On the Green Interest Rate.

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Appendix

The supplementary appendix contains three sections. It begins by deriving the household labor supply function. The second section explores transmission of the interest rates. The third subsection considers a utility function defined over both consumption of market goods and environmental quality.

I. Household Labor Supply.

Households maximize utility which is defined over consumption and the disutility of work at which workers earn a nominal wage (W_t).

$$U_t(c_t, n_t) = \frac{{c_t}^{1-\eta}}{1-\eta} - \frac{n_t^{1+\psi}}{1+\psi} \tag{1}$$

The intertemporal budget constraint is shown in (2), now introducing prices for consumption goods.

$$(p_1 c_1) + \left(\frac{1}{1+r}\right)(p_1 c_1) \leq W_1 n_1 + \left(\frac{1}{1+r}\right) W_1 n_1 \tag{2}$$

And the time budget constraint for period (t) is:

$$l_t + n_t \leq 1 \tag{3}$$

The first order conditions for utility maximization across leisure (defined as $1 - n_t$ from (3)) and consumption is shown in (4), assuming log utility in consumption and leisure:

$$\frac{W_t}{P_t} = c_t(1 - n_t). \tag{4}$$

Rearranging yields: $n_t = (1 - w_t/c_t)$, where $w_t = \frac{W_t}{P_t}$.

II. Transmission.

To evaluate how the green interest rate transmits information pertaining to pollution damage through the economy, the analysis begins by examining its effect on consumption in the present and the future. Because the central bank (jurisdiction) does not directly force internalization of pollution damage through conventional means such as a Pigouvian tax, the effect of the green interest rate on consumption of market goods and services is analyzed. That is, households are assumed to only differ in consumption choices based on the interest rate that they face rather than perceiving that potential income has been adjusted or reduced by the deduction of pollution damage.

Period 1 consumption defined in terms of market goods and services is shown in (5) when facing the green interest rate:

$$c_1^g = \frac{(1+\rho)}{(1+r_g)}\left(\frac{a_2}{1+\mu_2-\varphi}\right) \tag{5}$$

And period 2 consumption is:

$$c_2 = \frac{(1+r_g)}{(1+\rho)}\left(\frac{a_1}{1+\mu_1-\varphi}\right) \tag{6}$$

The growth rate in consumption from period 1 to period 2 is:

$$\left(\frac{a_2}{a_1}\right)\left(\frac{1+\mu_1-\varphi}{1+\mu_2-\varphi}\right)\left(\frac{1-(\alpha_2-\beta_2\gamma_2)}{1-(\alpha_1-\beta_1\gamma_1)}\right)^2 \tag{7}$$

The growth rate in consumption when households face (r^*) is:

$$\left(\frac{a_2}{a_1}\right)\left(\frac{1+\mu_1-\varphi}{1+\mu_2-\varphi}\right) \tag{8}$$

The consumption growth rate when households face (r_g) differs from (r^*) by the $\left(\frac{1-(\alpha_2-\beta_2\gamma_2)}{1-(\alpha_1-\beta_1\gamma_1)}\right)^2$ term. The consumption growth rate induced by (r_g) exceeds that from (r^*) if pollution intensity is falling. This stems directly from the conclusions about the relative rates above; if pollution damage intensity falls, (r_g) > (r^*). And, a higher rate urges households to save in period (1) and consume in period (2). Hence, consumption growth rises if damages fall as (r_g) reallocates consumption to the lower damage period. The opposite effect manifests if damage intensity rises. In that case, (r_g) < (r^*), and period (1) savings falls. Consumption moves from the future to the present, and consumption growth decreases. Again, (r_g) reallocates consumption to the lower damage period.

Next, the implications of shifts in consumption according to (r_g) are explored through labor supply, wages, the markup and inflation. Beginning with labor supply, note that the partial effect of consumption shifts on labor supply is: $\left(\frac{\partial n_t}{\partial c_t} = -\frac{1}{w_t}\right)$. Inward (outward) shifts in consumption yield increases (decrease) in labor supply, holding the wage fixed.

