Online Appendix for "Exchange Rates and Monetary Policy with Heterogeneous Agents: Sizing up the Real Income Channel"

A Model details

A.1 Model setup

Here we provide additional details on the setup of the model in section 2.

Preferences across goods. In our baseline model, consumption c_{it} of any agent i living in any country aggregates their home good H and a composite foreign good F with elasticity η,

$$c_{it} = \left[(1-\alpha)^{\frac{1}{\eta}} \left(c_{iHt}\right)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} \left(c_{iFt}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

home consumption aggregates goods j produced at home, while foreign consumption aggregates goods produced in a continuum of countries k:

$$c_{iHt} = \left(\int_0^1 c_{iHt}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \qquad c_{iFt} = \left(\int_0^1 c_{ikt}^{\frac{\gamma-1}{\gamma}} dk \right)^{\frac{\gamma}{\gamma-1}}$$

with $\epsilon > 1$, $\gamma > 0$ and $\eta > 0$. In turn, consumption from country k aggregates goods produced there with the same elasticity ϵ as that used to aggregate goods produced at home,

$$c_{ikt} = \left(\int_0^1 c_{ikt}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}$$

The agent's budget constraint is

$$\int_0^1 P_{Ht}(j)\, c_{iHt}(j)\, dj + \int_0^1 \int_0^1 P_{kt}(j)\, c_{ikt}(j)\, dj dk + a_{it+1} \le \left(1 + r_t^p\right) a_{it} + e_{it} \frac{W_t}{P_t} N_t$$

hence, consumer i's demand for good j in country k is

$$c_{ikt}(j) = \alpha \left(\frac{P_{kt}(j)}{P_{kt}} \right)^{-\epsilon} \left(\frac{P_{kt}}{P_{Ft}} \right)^{-\gamma} \left(\frac{P_{Ft}}{P_t} \right)^{-\eta} c_{it}$$

while their demand for good j in the home country is

$$c_{iHt}(j) = (1-\alpha) \left(\frac{P_{Ht}(j)}{P_{Ht}} \right)^{-\epsilon} \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} c_{it}$$

Applying this demand system to the heterogeneous agents at home, indexed by their state (a, e), delivers equations (4) and (5). Applying this demand system to the representative foreign agents, noting that all foreign countries are symmetric and prices are flexible abroad so that $P^*_{Ft} = P^*_t$, delivers

$$C_{Ht}^{*} = \alpha \left(\frac{P_{Ht}^{*}}{P_t^{*}} \right)^{-\gamma} C_t^{*}$$

which is equation (7).

Financial sector. Let $\mathcal{A}_t$ denote the nominal liabilities of the domestic mutual fund (the amount of shares it sold to households), and i_t^p denote the nominal return on these assets. Let s_t^H (resp. s_t^F) denote the mutual fund's holdings of domestic (resp. foreign) shares, and B_t^H (resp. B_t^F) denote its holdings of domestic (resp. foreign) bonds. At the beginning of period t, the liquidation value of the intermediary's liabilities is equal to the liquidation value of its assets, so that

$$\left(1 + i_t^p\right) \mathcal{A}_{t-1} = \left(D_t + p_t\right) P_t s_{t-1}^H + \left(1 + i_{t-1}\right) B_{t-1}^H + \left(D_t^* + p_t^*\right) P_t^* \mathcal{E}_t s_{t-1}^F + \left(1 + i_{t-1}^*\right) \mathcal{E}_t B_{t-1}^F \quad \text{(A.1)}$$

At the end of the period, the value of newly purchased assets must be

$$\mathcal{A}_t = p_t P_t s_t^H + B_t^H + p_t^* P_t^* \mathcal{E}_t s_t^F + \mathcal{E}_t B_t^F \quad \text{(A.2)}$$

Define the ex-post real return to the mutual fund as

$$1 + r_t^p = \left(1 + i_t^p\right) \frac{P_{t-1}}{P_t} \quad \text{(A.3)}$$

The mutual fund maximizes the (expected) real return on its liabilities, r_{t+1}^p, for all $t \geq 0$, which here is equivalent to maximizing the (expected) nominal return i_{t+1}^p. Optimality requires that the nominal returns on all assets is equalized,

$$\frac{\left(D_{t+1} + p_{t+1}\right) P_{t+1}}{p_t P_t} = 1 + i_t = \frac{\left(D_{t+1}^* + p_{t+1}^*\right) P_{t+1}^* \mathcal{E}_{t+1}}{p_t^* P_t^* \mathcal{E}_t} = \frac{\left(1 + i_t^*\right) \mathcal{E}_{t+1}}{\mathcal{E}_t} \quad \text{(A.4)}$$

Combining (A.1) and (A.4), we find that, for all $t \geq 0$,

$$1 + i_{t+1}^p = 1 + i_t \quad \text{(A.5)}$$

Further define the ex-ante real risk free rate as

$$1 + r_t \equiv \left(1 + i_t\right) \frac{P_t}{P_{t+1}} \quad \text{(A.6)}$$

This is equation (14). Using the definition of the real exchange rate in (6), real exchange rate depreciation between time $t-1$ and t can be expressed as

$$\frac{Q_t}{Q_{t-1}} = \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \frac{P_{t-1}}{P_t} \quad \text{(A.7)}$$

Using (A.3), (A.4), (A.6) and (A.7), together with the fact that $P_t^* = 1$ always, it follows that, for all $t \geq 0$,

$$1 + r_{t+1}^p = 1 + r_t = \frac{D_{t+1} + p_{t+1}}{p_t} = \frac{\left(D_{t+1}^* + p_{t+1}^*\right) Q_{t+1}}{p_t^* Q_t} = \frac{\left(1 + i_t^*\right) Q_{t+1}}{Q_t} \quad \text{(A.8)}$$

which delivers equations (15) and (16).

Finally, let the real asset position of the mutual fund be defined as

$$A_t \equiv \frac{\mathcal{A}_t}{P_t} \tag{A.9}$$

and define the net foreign asset position to be the difference between A_t and the total value of assets in net supply domestically,

$$\text{nfa}_t \equiv A_t - p_t$$

which is equation (17).

Using (A.9), equation (A.1) rewrites

$$\left(1 + r_t^p\right) A_{t-1} = \left(D_t + p_t\right) s_{t-1}^H + \left(1 + i_{t-1}\right) \frac{P_{t-1}}{P_t} \frac{B_{t-1}^H}{P_{t-1}} + \left(D_t^* + p_t^*\right) Q_t s_{t-1}^F + \left(1 + i_{t-1}^*\right) Q_t \frac{B_{t-1}^F}{P^*} \tag{A.10}$$

while (A.2) rewrites

$$A_t = p_t s_t^H + \frac{B_t^H}{P_t} + p_t^* Q_t s_t^F + Q_t \frac{B_t^F}{P^*} \tag{A.11}$$

From (A.10), we can calculate $1 + r_0^p$ given the initial mutual fund portfolio s_{-1}^H, s_{-1}^F, $\frac{B_{-1}^H}{P_{-1}}$ and $\frac{B_{-1}^H}{P^*}$, whose value in steady state, given (A.11), must add up to A_{ss},

$$A_{ss} = p_{ss} s_{-1}^H + \frac{B_{-1}^H}{P_{-1}} + p_{ss}^* Q_{ss} s_{-1}^F + Q_{ss} \frac{B_{-1}^F}{P^*}$$

In our baseline calibration, we assume that the mutual fund holds all the domestic assets and no other assets, ie $s_{-1}^H = 1$ and $\frac{B_{-1}^H}{P_{-1}} = s_{-1}^F = \frac{B_{-1}^F}{P^*} = 0$. This ensures that $A_{ss} = p_{ss}$, and therefore that $\text{nfa}_{-1} = 0$.

Foreign agents. All foreign countries are symmetric. In each lives a representative foreign agent with utility

$$\sum_{t=0}^{\infty} \left(\beta^*\right)^t \mathcal{B}_t \left\{u\left(C_t^*\right) - v\left(N_t^*\right)\right\} \tag{A.12}$$

where β^* is the foreign discount factor, and $\mathcal{B}_t$ is a utility modifier capturing time-varying patience for the foreign household. We assume that $\mathcal{B}_t$ has initial value $\mathcal{B}_{-1} = 1$, is nonnegative and bounded, $\mathcal{B}_t \in \left(0, \overline{B}\right)$ for $\overline{B} > 0$, and reverts to 1 in the long run: $\lim_{t \to \infty} \mathcal{B}_t = 1$.

Foreign countries produce their own good under constant returns to scale with production function

$$Y_t^* = Z^* N_t^*$$

Prices and wages are flexible abroad, so that

$$P_t^* = \mu W_t^*$$

The home country is infinitesimal, so that market clearing for the composite foreign good is

$$C_t^* = Y_t^*$$

The first order conditions for a representative foreign agent are

$$\frac{v'\left(N_t^*\right)}{u'\left(C_t^*\right)}=\frac{W_t^*}{P_t^*}$$

and

$$\mathcal{B}_t\left(C_t^*\right)^{-\sigma}=\beta^*\left(1+r_t^*\right)\mathcal{B}_{t+1}\left(C_{t+1}^*\right)^{-\sigma} \tag{A.13}$$

where r_t^* denotes the foreign interest rate. It follows that the world equilibrium features a constant level of consumption C^* (and output $Y^*=C^*$) given by

$$\frac{v'\left(C^*/Z^*\right)}{u'\left(C^*\right)}=\frac{1}{\mu}$$

and that the real interest rate r_t^* is given by (A.13) when $C_t^*=C_{t+1}^*=C^*$. The central bank targets a constant price index P^*, which it achieves by setting the foreign interest rate according to price-level targeting rule with the natural rate r_t^* as an intercept, $i_t^*=r_t^*+\phi\log\left(P_t^*/P^*\right)$. In equilibrium, the foreign nominal and real interest rates are equal, and relate to the discount factor shocks $\mathcal{B}_t$ according to

$$1+i_t^*=1+r_t^*=\frac{1}{\beta^*}\frac{\mathcal{B}_t}{\mathcal{B}_{t+1}} \tag{A.14}$$

The primitive shocks in our economy are the sequence of $\mathcal{B}_t$'s. Alternatively, given (A.14), we can construct this sequence for a given exogenous sequence of foreign interest rates i_t^*'s and the fact that $\lim\mathcal{B}_t=1$, from

$$\mathcal{B}_t=\prod_{s\geq t}\left(\frac{1+i_s^*}{1/\beta^*}\right)$$

so that high $\mathcal{B}_t$ corresponds to high current or future foreign interest rate i_t^* relative to the steady state foreign interest rate $1/\beta^*$.

A.2 Version of the model with UIP deviations

We consider a version of our model in which we allow for deviations in the UIP condition. In particular, we assume that mutual funds cannot directly hold positions in foreign bonds. Instead, there exist foreign intermediaries that can trade in both foreign and domestic bond markets. Denote the end-of-period positions of these intermediaries by b_t^I. We assume that foreign intermediaries have an imperfectly elastic demand for domestic bonds, that is,

$$b_t^I=\frac{1}{\Gamma}\left[(1+i_t)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right)-(1+i_t^*)\right]$$

Such imperfect elasticity can be microfounded by assuming that there is only a limited number of foreign intermediaries (of similar measure as the small open economy itself) and that foreign intermediaries face limited commitment (Gabaix and Maggiori 2015), risk (Itskhoki and Mukhin 2020) or adjustment costs (Alvarez, Atkeson and Kehoe 2009, Fanelli and Straub 2020). These microfoundations are identical for our purposes.

In addition to foreign intermediaries, we also allow for noise traders with exogenous demand ξ_t for domestic bonds as in Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2020). Together, foreign intermediaries and noise traders hold the inverse of the country's net foreign asset position,

$$b_t^I + \xi_t = -\text{nfa}_t$$

Rearranging, this implies that the UIP condition (13) no longer holds whenever $\Gamma > 0$

$$(1+i_t)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) = 1 + i_t^* - \Gamma\left(\xi_t + \text{nfa}_t\right) \tag{A.15}$$

The dependence on the NFA in (A.15) captures the idea that the country has to pay a premium when it is a net borrower $\text{nfa}_t < 0$, in terms of a greater interest rate i_t. The dependence on ξ_t captures the idea that noise shocks can also move exchange rates.[44]

In the limit where $\Gamma \to 0$, we recover the UIP condition (13). On the other hand if, as in Itskhoki and Mukhin (2020), we simultaneously assume $\Gamma \to 0$ but $\Gamma \xi_t \not\to 0$, we obtain a version of (A.15) with exogenous UIP shocks

$$(1+i_t)\left(\frac{\mathcal{E}_t}{\mathcal{E}_{t+1}}\right) = 1 + i_t^* - \Gamma \xi_t$$

Observe that those shocks enter in exactly the same way as the world interest rate shocks i_t^* that we introduced in section 2. In that sense, our analysis for i_t^* shocks carries over to exogenous UIP shocks, by simply redefining $i_t^* \equiv i_t^* - \Gamma \xi_t$.

We study the effects of endogenous UIP deviations (A.15) with $\xi_t = 0$ in section 5.7.

A.3 Current account identity

Start by aggregating up household budgets in equation (1), using $\mathbb{E} e_{it} = 1$,

$$\frac{P_{Ft}}{P_t} C_{Ft} + \frac{P_{Ht}}{P_t} C_{Ht} + A_t = \left(1 + r_t^p\right) A_{t-1} + w_t N_t$$

Use the definition of the NFA (17), noting that we can always write the aggregate expost return as

$$\begin{aligned}
\left(1 + r_t^p\right) A_{t-1} &= (1 + r_{t-1}) A_{t-1} + \left(r_t^p - r_{t-1}\right) A_{t-1} \\
&= (1 + r_{t-1}) (p_{t-1} + \text{nfa}_{t-1}) + \left(r_t^p - r_{t-1}\right) A_{t-1} \\
&= D_t + p_t + (1 + r_{t-1}) \text{nfa}_{t-1} + \left(r_t^p - r_{t-1}\right) A_{t-1}
\end{aligned} \tag{A.16}$$

to obtain

$$\frac{P_{Ft}}{P_t} C_{Ft} + \frac{P_{Ht}}{P_t} C_{Ht} + p_t + \text{nfa}_t = D_t + p_t + (1 + r_{t-1}) \text{nfa}_{t-1} + \left(r_t^p - r_{t-1}\right) A_{t-1} + w_t N_t$$

Substitute in the value of dividends D_t in (12),

$$\underbrace{\frac{P_{Ft}}{P_t} C_{Ft} + \frac{P_{Ht}}{P_t} C_{Ht}}_{\equiv C_t} + \text{nfa}_t = \frac{P_{Ht}}{P_t} \underbrace{\left(C_{Ht} + C_{Ht}^*\right)}_{\equiv Y_t} + (1 + r_{t-1}) \text{nfa}_{t-1} + \left(r_t^p - r_{t-1}\right) A_{t-1}$$

[44] An alternative way to think of noise shocks is as movements in risk premia.

Hence,

$$\text{nfa}_t - \text{nfa}_{t-1} = \underbrace{\frac{P_{Ht}}{P_t} Y_t - C_t}_{NX_t} + r_{t-1}\text{nfa}_{t-1} + \left(r_t^p - r_{t-1}\right) A_{t-1} \tag{A.17}$$

which is the current account identity. Note that we have defined net exports NX_t (the trade balance) in units of the consumer price index, so that

$$NX_t \equiv \frac{P_{Ht}}{P_t} Y_t - C_t = \frac{P_{Ht}}{P_t} C_{Ht}^* - \frac{P_{Ft}}{P_t} C_{Ft} \tag{A.18}$$

A.4 Characterizing steady states

Combining the goods market clearing condition (21) with the equations for demand for domestic goods (5) and (7), and the PCP price-setting condition (8), we see that domestic output is always given by

$$Y_t = (1-\alpha) \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} C_t + \alpha \left(\frac{P_{Ht}}{\mathcal{E}_t}\right)^{-\gamma} C^* \tag{A.19}$$

Combining instead the equations for net exports (A.18) with these same equations, together with $P_{Ft} = \mathcal{E}_t$, we see that the trade balance is always given by

$$NX_t = \alpha \frac{P_{Ht}}{P_t} \left(\frac{P_{Ht}}{\mathcal{E}_t}\right)^{-\gamma} C^* - \alpha \frac{\mathcal{E}_t}{P_t} \left(\frac{\mathcal{E}_t}{P_t}\right)^{-\eta} C_t \tag{A.20}$$

We next relate all relative prices in equations (A.19) and (A.20) to the real exchange rate $Q_t = \frac{\mathcal{E}_t}{P_t}$. Manipulating the price index equation (3), we see that Q_t is related to the relative price of home goods P_{Ht}/P_t through

$$1 = \left[(1-\alpha) \left(\frac{P_{Ht}}{P_t}\right)^{1-\eta} + \alpha Q_t^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{A.21}$$

Denote by $p_H(Q)$ the mapping between $\frac{P_H}{P}$ and Q implicit in equation (A.21). We can also rewrite (A.21) to relate the real exchange rate Q_t to the relative price of home and foreign goods (the terms of trade) $S_t \equiv \mathcal{E}_t / P_{Ht}$ via

$$Q_t^{-1} = \left[(1-\alpha) \left(\left(\frac{\mathcal{E}_t}{P_{Ht}}\right)^{-1}\right)^{1-\eta} + \alpha\right]^{\frac{1}{1-\eta}} \tag{A.22}$$

We let $s(Q)$ denote the mapping between the terms of trade $\mathcal{E}/P_H$ and the real exchange rate Q implicit in this equation.

