# Online Appendix for

# "In Search of the Origins of Financial Fluctuations: The Inelastic Market Hypothesis"

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# C Data Sources and Construction

## C.1 Sector-level data from the Flow of Funds

We summarize the adjustments we make to the FoF data and the precise mapping to our model.

### C.1.1 Data items

We use Corporate Equities (Table 223) for equities. For fixed income, we take the sum of Treasury Securities (Table 210) and Corporate and Foreign Bonds (Table 213). We use unadjusted flows (FU) and, for the levels, we use the unadjusted market values when available (LM) and otherwise the estimated level (FL). Net issuances are equal to the total aggregate flow. The Flow of Funds revises historical data every quarter and we use the June 2019 vintage of the data.

### C.1.2 Notation and data definitions

Sectors are indexed by $i = 1, ..., I$, where $i = Foreign$ refers to the foreign sector. We observe holdings of equities, $W^{\mathcal{E}}_{it}$, Treasuries, $Tr_{it}$, and corporate bonds, $C_{it}$. We refer to the sum of Treasuries and corporate bonds as bonds, $B_{it} = Tr_{it} + C_{it}$. The flows corresponding to each asset class are denoted by $\Delta F^a_{it}$, $a = W^{\mathcal{E}}, Tr, C, B$. Aggregate levels and flows omit the subscript $i$, implying, for instance, for equities $\sum_i W^{\mathcal{E}}_{it} = W^{\mathcal{E}}_t$ and for bonds $B_t = \sum_i B_{it}$. Lastly, the gross capital gain for equities is denoted by $R^X_t$ and the return inclusive of dividend payments is denoted by $R_t$. We define the total assets of sector $i$ as $W_{it} = W^{\mathcal{E}}_{it} + B_{it}$. Net issuances, $ni_t = \frac{NI_t}{W^{\mathcal{E}}_{t-1}}$, are based on equity markets.

In the FoF, equity flows are defined by $\Delta F^{\mathcal{E}}_{it} = W^{\mathcal{E}}_{it} - W^{\mathcal{E}}_{i,t-1} R^X_t$.[75] We assume in what follows that the securities are adjusted at the end of the quarter, $\Delta F^a_{it} = \Delta F^a_{it} = (\Delta Q^a_{it}) P^a_t, a = \mathcal{E}, B$. The total per-period flow is $\Delta F_{it} = \Delta F^{\mathcal{E}}_{it} + \Delta F^B_{it}$ and in relative terms $\Delta f^{\mathcal{E}}_{it} = \frac{\Delta F^{\mathcal{E}}_{it}}{W^{\mathcal{E}}_{i,t-1}}$ for equities

[75]When possible, the FoF also follows this definition in other classes and has moved to market values for fixed income securities as well. However, in some cases, investors report holdings at book value for fixed income and no direct data on purchases are available, in which case flows are impacted by valuation effects.

and $\Delta f_{it}^B = \frac{\Delta F_{it}^B}{B_{i,t-1}}$ for bonds. The proportional per-period total flow is given by $\Delta f_{it} = \frac{\Delta F_{it}}{W_{i,t-1}}$. The equity shares are $S_{it} = \frac{W_{it}^{\mathcal{E}}}{W_t^{\mathcal{E}}}$. The relative change in equity demand, adjusted for price effects, is given by $\Delta q_{it}^{\mathcal{E}} = \Delta f_{it}^{\mathcal{E}} \left(R_t^X\right)^{-1} = \frac{\Delta Q_{it}^{\mathcal{E}}}{Q_{i,t-1}^{\mathcal{E}}}$. The aggregate per-period flow measure is defined as $\Delta f_{St} = \sum_i S_{i,t-1} \Delta f_{it}$.

In the remainder of this subsection, we summarize the adjustments we make to the raw data to account for measurement challenges in the data. In every step, we make sure that the market clearing conditions hold for both levels and flows.

### C.1.3 Adjustment for foreign holdings of equity and corporate bonds

The FoF reports total flows and holdings of corporate equities and corporate bonds, including foreign assets held by US investors. As we are interested in measuring the flow into the US equity market, we adjust the holdings and flows for foreign positions. Unfortunately, we do not know the holdings and flows of foreign assets by sector, but we do know the aggregate positions across investors. We discuss our measurement approach in the context of equities, but we apply the same procedure to corporate bonds.[76]

Let $W_{it}^{\mathcal{E},j}$ be the equity holdings of sector $i$ in period $t$ for $j = D, F, T$, that is, the investment in domestic ($D$) and in foreign ($F$) securities as well as their total ($T$). We define the set of all US institutions by $US$. We define $x_{US,t} := \sum_{i \in US} x_{it}$ for $x = W^{\mathcal{E}}, \Delta F^{\mathcal{E}}$, that is, for equity levels and equity flows.

We start from the following identities, for $j = D, F, T$,

$$x_{it}^D + x_{it}^F = x_{it}^T, \tag{68}$$

$$W_{it}^{\mathcal{E},j} = W_{i,t-1}^{\mathcal{E},j} R_t^{X,j} + \Delta F_{it}^{\mathcal{E},j}, \tag{69}$$

where $R_t^{X,j}$ is the capital gain as before. We observe $x_t^D$, $x_t^F$, $x_{it}^T$ for $x = W^{\mathcal{E}}, \Delta F^{\mathcal{E}}$. We assume that the capital gain that different investors earn in the US is the same across investors (that is, $R_{it}^{X,D} = R_t^{X,D}$), and we make the same assumption for the capital gain on foreign investments (that is, $R_{it}^{X,F} = R_t^{X,F}$).

We assume for all US institutions, $i \in US$, that their equity holdings are split in the same way across foreign and domestic equities:

$$W_{it}^{\mathcal{E},D} = \phi_t W_{it}^{\mathcal{E},T}, \forall i \in US.$$

It then follows that

$$\phi_t = \frac{W_{US,t}^{\mathcal{E},D}}{W_{US,t}^{\mathcal{E},T}} = 1 - \frac{W_{US,t}^{\mathcal{E},F}}{W_{US,t}^{\mathcal{E},T}},$$

where $W_{US,t}^{\mathcal{E},F}$ and $W_{US,t}^{\mathcal{E},T}$ can directly be observed in the FoF. This measures $\phi_t$.

For flows, we assume that

$$\Delta F_{it}^{\mathcal{E},D} = W_{i,t-1}^{\mathcal{E},D} \eta_t^D + \phi_{t-1} \Delta F_{it}^{\mathcal{E},T},$$

[76] As US Treasuries are only issued in the US, obviously no adjustment is required for US Treasuries.

where $\eta_t^D$ is a taste shock that we assume to be common across investors and it impacts investors in proportion to their position in the previous period. Aggregating across all US institutions implies

$$\Delta F_{US,t}^{\mathcal{E},D} = W_{US,t-1}^{\mathcal{E},D}\eta_t^D + \phi_{t-1}\Delta F_{US,t}^{\mathcal{E},T},$$

implying

$$\eta_t^D = \frac{\Delta F_{US,t}^{\mathcal{E},D} - \phi_{t-1}\Delta F_{US,t}^{\mathcal{E},T}}{W_{US,t-1}^{\mathcal{E},D}} = \frac{(1-\phi_{t-1})\,\Delta F_{US,t}^{\mathcal{E},T} - \Delta F_{US,t}^{\mathcal{E},F}}{W_{US,t-1}^{\mathcal{E},T} - W_{US,t-1}^{\mathcal{E},F}},$$

which can be computed directly from the FoF. With $\eta_t$ and $\phi_t$ in hand, we compute the estimate of domestic equity holdings and flows. We also adjust aggregate flows and levels to ensure that market clearing holds.

### C.1.4 The impact of the 2008-2009 financial crisis

Three sectors have non-zero equity holdings only since the 2008-2009 financial crisis: the Federal Government (sector 31), the Monetary Authority (sector 50), and Funding Corporations (sector 71). These positions are all associated with the federal financial stabilization programs. We describe the adjustments we make to these series.

The holdings of the Federal Government are derived from "corporate equities issued by commercial banking under the federal financial stabilization programs," "corporate equities issued by funding corporations (AIG) under the federal financial stabilization programs," "corporate equities issued by bank-holding companies (GMAC) under the federal financial stabilization programs," and "corporate equities issued by GSEs under the federal financial stabilization programs." From 2009.Q4 - 2011.Q1, Funding Corporations and the Monetary Authority record the exact same equity holdings. Their holdings are zero elsewhere. It is only a small position, and it comes from the way the AIG bailout was structured (per correspondence with economists at the FoF). The holdings are described as "Federal Reserve Bank of New York's Preferred Interests in AIA Aurora LLC and ALICO Holdings LLC." Both are life insurance subsidiaries of AIG.

The dynamics of the levels are plotted in the left panel of Figure C.1 from 1993.Q1 to 2018.Q4. The dynamics of net issuances alongside the flows associated with the three sectors are plotted in the right panel of Figure C.1 from 2008.Q1 to 2014.Q4. The flows from funding corporations and the monetary authority are identical and cannot be distinguished visually. As can be seen from the graph, the stabilization programs created a spike in net issuances and these issuances are not absorbed by the typical investor sectors.

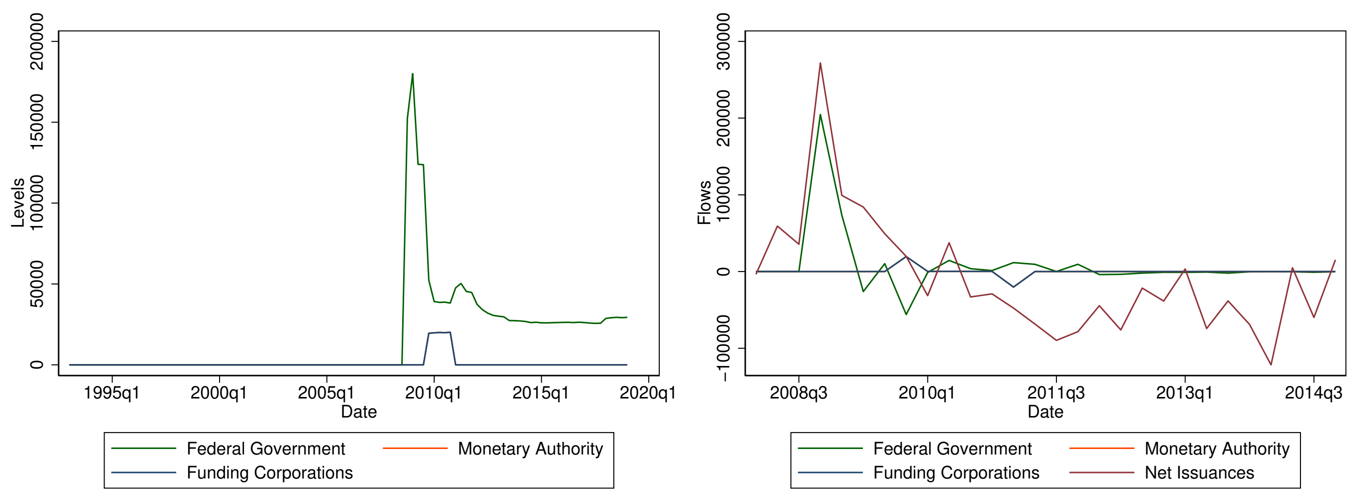
We aggregate the flows of these three sectors and subtract them from net issuances. We adjust the levels as well, and then remove these three sectors from our analysis.

### C.1.5 Foreign banking offices in the US and non-financial corporate business holdings

We make adjustments for two additional sectors. First, the sector Foreign Banking Offices in the US (sector 75) has zero flows since 1993 in most periods, see the left panel of Figure C.2. Second, for the asset holdings of non-financial corporate businesses (sector 10), the quarterly flows are poorly measured, see the right panel of Figure C.2 showing the series from 1993.Q1 to 2018.Q4. The reason is that the FoF interpolates annual flows.[77] These flows and holdings reflect firms' holdings of other firms' equity, for instance for strategic or speculative reasons. Prior to the September 2018

[77]For details of the current procedure, please see here.

Figure C.1: Equity levels and flows around the 2008-2009 financial crisis. The left panel shows the levels for the Federal government, the monetary authority, and funding corporations. The last two sectors have identical holdings and flows, and are therefore visually indistinguishable. The sample is from 1993.Q1 to 2018.Q4. The right panel reports the flows associated with the same sectors as well as net issuances from 2008.Q1 to 2014.Q4. Levels and flows are in expressed in millions of nominal dollars.