To see how r_g affects labor supply choices across time, (2) is combined with (4) and the expression for the real wage to depict equilibrium labor supply when firms face (r^*): $n_1^* =$

$\frac{1-\varphi}{1+\mu_1-\varphi}$. Labor supply varies with the markup and curvature in the production function. The difference in labor supply when consumers face (r_g) stems from the consumption shift, which operates through the numerator in the second term in (2). Hence, period (1) equilibrium labor supply when firms face (r_g) is $n_1^g = \frac{1-\varphi}{1+\left(\frac{1-\delta_1}{1-\delta_2}\right)\mu_1-\varphi}$. In period (2), $n_2^g = \frac{1-\varphi}{1+\left(\frac{1-\delta_2}{1-\delta_1}\right)\mu_2-\varphi}$. In both periods, the adjustment from pollution damage permutes labor supply through the markup. If damage intensity falls as the economy cleans up, (r_g) > (r^*), labor supply increases in the present and falls in the future. Conversely, if the economy becomes more polluted, (r_g) < (r^*), consumption reallocates to the present and labor supply decreases in the present and rises in the future.

A Pigouvian tax reducing net productivity would not affect equilibrium labor supply because it would affect consumption and wages proportionally. In contrast, the consumption shift from (r_g) does not arise from a change in marginal productivity of labor. Rather, it manifests because of the interest rate. Thus, the wage is not directly affected through a productivity channel as it would in the presence of a Pigouvian tax. This asymmetry results in the changes to labor supply discussed above.

Next, (2) from the main text of the paper is rearranged to characterize the wage as a function of labor supply and consumption: $w_t = \frac{c_t}{1-n_t}$. Because both consumption and labor supply are affected by (r_g), both (n_1^*) and (c_1^*) are substituted in the expression for w_1 to find w_1^*, and (n_1^g) and (c_1^g) are substituted in to find w_1^g. Then taking the ratio of w_1^* to w_1^g yields: $\frac{1+\mu_1-\varphi}{1+\left(\frac{1-\delta_1}{1-\delta_2}\right)\mu_1-\varphi}$. A downward trajectory of damages implies that $\left(\frac{1-\delta_1}{1-\delta_2}\right) > 1$, and $\frac{w_1^*}{w_1^g} > 1$, and, in period (2), declining damages means that $\frac{w_2^*}{w_2^g} < 1$. With households facing (r_g), falling damages results in upward pressure on wage growth. Conversely, rising pollution suppresses wage growth.

Labor supply and consumption responses to (r_g) move in opposite directions. As such, a priori, the effect of (r_g) on the wage is ambiguous. The fact that falling damages results in upward pressure on wage growth, indicates that the consumption channel dominates the labor supply channel in terms of the effect of (r_g) on wages.

The effect on the markup follows from the consumption-wage channel. The direction of the impact on the markup hinges on whether damages are rising or falling. If the economy becomes less polluted, and households face (r_g), consumption and wage growth rise. This compresses the markup $\left(\frac{P_t}{MC_t}\right)$ since marginal cost comprises the wage over the marginal product of labor. And if the economy grows more polluted, consumption and wage growth diminish, which expands the markup because marginal cost falls.

III. Substitution between environmental damage and consumption.

An important issue in the context of the present paper is substitution between consumption of market goods and environmental quality, or damage. The simple, univariate power utility function employed in section II. of the main body of the paper is easily augmented to include environmental quality in a separable fashion. For example, Hoel and Sterner (2007) propose a utility function of the form:

$$U_t(c_t, e_t) = \frac{1}{1-\eta}\left((1-\theta)c_t^{1-\frac{1}{\sigma}} + (\theta)\big(c_t(\alpha_t - \beta_t\gamma_t)\big)e_t^{1-\frac{1}{\sigma}}\right)^{\frac{(1-\eta)\sigma}{\sigma-1}} \qquad (9)$$

where: $\sigma =$ constant elasticity of substitution between environmental quality (e_t) and market goods (ct).

$\theta =$ scaling parameter proportional to consumption shares of environmental quality (e_t) and market goods (c_t).

One central question is whether, conditional on the form shown in (9), the first order condition with respect to consumption of market goods is affected. Recall from section II., that this first order condition plays a key role in the conceptual analysis which solves for the green interest rate.

