For Online Publication: Appendix to Common Agent or Double Agent? Pharmacy Benefit Managers in the Prescription Drug Market

Rena Conti,[*] Brigham Frandsen,[†] Michael Powell,[‡] James B. Rebitzer[§]

May 11, 2021

Appendix A: Proofs of theoretical results

Proof of Lemma 1. First consider the consumer's drug purchasing decision. The consumer receives zero utility when not purchasing a drug. The consumer whose medical condition is $D = d$ receives $V - \bar{p}$ when purchasing drug d out of pocket and $V - c_d$ when purchasing it through the formulary if she purchased insurance. The consumer's utility is therefore maximized by purchasing the drug if and only if $V \geq \min\{c_d, \bar{p}\}$, and, if the consumer purchases the drug, she will do so at the lower price: through the formulary if $c_d \leq \bar{p}$ and out of pocket otherwise. If she did not enroll in insurance she purchases the drug if and only iff $V \geq \bar{p}$. Now consider the consumer's insurance enrollment decision. Her net utility if not enrolling is U_0 and if enrolling is $U_1 - p_0$. She therefore enrolls if and only if $p_0 \leq U_1 - U_0$. ■

Proof of Lemma 2. First consider the payer's premium choice. The payer can guarantee zero profit by setting $p_0 > U_1 - U_0$. Subject to the consumer enrolling in insurance, the payer's profit is increasing in the premium, p_0, and so it will set the premium as high as possible subject to the consumer's enrollment constraint:

$$p_0 = U_1 - U_0,$$

provided $\pi_{\text{payer}}(U_1 - U_0, c_1, c_2, p_1, p_2) \geq 0$, and any value $p_0 > U_1 - U_0$ otherwise. Now consider the payer's tier-assignment choice. The payer assigns drug 1 to the

[*]Boston University
[†]Brigham Young University
[‡]Northwestern University
[§]Boston University

preferred tier if and only if the profit from doing so is greater than or equal to the profit from assigning drug 2 to the preferred tier:

$$\pi_{\text{payer}}\left(p_0, c_L, c_H, p_1, p_2\right) \geq \pi_{\text{payer}}\left(p_0, c_H, c_L, p_1, p_2\right).$$

Substituting in the expressions for the payer's profit, and noting that $c_L \leq c_H$ by definition, this condition simplifies to

$$p_1 \leq p_2.$$

■

Proof of Lemma 3. Note that the formulary contest is a 2-player, 2-prize all-pay auction with complete information of the sort analyzed by Barut and Kovenock (1998). We refer to their Theorem 2, Part A to establish that there is a unique equilibrium in which drug makers randomize continuously over a closed interval. Note also that the upper bound of the support is $\bar{p}$, because at the upper bound the drug maker loses with probability one, and drug maker profit given loss is maximized at $\bar{p}$. Let F be the cdf corresponding to drug maker 2's equilibrium strategy. First, note that Drug maker 1's expected profit at support point p is

$$E\left[\pi_1\left(p\right)\right] = \left(1 - F\left(p\right)\right) pq\left(c_L\right) + F\left(p\right) pq\left(c_H\right).$$

Noting that $F\left(\bar{p}\right) = 1$, the equilibrium condition that profit be equal at all points in the support of F means

$$\bar{p}q\left(c_H\right) = \left(1 - F\left(p\right)\right) pq\left(c_L\right) + F\left(p\right) pq\left(c_H\right).$$

Solving this condition for $F\left(p\right)$ yields

$$F\left(p\right) = \frac{q\left(c_L\right) - \frac{\bar{p}}{p}q\left(c_H\right)}{q\left(c_L\right) - q\left(c_H\right)}.$$

The lower bound of the support occurs where F equals zero:

$$p = \bar{p}\frac{q\left(c_H\right)}{q\left(c_L\right)}.$$

■

Proof of Lemma 4. Recall that the equilibrium net price cdf is

$$F\left(p; c_L, c_H\right) = \frac{q\left(c_L\right) - \frac{\bar{p}}{p}q\left(c_H\right)}{q\left(c_L\right) - q\left(c_H\right)}.$$

The derivative of the cdf with respect to c_L is

$$\frac{\partial F\left(p; c_L, c_H\right)}{\partial c_L} = \frac{q\left(c_H\right)\left(\frac{\bar{p}}{p} - 1\right)}{\left(q\left(c_L\right) - q\left(c_H\right)\right)^2} q'\left(c_L\right) < 0.$$

The derivative of the cdf with respect to c_H is

$$\frac{\partial F\left(p; c_L, c_H\right)}{\partial c_H} = -\frac{q\left(c_L\right)\left(\frac{\bar{p}}{p} - 1\right)}{\left(q\left(c_L\right) - q\left(c_H\right)\right)^2} q'\left(c_H\right) > 0.$$

■

Proof of Lemma 5. The payer's expected profit is

$$E\left[\pi_{\text{payer}}\left(c_L, c_H\right)\right] = TS\left(c_L, c_H\right) - CS - \bar{p}q\left(c_H\right),$$

where $TS\left(c_L, c_H\right)$ is total surplus:

$$TS\left(c_L, c_H\right) = \frac{1}{2}\left(E\left[1\left(V > c_H\right)V\right] + E\left[1\left(V > c_L\right)V\right]\right),$$

and CS is consumer surplus:

$$CS = E\left[\left(V - \bar{p}\right)1\left(V > \bar{p}\right)\right],$$

which holds by an implication of Lemma 2. By the proof of Lemma 3, $\bar{p}q\left(c_H\right)$ is the sum of the drug makers' expected profit. Note that $E\left[\pi_{\text{payer}}\left(c_L, c_H\right)\right]$ is clearly decreasing in c_L, so the profit maximizing choice is $c_L = 0$. Payer profit is increasing in c_H:

$$\frac{\partial E\left[\pi_{\text{payer}}\left(c_L, c_H\right)\right]}{\partial c_H} = -q'\left(c_H\right)\left(\bar{p} - \frac{1}{2}c_H\right) > 0,$$

where the equality follows from Leibniz' rule and the fact that the slope of the demand curve is the opposite of the density of willingness to pay. The inequality follows from the assumption that demand is strictly downward sloping and the constraint that $c_H \leq \bar{p}$. The profit maximizing choice of c_H is therefore $\min\left\{q^{-1}\left(0\right), \bar{p}\right\}$, which is $\bar{p}$ by assumption. ■

Proof of Proposition 1. Given a common exogenous list price $\bar{p}$, expected utility without insurance is unchanged from the baseline model:

$$U_0 = E\left[\left(V - \bar{p}\right)1\left(V > \bar{p}\right)\right].$$

Expected utility with insurance is

$$\begin{aligned} U_1 &= \frac{1}{2}\left(1 + \tau\right)E\left[\left(V - \min\left\{c_L, \bar{p}\right\}\right)1\left(V > \min\left\{c_L, \bar{p}\right\}\right)\right] \\ &\quad + \frac{1}{2}\left(1 - \tau\right)E\left[\left(V - \min\left\{c_H, \bar{p}\right\}\right)1\left(V > \min\left\{c_H, \bar{p}\right\}\right)\right]. \end{aligned}$$

As before, consumers purchase insurance if $U_1 \geq U_0 + p_0$. The premium will therefore be set to $p_0 = U_1 - U_0$.

The intermediary's tier assignment is also the same as the baseline model: the payer will assign drug 1 to the preferred tier if

$$\pi_{\text{payer}}\left(p_0, c_L, c_H, p_1, p_2\right) \geq \pi_{\text{payer}}\left(p_0, c_H, c_L, p_1, p_2\right),$$

which is equivalent to $p_1 \leq p_2$.

