*(i,s_m)di}{\int_0^\infty f(i,w)\phi^*(i,s_w)di}$

where $f(i, d)$ is the distribution of infraction severity by motorist type. Holding majority motorist stop costs fixed, discrimination (or an increase in discrimination) can be represented

[6] We note that based on Assumption 1.4 $u(i) - s_d$ is always greater than zero for positive i.

as a decrease in minority stop costs. Proposition 1 is consistent with the typical assumption that discrimination increases the relative stop rate of minority motorists (K_f).

Proposition 1. *A decrease in the stop costs of minority motorists, s_m, will increase the relative stop rate of minority motorists, K_f.*

Proof of Proposition 1. The theorem is established by taking the derivative of K_f with respect to s_m.

$$\frac{dK_f}{ds_m} = \frac{1}{p[w|stopped, s_w, f(i,w)]} \int_0^\infty f(i,m) \frac{\partial \phi^*}{\partial s_m} di < 0$$

The derivative is negative based on part ii of Lemma 1. **QED**

While the model implies that the marginal offenders are those who commit the least serious infractions, the composition effect that leads to an ambiguity in the VOD test will arise if any minority motorists exit the population of those committing infractions in the presence of discrimination, even if for example they exit this population by not driving.

A.2. The Motorist's Problem

The motorist problem can be characterized as a trade-off between the benefit of committing an infraction $b(i,c)$, which depends on motorist preferences c, e.g. recklessness, criminality, stress, timing of trip, sleep deprivation, etc. and the expected cost of being stopped, or

$$\max_{i'(c,s_d)} b(i,c) - \tau(i)\phi^*(i,s_d) \tag{4}$$

Where benefit of committing an infraction $b(i,c)$ depends on motorist preferences c and cost of being stopped for committing an infraction $\tau(i)$ times the probability of being stopped ϕ^*.

We make the following assumptions about motorist's constraints and preferences

Assumption 2.1 b is a continuous, twice differentiable, non-negative function, $\frac{\partial b}{\partial i} > 0$ and $\frac{\partial^2 b}{\partial i^2} < 0 \; \forall \; c$ and $i \geq 0$, $b(0,c) = 0$, and $\lim_{c \to -\infty} b(i,c) = 0 \; \forall \; i$;

Assumption 2.2 $\frac{\partial b}{\partial c} > 0$ and $\frac{\partial^2 b}{\partial c \partial i} \geq 0 \; \forall \; c$ and for $i \geq 0$;

Assumption 2.3 τ is a continuous, twice differentiable, positive function, $\frac{d\tau}{di} > 0$ and $\frac{d^2\tau}{di^2} > 0$ for $i \geq 0$, and $\tau(0) > 0$;

Assumption 2.4 $\frac{\partial b}{\partial i}|_{i=0} \geq \frac{d\tau}{di}|_{i=0} h^{-1}(u_0 - s_d) + \tau(0) h^{-1'}(u_0 - s_d)\ \forall\ c$ and $\lim_{i\to\infty} \frac{d\tau}{di} > \frac{db}{di}$

Assumption 2.5 $\frac{\frac{d^2u}{di^2}}{\frac{du}{di}} \geq \frac{-h^{-1''}}{h^{-1'}} \frac{\partial u}{\partial i}$ and $\frac{\frac{\partial \tau}{\partial i}}{\tau(i)} > \frac{-h^{-1''}}{h^{-1'}} \frac{\partial u}{\partial i} = \frac{\partial^2 \phi^*}{\partial i \partial s_d} \left(-\frac{\partial \phi^*}{\partial s_d}\right)^{-1}$ for $i \geq 0$

Assumptions 2.1-2.4 are relatively standard assumptions. In Assumption 2.1, we assume that the motorist benefit or pay-off is an increasing function of infraction severity and that marginal benefit is diminishing. In Assumption 2.2, we assume that both the benefit and the marginal benefit of infracting rise with c, which simply initializes the effect direction of the preference parameter. In Assumption 2.3, we assume that the motorist's cost and marginal cost are increasing in infraction severity. In the last part of Assumption 2.3, we assume that motorist's cost is bounded away from zero for small infraction levels, consistent with fine schedules. This last assumption combined with Lemma 1 allows for the existence of inframarginal motorists who do not commit an infraction (modeling requirement 2). To assure an interior optimal infraction level for motorists who choose to commit an infraction, Assumption 2.4 requires that the slope of the cost function is less than the slope of the benefit function when i equals zero and greater than the slope of the benefit function at large i.

Assumption 2.5 imposes two technical assumptions that the curvature (relative to the slope) of the officer's utility function and the relative slope of the cost function both exceed in magnitude the cross partial derivative of ϕ^* relative to the first derivative of ϕ^* with respect to s_d. Effectively, this restriction places a limit on how quickly the negative relationship between the probability of a stop and stop costs can fall as infraction severity increases. In terms of the primitives, the positive slope of h^{-1} cannot decrease too quickly, or equivalently the positive relationship between circumstances and stop costs cannot increase too quickly in percentage terms. The first restriction allows us to sign the second order condition of the motorist's problem assuring a unique, interior optimum infraction level.[7] The second restriction assures that infraction severity responds to stop costs in the expected manner, i.e. increasing when police find it more costly to stop motorists.

[7] As shown in the proof of Lemma 2, this assumption is only required to establish uniqueness, not existence.

Based on these assumptions, we derive the properties of the optimal motorist infraction level.

Lemma 2. (*i) There exists a unique optimal infraction level* i' *on* R^+ *for a motorist of type* $\{c, d\}$. *(ii) The optimal infraction level is increasing in preferences* c, *increasing in stop costs* s_d, *and the first derivatives of this infraction level function are finite.*

Proof of Lemma 2. (i) The motorist can chooses an infraction level that satisfies the following first-order condition

$$\text{FOC} \equiv \frac{\partial b(i, c)}{\partial i} - \frac{d\tau(i)}{di} \phi^*(i, s_d) - \tau(i) \frac{\partial \phi^*(i, s_d)}{\partial i} = 0 \tag{6}$$

By Assumption 2.1, the first term in Equation (6) is positive on R^+, and by Assumption 2.3 and Lemma 1 the second and third terms are negative when including the subtraction signs. The first part of Assumption 2.4 implies that the right-hand side of Equation (6) is positive at $i = 0$. Turning back to the officer's problem, we know that $\lim_{i \to \infty} u(i) = \infty$ due to $u(i)$ having a positive slope and a non-negative second derivative (Assumption 1.1), and by Assumption 1.3 $\lim_{\omega \to \infty} h^{-1}(\omega) = 1$. Therefore, based on Equation (3), $\lim_{i \to \infty} \phi^*(i, s_d) = 1$, and so by the second part of Assumption 2.4 the negative second term becomes larger in magnitude than the first term as i limits to infinity. These results imply that the FOC is negative for some positive values of i. Therefore, by continuity of all functions over R^+, a positive FOC value at zero and negative FOC value as infinity is approached, solutions i' to Equation (6) must exist on R^+ and an odd number of those solutions must maximize the objective function in Equation (5).

In order to assure a unique solution over R^+, we examine the second-order condition of the motorist's problem

$$\text{SOC} \equiv \frac{\partial^2 b(i, c)}{\partial i^2} - \frac{d^2\tau(i)}{di^2} \phi^*(i, s_d) - 2 \frac{d\tau(i)}{di} \frac{\partial \phi^*(i, s_d)}{\partial i} - \tau(i) \frac{\partial^2 \phi^*(i, s_d)}{\partial i^2} > 0 \tag{7}$$

The first term in Equation (7) is negative based on Assumption 2.1, the second and third terms (again including the minus signs) are negative based on Assumption 2.3 and Lemma 1. If the final term is negative, the SOC is unambiguously negative. In order to show why the final term is negative, we draw on the solution of the officer's problem and the monotonicity of $h^{-1}(x)$. Recall that $\phi^*(i, s_d) = h^{-1}[u(i) - s_d]$; we use this expression to expand the second derivative of ϕ^* from Equation (3)

$$\frac{\partial^2 \phi^*(i, s_{v,d})}{\partial i^2} = \left(\frac{du(i)}{di}\right)^2 h^{-1''}(u(i) - s_d) + \frac{d^2u(i)}{di^2} h^{-1'}(u(i) - s_d) \geq 0.$$

The first term is ambiguous and the second term is positive. If the first term is negative, the second term is at least as large in magnitude as the first term based on Assumption 2.5. Therefore, the last term in Equation (7) is negative, and there exists a unique positive value of i' that maximizes motorist payoff over R^+. Finally, by the continuity of all functions, this solution varies continuously with c and s_d. The continuity of i' assures the derivatives are finite.

(ii) Next, we turn to signing the derivatives of i'. By total differentiation of the first order condition in Equation (6), we show that the optimal infraction level i' is increasing in criminality. Specifically,

$$\frac{di'}{dc} = -\frac{1}{SOC}\frac{\partial(FOC)}{\partial c} = -\frac{\frac{\partial^2 b}{\partial c\,\partial i}}{SOC} > 0\ \forall\ c \text{ and } s_d > 0,$$

where the sign of the numerator is positive based on Assumption 2.2 and the SOC is the expression for the second-order condition in Equation (7) and is negative at positive infraction level where motorists are maximizing their net benefits from infracting on R^+.

A similar exercise signs the derivative with respect to stop costs s_d where the derivative of the FOC or the numerator is

$$\frac{di'}{ds_d} = -\frac{1}{SOC}\frac{\partial(FOC)}{\partial s_d} = \frac{1}{SOC}\left(\frac{\partial \tau}{\partial i}\frac{\partial \phi^*}{\partial s_d} + \tau(i)\frac{\partial^2 \phi^*}{\partial i\,\partial s_d}\right) > 0$$

The first term in parentheses is negative by Assumption 2.3 and Lemma 1, but the second term is ambiguous in sign. Rearranging the expression in the second part of Assumption 2.5 demonstrates that the first term is larger in magnitude than the second term. The negative sign of the SOC implies that the total derivative is positive. **QED**

The curvature restrictions imposed on h^{-1} by Assumption 2.5 are required to establish Lemma 2 because motorists are making decisions based on the expected cost of committing an infraction, $\tau(i)\phi^*(i, s_d)$. As i becomes large, the curvature of $\tau(i)$ dominates as $\phi^*(i, s_d)$

approaches a constant, but at low infraction levels rapid changes in the relationship between stop probability and infraction level as stop costs change can dominate the changes in the infraction penalty function $\tau(i)$. Without the curvature assumptions, motorists could decrease their infraction level as stop costs rise and the likelihood of stop falls, creating the possibility of multiple interior, infraction-level optima.

Next, we define i^{**} as the actual infraction level of the motorist. If the pay-off from the interior, optimal infraction level is positive then $i^{**} = i'$, but if negative then $i^{**} = 0$ and if zero motorists are indifferent between infracting and not. Then, motorists with sufficiently low values of c will choose not to commit an infraction (modeling requirement 2).

Lemma 3. (*i) As long as some motorists chose to commit infractions at finite* c*, there exists a threshold* c^* *on* R *above which motorists commit a traffic infraction at the optimal level* i' *and below which motorists do not commit an infraction or* $i' = 0$*. (ii)* $lim_{c \to c^{*+}} i^{**} > 0$ *where the plus sign indicates the limit from above. (iii) If* c^* *exists, it is decreasing in* s_d*.*

Proof of Lemma 3. (i) The last part of Assumption 2.1, the last part of Assumption 2.3 and part (iii) of Lemma 1 implies that $\lim_{c \to -\infty} (b(i', c) - \tau(i')\phi^*(i', s_d)) < 0$ since benefits limit to zero regardless of the optimal infraction level i' and stop costs and stop probability are bounded above zero for any positive i. If some motorists infract, then there exist values of c for which $(b(i', c) - \tau(i')\phi^*(i', s_d)) > 0$, and by the continuity of i' over c this establishes the existence of a c^* where $(b(i', c^*) - \tau(i')\phi^*(i', s_d)) = 0$.