Taken together, equations (A.19)–(A.22) imply that the level of the real exchange rate Q_t and aggregate domestic spending C_t uniquely determine the level of domestic output on the one hand,

$$Y_t = (1-\alpha) \left(p_H(Q_t)\right)^{-\eta} C_t + \alpha \left(s(Q_t)\right)^{\gamma} C^* \tag{A.23}$$

and the trade balance on the other,

$$NX_t = \alpha p_H(Q_t) \left(p_H^*(Q_t)\right)^{-\gamma} C^* - \alpha (Q_t)^{1-\eta} C_t \tag{A.24}$$

Steady state. Consider a steady state of our model, with a constant level of all aggregates $\{C, C_H, C_F, Y, A, p, D, \text{nfa}\}$ and relative real prices $\{Q, P_H/P, W/P, r, i^*\}$, for given constant foreign discount factor shocks $\mathcal{B}$ and productivity Z. The real UIP condition (16) together with the foreign Euler equation (A.14) implies that the domestic real interest rate is $r = i^* = (\beta^*)^{-1} - 1$. Equations (29) and (30) imply that the long-run real wage and dividends are given, respectively, by

$$\begin{aligned} \frac{W}{P} &= \frac{1}{\mu}\frac{P_H}{P}Z = \frac{1}{\mu}p_H(Q)Z \\ D &= \left(1 - \frac{1}{\mu}\right)p_H(Q)Z \end{aligned}$$

The asset pricing equation (15) then implies that the domestic stock price is

$$p = \frac{1}{r}\left(1 - \frac{1}{\mu}\right)p_H(Q)Z$$

Households accumulate aggregate assets A so that the steady state net foreign asset position is

$$\text{nfa} = A - p$$

where $A = A^{HA}(r, p_H(Q)Z)$ is only a function of the steady state real interest rate r and real income $p_H(Q)Z$. Equivalently, their aggregate consumption C must, on the one hand, satisfy

$$C = C^{HA}(r, p_H(Q)Z) \tag{A.25}$$

and on the other hand, satisfy equation (A.17),

$$r \cdot \text{nfa} = -NX = C - p_H(Q)Y \tag{A.26}$$

Moreover, from (A.23), aggregate output must be

$$Y = (1-\alpha)(p_H(Q))^{-\eta}C + \alpha(p_H^*(Q))^{-\gamma}C^* \tag{A.27}$$

Combining (A.26) with (A.27) implies the steady state net export equation (A.24). Finally, the wage Phillips curve (18), together with the production function $Y = ZN$, implies that long-run wage (and price) inflation rate is equal to

$$\pi_w = \frac{1}{1-\beta}\kappa_w\left(\frac{v'(Y/Z)}{\frac{1}{\mu_w}\frac{1}{\mu}p_H(Q)Zu'(C)} - 1\right) \tag{A.28}$$

A *steady state* in our model is characterized by a 4-tuple (Y, Q, C, nfa) for output, the real exchange rate, consumption and the net foreign position, that simultaneously satisfies equations (A.25), (A.26) and (A.27). A *zero-inflation steady state* additionally restricts this tuple to satisfy equation (A.28) with $\pi_w = 0$. Hence, there always is a unique no-inflation steady state.

We always start from such a steady state. We pick β^* to deliver our target for the real interest rate r, and normalize $C^* = Q = 1$ so that all relative prices are 1. Then, equation (A.26) implies that $C = Y + r\text{nfa}$, while equation (A.27) implies that $Y = (1-\alpha)\,C + \alpha$, so that

$$Y = 1 + \frac{1-\alpha}{\alpha} \cdot r \cdot \text{nfa} \qquad \text{and} \qquad C = 1 + \frac{1}{\alpha} \cdot r \cdot \text{nfa} \tag{A.29}$$

We finally set $Z = 1$ and solve for the scaling parameter in labor disutility v' such that equation (A.28) holds for these values of Y and C, given our choice for μ_w, μ, and the initial steady state nfa. In our baseline calibration we set nfa $= 0$, so that these normalizations imply $Y = C = 1$.

Unique steady state with $Q = 1$. After transitory monetary policy or foreign interest rate shocks, the model always returns to a steady state. There is a one-dimensional set of such steady states, characterized by the 4-tuples (Y, Q, C, nfa) such that (A.25), (A.26) and (A.27) simultaneously hold. The unique steady state with $Q = 1$ is therefore the initial steady state, which in our baseline features $Y = C = 1$ and nfa $= 0$.

A.5 Representative-agent complete-market model

In the representative-agent complete markets model, all agents can buy and sell Arrow-Debreu securities for each aggregate state s^t and in each country. From an "ex-ante" perspective, before any realization of shocks, the domestic agents maximizes

$$\sum_{t=0}^{\infty} \sum_{s^t} \Pr(s^t) \beta^t \left\{ u\left(C_t\right) - v\left(N_t\right) \right\} \tag{A.30}$$

where C_t is the consumption basket in (2), subject to the sequence of budget constraints at each t,

$$P_t C_t + \sum_{s_{t+1}} M_{t,t+1}\left(s_{t+1}\right) B_{t+1}\left(s_{t+1}\right) + \mathcal{E}_t \sum_{s_{t+1}} M^*_{t,t+1}\left(s_{t+1}\right) B^*_{t+1}\left(s_{t+1}\right) \leq W_t N_t + D_t + B_t + \mathcal{E}_t B^*_t$$

where $B_{t+1}\left(s_{t+1}\right)$ denotes holdings of domestic Arrow securities, paying a unit of domestic currency in state s_{t+1}, $B^*_{t+1}\left(s_{t+1}\right)$ denotes holdings of foreign Arrow securities paying a unit of foreign currency in that state, and $M_{t,t+1}\left(s_{t+1}\right)$ (respectively $M^*_{t,t+1}\left(s_{t+1}\right)$) denotes the domestic-currency (respectively foreign-currency) price at time t of these securities. The first order conditions for the choices of B_{t+1} are

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = M_{t,t+1} \tag{A.31}$$

and those for the choices of B^*_{t+1} imply

$$\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = M^*_{t,t+1} \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) \tag{A.32}$$

Combining these two equations implies the standard equilibrium condition linking Arrow-Debreu state prices in different countries and exchange rates,

$$M_{t,t+1} \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t}\right) = M^*_{t,t+1}$$

In particular, since $\frac{1}{1+i_t} = \sum_{s_{t+1}} M_{t,t+1}\left(s_{t+1}\right)$ and $\frac{1}{1+i^*_t} = \sum_{s_{t+1}} M^*_{t,t+1}\left(s_{t+1}\right)$, we recover the UIP condition (13).

The foreign agent maximizes its utility function (A.12) subject to the budget constraint

$$P_t^* C_t^* + \sum_{s_{t+1}} M_{t,t+1}^* (s_{t+1}) B_{t+1}^* (s_{t+1}) + \frac{1}{\mathcal{E}_t} \sum_{s_{t+1}} M_{t,t+1} (s_{t+1}) B_{t+1} (s_{t+1}) \leq W_t^* N_t^* + D_t^* + \frac{B_t}{\mathcal{E}_t} + B_t^*$$

That agent's first order conditions for the choices of B_{t+1} are

$$\beta^* \frac{\mathcal{B}_{t+1}}{\mathcal{B}_t} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left(\frac{P_t^*}{P_{t+1}^*} \right) = M_{t,t+1} \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \tag{A.33}$$

with a similar equation for the choice of B_{t+1}^*. Combining (A.31) and (A.33) we obtain

$$\beta \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) = \beta^* \frac{\mathcal{B}_{t+1}}{\mathcal{B}_t} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left(\frac{P_t^*}{P_{t+1}^*} \right) \tag{A.34}$$

Stationarity requires that the discount factors of of both agents are equal, $\beta = \beta^*$. Since, in addition, $P_t^* = P_{t+1}^* = 1$ and $C_t^* = C_{t+1}^* = C^*$, we can rewrite equation (A.34) as

$$\frac{\mathcal{B}_t}{\mathcal{B}_{t+1}} \left(\frac{Q_{t+1}}{Q_t} \right) \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} = 1$$

This equation holds in every state s^t, hence, we obtain the Backus-Smith condition modified for discount factor shocks,

$$\frac{Q_t}{\mathcal{B}_t} C_t^{-\sigma} = \text{cst} = C_{ss}^{-\sigma}$$

where the last line follows from the fact that $\mathcal{B}_{-1} = Q_{-1} = 1$. This is equation (23).

Finally, taking the sum of (A.31) across all states, using $\frac{1}{1+i_t} = \sum_{s_{t+1}} M_{t,t+1} (s_{t+1})$ and the Fisher equation $\frac{1}{1+i_t} \frac{P_{t+1}}{P_t} = \frac{1}{1+r_t}$, we recover the Euler equation

$$C_t^{-\sigma} = \beta (1 + r_t) C_{t+1}^{-\sigma}$$

which is equation (24).

Steady states. Steady states in this model are characterized by 4-tuples (Y, Q, C, nfa) that satisfy the same equations (A.26) and (A.27) as in the representative-agent model, while equation (A.25) is replaced by the steady state version of equation (23),

$$QC^{-\sigma} = C_{ss}^{-\sigma}$$

As in section A.4, there is a unique steady state with $Q = 1$, and this is the initial steady state.

Equivalence to Gali-Monacelli. Here, we log-linearize the equilibrium conditions in the spirit of Galí and Monacelli (2005). Denote $b_t \equiv d \log \frac{\mathcal{B}_t}{\mathcal{B}_{ss}}$, $c_t \equiv d \log \frac{C_t}{C_{ss}}$, $q_t \equiv d \log \frac{Q_t}{Q_{ss}}$, $e_t \equiv d \log \frac{\mathcal{E}_t}{\mathcal{E}_{ss}}$ and so on, for log deviations of aggregate variables from their steady state. The equation for the price index (3) log-linearizes as

$$p_t = \alpha p_{Ft} + (1 - \alpha) p_{Ht}$$

so, using that $p_{Ft} = e_t$ and $q_t = e_t - p_t$, we obtain

$$p_{Ht} - p_t = -\frac{\alpha}{1-\alpha} q_t \tag{A.35}$$

in particular, the difference between PPI inflation $\pi_{Ht} = p_{Ht} - p_{Ht-1}$ and CPI inflation $\pi_t = p_t - p_{t-1}$ is given by

$$\pi_{Ht} - \pi_t = -\frac{\alpha}{1-\alpha} \Delta q_t \tag{A.36}$$

where $\Delta q_t = q_t - q_{t-1}$. Log-linearizing the demand equation (26), we have

$$y_t = (1-\alpha)\left(\eta \frac{\alpha}{1-\alpha} q_t + c_t\right) + \alpha\gamma \frac{1}{1-\alpha} q_t$$

and using (A.35), together with the definition of χ in (9), we find that output is

$$y_t = \frac{\alpha}{1-\alpha}\left(\underbrace{(1-\alpha)\eta + \gamma}_{\chi}\right) q_t + (1-\alpha) c_t \tag{A.37}$$

Log-linearizing the Euler equation (24), we obtain

$$c_t = c_{t+1} - \frac{1}{\sigma}(i_t - \pi_{t+1} - \rho) \tag{A.38}$$

where $\rho = -\log \beta$, while the log-linearized Backus-Smith condition (23) reads

$$c_t = \frac{1}{\sigma}(q_t - b_t) \tag{A.39}$$

Finally, to derive the Phillips curve, combine equations (18) and (11) to obtain that $\pi_{Ht} = \pi_{wt}$, with

$$\pi_{Ht} = \kappa_w \left(\frac{v'(N_t)}{\frac{Z}{\mu\mu_w} \frac{P_{Ht}}{P_t} u'(C_t)} - 1 \right) + \beta \pi_{Ht+1}$$

Log-linearizing around the steady state with zero inflation so that $\frac{Z}{\mu\mu_w} \frac{P_H}{P} \frac{u'(C)}{v'(N)} = 1$, this results in

$$\pi_{Ht} = \kappa_w (\varphi n_t + \sigma c_t - (p_{Ht} - p_t)) + \beta \pi_{t+1}$$

Combining this equation with production (10),

$$y_t = n_t$$

and the expression for the relative price of home goods in (A.35), we obtain

$$\pi_{Ht} = \kappa_w \left(\varphi y_t + \sigma c_t + \frac{\alpha}{1-\alpha} q_t \right) + \beta \pi_{Ht+1} \tag{A.40}$$

This is exactly equations (32) and (33) in Gali-Monacelli (since we do not consider productivity or foreign spending shocks, and the terms of trade in their notation is $s_t = e_t - p_{Ht} = q_t + p_t - p_{Ht} = q_t + \frac{\alpha}{1-\alpha} q_t = \frac{1}{1-\alpha} q_t$). In particular there was no loss of generality in considering sticky wages rather than sticky prices. The equivalence between the slopes of the Phillips curves obtains provided that we set

$$\kappa_w = \frac{(1-\beta\theta)(1-\theta)}{\theta} \tag{A.41}$$

where θ is the Calvo probability of keeping a domestic price fixed in Gali-Monacelli.

Equations (A.36), (A.37), (A.38), (A.39), and (A.40), characterize the log-linear model, delivering $\pi_t, \pi_{Ht}, y_t, c_t, q_t$ as a function of the foreign preference shock b_t for a given monetary policy rule for i_t.

Reduced-form two-equation system. To see the connection with the equations derived in Galí and Monacelli (2005), observe that we can use the Backus-Smith condition (A.39) and the market clearing condition (A.37) to solve for c_t and q_t as a function of y_t and the exogenous variable b_t. This gives us

$$\begin{pmatrix} 1 & -\frac{1}{\sigma} \\ 1-\alpha & \frac{\alpha}{1-\alpha}\chi \end{pmatrix} \begin{pmatrix} c_t \\ q_t \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sigma} b_t \\ y_t \end{pmatrix}$$

which can be inverted to read

$$\begin{pmatrix} c_t \\ q_t \end{pmatrix} = (1-\alpha)\,\sigma_\alpha \begin{pmatrix} -\frac{\alpha}{1-\alpha}\frac{\chi}{\sigma} b_t + \frac{1}{\sigma} y_t \\ \frac{1-\alpha}{\sigma} b_t + y_t \end{pmatrix}$$

where we have defined, as in in Gali-Monacelli,

$$\sigma_\alpha \equiv \frac{\sigma}{1-\alpha+\alpha\omega} \quad \text{with} \quad \omega \equiv \sigma\chi - (1-\alpha)$$

This implies that

$$\sigma c_t + \frac{\alpha}{1-\alpha} q_t = \sigma_\alpha \left(y_t - \frac{\alpha}{\sigma}\omega b_t \right) \tag{A.42}$$

Since substituting (A.36) into the Euler equation (A.38) implies

$$\sigma \Delta c_{t+1} + \frac{\alpha}{1-\alpha}\Delta q_{t+1} = (i_t - \pi_{Ht+1} - \rho)$$

we have, using (A.42), that

$$\sigma_\alpha \Delta y_{t+1} = i_t - \pi_{Ht+1} - \rho + \alpha\frac{\sigma_\alpha}{\sigma}\omega\Delta b_{t+1} \tag{A.43}$$

Next, substituting (A.42) into (A.40), the Phillips curve is

$$\pi_{Ht} = \kappa_w \left((\varphi + \sigma_\alpha) y_t - \alpha\frac{\sigma_\alpha}{\sigma}\omega b_t \right) + \beta\pi_{Ht+1}$$

Under flexible wages, output solves

$$(\varphi + \sigma_\alpha) y_t^n = \alpha\frac{\sigma_\alpha}{\sigma}\omega b_t \tag{A.44}$$

hence, the Phillips curve is

$$\pi_{Ht} = \kappa_w (\varphi + \sigma_\alpha)(y_t - y_t^n) + \beta \pi_{Ht+1} \tag{A.45}$$

which is exactly equation (36) in Galí and Monacelli (2005) augmented to include foreign discount factor shocks, which affect the natural output level y_t^n according to (A.44). Finally, equation (A.43) implies that, defining the natural real interest rate as

$$r_t^n \equiv \rho + \sigma_\alpha \Delta y_{t+1}^n - \alpha \frac{\sigma_\alpha}{\sigma} \omega \Delta b_{t+1} \tag{A.46}$$

the Euler equation reads

$$\Delta y_{t+1} = \sigma_\alpha^{-1} (i_t - \pi_{Ht+1} - r_t^n) \tag{A.47}$$

which is the counterpart of equation (37) in Galí and Monacelli (2005), where the natural PPI-based real rate of interest is affected by foreign discount factor shocks per equation (A.44).