By the same argument as Lemma 3, equilibrium involves a mixed strategy F whose support has upper bound $\bar{p}$. Drug maker 1's expected profit given own price p and that drug maker 2 is mixing with F is

$$E\left[\pi_1(p)\right] = \left(1 - F(p)\right) p \left(1+\tau\right) q\left(c_L\right) + F(p)\, p \left(1-\tau\right) q\left(c_H\right).$$

Noting that $F(\bar{p}) = 1$, indifference between p and $\bar{p}$ implies that the equilibrium net price distribution is

$$F(p) = \frac{\left(1+\tau\right) q\left(c_L\right) - \frac{\bar{p}}{p}\left(1-\tau\right) q\left(c_H\right)}{\left(1+\tau\right) q\left(c_L\right) - \left(1-\tau\right) q\left(c_H\right)}.$$

This distribution is stochastically decreasing in the degree of substitutability, τ:

$$\frac{\partial}{\partial \tau} F(p) = \frac{2\left(\frac{\bar{p}}{p} - 1\right) q\left(c_L\right) q\left(c_H\right)}{\left(\left(1+\tau\right) q\left(c_L\right) - \left(1-\tau\right) q\left(c_H\right)\right)^2} > 0,$$

meaning that more substitution lowers the net price distribution in a stochastically dominant sense, as required.

Drug maker total expected profit is

$$\bar{p}\left(1-\tau\right) q\left(c_H\right),$$

as required.

Now consider the intermediary's choice of copays. The payer's expected profit is

$$\begin{aligned} \pi_{\text{payer}} &= TS - CS - \bar{p}\left(1-\tau\right) q\left(c_H\right), \\ &= -\bar{p}\left(1-\tau\right) q\left(c_H\right). \end{aligned}$$

where total surplus is

$$TS = \frac{1}{2}\left(\left(1+\tau\right) E\left[V 1\left(V \geq c_L\right)\right] + \left(1-\tau\right) E\left[V 1\left(V \geq c_H\right)\right]\right)$$

and consumer surplus is

$$CS = E\left[\left(V - \bar{p}\right) 1\left(V > \bar{p}\right)\right].$$

Note that total surplus is higher than in the baseline case when $\tau > 0$. Since c_L only enters TS, and TS is decreasing in c_L, the optimal choice is $c_L = 0$ as in the baseline model. Intermediary profit is increasing in c_H :

$$\frac{\partial \pi_{\text{payer}}}{\partial c_H} = -\left(1-\tau\right) q'\left(c_H\right)\left(\bar{p} - \frac{1}{2} c_H\right) > 0,$$

so $c_H = \bar{p}$ also as in the baseline model. ■

Lemma 1 (Tier assignment with m drugs) *The equilibrium tier assignment t sorts drugs by net prices; that is, t is the permutation on $\{1,\ldots,m\}$ such that for any i and i' in $\{1,\ldots,m\}$,*

$$p_i < p_{i'} \Longrightarrow t(i) < t(i'),$$

and ties are broken randomly.

Proof of Lemma 1. Let i and i' index two drugs such that $p_i < p_{i'}$. Fix the tier assignments for all other drugs at $\{t_j\}_{j\neq i,i'}$. Without loss of generality, let c_a and c_b be the copays corresponding to the tiers to be assigned to either i or i', where $c_a \leq c_b$. The portion of the payer's revenue that depends on the net prices and tier assignments of drugs i and i' is:

$$B\left(c_{t(i)}, c_{t(i')}\right) = \frac{1}{m}\left[\left(c_{t(i)} - p_i\right) q\left(c_{t(i)}\right) + \left(c_{t(i')} - p_{i'}\right) q\left(c_{t(i')}\right)\right].$$

It suffices to show that $p_i < p_{i'}$ implies $B(c_a, c_b) \geq B(c_b, c_a)$ with the inequality strict when $c_a < c_b$. The condition $B(c_a, c_b) \geq B(c_b, c_a)$ simplifies to

$$(p_{i'} - p_i)\left(q(c_a) - q(c_b)\right) \geq 0,$$

which holds with strict inequality with $c_a < c_b$. ■

Lemma 2 (Equilibrium net-price distribution with m drugs) *There exists a symmetric equilibrium. Any symmetric equilibrium involves continuously mixed strategies with an interval support $\left[\underline{p}, \bar{p}\right]$ for some $\underline{p} < \bar{p}$, where $\bar{p}$ is the list price.*

Proof of Lemma 2. First, we will argue that there are no mass points. Suppose there was a mass point at some price p. Then with strictly positive probability, all drug makers will simultaneously choose price p, and at that price, they would have an equal chance of winning each of the prizes. One of the drug makers could deviate by allocating all that mass instead to price $p - \varepsilon$ for ε arbitrarily small and for sure win $q(c_1)(p - \varepsilon)$, so there is a profitable deviation. So there cannot be any mass points.

Next, suppose the upper bound of the price distribution is $\tilde{p} < \bar{p}$. Then since there are no mass points, by choosing price $p_i = \tilde{p}$, drug maker i can only get revenues of $q(c_i)\tilde{p}$. It could obtain $q(c_i)\bar{p}$ by bidding $p_i = \bar{p}$ and would get strictly higher profits since $\bar{p} > \tilde{p}$. So the upper bound of the price distribution must be $\bar{p}$. A similar argument establishes that the support of the distribution is an interval: suppose $\hat{p}^1$ and $\hat{p}^2 > \hat{p}^1$ are in the support of the distribution, but there is a gap in the support between these two points. Then drug maker i would strictly prefer to choose price $\hat{p}^2$ over $\hat{p}^1$, which again is a contradiction. So any symmetric equilibrium price distribution is continuous and has support $\left[\underline{p}, \bar{p}\right]$ for some $\underline{p} < \bar{p}$.

The preceding shows that if there is a symmetric equilibrium price distribution F^*, it is continuous and has support $\left[\underline{p}, \bar{p}\right]$. Let $F(p)$ be the cdf of a candidate equilibrium mixing distribution. Let

$$F^{k,m-1}(p) = \binom{m-1}{k} F(p)^k \left(1 - F(p)\right)^{m-1-k}$$

be the probability that exactly k of the other $m-1$ prices is less than p if all drug makers mix with continuous distribution F on $\left[\underline{p}, \bar{p}\right]$. Then the expected profit for firm i if it chooses net price p_i is

$$\pi\left(p_i\right) = \sum_{k=0}^{m-1} F^{k,m-1}\left(p_i\right) q\left(c_{k+1}\right) p_i.$$

Let F^* be such that

$$\pi\left(p\right) = \sum_{k=0}^{m-1} \binom{m-1}{k} F^*\left(p\right)^k \left(1 - F^*\left(p\right)\right)^{m-1-k} q\left(c_{k+1}\right) p$$

is constant on $\left[\underline{p}, \bar{p}\right]$. Then F^* is an equilibrium price distribution.

To see why such an F^* exists, we will show that for any strictly decreasing and differentiable function $Q\left(p\right)$ satisfying $Q\left(\underline{p}\right) = q\left(c_1\right)$ and $Q\left(\bar{p}\right) = q\left(c_m\right)$, there exists a CDF $F\left(p\right)$ such that $Q\left(p\right) = \sum_{k=0}^{m-1} F^{k,m-1}\left(p\right) q\left(c_{k+1}\right)$ for all p.