We can differentiate the motorist net benefits expression (NB) from Equation (5) at any c. We then cancel out derivative terms involving i' since the FOC is zero at the optimal infraction level (envelope theorem), and show that

$$\frac{dNB}{dc} = \frac{\partial}{\partial c}(b(i', c) - \tau(i')\phi^*(i', s_d)) = \frac{\partial b(i', c)}{\partial c} > 0$$

Therefore, with NB of zero at c^*, NB must be negative for $c < c^*$ and positive for $c > c^*$

(ii) In the proof of Lemma 2, we show that the optimal infraction level i' is positive for all c and that the function i' is continuous and monotonically increasing in c. Therefore, if c^* exists for a given equilibrium based on part (i) above, i' is positive for c equal to c^*, and the continuity of i' implies that the optimal infraction level at c must approach that positive value as c approaches c^* from above or equivalently $\lim_{c \to c^{*+}} i^{**} > 0$.

(iii) We calculate the total derivative of the equation that defines c^*, $NB = 0$, with respect to s_d and c^*. We again exploit the envelop theorem cancelling out terms that involve the derivative of i' at the optimal infraction level.

$$\left(\frac{d}{dc}(b(i',c) - \tau(i')\phi^*(i',s_d))dc^* + \frac{d}{ds_d}(b(i',c) - \tau(i')\phi^*(i',s_d))ds_d\right)_{c=c^*} = 0$$

Accordingly,

$$\left(\frac{\partial b}{\partial c}dc^* - \tau(i')\frac{\partial \phi^*}{\partial s_d}ds_d\right)_{c=c^*} = 0 \text{ or } \frac{dc^*}{ds_d} = \tau(i')\left(\frac{\partial \phi^*}{\partial s_d}\right)\left(\frac{\partial b}{\partial c}\right)^{-1}_{c=c^*} < 0$$

where the terms in parentheses are evaluated at c^* and $i'(c^*, s_d)$. Finally, c^* falls with s_d based on Lemma 1 part (ii) and Assumption 2.2. **QED**

The non-convexity in the police pay-off and motorist penalty at $i = 0$ leads to a situation where the motorist benefit at the optimal, positive infraction level can be smaller than the expected cost of stop. Figure 1 illustrates the optimization problem presenting benefits and costs over infraction level for different values of the preference parameter.[8] Starting on the left with a low value of $c = -2$, the benefit curve lies below the expected cost curve and motorists choose not to infract. As c increases, the benefit function increases and crosses the expected cost function yielding a positive optimal infraction level above a threshold c^*.

As above, discrimination arises when police officers have a lower cost of stopping a minority $s_m < s_w$. However, the standard statistic for racial discrimination in police stops (share of stop motorist who are minority) can now be written utilizing the distribution of motorists over preferences $g(c, d)$.

Definition 2. $K_g \equiv \frac{p[m|stopped, s_m, g(c,m)]}{p[w|stopped, s_w, g(c,w)]} = \frac{\int_{c^*(s_m)}^{c_h} g(c,m)\tilde{\phi}(c,s_m)di}{\int_{c^*(s_w)}^{c_h} g(c,w)\tilde{\phi}(c,s_w)di}$

where $\tilde{\phi}(c, s_d) \equiv \phi^*(i'(c, s_d), s_d)$ and g(c,d) is the distribution of motorists.

[8] Note that the data used to generate this figure and the two figures that follow comes from the calibrated simulation of the model for Massachusetts that is described in Section 5.

Unlike the ϕ^*, the derivative of $\tilde{\phi}$ is ambiguous in sign

$$\frac{d\tilde{\phi}}{ds_d} = \frac{\partial \phi^*}{\partial s_d} + \frac{\partial \phi^*}{\partial i}\frac{\partial i'}{\partial s_d} <> 0 \tag{8}$$

Proposition 2. *Given the general motorist and officer problems defined above, equilibria exist where a decrease in* s_m *leads to a decrease in* K_g.

Proof of Proposition 2. As in Proposition 1, we examine the impact of decreasing s_m.

$$\frac{dK_g}{ds_m} = \frac{1}{p[w|\text{stopped}, s_w, g(c,w)]}\left(-\left(\frac{dc^*}{ds_m}\right)_{c=c^*} g(c^*, m)\tilde{\phi}(c^*, s_m) + \int_{c^*}^{\infty} g(c,m)\frac{d\tilde{\phi}}{ds_m}dc\right)$$

A positive derivative is consistent with the existence of equilibria that satisfy Proposition 2.

The first term in paratheses is positive by Lemma 3 part (i) as stop costs rise new motorists with lower values of c begin to commit infractions raising minority motorists' share in the population of stops. The second term is generally ambiguous. The proposition will hold if equilibria exist when the inequality below is satisfied.

$$-\left(\frac{dc^*}{ds_m}\right)_{c=c^*} g(c^*, m)\tilde{\phi}(c^*, s_m) > -\int_{c^*}^{\infty} g(c,m)\frac{d\tilde{\phi}}{ds_m}dc$$

The rest of the proof will proceed by constructing an example of an equilibrium by selecting primitives where the inequality above holds. We can bound the integral on the right hand side of the inequality from above by first exploiting the fact that the partial derivative of ϕ^* with respect to s_m must be less than the total derivative of $\tilde{\phi}$ with respect to s_m because the second term in Equation (8) is always positive.

$$-\int_{c^*}^{\infty} g(c,m)\frac{\partial \phi^*}{\partial s_m}di > -\int_{c^*}^{\infty} g(c,m)\frac{d\tilde{\phi}}{ds_m}di$$

Second, select h so that the second derivative of h^{-1} is always negative. Now, we can bound the resulting expression from above because the negative second derivative of h^{-1} implies that the derivative of ϕ^* with respect to s_m is always increasing in c. Specifically, Equation (3) replacing i with $i'(c, s_d)$ yields.

$$\frac{\partial^2 \phi^*}{\partial c\, \partial s_d} = -h^{-1\prime\prime}\frac{du}{di}\frac{\partial i'}{\partial c} > 0$$

or equivalently that the negative derivative of ϕ^* with respect to s_d is falling in magnitude with c. Therefore, the partial derivative of ϕ^* takes its maximum value within the intergral at c^*, and so this derivative can be replaced by a constant equal to its value at $i'(c^*, s_m)$ and then factored out of the integral.

$$-\left(\frac{\partial\phi^*}{\partial s_m}\right)_{i=i'(c^*,s_m)}\left(1 - G(c^*, m)\right) \geq -\int_{c^*}^{\infty} g(c, m)\frac{\partial\phi^*}{\partial s_m}di$$

where $G(c^*, m)$ is the cumulative distribution function of $g(c, m)$ at c^*.

Using this inequality, we replace the right-hand side of the inequality required for Proposition 2 to hold yielding a sufficient condition for a positive derivative of K_g.

$$-\left(\frac{dc^*}{ds_m}\right)_{c=c^*} g(c^*, m)\tilde{\phi}(c^*, s_m) > -\left(\frac{\partial\phi^*}{\partial s_m}\right)_{i=i'(c^*,s_m)}\left(1 - G(c^*, m)\right)$$

Next, we replace the derivative of c^* using the equation from the proof of Lemma 3 part (iii)

$$\frac{dc^*}{ds_d} = \tau(i')\left(\frac{\partial\phi^*}{\partial s_d}\right)\left(\frac{\partial b}{\partial c}\right)_{c=c^*}^{-1}$$

then the proposition holds if

$$g(c^*, m)\tilde{\phi}(c^*, s_m) > \frac{1}{\tau(i')}\frac{\partial b}{\partial c}\left(1 - G(c^*, m)\right)$$

where the negative of the derivatives of ϕ^* with respect to s_m on both sides of the inequality were evaluated at $i'(c^*, s_m)$ and so cancel out of the expression, and τ and the derivative of b are evaluated at $i'(c^*, s_m)$ and c^*.

Now, let $g(c, m)$ be a symmetric, unimodal probability distribution centered on c^* with a maximum density of $\bar{g}$ at c^* and rewrite the inequality based on this distribution.

$$\bar{g}\,\tilde{\phi}(c^*, s_m) > \frac{1}{\tau(i')}\frac{\partial b}{\partial c}\frac{1}{2}$$

The solutions for c^*, $i'(c^*, s_m)$, $\tilde{\phi}(c^*, s_m)$ and the derivative of b do not depend upon the probability distribution, and $\tilde{\phi}(c^*, s_m)$ is bounded away from zero. By construction, $\bar{g}$ must limit to infinity as the variance of the distribution of c limits to zero. Therefore, by continually reducing the variance of the distribution, we can obtain a density $\bar{g}$ that is sufficiently large to satisfy the inequality above. **QED**

As with Proposition 1, this proposition is established by examining the derivative of K_g with respect to s_m. A decrease in stop costs will lead to a direct change in the equilibrium stop probability that likely raises the share of minorities stopped, as well as decreasing the share of minority motorists who commit infractions and are at risk of being stopped. This second negative effect can dominate the direct effect if either the density of inframarginal motorists at c^* or the change in c^* with stop cost is large enough to counteract changes in stop probabilities. Any parameters that change the responsiveness of c^* to stop costs also influence stop probabilities, and so the proof in the appendix creates a counterexample by modifying the density of motorists at c^*. Notably, this counterexample is constructed without imposing any restrictions on the preferences of majority motorists, and while holding the mean of preferences for minority motorists fixed. Figure 2 illustrates the response of motorists to discrimination using daylight stop costs calculated from the model calibrations presented later in the paper. Lower stop costs lead to a large increase in the threshold for committing infractions and a modest decline in severity for motorists who commit infractions.

A.3. Equilibrium Distribution of Infraction Levels

Finally, we examine the infraction distribution of stopped motorists. We demonstrate that discrimination shifts the distribution of stopped motorist infraction severity downwards to less severe infractions above a certain percentile threshold. We rely on this property of our model for our empirical analyses of the speed distribution of stopped motorists. For convenience, we suppress the minority indicator on the probability distribution $g(c, m)$.

We characterize changes in the observed infraction severity distribution by examining the effect of a change in s_m on severity level i_x of motorists at a specific percentile x in the speed distribution of stopped motorists. Conditional on s_m and motorist preference $c \geq c^*(s_m)$, we write a stopped motorist percentile by integrating over the product of the pdf of c and the equilibrium probability of stop $\tilde{\phi}(c, s_m) = \phi^*(i'(c, s_m), s_m)$, or

$$x(c, s_m) = \frac{\int_{c^*(s_m)}^{c} g\,(c')\phi^*(i'(c', s_m), s_m)\,dc'}{\int_{c^*(s_m)}^{\infty} g\,(c')\phi^*(i'(c', s_m), s_m)\,dc'}$$

where the numerator captures the mass of stopped motorists below c and the denominator captures all stopped motorists. Similarly, we can pick a percentile x and write the preference parameter of the motorist as an implicit function c_x of the percentile.

$$\int_{c^*(s_m)}^{c_x(x,s_m)} g\,(c')\phi^*(i'(c',s_m),s_m)\,dc' = x\int_{c^*(s_m)}^{\infty} g\,(c')\phi^*(i'(c',s_m),s_m)\,dc' \tag{9}$$

Finally, we define the equilibrium infraction level of stopped motorists at each percentile by substituting c_x into i'.

Definition 3. $i_x(x,s_m) \equiv i'(c_x(x,s_m),s_m)$

Next, we impose several assumptions to assure that the motorist problem is well behaved as x limits to one. If the density of c is positive over R, c limits to infinity as x limits to one, $x < 1$ for all finite c, and infraction level i may limit to infinity as x limits to one. So, we strengthen the second part of Assumption 2.5 on the relative curvature of h^{-1}.

Assumption 3.1 $\lim_{i\to\infty}\left(\frac{\partial\tau}{\partial i}h^{-1'} + \tau(i)h^{-1''}\frac{\partial u}{\partial i}\right) = L > 0$ where L is finite and the derivatives of h^{-1} are evaluated at $(u(i)-s_m)$.