A.6 Representative-agent incomplete-market model

In the incomplete-market representative-agent model (henceforth "RA-IM"), the domestic agent again maximizes the utility function (A.30), but now only has access to nominal bonds in both countries that cannot be indexed to aggregate shocks. As in the main text, we appeal to certainty equivalence for small shocks and solve this model from an ex-post perspective. Then, the representative agent solves

$$\sum_{t=0}^{\infty} \beta^t \{u(C_t) - v(N_t)\}$$

where C_t is the consumption basket in (2), subject to the sequence of budget constraints

$$P_t C_t + \frac{1}{1+i_t} B_{t+1} + \mathcal{E}_t \frac{1}{1+i_t^*} B_{t+1}^* \leq W_t N_t + D_t + B_t + \mathcal{E}_t B_t^*$$

with initial portfolio $B_0 = B_0^* = 0$. Optimality immediately implies the UIP condition (13). Defining real wealth as $a_t \equiv \frac{B_t + \mathcal{E}_t B_t^*}{P_t}$, we then have

$$C_t + \frac{1}{1+i_t} \frac{P_{t+1}}{P_t} a_{t+1} \leq \frac{W_t}{P_t} N_t + \frac{D_t}{P_t} + a_t$$

using the Fisher equation (14), this rewrites as

$$C_t + \frac{1}{1+r_t} a_{t+1} \leq \frac{W_t}{P_t} N_t + \frac{D_t}{P_t} + a_t$$

Iterating on this equation, which clearly holds with equality, starting from $a_0 = 0$ delivers the country's present value budget constraint,

$$\sum_{s \geq 0} q_s C_s = \sum_{s \geq 0} q_s \left(\frac{W_s}{P_s} N_s + \frac{D_s}{P_s} \right)$$

where $q_s \equiv \Pi_{u=0}^{s-1} \frac{1}{1+r_u}$. Finally, using (29) and (30), we obtain the incomplete-market present value budget constraint, stating that the present value of the country's consumption must be equal to the present value of its real income,

$$\sum_{s\geq 0} q_s C_s = \sum_{s\geq 0} q_s \frac{P_{Hs}}{P_s} Y_s \tag{A.48}$$

The first order condition for the choice of consumption delivers the standard Euler equation (24), and combining the usual equations we obtain the real UIP condition (16).

Steady states. In this model, a steady state is characterized by a 4-tuple (Y, Q, C, nfa) that simultaneously satisfies equations (A.23) and (22), together with a long-run equation for the nfa ,

$$\text{nfa} = \lim_{t\to\infty} \frac{1}{q_t} \left(\sum_{s=0}^{t} q_s \left(p_H \left(Q_s \right) Y_s - C_s \right) \right) \tag{A.49}$$

where, from (24),

$$C_s = \prod_{s\geq t} \left(\frac{1+r_s}{1/\beta} \right)^{-\frac{1}{\sigma}} C \tag{A.50}$$

and, from (16),

$$Q_s = \prod_{s\geq t} \left(\frac{1+i_s^*}{1+r_s} \right) Q \tag{A.51}$$

Hence, the nfa depends on the entire sequence of shocks, contrary what happens to the HA model. This reflects the non-stationarity of the model. There is still a unique equilibrium with $Q = 1$, but in contrast to the HA and complete market models, in general the long level of (Y, C, nfa) differs from the initial steady state.

A.7 Two-agent complete-market model

For the two-agent complete-market model (henceforth "TA model"), we assume that the household side of the model consists of a share $1-\lambda$ of agents with unfettered access to financial markets, denoted by superscript u (for "unconstrained"), and a share λ of agents without any access to financial markets, denoted by superscript c (for "constrained").

The consumption behavior of unconstrained agents is determined just like the behavior of the representative agent in section A.5. We again take an "ex-ante" perspective, before any realization of shocks. Unconstrained agents solve

$$\max \sum_{t=0}^{\infty} \sum_{s^t} \Pr(s^t) \cdot \beta^t \left\{ u\left(c_t^u\right) - v\left(N_t^u\right) \right\}$$

subject to the sequence of budget constraints

$$P_t c_t^u + \sum_{s_{t+1}} M_{t,t+1}\left(s_{t+1}\right) B_{t+1}\left(s_{t+1}\right) + \mathcal{E}_t \sum_{s_{t+1}} M_{t,t+1}^*\left(s_{t+1}\right) B_{t+1}^*\left(s_{t+1}\right) \leq W_t N_t^u + D_t + B_t + \mathcal{E}_t B_t^*$$

As in section A.5, this determines consumption of unconstrained agents according to the Backus-Smith condition (23), that is

$$\left(c_t^u\right)^{-\sigma} = \left(c_{ss}^u\right)^{-\sigma} \frac{\mathcal{B}_t}{Q_t} \tag{A.52}$$

where c_{ss}^{u} denotes steady state consumption of unconstrained agents, as well as the Euler equation,

$$\left(c_t^u\right)^{-\sigma} = \beta\left(1 + r_t\right)\left(c_{t+1}^u\right)^{-\sigma} \tag{A.53}$$

Constrained agents do not have access to financial markets. Following similar notation, they solve

$$\max \sum_{t=0}^{\infty} \beta^t \left\{u\left(c_t^c\right) - v(N_t^c)\right\}$$

subject to

$$P_t c_t^c \leq W_t N_t^c$$

This pins down their consumption bundle as

$$c_t^c = \frac{W_t}{P_t} N_t^c \tag{A.54}$$

Aggregate consumption is given by

$$C_t = (1 - \lambda)\, c_t^u + \lambda c_t^c$$

We assume the same equal rationing among the two types of agents that we assume among heterogeneous agents in our heterogeneous agent model of section 2. Thus,

$$N_t^u = N_t^c = N_t$$

This implies that steady state consumption for the two agents is given by

$$\begin{aligned} c_{ss}^u &= \frac{1}{1-\lambda} D_{ss} + \frac{W_{ss}}{P_{ss}} N_{ss} = \frac{1-\mu^{-1}}{1-\lambda} + \frac{1}{\mu} \\ c_{ss}^c &= \frac{W_{ss}}{P_{ss}} N_{ss} = \frac{1}{\mu} \end{aligned}$$

B Proofs

B.1 Proof of proposition 1

Linearizing the goods market clearing condition (26) around the steady state with $C = Y = Q = P_H = P_F = \mathcal{E} = 1$, we find

$$dY_t = (1-\alpha)\,(-\eta)\,(dP_{Ht} - dP_t) + \alpha\,(-\gamma)\,(dP_{Ht} - d\mathcal{E}_t) + (1-\alpha)\, dC_t \tag{A.55}$$

Linearizing the equation defining the CPI (3), we have $dP_t = (1-\alpha)\, dP_{Ht} + \alpha d\mathcal{E}_t$. Linearizing the equation defining the real exchange rate (6), we have $dQ_t = d\mathcal{E}_t - dP_t$. Combining, we find

$$dP_{Ht} = \frac{dP_t - \alpha d\mathcal{E}_t}{1-\alpha} = dP_t - \frac{\alpha}{1-\alpha} dQ_t \tag{A.56}$$

Substituting (A.56) into (A.55), we find

$$
\begin{aligned}
dY_t &= \alpha \eta dQ_t + \frac{\alpha}{1-\alpha}\gamma dQ_t + (1-\alpha)\, dC_t \\
&= \frac{\alpha}{1-\alpha}\left(\eta\,(1-\alpha)+\gamma\right) dQ_t + (1-\alpha)\, dC_t \\
&= \frac{\alpha}{1-\alpha}\chi dQ_t + (1-\alpha)\, dC_t
\end{aligned}
\tag{A.57}
$$

where χ is defined in (9). Finally, in the representative-agent model, we know from the Backus-Smith condition and the assumption that $Q_\infty = 1$ that $dC_t = 0$. This delivers equation (28).

B.2 Proof of propositions 2 and 3

In this section, we derive the "international Keynesian Cross" in its general form (38), which nests (32), allowing for shocks to foreign and domestic interest rates, di_t^* and dr_t.

To derive (38), we start from substituting the domestic and foreign demand equations (5) and (7) into the goods market clearing condition (21), using the price-setting equation for home goods abroad (8), and aggregating up,

$$
Y_t = (1-\alpha)\left(\frac{P_{Ht}}{P_t}\right)^{-\eta} \mathcal{C}_t\left(\left\{r_0^p, r_s, \frac{W_s}{P_s}N_s\right\}\right) + \alpha\left(\frac{P_{Ht}}{\mathcal{E}_t}\right)^{-\gamma\epsilon} C^* \tag{A.58}
$$

In (A.58), we have made explicit the fact that aggregate demand for consumption C_t depends only on the initial ex-post return r_0^p, reflecting valuation effects, the time path of real interest rates r_s for $s \geq 0$ set by monetary policy (since, by (15), $r_{t+1}^p = r_t$ for all $t \geq 1$), and the path of real wages $w_s N_s$ for $s \geq 0$. We denote this general "consumption function" by $\mathcal{C}_t$. Note that $\mathcal{C}_t$ here is a more general object compared to our usage in Section 3.2.[45]

Substituting these mappings into (A.58) and using the value of real labor income in (29), we have

$$
Y_t = (1-\alpha)\left(p_H\left(Q_t\right)\right)^{-\eta} \mathcal{C}_t\left(\left\{r_0^p, r_s, \frac{1}{\mu}p_H\left(Q_s\right)Y_s\right\}\right) + \alpha\left(p_H^*\left(Q_t\right)\right)^{-\gamma\epsilon} C^* \tag{A.59}
$$

Moreover, the valuation equation for assets, combined with (12), implies that share prices are

$$
p_t = \frac{D_{t+1}+p_{t+1}}{1+r_t} = PDV\left(\left\{r_s, \left(1-\frac{1}{\mu}\right)p_H\left(Q_s\right)Y_s\right\}\right) \tag{A.60}
$$

so that the initial revaluation r_0^p also only depends on the path of r_s, Q_s, and Y_s.

We next differentiate (A.59) around the steady state with $C = C^* = Y = 1$, and use the fact that (A.21) and (A.22) respectively imply $\frac{d(P_{Ht}/P_t)}{P_{Ht}/P_t} = -\frac{\alpha}{1-\alpha}\frac{dQ_t}{Q_t}$ and $\frac{d(P_{Ht}/\mathcal{E}_t)}{P_{Ht}/\mathcal{E}_t} = -\frac{1}{1-\alpha}\frac{dQ_t}{Q_t}$ to find

$$
d\mathbf{Y} = (1-\alpha)\left(\eta\frac{\alpha}{1-\alpha}d\mathbf{Q} + \mathbf{C}_v dr_0^p + \mathbf{C}_r d\mathbf{r} + \mathbf{C}_Y\frac{1}{\mu}\left(d\mathbf{Y} - \frac{\alpha}{1-\alpha}d\mathbf{Q}\right)\right) + \frac{\alpha}{1-\alpha}\gamma\epsilon d\mathbf{Q}
$$

where $\mathbf{C}_Y$ is the Jacobian of aggregate consumption with respect to the path of output Y_s, $\mathbf{C}_r$ is the Jacobian of aggregate consumption with respect to real interest rates r_s, $\mathbf{C}_v$ is the column vector

[45] For details on why a function $\mathcal{C}_t$ exists, see Auclert, Rognlie and Straub (2018).

representing the increased spending from revaluation r_0^p, and from (A.60) we have

$$dr_0 = \mathbf{J}_r' d\mathbf{r} + \left(1 - \frac{1}{\mu}\right) \mathbf{J}_d' \left(d\mathbf{Y} - \frac{\alpha}{1-\alpha} d\mathbf{Q}\right)$$

where $\mathbf{J}_r$, $\mathbf{J}_d$ are the Jacobians of the present discounted value with respect to the discount rate r and dividends, respectively.[46] Substituting in these valuation effects, we have

$$\begin{aligned} d\mathbf{Y} &= (1-\alpha)\left(\eta \frac{\alpha}{1-\alpha} d\mathbf{Q} + \mathbf{C}_v \mathbf{J}_r' d\mathbf{r} + \left(1 - \frac{1}{\mu}\right) \mathbf{C}_v \mathbf{J}_d' d\mathbf{Y}\right) \\ &+ (1-\alpha)\left(-\left(1 - \frac{1}{\mu}\right) \mathbf{C}_v \mathbf{J}_d' \frac{\alpha}{1-\alpha} d\mathbf{Q} + \mathbf{C}_v d\mathbf{r} + C_Y \frac{1}{\mu}\left(d\mathbf{Y} - \frac{\alpha}{1-\alpha} d\mathbf{Q}\right)\right) \\ &+ \frac{\alpha}{1-\alpha} \gamma \epsilon d\mathbf{Q} \end{aligned}$$

Hence

$$\begin{aligned} d\mathbf{Y} &= \underbrace{(1-\alpha)\left(\mathbf{C}_r + \mathbf{C}_v \mathbf{J}_r'\right) d\mathbf{r}}_{\text{intertemporal subst. + valuation}} + \alpha \underbrace{\left(\eta + \frac{\gamma\epsilon}{1-\alpha}\right) d\mathbf{Q}}_{\text{Expenditure switching}} \\ & \quad \underbrace{- \alpha \left(\frac{1}{\mu}\mathbf{C}_Y + \left(1 - \frac{1}{\mu}\right) \mathbf{C}_v \mathbf{J}_d'\right)}_{\text{real income}} d\mathbf{Q} + \underbrace{(1-\alpha)\left(\frac{1}{\mu}\mathbf{C}_Y + \left(1 - \frac{1}{\mu}\right) \mathbf{C}_v \mathbf{J}_d'\right) d\mathbf{Y}}_{\text{Multiplier effect}} \end{aligned}$$

We next define

$$\mathbf{M} \equiv \frac{1}{\mu}\mathbf{C}_Y + \left(1 - \frac{1}{\mu}\right) \mathbf{C}_v \mathbf{J}_d'$$

for the overall Jacobian of spending with respect to income $\mathbf{Y}$. In the notation of section 3.2, $\mathbf{M}$ corresponds to the Jacobian of $\mathcal{C}_t$ with respect to $\{\mathcal{Y}_s\}$. Also, define

$$\mathbf{M}^r \equiv \mathbf{C}_r + \mathbf{C}_v \mathbf{J}_r'$$

for the overall Jacobian of consumption to interest rates $\mathbf{r}$. Using the expression for the trade elasticity in (9), we have that

$$d\mathbf{Y} = \underbrace{(1-\alpha)\mathbf{M}^r d\mathbf{r}}_{\text{int. subst + valuation}} + \underbrace{\frac{\alpha}{1-\alpha}\chi d\mathbf{Q}}_{\text{D+F expenditure switching}} - \underbrace{\alpha\mathbf{M}}_{\text{real income}} d\mathbf{Q} + \underbrace{(1-\alpha)\mathbf{M}}_{\text{Multiplier effect}} d\mathbf{Y} \tag{A.61}$$

where the response of the real exchange rate is given by

$$dQ_t = \sum_{s \ge t} \frac{di_s^* - dr_s}{(1+r)^{s-t+1}} \tag{A.62}$$

Equation (A.61) is the international Keynesian cross. It is a fixed point equation for the sequence of output changes $d\mathbf{Y}$. The Jacobians $\mathbf{M}^r$ and $\mathbf{M}$ are infinite-dimensional matrices. By construction

[46] These have the simple expressions $\mathbf{J}_d' = \frac{r}{D}\left(1 \; \frac{1}{1+r} \; \left(\frac{1}{1+r}\right)^2 \cdots\right)$ and $\mathbf{J}_r' = -\left(\frac{1}{1+r} \; \left(\frac{1}{1+r}\right)^2 \cdots\right)$.

$\mathbf{M}^r d\mathbf{r}$ has finite elements. We have assumed (and numerically verified) that $\mathbf{M}$ is positive. Since the columns of $\mathbf{M}$ have a present value of 1, this implies that $\mathbf{M}d\mathbf{Y}$ are finite for any present-value summable $d\mathbf{Y}$ since

$$|\mathbf{M}d\mathbf{Y}| \leq \mathbf{M}|d\mathbf{Y}| \leq \mathbf{q}'\mathbf{M}|d\mathbf{Y}| \leq \mathbf{q}'|d\mathbf{Y}| \leq \sum_{t=0}^{\infty}(1+r)^{-t}|dY_t| < \infty$$

where we defined $\mathbf{q} \equiv \left((1+r)^{-t}\right)_{t\geq 0}$. A similar argument establishes that $\mathbf{M}d\mathbf{Q}$ has finite elements. Thus, (A.61) is well-defined.