Towards this end, define $\tilde{F}^{k,m-1}\left(p\right) = \sum_{j=k}^{m-1} F^{j,m-1}\left(p\right)$ to be the probability that at least k of the other $m-1$ prices is less than p, so that if drug maker i sets price p_i, the probability it will be placed in a tier with a copay at least as bad as c_{k+1} is $\tilde{F}^{k,m-1}\left(p_i\right)$. While $F^{k,m-1}\left(p\right)$ is not necessarily monotonic in $F\left(p\right)$, $\tilde{F}^{k,m-1}\left(p\right)$ is a monotonically increasing in $F\left(p\right)$, as it corresponds to one minus the cdf of a binomial distribution with success probability $F\left(p\right)$ and $m-1$ trials evaluated at $k-1$, which is increasing in the success probability in the first order stochastic dominance sense. Let

$$\phi\left(F\left(p\right)\right) = \sum_{k=1}^{m-1} \tilde{F}^{k,m-1}\left(p\right)\left(q\left(c_k\right) - q\left(c_{k+1}\right)\right).$$

Then ϕ is a strictly increasing and continuous function that satisfies $\phi\left(0\right) = 0$ and $\phi\left(1\right) = q\left(c_1\right) - q\left(c_m\right)$, so it is invertible on the domain $[0,1]$. Given any arbitrary strictly decreasing and continuous function $Q\left(p\right)$ satisfying $Q\left(\underline{p}\right) = q\left(c_1\right)$ and $Q\left(\bar{p}\right) = q\left(c_m\right)$, define $\hat{F}$ to satisfy $\hat{F}\left(p\right) = \phi^{-1}\left(q\left(c_1\right) - Q\left(p\right)\right)$ for all p. Then there exists a symmetric equilibrium price distribution in which each drug maker chooses a continuous mixing distribution $F^*\left(p\right) = \hat{F}\left(p\right)$ for $Q\left(p\right)$ satisfying $pQ\left(p\right) = \bar{p}q\left(c_m\right)$ on support $\left[\underline{p}, \bar{p}\right]$. ■

Proof of Proposition 2. The payer's expected profit is equal to total surplus (TS) minus consumer surplus (CS) minus drug makers' profit. As a function of copays $c_1, \ldots, c_m$, total surplus and consumer surplus are

$$\begin{aligned} TS\left(c_1, \ldots, c_m\right) &= \frac{1}{m}\sum_{i=1}^{m} E\left[1\left(V > c_i\right) V\right], \\ CS &= E\left[\left(V - \bar{p}\right) 1\left(V > \bar{p}\right)\right]. \end{aligned}$$

Expected drug maker profit is $\bar{p}q(c_m)$, as shown in the proof of Lemma 2. The payer's expected profit is therefore

$$E[\pi_{\text{payer}}(c_1,\dots,c_m)] = TS(c_1,\dots,c_m) - CS - \bar{p}q(c_m).$$

Note that this is clearly decreasing in $c_1,\dots,c_{m-1}$, so the profit maximizing choice of the first $m-1$ copays is $c_1 = c_2 = \cdots = c_{m-1} = 0$. Payer profit is increasing in c_m:

$$\frac{\partial E[\pi_{\text{payer}}]}{\partial c_m} = -q'(c_m)\left(\bar{p} - \frac{1}{m}c_m\right) > 0,$$

where the equality follows from Leibniz' rule and the fact that the slope of the demand curve is the opposite of the density of willingness to pay. The inequality follows from the assumption that demand is strictly downward sloping and the constraint that $c_m \le \bar{p}$. The profit maximizing choice of c_m is therefore $\bar{p}$. This proves the proposition's first result.

As a function of copays, total surplus is

$$TS(c_1,\dots,c_m) = \frac{1}{m}\sum_{i=1}^{m} E[1(V > c_i)V].$$

Plugging in the equilibrium copays established in the first result, this becomes

$$\begin{aligned} TS(0,\dots,\bar{p}) &= \frac{1}{m}((m-1)E[V] + E[1(V > \bar{p})V]) \\ &= E[V] - \frac{1}{m}E[1(V \le \bar{p})V]. \end{aligned}$$

■

Proof of Proposition 3. We will first establish that $c_L^j = 0$, $j = 1,2$, cannot be part of an equilibrium by supposing that it is and showing that payer 1 can profitably deviate to some $c_L^1 > 0$. Payer 1's expected profit is equal to total surplus (TS) minus consumer surplus (CS) minus the other payer's profit minus drug makers' profit:

$$\pi_1 = TS(\mathbf{c}_L,\mathbf{c}_H) - CS - \pi_2 - \overline{pq}(\mathbf{c}_H),$$

where total surplus and consumer surplus are

$$\begin{aligned} TS(\mathbf{c}_L,\mathbf{c}_H) &= \frac{1}{2}\sum_{j=1}^{2}\frac{1}{2}\left(E\left[1\left(V > c_L^j\right)V\right] + E\left[1\left(V > c_H^j\right)V\right]\right), \\ CS &= E[(V-\bar{p})1(V > \bar{p})], \end{aligned}$$

and payer 2's profit is

$$\pi_2 = \frac{1}{2}\left[p_0^2\left(c_L^2, c_H^2\right) + \frac{1}{2}\left(\left(c_L^2 - E[p_L]\right)q\left(c_L^2\right) + \left(c_H^2 - E[p_H]\right)q\left(c_H^2\right)\right)\right].$$

Note that neither consumer surplus nor drug maker 2's profit depends on c_L^1. To see that payer 1 can profitably deviate from $c_L^1 = 0$, note that

$$\left.\frac{\partial \pi_1}{\partial c_L^1}\right|_{c_L^1=0} = \left.\frac{\partial TS}{\partial c_L^1}\right|_{c_L^1=0} - \left.\frac{\partial \pi_2}{\partial c_L^1}\right|_{c_L^1=0}$$
$$= \frac{1}{4}\left(\frac{\partial E[p_L]}{\partial c_L^1} q\left(c_L^2\right) + \frac{\partial E[p_H]}{\partial c_L^1} q\left(c_H^2\right)\right) > 0,$$

where $\partial TS/\partial c_L^1|_{c_L^1=0} = 0$ because a zero copay maximizes total surplus, and the final inequality follows from Lemma 4. Therefore, $c_L^1 = 0$ cannot be part of an equilibrium, and any symmetric equilibrium will involve $c_L^1 = c_L^2 = c_L > 0$.

Next, we will establish that in any symmetric equilibrium, $c_H^j = \overline{p}$ for $j = 1, 2$. Let $q_L = q\left(c_L^1\right) + q\left(c_L^2\right)$ be the total sales for a drug assigned to the preferred tier and $q_H = q\left(c_H^1\right) + q\left(c_H^2\right)$ be the total sales for a drug assigned to the non-preferred tier. Define the expected drug expenditures for payer j to be $E\left[C^j\right] = q\left(c_L^j\right) E[p_L] + q\left(c_H^j\right) E[p_H]$ and the expected total drug expenditures to be $E[C] = q_L E[p_L] + q_H E[p_H]$. Using the endogenous price distributions described in Lemma 3, we can compute the following objects, which will be helpful: $(i.)$ $E[C] = 2\overline{p} q_H$; $(ii.)$ $q_L \frac{\partial E[p_L]}{\partial q_L} + q_H \frac{\partial E[p_H]}{\partial q_L} = -E[p_L]$; and $(iii.)$ $q_L \frac{\partial E[p_L]}{\partial q_H} + q_H \frac{\partial E[p_H]}{\partial q_H} = \frac{q_L}{q_H} E[p_L]$.

Given c_L^2 and c_H^2, payer 1's problem is to choose c_L^1 and c_H^1 to maximize π_1. Payer 1's optimality conditions, if $0 < c_L^{1*}, c_H^{1*} < \overline{p}$, satisfy

$$c_L^{1*} = E[p_L] + \frac{\partial E\left[C^1\right]}{\partial q\left(c_L^1\right)}$$
$$c_H^{1*} = E[p_H] + \frac{\partial E\left[C^1\right]}{\partial q\left(c_H^1\right)},$$

that is, the optimal low copay is equal to the expected marginal cost of the low-net-price drug plus a term that captures the impact of an increase in the low copay on the net-price distribution, and similarly for the high copay. These optimality conditions immediately imply that $c_H^{1*} = c_H^{2*} = c_H^*$ and $c_L^{1*} = c_L^{2*} = c_L^*$ in equilibrium. Next, note that $\frac{\partial E\left[C^1\right]}{\partial q\left(c_H^1\right)} = \frac{1}{2}\frac{\partial E[C]}{\partial q_H} = \overline{p}$, so the optimality conditions above give us that $c_H^* = E[p_H] + \overline{p}$, which is not interior. We therefore have that $c_H^* = \overline{p}$.