To explore this, the marginal utility from market consumption is derived for t = 1 and t = 2. Here the assumption of log utility is maintained for simplicity. This assumption yields (10).

$$U_t(c_t, e_t) = \frac{\sigma}{\sigma-1} ln\left((1-\theta)c_t^{1-\frac{1}{\sigma}} + (\theta)\big(c_t(\alpha_t - \beta_t\gamma_t)\big)e_t^{1-\frac{1}{\sigma}}\right) \quad (10)$$

Setting marginal utilities from consumption of market goods in periods 1 and 2 equal and rearranging yields (11) which is identical to (1) in the main text, for the case of log utility.

$$\frac{c_2}{c_1} = \frac{1+r}{1+\rho} \quad (11)$$

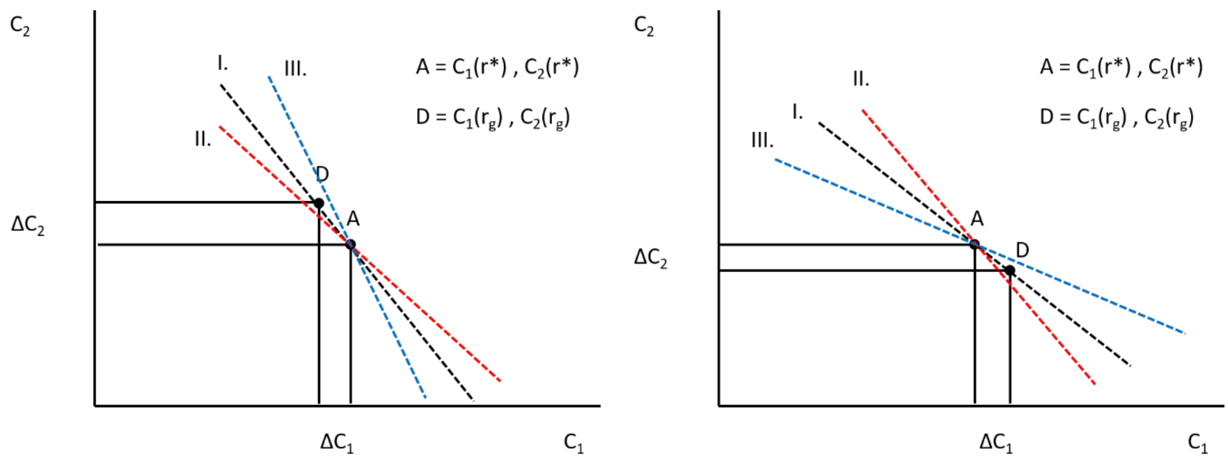
Thus, with power utility and with $\eta = 1$, the key form used to derive interest rates is unaffected by allowing arbitrary substitution between market goods and environmental quality in the representative consumer's utility function.

It is important to note that environmental quality does not enter into the utility function as employed into the main body of the paper. Rather, environmental pollution damage is, in effect, deducted from *potential income*. This treatment reflects guidance from the environmental accounting literature (Nordhaus, 2006; Abraham and Mackie, 2006). This tack does imply substitution between damage and consumption (through the adjustment of potential income).

This assumption that damages are deducted from potential income manifests through the intercepts of the intertemporal budget constraint as shown in figure 1.