Let

$$\mathbf{G} \equiv (\mathbf{I} - (1-\alpha)\mathbf{M})^{-1} = I + (1-\alpha)\mathbf{M} + (1-\alpha)^2\mathbf{M}^2 + \cdots \tag{A.63}$$

which is a well-defined infinite-dimensional, non-negative, bounded matrix (as $\mathbf{M}$ is non-negative and bounded). Then, the unique bounded solution to (A.61) must satisfy

$$\begin{aligned} d\mathbf{Y} &= (\mathbf{I} - (1-\alpha)\mathbf{M})^{-1} \cdot \left\{(1-\alpha)\mathbf{M}^r d\mathbf{r} + \alpha\left(\frac{\chi}{1-\alpha}\mathbf{I} - \mathbf{M}\right)d\mathbf{Q}\right\} \\ &= (1-\alpha)\mathbf{G}\mathbf{M}^r d\mathbf{r} + \alpha\frac{\chi}{1-\alpha}\mathbf{G}d\mathbf{Q} - \alpha\mathbf{G}\mathbf{M}d\mathbf{Q} \end{aligned} \tag{A.64}$$

B.3 Proof of propositions 4 and 5

Proof of proposition 4. We start with the solution to the international Keynesian cross (A.64). Without monetary policy shocks, $d\mathbf{r} = 0$, we can rewrite the solution as

$$d\mathbf{Y} = \frac{\alpha}{1-\alpha}\mathbf{G}(\chi\mathbf{I} - (1-\alpha)\mathbf{M})d\mathbf{Q}$$

which simplifies to

$$d\mathbf{Y} = \frac{\alpha}{1-\alpha}d\mathbf{Q} + \frac{\alpha}{1-\alpha}(\chi - 1)\mathbf{G}d\mathbf{Q} \tag{A.65}$$

where the first term is just the output response in the complete-market RA model when $\chi = 1$. Equation (A.65) immediately proves the neutrality result for $\chi = 1$. The ranking for $\chi \gtrless 1$ for depreciations $d\mathbf{Q} \geq 0$ follows from the non-negativity of $\mathbf{G}$ (which itself is a consequence of the non-negativity of $\mathbf{M}$, see our discussion around (A.63)). The ranking for appreciations $d\mathbf{Q} \leq 0$ is flipped.

To obtain the equation for the trade balance (34), we linearize equation (A.24), obtaining

$$dNX_t = \alpha\left(-\frac{\alpha}{1-\alpha}dQ_t + \frac{\gamma}{1-\alpha}dQ_t\right) - \alpha\left((1-\eta)dQ_t + dC_t\right)$$

Here, we used the facts that $dp_{Ht} = -\frac{\alpha}{1-\alpha}dQ_t$ and $dp^*_{Ht} = -\frac{1}{1-\alpha}dQ_t$. Simplifying, we find

$$dNX_t = \frac{\alpha}{1-\alpha}(\chi - 1)dQ_t - \alpha dC_t$$

which is (34). □

Proof of proposition 5. To derive the result on present values, denoted by $PV(\cdot)$, we first point out that the columns of $\mathbf{M}$ each have a present value of 1, so that

$$PV(\mathbf{M}d\mathbf{Q}) = PV(d\mathbf{Q})$$

A direct consequence is that

$$PV(\mathbf{G}d\mathbf{Q}) = \frac{1}{\alpha}PV(d\mathbf{Q})$$

Using this result in conjunction with (A.65), we find

$$PV(d\mathbf{Y}) = \frac{\chi - (1-\alpha)}{1-\alpha}PV(d\mathbf{Q})$$

This shows that $PV(d\mathbf{Y}) < 0$ in response to a depreciation $dQ_t \geq 0$ (with strict inequality in at least a single period).

When is $dY_0 < 0$, i.e. the output response to the depreciation is contractionary on impact? Again, building on (A.65), we find

$$dY_0 = \frac{\alpha}{1-\alpha}dQ_0 + \frac{\alpha}{1-\alpha}(\chi - 1)\sum_{t=0}^{\infty} G_{0,t}dQ_t$$

The threshold χ^* for χ below which dY_0 is negative can simply be determined to be

$$\chi^* = 1 - \frac{dQ_0}{\sum_{t=0}^{\infty} G_{0,t}dQ_t}$$

To bound this threshold, we can use (A.63) to find that

$$\sum_{t=0}^{\infty} G_{0,t}dQ_t \geq G_{0,0}dQ_0 \geq (1 - (1-\alpha)M_{0,0})^{-1} dQ_0$$

Thus,

$$\chi^* \geq (1-\alpha)M_{0,0}$$

□

B.4 Dollar currency pricing

In this section, we derive (36). Consider a one-time depreciation dQ_0. This affects real income in period s by

$$d\left(\frac{P_{Hs}}{P_s}Y_s\right) = -\frac{\alpha}{1-\alpha}1_{\{s=0\}}dQ_0 + dY_s$$

The effects on labor income is then simply

$$d\left(\frac{W_s}{P_s}N_s\right) = \frac{1}{\mu}d\left(\frac{P_{Hs}}{P_s}Y_s\right) = -\frac{\alpha}{1-\alpha}1_{\{s=0\}}\frac{1}{\mu}dQ_0 + \frac{1}{\mu}dY_s$$

while the effect on dividends (12) is given by

$$dD_s = -\frac{\alpha}{1-\alpha}1_{\{s=0\}}\left(1-\frac{1}{\mu}\right)dQ_0 + \left(1-\frac{1}{\mu}\right)dY_s + \frac{\alpha}{1-\alpha}1_{\{s=0\}}dQ_0$$

where the last term captures increased markups on exports that emerge with DCP. Following the steps in section B.2, we see that, by definition of $M_{t,s}$, the total date-0 consumption response to $\{dY_s\}$ is given by $\sum_{s\geq 0} M_{0,s} dY_s$.

To obtain the date-0 consumption response to dQ_0 (the real income channel at date 0), denote by MPC_i agent i's date-0 MPC out of a transitory date-0 transfer. Note that agent i's exposure to dQ_0 depends on its initial income state e_{i0} (its share of labor income) and initial share of wealth $\frac{a_{i0}}{A_{ss}}$, multiplied by the aggregate changes in labor income and dividends respectively. We collect these terms in an object we call *net exchange rate exposure* NXE_i,

$$\frac{\alpha}{1-\alpha} NXE_i \equiv \frac{a_{i0}}{A_{ss}} \cdot \frac{1}{\mu} \frac{\alpha}{1-\alpha} - e_{i0} \cdot \frac{1}{\mu} \frac{\alpha}{1-\alpha} = \left(\frac{a_{i0}}{A_{ss}} - e_{i0} \right) \cdot \frac{1}{\mu} \frac{\alpha}{1-\alpha}$$

The total date-0 consumption response is then

$$dC_0 = \frac{\alpha}{1-\alpha} \mathbb{E} \left[MPC_i \cdot NXE_i \right] \cdot dQ_0 + \sum_{s \geq 0} M_{0,s} dY_s$$

Observe that net exchange rate exposures average to zero, $\mathbb{E}[NXE_i] = 0$, so that $\mathbb{E} \left[MPC_i \cdot NXE_i \right] = Cov \left(MPC_i, NXE_i \right)$. Substituting dC_0 into the linearized goods market clearing condition (A.57), we find (36).

B.5 Proof of proposition 6

Proof of the non-linear neutrality result for $\eta = \gamma = 1$. We start by proving the neutrality result non-linearly. To do so, we state an important property that comes out of the closed economy result in Werning (2015) based on its proof in Appendix A of Auclert, Rognlie and Straub (2020). The consumption function that we introduce in Section 4, $C_t = \mathcal{C}_t \left(r_t, \mathcal{Y}_t \right)$, where we abbreviate real income by $\mathcal{Y}_t \equiv \frac{P_{Ht}}{P_t} Y_t$, has the following property,

$$\mathcal{C}_t \left(\{ r_s, \mathcal{Y}_s \} \right) = \mathcal{Y}_t \cdot \mathcal{C}_t \left(\left\{ (1 + r_s) \cdot \frac{\mathcal{Y}_s}{\mathcal{Y}_{s+1}} - 1, 1 \right\} \right) \tag{A.66}$$

In particular, this implies that if an aggregate Euler equation relationship between $\mathcal{Y}_t$ and r_t holds, that is, $\frac{1}{\mathcal{Y}_s} = \frac{1+r_s}{1+r_{ss}} \cdot \frac{1}{\mathcal{Y}_{s+1}}$, this simplifies to

$$\mathcal{C}_t \left(\{ r_s, \mathcal{Y}_s \} \right) = \mathcal{Y}_t \cdot C_{ss} \tag{A.67}$$

where $C_{ss} = \mathcal{C}_t \left(r_{ss}, 1 \right)$ denotes steady state consumption (normalized to 1 in our model).

With this in mind, we now prove the non-linear equivalence result with $\eta = \gamma = 1$, that is, $Y_t = Y_t^{RA}$ in response to an arbitrary monetary policy shock $\{ r_t \}$. We begin by deriving C_t^{RA}, Y_t^{RA} and the path of aggregate real income $\mathcal{Y}_t^{RA} \equiv \frac{P_{Ht}}{P_t} Y_t^{RA}$ in the RA model.

Since we are considering the Cole-Obstfeld case $\eta = 1$, the CPI (3) is

$$P_t = P_{Ht}^{1-\alpha} \mathcal{E}_t^{\alpha}$$

and the real exchange rate (6) is

$$Q_t = \frac{\mathcal{E}_t}{P_t} = \frac{\mathcal{E}_t}{P_{Ht}^{1-\alpha} \mathcal{E}_t^{\alpha}} = \left(\frac{\mathcal{E}_t}{P_{Ht}} \right)^{1-\alpha}$$

In particular, the relative price of home goods in units of the CPI is

$$\frac{P_{Ht}}{P_t} = \frac{P_{Ht}}{P_{Ht}^{1-\alpha}\mathcal{E}_t^{\alpha}} = \left(\frac{P_{Ht}}{\mathcal{E}_t}\right)^{\alpha} = (Q_t)^{-\frac{\alpha}{1-\alpha}}$$

Home output is therefore given by

$$Y_t^{RA} = (1-\alpha)\, Q_t^{\frac{\alpha}{1-\alpha}} C_t^{RA} + \alpha Q_t^{\frac{1}{1-\alpha}} \tag{A.68}$$

Observe that if the Backus-Smith condition $C_t^{RA} = Q_t$ holds, then (A.68) implies that aggregate real income is simply given by

$$\mathcal{Y}_t^{RA} = \frac{P_{Ht}}{P_t} Y_t^{RA} = Q_t^{-\frac{\alpha}{1-\alpha}} Y_t^{RA} = C_t^{RA} \tag{A.69}$$

Moreover, aggregate consumption satisfies the Euler equation (24),

$$\frac{1}{C_t^{RA}} = \frac{1+r_t}{1+r_{ss}} \frac{1}{C_{t+1}^{RA}} \tag{A.70}$$

Combining (A.69) and (A.70), we see that $\mathcal{Y}_t^{RA}$ satisfies the same Euler equation,

$$\frac{1}{\mathcal{Y}_t^{RA}} = \frac{1+r_t}{1+r_{ss}} \frac{1}{\mathcal{Y}_{t+1}^{RA}} \tag{A.71}$$

To verify that $Y_t^{RA}, \mathcal{Y}_t^{RA}, C_t^{RA}$ are identical in the HA model, we need to show that

$$C_t = \mathcal{C}_t\left(\left\{r_s, \mathcal{Y}_s^{RA}\right\}\right) = C_t^{RA} \tag{A.72}$$

But (A.72) follows directly from property (A.66) of the consumption function, which simplifies to (A.67) here given (A.71). Since the other aggregate equations of the model are the same, the result holds under any monetary policy rule and applies to all aggregate prices and quantities. This concludes our proof. □

Proof to first-order for general η, γ such that $(1-\alpha)\eta + \gamma = 2 - \alpha$. We proceed in two steps. First we prove a helpful lemma.

Lemma 1. *For our heterogeneous-agent model with $\sigma = 1$, we have that*

$$\mathbf{M}^r = -(\mathbf{I} - \mathbf{M})\,\mathbf{U} \tag{A.73}$$

Proof to lemma 1. This result is the differential version of (A.66). To see this, construct, for any given path $\{r_t\}$ a path of real income $\{\mathcal{Y}_t\}$ defined recursively by

$$\mathcal{Y}_t = \frac{1+r_{ss}}{1+r_t}\mathcal{Y}_{t+1}$$

To first order, this equation implies that

$$d\mathcal{Y} = -\mathbf{U} d\mathbf{r} \tag{A.74}$$

where

$$\mathbf{U} \equiv \begin{pmatrix} 1 & 1 & 1 & \cdots \\ 0 & 1 & 1 & \cdots \\ 0 & 0 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \tag{A.75}$$

Now we linearize (A.66). We find

$$d\mathbf{C} = \mathbf{M}^r d\mathbf{r} + \mathbf{M} d\mathcal{Y} = d\mathcal{Y}$$

Substituting $d\mathcal{Y}$ with (A.74), this can be restated as

$$\mathbf{M}^r d\mathbf{r} - \mathbf{M}\mathbf{U} d\mathbf{r} = -\mathbf{U} d\mathbf{r}$$

As this holds for an arbitrary path $d\mathbf{r}$, we find

$$\mathbf{M}^r - \mathbf{M}\mathbf{U} = -\mathbf{U}$$

which is equivalent to (A.73). □

We use lemma 1 to restate the generalized Keynesian cross (38), now with arbitrary η, γ as

$$d\mathbf{Y} = \left(\frac{\alpha}{1-\alpha}\chi + 1 - \alpha \right) d\mathbf{Q} - \mathbf{M} d\mathbf{Q} + (1-\alpha)\, \mathbf{M} d\mathbf{Y} \tag{A.76}$$

where $d\mathbf{Q}$ continues to be given by (A.62) as the implied real exchange rate response to the monetary policy shock. Solving (A.76) as in section B.2, we find

$$d\mathbf{Y} = d\mathbf{Y}^{RA} + \alpha \mathbf{G} \left(\chi - (2-\alpha) \right) d\mathbf{Q}$$

Following the same steps as in section B.3, this allows us to sign the magnitude of the output response relative to the RA solution. For example, in response to monetary easing, inducing an exchange rate depreciation $d\mathbf{Q} \geq 0$, a value $\chi < 2 - \alpha$ results in an output response $d\mathbf{Y}$ that lies below the RA model's output response. □

B.6 Proof of proposition 7

Proof. If the economy starts with an NFA $dNFA$, the consolidated household budget constraint implies that

$$PV(d\mathbf{C}) = -dNFA + PV(d\mathbf{Y}) \tag{A.77}$$

Moreover, from section B.2, we know that when $d\mathbf{Q} = 0$ in the international Keynesian cross, we simply obtain

$$d\mathbf{Y} = (1-\alpha)\, d\mathbf{C} \tag{A.78}$$

Combining (A.77) and (A.78) yields

$$\begin{aligned} PV(d\mathbf{C}) &= -\frac{1}{\alpha} dNFA \\ PV(d\mathbf{Y}) &= -\frac{1-\alpha}{\alpha} dNFA \end{aligned}$$

which is the desired result. □

Proof. We also prove the statement here, that the present value of the output response to monetary policy is negative whenever $\chi < 1 - \alpha$. To do so, consider equation (A.76) and take present values on both sides. Since $\mathbf{M}$ preserves present values, we find

$$PV(d\mathbf{Y}) = \left(\frac{\alpha}{1-\alpha}\chi + 1 - \alpha - 1\right) PV(d\mathbf{Q}) + (1-\alpha)\, PV(d\mathbf{Y}) = \frac{\chi - (1-\alpha)}{1-\alpha} PV(d\mathbf{Q})$$

This shows that the present value is negative, due to "stealing demand from the future", whenever $\chi < 1 - \alpha$. □

B.7 Representative-agent incomplete-market (RA-IM) model

The RA-IM model is a special case of the derivations in appendices B.2–B.5, with specific $\mathbf{M}$ and $\mathbf{M}^r$. To derive the two matrices in this case, note first that a representative agent spends any marginal increase in the present value of its income stream by increasing consumption equally in all periods. That is, the Jacobians of consumption with respect to labor income $\mathbf{C}_Y$ and ex-post returns $\mathbf{C}_v$ are given by

$$\mathbf{C}_Y = \begin{pmatrix} 1-\beta & (1-\beta)\,\beta & (1-\beta)\,\beta^2 & \cdots \\ 1-\beta & (1-\beta)\,\beta & (1-\beta)\,\beta^2 & \cdots \\ 1-\beta & (1-\beta)\,\beta & (1-\beta)\,\beta^2 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad \mathbf{C}_v = A \begin{pmatrix} 1-\beta \\ 1-\beta \\ 1-\beta \\ \vdots \end{pmatrix}.$$

Since total steady state wealth is given by $A = \frac{D}{r}$ and $\mathbf{J}_d' = \frac{1}{A}\left(1\;\beta\;\beta^2\;\cdots\right)$, we have that

$$\mathbf{C}_v \mathbf{J}_d' = \mathbf{C}_Y$$

and therefore that

$$\mathbf{M} = \mathbf{C}_Y = (1-\beta) \cdot \mathbf{1}\mathbf{q}'$$

where we use $\mathbf{q} = \left((1+r)^{-t}\right)_{t\geq 0} = \left(\beta^t\right)_{t\geq 0}$ as in appendix B.2. Observe that $\mathbf{q}'\mathbf{1} = (1-\beta)^{-1}$, which is why $\mathbf{M}$ is idempotent in the RA-IM model, that is, $\mathbf{M}^2 = \mathbf{M}$. From (A.63), we can then explicitly compute

$$\mathbf{G} = \mathbf{I} + \frac{1-\alpha}{\alpha}\mathbf{M}$$

To derive the matrix $\mathbf{M}^r$, observe that by the Euler equation (24), the consumption response to small interest rate changes dr_t is given by

$$dC_t = dC_\infty - \frac{1}{\sigma}\sum_{s\geq t}\frac{dr_s}{1+r}$$

where, due to a zero NFA, the consumption response at infinity dC_∞ is pinned down by the requirement that the present value of dC_t adds to 0, and so

$$dC_\infty = \sum_{s=0}^{\infty}(1-\beta)\,\beta^s \frac{1}{\sigma}\sum_{s\geq t}\frac{dr_s}{1+r}$$

In matrix notation, we can then write

$$dC = \underbrace{-\frac{1}{\sigma}(\mathbf{I}-\mathbf{M})\,\mathbf{U}}_{\equiv \mathbf{M}^r} d\mathbf{r}$$

where $\mathbf{U}$ is given by (A.75) as before. When $\sigma = 1$, this is a special case of (A.73).