Assumption 2.5 assures that this expression is positive on R^+, and Assumption 3.1 extends this condition on the curvature of h^{-1} so that this expression does not limit to zero as infraction level increases. Next, we impose assumptions on the police and motorist problems as c and $i'(c,s_m)$ limit to infinity.

Assumption 3.2 $\lim_{i\to\infty}\frac{d^2u}{di^2} = 0$, $\lim_{i\to\infty}\frac{d^2\tau}{di^2} > 0$, $\lim_{c\to\infty}\frac{\partial^2 b}{\partial i^2} \geq 0$, $\lim_{i\to\infty}\left(\tau(i)h^{-1'}\right) \neq \infty$ where $h^{-1'}$ is evaluated at $(u(i)-s_m)$, $\lim_{c\to\infty}\frac{\partial^2 b}{\partial c\partial i} \geq 0$, and all limits listed in the assumption plus $\lim_{i\to\infty} h^{-1''}$ exist and are finite. [9]

The restriction on the second derivative of u assures that the limit of the first and second derivatives of ϕ^* are both zero, consistent with ϕ^* asymptotically approaching one or some

[9] The existence requirement of assumption 3.2 eliminates situations where the second derivative of functions could oscillate between positive and negative. Such oscillation creates the possibility that the first derivative can limit to zero even though the second derivative does not exist. The classic example of this type of problem is $f'(x) = 1 + \sin(x^2)/x$ where $\lim_{x\to\infty} f(x) = 1$, a horizontal asymptote, but $f''(x) = 2cos(x^2) - \sin(x^2)/x^2$ and so the limit of the second derivative does not exist.

upper limit as i approaches infinity and assuring that stop is never certain for a finite i. The restrictions on the limits of the second derivatives of τ and b and on the limit of $\tau(i)h^{-1'}$ are required so that the limit of the second order condition is finite and non-zero as i increases. Note that a finite, non-zero second derivative of τ implies that the first derivative of τ limits to infinity based on a finite, non-zero rate of change. Therefore, we also restrict the cross-partial derivative of b to be finite so that the first derivative of b will also limit to infinity with c based on a finite rate of change. So, in cases where i' limits to infinity with c, the marginal costs and benefits of the first order condition from the motorist's problem will both move together.

Lemma 4. *(i)* $\lim_{i\to\infty}\frac{\partial\phi^*}{\partial i} = 0$ *and* $\lim_{i\to\infty}\frac{\partial^2\phi^*}{\partial i^2} = 0$, *(ii) if* $\lim_{c\to\infty}\frac{\partial^2 b}{\partial c\partial i} = 0$ *then* $\lim_{c\to\infty} i'(c, s_d) = I(s_d)$, *while if* $\lim_{c\to\infty}\frac{\partial^2 b}{\partial c\partial i} > 0$ *then* $\lim_{c\to\infty} i'(c, s_d) = \infty$ *, (iii).* $\lim_{c\to\infty}(SOC)_{i=i'} \neq 0$ *and finite.*

Proof of Lemma 4. (i) Since the second derivative of u limits to zero, the first derivative of u must approach a horizontal asymptote and so be finite. Using Equation (3),

$$\lim_{\mathrm{i}\to\infty}\frac{\partial\phi^*}{\partial \mathrm{i}} = \lim_{\mathrm{i}\to\infty}\mathrm{h}^{-1'}\frac{\mathrm{du}}{\mathrm{di}} = 0$$

The first term of the product limits to zero based on the definition of h in Assumption 1.3 and the first derivative of u is finite as noted above and so the limit of the derivative equals zero. Next, we can write

$$\lim_{\mathrm{i}\to\infty}\frac{\partial^2\phi^*}{\partial \mathrm{i}^2} = \lim_{\mathrm{i}\to\infty}\left(\mathrm{h}^{-1''}\left(\frac{\mathrm{du}}{\mathrm{di}}\right)^2 + \mathrm{h}^{-1'}\frac{\mathrm{d}^2\mathrm{u}}{\mathrm{di}^2}\right) = 0$$

The second term limits to zero based on the definition of h and Assumption 3.2. Turning to the first term, the fact that $\mathrm{h}^{-1'}$ limits to zero requires that $\mathrm{h}^{-1''}$ also limit to zero under the assumption that its limit exists. Specifically, if $\mathrm{h}^{-1''}$ limits to a negative value, there exists an i large enough that $\mathrm{h}^{-1''}$ will always be within ε of that limiting value. Then, for any finite, positive value of $\mathrm{h}^{-1'}$ at this i, we can divide this positive value by the lower bound of the magnitude of $\mathrm{h}^{-1''}$ (its current value at i plus ε) and increasing i by this amount leads to a negative value of $h^{-1'}$ and a contradiction. Therefore, $h^{-1''}$ must limit to zero, and since the limit of the derivative of u is finite the first term of the expression above also limits to zero.

(ii) If the cross-partial derivative of b limits to zero with c, then the limit of the first derivative of b in the first order condition must limit to a constant with c holding i fixed. Further, because the second derivative of b with respect to i is negative, this limit must be larger than the limit that arises when the limit of the derivative is evaluated for $i'(c)$ so that i increases as c increases, and so the limit of the first derivative of b evaluated at $i'(c)$ is also finite

$$\lim_{c\to\infty}\frac{\partial b}{\partial i} = B(i) > \lim_{c\to\infty}\left(\frac{\partial b}{\partial i}\right)_{i=i'} = B$$

Therefore,

$$\lim_{c\to\infty} FOC = B - \lim_{c\to\infty}\left(\frac{d\tau}{di}\phi^*(i, s_m) + \tau(i)\frac{\partial\phi^*}{\partial i}\right)_{i=i'} = 0$$

The non-zero second derivative of τ implies that the second term in the FOC limits to infinity as i limits to infinity because the first derivative of τ is always increasing with i by some value that is bounded away from zero. Therefore, since the first term is finite in the limit at B, the FOC can only be satisfied if i' limits to a finite value as c limits to infinity, $\lim_{c\to\infty} i'(c, s_m) = I(s_m)$.

If the cross-partial of b limits to a positive value, then the first derivative of b must limit to infinity with c. Now, rewriting the limit of the FOC

$$\lim_{c\to\infty} FOC = \lim_{c\to\infty}\left(\frac{\partial b}{\partial i}\right)_{i=i'} - \lim_{c\to\infty}\left(\frac{d\tau}{di}\phi^*(i, s_m) + \tau(i)\frac{\partial\phi^*}{\partial i}\right)_{i=i'} = 0$$

It is clear by inspection that the first order condition can only be satisfied in the limit if the second term limits to infinity and this will only occur if $\lim_{c\to\infty} i'(c, s_d) = \infty$.

(iii) The second order condition based on primitive functions is

$$SOC \equiv \frac{\partial^2 b(i,c)}{\partial i^2} - \frac{d^2\tau(i)}{di^2}\phi^*(i, s_d) - 2\frac{d\tau(i)}{di}h^{-1'}\frac{\partial u}{\partial i} - \tau(i)\left(h^{-1''}\left(\frac{du}{di}\right)^2 + h^{-1'}\frac{d^2u}{di^2}\right)$$

If $\lim_{c\to\infty} i'(c, s_m) = I(s_m)$, then all of the terms in the SOC are evaluated in the limit for a finite value of i. The first term is finite based on Assumption 3.2 and all other terms are finite based on the finite value of i. Similarly, all terms except for the first term are non-zero at any finite i.

If $\lim_{c\to\infty} \mathrm{i}'(\mathrm{c}, \mathrm{s_m}) = \infty$, then we must evaluate each term in the SOC individually. The first term is zero. In order to see this, remember that the first derivative is unambiguously positive and the second derivative is unambiguously negative for any finite i and c. As i limits to infinity for any finite c, the second derivative as long as it exists must limit to zero for any finite c. Otherwise, we could find a value of i large enough that the second derivative is within ε of its limiting negative value, and then an increase of i by the current value of the first derivative divided by the lower bound of the second derivative (the limiting value plus epsilon) will result in a negative first derivative and a contradiction. If the first term limits to zero for any finite c, then it must limit to zero as c and $\mathrm{i}'(\mathrm{c})$ limit to infinity. The second term is finite and non-zero based directly on Assumption 3.2. The third term is finite and non-zero because the first derivative of u is finite and non-zero and Assumption 3.1 implies that the first two terms in this product are finite and non-zero. The fourth term is zero because Assumption 2.5 implies that the second half of this term dominates the first half and Assumption 3.2 implies that $\tau(\mathrm{i})\mathrm{h}^{-1'}$ is finite and that the second derivative of u limits to zero. **QED**

Finally, we impose a key restriction on the distribution of c. The intuition behind the proposition below is based on fact that adding population to the bottom of a distribution has a much larger effect on the bottom of the distribution than on the top. For example, increasing the total population by 11 percent by adding people to the bottom will shift the person who was originally at the bottom to the 10th percentile, while only moving someone originally at the 90th percentile to about the 91st percentile. The difficulty arises if the density over the preference parameter approaches zero as the preference parameter becomes large requiring larger and larger changes in c to move the percentile as c approaches infinity. Then, small percentile changes at the top of the distribution could have large impacts on preferences and infraction levels. To rule this out, we first require the distribution be continuous, and then place restrictions on how quickly the probability density can limit to zero.

A**ssumption 3.3** The domain of the non-zero values of the probability distribution of c is continuous, or equivalently for any c where $g(c) \neq 0$ if there exists $c_h > c$ where $g(c_h) = 0$

then $g(c') = 0$ for all $c' > c_h$ and if there exists $c_l < c$ where $g(c_l) = 0$ then $g(c') = 0$ for all $c' < c_l$. Given this continuity assumption, if the domain of g is not bounded above, i.e. there exists a c_l such that $g(c) \neq 0$ for all $c > c_l$, then $\lim_{c\to\infty} {(1 - G(c))}/{g(c)} = 0$. On the other hand, if the non-zero domain of g ends at c_h, i.e. there exists a c_h such that $G(c) \neq 0$ for $c_l < c < c_h$ for some $c_l \neq c_h$ and $G(c) = 0$ for $c > c_h$, then either $g(c_h) \neq 0$ or $\lim_{c\to c_h} {(1 - G(c))}/{g(c)} = 0$.

One can verify manually that this assumption encompasses several well-known probability distributions by applying L'hopital's rule to the limit in Assumption 3.3

$$\lim_{c\to\infty} {(1 - G(c))}/{g(c)} = \lim_{c\to\infty} {-g(c)}/{g'(c)} = 0.$$

The generalized normal distribution $g(c) = k(\beta, \sigma)e^{-\sigma^{-1}|c|^\beta}$ satisfies these requirements for all $\beta > 1$ including the normal distribution, but excluding the Laplace distribution where $\beta = 1$. The assumption is also satisfied for the skew normal distribution $g(c) = 2(2\pi\sigma)^{-1}e^{-\sigma^{-1}c^2}\Phi(c)$ where Φ is the CDF of the normal distribution, and the generalized gamma distribution $g(c) = l(\beta, \sigma, \delta)c^{\delta-1}e^{-b(c/\sigma)^\beta}$ for $\beta > 1$ including the Weibull distribution where $\delta = \beta$ if $\beta > 1$, but excluding the gamma distribution where $\beta = 1$. Assumption 3.3 tends to hold for probability distributions that include an exponential function and have a light tail, but does include distributions with heavier tails than the normal. However, the condition fails for distributions that contain an exponential that is linear in c, such as the Laplace or gamma distributions, or for distributions based only on powers of c, such as the pareto or Cauchy distributions.

Under these assumptions, discrimination will decrease the infraction levels of stopped motorists above some percentile $\tilde{x}$ of the infraction level distribution.

Proposition 3. *For all* s_m *there exists* $\tilde{x}$ *such that* $\frac{di_x}{ds_m} > 0$ *for all* $x > \tilde{x}$.