Finally, note that the symmetric equilibrium values $c_L^* > 0$ and $c_H^* = \overline{p}$ were in the payer's choice set in the one-payer model but were dominated by $c_L^* = 0$ and $c_H^* = \overline{p}$. Therefore total payer profit is reduced when $n = 2$. Note also that total surplus is strictly decreasing in c_L and c_H. Because c_L is strictly higher when $n = 2$, and c_H is the same, total surplus is also reduced. ■

Proof of Lemma 6. The PBM's profit as a function of copays and reimbursement prices, taking net prices as given is

$$\pi_{PBM}\left(c_1, c_2, r_1, r_2; p_1, p_2\right) = \frac{1}{2}\sum_{i=1}^{2} q\left(c_i\right)\left(r_i - p_i\right).$$

Profit is increasing in the reimbursement prices r_1 and r_2. The PBM will therefore set them so that the payers' zero profit condition binds. Profit for payer j is

$$\pi_j = \frac{1}{2}\left(p_0 + \frac{1}{2}\sum_{i=1}^{2} q\left(c_i\right)\left(c_i - r_i\right)\right).$$

Setting this equal to zero gives the following expression for the weighted average reimbursement price:

$$\frac{1}{2}\sum_{i=1}^{2} q\left(c_i\right) r_i = p_0 + \frac{1}{2}\sum_{i=1}^{2} q\left(c_i\right) c_i,$$

as stated in the lemma.

Substituting this condition into the PBM's profit function gives

$$\pi_{PBM}\left(c_1, c_2; p_1, p_2\right) = p_0 + \frac{1}{2}\sum_{i=1}^{2} q\left(c_i\right)\left(c_i - p_i\right),$$

which is identical to the payer's profit function in the one-payer case. Tier assignment is therefore identical to the one-payer case established in Lemma 2. ■

Proof of Proposition 4. As established in the proof to Lemma 6, the PBM's profit function after substituting in the profit-maximizing choice of reimbursement prices, is identical to the payer's profit in the one-payer case. Therefore the PBM's equilibrium choice of copays and the drug makers' net price strategies coincide with the one-payer case. The result of Proposition 3 therefore means that total surplus and joint PBM and payer profit is higher with a PBM than when each payer acts as its own intermediary. ■

Proof of Lemma 7. The intermediary's profit as a function of copays assignments c_1 and c_2, taking net prices as given, is

$$\pi\left(c_1, c_2; p_1, p_2\right) = p_0 + \frac{1}{2}\sum_{i=1}^{2} q\left(\min\left\{c_i, \bar{p}_i\right\}\right)\left(\min\left\{c_i, \bar{p}_i\right\} - p_i\right),$$

where $p_0 = U_1 - U_0$ and

$$\begin{aligned} U_0 &= \frac{1}{2}\sum_{i=1}^{2} E\left[\left(V - \bar{p}_i\right) 1\left(V > \bar{p}_i\right)\right], \\ U_1 &= \frac{1}{2}\sum_{i=1}^{2} E\left[\left(V - \min\left\{c_i, \bar{p}_i\right\}\right) 1\left(V > \min\left\{c_i, \bar{p}_i\right\}\right)\right]. \end{aligned}$$

The intermediary assigns $c_1 = c_L$ and $c_2 = c_H$ if its profit from doing so is greater than its profit from doing otherwise; that is, it assigns drug 1 to the preferred tier if and only if $\pi(c_L, c_H; p_1, p_2) \geq \pi(c_H, c_L; p_1, p_2)$. In Case 1 ($c_H \leq \min\{\bar{p}_1, \bar{p}_2\}$) this condition becomes

$$p_1 \leq p_2,$$

as in Lemma 2. In Case 2 ($\bar{p}_1 < c_H \leq \bar{p}_2$), the condition becomes

$$p_2 \geq \frac{q(c_L) - q(\bar{p}_1)}{q(c_L) - q(c_H)} p_1 + \frac{E[V|V > \bar{p}_1] q(\bar{p}_1) - E[V|V > c_H] q(c_H)}{q(c_L) - q(c_H)}.$$

■

Proof of Proposition 5. The equilibrium list price vector $(\bar{p}_1^*, \bar{p}_2^*)'$ is a fixed point of the drug makers' best response function:

$$BR\left(\left(\begin{array}{c}\bar{p}_1\\ \bar{p}_2\end{array}\right)\right) = \left(\begin{array}{c}\arg\max_x \pi_D(\mathbf{c}^*(x, \bar{p}_2), x, \bar{p}_2)\\ \arg\max_x \pi_D(\mathbf{c}^*(\bar{p}_1, x), x, \bar{p}_1)\end{array}\right),$$

where $\pi_D(\mathbf{c}, \bar{p}_i, \bar{p}_{-i})$ is drug maker i's expected profit given formulary copays $\mathbf{c} = (c_L, c_H)'$, own list price $\bar{p}_i$ and the other drug maker's list price $\bar{p}_{-i}$. The function $\mathbf{c}^*(\bar{p}_1, \bar{p}_2)$ gives the intermediary's equilibrium choice of $(c_L, c_H)'$ given list prices:

$$\mathbf{c}^*(\bar{p}_1, \bar{p}_2) = \arg\max_{\mathbf{c}} \pi_I(\mathbf{c}, \bar{p}_1, \bar{p}_2).$$

The intermediary's expected profit in turn is given by total surplus minus consumer surplus minus combined drug maker profit:

$$\pi_I(\mathbf{c}, \bar{p}_1, \bar{p}_2) = TS(\mathbf{c}, \bar{p}_1, \bar{p}_2) - CS(\bar{p}_1, \bar{p}_2) - (\pi_D(\mathbf{c}, \bar{p}_1, \bar{p}_2) + \pi_D(\mathbf{c}, \bar{p}_2, \bar{p}_1)).$$

Total surplus is

$$\begin{aligned} TS(\mathbf{c}, \bar{p}_1, \bar{p}_2) &= \frac{1}{2}\sum_{i=1}^{2} E[E[V|V > c_i] q(c_i)] \\ &= \frac{1}{2}\left(1 - \frac{1}{2}E\left[c_1^2 + c_2^2\right]\right), \end{aligned}$$

where the second equality follows from the assumed linear demand function, and c_1 and c_2 are the copays corresponding to the tier to which drugs 1 and 2 (respectively) are assigned. These are random quantities because the tier assignment depends on the net prices, which in equilibrium are drawn from a mixed strategy. The distribution of net prices and thus the distribution of $c_i, i = 1, 2$ is derived below, and depends on $\mathbf{c}$, $\bar{p}_1$, and $\bar{p}_2$. Consumer surplus is determined by the list prices:

$$\begin{aligned} CS(\bar{p}_1, \bar{p}_2) &= \frac{1}{2}\sum_{i=1}^{2} E[(V - \bar{p}_i) 1(V > \bar{p}_i)] \\ &= \frac{1}{4}\sum_{i=1}^{2} (1 - \bar{p}_i)^2, \end{aligned}$$

where the second equality follows from the linear demand function. The final component of $\pi_I(\mathbf{c}, \bar{p}_1, \bar{p}_2)$ is drug maker profit, which Lemma 4 shows is as follows. Drug maker 1's profit is

$$\pi_D(\mathbf{c},\bar{p}_1,\bar{p}_2) = \begin{cases} \frac{1}{2}(1-c_L)\min\left\{\bar{p}_1, \phi^{-1}\left(\frac{1-c_H}{1-c_L}\bar{p}_2\right)\right\} & , \quad \text{Case 1,2a} \\ \frac{1}{2}(1-\bar{p}_1)\bar{p}_1 & , \quad \text{Case 2b} \end{cases},$$