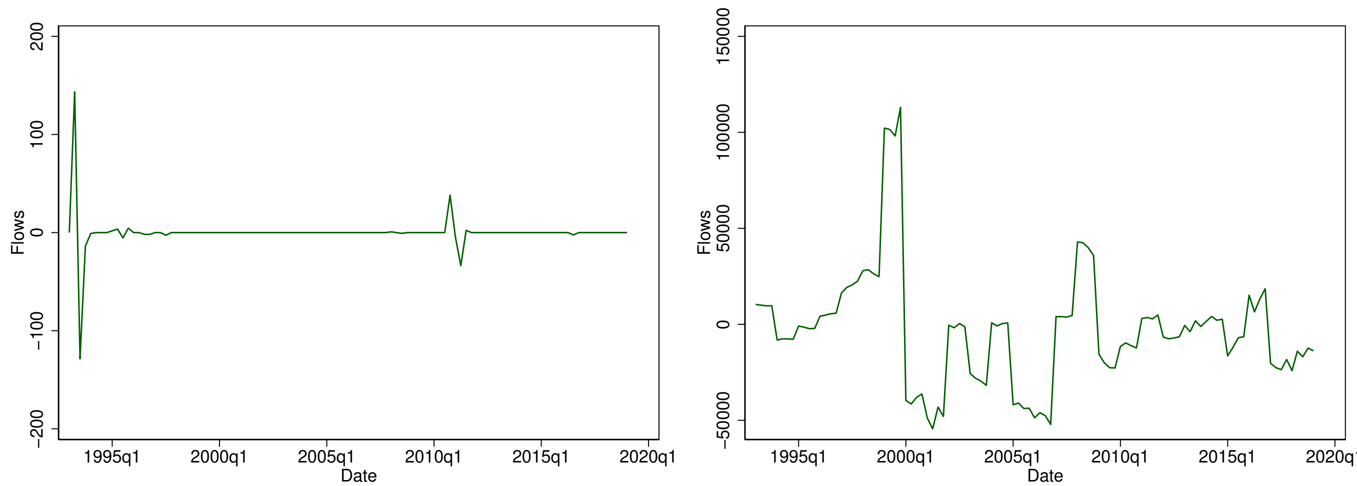

publication, the FoF showed the equity liability of the non-financial corporate sector net of these inter-corporate equity investments. The current release added the inter-corporate holdings as an asset and a liability. We undo this adjustment. For both sectors, we subtract the flows from net issuances and adjust the levels accordingly.

### C.1.6 Examples of measurement issues

Even though the FoF data are the best data to use for both equity and fixed income holdings, certain measurement issues remain. We list them here and perhaps future research can refine some of our calculations. First, in the FoF, shares issued by ETFs, closed-end funds, and real estate investment trusts (REITS) are included in the corporate equities instrument category. This may impact the net issuance statistics, for instance. Most investor sectors do not separately report on holdings of ETFs, for instance, versus direct investments. As a result, we cannot adjust the holdings. Similarly, the total holdings include closely held equity. While the supply side is separated, we do not have disaggregated holdings, which implies we cannot adjust for this on the demand side.

We start our sample in 1993. In part, this starting date is driven by the fact that institutional ownership has been rising, and this allows us to disaggregate a large fraction of the holdings. Also, the dynamics of equity flows, $\Delta q$, also looks more erratic in the earlier years. In Figure C.3, we plot the dynamics of equity flows across sectors to illustrate this issue. Lastly, ETFs become available in 1993. ETFs have been growing since then, in part to replace mutual funds, and we merge ETFs and mutual funds for parts of our analysis.

Figure C.2: Flows of Foreign Banking Offices in the US and Non-financial Corporate Businesses. The left panel shows the flows for Foreign Banking Offices in the US and the right panel for Non-financial Corporate Businesses. The sample is from 1993.Q1 to 2018.Q4. Flows are in expressed in millions of nominal dollars.




### C.1.7 Sample construction of the data for the GIV estimation

Before implementing the GIV procedure, we make two adjustments to the data to mitigate the impact of outliers. First, we merge the mutual fund and ETF sectors. ETFs were introduced in 1993, which is the start of our sample. The initial flows are very volatile, but their share of the overall market was small. This volatility gradually dissipates as the sector grows, in part at the expense of mutual funds. The volatility of the combined sector is much more stable over time.

Second, we winsorize the data by first removing the time-series median of each series, which is a robust way to remove differences in the levels of the series. We then winsorize the data across time and sectors at the 5%- and 95%-percentiles for the period from 1993.Q1 to 2006.Q4 to mitigate the influence of outliers. This avoids the need to winsorize the data during the financial crisis and the larger, as well as in the case of the more volatile flows happening during the earlier part of the sample. Winsorizing the data unconditionally does not impact our results much as we show in Section D.4. We exclude the corporate sector in winsorizing the data.

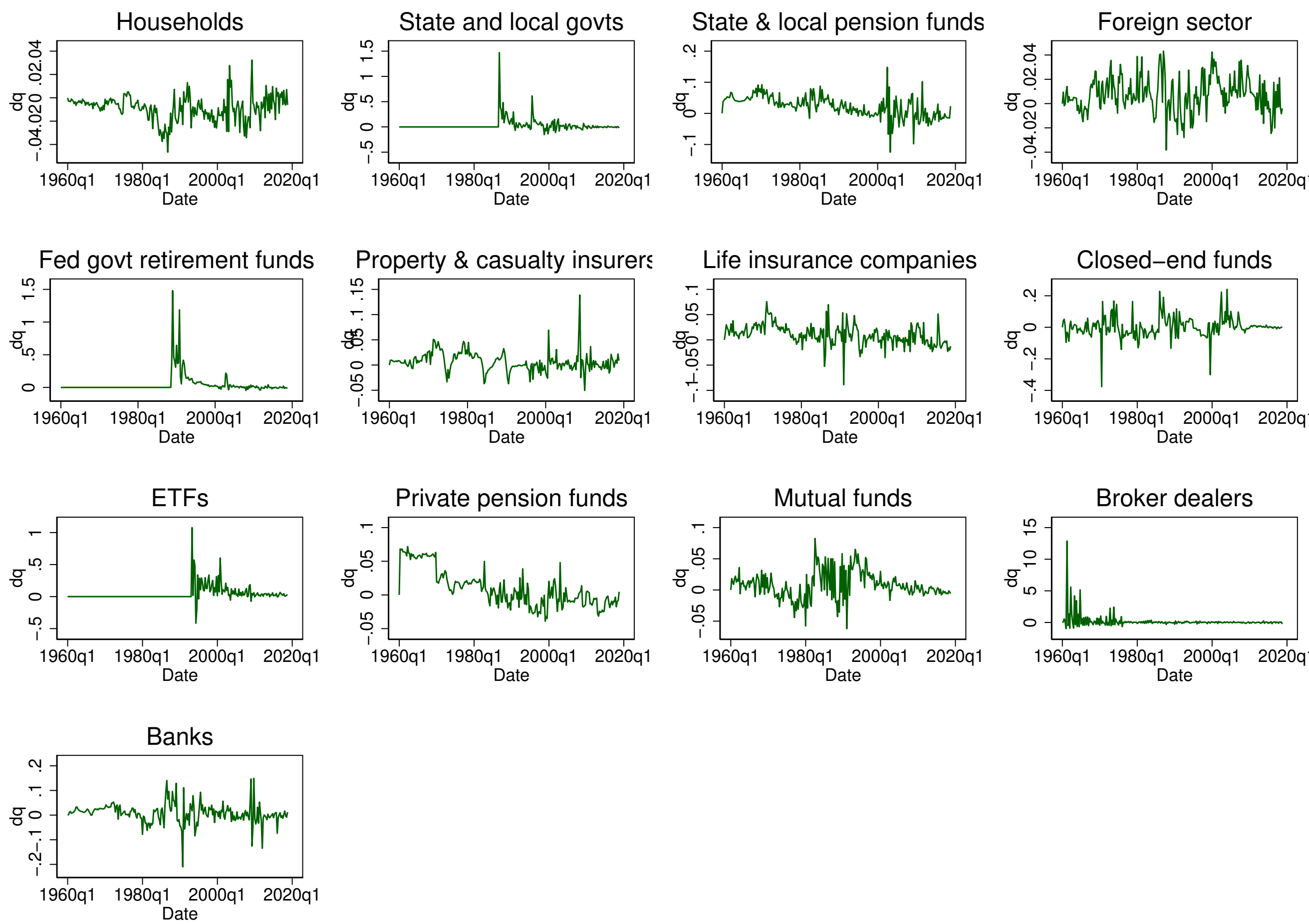
## C.2 Investor-level data from 13F filings

We summarize the construction of the 13F data in this section.

### C.2.1 Data construction

We source the 13F data from FactSet following Koijen et al. (2019). We start from the holdings data (table `own_inst_13f_detail_eq`) and we aggregate the holdings by roll-up entity (using table `own_ent_13f_combined_inst`). This combines filers that FactSet assigns as subsidiaries of the same investor. In addition, we aggregate the subsidiaries of BlackRock based on their names into a single entity. We identify an investor's type using table `own_ent_institutions`. We compute the market capitalization and holdings using the adjusted variables in FactSet (variables `adj_price`,

Figure C.3: Dynamics of equity flows across sectors. The figure shows the equity flows, $\Delta q_{it}^{\mathcal{E}}$, for the final 13 sectors in our sample from 1960.Q1 to 2018.Q4.



`adj_shares_outstanding`, and `adj_mv`). In some rare cases, the total holdings exceeds the shares outstanding, which may be due to short-selling activity or filing errors. In these cases, we scale the holdings of all investors for a particular security to ensure that the market clearing condition holds. We then merge these data with the CRSP-Compustat merged data using CUSIPs. We select the securities with share code 10 or 11 and an exchange code equal to 1, 2 or 3.

### C.2.2 Measuring changes in equity demand

We first discuss how we construct changes in equity demand, $\Delta q_{it}$. We denote by $H_{iat} = Q_{iat}P_{at}$ investor $i$'s dollar holdings of security $a$ at time $t$, where time $t$ corresponds to the last day of the quarter. Total equity holdings are given by $\mathcal{E}_{it} = \sum_a H_{iat}$. We also define $\mathcal{E}_{it}^{-} = \sum_a H_{iat}^{-}$, where $H_{iat}^{-} = \frac{H_{iat}}{1+R_{at}^X}$, where $R_{at}^X$ is the capital gain. In the absence of (reverse) splits, it holds $H_{iat}^{-} = Q_{iat}P_{a,t-1}$. We now define the change in equity demand as

$$\Delta q_{it} = \frac{\mathcal{E}_{it}^{-} - \mathcal{E}_{i,t-1}}{\mathcal{E}_{it}^{*}},$$

where $\mathcal{E}_{it}^{*} = \frac{1}{2}\left(\mathcal{E}_{it}^{-} + \mathcal{E}_{i,t-1}\right)$, which implies $\Delta q_{it} \in [-2, 2]$. This measure of flows is less sensitive to outliers than the alternative measure that uses only $\mathcal{E}_{i,t-1}$ in the denominator, as in e.g. Davis and Haltiwanger (1992).

### C.2.3 Constructing characteristics

Next, we discuss how we construct the characteristics that we use in Section B.2. We define the following characteristics

1. Log investor size, $\ln \mathcal{E}_{it}^{*}$.

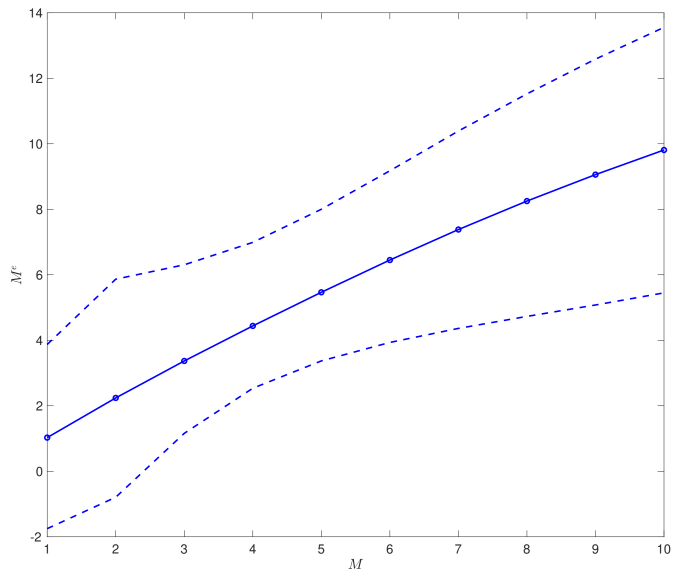
2. Active share, which is defined as
$$\frac{1}{2}\sum_a \left|\theta_{iat} - \theta_{iat}^{m}\right|,$$
where $\theta_{iat}$ is the portfolio share and $\theta_{iat}^{m}$ the market-weighted portfolio of securities held by investor $i$ at time $t$.

These characteristics define $x_{it}$, and we use their lagged values, $x_{i,t-1}$, to extract the factors.

### C.2.4 Sample selection

We use 13F data to extract common factors, $\eta_t$, based on investors outside of the mutual fund industry using the same assignment of investor types as in Koijen et al. (2019). To mitigate the impact of outliers, we focus on the largest 1,000 investors in each period. Also, we remove fund-quarter observations for which $|\Delta q_{it}| > 3\sigma_i$, where $\sigma_i = IQR_i/1.35$, where $IQR_i$ is the interquartile range, a robust estimator of the standard deviation. We compute $IQR_i$ using the full sample for a given investor. Lastly, we keep investors for which we have at least 20 observations after imposing these screens.

Figure D.4: Simulation results. The horizontal axis shows the multiplier in the data generating process and the vertical axis the average estimated multiplier, alongside the 2.5% and 97.5% percentiles, across 50,000 replications. We use the same sample size as in the empirical application in the next section. The text provides further details.