Proof of Proposition 3. Differentiation of $\mathrm{i_x(x, s_m)}$ in Definition 3 yields

$$\frac{\mathrm{di_x}}{\mathrm{ds_m}} = \frac{\mathrm{di'}}{\mathrm{ds_m}} + \frac{\mathrm{di'}}{\mathrm{dc}}\frac{\mathrm{dc_x}}{\mathrm{ds_m}}$$

The derivative of $c_x(x, s_m)$ can be found by differentiating Equation (9) with respect to s_m and replacing x with $G(c_x)$.

$$\frac{dc_x}{ds_m} g(c_x)\phi^*(i'(c_x, s_m), s_m) = (1 - G(c_x))\frac{dc^*}{ds_d} g(c^*)\phi^*(i'(c^*, s_m), s_m) +$$
$$G(c_x)\int_{c^*(s_m)}^{c_h} g(c')\frac{d\phi^*}{ds_m} dc' - \int_{c^*(s_n)}^{c_x} g(c')\frac{d\phi^*}{ds_m} dc'$$

where c_h is the maximum value of c within the domain of the probability distribution, which could be positive infinity.

The first term on the right-hand side of the equation above is negative based on Lemma 3 leading to an ambiguous derivative of i_x. This first term represents the same source of ambiguity discussed in Proposition 2. As stop costs increase, c^* falls and more minority motorists commit infractions. These new infracting motorists have lower values of c shifting the distribution of infracting motorists to lower infraction levels.

However, as we increase c and move to higher percentiles (x or $G(c_x)$ approaches 1), the first term goes to zero. Further, as c_x approaches c_h, $G(c_x)$ approaches 1, and the second and third terms exactly cancel out when $c_x = c_h$. Therefore, if $g(c_h) \neq 0$, then the derivative of c_x with respect to s_m is zero at $x = 1$.

If $\lim_{c \to c_h} g(c) = 0$ whether c_h is finite or infinite, we must evaluate the limit of the derivative of c_x.

$$\lim_{c_x \to c_h} \frac{dc_x}{ds_m} = \lim_{c_x \to c_h} \frac{1}{\phi^*(i'(c_x, s_m), s_m)g(c_x)}\Bigg((1 - x)\frac{dc^*}{ds_d} g(c^*)\phi^*(i'(c^*, s_m), s_m) +$$
$$x\int_{c^*(s_m)}^{c_h} g(c')\frac{d\tilde{\phi}}{ds_m} dc' - \int_{c^*(s_n)}^{c_x} g(c')\frac{d\tilde{\phi}}{ds_m} dc'\Bigg)$$

Now, we can rewrite the last two terms in parentheses by extending the limit of the second integral from c_x to c_h and adding a new term to offset that extentions.

$$\lim_{c_x \to c_h} G(c_x)\int_{c^*(s_m)}^{c_h} g(c')\frac{d\tilde{\phi}}{ds_m} dc' - \int_{c^*(s_n)}^{c_x} g(c')\frac{d\tilde{\phi}}{ds_m} dc'$$
$$= \lim_{c_x \to c_h} -(1 - G(c_x))\int_{c^*(s_m)}^{c_h} g(c')\frac{d\tilde{\phi}}{ds_m} dc' + \int_{c_x}^{c_h} g(c')\frac{d\tilde{\phi}}{ds_m} dc'$$

Since the derivative of $\tilde{\phi}$ is finite, we can bound the magnitude of the last term by replacing this derivative with the maximum of its absolute value and factoring this out of the integral.

$$\lim_{c_x \to c_h} \left| \int_{c_x}^{c_h} g(c') \frac{d\tilde{\phi}}{ds_m} dc' \right| < \lim_{c_x \to c_h} \max_{c} \left| \frac{d\tilde{\phi}}{ds_m} \right| |(1 - G(c_x))|$$

As a result, the first term and revised second term only depend upon c_x through a linear function of $(1 - G(c_x))$ and the revised third term is bounded by a function that also depends linearly on $(1 - G(c_x))$. Based on Assumption 3.3, the limit of the ratio of $(1 - G(c))$ to $g(c)$ is zero and so the derivative of c_x with respect to s_m limits to zero as c limits to c_h even if $g(c)$ limits to zero.

Using the equations for the derivatives from part (ii) of Lemma 2, we note that

$$\frac{di'}{dc} = -\frac{\frac{\partial^2 b}{\partial c\, \partial i}}{SOC}$$

$$\frac{di'}{ds_m} = \frac{1}{SOC}\left(\frac{\partial \tau}{\partial i}\frac{\partial \phi^*}{\partial s_m} + \tau(i)\frac{\partial^2 \phi^*}{\partial i\, \partial s_m}\right) = \frac{-1}{SOC}\left(\frac{\partial \tau}{\partial i} h^{-1'} + \tau(i) h^{-1''} \frac{\partial u}{\partial i}\right)$$

If the cross-partial of b limits to zero and based on Lemma 4 $\lim_{c \to \infty} i'(c, s_d) = I(s_d)$ or alternatively if the probability distribution of c has zero density above some finite value of c_h, then $i'(c)$ is finite in the limit. As a result, both derivatives of i' are positive and finite. Therefore, the second term in the derivative of i_x limits to zero and the first term is finite so that the derivative of i_x must limit to a positive value as x approaches one.

On the other hand, if $\lim_{c \to \infty} i'(c, s_d) = \infty$ and the density is non-zero for any finite c, we must evaluate these two derivatives in the limit as i approaches infinity. Assumption 3.2 assures that the limit of the derivative of i' with respect to c is finite because the limit of the cross-partial of b is finite and based on Lemma 4 the SOC does not limit to zero. Assumption 3.1 assures that the limit of the derivative of i' with respect to s_d is bounded away from zero. Therefore, the first term in the derivative of i_x limits to a positive value and the second term limits to zero. **QED**

As seen above, the proof in the appendix proceeds by differentiating $i_x(x, s_m)$ in Definition 3

$$\frac{di_x}{ds_m} = \frac{di'}{ds_m} + \frac{di'}{dc}\frac{dc_x}{ds_m}.$$

Assumptions 2.5 and 3.1 imply that optimal motorist infraction level increases as stop costs rise. However, changes in the distribution of infraction severity are ambiguous because additional motorists who had chosen not to infract due to weak preferences may now choose to commit an infraction given higher stop costs and c_x falls as those additional motorists are added to the bottom of the distribution.

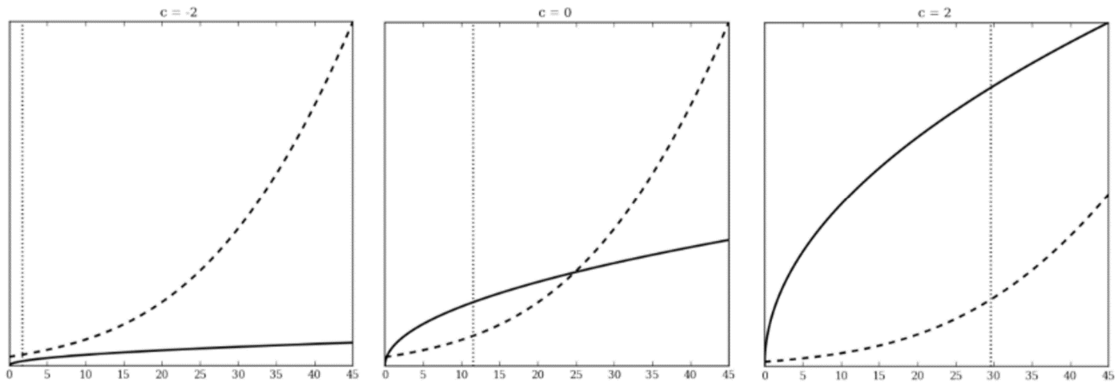
However, this phenomenon grows weaker as we move further out in the speed distribution. Additional infracting motorists added at the bottom of the distribution result in only a fraction of motorists at a fixed preference level c being shifted across any percentile. As the percentile x approaches one (top of the speed distribution), the first term in the derivative of i_x (the partial derivative of i') remains bounded away from zero, while the share shifted across the percentile, i.e. the derivative of c_x, approaches zero.

$$\frac{dc_x}{ds_m} = \frac{1}{\phi^*(i'(c_x, s_m), s_m)g(c_x)}\Bigg((1-x)\frac{dc^*}{ds_d}g(c^*)\phi^*(i'(c^*, s_m), s_m) + -(1-x)\int_{c^*(s_m)}^{c_h} g(c')\frac{d\tilde{\phi}}{ds_m}\,dc' + \int_{c_x}^{c_h} g(c')\frac{d\tilde{\phi}}{ds_m}\,dc'\Bigg)$$

The first two terms in parentheses are proportional to $(1-x)$ and the last term is shown in the proof of the proposition to be bounded by an expression that is proportional to $(1-x)$, and so the derivative limits to zero. As a result, any significant increase in the speed of stopped minority motorists near the top of the speed distribution is suggestive that minority motorists may be responding to real or perceived discrimination.

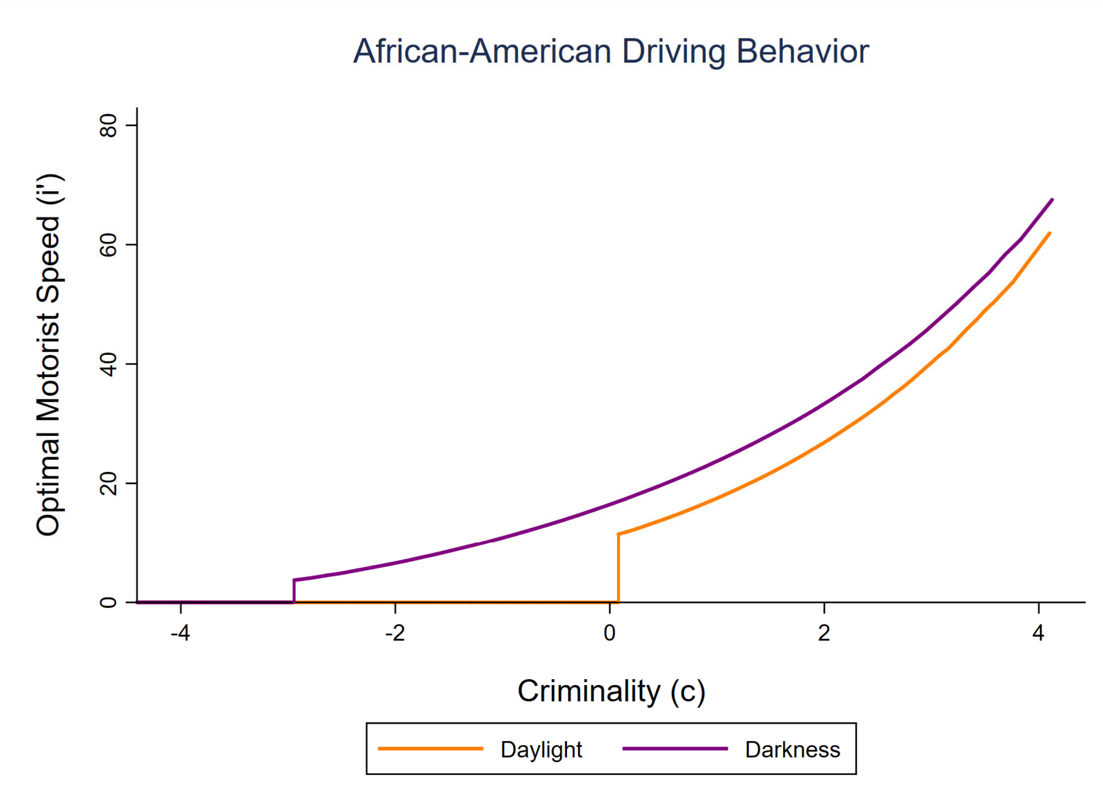
Note that the effects discussed above are driven primarily by the selection of motorists into committing infractions, rather than selection into stop. Figure 3 illustrates this by plotting the empirical distribution of minority speeders (solid lines) and minority motorists stopped for speeding (dashed lines) with discrimination (daylight) and without (darkness) using the model calibration for Massachusetts from below. The speed distribution is substantially slower with discrimination whether based on all speeders or stopped motorists only.