and drug maker 2's expected profit is

$$\pi_D(\mathbf{c},\bar{p}_2,\bar{p}_1) = \begin{cases} \frac{1}{2}(1-c_H)\bar{p}_2 & , \quad \text{Case 1,2a} \\ \frac{1}{2}(1-c_L)\phi\left(\frac{1-\bar{p}_1}{1-c_L}\bar{p}_1\right) & , \quad \text{Case 2b} \end{cases},$$

where Cases 1 and 2 are defined in Lemma 7, and within Case 2, Case 2a obtains when

$$\frac{1-c_H}{1-c_L}\bar{p}_2 \geq \frac{\bar{p}_1 - c_L}{c_H - c_L}\frac{1-\bar{p}_1}{1-c_L}\bar{p}_1 + \frac{1}{2}\frac{(c_H^2 - \bar{p}_1^2)}{c_H - c_L}$$

is satisfied and Case 2b otherwise. In the case of a non-degenerate contest, drug maker 1's profit becomes

$$\pi_D(\mathbf{c},\bar{p}_1,\bar{p}_2) = \underline{\pi}_1(\mathbf{c},\bar{p}_1,\bar{p}_2) + \frac{1}{2}(1-c_L)x_1,$$

where the default payoff is

$$\underline{\pi}_1(\mathbf{c},\bar{p}_1,\bar{p}_2) = \frac{1}{2}(1-\min\{\bar{p}_1, c_H\})\bar{p}_1$$

and the excess reach is

$$x_1 = \max\left\{\phi^{-1}\left(\frac{1-c_H}{1-c_L}\bar{p}_2\right) - \frac{1-\min\{\bar{p}_1, c_H\}}{1-c_L}\bar{p}_1, 0\right\}.$$

In a non-degenerate contest drug maker 2's profit becomes

$$\pi_D(\mathbf{c},\bar{p}_2,\bar{p}_1) = \underline{\pi}_2(\mathbf{c},\bar{p}_1,\bar{p}_2) + \frac{1}{2}(1-c_L)x_2,$$

where the default payoff is

$$\underline{\pi}_2(\mathbf{c},\bar{p}_2,\bar{p}_1) = \frac{1}{2}(1-c_H)\bar{p}_2$$

and the excess reach is

$$x_2 = \max\left\{\phi\left(\frac{1-\min\{\bar{p}_1, c_H\}}{1-c_L}\bar{p}_1\right) - \frac{1-c_H}{1-c_L}\bar{p}_2, 0\right\}.$$

It remains only to find $E\left[c_1^2 + c_2^2\right]$. In Case 1, where both list prices are higher than c_H, $c_1^2 + c_2^2$ is non-random because the identity of the "winning" drug does not affect the value of the higher copay, so

$$E\left[c_1^2 + c_2^2\right] = c_L^2 + c_H^2 \text{ (Case 1).}$$

In Case 2, the value of the higher copay depends on which drug wins the formulary contest, and this is random. We therefore have:

$$E\left[c_1^2 + c_2^2\right] = c_L^2 + c_H^2 \gamma\left(\mathbf{c}, \bar{p}_1, \bar{p}_2\right) + \bar{p}_1^2 \left(1 - \gamma\left(\mathbf{c}, \bar{p}_1, \bar{p}_2\right)\right) \text{ (Case 2),}$$

where $\gamma\left(\mathbf{c}, \bar{p}_1, \bar{p}_2\right)$ is the probability that drug maker 1 wins the formulary contest:

$$\begin{aligned} \gamma\left(\mathbf{c}, \bar{p}_1, \bar{p}_2\right) &= \Pr\left(\phi\left(p_1\right) \leq p_2\right) \\ &= \int F_1\left(p_2; \mathbf{c}, \bar{p}_1, \bar{p}_2\right) dF_2\left(p_2; \mathbf{c}, \bar{p}_1, \bar{p}_2\right), \end{aligned}$$

and F_1 and F_2 are the equilibrium mixed strategy distributions given $\mathbf{c}, \bar{p}_1, \bar{p}_2$, shown in Lemma 3 to be, for Case 2a,

$$\begin{aligned} F_1\left(p\right) &= \begin{cases} \frac{\left(1-c_L\right)\left(1-\frac{\underline{p}_2}{p}\right)}{c_H - c_L} &, \quad \underline{p}_2 < p < \phi\left(\bar{p}_1\right) \\ 1 &, \quad p = \phi\left(\bar{p}_1\right) \end{cases}, \\ F_2\left(p\right) &= \begin{cases} \frac{\left(1-c_L\right)\left(1-\frac{\phi^{-1}\left(\underline{p}_2\right)}{\phi^{-1}\left(p\right)}\right)}{\bar{p}_1 - c_L} &, \quad \underline{p}_2 < p < \phi\left(\bar{p}_1\right) \\ 1 &, \quad p = \bar{p}_2 \end{cases}, \end{aligned}$$

and, for Case 2b

$$\begin{aligned} F_1\left(p\right) &= \begin{cases} \frac{1-c_L}{c_H - c_L}\left(1 - \frac{\underline{p}_1}{p}\right) &, \quad \underline{p}_1 < p < \phi\left(\bar{p}_1\right) \\ 1 &, \quad p = \phi\left(\bar{p}_1\right) \end{cases}, \\ F_2\left(p\right) &= \begin{cases} \frac{1-c_L-\left(1-\bar{p}_1\right)\frac{\bar{p}_1}{\phi^{-1}\left(p\right)}}{\bar{p}_1 - c_L} &, \quad \underline{p}_1 < p \leq \phi\left(\bar{p}_1\right) \end{cases}, \end{aligned}$$

where

$$\begin{aligned} \underline{p}_2 &= \frac{1-c_H}{1-c_L}\bar{p}_2, \\ \underline{p}_1 &= \phi\left(\frac{1-\bar{p}_1}{1-c_L}\bar{p}_1\right). \end{aligned}$$

Drug companies' best response functions now depend on no unknowns and can be inspected directly. Figure 4 plots the best responses. By inspection, there is a single fixed point such that $\bar{p}_1 \leq \bar{p}_2$ located at $\bar{p}_1^* = 1/2$ and $\bar{p}_2^* = 1$. ■

Lemma 3 (Drug net price equilibrium distribution with endogenous list prices) *Suppose demand is linear. Then given* (c_L, c_H) *and* $\bar{p}_1 \leq \bar{p}_2$*, the unique equilibrium mixed strategy net price distributions are, for Case 2a,*

$$
\begin{aligned}
F_1(p) &= \begin{cases} \frac{(1-c_L)\left(1-\frac{\underline{p}_2}{p}\right)}{c_H-c_L} & , \quad \underline{p}_2 < p < \phi(\bar{p}_1) \\ 1 & , \quad p = \phi(\bar{p}_1) \end{cases} , \\
F_2(p) &= \begin{cases} \frac{(1-c_L)\left(1-\frac{\phi^{-1}\left(\underline{p}_2\right)}{\phi^{-1}(p)}\right)}{\bar{p}_1-c_L} & , \quad \underline{p}_2 < p < \phi(\bar{p}_1) \\ 1 & , \quad p = \bar{p}_2 \end{cases} ,
\end{aligned}
$$

and, for Case 2b

$$
\begin{aligned}
F_1(p) &= \begin{cases} \frac{1-c_L}{c_H-c_L}\left(1-\frac{\underline{p}_1}{p}\right) & , \quad \underline{p}_1 < p < \phi(\bar{p}_1) \\ 1 & , \quad p = \phi(\bar{p}_1) \end{cases} , \\
F_2(p) &= \begin{cases} \frac{1-c_L-(1-\bar{p}_1)\frac{\bar{p}_1}{\phi^{-1}(p)}}{\bar{p}_1-c_L} & , \quad \underline{p}_1 < p \leq \phi(\bar{p}_1) \end{cases} .
\end{aligned}
$$

where

$$
\begin{aligned}
\underline{p}_2 &= \frac{1-c_H}{1-c_L}\bar{p}_2, \\
\underline{p}_1 &= \phi\left(\frac{1-\bar{p}_1}{1-c_L}\bar{p}_1\right).
\end{aligned}
$$

Proof. Note that the upper bound on the support of F_1 is $\phi(\bar{p}_1)$ and the upper bound on the support of F_2 is $\bar{p}_2$, because net prices cannot exceed list prices. Note also that a lower bound on the support of F_i is $\max\{1-T, 1-a_i\}$.