# D Additional Empirical Results

## D.1 Performance of the GIV Estimator: Simulations

We illustrate the performance of the GIV estimator in an environment that closely mimics our empirical setting. We refer to Gabaix and Koijen (2020) for a more extended analysis of the GIV estimator. In particular, we proceed as follows to construct a realistic set of simulations. We start from our original sample and pick a value of $\zeta$ that we vary from $\zeta = 1$ (a multiplier of 1, a typical estimate for the micro elasticity) to $\zeta = 0.1$ (a multiplier of 10). For each value of $\zeta$, we construct $f_t^\nu = \Delta q_t + \zeta \Delta p_t$. We then assume that the data follow a one-factor factor model and estimate $f_t^\nu = \lambda \eta_t + u_t$ using principal components analysis. This provides us with an estimate of $\lambda$ and $\sigma = \sigma(u_t)$. These are the estimates one would obtain given the data we observed historically and if the true elasticity were equal to the assumed value. It tells us the volatility of aggregate shocks ($\mu_\lambda = \iota' \lambda / N$), the average volatility of idiosyncratic shocks ($\mu_{\ln \sigma} = \iota' \ln \sigma / N$), and the dispersion in these parameters across investors ($\sigma_\lambda = \sigma(\lambda)$ and $\sigma_{\ln \sigma} = \sigma(\ln \sigma)$).

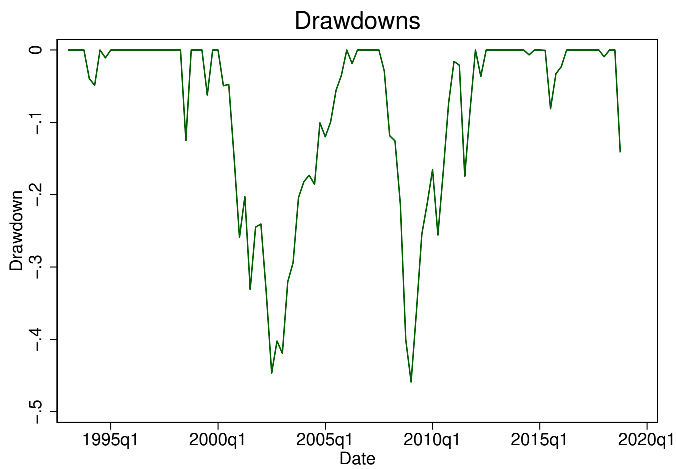
To simulate the data, we assume that $\lambda \sim N\left(\mu_\lambda, \sigma_\lambda^2\right)$, $\ln \sigma \sim N\left(\mu_{\ln \sigma}, \sigma_{\ln \sigma}^2\right)$, and that the shocks are normally distributed. In doing so, we ensure that the volatility of prices is the same as in the data. Throughout, we use the same size distribution across sectors, which on average equals the one that we observe empirically. We then follow the standard procedure to estimate the multiplier.

We consider 50,000 replications for each value of $\zeta = 0.1, 0.2, \ldots, 1$ and report the average estimate alongside the 2.5% and 97.5% percentiles across all replications in Figure D.4. We report the multiplier $M$ corresponding to the true data-generating process on the horizontal axis and the distribution of the estimated multipliers, $M^e$, on the vertical axis. The key takeaway is that our estimates uncover the true multipliers accurately with the dimensions of $N$ and $T$ that we observe empirically.

## D.2 Drawdown dynamics

In Figure D.5, we plot the drawdowns, defined as the decline in the cumulative stock market index relative to its maximum so far, of the CRSP value-weighted index. We use these drawdowns to date recessions that we study in Section 2.

Figure D.5: The figure illustrates the drawdowns of the US stock market from 1993.Q1 to 2018.Q4. Drawdowns are defined as the ratio of the cumulative return index relative to its running maximum minus one.



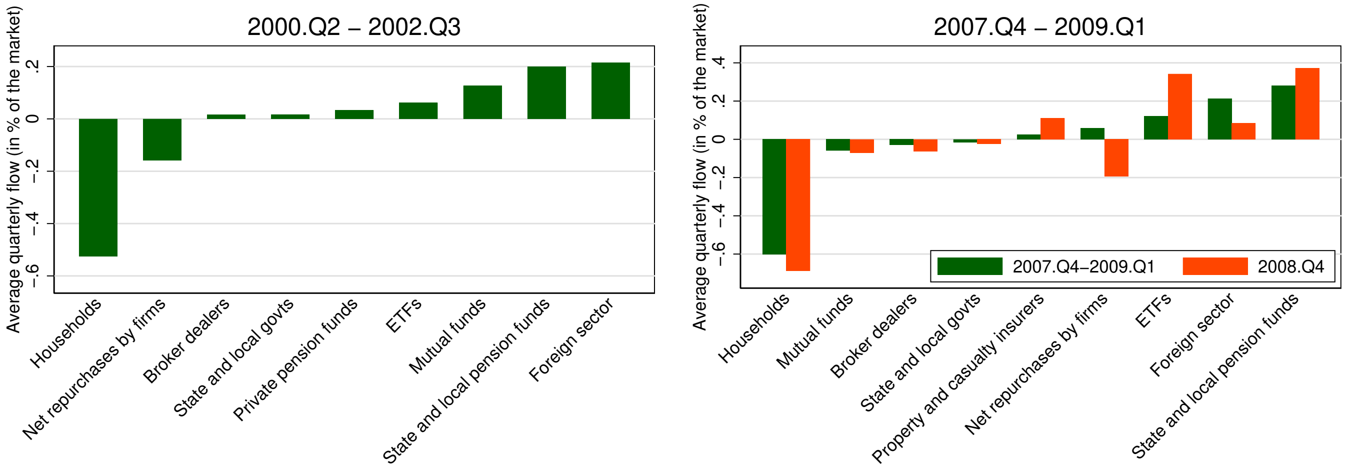

## D.3 Flows across investor classes are small

If market fluctuations are the result of small demand or flow shocks hitting macro inelastic markets, studying extreme episodes may provide a hint as to which investor sectors have volatile demand and flow shocks and which investor sectors provide elasticity to the market. We therefore consider a case study of the two largest equity downturns in our sample, namely from 2000.Q2 to 2002.Q3 (the technology crash) and from 2007.Q4 to 2009.Q1 (the 2008 global financial crisis), as shown in Appendix D.2. To measure equity flows, we scale the dollar equity flows for each sector $j$, $\Delta F_{jt}^{\mathcal{E}}$, by the size of the aggregate market in the previous quarter, $\mathcal{E}_{t-1}$, $\frac{\Delta F_{jt}^{\mathcal{E}}}{\mathcal{E}_{t-1}}$. We remove the mechanical effects due to revaluation, so that we show only the "active" flows. We then average the percent flow by sector across quarters for a given downturn.

The left panel of Figure D.6 corresponds to the tech crash and the right panel to the 2008 financial crisis. In the case of the 2008 financial crisis, we separately report the results for 2008.Q4, which is the worst quarterly return in our sample. In both cases, we select the eight sectors with the largest absolute flows as well as the corporate sector. While the total equity risk reallocation, on average per quarter, remains small, households sell about 0.5% of the market per quarter.[78]

[78]We emphasize once more that the household sector in the FoF includes institutional investors such as hedge funds and non-profits (e.g., endowments), as it is computed as a residual.

Figure D.6: The figure illustrates the rebalancing of investors during drawdowns of the US stock market from 1993.Q1 to 2018.Q4. The left panel summarizes the data from the tech crash (from 2000.Q2 to 2002.Q3) and the right panel from the 2008 global financial crisis (from 2007.Q4 to 2009.Q1). We plot the average quarterly rebalancing by sector expressed as a fraction of the total market capitalization (expressed in %). In the right panel, we also replicate the calculation for the fourth quarter of 2008, which is the most negative quarterly return in our sample. In all cases, we select the eight sectors with the largest absolute flows as well as the corporate sector.




During the 2008 financial crisis, net repurchases by firms fell (as firms cut their share buybacks in bad times) and indeed turned negative, implying that they issued equity. If we zoom in on 2008.Q4, we see large issuances (for instance by financial firms, in part forced by the government to issue shares),[79] which may have further amplified the market decline if the market is inelastic.

Who is providing elasticity to the market during these episodes? Quite surprisingly, the foreign sector as well as state and local pension funds are the sectors purchasing the most during each of the episodes. For the pension funds, this may reflect their mandate to maintain a fixed-share strategy instead of a conscious effort to time the market (see Proposition 2).[80]

The flows across sectors are not only small during downturns, but also on average. To assess the magnitude of equity risk reallocation across sectors, we compute $y_t^{Gross} = \frac{\sum \left|\Delta F_{jt}^{\mathcal{E}}\right| + \left|\Delta F_t^{Firm}\right|}{2\mathcal{E}_{t-1}}$, where $\Delta F_t^{Firm}$ denotes net issuances of equity by firms. We divide the measure by two as for every buyer of \$1 of equity, there is a seller of the same amount. As some of the flows are associated with net repurchases, we separately measure the equity risk "creation" and "redemption" as a result of such corporate actions via $y_t^{AbsNet} = \frac{\left|\Delta F_t^{Firm}\right|}{\mathcal{E}_{t-1}}$, which we will refer to as absolute net flows.

The average absolute net flow equals 0.30% per quarter and the average gross flows average to 0.87% per quarter for the period from 1993.Q1 to 2018.Q4. The standard deviations are 0.26% and 0.37%, respectively. The difference between the series measures the risk reallocation in equity

[79]During this period, several firms received support from the government. In the FoF, new sectors were created that otherwise hold no equity positions. We adjust net repurchases and flows for these sectors in order not to distort our calculations; see Appendix C for details.

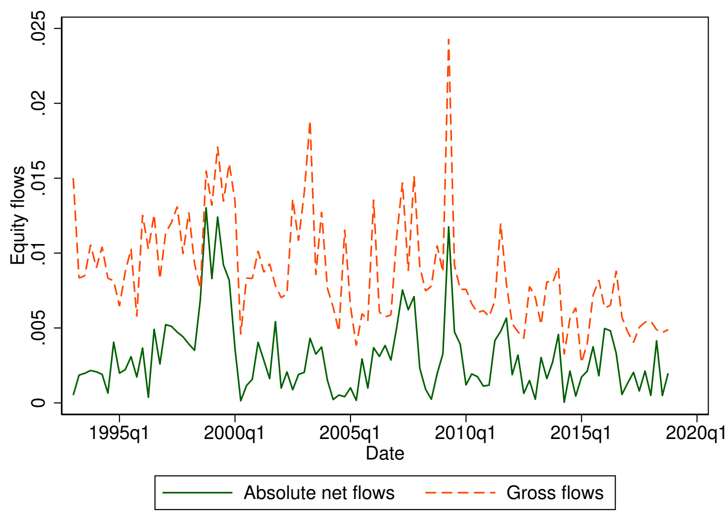
[80]Relatedly, Timmer (2018) finds that in German data, banks (broker dealers) sell when stock prices fall, and pension funds buy.

markets across institutional sectors, which averages to approximately 0.6% per quarter.[81] We plot the time series of both measures in Figure D.7 for the period from 1993.Q1 to 2018.Q4. The key takeaway is that the amount of equity risk that gets reallocated across sectors is small. These small flows contrast with the high levels of trading volume that are observed. However, much of this trading activity is at the single stock level, that is, exchanging stock A for stock B, instead of movements in or out of the stock market.

Small flows are not necessarily inconsistent with elastic markets. Many modern asset pricing models do not feature any trade. However, in the presence of volatile preference or belief shocks, this evidence implies that investors must experience the same shocks to preferences or beliefs, and have virtually the same exposure to these shocks, as otherwise we would see large flows across sectors.

In addition to quantities alone, Appendix J.2 also provides some additional first evidence on the link between flows and prices. Indeed, the demand by households (including mutual funds and ETFs) is positively correlated with price changes while the demand of the other sectors is strongly negatively correlated with price changes. This is consistent with the inelastic markets hypothesis in which shocks from the household sector, as defined by the FoF, lead to volatile prices as market are inelastic.

Figure D.7: The figure illustrates the reallocation of equity risk across various institutional sectors. The gross and net flow are defined in the main text. The sample is from 1993.Q1 to 2018.Q4.




## D.4 GIV estimates using FoF data: Robustness

In this section, we illustrate the robustness of our estimate of the multiplier, $M$. Given the small number of sectors in the FoF data, we focus on the case with a single principal component (in addition to a factor with uniform loadings). In Table D.8, we first repeat the benchmark estimate as a point of reference in the first column. In the second column, we omit all principal components.

[81]This number is an upper bound to the extent that we care about the aggregate market elasticity as some of the flows between sectors are low-frequency time trends such as the shift from pension funds to mutual funds in the nineties or the shift from mutual funds to ETFs during the last twenty years.

In the third column, we do not merge mutual funds and ETFs. In the fourth column, we do merge mutual funds and ETFs again, but now winsorize over the full sample from 1993.Q1 to 2018.Q4 instead of from 1993.Q1 to 2006.Q4 (and hence omit the financial crisis). In column five, we omit the time trend in (63). In column six, we control for the lagged value of $\Delta q_{jt}$ in (63) and we allow for heterogeneous slope coefficients across sectors on $\Delta q_{j,t-1}$. In the seventh column, we start the sample in 2000.Q1. In the last two columns, we change the winsorization of the regression weights to $\frac{1.25}{N_t}$ (Column 8) or $\frac{1.75}{N_t}$ (Column 9), see the first step of the GIV algorithm. The main takeaway is that the multiplier estimates are quite stable across the various specifications. The estimates of the multiplier vary between 5.1 and 8.0.