The solution (A.64) to the international Keynesian cross is then given by

$$d\mathbf{Y} = -(1-\alpha)\frac{1}{\sigma}(\mathbf{I}-\mathbf{M})\,\mathbf{U}d\mathbf{r} + \frac{\alpha}{1-\alpha}\chi d\mathbf{Q} + (\chi-1)\,\mathbf{M}d\mathbf{Q} \tag{A.79}$$

Exchange rate shocks. Directly from (A.79), we obtain the following two results without monetary policy shocks, $dr_t = 0$. First, when $\chi = 1$, the output response reduces to the RA solution (28). This is an incarnation of our general neutrality result in proposition 4. When $\chi \neq 1$, the RA-IM model has a non-trivial consumption response

$$(1-\alpha)\,d\mathbf{C} = (\chi-1)\,\mathbf{M}d\mathbf{Q} = (1-\beta)(\chi-1)\,\mathbf{1}\mathbf{q}'d\mathbf{Q}$$

which accounts for the difference to our benchmark RA-CM model.

Second, when dQ_t is decreasing towards zero, with AR(1) persistence $\rho \in (0,1)$, the entire output response dY_t is negative whenever

$$\chi < (1-\alpha)\frac{1}{1+\alpha\beta\frac{1-\rho}{1-\beta}} \equiv \chi^*$$

This is a special case of proposition 5. The threshold χ^* lies below $1-\alpha$, potentially by a lot given that standard quarterly calibrations for ρ are in the neighborhood of 0.85, while calibrations for β are in the neighborhood of 0.99. For our baseline calibration,

$$\chi^* \approx 0.14 \cdot (1-\alpha) = 0.057$$

This implies that contractionary depreciations are much less likely to be obtained in a RA-IM model than in a HA model.

Monetary policy shocks. When there are only monetary policy shocks, i.e. $dQ_t = -\sum_{s\geq t}\frac{dr_s}{1+r}$, the solution simplifies to

$$d\mathbf{Y} = \frac{1}{1-\alpha}d\mathbf{Q} + \frac{1}{1-\alpha}\left(\alpha\left(\chi-(2-\alpha)\right)+(1-\alpha)^2\left(\frac{1}{\sigma}-1\right)\right)d\mathbf{Q} + \left(\chi-1-(1-\alpha)\frac{1}{\sigma}\right)\mathbf{M}d\mathbf{Q}$$

which immediately shows that, if $\sigma = 1$ and $\chi = 2-\alpha$, the solution collapses to the RA-CM model's solution, in line with our general result in proposition 6. More generally, neutrality is achieved if $\chi = 1 + \frac{1-\alpha}{\sigma}$.

B.8 Two agent complete-markets (TA) model

Here we analytically derive the solution to the two-agent complete-market model, and show how it fits into the general framework of the paper with a particular diagonal $\mathbf{M}$ matrix.

We start from equation (A.57), which holds in any model. To obtain dC_t, observe that from (A.53), we have

$$dc_t^u = -\frac{1}{\sigma} c_{ss}^u \sum_{s \geq t} \frac{dr_s}{1+r} \tag{A.80}$$

From (A.54), we find that

$$dc_t^c = d\left(\frac{W_t}{P_t} N_t\right) = \frac{1}{\mu} d\left(\frac{P_{Ht}}{P_t} Y_t\right) = \frac{1}{\mu}\left(-\frac{\alpha}{1-\alpha} dQ_t + dY_t\right) \tag{A.81}$$

where we have used (A.56). This implies

$$dC_t = -\left(1 - \frac{\lambda}{\mu}\right) \frac{1}{\sigma} \sum_{s \geq t} \frac{dr_s}{1+r} + \frac{\lambda}{\mu}\left(-\frac{\alpha}{1-\alpha} dQ_t + dY_t\right)$$

Substituting dC_t into (A.57), we arrive at

$$dY_t = -(1-\alpha)\left(1 - \frac{\lambda}{\mu}\right) \frac{1}{\sigma} \sum_{s \geq t} \frac{dr_s}{1+r} + \frac{\alpha}{1-\alpha} \chi dQ_t - \alpha \frac{\lambda}{\mu} dQ_t + (1-\alpha) \frac{\lambda}{\mu} dY_t \tag{A.82}$$

Solving (A.82) for dY_t, we find that

$$dY_t = -\frac{(1-\alpha)\left(1 - \frac{\lambda}{\mu}\right)}{1 - (1-\alpha)\frac{\lambda}{\mu}} \frac{1}{\sigma} \sum_{s \geq t} \frac{dr_s}{1+r} + \frac{\alpha}{1-\alpha} \frac{\chi - (1-\alpha)\frac{\lambda}{\mu}}{1 - (1-\alpha)\frac{\lambda}{\mu}} dQ_t \tag{A.83}$$

Here, the factor $1/\left(1 - (1-\alpha)\frac{\lambda}{\mu}\right)$ is the Keynesian multiplier that hand-to-mouth agents induce, see Bilbiie (2020) and Auclert, Rognlie and Straub (2018) for similar multipliers in the context of fiscal policy. We now use equation (A.83) to derive analytically the response of the two agent model to exchange rate shocks and monetary policy shocks.

Exchange rate shocks. For exchange rate shocks, $dr_t = 0$. Equation ((A.82)) is then a special case of equation (32), with $\mathbf{M} = \frac{\lambda}{\mu}\mathbf{I}$. Hence, proposition (2) holds with that $\mathbf{M}$ matrix. Comparing (33) with (A.83), we see that Proposition (3) also holds (here, $\mathbf{M} > 0$ iff $\lambda > 0$).

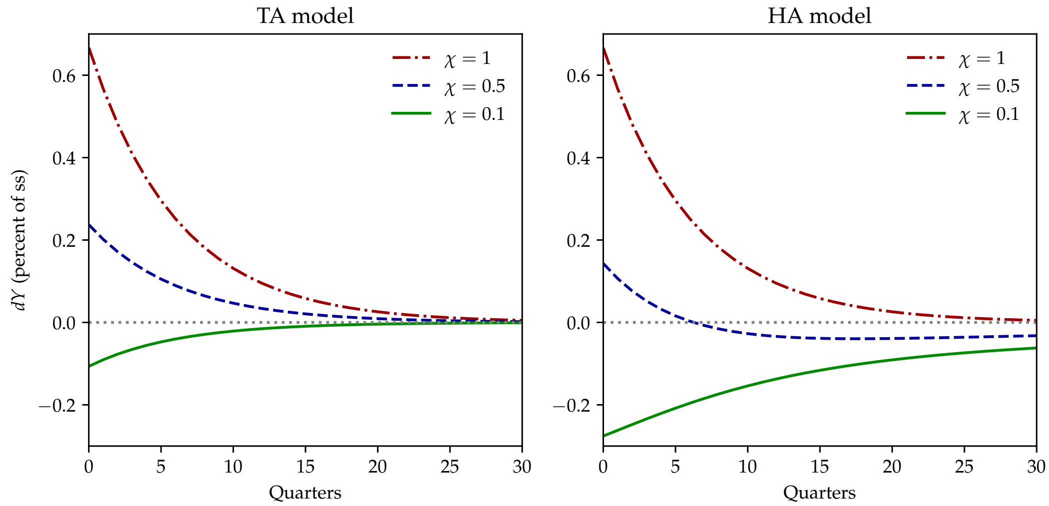
Moreover, observe from (A.83) that, when $\chi = 1$, the value of λ/μ is irrelevant: the fraction $\frac{\chi-(1-\alpha)\frac{\lambda}{\mu}}{1-(1-\alpha)\frac{\lambda}{\mu}}$ collapses to 1, and we obtain the RA solution $dY_t = \frac{\alpha}{1-\alpha} dQ_t$ for any λ/μ. The cross-partial with respect to χ and λ/μ is positive, implying that for $\chi > 1$, the TA model's output response lies above the RA model's, while the opposite is true for $\chi < 1$. Hence, Proposition 4 holds.

Finally, in response to a depreciation $dQ_t \geq 0$, the entire output response dY_t is negative for values of χ below $(1-\alpha)\frac{\lambda}{\mu}$. This shows that Proposition 5 holds, with $\chi^* = (1-\alpha)\frac{\lambda}{\mu}$. [47]

We illustrate both of these results in figure A.1. We choose $\lambda/\mu = 0.384$ to match the quarterly MPC of our baseline model. The red dash-dotted line with $\chi = 1$ illustrates the neutrality result: the output responses are identical across the TA and HA models. For lower values of χ, the TA model also generates a contraction, albeit a significantly more modest one than the HA model.

[47] Note that the result in proposition 5 on the present value turning negative for $\chi < 1 - \alpha$ does not carry over to the TA model, due to the presence of complete markets between unconstrained agents and the rest of the world.

Figure A.1: Output effect of exchange rate shocks by χ



Note: impulse response in the TA and HA models to the shock to i_t^* displayed in Figure 1. The share of constrained agents in the TA model λ is calibrated to match the quarterly MPC of the HA model.

The intuition for this is that $\mathbf{M} = \frac{\lambda}{\mu}\mathbf{I}$ and hence misses all the off-diagonal entries that populate the intertemporal MPC matrix of the HA model.

Monetary policy shocks. With $\sigma = 1$, but allowing for monetary policy shocks, equation (A.82) is a special case of (38), with $\mathbf{M} = \frac{\lambda}{\mu}\mathbf{I}$ and

$$\mathbf{M}^r = -(\mathbf{I} - \mathbf{M})\,\mathbf{U} = -\left(1 - \frac{\lambda}{\mu}\right)\mathbf{U} \tag{A.84}$$

where $\mathbf{U}$ is defined in (A.75) the triangular matrix with 1's above and on the diagonal, and zeros elsewhere. Observe that (A.84) is a special case of our general result in (A.73).

When there are only monetary policy shocks, i.e. $dQ_t = -\sum_{s\geq t}\frac{dr_s}{1+r}$, the solution simplifies to

$$dY_t = -\frac{1}{1-\alpha}\left(1 + \frac{\alpha\left(\chi - (2-\alpha)\right) + (1-\alpha)^2\left(\frac{1}{\sigma} - 1\right)\left(1 - \frac{\lambda}{\mu}\right)}{1 - (1-\alpha)\frac{\lambda}{\mu}}\right)\sum_{s\geq t}\frac{dr_s}{1+r} \tag{A.85}$$

When $\sigma = 1$ and $\chi = 2 - \alpha$, the big fraction collapses to 0, and we obtain the RA model's output response, irrespective of λ/μ. This is a special case of the neutrality result in proposition 6. The cross-partial with respect to χ and λ/μ is positive again (for accommodative monetary policy shocks), so that for $\chi > 2 - \alpha$, the TA model's output response lies above the RA model's, in line with our general result in proposition 6.

We can also say what happens for $\sigma \neq 1$ in (A.85). When $\chi = 2 - \alpha$, and $\sigma > 1$, the neutrality result fails in that the output response rises in the income share of constrained agents λ/μ. The opposite holds for $\sigma < 1$. One can also derive a σ-dependent neutrality threshold, for which dY_t is independent of λ/μ, namely

$$\chi = 1 + (1-\alpha)\frac{1}{\sigma}$$

In the special case of $\sigma = 1$, this collapses to $\chi = 2 - \alpha$.

B.9 Taylor rules

In this section, we explore how the HA and RA models respond to shocks when the central bank follows a Taylor rule, as in (20), rather than the real interest rate rule (19). We first revisit the neutrality results for exchange rate depreciations and monetary policy, and then consider the response to productivity shocks as well.

Exchange rate depreciations. A depreciation $dQ_t \geq 0, di_t^* \geq 0$, affects the demand for home goods and thus PPI inflation $\pi_{Ht} = \pi_{wt}$ through the Phillips curve (18). CPI inflation is then determined by

$$\pi_t = \pi_{Ht} + \frac{\alpha}{1-\alpha}\left(dQ_t - dQ_{t-1}\right) \tag{A.86}$$

and the real interest rate path by

$$dr_t = di_t - \pi_{t+1} = \phi\pi_{Ht} - \pi_{Ht+1} - \frac{\alpha}{1-\alpha}\left(dQ_{t+1} - dQ_t\right) \tag{A.87}$$

As the Phillips curve involves endogenous variables, that themselves depend on dr_t, this situation is significantly less tractable than the case with a fixed real interest rate. Still, we can make progress by focusing on AR(1) shocks to i_t^*, with some fixed persistence $\rho \in (0,1)$. We show below that, in this case, there still exists a threshold value for χ,

$$\chi = 1 - \alpha\frac{1+\zeta\varphi}{1+\zeta\left(\varphi+\sigma\right)} \in \left(1-\alpha, 1\right), \qquad \zeta \equiv \frac{\kappa_w\left(\phi-\rho\right)}{\left(1-\rho\right)\left(1-\beta\rho\right)} > 0$$

for which the responses of all aggregate variables, such as output, employment, and consumption are independent of $\mathbf{M}$, and hence the same for the RA and the HA model.[48]

Monetary policy shocks. The path of real rates is now no longer just a function of the path of monetary policy shocks ϵ_t; it also depends on the response of inflation, and thereby also on the endogenous response of aggregate variables. However, the international Keynesian cross (38) still holds, and thus, for $\chi = 2 - \alpha$, the response of aggregates to the shock is still independent of $\mathbf{M}$ *conditional* on a given real interest rate path. This is why our neutrality result for monetary policy goes through unchanged for the Taylor rule (20).

Productivity shocks. The exact same logic applies to the case of productivity shocks Z_t. Those shocks affect home prices according to $P_{Ht} = \mu W_t/Z_t$. This causes shifts in wage inflation via the Phillips curve (18) and thus shifts in real interest rates via (A.87). Since the responses in RA and HA models to changes in real interest rates are still identical when $\chi = 2 - \alpha$, the responses to productivity shocks in both models are also identical for this choice of χ.