Take Case 2a first, where drug maker 2 is marginal. This means that drug maker 2's reach, where it is indifferent between winning and settling for a loss, determines the threshold price, which is a lower bound on the support of F_2:

$$\underline{p}_2 = \frac{1-c_H}{1-c_L}\bar{p}_2.$$

Suppose first that $\underline{p}_2 > \phi(\bar{p}_1)$ (that is, drug maker 2's lower bound is greater than drug maker 1's maximum possible bid). Then F_1 will be degenerate at $\phi(\bar{p}_1)$ and F_2 will be degenerate at $\bar{p}_2$ and drug maker 1 wins with probability one. Now suppose $\underline{p}_2 \leq \phi(\bar{p}_1)$. The supports of both F_1 and F_2 will then have a lower bound of $\underline{p}_2$. Note that $\phi(\bar{p}_1)$ is an upper bound on the support of F_1. Any continuous portion of F_2 will therefore have an upper bound of $\phi(\bar{p}_1)$. We first derive the continuous portion (if any) of F_2 by considering drug company 1's profit as a function of some bid p in the continuous portion of the support of F_2:

$$\pi_1(p) = \frac{1}{2}(1-\bar{p}_1)\phi^{-1}(p)F_2(p) + \frac{1}{2}(1-c_L)\phi^{-1}(p)(1-F_2(p)).$$

Equilibrium requires that $\pi_1(p)$ be equal to the profit derived above using Siegel (2009), which allows us to solve for the the continuous portion of F_2:

$$F_2^{\text{cont}}(p) = \frac{(1-c_L)\left(1-\frac{\phi^{-1}\left(\underline{p}_2\right)}{\phi^{-1}(p)}\right)}{\bar{p}_1 - c_L}.$$

We can similarly find the continuous portion of F_1 by considering drug maker 2's profit at some net price p in the continuous portion of the support of F_1:

$$\pi_2(p) = \frac{1}{2}(1-c_H)\,pF_1(p) + \frac{1}{2}(1-c_L)\,p\,(1-F_1(p)).$$

Again, using the profit derived above for drug maker 2, indifference determines the continuous portion of F_1:

$$F_1^{\text{cont}}(p) = \frac{(1-c_L)\left(1-\frac{\underline{p}_2}{p}\right)}{c_H - c_L}.$$

We now determine any mass points in the distributions. Note that F_1^{cont} and F_2^{cont} are both zero at $\underline{p}_2$, meaning there is no mass point at the lower end of the support. Note also that $\phi(\bar{p}_1)$ is an upper bound on the support of F_1 and thus also on the continuous portion of F_2. Because $F_1^{\text{cont}}(\phi(\bar{p}_1)) < 1$, F_1 has a mass point of $\lambda_1 = 1 - F_1^{\text{cont}}(\phi(\bar{p}_1))$ at $\phi(\bar{p}_1)$. Because $F_2^{\text{cont}}(\phi(\bar{p}_1)) < 1$ (which is true because we are in Case 2a), Drug maker 2 will put all remaining mass above $\phi(\bar{p}_1)$ at $\bar{p}_2$, for a mass point of $\lambda_2 = 1 - F_2^{\text{cont}}(\phi(\bar{p}_1))$ at $\bar{p}_2$. In summary, in Case 2a, the equilibrium distributions are

$$F_1(p) = \begin{cases} \frac{(1-c_L)\left(1-\frac{\underline{p}_2}{p}\right)}{c_H-c_L} & , \quad \underline{p}_2 < p < \phi(\bar{p}_1) \\ 1 & , \quad p = \phi(\bar{p}_1) \end{cases},$$

$$F_2(p) = \begin{cases} \frac{(1-c_L)\left(1-\frac{\phi^{-1}\left(\underline{p}_2\right)}{\phi^{-1}(p)}\right)}{\bar{p}_1-c_L} & , \quad \underline{p}_2 < p < \phi(\bar{p}_1) \\ 1 & , \quad p = \bar{p}_2 \end{cases},$$

if $\underline{p}_2 \leq \phi(\bar{p}_1)$, and degenerate at $\phi(\bar{p}_1)$ and $\bar{p}_2$ otherwise.

Now take Case 2b, where drug maker 1 is marginal. Now the threshold price is determined by drug maker 1's reach, where it is indifferent between winning and settling for losing. This establishes a lower bound on the support of F_1:

$$\underline{p}_1 = \phi\left(\frac{1-\bar{p}_1}{1-c_L}\bar{p}_1\right).$$

Note that $\underline{p}_1 < \bar{p}_2$, so $\underline{p}_1$ is also a lower bound on the support of F_2. As before, we can determine the continuous portion of F_2 by looking at drug maker 1's profit as a function of some bid p in the continuous portion of the support of F_2:

$$\pi_1(p) = \frac{1}{2}(1-\bar{p}_1)\,\phi^{-1}(p)\,F_2(p) + \frac{1}{2}(1-c_L)\,\phi^{-1}(p)\,(1-F_2(p)).$$

Using the profit for drug maker 1 derived above for Case 2b, indifference determines the continuous portion of F_1:

$$F_2^{\text{cont}}(p) = \frac{1 - c_L - (1 - \bar{p}_1)\frac{\bar{p}_1}{\phi^{-1}(p)}}{\bar{p}_1 - c_L}.$$

Similarly, we determine the continuous portion of F_1 by looking at drug maker 2's profit at some bid b in the continuous portion of the support of F_1:

$$\pi_2(p) = \frac{1}{2}(1 - c_H)\,pF_1(p) + \frac{1}{2}(1 - c_L)\,p\,(1 - F_1(p)).$$

Using the profit for drug maker 1 derived above for Case 2b, indifference determines the continuous portion of F_2:

$$F_1^{\text{cont}}(p) = \frac{1 - c_L}{c_H - c_L}\left(1 - \frac{\underline{p}_1}{p}\right).$$

Now we determine mass points. Note F_1^{cont} and F_2^{cont} are both zero at $\underline{p}_1$, meaning neither distribution has a mass point at the lower end of the support. Note that F_1^{cont} reaches one at $\frac{1-c_L}{1-c_H}\underline{p}_1 > \phi(\bar{p}_1)$, meaning F_1 has a mass point at $\phi(\bar{p}_1)$ equal to $\lambda_1 = 1 - F_1^{\text{cont}}(\phi(\bar{p}_1))$. Because $F_2^{\text{cont}}(\phi(\bar{p}_1)) = 1$, F_2 has no mass point. In summary, in Case 2b, the equilibrium distributions are

$$\begin{aligned}
F_1(p) &= \begin{cases} \frac{1-c_L}{c_H-c_L}\left(1 - \frac{\underline{p}_1}{p}\right) &, \quad \underline{p}_1 < p < \phi(\bar{p}_1) \\ 1 &, \quad p = \phi(\bar{p}_1) \end{cases}, \\
F_2(p) &= \left\{ \frac{1-c_L-(1-\bar{p}_1)\frac{\bar{p}_1}{\phi^{-1}(p)}}{\bar{p}_1-c_L} \right. , \quad \underline{p}_1 < p \leq \phi(\bar{p}_1) .
\end{aligned}$$