Figure A1: Motorist Benefits and Expected Costs by the Preference Parameter



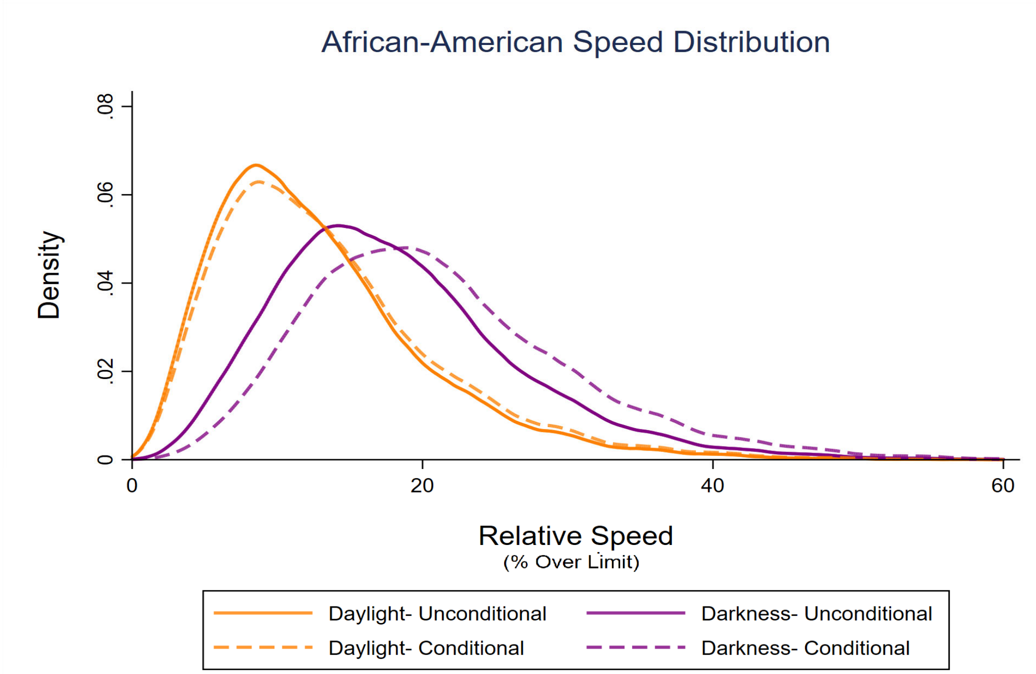
Notes: The dashed line represents the product of the probability of stop and the cost of stop at different speeds over the speed limit and the solid line represents the benefits from the infraction. The left most panel is for motorists with very weak preferences, c=-2, who are inframarginal and do not commit an infraction, the middle panel is for moderate preferences, c=0, and the last panel is for strong preference, c=2. The vertical line represents the optimal infraction level if a motorist chooses to infract.

Figure A2: Speeding Violations of Motorists by Preference Parameters and Visibility



Notes: The dark line represents the infraction level for minority motorists in darkness for different values of the preference parameter c. The light line represents the infraction level in daylight when stop costs of minority motorists are lower. Both curves lie on the horizontal axis or at zero for preference levels where motorists choose not to commit an infraction.

Figure A3. Speed Distribution of Motorists who Commit Infractions by Visibility



Note: The dark line represents the probability density function of minority motorists who commit infractions over speed in darkness, while the light line represents the density function in daylight when stop costs are lower. The solid lines represent all motorists who commit infractions, while the dashed lines represent the distribution for stopped motorists, The distributions for stopped motorists are shifted to the right based on higher stop probabilities at higher speeds. The speed distribution for all motorists is positive over all positive speeds even as zero is approached, because the density of the preference parameter is positive over the real line.

Appendix B. Empirical Appendix

Table B1: Descriptive Statistics for the FARS Accident Data

Total Accidents		615,826		
Fatal Accidents		282,924		
Inter-Twilight		39,076		
Sample		All	AA	White
Daylight		53.44%	49.93%	53.95%
Motorist	African-American	12.83%	100.00%	0.00%
	Male	67.67%	72.22%	66.99%
	Young	42.74%	38.92%	43.31%
Auto.	Domestic	66.36%	62.25%	66.97%
	Old	22.05%	19.10%	22.48%
Day of Week	Sunday	14.03%	15.19%	13.86%
	Monday	13.49%	12.96%	13.57%
	Tuesday	12.91%	11.49%	13.11%
	Wednesday	13.50%	12.90%	13.59%
	Thursday	14.01%	13.52%	14.08%
	Friday	16.52%	16.15%	16.58%
	Saturday	15.54%	17.79%	15.21%
Hour of Day	4:00 PM	5.70%	3.23%	6.07%
	5:00 PM	22.97%	21.79%	23.14%
	6:00 PM	24.83%	24.83%	24.83%
	7:00 PM	21.53%	23.83%	21.19%
	8:00 PM	18.06%	19.64%	17.82%
	9:00 PM	4.87%	3.93%	5.01%
States + DC		49	49	49

Note: The overall sample includes only traffic stops involving African-American or Non-Hispanic white motorists.

Table B2: Estimated Change in the Accidents Rate for Minority Motorists in Daylight, USNO Daylight Definition

LHS: African-American		(1)	(2)	(3)	(4)
Baseline					
Daylight		-0.01107*** (0.00413)	-0.01019*** (0.00389)	-0.00986*** (0.00392)	-0.00960*** (0.00391)
Observations		39076	39076	39076	39076
Interaction – Black-White Police Shootings Odds Ratio					
Daylight x Police Shootings		-0.00268* (0.00152)	-0.00399*** (0.00146)	-0.00437*** (0.00152)	-0.00451*** (0.00151)
Observations		39063	39063	39063	39063
Interaction – Google Search Racism Index					
Daylight x Racism Index		-0.00779** (0.00348)	-0.01167*** (0.00337)	-0.01125*** (0.00347)	-0.01196*** (0.00345)
Observations		39063	39063	39063	39063
VOD Inconclusive States					
Daylight		-0.04334*** (0.01162)	-0.03245*** (0.01052)	-0.03235*** (0.01037)	-0.03277*** (0.01031)
Observations		6587	6587	6587	6587
Controls	Hour of Day	X	X	X	X
	Day of Week	X	X	X	X
	Year	X	X		
	State		X		
	State x Year			X	X
	Motorist/Vehicle				X

Table B3: Estimated Change in the Accidents Rate for Minority Motorists in Daylight, Fatality Risk Weighted

LHS: African-American		(1)	(2)	(3)	(4)
Baseline					
Daylight		-0.00879 (0.00690)	-0.01296** (0.00634)	-0.01328** (0.00648)	-0.01353** (0.00650)
Observations		39076	39076	39076	39076
Interaction – Black-White Police Shootings Odds Ratio					
Daylight x Police Shootings		-0.00107 (0.00156)	-0.00240 (0.00157)	-0.00312* (0.00164)	-0.00323** (0.00163)
Observations		39063	39063	39063	39063
Interaction – Google Search Racism Index					
Daylight x Racism Index		-0.00937*** (0.00381)	-0.01138*** (0.00373)	-0.01056*** (0.00389)	-0.01109*** (0.00385)
Observations		39063	39063	39063	39063
VOD Inconclusive States					
Daylight		-0.04623*** (0.01242)	-0.03683*** (0.01082)	-0.03601*** (0.01077)	-0.03640*** (0.01072)
Observations		6587	6587	6587	6587
Controls	Hour of Day	X	X	X	X
	Day of Week	X	X	X	X
	Year	X	X		
	State		X		
	State x Year			X	X
	Motorist/Vehicle				X

Table B4: Descriptive Statistics for Massachusetts and Tennessee Traffic Stop Data

		MA		East TN		West TN	
Total Stops		401,408		489,313		1,658,611	
Speeding Stops		80,471		143,014		541,667	
Inter-Twilight		10,203		23,515		102,054	
Sample		AA	White	AA	White	AA	White
	Daylight	71.05%	65.78%	67.59%	68.63%	63.15%	65.03%
Speed	Pct over limit	44.61%	39.85%	30.92%	34.90%	28.88%	29.44%
	MPH over limit	15.13	17.38	17.38	17.71	17.02	16.13
Motorist	African-American	100.00%	0.00%	100.00%	0.00%	100.00%	0.00%
	Male	69.82%	73.42%	58.54%	62.07%	73.33%	65.21%
	Young	50.62%	52.27%	-	-	-	-
Auto.	Domestic	28.62%	33.75%	34.12%	36.77%	30.61%	31.85%
	Old	50.62%	49.54%	-	-	-	-
	Red	11.88%	10.01%	-	-	-	
Day of Week	Sunday	13.48%	14.99%	16.04%	12.98%	14.67%	11.85%
	Monday	12.04%	14.24%	12.96%	13.30%	16.04%	14.17%
	Tuesday	15.78%	14.84%	11.50%	13.03%	10.54%	12.69%
	Wednesday	13.43%	13.51%	11.48%	13.54%	11.82%	13.34%
	Thursday	15.84%	13.70%	12.99%	14.10%	11.73%	13.71%
	Friday	12.68%	14.66%	18.92%	19.58%	19.62%	20.50%
	Saturday	16.75%	14.05%	16.11%	13.48%	15.58%	13.73%
Hour of Day	5:00 PM	33.87%	37.51%	22.69%	23.97%	24.84%	26.44%
	6:00 PM	38.26%	33.05%	27.29%	28.94%	21.91%	23.61%
	7:00 PM	17.50%	16.02%	22.57%	21.90%	20.81%	20.60%
	8:00 PM	10.38%	13.43%	15.65%	14.53%	19.62%	17.27%
	9:00 PM			11.79%	10.67%	12.83%	12.08%
Counties/Towns		18		13		44	

Note: The overall sample includes only traffic stops involving African-American or Non-Hispanic white motorists. MA is used in this and the following tables as an abbreviation for Massachusetts and TN is used as an abbreviation for Tennessee.

Table B5: Canonical Veil of Darkness Estimates, Logit

LHS: African-American		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
		MA		East TN			West TN		
Daylight		0.409***	0.416***	0.0104	-0.0150	0.00300	0.0706**	0.0637**	0.0817***
		(0.0703)	(0.0989)	(0.0958)	(0.0943)	(0.0981)	(0.0289)	(0.0288)	(0.0286)
Controls	Day of Week	X	X	X	X	X	X	X	X
	Time of Day	X	X	X	X	X	X	X	X
	County (or Town)	X	X	X	X		X	X	
	Year			X	X		X	X	
	Motorist/Vehicle		X		X	X		X	X
	County x Year					X			X
Observations		10203	10203	23515	23515	23515	102054	102054	102054

Notes: Coefficient estimates are presented where * represents a p-value .1, ** represents a p-value .05, and *** represents a p-value .01 level of significance. Standard errors are clustered on county by year (TN) and patrol districts (MA) but robust to clustering on county and year separately (TN), patrol district (TN), or town (MA). The sample includes only traffic stops involving African-American or Non-Hispanic white motorists. The two Tennessee samples also include controls for year in the first two specifications of each panel.