Proof of the result on exchange rate depreciations with a Taylor rule. We focus on AR(1) shocks here, such that $dQ_t = \rho dQ_{t-1}$ for some $\rho \in (0,1)$. The output response with a Taylor rule is determined by the following system of equations. The first is the international Keynesian cross (38),

$$d\mathbf{Y} = -\left(1-\alpha\right)\left(\mathbf{I}-\mathbf{M}\right)\mathbf{U}d\mathbf{r} + \alpha\frac{\chi}{1-\alpha}d\mathbf{Q} - \alpha\mathbf{M}d\mathbf{Q} + \left(1-\alpha\right)\mathbf{M}d\mathbf{Y} \tag{A.88}$$

[48] The response is also the same in the incomplete markets RA and the TA model.

simplified using (A.73). The second is the Phillips curve for wage inflation (18), in linearized terms given by

$$\pi_{wt} = \kappa_w \left(\varphi dN_t + \sigma dC_t\right) + \beta \pi_{wt+1} \tag{A.89}$$

The third is the determination of the real interest rate through the Taylor rule (A.87),

$$dr_t = di_t - \pi_{t+1} = \phi \pi_{wt} - \pi_{wt+1} - \frac{\alpha}{1-\alpha}\left(dQ_{t+1} - dQ_t\right) \tag{A.90}$$

and the forth is the determination of the real exchange rate as

$$dQ_t = \sum_{s \geq t} \frac{di_s^* - dr_s}{1+r} \tag{A.91}$$

We guess and verify that all variables are exponentially decaying with the same persistence ρ. In this case,

$$d\mathbf{r} = k \cdot d\mathbf{i}^*$$

with an unknown $k = \frac{dr_0}{di_0^*}$. From (A.91), we then get that

$$d\mathbf{Q} = \frac{1}{1-\rho}\left(d\mathbf{i}^* - d\mathbf{r}\right) = \frac{k^{-1}-1}{1-\rho} d\mathbf{r} \tag{A.92}$$

So we can rewrite (A.88) as

$$d\mathbf{Y} = \left(\alpha \frac{\chi}{1-\alpha} - (1-\alpha)\frac{k}{1-k}\right) d\mathbf{Q} - \mathbf{M}\left(\alpha - (1-\alpha)\frac{k}{1-k}\right) d\mathbf{Q} + (1-\alpha)\mathbf{M} d\mathbf{Y} \tag{A.93}$$

Rearranging,

$$d\mathbf{Y} - \left(\alpha \frac{\chi}{1-\alpha} - (1-\alpha)\frac{k}{1-k}\right) d\mathbf{Q} = (1-\alpha)\mathbf{M}\left(d\mathbf{Y} - \frac{\alpha - (1-\alpha)\frac{k}{1-k}}{1-\alpha} d\mathbf{Q}\right)$$

we see that the solution is independent of $\mathbf{M}$ precisely if and only if

$$\alpha \frac{\chi}{1-\alpha} - (1-\alpha)\frac{k}{1-k} = \frac{\alpha - (1-\alpha)\frac{k}{1-k}}{1-\alpha}$$

which is equivalent to

$$\chi = 1 - (1-\alpha)\frac{k}{1-k}$$

For $k = 0$, we recover neutrality at $\chi = 1$. For $k \to \infty$, the monetary response dominates the output response, so the threshold converges to $\chi \to 2 - \alpha$. In case the neutrality result holds, the output response is given by

$$d\mathbf{Y} = \left(\frac{\alpha}{1-\alpha} - \frac{k}{1-k}\right) d\mathbf{Q}$$

Given the linear production function, this is equivalent to the employment response, $d\mathbf{N} = d\mathbf{Y}$. The aggregate consumption response is

$$d\mathbf{C} = -\frac{k}{1-k} d\mathbf{Q}$$

Substituting these formulas into the linearized wage Phillips curve (A.89), we find that

$$\pi_{wt} = \frac{1}{1-\beta\rho}\kappa_w \left(\varphi dN_t + \sigma dC_t\right) = \frac{1}{1-\beta\rho}\kappa_w \left(\varphi\left(\frac{\alpha}{1-\alpha} - \frac{k}{1-k}\right) - \sigma\frac{k}{1-k}\right) dQ_t$$

and therefore that

$$dr_t = \left\{\frac{\phi-\rho}{1-\beta\rho}\kappa_w \left(\varphi\left(\frac{\alpha}{1-\alpha} - \frac{k}{1-k}\right) - \sigma\frac{k}{1-k}\right) + \frac{\alpha}{1-\alpha}(1-\rho)\right\} dQ_t$$

Comparing this with (A.92), we find an equation for $k/(1-k)$,

$$(1-\rho)\frac{k}{1-k} = \frac{\phi-\rho}{1-\beta\rho}\kappa_w \left(\varphi\left(\frac{\alpha}{1-\alpha} - \frac{k}{1-k}\right) - \sigma\frac{k}{1-k}\right) + \frac{\alpha}{1-\alpha}(1-\rho)$$

We solve this equation for $k/(1-k)$

$$\left(1-\rho + \frac{\phi-\rho}{1-\beta\rho}\kappa_w(\varphi+\sigma)\right)\frac{k}{1-k} = \frac{\phi-\rho}{1-\beta\rho}\kappa_w\varphi\frac{\alpha}{1-\alpha} + \frac{\alpha}{1-\alpha}(1-\rho)$$

$$\left(1-\rho + \frac{\phi-\rho}{1-\beta\rho}\kappa_w(\varphi+\sigma)\right)\frac{k}{1-k} = \frac{\alpha}{1-\alpha}\left(1-\rho + \frac{\phi-\rho}{1-\beta\rho}\kappa_w\varphi\right)$$

$$\frac{k}{1-k} = \frac{\alpha}{1-\alpha}\cdot\frac{1+\frac{\kappa_w(\phi-\rho)}{(1-\rho)(1-\beta\rho)}\varphi}{1+\frac{\kappa_w(\phi-\rho)}{(1-\rho)(1-\beta\rho)}(\varphi+\sigma)}$$

Note that the solution lies in $k \in (0,\alpha)$. The neutrality threshold is then given by

$$\chi = 1-\alpha\frac{1+\frac{\kappa_w(\phi-\rho)}{(1-\rho)(1-\beta\rho)}\varphi}{1+\frac{\kappa_w(\phi-\rho)}{(1-\rho)(1-\beta\rho)}(\varphi+\sigma)} \in (1-\alpha, 1)$$

C Quantitative model details

C.1 Non-homothetic demand

The Bellman equation (1) that incorporates the preferences in (39) reads

$$\begin{aligned} V_t(a,e) \quad &= \quad \max_{c_F,c_H,a'} u\left(\left[\alpha^{1/\eta}\left(c_F-\underline{c}\right)^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta}c_H^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}\right) \\ &\qquad -v(N_t) + \beta\mathbb{E}_t\left[V_{t+1}\left(a',e'\right)\right] \\ \text{s.t.} \quad &\quad \frac{P_{Ft}}{P_t}c_F + \frac{P_{Ht}}{P_t}c_H + a' = \left(1+r_t^p\right)a + e\frac{W_t}{P_t}N_t \\ &\quad a' \geq \underline{a} \end{aligned}$$

Notice that, by relabeling $\widetilde{c_F} \equiv c_F - \underline{c}$, this is equivalent to

$$\begin{aligned} V_t(a,e) &= \max_{\widetilde{c_F}, c_H, a'} u\left(\left[\alpha^{1/\eta}\left(\widetilde{c_F}\right)^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta} c_H^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}\right) \\ &\quad -v(N_t) + \beta \mathbb{E}_t\left[V_{t+1}(a',e')\right] \\ \text{s.t.} &\quad \frac{P_{Ft}}{P_t}\widetilde{c_F} + \frac{P_{Ht}}{P_t}c_H + a' = \left(1+r_t^p\right)a + e\frac{W_t}{P_t}N_t - \frac{P_{Ft}}{P_t}\underline{c} \\ &\quad a' \geq \underline{a} \end{aligned}$$

or, defining $\widetilde{c} = \left[\alpha^{1/\eta}\left(\widetilde{c_F}\right)^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta} c_H^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}$, as simply

$$\begin{aligned} V_t(a,e) &= \max_{\widetilde{c}, a'} u(\widetilde{c}) - v(N_t) + \beta \mathbb{E}_t\left[V_{t+1}(a',e')\right] \\ \text{s.t.} &\quad \widetilde{c} + a' = \left(1+r_t^p\right)a + e\frac{W_t}{P_t}N_t - \frac{P_{Ft}}{P_t}\underline{c} \\ &\quad a' \geq \underline{a} \end{aligned}$$

where P_t is the standard CES price index (3). Hence, this is the standard consumption-saving problem, only with a modified income process that subtracts $-\frac{P_{Ft}}{P_t}\underline{c}$ to real income in every state of the world. The policy functions $\widetilde{c}_t(a,e)$, $a_{t+1}(a,e)$ for this problem, as well as the aggregates $\widetilde{C_F}$ and A_{t+1} of policies integrated against the time varying distribution, can be obtained from the sequence of inputs $\left\{r_t^p, \frac{W_t}{P_t}N_t, \frac{P_{Ft}}{P_t}\underline{c}\right\}$ using standard tools. Since every agent's policy for $c_{Ft}(a,e) = \underline{c} + \widetilde{c_{Ft}}(a,e)$, it follows that aggregate spending on foreign goods is simply

$$\begin{aligned} C_{Ft} &= \underline{c} + \widetilde{C_{Ft}} \\ &= \underline{c} + \alpha\left(\frac{P_{Ft}}{P_t}\right)^{-\eta}\widetilde{C}_t \end{aligned} \tag{A.94}$$

while aggregate spending on domestic goods is

$$C_{Ht} = (1-\alpha)\left(\frac{P_{Ht}}{P_t}\right)^{-\eta}\widetilde{C}_t \tag{A.95}$$

Note that equations (A.94) and (A.95) only require the standard CES prices index P_t, which is the price index of an infinitely wealthy agent, and does not require the ideal price indices for agents at different points in the distribution (a,e).

C.2 Delayed substitution model

We introduce delayed substitution as a modification of the household side of the model, both in the domestic economy as well as in the rest of the world. We provide a detailed description of the former for the HA model; the latter is similar.

Delayed substitution in the domestic economy. We assume that instead of being able to flexibly adjust their relative consumption of home and foreign goods each period, domestic agents can only do so with a certain probability $1-\theta$. With probability θ, they are forced to keep the ratio of foreign good to home good consumption constant. Crucially, while relative consumption choices are restricted in that case, agents are still able to adjust their overall expenditure.

To describe this behavior more formally, consider a given household with fixed relative consumption of $x = \frac{c_F}{c_H}$. For a given total nominal expenditure $z = P_H c_H + P_F c_F$, the household therefore consumes $c_{Ht} = z/(P_H + xP_F)$ home goods and $c_F = xz/(P_H + xP_F)$ foreign goods. When expenditure is given in real terms, in units of the aggregate CPI, $\tilde{c} = z/P$, consumption is given by $c_H = P\tilde{c}/(P_H + xP_F)$ and $c_F = xP\tilde{c}/(P_H + xP_F)$. Consumption of the bundle is then

$$\begin{aligned} c &= \left[\alpha^{1/\eta} c_F^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta} c_H^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)} \\ &= \tilde{c} \cdot \underbrace{\frac{P}{P_H + xP_F}\left[\alpha^{1/\eta} x^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta}\right]^{\eta/(\eta-1)}}_{\equiv 1/\overline{p}(x)} \end{aligned} \tag{A.96}$$

The effect of a fixed ratio of relative consumption x is that it distorts the price index: For each unit of the optimal consumption bundle $\tilde{c}$, only $1/\overline{p}(x)$ units of the actual, distorted consumption bundle c can be purchased. Thus, $\overline{p}(x)$ acts like the relative price of the actual consumption bundle in terms of the optimal bundle. As such, $\overline{p}(x)$ is equal to one precisely for the optimal ratio $x = \frac{\alpha}{1-\alpha}\left(\frac{P_F}{P_H}\right)^{-\eta}$, and strictly greater than 1 for any other x. In the following, we denote the relative price by $\overline{p}_t(x)$ to emphasize that P_{Ht}, P_{Ft}, P_t are all time-dependent.

Having defined $\overline{p}_t(x)$, the household problem is now given by

$$\begin{aligned} V_t(a,e,x) &= \max_{c,a'} u(c) - v(N_t) + \beta\theta\mathbb{E}_t\left[V_{t+1}(a',e',x)\right] + \beta(1-\theta)\mathbb{E}_t\left[\max_{x'} V_{t+1}(a',e',x')\right] \\ & \overline{p}_t(x)c + a' = \left(1 + r_t^p\right)a + e\frac{W_t}{P_t}N_t \qquad a' \geq \underline{a} \end{aligned}$$

where, with some abuse of notation, we write $u(c)$ for the utility function over bundles, $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. A household entering a period with a fixed ratio x needs to pay a relative price $\overline{p}_t(x)$ for her consumption c. Her consumption ratio remains at x with probability θ and she is able to reoptimize the ratio at the beginning of the subsequent period with probability $1-\theta$.

The goal of the re-optimization is to minimize the expenditures $\overline{p}_{t+s}(x)$ paid in future periods. This is reflected in the first order condition for the optimal choice $\hat{x}_t$ at the beginning of period t,

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta)^s u'(c_{t+s}) c_{t+s} \frac{\partial \log \overline{p}_{t+s}(\hat{x}_t)}{\partial x} = 0 \tag{A.97}$$

Importantly, c_{t+s} is household-specific implying that the optimal ratios $\hat{x}_t$ differ across households in general. An instructive special case is given by $\sigma = 1$, log preferences, in which $\hat{x}_t$ is equal across households

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta)^s \frac{\partial \log \overline{p}_{t+s}(\hat{x}_t)}{\partial x} = 0$$

First order solution. Equation (A.97) can be log-linearized around the steady state, in which

$x^* = x_{ss} = \frac{\alpha}{1-\alpha}$. We focus on the case of $\sigma = 1$, where we find

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\theta)^s \left(\frac{\partial^2 \log \overline{p}\,(x_{ss})}{\partial \log x^2} d \log \hat{x}_t + \frac{\partial^2 \log \overline{p}_{t+s}\,(x_{ss})}{\partial \log x \partial \log P_{Ht+s}} d \log P_{Ht+s} + \frac{\partial^2 \log \overline{p}_{t+s}\,(x_{ss})}{\partial \log x \partial P_{Ft+s}} d \log P_{Ft+s} \right) = 0 \tag{A.98}$$

Rewriting (A.98), we find a simple recursive relationship characterizing deviations in the optimal ratio $d \log \hat{x}_t$

$$d \log \hat{x}_t = \eta(1-\beta\theta) d \log \frac{P_{Ht}}{P_{Ft}} + \beta\theta d \log \hat{x}_{t+1} \tag{A.99}$$

Equation (A.99) has a similar logic to a New Keynesian Phillips curve. The response of the optimal consumption ratio $d \log \hat{x}_t$ is a weighted average over future relative prices, multiplied by the elasticity, $\eta \times d \log \frac{P_{Ht}}{P_{Ft}}$. The more persistent consumption ratios are, that is, the greater θ, the more weight is put on farther out price changes.

Having characterized the optimal individual choices of $\hat{x}_t$ in (A.99), we aggregate them in order to find the behavior of $d \log x_t$

$$d \log x_t = (1-\theta) d \log \hat{x}_t + \theta d \log x_{t-1} \tag{A.100}$$

An instructive special case of (A.99) and (A.100) is that of an unanticipated permanent increase in the relative price of home vs foreign goods by some $d \log \frac{P_H}{P_F}$. Since (A.99) is perfectly forward-looking, the optimal consumption ratio adjusts immediately to the new level, $d \log \hat{x}_t = \eta d \log \frac{P_H}{P_F}$, with elasticity η. If adjustment was instantaneous, $\theta = 0$, $d \log x_t$ would immediately jump to the same level. In the case where adjustment is sluggish, $\theta > 0$, however, $d \log \hat{x}_t$ slowly increases over time. On impact it is equal to $(1-\theta) \times \eta d \log \frac{P_H}{P_F}$. In the long run, it converges to $\eta d \log \frac{P_H}{P_F}$. While η remains the long-run elasticity in the model, the short-run elasticity is reduced.

Once $d \log x_t$ is computed using (A.99) and (A.100), demand for home goods is given by

$$dC_{Ht} = (1-\alpha)\, dC_t - \alpha\,(1-\alpha)\, d \log x_t$$

In the frictionless case, $\theta = 0$ and $d \log x_t = -\frac{1}{1-\alpha} \eta dQ_t$, reproducing the first term of (28).

One feature of this model is that the elasticities are not just time dependent, but also shock dependent, in the sense that the persistence of the shock to $d \log \frac{P_{Ht}}{P_{Ft}}$ factors into the elasticity with which $d \log x_t$ is affected. The more persistent a shock is, the greater the elasticity will be. This can explain, for instance, why permanent tariff changes can have different effects on export volumes than shocks to exchange rates due to capital flows, as estimated in Fitzgerald and Haller (2018) and Cavallo, Gopinath, Neiman and Tang (2021).

Delayed substitution in the rest of the world. We follow a similar logic for the rest of the world. There, we assume that foreign households are slow to substitute between the varieties produced by the different countries. Denoting by x_t^* and $\hat{x}_t^*$ the current and optimal ratios of spending on home goods vs foreign goods, a suitable extension of the above yields the following expressions. The optimal ratio $\hat{x}_t^*$ is determined by

$$d \log \hat{x}_t^* = \gamma(1-\beta\theta) d \log \frac{P_{Ht}}{P_{Ft}} + \beta\theta d \log \hat{x}_{t+1}^*$$

which then enters the evolution of the actual ratio x_t^*

$$d\log x_t^* = (1-\theta) d\log \hat{x}_t^* + \theta d\log x_{t-1}^*$$

Ultimately, x_t^* pins down demand for home goods,

$$dC_{Ht}^* = -\alpha\left(1-\alpha\right) d\log x_t^*$$

C.3 Calibrating openness and price pass-through

This appendix provides data from a representative set of countries that experienced a large depreciation. This includes Mexico, which we use as our main calibration target, as well as eight other countries with a depreciation episode studied in Burstein and Gopinath (2015).