Table D.8: Robustness of the GIV estimates. The table reports estimates of the multiplier under different measurement assumptions. We first repeat the benchmark estimate as a point of reference in the first column from 1993.Q1 to 2018.Q4. In the second column, we omit all principal components. In the third column, we do not merge mutual funds and ETFs. In the fourth column, we do merge mutual funds and ETFs again, but now winsorize over the full sample from 1993.Q1 to 2018.Q4 instead of from 1993.Q1 to 2006.Q4. In column five, we omit the time trend. In column six, we control for the lagged value of $\Delta q_{jt}$ in (63) and we allow for heterogeneous persistence coefficients across sectors on $\Delta q_{j,t-1}$. In the seventh column, we start the sample in 2000.Q1. In the last two columns, we change the winsorization of the regression weights to $\frac{1.25}{N_t}$ (Column 8) or $\frac{1.75}{N_t}$ (Column 9). Standard errors that account for autocorrelation are reported in parentheses.

| | $\Delta p$ | $\Delta p$ | $\Delta p$ | $\Delta p$ | $\Delta p$ | $\Delta p$ | $\Delta p$ | $\Delta p$ | $\Delta p$ |
|---|---|---|---|---|---|---|---|---|---|
| Z | 7.08 | 8.00 | 6.94 | 7.65 | 6.63 | 6.77 | 6.79 | 7.17 | 5.12 |
| | (1.86) | (1.24) | (1.48) | (1.37) | (1.17) | (2.18) | (2.02) | (1.18) | (2.33) |
| GDP growth | 5.99 | 5.99 | 6.06 | 6.02 | 6.14 | 6.01 | 6.20 | 5.99 | 5.97 |
| | (0.69) | (0.59) | (0.66) | (0.75) | (0.69) | (0.73) | (0.74) | (0.62) | (0.80) |
| $\eta_1$ | 21.06 | | 22.09 | 15.59 | 32.54 | 25.24 | -30.65 | 14.44 | 26.23 |
| | (13.58) | | (11.11) | (11.58) | (6.40) | (13.64) | (15.65) | (10.71) | (15.45) |
| Constant | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) | (0.01) |
| Observations | 104 | 104 | 104 | 104 | 104 | 103 | 76 | 104 | 104 |

In Table D.9, we replicate Table 2, but now adding the lagged value of $Z_t$. Across various specifications, $Z_t$ has a small positive autocorrelation of approximately 10-15%. As is clear by comparing both sets of estimates, including a lag does not change the estimates in a meaningful way, and the lagged value of $Z_t$ is in all cases insignificant.

In Table D.10, we start from the benchmark results in the previous table and add additional principal components. Given that the cross-section is small (we only have 12 sectors once we merge mutual funds and ETFs), the data are not well suited to go beyond one or two principal components, unfortunately. Nevertheless, for transparency, we show the results up to five principal components for completeness in Table D.10. By adding additional principal components, idiosyncratic shocks will end up as factors, which makes it more challenging for us to identify the multiplier precisely. If we go beyond two principal components, the multiplier declines somewhat from 5.3 with two principal components to a range from 3.5 to 4.2 with three to five principal components.

Table D.9: Robustness of the GIV estimates for the Flow of Funds: Persistence in $Z$. The table replicates Table 2, but now adding the lagged value of $Z_t$. The sample is from 1993.Q1 to 2018.Q4. Standard errors are reported in parentheses.

| | $\Delta p$ | $\Delta p$ | $\Delta q_E$ | $\Delta q_E$ | $\Delta q_C$ | $\Delta q_C$ |
|---|---|---|---|---|---|---|
| Z | 7.41 | 5.62 | | | | |
| | (1.96) | (1.21) | | | | |
| $\Delta p$ | | | -0.12 | -0.16 | -0.01 | -0.01 |
| | | | (0.03) | (0.04) | (0.01) | (0.02) |
| Z (lag) | -1.59 | -1.75 | -0.30 | -0.40 | -0.02 | -0.02 |
| | (1.84) | (1.46) | (0.26) | (0.29) | (0.09) | (0.08) |
| GDP growth | 6.41 | 6.42 | 0.59 | 0.86 | 0.22 | 0.24 |
| | (0.93) | (0.90) | (0.25) | (0.24) | (0.13) | (0.12) |
| $\eta_1$ | 21.64 | 24.36 | 3.88 | 5.24 | -0.72 | -0.64 |
| | (13.74) | (13.10) | (1.58) | (2.42) | (0.62) | (0.66) |
| $\eta_2$ | | 30.11 | | 5.13 | | 0.28 |
| | | (6.02) | | (1.16) | | (0.80) |
| Constant | -0.01 | -0.02 | 0.00 | 0.00 | -0.00 | -0.00 |
| | (0.01) | (0.01) | (0.00) | (0.00) | (0.00) | (0.00) |
| Observations | 103 | 103 | 103 | 103 | 103 | 103 |

## D.5 The volatility of idiosyncratic demand shocks

In Table D.11, we report the standard deviation of sector-specific demand shocks, $u_{it}$.

## D.6 Dynamics of mutual fund flows

In Table D.12, we report the estimates of the dynamic model of mutual fund flows that we use in Section 4.3.

## D.7 The impact of mutual fund flows at longer horizons

In Figure D.8, we repeat the analysis as in Section 4.2 but now using mutual fund flow innovations as in Section 4.3. We estimate the same regression as in (38), but now $Z_t$ is constructed using data on mutual fund flows and $\eta_t$ is estimated using 13F data. We present the results in Figure D.8. As before the impact of flow shocks on prices is persistent although the confidence interval is wide at longer horizons of one year.

Table D.10: Robustness of the GIV estimates for the Flow of Funds: Additional principal components. The table adds principal components starting from a single principal component. The sample is from 1993.Q1 to 2018.Q4. Standard errors are reported in parentheses.

| | $\Delta p$ | $\Delta p$ | $\Delta p$ | $\Delta p$ | $\Delta p$ |
|---|---|---|---|---|---|
| Z | 7.08 | 5.28 | 3.89 | 4.23 | 3.46 |
| | (1.86) | (1.10) | (0.92) | (0.60) | (0.54) |
| GDP growth | 5.99 | 5.97 | 5.96 | 5.96 | 5.96 |
| | (0.69) | (0.67) | (0.64) | (0.51) | (0.51) |
| $\eta_1$ | 21.06 | 23.72 | 25.76 | 25.26 | 26.39 |
| | (13.58) | (12.79) | (7.26) | (7.66) | (8.36) |
| $\eta_2$ | | 29.95 | 32.56 | 31.92 | 33.36 |
| | | (6.54) | (5.37) | (5.35) | (5.93) |
| $\eta_3$ | | | -25.57 | -25.06 | -26.21 |
| | | | (5.57) | (5.20) | (5.16) |
| $\eta_4$ | | | | 16.34 | 15.93 |
| | | | | (8.34) | (6.68) |
| $\eta_5$ | | | | | -18.10 |
| | | | | | (6.20) |
| Constant | -0.01 | -0.01 | -0.01 | -0.01 | -0.01 |
| | (0.01) | (0.01) | (0.01) | (0.01) | (0.00) |
| Observations | 104 | 104 | 104 | 104 | 104 |

## D.8 Screening capital flows

In Table D.13, we report the slope estimates, $\beta_j$, of the regression in equation (40). In the first column we list the sector, in the second column whether we consider a flow to be mismeasured, and in the third column the estimate of $\beta_j$ for that particular sector.

Table D.11: Volatility of idiosyncratic demand shocks by sector of the Flow of Funds. The table reports the volatility of idiosyncratic demand shocks by sector. We follow the procedure outlined in Section 4.2 to estimate the demand shocks. The sample is from 1993.Q1 to 2018.Q4.

| Sector | $\overline{S}_{it}\sigma(u_{it})$ | $\sigma(u_{it})$ |
|---|---|---|
| Households | 0.43 | 1.05 |
| Mutual funds | 0.22 | 0.92 |
| Foreign sector | 0.19 | 1.44 |
| State & local pension funds | 0.13 | 1.69 |
| Private pension funds | 0.12 | 1.22 |
| Broker dealers | 0.04 | 7.24 |
| Life insurers | 0.03 | 1.45 |
| Property & casualty insurers | 0.02 | 1.66 |
| State and local govts | 0.02 | 3.18 |
| Closed-end funds | 0.01 | 2.99 |
| Fed govt retirement funds | 0.01 | 2.20 |
| Banks | 0.01 | 3.17 |

Table D.12: Dynamics of mutual fund flows. The table reports the dynamics of fund flows that we use in Section 4.3. We consider an AR(1), in Column 1, to AR(4) model of flows, in Column 4. In all cases we include a time trend. The standard errors, which correct for autocorrelation, are reported in parentheses.

| | f | f | f | f |
|---|---|---|---|---|
| L.f | 0.56 | 0.44 | 0.41 | 0.40 |
| | (0.04) | (0.04) | (0.04) | (0.04) |
| L2.f | | 0.22 | 0.18 | 0.17 |
| | | (0.03) | (0.04) | (0.04) |
| L3.f | | | 0.11 | 0.11 |
| | | | (0.04) | (0.04) |
| L4.f | | | | 0.03 |
| | | | | (0.06) |
| t | -0.19 | -0.15 | -0.13 | -0.12 |
| | (0.03) | (0.03) | (0.02) | (0.02) |
| Constant | 0.00 | 0.00 | 0.00 | 0.00 |
| | (0.00) | (0.00) | (0.00) | (0.00) |
| Observations | 321 | 320 | 319 | 318 |
| $R^2$ | 0.692 | 0.702 | 0.703 | 0.701 |

Figure D.8: Estimates of the aggregate multiplier $M = \frac{1}{\zeta}$ by horizon. The figure plots the multi-period impact of demand shocks: a demand shock of $f_t$ at date $t$ increases the (log) price of equities from $t-1$ to $t+h$ by $Mf_t$. We use the GIV for instrumentation, see (38). The horizontal axis indicates the horizon in quarters, from zero (that is, the current) to four quarters. Standard errors are adjusted for autocorrelation. The sample is from 2000.Q1 to 2019.Q4.

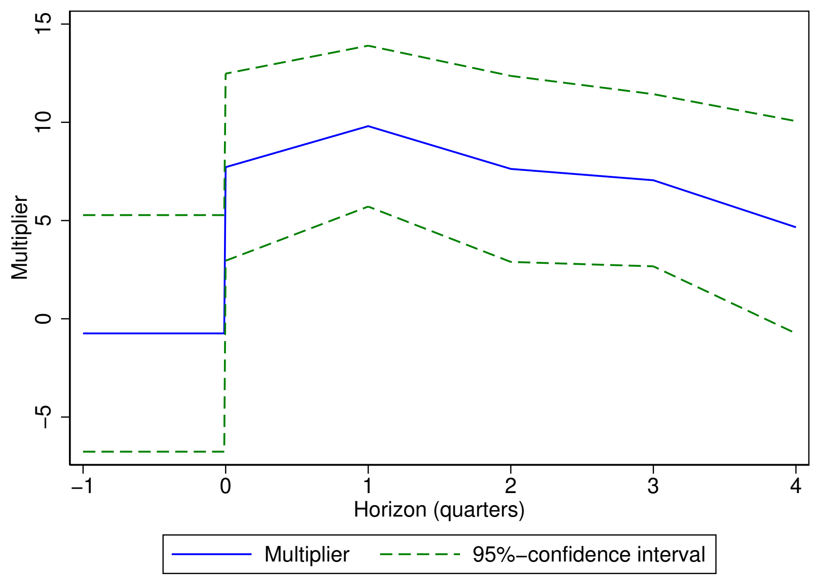



Table D.13: Assessing the mismeasurement of capital flows. The table reports the slope coefficient of the regression in equation (40) to assess whether capital flows are mismeasured. We consider a flow to be correctly measured when $\beta_j$ is significantly different from one. The sample is from 1993.Q1 to 2018.Q4.

| Sector | Flows included | $\beta_j$ | T-statistics for $H_0 : \beta_j = 1$ |
|---|---|---|---|
| Households | 0 | 0.47 | 9.11 |
| State and local govts | 1 | 0.49 | 1.95 |
| State & local pension funds | 1 | 1.04 | 0.73 |
| Foreign sector | 0 | 0.35 | 6.90 |
| Fed govt retirement funds | 0 | -0.03 | 12.08 |
| Property & casualty insurers | 0 | 0.55 | 3.30 |
| Life insurance companies | 0 | 0.61 | 2.61 |
| Closed-end funds | 1 | 1.08 | 0.85 |
| ETFs | 1 | 1.01 | 0.93 |
| Private pension funds | 1 | 1.10 | 1.49 |
| Mutual funds | 1 | 0.98 | 0.53 |
| Broker dealers | 0 | 0.07 | 22.78 |
| Banks | 0 | -0.06 | 10.67 |