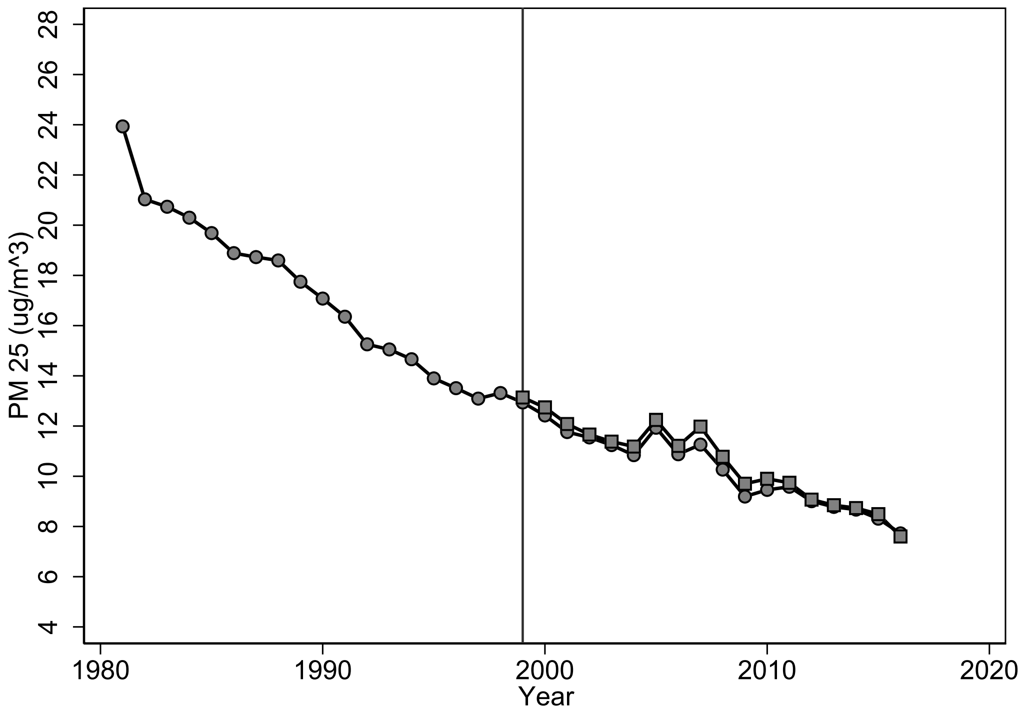
Supplementary Figures.

Figure A.1: Comparison of consumption, damage, and net consumption with r* and r_g.



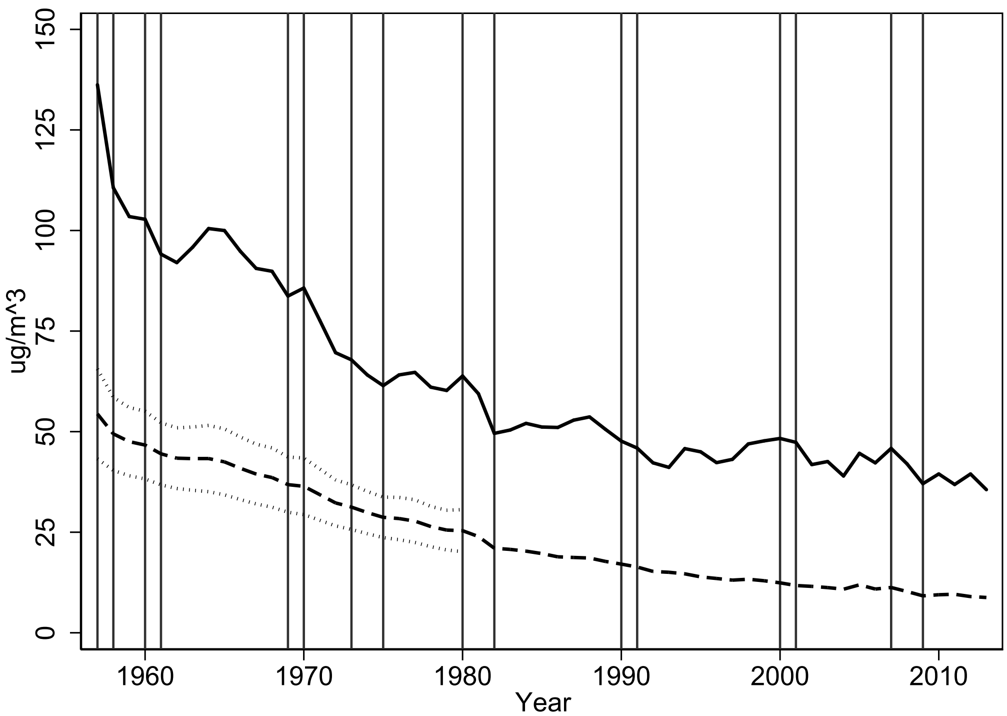
Left panel: damage intensity falling
Right panel: damage intensity rising.
I. = intertemporal budget constraint when representative household faces r_g. Slope = $-\left(\frac{1+r_g}{1+\rho}\right) = \frac{\Delta c_2}{\Delta c_2}$. II. = rate of change in damage as household reallocates consumption due to r_g. Slope = $\frac{\Delta c_2}{\Delta c_2}\frac{(\alpha_2-\beta_2\gamma_2)}{(\alpha_1-\beta_1\gamma_1)}$. III. = rate of change in net consumption as household reallocates consumption due to r_g. Slope = $\frac{\Delta c_2}{\Delta c_2}\frac{(1-(\alpha_2-\beta_2\gamma_2))}{(1-(\alpha_1-\beta_1\gamma_1))}$.