Calibrating α. We start by providing recent data on the import/GDP ratio from the IMF International Financial Statistics in the top panel of Table A.1. The import-GDP ratio informs the choice of α in our benchmark model, or of the aggregate $\frac{C_F}{C}$ in our quantitative model with non-homothetic demand. This justifies our calibration to $\alpha = 0.4$ for Mexico.

Calibrating θ_F. To calibrate price stickiness parameters, we use information from the country's large devaluation episode to inform our choice of exchange rate pass-through. For this exercise, we proceed as follows. We start from the equations describing the dynamics of import prices P_{Ft} in response to an exchange rate change, (41). Note that this equation delivers price dynamics as a pure function of the exchange rate path $\mathcal{E}_t$ and parameters θ_F and r, independently of the rest of the model. In particular, (41) implies that the linearized price dynamics of $p_{Ft} = \log P_{Ft}$ in response to an impulse to the exchange rate of $e_t = \log \mathcal{E}_t$ are

$$p_{Ft} - p_{Ft-1} = \frac{\left(1 - \frac{1}{1+r}\theta_F\right)\left(1-\theta_F\right)}{\theta_F}\left(e_t - p_{Ft}\right) + \frac{1}{1+r}\left(p_{Ft+1} - p_{Ft}\right) \tag{A.101}$$

We conceptualize the exchange rate depreciations experienced by each country in our case study as a one-time permanent shock to the exchange rate, from its initial level of $e_{-1} = 0$ to $e_t = \bar{e}$ for $t \geq 0$. Though stylized, this provides a useful approximation to the behavior of the nominal exchange rate in these episodes (see e.g. Burstein, Eichenbaum and Rebelo 2005, figure 1). It is easy to verify that the solution for p_{Ft} under this particular path for e_t in (A.101) is:

$$p_{Ft} = \bar{e}\left(1 - \theta_F^t\right) \tag{A.102}$$

Equation (A.102) delivers a simple way to back out the Calvo price rigidity coefficient as a function of the import price pass-through as of date t,

$$\theta_F = \left(1 - \frac{p_{Ft}}{\bar{e}}\right)^{\frac{1}{t}} \tag{A.103}$$

To perform this calculation for each of our countries, we need a measure of the pass-through to the *retail price of imported goods*, $p_{Ft}/\bar{e}$ at some date t following the depreciation at $t = 0$. Burstein, Eichenbaum and Rebelo (2005) (henceforth BER) measured pass-through at 24 months, corresponding to $t = 8$, but only for dock prices, tradable retail prices and nontradable retail prices. We convert this information into a measure of the retail price of imported goods following BER's

	Mexico	Argentina	Brazil	Korea	Thailand	Finland	Sweden	Italy	UK
Latest data	*2019*	*2019*	*2019*	*2019*	*2019*	*2019*	*2019*	*2019*	*2019*
Imports/GDP	40%	15%	14%	37%	51%	40%	44%	29%	32%
Depreciation year	*1994*	*2001*	*1998*	*1997*	*1997*	*1992*	*1992*	*1992*	*1992*
Dock PT	107%	87%	126%	60%	68%	116%	76%	63%	141%
Tradable retail PT	82%	36%	36%	30%	28%	64%	29%	32%	22%
Nontradable PT	30%	7%	11%	11%	25%	6%	14%	19%	41%
Imported retail PT	122%	63%	52%	46%	28%	102%	34%	38%	-15%
Implied θ_F	0.00	0.78	0.91	0.93	0.96	0.00	0.95	0.94	1.00
dY_0	- 0.35	- 0.03	0.00	- 0.01	0.00	- 0.35	0.00	- 0.01	- 0.01
$\sum_{t=0}^{7} dY_t$	- 1.81	- 0.42	- 0.10	- 0.11	0.03	- 1.81	- 0.02	- 0.08	0.01

Notes: data on dock and retail pass-through are taken from Table 7.5 in Burstein and Gopinath (2015), using the ratio of the increase in dock import prices and retail prices at 24 month to the trade weighted exchange rate at 24 month. Data on imports and international investment position (assets, liabilities and NIP) are taken from the IMF International Financial Statistics. The bottom two rows replicate the exercise of Table 2 for these countries.

Table A.1: Imported price pass-through, openness, and international positions for selected countries

framework. Specifically, we assume that (log) traded goods prices p_{Tt} are made up of imported goods prices and local goods, whose price is well proxied by the price of non-traded goods, so that

$$p_{Tt} = (1 - \phi)\, p_{Ft} + \phi p_{Nt} \tag{A.104}$$

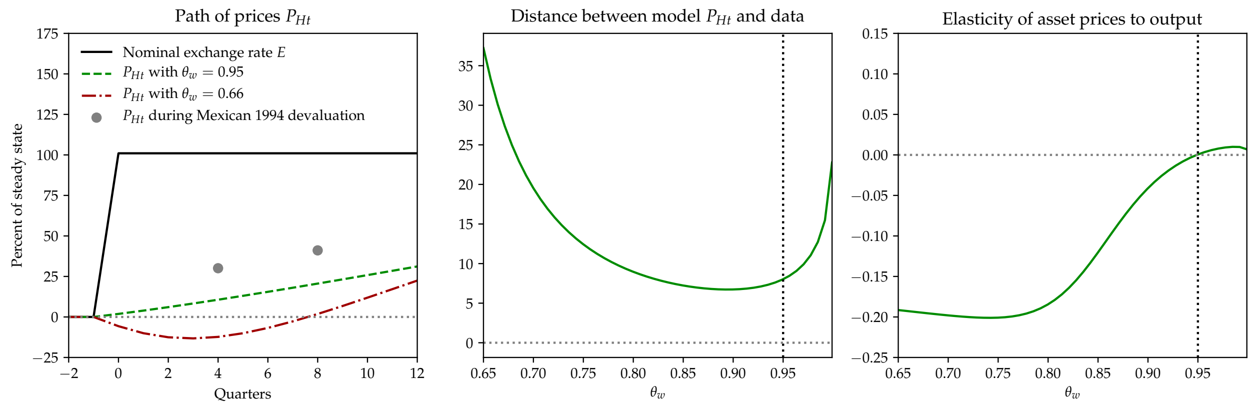
Following BER, we assume $\phi = 1/2$, and use equation (A.104) to back out p_{Ft}.[49]

The bottom row of Table A.1 reports the result of this exercise. The first two rows report $p_{Dt}/\bar{e}$, $p_{Tt}/\bar{e}$ and $p_{Nt}/\bar{e}$ for $t = 8$ quarters. The next row reports $p_{Ft}/\bar{e}$ backed out from (A.104), and the final row reports the implied quarterly θ_F from equation (A.103). As can be seen, the devaluations suggest a lot of heterogeneity in imported price pass-through in each episode. In Mexico, this procedures infers full price pass-through, given the large movements in tradable retail prices and limited movement in nontradable prices. This is consistent with the large amount of price pass-through observed at the dock in that episode. In other episodes tradable retail prices move a lot less, so our procedure infers much more limited import price pass-through.

Calibrating θ_w. We also use information from Table A.1 to calibrate the wage stickiness parameter θ_w . We use information from the time path of non-traded good prices, which in our model correspond to p_{Ht}, in order to discipline that parameter. Note that, in contrast to θ_F, which is identified directly from the price pass-through data, θ_w depends on the entire structure of the model, and in particular on the relationship between monetary policy and domestic economic activity. Moreover, the path of p_{Ht} does not separately identify the stickiness of wages θ_w and the stickiness of prices θ_H. We therefore follow the standard in the literature and set θ_H to imply a

[49] We obtain similar results if we assume that the retail price of imported goods is a mix $p_{Ft} = (1 - \phi)\, p_{Dt} + \phi p_{Nt}$ of dock prices and non-traded goods prices.

Figure A.2: Calibrating θ_w



Note: The left panel shows the impulse response of home goods prices P_{Ht}following a permanent change in the nominal exchange rate E_t.The data is taken from Table 7.5 in Burstein and Gopinath (2015). The middle panel plots the sum of square distance between model and data at $t = 4$ and $t = 8$, for different values of the calvo parameter θ_w. The right panel shows the reduced-form elasticity of the asset price to output, for different values of the Calvo parameter θ_w.

price duration of 3 quarters. We then find θ_w to match two features of the data. First, we look for the best fit in terms of the path of prices in response to the pure devaluation shock as described above. Second, we look for a coefficient that implies a reasonable degree of cyclicality of stock prices in response to a monetary policy shock.

Figure A.2 shows the outcome of that exercise. The left panel plots the path of prices P_{Ht} after the devaluation in the model and in our data, the middle panel plots the sum of square distance between model and data at $t = 4$ and $t = 8$, and the right panel plots the reduced-form elasticity of asset prices induced by a contractionary shock to capital flows. When wages are relatively flexible ($\theta_w = 0.66$), home goods prices initially go *down* in response to the devaluation shock, because this shock induces a recession in the short run. This does not match the path of prices that we observed in the Mexican devaluation. Our model infers that wages are stickier than this, in the range of $\theta_w = 0.85$ or above. Second, at $\theta_w = 0.85$, stock prices are still countercyclical, going up after a contractionary capital outflow. We pick $\theta_w = 0.95$ because it hits a tradeoff between making asset prices roughly acyclical in response to capital flow shocks, and getting a path of prices p_{Ht} that lines up well with the data.

We use the calibration in this section to consider the effect of depreciations in other countries than our benchmark of Mexico, which featured high openness and full import pass-through. We recalibrate our model to hit their import-GDP ratio and their degree of import price pass-through, but keeping the MPCs the same. The bottom two rows of the table illustrates that the effects of the same exchange rate shock are very heterogeneous across countries. Countries with lower import price pass-through have much less of an immediate impact on output, since the real income effect is really muted. Openness has a non-monotonic effect, since the immediate effect of an exchange rate depreciation is not as large in an economy that is more closed, but the general equilibrium effect of any open international position is much larger, through the logic of Proposition 7.

C.4 Currency mismatch in balance sheets

To incorporate currency mismatch in the net foreign asset position, we proceed as follows. We expand the setting in section A.1 by allowing countries to invest in *long-duration* foreign currency assets, modeled as bonds with nominal coupons that exponentially decay at a rate δ, and a foreign currency price of $\mathcal{Q}_t^*$, where δ is calibrated to empirical duration data.

Investment through the mutual fund. We first assume that foreign currency exposures are held through the mutual fund. Suppressing the choice of domestic nominal bonds and foreign stocks for simplicity, the beginning-of-period valuation equation for the mutual fund (A.1) becomes

$$\left(1+i_t^p\right)\mathcal{A}_{t-1} = \left(D_t+p_t\right)P_t s_{t-1}^H + \mathcal{E}_t\left(1+\delta\mathcal{Q}_t^*\right)\Lambda_{t-1}^F$$

where Λ_t^F is the number of foreign-currency asset coupons held in period t, while the end-of period valuation equation A.2 is

$$\mathcal{A}_t = p_t P_t s_t^H + \mathcal{E}_t \mathcal{Q}_t^* \Lambda_t^F$$

Optimal investment now requires, in addition to the equations in (A.4), that

$$1+i_t^* = \frac{1+\delta\mathcal{Q}_{t+1}^*}{\mathcal{Q}_t^*} \tag{A.105}$$

which gives the valuation equation for the foreign currency bond price $\mathcal{Q}_t^*$. The real value of mutual fund assets (A.9) is now

$$\left(1+r_t^p\right)A_{t-1} = \left(D_t+p_t\right)s_{t-1}^H + \left(1+i_{t-1}^*\right)Q_t\frac{\Lambda_{t-1}^F}{P^*} \tag{A.106}$$

In the steady state, this now reads

$$A_{ss} = p_{ss}s_{-1}^H + \left(1+\delta\mathcal{Q}_{ss}^*\right)\Lambda_{-1}^F = p_{ss}s_{-1}^H + \underbrace{\left(1+i_{ss}^*\right)\mathcal{Q}_{ss}^*\Lambda_{-1}^F}_{\equiv f_Y} \tag{A.107}$$

We continue to calibrate the steady state of the model so that $\text{nfa}_{-1}=0$, and hence $A_{ss}=p_{ss}$, but now allow for a gross currency mismatch, where the country has a share f_Y of foreign currency assets in excess of foreign currency liabilities, relative to its GDP. Combining (A.107) with $A_{ss} = p_{ss}$, we have:

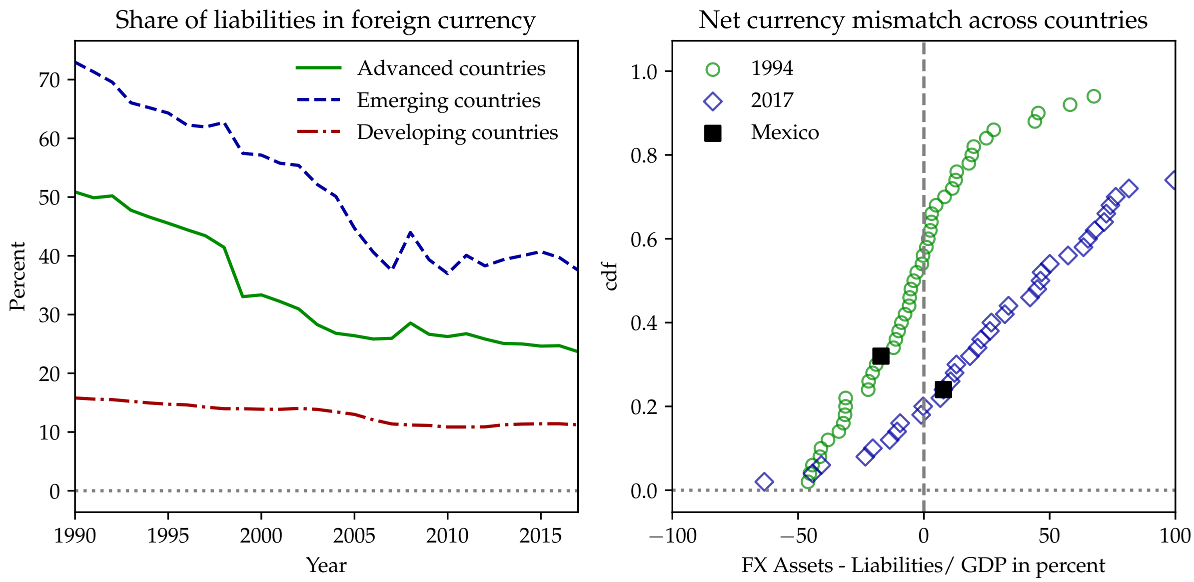
$$f_Y = p_{ss}\left(1-s_{-1}^H\right)$$

In other words, if $f_Y>0$, then the country has foreign currency assets that are offset by foreign direct investment in the domestic stock market ($s_{-1}^H<1$), while if $f_Y<0$, the country has borrowed in foreign currency and holds a levered position in its domestic stock market, $s_{-1}^H>1$.

After a depreciation induced by a change in the path of i_t^* for $t\geq 0$, the country experiences an adverse valuation effect to its liabilities. We use equation (A.105) to calculate the induced new bond price $\mathcal{Q}_0^*$ (note that when $\delta>0$, the increase in foreign interest rates reduces the present value of liabilities in foreign currency term), and (16) to calculate the new real exchange rate Q_0. Together with our calibration of f_Y, this determines r_0^p via equation (A.106) and therefore the magnitude of the valuation effect.

Investment through the government balance sheet. When foreign currency investments are done through the mutual fund, the valuation effects are distributed in the population according to their holdings of mutual fund shares. To quantify the effect of alternative distribution rules, we now assume that the government holds the country's gross currency exposure, and rebates it according to various schemes. We add a government, with shares s_t^G in domestic assets and Λ_{t-1}^G

Figure A.3: Calibrating f_Y



Note: The left panel shows the fraction of liabilities denominated in foreign currency as a share of total liabilities, on average across advanced, emerging and developing countries, respectively. The right panel shows the distribution, in a set of 50 countries, of the difference between foreign currency assets and liabilities ("gross currency mismatch"), as a share of GDP. Source: Bénétrix et al. (2020), updating an earlier study by Lane and Shambaugh (2010).

coupons in foreign bonds, and budget constraint

$$B_t = (1 + r_{t-1}) B_{t-1} - T_t + (1 + i^*_{t-1}) Q_t \frac{\Lambda^G_{t-1}}{P^*} + s^G_t (D_t + p_t) \tag{A.108}$$

where T_t are aggregate taxes. We distribute those taxes by modifying the household budget constraint (1) to read

$$\frac{P_{Ft}}{P_t} c_F + \frac{P_{Ht}}{P_t} c_H + a' = \left(1 + r^p_t\right) a + e \frac{W_t}{P_t} N_t - T_t \frac{e^{1-\lambda}}{\mathbb{E}\left[e^{1-\lambda}\right]}$$

$\lambda = 1$ represents lump-sum taxes, while $\lambda = 0$ are proportional taxes. The higher λ, the more regressive the tax system is. Finally we assume the fiscal rule

$$B_t = (\rho_B)^t B_{t-1}$$

When $\rho_B = 0$, through (A.108), the government immediately must adjust taxes to shore up its balance sheet loss from foreign liabilities after a depreciation. When $\rho_B > 0$, the government builds up debt and taxes later, which mitigates the immediate effect on spending.