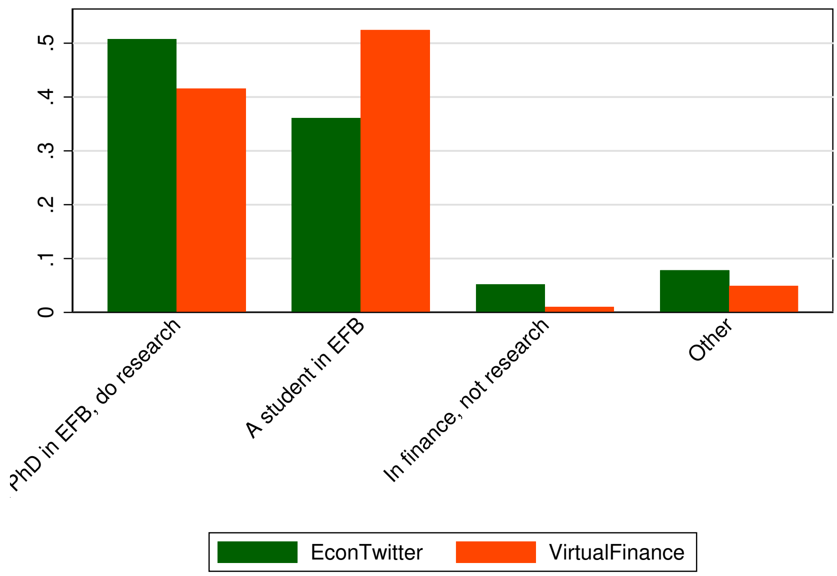
# E Survey Details

We conducted three surveys. The first survey by putting out a request via Twitter (using the `#EconTwitter` tag) to complete an online survey. In addition, we asked participants of an online seminar at `VirtualFinance.org` to complete the same survey – this latter audience being naturally more representative of the population of academic researchers in finance. Both surveys were conducted before the paper was available online and before the seminar on May 8, 2020. We launched the Twitter survey on May 7, 2020. We asked four questions:

1. If a fund buys $1 billion worth of US equities (permanently; it sells bonds to finance that position), slowly over a quarter, how much does the aggregate market value of equities change?
2. In response to the fund buying $1 billion over the quarter, some other investors need to sell. Who are the likely investors (by type) to sell their positions? (Pick at most two investor types).
   - Potential answers:
     (a) Hedge funds.
     (b) Mutual funds or ETFs.
     (c) Long-term investors such as pension funds and insurance companies.
     (d) Broker dealers.
     (e) Households.
     (f) Foreign investors (of any type).
     (g) Firms issuing new equity
     (h) Other [open text box]. We received hardly any additional sectors and will omit it from the discussion.
3. Since December 2019, did the equity risk premium:[82]
   - Potential answers:
     (a) Increase by more than 2.5%.
     (b) Increase between 0% and 2.5%.
     (c) Decrease between 0% and 2.5%.
     (d) Decrease by more than 2.5%.
4. Can you tell us a bit about yourself
   - Potential answers:
     (a) I am a student in economics / finance / business.
     (b) I have a PhD / doctorate in economics / finance / business, and do research.
     (c) None of the above.

[82] For the sake of brevity, we do not report the results for this question. It shows a significant amount of disagreement across respondents. In some models, such uncertainty about the exact value of the equity risk premium gives rise to inaction and therefore inelastic demand.

Figure E.9: Composition of survey respondents. The figure plots the distribution of respondents across two surveys; one conducted via Twitter (using the hashtag `#EconTwitter`) and one conducted at the beginning of a `VirtualFinance.org` seminar. The abbreviation EFB stands for Economics, Finance, and Business.




We also presented the paper a week later in the Virtual Macro Seminars (VMACS) on May 14. We repeated only the first and the last question, but attendees may have already seen the earlier presentation or have seen the slides. While the results are comparable, we consider it to be slightly polluted and focus on the earlier two surveys as a result. We remove responses that only signed the effect (e.g., "positive" or "negative" or ">0"). We received 192 responses via EconTwitter and 102 responses via `VirtualFinance.org`. In Figure E.9, we summarize the composition of responses. The abbreviation EFB stands for Economics, Finance, and Business. At least 85% of respondents are EFB students or have a PhD in EFB and do research.

In Table E.14, we summarize the responses about the multiplier, $M$. The main takeaway is that the profession views the aggregate stock market as highly elastic. Only 3% expects the multiplier to be larger than one and, in fact, fewer than 50% of the respondents expects a positive multiplier in each of the surveys. As a result the median multiplier estimate is zero in both surveys, and the mean is about 0.1. Note that this is even an order of magnitude smaller than the recent estimates of the micro elasticity of demand.

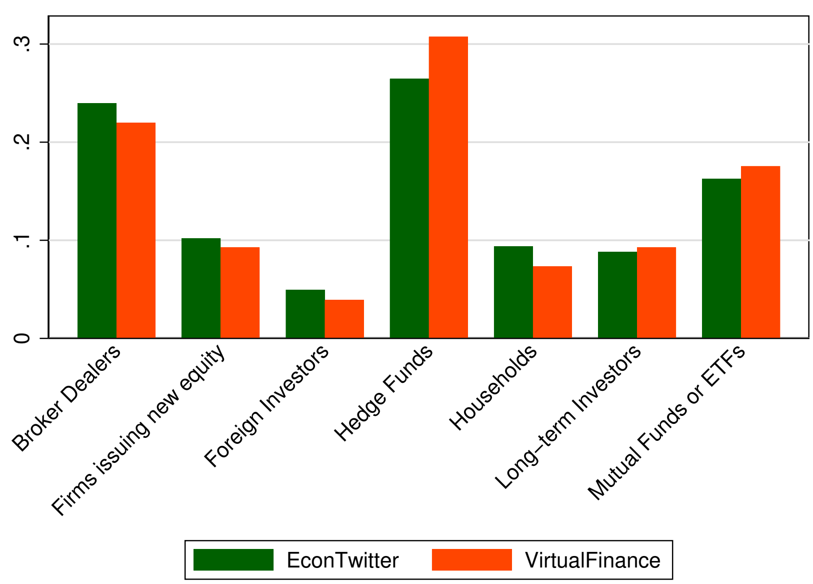
Given this feedback, it is interesting to explore the mechanism that may give rise to such high elasticities.[83] In Figure E.10, we provide the results to the third question, which points to hedge funds and broker dealers. We will explore these sectors in more detail in the paper. However, the bottom line is that broker dealers are fairly small as a sector and hedge funds do not appear to provide elasticity, and in particular not during economic downturns when the equity premium tends to rise sharply.

[83] This approach to "testing the mechanism" is similar in spirit to the ideas in Chinco et al. (2020) .

Table E.14: Survey responses regarding the multiplier. The table summarizes the distribution of survey responses about the multiplier, $M$. The data are from two surveys; one conducted via Twitter (using the hashtag `#EconTwitter`) and one conducted at the beginning of a `VirtualFinance.org` seminar. The first column reports the number of respondents. Columns 2 to 6 report the fraction of respondents who consider the multiplier to exceed one, be greater or equal to one, to exceed zero, equal to zero, or negative.

| | | Fraction of responses for which | | | | | |
|---|---|---|---|---|---|---|---|
| Survey | No. obs. | $M > 1$ | $M \geq 1$ | $M > 0$ | $M = 0$ | $M < 0$ | |
| VirtualFinance | 102 | 2.9% | 5.9% | 47.1% | 52.0% | 1.0% | |
| EconTwitter | 192 | 3.1% | 5.2% | 29.5% | 67.9% | 2.6% | |
| | | Percentiles | | | | | |
| Survey | Mean | 5 | 25 | 50 | 75 | 95 | Max |
| VirtualFinance | 0.13 | 0 | 0 | 0 | 0.01 | 1 | 5 |
| EconTwitter | 0.11 | 0 | 0 | 0 | 0 | 0.75 | 10 |

Figure E.10: Who provides elasticity to the market? The figure reports the fraction of respondents pointing to a particular sector as providing elasticity when an investors wants to sell \$1bn worth of equities. The data are from two surveys; one conducted via Twitter (using the hashtag `#EconTwitter`) and one conducted at the beginning of a `VirtualFinance.org` seminar.



# F Omitted Proofs

## F.1 Derivation of (18)

$$\begin{aligned}
1+r_f+\bar{\pi}+\hat{\pi}_t &= 1+r_f+\pi_t \\
&= \frac{\mathbb{E}_t\left[P_{t+1}+D_{t+1}\right]}{P_t} \\
&= \frac{\mathbb{E}_t\left[\bar{P}_{t+1}\left(1+p_{t+1}\right)+\bar{D}_{t+1}\left(1+d_{t+1}\right)\right]}{\bar{P}_t\left(1+p_t\right)} \\
&= \mathbb{E}_t\left[\frac{\bar{P}_{t+1}}{\bar{P}_t}\left(1+p_{t+1}-p_t\right)+\frac{\bar{D}_{t+1}}{\bar{D}_t}\frac{\bar{D}_t}{\bar{P}_t}\left(1+d_{t+1}-p_t\right)\right] \\
&= \mathbb{E}_t\left[\left(1+g\right)\left(1+p_{t+1}-p_t\right)+\left(1+g\right)\delta\left(1+d_{t+1}-p_t\right)\right] \qquad (70) \\
&= \left(1+g\right)\left(1+\delta\right)+\left(1+g\right)\mathbb{E}_t\left[\Delta p_{t+1}+\delta\left(d_{t+1}-p_t\right)\right] \qquad (71)
\end{aligned}$$

The zero-th order term gives $1+r_f+\bar{\pi}=(1+g)(1+\delta)$, which is the Gordon growth formula, $r_f+\bar{\pi}-g=(1+g)\delta=\frac{\mathbb{E}_t[D_{t+1}]}{P_t}$. The next order term gives

$$\hat{\pi}_t=\left(1+g\right)\mathbb{E}_t\left[\Delta p_{t+1}+\delta\left(d_{t+1}-p_t\right)\right] \qquad (72)$$

In the text, to reduce the notational clutter, we take (18), which is this expression using the definition of $\delta$ as the baseline (i.e., frictionless) value of $\frac{\mathbb{E}[D_{t+1}]}{P_t}$ rather than of $\frac{D_t}{P_t}$. This can be also interpreted as this expression in the limit of small time intervals, or when the trend growth rate $g$ is 0, or by changing $\kappa$ as $\kappa(1+g)$.

## F.2 Proof of Proposition 6

The values of $p_t$, $\hat{\pi}_t$ and so on, were derived in (26). We need to derive the flow $f_t$, the interest rate $r_f$, and the average risk premium $\bar{\pi}$.

As $\bar{W}_t=\frac{1}{\theta}\frac{\mathcal{D}_t}{\delta}$ the baseline value of the mixed fund (when the behavioral friction is 0), (17) and (45) give: $f_t=\frac{F_t-\bar{F}_t}{\bar{W}_t}=\frac{b_t\frac{\mathcal{D}_t}{\delta}}{\frac{1}{\theta\delta}\mathcal{D}_t}=\theta b_t$.

Next, we derive the risk-free rate. The consumer's first order condition gives the Euler equation $1=\beta R_{f,t}\mathbb{E}_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\right]$. As in equilibrium $C_t=Y_t$, the interest rate satisfies the Euler equation for bonds.

Now, we move to stocks. The average allocation in equities maximizes a risk-adjusted return, $\mathbb{E}_t\left[V^p\left(R_{t+1}\right)\right]$, with $V^p(R)=\frac{R^{1-\gamma}-1}{1-\gamma}$. Then, approximately, the allocation in equities is $\bar{\theta}^{\mathcal{E}}=\frac{\bar{\pi}}{\gamma\sigma_r^2}$. Given that in equilibrium all the wealth comes from equity, the equity premium is $\bar{\pi}=\gamma\sigma_r^2$.

## F.3 Proof of Proposition 7

Calling $\mathcal{D}_1$ the aggregate dividend, the dividend per share goes from $D_1=\frac{\mathcal{D}_1}{Q_0}$ to $D_1'=\frac{\mathcal{D}_1}{Q_0'}=\frac{D_1}{1-b}$. So, the time-1 dividend per share increases by a fraction $d=b$.

*Let us first consider a frictionless, elastic / rational model.* The price per share increase by the same fraction as the time-1 dividend per share, i.e. $p=b$. Calling $v=\Delta\ln(PQ)=p+q^S$ the change in the market value of the firm, and $r$ the excess return created by the buyback, we have:

$$\text{Frictionless model: } q^S = -b, \qquad d = b, \qquad p = b, \qquad v = 0, \qquad r = 0. \tag{73}$$

The market value does not change: the lowering of the number of share outstanding by $b$ is compensated by the increase in the price per share by a fraction, which is the same $b$.

*Let us next consider an inelastic model.* The buyback decreases the total dividend payout from $\mathcal{D}_0$ to $\mathcal{D}_0 - P_0 Q_0 b$. So the households experience a change in dividend received $\Delta\mathcal{D}_0 = -P_0 Q_0 b$ (recall that the all dividends are passed on by the fund to the consumers)[84] and a capital gain $Q_0 \Delta P = P_0 Q_0 p$. Recall that we said that if the extra dividend (respectively extra capital gain) is $X$ dollars, consumers will "remove from the mixed fund" $\mu^D X$ (respectively $\mu^G X$) dollars. This means that the flow is

$$\Delta F_0 = \left(1 - \mu^D\right) \Delta\mathcal{D}_0 - \mu^G Q_0 \Delta P \tag{74}$$

Indeed, if the extra dividend received is $\Delta\mathcal{D}_0$, there is a counteracting flow of $\left(1 - \mu^D\right) \Delta\mathcal{D}_0$, so that the total dividend change removed from the mixed fund is $\mu^D \Delta\mathcal{D}_0$. Likewise $\mu^G Q_0 \Delta P$ is "removed" from the mixed fund. This means that the flow is

$$f = \frac{\Delta F_0}{W_0} = \frac{Q_0 P_0}{W_0} \frac{\left(1 - \mu^D\right) \Delta\mathcal{D}_0 - \mu^G Q_0 \Delta P}{Q_0 P_0} = \theta \left[\left(1 - \mu^D\right)(-b) - \mu^G p\right]$$

The total demand change by the mixed fund is then $q = -\zeta p + \kappa\delta d + f$, and should be equal to the supply change $q^S = -b$. So

$$0 = q - q^S = -\zeta p + \kappa\delta d + f + b = -\zeta p + \kappa\delta b - \theta\left(1 - \mu^D\right) b - \theta\mu^G p + b$$

and the share price change is:

$$p = \frac{\zeta + \mu^D\theta}{\zeta + \mu^G\theta} b. \tag{75}$$

This yields (56) and implies that $p > b$ if $\mu^D > \mu^G$. A share buyback increases the market value by $v = p + q^S = p - b > 0$.