Table B6: Estimated Change in Speed Distribution for Stopped Minority Motorists in Daylight, Demographic Controls and County by Year Fixed Effects for Tennessee

LHS: Rel. Speed		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		10 pct	20 pct	30 pct	40 pct	50 pct	60 pct	70 pct	80 pct	90 pct
MA	Daylight	0.0811	0.196	1.938	0.727	-0.318	-0.0289	0.482	-0.560	-1.260
		(1.117)	(1.174)	(1.260)	(0.931)	(1.128)	(1.251)	(1.389)	(2.252)	(2.918)
	African-American	0.712	0.555	2.479*	2.164***	1.467**	1.601*	1.262	5.639***	6.154**
		(1.054)	(1.053)	(1.243)	(0.738)	(0.687)	(0.860)	(1.669)	(1.801)	(2.891)
	Daylight*African-American	-0.324	-0.221	-1.683	-2.230**	-5.000**	-6.748**	-7.616**	-10.79***	-11.93**
		(1.308)	(1.311)	(1.392)	(1.011)	(1.974)	(2.624)	(2.680)	(2.739)	(4.133)
	Obs.	10203	10203	10203	10203	10203	10203	10203	10203	10203
East TN	Daylight	0.372	0.327	0.0145	0.00964	0.0284	0.0854	0.374	0.459	-0.100
		(0.695)	(0.454)	(0.274)	(0.273)	(0.318)	(0.354)	(0.461)	(0.699)	(0.991)
	African-American	-2.008	-1.807**	-1.190**	-0.877	-0.780	-0.989	-0.180	-1.187	-1.958
		(1.236)	(0.868)	(0.551)	(0.535)	(0.614)	(0.711)	(0.892)	(1.322)	(1.682)
	Daylight*African-American	-0.822	-1.023	-0.812	-0.700	-1.113	-1.324*	-2.757***	-1.697	-1.684
		(1.334)	(0.980)	(0.629)	(0.574)	(0.681)	(0.738)	(0.989)	(1.486)	(2.028)
	Obs.	23515	23515	23515	23515	23515	23515	23515	23515	23515
West TN	Daylight	0.153	0.277*	0.0104	-0.0326	-0.0999	0.0761	0.138	-0.0119	-0.0466
		(0.108)	(0.156)	(0.108)	(0.146)	(0.121)	(0.171)	(0.235)	(0.296)	(0.397)
	African-American	0.193	0.600***	0.665***	0.685***	0.331*	0.738***	0.754**	0.637*	0.319
		(0.131)	(0.164)	(0.135)	(0.200)	(0.180)	(0.240)	(0.309)	(0.347)	(0.502)
	Daylight*African-American	-0.115	-0.214	-0.538***	-0.865***	-0.543***	-0.854***	-1.015***	-0.781**	-0.903
		(0.146)	(0.193)	(0.153)	(0.218)	(0.184)	(0.265)	(0.356)	(0.395)	(0.572)
	Obs.	102054	102054	102054	102054	102054	102054	102054	102054	102054

Notes: Coefficient estimates are presented such that * represents a p-value .1, ** represents a p-value .05, and *** represents a p-value .01 level of significance. Standard errors are clustered on county by year in East and West Tennessee (TN) and patrol districts in Massachusetts (MA). Bootstrapping one-thousand random samples, we find that the p-value for a one-sided permutation test of joint significance on all nine quantiles is equal to 1.4 percent for Massachusetts, 0.4 percent for East Tennessee, and 0.1 percent for West Tennessee. The sample includes only traffic stops involving African-American or Non-Hispanic white motorists. Controls include observed motorist and vehicle attributes, time of day, day of week, and geographic location fixed-effects. The two Tennessee samples also include controls for county by year fixed effects. Relative speed is calculated as speed relative to the speed limit and multiplied by one hundred.

Table B7: Falsification Test over Gender (Panel 1) and over Vehicle Type (Panel 2) with White Motorists

		Motorist Gender								
LHS: Rel. Speed		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		10 pct	20 pct	30 pct	40 pct	50 pct	60 pct	70 pct	80 pct	90 pct
MA	Daylight*Male	-1.265	-0.516	-0.267	-0.752	-0.226	0.0118	0.850	0.179	0.348
		(0.847)	(0.789)	(0.801)	(1.061)	(1.293)	(1.628)	(1.842)	(2.076)	(4.474)
	Obs.	8334	8334	8334	8334	8334	8334	8334	8334	8334
		(0.602)	(0.359)	(0.254)	(0.227)	(0.189)	(0.267)	(0.284)	(0.646)	(1.202)
East TN	Daylight*Male	-0.471	-0.186	0.260	0.314*	0.110	-0.0347	-0.265	-0.306	-1.004
		(0.493)	(0.353)	(0.274)	(0.158)	(0.336)	(0.431)	(0.513)	(0.841)	(1.314)
	Obs.	22424	22424	22424	22424	22424	22424	22424	22424	22424
		(0.0957)	(0.162)	(0.137)	(0.145)	(0.125)	(0.172)	(0.171)	(0.254)	(0.319)
West TN	Daylight*Male	-0.00342	0.0480	-0.00976	-0.0928	-0.0347	-0.307	-0.285	-0.779**	-0.285
		(0.151)	(0.197)	(0.193)	(0.193)	(0.160)	(0.187)	(0.219)	(0.333)	(0.452)
	Obs.	83076	83076	83076	83076	83076	83076	83076	83076	83076
		Domestic vs. Imported Vehicle								
LHS: Rel. Speed		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		10 pct	20 pct	30 pct	40 pct	50 pct	60 pct	70 pct	80 pct	90 pct
MA	Daylight*Domestic	-0.147	-0.554	-0.910	0.118	0.654	0.576	0.280	1.457	-1.389
		(1.158)	(1.234)	(0.914)	(1.098)	(1.007)	(0.942)	(1.302)	(2.182)	(2.221)
	Obs.	8334	8334	8334	8334	8334	8334	8334	8334	8334
		(0.411)	(0.490)	(0.238)	(0.245)	(0.251)	(0.316)	(0.538)	(0.865)	(1.489)
East TN	Daylight*Domestic	0.104	-0.655	-0.338	0.00464	-0.192	-0.146	-0.451	-0.556	-1.559
		(0.649)	(0.718)	(0.302)	(0.323)	(0.303)	(0.372)	(0.532)	(0.866)	(1.711)
	Obs.	22424	22424	22424	22424	22424	22424	22424	22424	22424
		(0.118)	(0.177)	(0.125)	(0.179)	(0.134)	(0.198)	(0.283)	(0.325)	(0.381)
West TN	Daylight*Domestic	0.131	0.0655	-0.0621	-0.119	-0.0466	0.0600	-0.105	0.0670	0.208
		(0.127)	(0.182)	(0.144)	(0.176)	(0.131)	(0.221)	(0.288)	(0.311)	(0.411)
	Obs.	83076	83076	83076	83076	83076	83076	83076	83076	83076

Coefficient estimates are presented such that * represents a p-value .1, ** represents a p-value .05, and *** represents a p-value .01 level of significance. Standard errors are clustered on county by year in East and West Tennessee (TN) and town or patrol districts in Massachusetts (MA. Bootstrapping one-thousand random samples, we find that the p-value for a one-sided permutation test of joint significance on all nine quantiles is equal to 85.7 percent for Massachusetts, 72.3 percent for East Tennessee, and 91.1 percent for West Tennessee. The sample includes only traffic stops for speeding violations involving Non-Hispanic white motorists. Controls include time of day, day of week, and geographic location fixed-effects. The two Tennessee samples also include controls for year. Relative speed is calculated as speed relative to the speed limit and multiplied by one hundred.

Table B8: Falsification Test over Motorist Age, Vehicle Age and Vehicle Color with White Motorists

Motorist under the Age of 30										
LHS: Rel. Speed		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		10 pct	20 pct	30 pct	40 pct	50 pct	60 pct	70 pct	80 pct	90 pct
MA	Daylight*Young Motorist	0.302	-0.113	0.626	0.539	-0.735	0.429	1.692	0.576	1.739
		(0.482)	(0.611)	(0.713)	(0.843)	(0.947)	(1.354)	(1.467)	(1.391)	(1.853)
	Obs.	8334	8334	8334	8334	8334	8334	8334	8334	8334
Vehicle is Older than Five Years										
LHS: Rel. Speed		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		10 pct	20 pct	30 pct	40 pct	50 pct	60 pct	70 pct	80 pct	90 pct
MA	Daylight*Old Vehicle	1.467*	-0.115	0.0108	1.323*	0.454	0.423	-0.229	-0.666	-0.336
		(0.771)	(0.742)	(0.782)	(0.643)	(0.654)	(0.792)	(1.079)	(1.656)	(2.345)
	Obs.	8334	8334	8334	8334	8334	8334	8334	8334	8334
Vehicle is Red										
LHS: Rel. Speed		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		10 pct	20 pct	30 pct	40 pct	50 pct	60 pct	70 pct	80 pct	90 pct
MA	Daylight*Red Vehicle	2.563	2.179	0.942	1.616	1.209	1.836	3.364	2.690	1.561
		(1.942)	(2.257)	(2.145)	(1.740)	(1.796)	(1.959)	(2.421)	(3.013)	(4.418)
	Obs.	8334	8334	8334	8334	8334	8334	8334	8334	8334

Notes: Coefficient estimates are presented such that * represents a p-value .1, ** represents a p-value .05, and *** represents a p-value .01 level of significance. Standard errors are clustered on counties (TN) and patrol districts (MA). Bootstrapping one-thousand random samples, we find that the p-value for a two-sided permutation test of joint significance on all nine quantiles is equal to 46.7 percent for MA-SP. Results estimated using absolute rather than relative results are generally robust and qualitatively similar to our primary estimates. The sample includes only traffic stops involving Non-Hispanic white motorists. Controls include time of day, day of week, and patrol location fixed-effects. The Tennessee sample also includes year indicators. Relative speed is calculated as speed above the limit relative to the limit and multiplied by one hundred

C. Calibration Appendix

C.1 Parameterization of the Model

In this section, we calibrate our model to the data on stopped motorists from Massachusetts and East and West Tennessee to calculate racial differences in police stop costs in daylight and darkness. We also use the darkness police stop costs to calculate counterfactual VOD test statistics that would have arisen if African-American motorists did not respond to increased scrutiny by police in daylight by driving more slowly. We note that we choose to conduct a macro-style calibration using the aggregate moments, rather than a structural estimation using micro data. This decision is based on computational demands given that each calibration takes several weeks to run.[1] Due to the use of calibration rather than structural estimation, we rely on the quantile regressions above for inference.

We assume that motorist preferences c follow a skew-normal distribution with skewness a, location e and scale w and separate parameters for whites and African-Americans:

$$f(t) = 2\phi(t)\Phi(at)$$

where ϕ and Φ are the normal PDF an CDF respectively, and $t = (x - e)/w$.

Next, we parameterize the probability of being stopped, $\phi^*(i, s_d)$, as a function of speed/infraction severity and police stop costs. We begin by specifying the police return from a stop as a monotonic function of motorist speed. Specifically,

$$u(i) = i^{\eta} + u_0 \quad \text{for } i > 1 \text{ and } u(0, s_d) = 0$$

where $\eta > 1$ allows the return to stop to increase non-linearly with infraction severity and $u_0 > \max(s_{v,d})$ for all $\{v, d\}$ assures that $u(i) > 0$ for all positive infraction levels.

Next, we need specify h^{-1} as a monotonic mapping from a priori net pay-off to stop probability ϕ^* between zero and one. Specifically,

$$\phi^*(i, s_d) = h^{-1}(u(i) - s_d) \text{ where } h^{-1}(\omega) = \begin{cases} \frac{\omega^a}{\omega^a + K} & \omega > 0 \\ 0 & otherwise \end{cases}, a > 0,\ K > 0.$$

The function limits to one as $\omega \to \infty$. If $a < 1$, the function has a negative second derivative for $h > 0$. Otherwise, the second derivative can change sign with i but is negative as $h \to \infty$.

[1] Beyond the increase in computation time required for just using micro data, we also approximate the relationship between infraction level and the preference parameter. This approximation represents most of the computational requirements for each optimization step. With aggregate moments, a relatively fine grid of 10,000 points provides reasonable accuracy, but micro data estimation implies the comparison of individual motorist speed levels to predicted speed levels at their percentile in the distribution requiring a much finer grid exploding computational requirements.

To specify the motorist problem, we assume a stop penalty of

$$\tau(i) = i^{\mu} + \tau_0 \quad for \;\; i > 0$$

where $\mu > 1$ and $\tau_0 > 0$ so costs are bounded away from zero and are convex in infraction level. The benefit function from committing the infraction depends on both i and c

$$b(i, c) = b_0 i^{\alpha_1} e^{\alpha_2 c}$$

where $b_0 > 0$, $0 < \alpha_1 < 1$ so that marginal returns are diminishing with infraction level, and the direction of the preference parameter is initialized by $\alpha_2 > 0$. The motorist solves

$$\max_{i'(c, s_d)} b(i, c) - \tau(i)\, \phi^*(i, s_d)$$

to find the optimal speed $i'(c, s_d)$.