Calibrating δ and f_Y. To calibrate the coupon δ, we note that the duration of a bond with price (A.105) is given by

$$D = \frac{1 + i^*}{1 + i^* - \delta}$$

We calibrate δ to hit a liability duration of $D = 18$ quarters, as implied by Doepke and Schneider (2006)'s estimates for the U.S.

We calibrate f_Y to data on from Lane and Shambaugh (2010) and Bénétrix et al. (2020). Lane and Shambaugh (2010) documented aggregate foreign currency exposures for 1994 to 2004 for a sample of 117 counties; Bénétrix et al. (2020) subsequently updated their data to 2017 for a sample of 50 countries. These studies measure foreign currency exposure as the difference between county i's gross foreign currency assets and gross foreign currency liabilities.[50] The right panel of figure A.3 shows the distribution of these currency exposures, normalized by GDP, from the most recent study by Bénétrix et al. (2020). As emphasized by these authors, countries have dramatically reduced the aggregate currency mismatch in their balance sheets since the 1990s: for instance, while Mexico used to have around 25% more foreign currency liabilities than assets as a share of its GDP, it now has around 5% more foreign currency assets than liabilities (the left panel illustrates that this has tended to happen via a reduction in the fraction of the share of liabilities that are in foreign currency.) In the latest 2017 data, only three countries in the dataset have foreign currency liabilities exceeding assets by more than 40% of GDP: Tunisia (-63%), Egypt (-44%) and Sri Lanka (-40%). In our exercise of section 5.6, we set $f_Y = -50\%$. This calibration therefore represents an upper bound on the size of valuation effects.

D Alternative models

This appendix presents three extensions of our baseline model, which we show can be reinterpreted as versions of our baseline model with different parameters. Appendix D.1 adds produced nontradable goods in addition to tradable goods. Appendix D.2 adds imported intermediate goods. Both of these can be directly reinterpreted as our baseline model with an appropriate reparameterization of the openness parameter α and the elasticity of substitution between home and foreign goods η. Appendix D.3 considers a tradable-nontradable model of a commodity exporter, which takes as given the price of exports. We show that in the standard case where a fixed quantity of tradables (commodities) is being produced each period, akin to a fixed endowment of tradable goods, this can be reinterpreted as our model with dollar currency pricing.

D.1 Nontradable goods

We first add nontradable goods to the model. Instead of (2), assume that household consumption is now an aggregate between tradable goods and (home-produced) nontradable goods,

$$c = \left[\phi^{1/\zeta} c_T^{(\zeta-1)/\zeta} + (1-\phi)^{1/\zeta} c_{H,NT}^{(\zeta-1)/\zeta}\right]^{\zeta/(\zeta-1)} \tag{A.109}$$

where the tradable bundle is a mix of imported tradables and home-produced tradable goods,

$$c_T = \left[\alpha^{1/\eta} c_F^{(\eta-1)/\eta} + (1-\alpha)^{1/\eta} c_{H,T}^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)} \tag{A.110}$$

Here, ϕ is the tradable share, while $1-\alpha$ is home bias *within* tradables; ζ is the elasticity of substitution between tradables and nontradables (which is plausibly quite low), while η is the elasticity

[50] Lane and Shambaugh (2010)'s headline measure of currency exposure for country i at time t, FX_{it}^{AGG}, is normalized by the sum of assets and liabilities, but the supplementary data in both Lane and Shambaugh (2010) and Bénétrix et al. (2020) report measures normalized by GDP, which correspond exactly to our f_Y.

of substitution between home and foreign goods within tradables. For this section, we assume that the production functions for tradables and nontradables are identical, so that they always have the same price, and that all that matters is the sum of $c_{H,T}$ and $c_{H,NT}$.

With this demand system, total demand for home goods coming domestic residents is

$$c_H \equiv c_{H,T} + c_{H,NT} = \left((1-\alpha) \left(\frac{P_H}{P_T} \right)^{-\eta} \phi \left(\frac{P_T}{P} \right)^{-\zeta} + (1-\phi) \left(\frac{P_H}{P} \right)^{-\zeta} \right) c \tag{A.111}$$

At the steady state where all prices are 1, the overall home and foreign shares of consumption are therefore

$$\frac{c_H}{c} = (1-\alpha)\phi + (1-\phi) \equiv 1-\bar{\alpha}; \qquad \frac{c_F}{c} = \alpha\phi \equiv \bar{\alpha} \tag{A.112}$$

In response to a shock to prices around the steady state, we log-linearize and find that this relative demand changes by

$$\hat{c}_H - \hat{c} = -\frac{(1-\alpha)\phi}{(1-\alpha)\phi + (1-\phi)} \left(\eta(\hat{p}_H - \hat{p}_T) + \zeta(\hat{p}_T - \hat{p}) \right) - \frac{1-\phi}{(1-\alpha)\phi + (1-\phi)} \zeta(\hat{p}_H - \hat{p}) \tag{A.113}$$

Noting that $\hat{p} = \phi\hat{p}_T + (1-\phi)\hat{p}_H$, we can write

$$\hat{p}_H - \hat{p}_T = \phi^{-1}(\hat{p}_H - \hat{p})$$
$$\hat{p}_T - \hat{p} = -\phi^{-1}(1-\phi)(\hat{p}_H - \hat{p})$$

and substitute these into (A.113) to obtain

$$\begin{aligned} -\frac{\hat{c}_H - \hat{c}}{\hat{p}_H - \hat{p}} &= \frac{(1-\alpha)\phi}{(1-\alpha)\phi + (1-\phi)} (\eta\phi^{-1} - \zeta\phi^{-1}(1-\phi)) + \frac{1-\phi}{(1-\alpha)\phi + (1-\phi)} \zeta \\ &= \frac{(1-\alpha)\eta + (1-\phi)\alpha\zeta}{(1-\alpha) + (1-\phi)\alpha} \equiv \bar{\eta} \end{aligned} \tag{A.114}$$

Note that the elasticity $\bar{\eta}$ in (A.114) is a weighted average of the primitive elasticities η and ζ.

Define the *consumption aggregator* function $\bar{c}(c_F, c_H)$ to maximize c subject to (A.109), (A.110) and $c_H = c_{H,T} + c_{H,NT}$. It is immediate that this has constant returns to scale. We can view total foreign and home consumption (as calculated above) as optimizing this function subject to prices P_F and P_H.

For steady-state $P_F = P_H = P = 1$, we found c_F and c_H in (A.112), with $\bar{\alpha} = \alpha\phi$ the steady-state foreign share and $1 - \bar{\alpha}$ the home share. In (A.114) we calculated the local elasticity of substitution of the consumption aggregator function, $\bar{\eta}$.

To first order in aggregate shocks, therefore, our model remains the same when nontradables are introduced; we need only replace the openness parameter α and elasticity of substitution between home and foreign goods η with their counterparts $\bar{\alpha}$ and $\bar{\eta}$ in (A.112) and (A.114).[51] The two implications of this equivalence mapping are the following.

First, the import-to-GDP ratio is now $\frac{c_F}{c} = \alpha\phi = \bar{\alpha}$. Hence, even in the presence of nontradables, it is appropriate to calibrate $\bar{\alpha}$ to that ratio.

Second, $\bar{\eta}$ is a weighted average of η (elasticity between home and foreign within tradables, which could be relatively high) and ζ (the elasticity between nontradable and tradable, which

[51] The problem of allocating within c_F between different countries' varieties is unchanged; the elasticity there remains γ.

could plausibly be much lower), with a larger weight on ζ when the nontradable share is higher. Hence, $\bar{\eta}$ itself could plausibly be relatively low.

D.2 Imported intermediates

We now return to the consumption basket in (2), but change the production structure to allow for imported intermediate goods. Specifically, suppose that the continuum of firms in each country now produce an intermediate good X using the technology $X = ZN$, and that the final good Y in each country is a CES aggregate of the country's own intermediate good and the foreign intermediate good.

Concretely, for the home country, suppose that production of the final good is given by

$$Y = \left[\phi^{1/\zeta} X_F^{(\zeta-1)/\zeta} + (1-\phi)^{1/\zeta} X_H^{(\zeta-1)/\zeta}\right]^{\zeta/(\zeta-1)} \tag{A.115}$$

where X_H is the home country's demand for the home intermediate, and X_F is the home country's demand for imported intermediates. Suppose further that X_F (analogous to c_F) is a CES aggregate of each other country's intermediate, with elasticity ν. As before, normalize all prices and quantities at the steady state to 1, and assume that foreign prices and quantities do not change. Note that a country's total value added, or GDP, equals its X.

It follows that total demand for the home country's intermediate X is

$$X = (1-\phi)\left(\frac{P_H^X}{P_H}\right)^{-\zeta} Y + \phi\left(\frac{P_H^X}{\mathcal{E}}\right)^{-\nu} Y^* \tag{A.116}$$

where demand for Y is the same as before

$$Y = (1-\alpha)\left(\frac{P_H}{P}\right)^{-\eta} C + \alpha\left(\frac{P_H}{\mathcal{E}}\right)^{-\gamma} C^* \tag{A.117}$$

Equations (29) and (30) continue to hold, replacing Y by X and P_H by P_H^X. Totally differentiating (A.116), we get

$$\begin{aligned} dX &= -(1-\phi)(1-\alpha)\eta(dP_H - dP) - (1-\phi)\alpha\gamma(dP_H - d\mathcal{E}) \\ &\quad -(1-\phi)\zeta(dP_H^X - dP_H) - \phi\nu(dP_H^X - d\mathcal{E}) + (1-\phi)(1-\alpha)dC \end{aligned} \tag{A.118}$$

As in appendix B.1, linearizing the CPI equation, we have $dP = (1-\alpha)dP_H + \alpha d\mathcal{E}$. Linearizing the price index corresponding to (A.115), we get $dP_H = (1-\phi)dP_H^X + \phi d\mathcal{E}$.

Writing all the relative prices in (A.118) in terms of the real exchange rate $dQ = d\mathcal{E} - dP$, we have

$$\begin{aligned} dP_H - dP &= -\frac{\alpha}{1-\alpha}dQ \\ dP_H - d\mathcal{E} &= -\frac{1}{1-\alpha}dQ \\ dP_H^X - dP_H &= \frac{\phi}{1-\phi}(dP_H - d\mathcal{E}) = -\frac{\phi}{1-\phi}\frac{1}{1-\alpha}dQ \\ dP_H^X - d\mathcal{E} &= \frac{1}{1-\phi}(dP_H - d\mathcal{E}) = -\frac{1}{1-\phi}\frac{1}{1-\alpha}dQ \end{aligned}$$

and can plug this into (A.118) to obtain

$$dX = \left((1-\phi)\alpha\eta + (1-\phi)\frac{\alpha}{1-\alpha}\gamma + \phi\zeta\frac{1}{1-\alpha} + \frac{\phi}{1-\phi}\nu\frac{1}{1-\alpha} \right) dQ + (1-\phi)(1-\alpha)dC \quad \text{(A.119)}$$

If we define $\bar{\alpha} \equiv 1-(1-\phi)(1-\alpha)$, and $\bar{\chi} \equiv \frac{1-\bar{\alpha}}{\bar{\alpha}} \left((1-\phi)\alpha\eta + (1-\phi)\frac{\alpha}{1-\alpha}\gamma + \phi\zeta\frac{1}{1-\alpha} + \frac{\phi}{1-\phi}\nu\frac{1}{1-\alpha} \right)$, then (A.119) becomes just

$$dX = \frac{\bar{\alpha}}{1-\bar{\alpha}}\bar{\chi}dQ + (1-\bar{\alpha})dC \quad \text{(A.120)}$$

which is identical to equation (A.57) in appendix B.1, but with $\bar{\alpha}$, $\bar{\chi}$, and dX replacing α, χ, and dY. With these substitutions, the International Keynesian Cross remains the same, and our analysis in the main body of the paper goes through. The two implications of this equivalence mapping are the following.

First, $\overline{\alpha} = \alpha + \phi - \alpha\phi$, while the import-to-GDP ratio is $\alpha + \phi$. Hence, provided α and ϕ are not too large, $\overline{\alpha}$ is close to the import-to-GDP ratio, though an ideal calibration would substract the reexported good-to-GDP ratio $\alpha\phi$.

Second, the trade elasticity $\bar{\chi}$ is now a more complex amalgam of four primitive elasticities: substitution between home and foreign final goods η, substitution between different countries' final goods γ, substitution between home and foreign intermediates ζ, and substitution between different countries' intermediates ν.

D.3 Commodity exporter model

As our last alternative model, we consider a model of a commodity exporter, who takes as given the price of tradable goods. We set this up as in Uribe and Schmitt-Grohé (2017), by assuming that the economy possesses a constant stream of tradable goods Y^T that it can sell in the world market at fixed prices. Vice versa, there are non-tradable goods that the economy does not export. We describe the main changes in this economy relative to the one in section 2 and argue that this model is identical to the DCP model in section 3.4 in which dollar prices of exports are fully rigid.

Households. Domestic households are assumed to behave as in section 2, except that they consume tradable and non-tradable goods, rather than foreign and domestic goods. P_{Tt} is the price of tradables and P_{Nt} is the price of non-tradables. The utility function $u(c_T, c_N)$ is the same as before, with c_T and c_N entering a CES basket with elasticity η and a consumption share of tradables of α, analogous to (2). The CPI is analogous to (3), individual demands analogous to (4)–(5). Foreign households elastically buy or sell tradables at a fixed dollar price $P_T^* = 1$.

Production. Non-tradables are produced using the linear production function

$$Y_{Nt} = Z_N N_{Nt}$$

and sold by a continuum of firms charging flexible prices at a markup μ. N_{Nt} is labor demand by non-tradable producers. Tradables are produced by the Leontief production function

$$Y_{Tt} = Z_T \min\{N_{Tt}, L\} \quad \text{(A.121)}$$

where $L > 0$ is a fixed factor the country is endowed with, such as the land on which natural resources can be found. Again we assume Y_{Tt} is sold by a continuum of firms charging flexible prices at a markup μ. N_{Tt} is labor demand by tradable producers.

We assume here that (A.121) is Leontief in line with the idea that tradables are basically an endowment of the economy, $Y_{Tt} = Z_T L = const$. The only reason why we do not outright assume that Y_{Tt} is an endowment is that in a heterogeneous-agent context, it matters *whose* endowment Y_{Tt} is. (A.121) provides us with a simple way to split the proceeds from selling Y_{Tt} into labor and profit income.

Rest of the model. All the remaining model ingredients are identical. For example, all firms' dividends (tradable and non-tradable alike)

$$D_t = \frac{P_{Nt} Y_{Nt} - W_t N_{Nt}}{P_t} + \frac{\mathcal{E}_t P_T^* Y_{Tt} - W_t N_{Tt}}{P_t}$$

are capitalized and traded, just like domestic firms' dividends before. Unions and wage stickiness, notation for exchange rates, market structure, and monetary policy are all identical.

Market clearing for non-tradable goods is given by

$$Y_{Nt} = (1-\alpha)\left(\frac{P_{Nt}}{P_t}\right)^{-\eta} C_t \tag{A.122}$$

essentially (26) without the second term, as non-tradable goods are not exported. We normalize all prices $\mathcal{E}_{ss}, Q_{ss}, P_{ss}, P_{Nss}, P_{Tss}$ to 1, and quantities $C_{ss} = 1, Y_{Nss} = 1-\alpha$, $Y_{Tss} = \alpha$ in the steady state of the model.

Consumption function. We can write consumption as function of real labor income and dividends $C_t = \mathcal{C}_t\left(\left\{\frac{W_s}{P_s} N_s, D_s\right\}\right)$ just like before.

Model analysis. Define real GDP as

$$Y_t \equiv \frac{P_{Tss}}{P_{ss}} Y_{Tt} + \frac{P_{Nss}}{P_{ss}} Y_{Nt} = Y_T + Y_{Nt}$$

We can write dividends as

$$D_t = \frac{P_{Nt} Y_t - W_t N_t}{P_t} + \frac{\mathcal{E}_t P_T^* - P_{Nt}}{P_t}\alpha \tag{A.123}$$

which is identical to (12) in the case of DCP with fully rigid dollar export prices, where $C_H^* = \alpha$ and $P_H^* = P_T^* = 1$. Just like before, aggregate labor income is given by

$$\frac{W_t}{P_t} N_t = \frac{1}{\mu} \frac{P_{Nt}}{P_t} Y_t \tag{A.124}$$

Rewriting (A.122), we find that

$$Y_t = (1-\alpha)\left(\frac{P_{Nt}}{P_t}\right)^{-\eta} C_t + \alpha \tag{A.125}$$

which is identical to the goods market clearing condition (26) with DCP. Given that (A.123)–(A.125) are the same as in the model with DCP (section 3.4), and the consumption function is unchanged, this proves that the model with tradable and non-tradable goods is isomorphic to the DCP model.