Figure A.2: Annual Average $PM_{2.5}$ Concentrations: Satellite-Monitor Comparison.



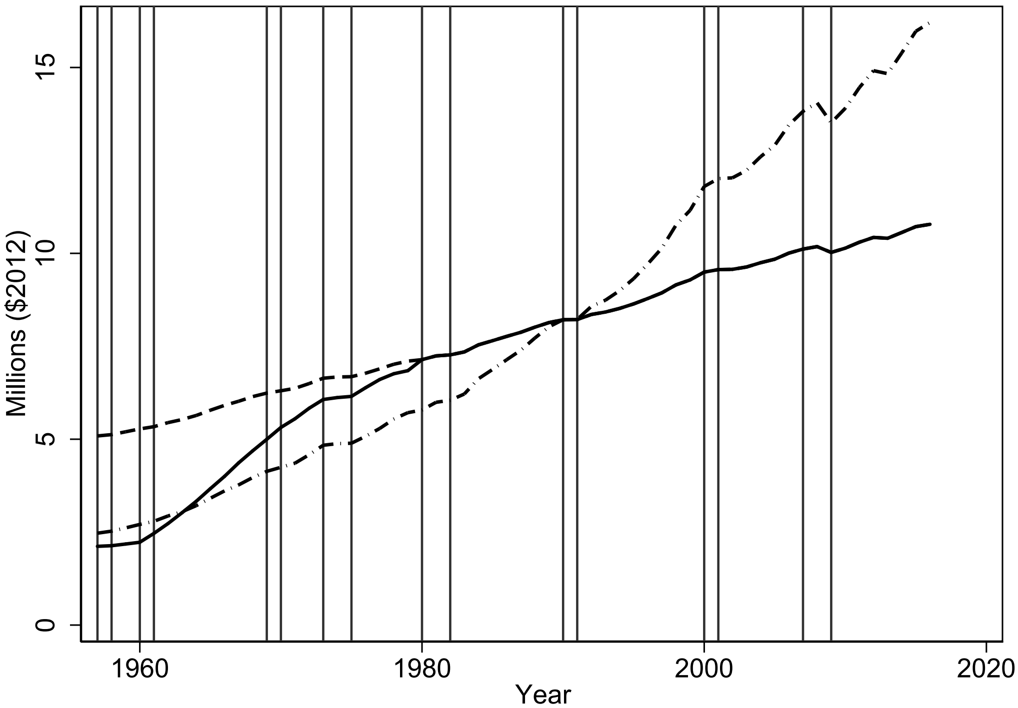
Squares = USEPA AQS Monitor Data, Circles = $PM_{2.5}$ Data from Meng et al., (2019).
Source: Muller (2019a).

Figure A.3: TSP and $PM_{2.5}$ National Average Concentrations.