## F.4 Traditional rational or behavioral models predict that markets are extremely price-elastic

In this section, we contrast our findings with the typical macro demand elasticities implied by most frictionless rational or behavioral models, and find that these are strongly inconsistent with the low price elasticities that we model and estimate empirically.

First, as a partial intuition, if agents were risk neutral and the equity premium were 0, any price discrepancy would lead to an arbitrage, and the price elasticity of demand would be infinite, $\zeta^r = \infty$. This is the intuition behind the most basic form of the efficient markets hypothesis, where the price is always equal to the present value of dividends (with a constant discount rate), independently of flows.

Second, let us examine the more sophisticated case with risk-averse agents. We model aggregate income $Y_t$ as going to the equity dividend as $D_t = \psi Y_t$, and the rest going to labor and other forms

[84] The outcome would be the same if the rule was different. The total dividend change removed from the mixed fund would still be $\mu^D \Delta\mathcal{D}_0$.

of business as $\Omega_t = (1-\psi) Y_t$. For simplicity, we consider the most classic case: the consumer has utility $\sum_t e^{-\rho t} \frac{C_t^{1-\gamma}}{1-\gamma}$ and the endowment $Y_t$ has i.i.d. growth, $Y_t = G_t Y_{t-1}$. The basic case is the lognormal one, $G_t = e^{g\Delta t + \sigma \varepsilon_t - \frac{\sigma^2}{2}\Delta t}$ (with $\varepsilon_t$ a standard Gaussian variable). We also consider a disaster model, where $G_t = e^{g\Delta t}$ if there is no disaster (which happens with probability $1 - p^D \Delta t$), and $G_t = e^{g\Delta t} B$ if there is a disaster (which happens with probability $p^D \Delta t$), so that if there is a disaster, the economy shrinks by a factor $B \in (0,1]$.

Suppose that for some reason the market value of equities is different from its rational level, permanently, by a fraction $p$ – that is, the price of equities is permanently $P_t = P_t^* (1+p)$, where $P_t^*$ is the rational price.[85] How much capital should flow into equities? The next proposition answers this (the proof is at the end of this subsection).

**Proposition 8.** (Market elasticity in frictionless rational or behavioral models) *We derive the price-elasticity of the demand for stocks in two classes of frictionless models. We suppose that all agents are frictionless (and with common beliefs, which can be rational or behavioral), with CRRA utility and with i.i.d. endowment growth. In the basic model in which growth rates are lognormal, the elasticity of demand for equities is:*

$$\zeta^r = \frac{1}{\pi} \frac{C}{W^{\mathcal{E}}}, \tag{76}$$

*where $\pi$ is the equity premium, $C$ is aggregate consumption and $W^{\mathcal{E}}$ is the stock market capitalization. In a disaster model where growth rates follow a jump process, the elasticity of demand for equities is $\zeta^{r,D} = \frac{1}{\pi} \frac{C}{W^{\mathcal{E}}} \frac{(1-B^\gamma)B}{\gamma(1-B)}$, where $B$ is the recovery rate of the endowment after a disaster.*

Take the calibrated values $C = 0.8Y$, where $Y$ is GDP, $W^{\mathcal{E}} = Y$ (as the typical market capitalization is roughly equal to GDP), and $\pi = 4\%$. Then (76) implies that the elasticity predicted by rational models is $\zeta^r = 20$. Hence, with a calibrated and empirical elasticity $\zeta = 0.2$, we find that the basic rational model predicts an elasticity of demand 100 times bigger than the empirical one:

$$\frac{\zeta^r}{\zeta} = 100. \tag{77}$$

Summing up, we find that *frictionless rational or behavioral models (of the common "wrong beliefs" type) predict an elasticity of demand 100 times bigger than the calibrated and empirical one.* Indeed, in a behavioral model agents may have wrong beliefs, but they strongly act on their beliefs, with the same elasticity as in rational models (replacing the equity premium $\pi$ in (76) by the perceived equity premium, but both are typically calibrated to have the same average value).

Now take the disaster model. Using the above calibration and the values $B = 2/3$, $\gamma = 4$ (Barro (2006), Gabaix (2012)), the elasticity in a disaster model given by Proposition 8 is $\zeta^{r,D} = 8$, so

[85] Johnson (2006) introduces a definition of market illiquidity that pertains to asset pricing models, whether or not there is trade between agents. His measure quantifies the equilibrium price change induced by a perturbation in asset supplies. Johnson (2006) examines this measure in the context of several rational setups, including a Lucas model, and this measure of illiquidity can be large and variable. We cannot use his results here because his definition of liquidity allows the interest rate to change when equity prices change, unlike our demand elasticity $\zeta$, which holds for a given interest rate. His notion of liquidity is generally lower than our elasticity, sometimes by an infinite factor. Indeed, take the case where there is no risk (the equity market is mispriced, so that the price is not the discounted value of the known dividend). In our model, the elasticity of demand is infinite (see Proposition 8), as it would be in most models with a riskless arbitrage opportunity, but Johnson's liquidity measure remains finite. In addition, we account for human capital, which is absent in Johnson's definition, and is quantitatively important.

that it is 40 times larger than the empirical one. In Section I we verify numerically that a similar reasoning works for long run risks models (Bansal and Yaron (2004)). We suspect that it would also apply to a habit formation model (Campbell and Cochrane (1999)).[86,87]

**Proof of Proposition 8** *Case 1: Gaussian risk.* We first deal with the case of Gaussian risk, for simplicity in the continuous-time limit. The desired holding of risky wealth is $\theta_t = \frac{\pi_t}{\gamma\sigma^2}$. Initially, that holding was $\theta_t = 1$: all wealth (human wealth and wealth capitalized in the stock market) is risky, with equal riskiness. This implies that $\gamma\sigma^2 = \pi$ initially. But after the change in the equity premium, the desired change in equity share is: $d\theta = \frac{d\pi}{\gamma\sigma^2}$, i.e.

$$d\theta = \frac{d\pi}{\pi}. \tag{78}$$

The consumer can sell his wealth for $P_t$, so that his market wealth is $W_t = QP_t$, where $Q$ is the total number of shares (of which $Q^{\mathcal{E}}$ are in equities, the rest in human wealth, i.e. promises to a stream of labor income). His dollar demand for risky assets is $W_t\theta_t$, so that in number of shares this is:

$$Q^D = \frac{W_t}{P_t}\theta_t = \frac{QP_t}{P_t}\theta_t = Q\theta_t = Q\left(1 + \frac{\Delta\pi}{\pi}\right).$$

All the trading is in the equity market, so that this net demand for equities is:

$$\Delta Q = Q\frac{\Delta\pi}{\pi}.$$

This flow, expressed as a fraction of the equity market (which has a number of shares $Q^{\mathcal{E}} = \psi Q$), is also:

$$\frac{\Delta Q}{Q^{\mathcal{E}}} = \frac{\Delta\pi}{\psi\pi}. \tag{79}$$

If the value of equity changes by $p$, the equity premium changes by $\Delta\pi = -\delta p$ (see (18)), so we have

$$\frac{\Delta Q}{Q^{\mathcal{E}}} = -\frac{\delta}{\psi\pi}p = -\zeta^r p,$$

where the rational elasticity is:

$$\zeta^r = \frac{\delta}{\psi\pi}.$$

Finally, consumption is $C_t = Y_t$, while aggregate stock dividends are only $D_t Q^{\mathcal{E}} = \psi Y_t$.[88] So,

$$\zeta^r = \frac{\delta}{\psi\pi} = \frac{\frac{D_t}{P_t}}{\frac{D_t Q^{\mathcal{E}}}{C_t}\pi} = \frac{C_t}{\left(P_t Q^{\mathcal{E}}\right)\pi} = \frac{C_t}{W_t^{\mathcal{E}}\pi},$$

[86] One could imagine other models, with idiosyncratic risk, but that would take us far afield.

[87] As a correlate, traditional models counterfactually predict very correlated flows and beliefs. Indeed, with several institutions and a demand $q_{it} = -\zeta^r p_t + f_{it}^{\nu}$, and $\zeta^r \simeq 20$, the term $\zeta^r p_t$ has an annual volatility of $\zeta^r \sigma_r = 20 \times 0.15 = 3$, or 300% per year. However, the annual volatility of equity holdings changes, we have seen, is about $\sigma_{q_i} \simeq 2\%$. Hence, to account for the empirical facts, we would need extremely volatile flows and demand changes $f_{it}^{\nu}$ of about 300%. In contrast, empirical flows $f_{it}$ (when they can be measured) are about 1%. Hence, we would need almost perfectly correlated news and taste shocks $\nu_{it}$, of 300% per year. All of this strikes us as quite implausible. It seems like a very difficult challenge to fit our facts with a traditional model.

[88] This is true in the logic of the Lucas tree model with no investment. In the calibration, we take the "fruit" to be consumption, which less than GDP as there is investment (in a closed economy), and indeed we take $C_t = 0.8Y_t$.

which is the announced expression.

*Case 2: Disaster risk.* The reasoning is the same, except that expression (78) is different with disaster risk. To derive it, observe that the value function must take the form $V(W_t) = K\frac{W_t^{1-\gamma}}{1-\gamma}$ for some constant $K$. Hence, calling $\tilde{R}_{t+1}$ the rate of return on stocks, the consumer's problem is:

$$\max_{C,\theta} u(C) + \beta \mathbb{E} V\left((W_t - C_t)\left(R_f + \theta\left(\tilde{R}_{t+1} - R_f\right)\right)\right)$$

It entails the following sub-problem for portfolio choice: $\max_\theta \mathbb{E}\left[\frac{\left(R_f+\theta\left(\tilde{R}_{t+1}-R_f\right)\right)^{1-\gamma}}{1-\gamma}\right]$. Calling $\tilde{r}_{t+1} = \frac{R_{t+1}}{R_f} - 1$ the normalized excess return on stocks, the problem is

$$\max_\theta \mathbb{E}\left[\frac{(1+\theta\tilde{r}_{t+1})^{1-\gamma}}{1-\gamma}\right],$$

so the FOC characterizing the equity share is:

$$\mathbb{E}\left[(1+\theta\tilde{r}_{t+1})^{-\gamma}\tilde{r}_{t+1}\right] = 0. \tag{80}$$

This expression holds for any i.i.d. excess return distribution $\tilde{r}_{t+1}$. In particular, it recovers the traditional expression $\theta = \frac{\pi}{\gamma\sigma^2}$ in the Gaussian case, $\tilde{r}_t = \pi\Delta t + \varepsilon_t$ (this is an exercise for the reader). Now take the disaster case,

$$\tilde{r}_t = \pi\Delta t - (1-B) J_t$$

where $\pi$ is the equity premium conditional on no disasters, where $J_t = 0$ if there is no disaster and 1 otherwise. Then (80) becomes

$$\left(1 - p^D\Delta t\right)(1+\theta\pi\Delta t)^{-\gamma}\pi\Delta t + p^D\Delta t\,(1+\theta(\pi\Delta t - (1-B)))^{-\gamma}(\pi\Delta t - (1-B)) = 0,$$

i.e. taking the small $\Delta t \to 0$ limit,

$$\pi = p^D(1-\theta(1-B))^{-\gamma}(1-B). \tag{81}$$

Taking logs on both sides and differentiating this expression (for small changes in $\pi$ and $\theta$) around $\theta = 1$ gives:

$$\frac{d\pi}{\pi} = d\ln\pi = d\ln\left[p^D(1-\theta(1-B))^{-\gamma}(1-B)\right] = \frac{\gamma(1-B)}{B}d\theta,$$

i.e.

$$d\theta = \frac{d\pi}{\pi}\frac{B}{\gamma(1-B)} \tag{82}$$

Finally, as $\pi = p^D B^{-\gamma}(1-B)$ by (81), the risk premium in a full sample (including an average number of disasters) is: $\bar{\pi} = \pi - p^D(1-B) = p^D(B^{-\gamma}-1)(1-B)$, so $\frac{\bar{\pi}}{\pi} = 1 - B^\gamma$. So (82) gives

$$d\theta = \frac{d\pi}{\bar{\pi}}\frac{B(1-B^\gamma)}{\gamma(1-B)}, \tag{83}$$

which is the disaster counterpart to (78): how the desired equity share changes as the equity premium changes.

The rest of the derivation is exactly as in the lognormal Case 1, replacing (78) by (83).

In a behavioral model agents may have wrong beliefs, but they strongly act on their beliefs, with the same elasticity as in rational models (replacing the equity premium $\pi$ by the perceived equity premium).

# G Theory Complements

## G.1 The model with many asset classes

We can easily extend the model to $K$ asset classes, indexed by $A \in \{1, \ldots, K\}$, such as stocks, long-term government bonds, and long-term corporate bonds. This way, we can study cross-market contagion effects, and the impact of those on real investment.