While a closed-form solution does not exist for $i'(c, s_d)$, we exploit the monotonicity of $i'(c, s_d)$ to define $c'(i, s_d) = i'^{-1}(i, s_d)$, and derive a closed-form solution for

$$c'(i, s_d) = \frac{1}{\alpha_2} ln \left(\mu \phi^*(i, s_d) i^{\mu - \alpha_1} + \frac{\partial \phi^*}{\partial i} \frac{(i^{\mu} + \tau_0)}{i^{\alpha_1 - 1}} \right) - \frac{\ln(\alpha_1 b_0)}{\alpha_2}$$

We calculate $c'(i, s_d)$ over a fine grid of values of i and create a piece-wise approximation of $i'(c, s_d)$ by linearly interpolating between the two nearest points in the grid.

For a given set of parameters, we can calculate the motorist's optimal speed for each c, and then solve for the value $c^*(s_d)$ where net benefits at the optimal speed are equal to zero. With $\phi^*(i, s_d)$, $i'(c, s_d)$, and $c^*(s_d)$, we can solve for the equilibrium speed distribution and the speed distribution of stopped motorists by drawing a large sample of motorists from the distribution of c and using the probability of stop as a weight. Assuming a common police stop cost $s_{\underline{v}}$ in darkness and separate daylight police stop costs for white and minority motorists, $s_{\overline{v},w} > s_{\underline{v}}$ and $s_{\overline{v},m} < s_{\underline{v}}$; we can use the same sample over c to simulate white and minority speed distributions in daylight and in darkness. Finally, we vary the share of minority motorists in the population by applying a weight to the minority distribution to calibrate the share of stops in daylight and darkness that involve minority motorists.

C.2 Calibration and Optimization Approach

To calibrate the model, we calculate six speed percentiles (20th, 40th, 60th, 80th, 90th, and 95th) in miles per hour over the speed limit for each combination of daylight/darkness and minority/non-minority, the fraction of motorists stopped during daylight who are minority, and the fraction of motorists stopped in darkness who are minority. Beyond the quintiles, we add moments for the 90th and 95th percentiles to help capture the skewed nature of the speed distribution. Further, to better fit the model, we calibrate using 12 moments associated with the speed distribution of white and minority motorists in daylight, 12 moments associated with the difference between the daylight and darkness speed at each percentile in the white and minority speed distributions. Similarly, we calibrate to one moment for the percentage (fraction times 100) of motorists stopped during the darkness who are minority and one moment for the VOD test statistic in Definition 3 again times 100. To assure that the speed moments are comparable to the estimations above, we remove the time of day, day of week and geographic fixed effects in our relative speed model and add the sample means back to the residuals yielding motorists with effectively common observables. Finally, we convert these relative speeds back to miles per hour using the mode speed limit in each sample. Given that the number of speed moments is arbitrary, we place a weight of 0.070 on the share minority stops and VOD test statistic moments and a smaller weight of approximately 0.036 on each speed distribution moments.[2]

The functional forms above contain ten parameters shared by both white and minority simulated motorists. The mean, variance, and skewness of our preference distribution, and daylight stop costs, must be determined separately for white and minority motorists. We initialize the darkness stop cost $s_{\underline{v}}$ to 44 allowing the daylight stop cost of both groups and the minimum return to a stop u_0 to vary relative to this fixed value. Finally, we must calibrate the fraction of minority motorists for the simulated population. Therefore, in total 18 free parameters are calibrated for each site.

Because the surface of this function is highly non-linear and appears to contain multiple local minima and inflections points, we first use a derivative-free Simplex-based optimization algorithm, Subplex (Rowan, 1990), to identify local minima for a variety of starting values. Once we have identified a local minimum, we use a second optimization routine based on quadratic approximations to the surface, BOBYQA (Powell 2009), to precisely locate that minimum and verify that the gradients over all parameters are approximately zero in this location. Finally, after identifying a specific local minimum that fits the data well, we will identify a global minimum using a modified evolutionary-based optimization routine, ESCH as described in da Silva Santos (2010) and accessed via an open source library for non-linear optimization (NLopt). The nature of evolutionary algorithms used for global optimization requires that limits be placed on the range of each parameter, and we use the

[2] We vary the weights later in this Appendix with little change to the calibrated model (Appendix Tables C4 and C5).

information generated from the various local optimizations to place these limits. The specific limits for each parameter are shown in Appendix Table C1.[3]

We calibrate the parameters separately for the moments from Massachusetts, East Tennessee, and West Tennessee samples in a series of stages using the results of each stage as initial values in the next stage.

1. First, we focus on matching the minority daylight speed distribution using the Simplex-based algorithm, while calibrating just distributional parameters, i.e. mean, variance, and skewness, but targeting an additional moment based on a specific positive fraction of minorities not infracting in daylight. Holding other parameters fixed at values that were found based on experimentation.
2. We next match both the daylight speed distribution and the difference between the daylight and darkness distributions for minorities additionally calibrating all motorist parameters that are common between groups plus the minority daylight stop cost, i.e. $\alpha_1, \alpha_2, b_0, \mu,$ and τ_0. At this stage, we also drop the target on the fraction of minorities not speeding, which was simply used to anchor the initial calibration.
3. We then target all 26 moments and calibrate all 18 parameters. We first identify the local minimum using the simplex-based algorithm, but as mentioned above, we locate the local minimum precisely using quadratic approximations to the surface.
4. We repeat the process outlined in steps 1-3 for initial fractions minority not infracting in daylight between 0.05 and 0.40 in increments of 0.05 typically identifying different local minima for each percent not infracting value (even though that moment restriction is removed starting in step 2). We then identify the local minimum arising from an initial fraction not infracting moment restriction in step 1 that results in the lowest overall Mean Squared Error in step 3. We also verify that this minimum is internal to the range of fractions considered.

[3] This second routine also requires that the analyst place limits on the parameter space, but this is a relatively non-restrictive process since we are simply refining an already identified local minimum. In practice, the search for the local minimum never crosses the bounds that we set on the parameters.

5. Finally, we use an evolutionary-based optimization routine using the best local optimum identified in step 4 and imposing parameter limits that were developed by observing the optimization over many possible local minima. Again, the quadratic approximation technique is used to precisely locate the minimum once the entropy-based routine has identified the minimum.

Note that the optimization also includes a penalty function starting below 2 percent of minority motorists not infracting in daylight in order to rule out corner solution equilibria where all motorists commit infractions. The final local and global optimums always imply a percent minority motorists not infracting above 2 percent so that the penalty function has no direct impact on the final optimum identified.

C.2 Calibration Results

Table 7 presents the results of the calibration with the first two columns presenting the empirical and the simulated moments for Massachusetts and the next four columns presenting the same results for East and West Tennessee (majority motorist moments are shown in Appendix Table C2). The model does a very good job of matching both the daylight speed distribution and the change in the speed distribution between daylight and darkness. The model also closely matches both the fraction of stops in darkness that involve minority motorists and the VOD test statistic. The results for East Tennessee are notable in that the model fits both the empirical VOD test statistic that is just below one, and the speed distribution with stopped minority motorists at upper speed percentiles driving substantially slower in daylight. At the bottom, the table also presents the share African-American motorists on the road as a fraction of white and African-American motorists, and the fraction not infracting for minority motorists in daylight and in darkness. For example in Massachusetts, while the fraction of stopped motorists in the inter-twilight window is 18.2, the predicted share African-American motorists on the road is calibrated to be 16.4, which is much closer to the fraction minority residents associated with our sample of 14.0.[4] In all states, but especially in Massachusetts, we observe a sizable change in the predicted share of African-American motorists not infracting between daylight and darkness. The calibrated parameters are shown in Appendix Table C3.

The higher share of African-Americans among stopped motorists in daylight and the large shift in the speed distribution in Massachusetts result in a calibration where the share of minority

[4] The town share African-American as a fraction of white and African-American residents is weighted by the share of stops in the towns included in our sample and we use state population share for stops made by the state police.

motorists not infracting increases by 50 percentage points between darkness and daylight. The more modest differences in East Tennessee imply a smaller 15 percentage point increase in the share not infracting. For comparison purposes, we turn to Gehrsitz (2017) who examines the effect of exposure to a one month license suspension penalty for severe speeding violations in Germany, which is imposed for a second speeding violation within one calendar year. He finds that exposure to this potential penalty reduces the likelihood of a second violation by between 5 and 8 percentage points over a base 25 percent recidivism rate.

Table 8 summarizes the impact of race on police stop behavior in the calibration. The first row presents the minority stop cost in daylight, which is 0.006 in Massachusetts, 30.113 in East Tennessee, and 37.753 in West Tennessee all in comparison to a darkness stop cost of 44.0. White stop costs in daylight are all near the darkness stop cost, consistent with the quantile regression estimates that showed no change in the speed distribution in daylight for white motorists. Consistent with previous studies and the large shift in the speed distribution, we find evidence of high levels of police prejudice in Massachusetts, i.e. a daylight stop cost far below the darkness stop cost. We observe higher levels of prejudice (lower stop costs) for East Tennessee than West Tennessee, based on the shift in the speed distribution in East Tennessee, even though the VOD test statistic for East Tennessee was near 1.0.

Further, we can use the calibrated parameters for police stop costs and $u(i)$ to compare the lower minority stop costs in daylight to the police pay-offs that arise from stopping a motorist whose speeding infraction is more severe. The next three rows show the change in return to a police stop if the speed of the motorist increases by ½, 2 or 5 standard deviations relative to the simulated mean level of infractions among stopped motorists. Specifically, we find the mean μ and standard deviation σ of the number of miles per hour over the speed limit within the simulation for motorists committing infractions, and calculate $(\mu + \alpha\sigma)^{\eta} - (\mu)^{\eta}$ where α takes on the values of ½, 2 and 5 and η is the exponent parameter in $u(i)$. Daylight raises the effective net returns to stopping minority motorists in Massachusetts by more than the effect of raising speed by five standard deviations above the mean. In East Tennessee, daylight raises the return to stopping minority motorists by an amount comparable to a 2 standard deviation increase in speed, but in West Tennessee where the speed distribution shift is smaller daylight raises the return by ½ a standard deviation.

The second panel of Table 8 presents the VOD test statistic from the calibration and a counterfactual VOD statistic that would arise if minority motorists did not change their infraction behavior in daylight, i.e. behaved in daylight as if they faced the police costs for stops in darkness. Following Grogger and Ridgeway (2006), the VOD test statistic is

Definition 4. $K_{VOD} \equiv \frac{p[m|stopped,\overline{v}]}{p[w|stopped,\overline{v}]}\frac{p[w|stopped,\underline{v}]}{p[m|stopped,\underline{v}]}$

We can calculate the alternative statistic K_{ADJ} by calculating the above probabilities in K_{VOD} except c^* and i' in daylight $\overline{v}$ are assumed to depend on the darkness $\underline{v}$ police stop cost.

Definition 5. $K_{ADJ} \equiv \frac{\int_{c^*(s_{\underline{v}})}^{c_h} g(c,m)\phi^*\left(i'\left(c,s_{\underline{v}}\right),s_{\overline{v},m}\right)di}{\int_{c^*(s_{\underline{v}})}^{c_h} g(c,w)\phi^*\left(i'\left(c,s_{\underline{v}}\right),s_{\overline{v},w}\right)di}\frac{\int_{c^*(s_{\underline{v}})}^{c_h} g(c,w)\phi^*\left(i'\left(c,s_{\underline{v}}\right),s_{\underline{v}}\right)di}{\int_{c^*(s_{\underline{v}})}^{c_h} g(c,m)\phi^*\left(i'\left(c,s_{\underline{v}}\right),s_{\underline{v}}\right)di}$

The counterfactual VOD test statistic increases the most in Massachusetts from 1.38 to 2.74, the next most in East Tennessee from 1.00 to 1.22, and by the smallest amount in West Tennessee to 1.17 from the calibrated value of 1.09. The results in Table 8 are repeated for alternative weights in Appendix Table C4, see Table C5 for calibration parameters.