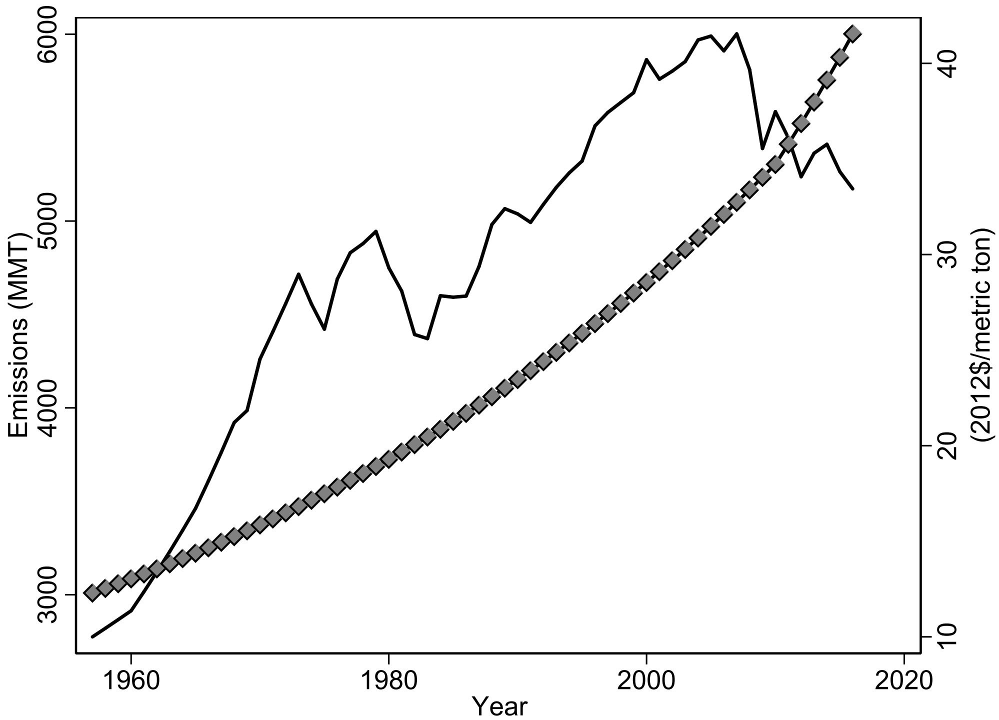
Dash = $PM_{2.5}$ (95% Confidence intervals on predicted values prior to 1980); Solid = TSP
Vertical lines demarcate NBER recessions.
Source: Muller (2019a).

Figure A.4: VSL under various assumptions.



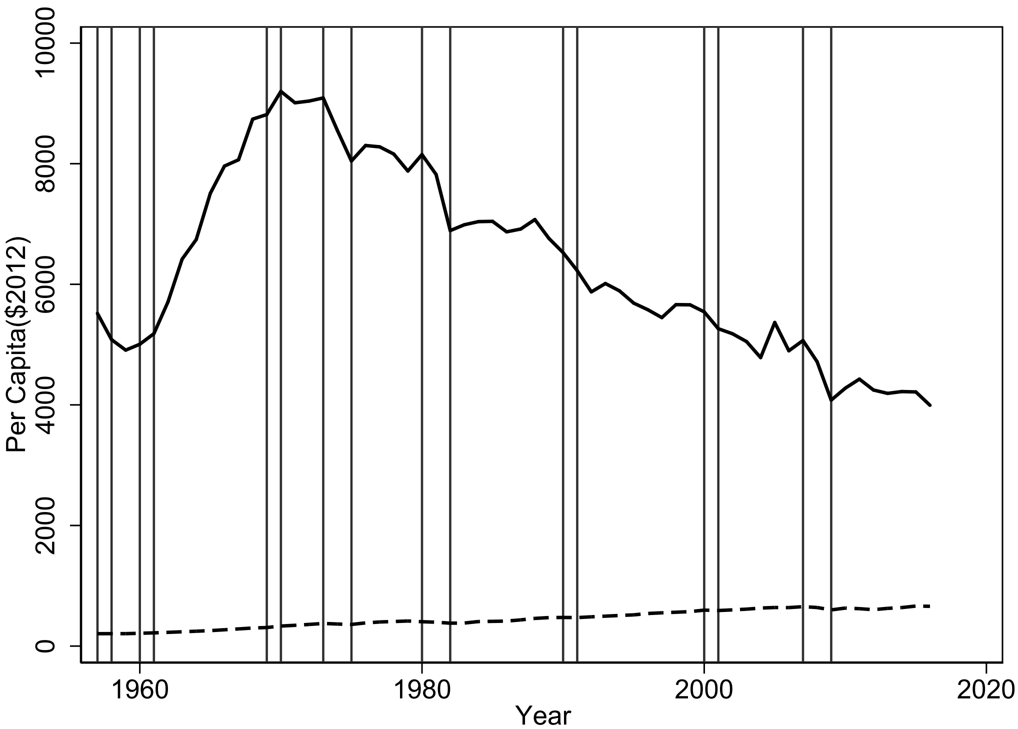
Solid: default VSL used in the present analysis.
Dash: VSL-income elasticity = 0.4
Dash-dot: VSL-income elasticity = 1.0
Source: Muller (2019a).

Figure A.5: U.S. Economy CO_2 Emissions and Social Cost of Carbon Estimates.