**Two-period model** We sketch this for the two-period model of Section 3.1.

The mandate leads to the following demand for asset $A$ (at least, for some small deviations from 0 in $d$ and $p$):

$$P_A Q_A^D = \theta_A W \exp\left(\sum_{B=1}^{K} \kappa_{AB}^D \left(d_B - p_B\right)\right).$$

For instance, if $\kappa_{AB}^D = 0$ the mixed fund seeks to keep a constant share $\theta_A$ in asset $A$. When $\kappa_{AB}^D$ is different from 0, a change in the risk premium in asset $B$ leads to a change in the amount allocated to asset $A$.

Suppose that there are shocks changing the prices and expected dividends for a given set of assets by fractions indexed as $p_B$ and $d_B$. Then, the value of the fund changes by $w = \frac{\Delta W}{W} = f + \sum_B \theta_B p_B$, so that the demand for a particular asset class $A$ changes by a fraction[89]

$$q_A^D = -\sum_B \zeta_{AB} p_B + f_A + \sum_B \kappa_{AB}^D d_B, \tag{84}$$

where the cross-elasticities of demand $\zeta_{AB}$ express how demand for asset $A$ changes with a change in the price of asset $B$: $\zeta_{AB} = 1_{A=B} - \theta_B + \kappa_{AB}^D$. In vector form, this gives

$$q = -\zeta p + f + \kappa^D d, \tag{85}$$

where now $q$, $p$, $d$, $f$ are vectors, and $\zeta$ and $\kappa^D$ are matrices, with dimension $K$. This generalizes Proposition 2. So, the equilibrium after a change in flows and expected dividends (but still constant asset supply is):

$$p = \zeta^{-1}\left(f + \kappa^D d\right). \tag{86}$$

In this paper we shall not measure, for example, how much the price of long-term bonds affects the demand for stocks. But one can readily contemplate a host of interesting cross-market effects. For instance, when investors sell stocks and invest in long-term bonds, bond yields will go down, which encourages firms to invest. Hence, we see an impact from stocks to corporate bonds, to real investment, and to GDP.

**Infinite horizon model** The formulas of the papers extend again, replacing scalars by matrices. For instance, the demand is

$$q_t = -\zeta p_t + f_t + \kappa^D d_t^e + \kappa \mathbb{E}_t\left[p_{t+1} - p_t\right],$$

[89]With just one fund, $f_A$ is the same across assets classes $A$. But with several funds, by aggregation, $f_A$ differs across asset classes so we use that more general notation.

where $q_t$,$f_t$, $d_t$, $p_t$ are vectors and $\kappa$, $\kappa^D$, $\zeta$ are matrices. In equilibrium, $q_t = 0$ at all dates, so that

$$p_t = (\zeta + \kappa)^{-1} \left(\kappa \mathbb{E}_t [p_{t+1}] + f_t + \kappa^D d_t^e\right).$$

Defining the matrix $\rho$:

$$\rho \coloneqq \kappa^{-1} \zeta, \tag{87}$$

we have

$$p_t = a \left(f_t + \kappa^D d_t^e\right) + (1 + \rho)^{-1} \mathbb{E}_t [p_{t+1}]$$

with $a = (\zeta + \kappa)^{-1}$. Solving forward, we have the multi-asset equivalent of (20):

$$p_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \frac{1}{(1+\rho)^{\tau-t}} \left(a f_\tau + a \kappa^D d_\tau^e\right). \tag{88}$$

In the AR(1) case where $f_t = (1 - \phi_f) f_{t-1} + \varepsilon_t^f$, with $\phi_f$ a matrix now, we have $p_t = b_f^p f_t$ with $b_f^p$ a matrix equal to:

$$b_f^p = \sum_{h=0}^{\infty} (1 + \rho)^{-h} a (1 - \phi_f)^h. \tag{89}$$

In general, there is no closed form, but $b_f^p$ can be computed iteratively as:

$$b_f^p = a + (1 + \rho)^{-1} b_f^p (1 - \phi_f). \tag{90}$$

## G.2 The model with time-varying market inelasticity

Here we study the model with a time-varying market elasticity.

Suppose we have a time-varying $\zeta_t$ and $\kappa_t$ but (for simplicity), a constant $\rho = \frac{\zeta_t}{\kappa_t}$. For simplicity, we assume $\mathbb{E}_t d_\tau^e = 0$. Then, we have the following variant of Proposition 5 (the derivation is similar):

$$p_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \frac{\rho}{(1+\rho)^{\tau-t+1}} \frac{f_\tau + \nu_\tau}{\zeta_\tau}. \tag{91}$$

To be concrete, we study the case

$$\frac{1}{\zeta_t} = \frac{1}{\zeta} (1 + \mathcal{M}_t), \qquad \mathbb{E}_t \mathcal{M}_{t+1} = (1 - \phi_\zeta) \mathcal{M}_t,$$

so that $\mathcal{M}_t$ is a temporary increase in market inelasticity, mean-reverting at a speed $\phi_\zeta$. We consider the impact of a permanent inflow, $f_\tau = f_0$ for $\tau \geq 0$. Then the price follows, at $t \geq 0$,

$$p_t = \frac{f_0}{\zeta} \left(1 + \frac{\rho}{\rho + \phi_\zeta} \mathcal{M}_t\right). \tag{92}$$

So, if the flow $f_0$ happens during a time of high market inelasticity $\frac{1}{\zeta_0}$ (i.e. high $\mathcal{M}_0$), then the price impact is higher, which makes sense. It is the average future value of the inelasticity shifter ($\frac{\rho}{\rho+\phi_\zeta} \mathcal{M}_0$) that matters, rather than the current inelasticity shifter ($\mathcal{M}_0$). In the scenario above, the price impact of $f_0$ mean-reverts at a speed $\phi_\zeta$.

More generally (if $\phi_\zeta > \phi_f$), this implies that returns that happened during a high-volatility period mean-revert faster.

*A tentative calibration.* With $\rho = 0.16/year$ and $\phi_\zeta = 0.15/year$, we have $\frac{\rho}{\rho+\phi_\zeta} \simeq 0.5$, so have then to get a price impact higher by a factor 0.5, we need $\mathcal{M}_t = \frac{0.5}{0.5} = 1$, i.e. a halving of $\zeta_t$. This effect might be detectable, though not easily.

## G.3 Micro versus macro elasticity: The cross-section of stocks

We generalize to a model with several stocks. This allows us to distinguish between the macro elasticity of demand for stocks, $\zeta$, and the micro-elasticity $\zeta^{\perp}$. The upshot is that the effects are the same, but with higher demand elasticity in the cross-section $\zeta^{\perp} > \zeta$ than in the aggregate. We recommend skipping this section at the first reading.

### G.3.1 Stock-level demands

We call $P_{at}$ the price of the stock, and $p_{at}$ its deviation from the baseline (as we did for the aggregate market). We define $p_a^{\perp} = p_a - p$ as the asset-$a$ specific price deviation. Likewise, all "perpendicular" terms are the deviation of stock $a$ from the aggregate stock market. We define $\pi_{at}^{\perp} = \pi_t^a - \beta_a \pi_t$ as the deviation of the equity premium of asset $a$ from the CAPM benchmark (this could be generalized of course), and $\hat{\pi}_{at}^{\perp} = \pi_{at}^{\perp} - \bar{\pi}_a^{\perp}$ as its deviation from the average.

We start from a model of stock-level demand for stock $a$ (as in *a*sset), which comes from a "tracking error" type of mandate: the fraction in equities allocated to asset $a$ is

$$\frac{P_{at}Q_{at}}{P_t Q_t} = \theta_a^{\mathcal{E}} e^{\kappa^{\perp}\hat{\pi}_{at}^{\perp} + \theta^{\perp} p_a^{\perp} + \nu_a^{\perp}}. \tag{93}$$

Indeed, $P_{at}Q_{at}$ is the dollar demand for asset $a$, and $P_t Q_t$ is the dollar demand for the aggregate stock market. On average, their ratio is $\theta_a^{\mathcal{E}}$. The term $\kappa^{\perp}$ is the micro-elasticity of demand with respect to the anomalous part of the equity premium $\hat{\pi}_{at}^{\perp}$. The term $\theta^{\perp}$ indicates a concern for tracking error: if the fraction allocated to asset $a$ is constant, then $\theta^{\perp} = 0$ (this is the baseline case). However, if the number of shares allocated to asset $a$ is constant, then $\theta^{\perp} = 1$.

Calling $q_{at} = \frac{Q_{at}}{\bar{Q}_a} - 1$ the deviation of the demand from the baseline, and $q_{at}^{\perp} = q_{at} - q_t$ how much asset $a$ deviates from the baseline, we obtain the following counterpart to Proposition 4 (the proof is in Appendix F).

**Proposition 9.** *(Demand for individual stocks in the infinite-horizon model) The demand change (compared to the baseline) for an individual asset a is $q_{at} = q_t + q_{at}^{\perp}$, where $q_t$ is the demand change for the aggregate stock market seen in Proposition 4, and $q_{at}^{\perp}$ is the asset-a specific demand change, given by*

$$q_{at}^{\perp} = -\zeta^{\perp} p_{at}^{\perp} + \kappa^{\perp}\delta d_{at}^{e,\perp} + \kappa^{\perp}\mathbb{E}_t\left[\Delta p_{a,t+1}^{\perp}\right] + \nu_{at}^{\perp} \tag{94}$$

*where $\zeta^{\perp}$ is the micro-elasticity of demand for individual stocks:*

$$\zeta^{\perp} = 1 - \theta^{\perp} + \kappa^{\perp}\delta. \tag{95}$$

*Proof.* Equation (93) implies

$$q_{at}^{\perp} = -\left(1 - \theta^{\perp}\right) p_{at}^{\perp} + \kappa^{\perp}\hat{\pi}_{at}^{\perp} + \nu_{at}^{\perp}. \tag{96}$$

Likewise, the analogue of (18) is $\hat{\pi}_{at}^{\perp} = \mathbb{E}_t\left[\Delta p_{a,t+1}^{\perp}\right] + \delta\left(d_{at}^{e,\perp} - p_a^{\perp}\right)$, with $d_{at}^{e,\perp} := \mathbb{E}_t\left[d_{a,t+1}^{\perp}\right]$. Combining the two gives the announced expression. □

This is exactly the same equation as the one for the aggregate stock market, but now in terms of stock-specific deviations. Hence, the economics of the aggregate stock market works for the individual stocks, but in "perpendicular space", i.e. replacing $\zeta$, $p_t$, $q_t$ by $\zeta^{\perp}$, $p_{at}^{\perp}$, $q_{at}^{\perp}$, and so on. For instance, the equilibrium value of the price $p_{at}^{\perp}$ is as in Proposition 5, replacing $\nu_t$ by $\nu_a^{\perp}$. We next spell this out and draw consequences. See Betermier et al. (2019) for an alternative demand-based model of the cross-section of stocks.Micro-elasticity of demand versus macro-elasticity of demand

Suppose that there is a "stock specific flow", whereby someone buys $\Delta F_a^{\perp}$ worth of stock $a$, while selling $\Delta F_a^{\perp}$ of the aggregate stock market, so that the total change in the demand for aggregate stocks is 0. The asset-$a$ specific fractional inflow is $f_a^{\perp} = \frac{\Delta F_a^{\perp}}{P_a Q_a}$, where $P_a Q_a$ is the (pre-flow) market value of stock $a$. As net demand is 0, we must have $q_{at}^{\perp} + f_a^{\perp} = 0$. So, the impact of a flow is:

$$p_a^{\perp} = \frac{f_a^{\perp}}{\zeta^{\perp}}, \tag{97}$$

where $\zeta^{\perp}$ is the price micro-elasticity of demand (95). We see that the price impact is $\frac{1}{\zeta^{\perp}}$, not $\frac{1}{\zeta}$.

**Calibration** Most papers have estimated the micro-elasticity of demand, $\zeta^{\perp}$ (Shleifer (1986), Wurgler and Zhuravskaya (2002), Duffie (2010), Chang et al. (2014), Koijen and Yogo (2019)), while the present paper is about the macro-elasticity of demand, $\zeta$. Indeed, the literature finds $\zeta^{\perp} \simeq 1$, with estimates in the 0.5 to 10 range. It makes sense that the macro-elasticity should be much smaller than the micro-elasticity, $\zeta \ll \zeta^{\perp}$. One way to rationalize this is to set $\theta^{\perp} \simeq 0.2$ for the inertia or concern for tracking error term, $\delta = 4\%$, and $\kappa^{\perp} = 5$.[90]

**Micro versus macro price impact** In the following illustrations, we take a micro elasticity $\zeta^{\perp} = 1$ and a macro elasticity $\zeta = 0.2$.

Consider what happens if an investor decides to buy \$1 worth of Apple shares, while selling \$1 worth of Google shares. Then, the market value of Apple goes up by \$1 (that is, \$1 $\times \frac{1}{\zeta^{\perp}}$), and that of Google falls by the same \$1. But the aggregate value of equities does not change, as the net demand for aggregate equities has not changed.

Next, suppose that an investor buys \$1 of a very small stock (selling \$1 worth of bonds), call it Peanut. Then, the market value of that Peanut stock goes up by \$1, and the market value of the aggregate stock market goes up by \$5 – so the aggregate market value of the other stocks increases by \$4.