■

Lemma 4 (Drug maker profit with endogenous list prices) *Suppose drug demand is linear. Then given formulary copays* $\mathbf{c} = (c_L, c_H)'$ *and list prices* $\bar{p}_1 \leq \bar{p}_2$*, drug maker 1's expected profit is*

$$\pi_D(\mathbf{c}, \bar{p}_1, \bar{p}_2) = \begin{cases} \frac{1}{2}(1 - c_L)\min\left\{\bar{p}_1, \phi^{-1}\left(\frac{1-c_H}{1-c_L}\bar{p}_2\right)\right\} &, \quad \textit{Case 1,2a} \\ \frac{1}{2}(1 - \bar{p}_1)\,\bar{p}_1 &, \quad \textit{Case 2b} \end{cases},$$

and drug maker 2's expected profit is

$$\pi_D(\mathbf{c}, \bar{p}_2, \bar{p}_1) = \begin{cases} \frac{1}{2}(1 - c_H)\,\bar{p}_2 &, \quad \textit{Case 1,2a} \\ \frac{1}{2}(1 - c_L)\,\phi\left(\frac{1-\bar{p}_1}{1-c_L}\bar{p}_1\right) &, \quad \textit{Case 2b} \end{cases},$$

where Cases 1 and 2 are defined in Lemma 7, and within Case 2, Case 2a obtains when

$$\frac{1 - c_H}{1 - c_L}\bar{p}_2 \geq \frac{\bar{p}_1 - c_L}{c_H - c_L}\frac{1 - \bar{p}_1}{1 - c_L}\bar{p}_1 + \frac{1}{2}\frac{(c_H^2 - \bar{p}_1^2)}{c_H - c_L}$$

is satisfied and Case 2b otherwise.

Proof. Drug maker profit $\pi_D(\mathbf{c}, \bar{p}_i, \bar{p}_{-i})$ is determined by the equilibrium of the formulary contest and is characterized as follows. Applying the linear demand function to the general result in Lemma 7, the intermediary awards the preferred tier to drug 1 if $p_2 \geq \phi(p_1)$, where

$$\phi(p) = \begin{cases} p & , \quad c_H \leq \min\{\bar{p}_1, \bar{p}_2\} \text{ (Case 1)} \\ \frac{\bar{p}_1 - c_L}{c_H - c_L} p + \frac{1}{2} \frac{\left(c_H^2 - \bar{p}_1^2\right)}{c_H - c_L} & , \quad \bar{p}_1 < c_H \leq \bar{p}_2 \text{ (Case 2)} \end{cases}.$$

The net price-setting game between the drug makers thus takes the form of an all-pay contest, in which drug maker 2 bids p_2 and drug maker 1 bids $\tilde{p}_1 = \phi(p_1)$. If drug maker 2 wins it receives payoff $q(c_L)p_2$ and if it loses it receives $q(c_H)p_2$. If drug maker 1 wins it receives payoff $q(c_L)p_1$ and if it loses it receives $q(\min\{c_H, \bar{p}_1\})p_1$. Equilibrium payoffs in contests like this are characterized in Siegel (2009). In the notation of Siegel (2009), the number of players is $n = 2$. The number of prizes (placement in the preferred tier) is $m = 1$. Because in Siegel's framework, higher scores win the contest, we define each player's score s_i as one minus the price bid, transformed by ϕ in the case of drug maker 1. Specifically, drug maker 1's score is $s_1 = 1 - \phi(p_1)$. Drug maker 2's score is $s_2 = 1 - p_2$. Drug maker 1 wins if $s_1 \geq s_2$. Drug makers have "initial scores" (lowest possible score they can choose): $a_1 = 1 - \phi(\bar{p}_1)$ and $a_2 = 1 - \bar{p}_2$, since net prices can be at most equal to the list price. Given $s = (s_1, s_2)'$, drug maker 1's payoff is

$$u_1(s) = 1(s_1 \geq s_2)\, v_1(s_1) - 1(s_1 < s_2)\, c_1(s_1),$$

where

$$v_1(s_1) = \frac{1}{2}(1 - c_L)\,\phi^{-1}(1 - s_1) - \frac{1}{2}(1 - \min\{c_H, \bar{p}_1\})\,\bar{p}_1$$

is drug maker 1's valuation for winning, which is defined to be net of the profit obtained by losing for sure, and

$$c_1(s_1) = -\left(\frac{1}{2}(1 - \min\{c_H, \bar{p}_1\})\,\phi^{-1}(1 - s_1) - \frac{1}{2}(1 - \min\{c_H, \bar{p}_1\})\,\bar{p}_1\right)$$

is drug maker 1's cost of losing, also defined to be net of the profit obtained by losing for sure. Given s, drug maker 2's payoff is

$$u_2(s) = 1(s_1 < s_2)\, v_2(s_2) - 1(s_1 \geq s_2)\, c_2(s_2),$$

where

$$v_2(s_2) = \frac{1}{2}(1 - c_L)(1 - s_2) - \frac{1}{2}(1 - c_H)\,\bar{p}_2$$

is drug maker 2's valuation for winning, and

$$c_2(s_2) = -\left(\frac{1}{2}(1 - c_H)(1 - s_2) - \frac{1}{2}(1 - c_H)\,\bar{p}_2\right)$$

is drug maker 2's cost of losing.

We now verify Siegel's (2009) Assumptions A1, A2, and A3. Assumption A1 is that v_i and $-c_i$ are continuous and nonincreasing. Noting that ϕ is an increasing function, this is true by inspection. Assumption A2 is that $v_i(a_i) > 0$ and $\lim_{s_i \to \infty} v_i(s_i) < c_i(a_i) = 0$. To see this, note that

$$v_1(a_1) = \frac{1}{2}\bar{p}_1 (\min\{c_H, \bar{p}_1\} - c_L),$$

which is greater than zero if $c_L < \min\{c_H, \bar{p}_1\}$, which is true by assumption. Also, note that

$$\begin{aligned} \lim_{s_1 \to \infty} v_1(s_1) &= \lim_{p_1 \to -\infty} \frac{1}{2}(1 - c_L) p_1 - \frac{1}{2}(1 - \min\{c_H, \bar{p}_1\}) \bar{p}_1 \\ &= -\infty, \end{aligned}$$

which is certainly less than

$$\begin{aligned} c_1(a_1) &= -\left(\frac{1}{2}(1 - \min\{c_H, \bar{p}_1\}) \bar{p}_1 - \frac{1}{2}(1 - \min\{c_H, \bar{p}_1\}) \bar{p}_1\right) \\ &= 0. \end{aligned}$$

For drug maker 2, note that

$$v_2(a_2) = \frac{1}{2}\bar{p}_2 (c_H - c_L) > 0,$$

and

$$\begin{aligned} \lim_{s_2 \to \infty} v_2(s_2) &= \frac{1}{2}(1 - c_L)(1 - s_2) - \frac{1}{2}(1 - c_H) \bar{p}_2 \\ &= -\infty, \end{aligned}$$

which is less than

$$\begin{aligned} c_2(a_2) &= -\left(\frac{1}{2}(1 - c_H) \bar{p}_2 - \frac{1}{2}(1 - c_H) \bar{p}_2\right) \\ &= 0. \end{aligned}$$

Assumption A3 is that $c_i(s_i) > 0$ if $v_i(s_i) = 0$. For drug maker 1, v_1 is zero at its *reach* (the highest score at which v_i is zero), which is:

$$r_1 = 1 - \phi\left(\frac{1 - \min\{c_H, \bar{p}_1\}}{1 - c_L} \bar{p}_1\right).$$

c_1 evaluated at this value is

$$c_1(r_1) = \frac{1}{2}(1 - \min\{c_H, \bar{p}_1\}) \bar{p}_1 \left(\frac{\min\{c_H, \bar{p}_1\} - c_L}{1 - c_L}\right),$$

which is positive as required. For drug maker 2, $v_2(s_2) = 0$ at

$$r_2 = 1 - \frac{1-c_H}{1-c_L}\bar{p}_2.$$

c_2 evaluated at this value is

$$c_2(r_2) = \frac{1}{2}\bar{p}_2(1-c_H)\left(\frac{c_H - c_L}{1-c_L}\right),$$

which is positive as required. Siegel's Assumptions A1-A3 are therefore satisfied in our setting.