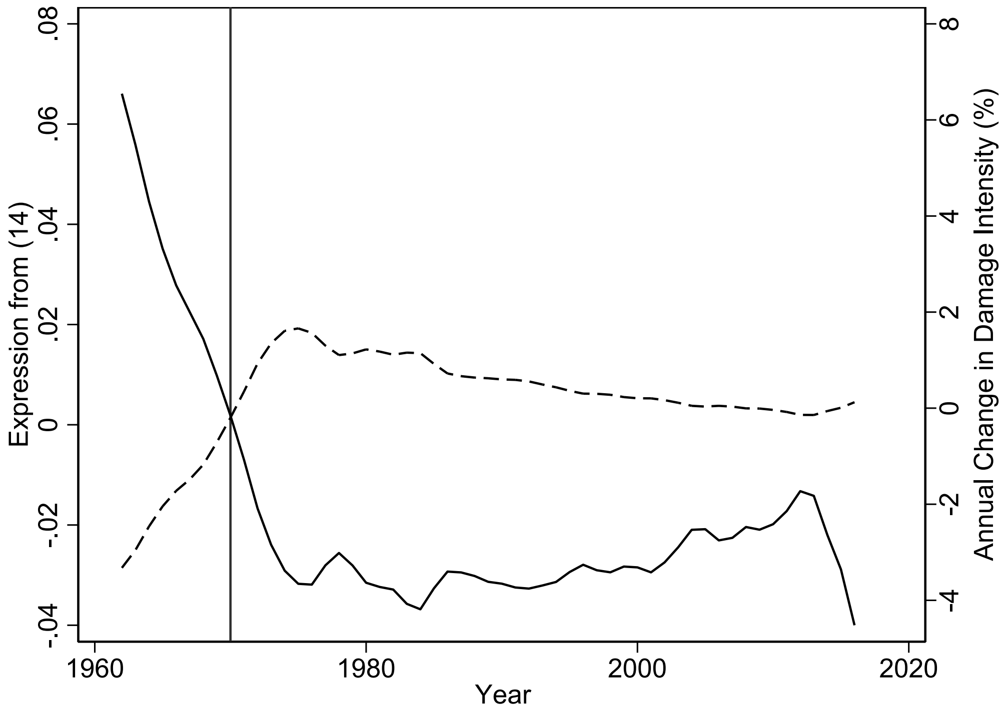
Solid line = CO_2 emission estimates.
Squares = SCC.
Source: USDOE, 2011; 2019; USFWG, 2016

Figure A.6: Real, Per Capita Air Pollution and CO_2 GED.



Solid = air pollution; Dash = CO_2.
Source: Author's calculations.

Figure A.7: Proportional Changes in Pollution Intensity.



Dash: $\left(\frac{((\alpha_{t-1}-\beta_{t-1}\gamma_{t-1})-(\alpha_t-\beta_t\gamma_t))}{(1-(\alpha_{t-1}-\beta_{t-1}\gamma_{t-1}))}\right)$.

Solid: $100 \times \left(\frac{(\alpha_t-\beta_t\gamma_t)}{(\alpha_{t-1}-\beta_{t-1}\gamma_{t-1})} - 1\right)$.

Vertical line = 1970.

Source: Author’s calculations.

Supplementary Tables.

Table A.1: Growth Rates and the Business Cycle.

	(1) Total sample	(2) Recession	(3) Expansion	(4) Recession Pre-1970	(5) Expansion Pre-1970	(6) Recession Post-1970	(7) Expansion Post-1970
GDP	1.90[A]	-0.35	2.81	0.06	3.81	-0.47	2.57
	(2.105)[B]	(2.134)	(1.249)	(2.732)	(1.209)	(2.034)	(1.153)
EVA	2.39	0.36	3.21	0.36	2.65	0.37	3.34
	(2.346)	(2.497)	(1.723)	(2.133)	(2.189)	(2.678)	(1.606)
GED	-0.55	-3.20	0.53	-0.49	6.10	-4.03	-0.79
	(5.338)	(5.742)	(4.832)	(5.266)	(5.153)	(5.815)	(3.751)
N	59	17	42	4	8	13	34

A = mean rate.
B = sd in parentheses.