C.4. Calibration Weights

Theory does not provide guidance for establishing the weights on the moments. Our simulation is matching 6 statistics: African-Americans and white daylight speed distribution, African-Americans and white daylight to darkness shift in the speed distribution, fraction stopped motorists minority in darkness and VOD test statistic. Equal weights with 6 statistics would imply a weight of 16.7 percent for each statistic. However, one might place more weight on the speed distribution statistics since they represent the sum of 6 individual moment squared deviations. On the other hand, we might limit the weight on these moments since the number of moments is arbitrary based on the number of speed percentiles considered. For our baseline calibration, we place three times the weight on the speed distribution statistics so that the weight on those four are 21.5 percent each, and the weight on the fraction stopped motorists minority and VOD test statistic are 7 percent each. We also run robustness tests where we use an equal weight of 16.7 percent, and where we place six times the weight based on the 6 moments of the speed distribution for a weight of 23.25 percent for the four speed distribution statistics and 3.5 percent for both percent minority stopped and the VOD test statistic.

We conduct a robustness test by modifying the weights. The first panel of Table C4 presents the results from Table 8. The second panel applies an equal weight of 16.7 percent to the four speed distribution components and two moments based on the percent minority stopped. The third panel assigns approximately six times the weight to the four speed distribution moments that have six components so each of those moments receive a weight of 23.25 percent and the percent minority stopped based moments receive a weight of 3.5 percent each.[5] The basic results are relatively robust with similar daylight minority stop costs across the three calibrations, and substantially larger VOD test statistics after adjusting for minority driver changes in behavior. The magnitude of the adjusted VOD test statistics is notably sensitive to the weights only for West Tennessee. The largest adjusted VOD test statistic arises for the third panel where a larger weight is placed on matching the speed distribution shift, which makes sense since the baseline calibration understated the speed distribution shift in West Tennessee. Surprisingly, placing lower weight on the speed distribution contributions also increases the West Tennessee adjusted VOD test statistic. A better match to the VOD test statistic, which now has higher weight, requires lower police stop costs for minorities in daylight, which appears to have increased the shift in the speed distribution even as the total fit of the speed distribution moments eroded due to having lower weight. The calibrated parameters based on the alternative weights are shown in Table C5.

[5] The calibrated parameters for these alternative weights are shown in Appendix Tables B4-B6 for the three sites.

Table C1: Minimum and Maximum Values for Parameters

Parameters	Min	Max
α_1	0	1
M	1	4
δ_0	0	50
A	0.8	1.5
α_2	0	3
K	100	500
H	1	1.5
b_0	0	200
τ_0	50	800
σ_m	0	3
σ_w	0	3
mean_m	-4	2
mean_w	-4	2
skew_m	-50	100
s_v	44	44
MA		
skew_w	-50	100
s_vm	0	15
s_vw	44	60
E TN		
skew_w	-50	600
s_vm	30	44
s_vw	44	50
W TN		
skew_w	-50	100
s_vm	30	44
s_vw	44	47

Notes. Table presents the bounds on parameter values used for the envolutionary based optimization selected based on the local optima identified during the initial stages of optimization. Most parameter limits are the same by site with the exception of the minority and white daylight stop costs which are influenced heavily by the empirical racial composition of stops, and for the white skewness where we observed unusually high levels of skewness in the white population in some of the initial calibrations for East Tennessee.

Table C2: Calibration Results

	Massachusetts		East Tennessee		West Tennessee	
	Data	Simulation	Data	Simulation	Data	Simulation
African-American Speed Distribution Daylight						
20th Percentile	13.3835	13.3344	12.1763	12.8364	11.5419	11.1629
40th Percentile	14.8568	15.5537	15.2878	14.9840	13.5632	13.4064
60th Percentile	18.3094	18.5776	17.8090	17.5080	15.7624	15.8768
80th Percentile	23.9418	23.6617	20.7542	21.5682	19.3593	19.5268
90th Percentile	28.4176	28.5276	25.3446	25.0375	22.8966	23.1396
95th Percentile	33.3009	33.1861	28.6582	28.4432	26.8071	26.5863
Difference Daylight and Darkness						
20th Percentile	-0.0899	0.2458	0.5273	-0.2437	0.0830	0.3135
40th Percentile	2.2735	2.3309	0.3049	0.3068	0.2845	0.4916
60th Percentile	3.8130	3.5311	0.7094	0.6952	0.4227	0.5726
80th Percentile	4.7673	4.6565	1.6052	0.9309	0.4644	0.6492
90th Percentile	5.6541	5.1206	1.2012	1.3029	0.7023	0.6702
95th Percentile	5.1922	5.3577	1.3253	1.4398	1.0512	0.7007
Minority Share of Stops						
Minority Share of Stops Darkness	0.1664	0.1665	0.0466	0.0466	0.1771	0.1771
VOD Test Statistic	1.3769	1.3793	0.9924	0.9973	1.0908	1.0899
Percent African-American						
Motorists	NA	0.1638	NA	0.0552	NA	0.1771
Not Infracting in Daylight	NA	0.4959	NA	0.3197	NA	0.0773
Not Infracting in Darkness	NA	0.0056	NA	0.1673	NA	0.0063

Notes: Empirical speed distribution in miles per hour based on regressing relative speed on day of week, time of day, geographic and for Tennessee year controls, calculating the residual, adding the means of controls back and then calculating miles per hour based on the mode speed limit of traffic stops for each site. The simulated moments arise from the global optimum identified by applying an evolutionary based optimization routine called ESCH and precisely located by applying a second optimization routine based on quadratic approximations to the surface BOBYQA. The calibrated parameters used to calculate these moments are shown in Appendix Table B2.

Table C3: Calibration Results White Motorists

	Massachusetts		East Tennessee		West Tennessee	
	Data	Simulation	Data	Simulation	Data	Simulation
White Speed Distribution Daylight						
20th Percentile	12.58119726	12.8990268	13.31786436	13.3819359	11.28808471	11.0761
40th Percentile	16.42617039	16.2500592	16.22345474	15.8291645	13.56950803	13.4798
60th Percentile	20.63785929	20.3829396	18.95081273	18.8376353	15.9148927	16.184
80th Percentile	26.69713667	26.8608619	23.30887568	23.5328263	19.74395994	20.0753
90th Percentile	33.27770052	33.3347123	27.7676207	27.9992266	23.69906791	23.7206
95th Percentile	41.03735374	40.9859405	33.42494972	33.2383482	28.02024315	27.7701
Difference Daylight and Darkness						
20th Percentile	-0.49062111	-0.2660177	0.00422909	-0.0000143	-0.11309278	0
40th Percentile	-0.38312866	-0.2971691	-0.11176114	0.0008881	-0.09190034	-0.0001
60th Percentile	0.2679906	-0.3117633	-0.03097062	0.0001257	-0.02815239	-0.0001
80th Percentile	-0.57652247	-0.3265926	-0.03718293	-0.0000601	0.03660167	0
90th Percentile	0.16650752	-0.34577	-0.07980587	0.0004908	0.26197307	-0.0001
95th Percentile	-1.70325014	-0.5461246	-0.2129078	0.0002214	0.46968762	0

Notes: Empirical speed distribution in miles per hour based on regressing relative speed on day of week, time of day, geographic and for Tennessee year controls, calculating the residual, adding the means of the controls back to the sample and then calculating the miles per hour based on the mode speed limit of traffic stops for each site. The simulated moments arise from the global optimum identified by applying an evolutionary based optimization routine called ESCH and precisely located by applying a second optimization routine based on quadratic approximations to the surface BOBYQA. The calibrated parameters used to calculate these moments are shown in Appendix Table 18.

Table C4: Calibrated Parameters

Parameters	Sites		
	MA	E TN	W TN
α_1	0.522029	0.509337	0.999519
M	1.55008	1.52118	2.18566
δ_0	5.14442	35.5046	3.0628
A	1.23914	1.1562	0.987308
α_2	0.509207	0.421155	1.4297
K	320.493	331.992	235.093
H	1.00387	1.00018	1.24943
b_0	16.7978	17.7386	128.356
τ_0	139.826	122.94	495.065
σ_m	1.23344	1.16092	0.513587
σ_w	1.66537	1.47121	0.53856
mean_m	-0.157625	-0.601036	-2.04262
mean_w	-1.24202	-1.02003	-2.08983
skew_m	0.269799	0.286773	2.5971
skew_w	11.551	3.47682	9.46006
s_vm	0.0057178	30.1313	37.7525
s_vw	44.9736	44.0005	44.0004
s_v	44	44	44
MSE	0.7483	0.638	0.2591

Notes. Each column of this table contains the calibrated parameters for one of the three sites for our baseline set of weights where the speed distribution components each have a weight of 21.5 and the share stops minority in darkness and the VOT test statistics (times 100) each have a weight of 3.5%. The parameters for the Massachusetts sample are in column 1 labelled MA. Column 2 contains parameters for East Tennessee labelled E TN, and column 3 is West Tennessee labeled W TN. The last row shows the mean squared error of the moments for each site.

Table C5: Calibration Results Related to Racial Differences in Police Stop Behavior

	Massachusetts	East Tennessee	West Tennessee
Original Weights			
Minority Stop Cost Diff	43.994	13.887	6.247
Simulated VOD Test	1.379	0.997	1.090
Adjusted VOD Test	2.736	1.223	1.173
Equal Weights			
Minority Stop Cost Diff	43.994	13.9996	10.125
Simulated VOD Test	1.38	0.994	1.091
Adjusted VOD Test	2.736	1.226	1.271
Speed Moments Times Six			
Minority Stop Cost Diff	43.979	12.348	12.38
Simulated VOD Test	1.38	1	1.09
Adjusted VOD Test	2.736	1.195	1.338

Notes. The first panel repeats the results from Table 12 using the original weights. The second panel presents results where the four speed distributions receive the same weight as each of the moments associated with percent minority stopped. The third panel presents results where the speed component contribution receives six times the weight because those components contain the mean squared error for six distinct moments.

Table C6: Calibrated Parameters for Massachusetts with Alternative Weights

Parameters	Massachusetts		East Tennessee		West Tennessee	
Weights	Equal	Speed*Six	Equal	Speed*Six	Equal	Speed*Six
α_1	0.522111	0.521889	0.509832	0.509053	0.999859	0.999519
M	1.55032	1.55012	1.52139	1.52119	2.18541	2.18367
δ_0	5.15151	5.14515	35.512	35.5046	3.48442	3.28581
A	1.23914	1.23901	1.1561	0.995303	0.987312	0.987276
α_2	0.509362	0.509277	0.423291	0.421159	1.42969	1.42973
K	320.355	320.48	330.47	331.94	235.036	233.854
H	1.00383	1.00385	1.00002	1.00073	1.25016	1.2497
b_0	16.7934	16.7978	17.7361	10.4695	129.092	128.877
τ_0	139.812	139.807	122.763	122.94	506.544	506.865
σ_m	1.23477	1.23406	1.16421	1.16237	0.516049	0.516706
σ_w	1.66587	1.66551	1.46317	1.47036	0.53923	0.538645
mean_m	-0.160836	-0.157642	-0.61939	-0.601093	-2.04261	-2.04212
mean_w	-1.24327	-1.24219	-1.0126	-1.0212	-2.09014	-2.08981
skew_m	0.268804	0.26982	0.293491	0.285781	2.59473	2.59168
skew_w	11.5521	11.5507	3.4744	3.4276	9.43347	9.47268
s_vm	0.104016	0.0209493	30.0004	31.6522	33.8754	31.6203
s_vw	44.9798	44.9742	44.0514	44.0331	44.0022	44.2422
s_v	44	44	44	44	44	44
MSE	0.5781	0.8044	0.4829	0.6164	0.168	0.2335

Notes. This table presents the calibrated parameters for different weights for the State of Massachusetts sample. The first column presents parameters for the baseline weights. The second column presents parameters for equal weights of 16.7% for the four speed components and the two components based on share minority stopped (share in darkness and VOD test), and the third column presents parameters for weights where the speed distribution components that contain 6 moments each have approximately 6 times the weight or 23.25% as the weight of 3.5% for the share stops minority in darkness and the VOD test statistic.