The following concepts in Siegel's framework help characterize equilibrium payoffs. The *marginal player* is the drug maker with the lower reach. In Case 1 ($c_H \leq \min\{\bar{p}_1, \bar{p}_2\}$), $r_2 \leq r_1$ so long as $\bar{p}_2 \geq \bar{p}_1$, which is true by definition. Therefore, drug maker 2 is marginal in Case 1. In Case 2 ($\bar{p}_1 < c_H \leq \bar{p}_2$), drug maker 2 is marginal if

$$\frac{1-c_H}{1-c_L}\bar{p}_2 \geq \frac{\bar{p}_1 - c_L}{c_H - c_L}\frac{1-\bar{p}_1}{1-c_L}\bar{p}_1 + \frac{1}{2}\frac{(c_H^2 - \bar{p}_1^2)}{c_H - c_L}.$$

This condition may or may not hold, depending on the values of $c_L, c_H, \bar{p}_1, \bar{p}_2$. We therefore consider both cases. In Case 2a, the above condition holds, so drug maker 2 is marginal. In Case 2b, the above condition does not hold, so drug maker 1 is marginal. In Cases 1 and 2a, therefore, drug maker 2 is marginal ($r_2 \leq r_1$), and in Case 2b, drug maker 2 is marginal ($r_2 > r_1$).

The contest's *threshold*, T, is the reach of the marginal player. Therefore, in Cases 1 and 2a, $T = 1 - \frac{1-c_H}{1-c_L}\bar{p}_2$. In case 2b, $T = 1 - \phi\left(\frac{1-\bar{p}_1}{1-c_L}\bar{p}_1\right)$.

Each drug maker's *power* is its valuation for winning at the threshold: $w_i = v_i(\max\{a_i, T\})$. By construction the power of the marginal player is zero. In Case 1, each drug maker's power is the following:

$$\begin{aligned} w_1 &= \frac{1}{2}(1-c_L)\min\left\{\bar{p}_1, \frac{1-c_H}{1-c_L}\bar{p}_2\right\} - \frac{1}{2}(1-c_H)\bar{p}_1, \\ w_2 &= 0. \end{aligned}$$

In Case 2a, the powers are the following:

$$\begin{aligned} w_1 &= \frac{1}{2}(1-c_L)\min\left\{\bar{p}_1, \phi^{-1}\left(\frac{1-c_H}{1-c_L}\bar{p}_2\right)\right\} - \frac{1}{2}(1-\bar{p}_1)\bar{p}_1 \\ w_2 &= 0. \end{aligned}$$

In Case 2b, where drug maker 1 is marginal, the powers are:

$$\begin{aligned} w_1 &= 0 \\ w_2 &= \frac{1}{2}(1-c_L)\phi\left(\frac{1-\bar{p}_1}{1-c_L}\bar{p}_1\right) - \frac{1}{2}(1-c_H)\bar{p}_2. \end{aligned}$$

Theorem 1 in Siegel (2009) tells us the expected payoff of each drug maker is equal to its power. Recall that payoffs here are defined net of the drug maker's profit if it loses for sure. Therefore we have that drug maker 1's expected profit is

$$\begin{aligned}\pi_D\left(\mathbf{c},\bar{p}_1,\bar{p}_2\right) &= w_1+\frac{1}{2}\left(1-\min\left\{c_H,\bar{p}_1\right\}\right)\bar{p}_1 \\ &= \begin{cases}\frac{1}{2}\left(1-c_L\right)\min\left\{\bar{p}_1,\phi^{-1}\left(\frac{1-c_H}{1-c_L}\bar{p}_2\right)\right\} & , \text{ Case 1,2a} \\ \frac{1}{2}\left(1-\bar{p}_1\right)\bar{p}_1 & , \text{ Case 2b}\end{cases}.\end{aligned}$$

Drug maker 2's expected profit is

$$\begin{aligned}\pi_D\left(\mathbf{c},\bar{p}_2,\bar{p}_1\right) &= w_2+\frac{1}{2}\left(1-c_H\right)\bar{p}_2 \\ &= \begin{cases}\frac{1}{2}\left(1-c_H\right)\bar{p}_2 & , \text{ Case 1,2a} \\ \frac{1}{2}\left(1-c_L\right)\phi\left(\frac{1-\bar{p}_1}{1-c_L}\bar{p}_1\right) & , \text{ Case 2b}\end{cases}\end{aligned}$$

■

Appendix B: Contingent rebates

In the baseline model drug makers each offer a single net price which the intermediary pays regardless of the drug maker's tier assignment. Another possibility is that drug makers offer contingent net prices, one contingent on preferred tier placement, and the other contingent on non-preferred placement. In this section we allow for this possibility by assuming that drug maker 1 offers the pair of contingent net prices $\left(p_1^L,p_1^H\right)$ and drug maker 2 offers the pair $\left(p_2^L,p_2^H\right)$.

Consumer choices are unchanged as they only depend on the premium, list price, and copays. Likewise the payer's choice of premium will be unchanged. Tier assignment, however, will be different. The intermediary maximizes profit by placing drug 1 in the preferred tier if the following condition holds:

$$q\left(c_L\right)\left(p_2^L-p_1^L\right)>q\left(c_H\right)\left(p_2^H-p_1^H\right). \tag{1}$$

The above condition is intuitive: the left hand side is the drug subsidy savings from placing drug 1 in the preferred tier. The right hand side is the drug subsidy savings from placing drug 1 in the non-preferred tier. Whichever saving is bigger dictates the tier assignment. If the condition holds with equality the intermediary is indifferent over tier assignments, and we assume it randomizes with equal probability.

Now consider the drug makers' net price equilibrium. Suppose drug maker 2 sets $p_2^L=\bar{p}q\left(c_H\right)/q\left(c_L\right)$ and $p_2^H=\bar{p}$. Condition (1) implies drug maker 1 wins preferred placement if

$$p_1^L<p_1^Hq\left(c_H\right)/q\left(c_L\right).$$

Increasing p_1^H unambiguously increases drug maker 1's profit because it increases the probability that drug maker 1 wins preferred placement, so drug maker 1's best response is $p_1^H = \bar{p}$. Its profit given that it loses is therefore $\bar{p}q\left(c_H\right)/2$. It earns strictly less profit if it sets $p_1^L < \bar{p}q\left(c_H\right)q\left(c_L\right)$ even though it wins with certainty. It is therefore a best response to set $p_1^L = \bar{p}q\left(c_H\right)/q\left(c_L\right)$. Thus the following net prices are an equilibrium:

$$\begin{aligned} p_i^L &= \frac{q\left(c_H\right)}{q\left(c_L\right)}\bar{p}, \\ p_i^H &= \bar{p}, \end{aligned}$$

for $i = 1, 2$. Drug makers earn total profit $\bar{p}q\left(c_H\right)$ as in the baseline model. Lemma 5 then implies that copays are also the same as in the baseline model, $c_L = 0$ and $c_H = \bar{p}$.

References

Yasar Barut and Dan Kovenock. The symmetric multiple prize all-pay auction with complete information. *European Journal of Political Economy*, 14(4):627 – 644, 1998.

Ron Siegel. All-pay contests. *Econometrica*, 77(1):71–92, 2009.