If the consumer buys \$1 of Apple, or any non-infinitesimal stock, the aggregate value of equities still increases by \$5, but the market value of Apple goes up by slightly more than \$1 (indeed, if Apple were the whole market, its would increase by \$5). To see all this analytically, consider a flow $f_a = \frac{\Delta F_a}{P_a Q_a}$ into just one asset $a$, which accounts for a fraction $\omega_a$ of the total equity capitalization. We do that in the two-period model, so we drop $t$ (this is equivalent to doing that for the infinite-horizon model, but assuming permanent inflows). The corresponding aggregate flow is $f = \omega_a f_a$, so that the impact on the aggregate market is $p = \frac{f}{\zeta}$, or

$$p = \frac{\omega_a f_a}{\zeta}.$$

The stock-specific flow to asset $a$ is $f_a^{\perp} = f_a - f = (1 - \omega_a) f_a$. Hence, the stock-specific impact is: $p_a^{\perp} = \frac{f_a^{\perp}}{\zeta^{\perp}} = \frac{1-\omega_a}{\zeta^{\perp}} f_a$. Hence, the total impact is $p_a = p + p_a^{\perp}$, or

$$p_a = \frac{f_a}{\zeta^{\perp}} + \left(\frac{1}{\zeta} - \frac{1}{\zeta^{\perp}}\right) \omega_a f_a. \tag{98}$$

[90]This ratio of price impact of roughly 1 to 5 is also consistent with Benzaquen et al. (2017).

For the other stocks $b \neq a$, we have $f_b^{\perp} = -f = -\omega_a f_a$, so the impact is:

$$p_b = \left(\frac{1}{\zeta} - \frac{1}{\zeta^{\perp}}\right) \omega_a f_a, \qquad \text{for a stock } b \neq a. \tag{99}$$

As $\zeta < \zeta^{\perp}$, the cross-impact is positive.

For instance, suppose that Apple's capitalization is $\omega_a = 5\%$ of the stock market. Then, if someone buys \$1 of Apple (selling bonds), the market value of Apple increases by \$1.2,[91] and the value of the aggregate equities still increases by \$5 – so, the aggregate value of all the other stocks increases by \$3.8. This is a moderate deviation from the above "small stock" Peanut benchmark.

**Infinite-horizon model for the cross-section** The infinite horizon model is exactly as above, but in "perpendicular" (asset-specific) space. We define $\rho^{\perp} = \frac{\zeta^{\perp}}{\kappa^{\perp}} = \frac{1-\theta^{\perp}}{\kappa^{\perp}} + \delta$ and $M^{D,\perp} = \frac{\kappa^{\perp}\delta}{1-\theta^{\perp}+\kappa^{\perp}\delta}$. The stock-specific deviation is given by (20) in asset-specific space:

$$p_{a,t}^{\perp} = \mathbb{E}_t \sum_{\tau=t}^{\infty} \frac{\rho^{\perp}}{(1+\rho^{\perp})^{\tau-t+1}} \left(\frac{f_{a\tau}^{\perp}}{\zeta^{\perp}} + M^{D,\perp} d_{a\tau}^{e\perp}\right). \tag{100}$$

**Conclusion: Aggregate versus cross-section** We conclude that the aggregate model extends well to the cross-section, and indeed is useful to think about the impact of flows in the cross-section and in the aggregate in a unified manner. While most prior work has been on the estimation of the cross-sectional elasticity $\zeta^{\perp}$, the main object of interest in this study is the aggregate elasticity $\zeta$.

## G.4 Short-term versus long-term elasticity when funds are inertial

The basic model describes price impacts and quantity adjustments assuming no inertia in funds' reactions. Here we study what happens if funds react with some inertia: this creates additional transitory dynamics.

We consider the case of a homogeneous type of fund, trading only the aggregate stock and a risk-free short-term bond. Total demand $q_t$ can change because of the inflow $f_t$ and via an "active" demand $q_t^a$:

$$q_t = q_t^a + f_t.$$

We model the actual active demand with inertia as:

$$\Delta q_t^a = \mu \Delta q_t^{a,\nu} + \phi \Delta t \left(q_{t-1}^{a,v} - q_{t-1}^a\right), \tag{101}$$

where $q_t^{a,v}$ is the "virtual active demand" – the one of a non-inertial fund:

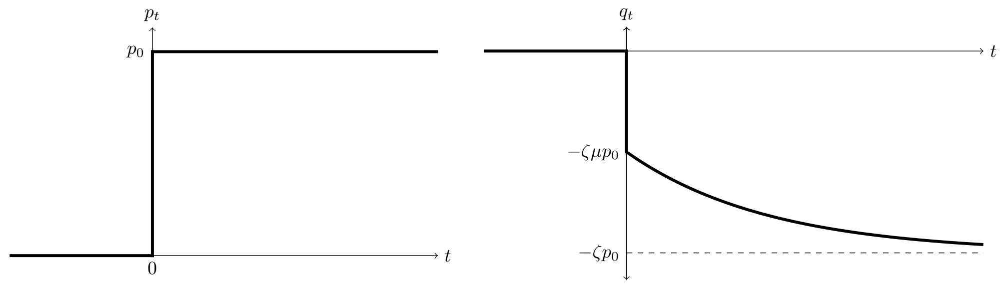
$$q_t^{a,v} = -(1-\theta)\, p_t + \kappa \hat{\pi}_t + \nu_t = -\zeta p_t + \kappa \left(\mathbb{E} p_{t+1} - p_t\right) + \kappa \delta d_t + \nu_t, \tag{102}$$

with $\mu \in [0,1]$ and $\phi \geq 0$. A frictionless investor has $\mu = 1$. The lower $\mu$ and $\phi$, the more frictional the investor. The adjustment to flows $f_t$ is instantaneous for simplicity, and as it does not require a "strategic" decision by the fund, which simply rescales its investment after an inflow.

We derive quantity adjustments (the proof is by plug and verify).

[91] Indeed, $\frac{1}{\zeta^{\perp}} + \left(\frac{1}{\zeta} - \frac{1}{\zeta^{\perp}}\right) \times \omega_a = 1 + (5-1) \times 5\% = 1.2.$

Figure G.11: This figure shows the quantity adjustment of an inert fund after a change in the aggregate stock price. It illustrates Proposition 10. If investors are inertial, there is a gradual adjustment of the quantity over time. When there is no inertia, $\mu = 1$ and the quantity adjustment is instantaneous.




(a) Impulse

(b) Fund's Quantity Response

**Proposition 10.** (Short-run versus long-run elasticity of demand) *Suppose a fund that exhibit inertia. Then, its short-run elasticity of demand is $\mu\zeta$, and its long-run elasticity of demand is $\zeta$. More precisely, suppose that the log price of equities jumps by $p_0$ at time 0, i.e. $p_t = 1_{t\geq 0} p_0$. Then, at $t \geq 0$, the fund's demand change is:*

$$q_t = -\zeta \left(1 + (\mu - 1)(1 - \phi)^t\right) p_0, \tag{103}$$

*while its virtual demand is $q_t^\nu = -\zeta p_0$.*

Figure G.11 illustrates the dynamics. The long run demand is $q_\infty^\nu = -\zeta p_0$, but the impact at 0 is only $\mu$ times that, $q_0^\nu = -\zeta \mu p_0$. In between there is an exponential relaxation at rate $\phi$.

The next proposition derives the price impact of a flow. We also assume $\phi < \mu$, which is automatically true in the limit of small time intervals.[92]

**Proposition 11.** (Price impact of an inflow when funds are inertial) *When funds exhibit inertia, the price impact of a permanent, unanticipated inflow $f_0$ at time 0 is (for $t \geq 0$),*

$$p_t = M_t f_0, \qquad M_t = \frac{1}{\zeta} + b(1 - \Phi)^t \tag{104}$$

*where $b = \frac{1-\mu}{\mu(\zeta + \kappa\Phi)}$* and $\Phi = \frac{\phi}{\mu}$. *So, the short run price impact is $M_0 = \frac{1}{\zeta} + b$, while the long run impact is $M_\infty = \frac{1}{\zeta}$.*

*Proof.* We conjecture a solution of the type (104). We normalize $f_0 = -1$. Plugging this in (102) gives

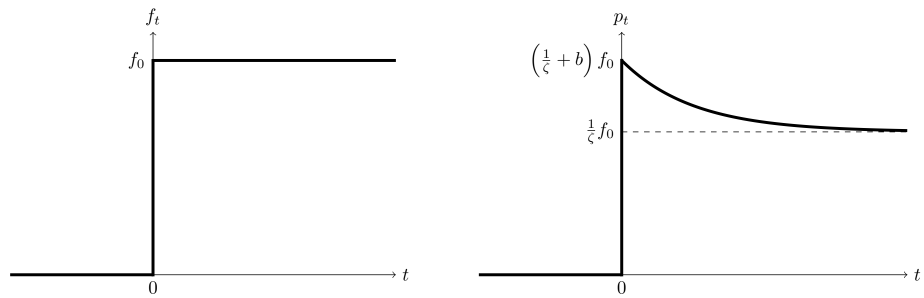
$$q_t^{a,v} = \left(1 + b\zeta(1 - \Phi)^t\right) + b\kappa\Phi(1 - \Phi)^t = 1 + b(\zeta + \kappa\Phi)(1 - \Phi)^t = 1 + c(1 - \Phi)^t.$$

For $t \geq 0$, equilibrium imposes $q_t^a + f_0 = 0$, i.e. $q_t^a = 1$. So (101) gives, for $t > 0$

$$0 = \Delta q_t^a = \mu \Delta q_t^{a,\nu} + \phi\left(q_{t-1}^{a,v} - q_{t-1}\right) = c(-\Phi\mu + \phi)(1 - \Phi)^{t-1},$$

[92] In the limit of small time intervals, we replace $(1 - \Phi)^t$ in (104) by $e^{-\Phi t}$.

Figure G.12: This figure shows the price dynamics caused by an unanticipated time-0 demand shock, when investors are inertial. It illustrates Proposition 11. The permanent demand shock $f_t$ creates a permanent price change $p_\infty = \frac{f_0}{\zeta}$. If investors are inertial, there is a small extra bump $b$ on impact, that decays exponentially over time. When investor are not inert, $b = 0$ and the price immediately jumps to its permanent value $p_\infty$.




(a) Impulse

(b) Price Response

which leads to $\Phi = \frac{\phi}{\mu}$. At time 0, (101) gives

$$q_0^a = \mu q_0^{a,v} = \mu \left(1 + b\left(\zeta + \kappa\Phi\right)\right).$$

As $q_0^a = 1$, this gives $b = \frac{1-\mu}{\mu(\zeta+\kappa\Phi)}$. □

Figure G.12 illustrates the dynamics of (104). An unanticipated, permanent inflow $f_0$ at time 0 has an immediate price impact $\left(\frac{1}{\zeta} + b\right) f_0$ that is bigger than the long-run price impact $\frac{f_0}{\zeta}$. The initial "excess reaction" $bf_0$ dies down at the exponential rate $\Phi$. When funds are not inert, $b = 0$. This echoes the findings in Duffie (2010), with a somewhat different model.

For $\mu < 1$, we have $\Phi > \phi$: surprisingly, the speed of price dynamics $\Phi$ is faster than the fund-level speed of adjustment of quantities $\phi$. This is because the movements of the equity premium creates an incentive for adjustment beyond the "mechanical" speed $\phi$.[93]

**Calibration** We discuss the model calibration.[94] For the fund-level inertia we take $\phi = 1/\text{year}$, so that the half-life is about 0.7 years. We also take the instantaneous sensitivity to events to be $\mu = 0.5$, where the calibration isn't too sensitive to that, provided that $\mu > 0.1$. So, the speed of mean-reversion coming from inertia is $\Phi = \frac{\phi}{\mu} = 2/\text{year}$, and the overshooting of flows on impact is, using $\zeta = 0.16$ (with $\zeta^M = 0.2$) and $\kappa = 1$ year for illustration:

$$b = \frac{1-\mu}{\mu\zeta + \phi\kappa} = \frac{1-0.5}{0.5 \cdot 0.16 + 1 \cdot 1} \simeq 0.45.$$

The immediate price impact is $\frac{1}{\zeta} + b = 5.4$, while the permanent price impact is $\frac{1}{\zeta} = 5$. So, the temporary bump $b \ll \frac{1}{\zeta}$ is pretty negligible in the big picture. As the price decays as $bf_t\left(1-\Phi\right)^t$,

[93] Imagine that the impulse is a positive inflow, which increases the price. First, the "active" part of the fund strategy wants to sell shares, as the price is high and the equity premium low. But a low "instantaneous share" $\mu$ creates a high initial price jump, so a very negative expected return, speeding up the selling of shares: hence, the smaller the $\mu$, the greater the price jump $p_0$, and the faster the price adjustment $\Phi$.

[94] We use a "continuous time" calibration: The expressions work in continuous time, which makes calibration easier, replacing expressions like $(1-\phi)^t$ by $e^{-\phi t}$.

the premium is $b\Phi f = 0.45 \cdot 2 \cdot 0.5\% = 0.4\%$ (if $f_t = 0.5\%$), again a small premium. A higher inertia (lower $\phi$) creates a bigger difference between long run and short term price impact. So, examining inertia across investor classes is a useful avenue for future research.

**Impact of anticipated and unanticipated flows when funds are partially inert** We next generalize the price as present value formula (20) and the price impact with inertia (104).

**Proposition 12.** (Price impact with inertial funds) *When funds exhibit inertia, the price impact of inflows $df_s$ is:*

$$p_t = \frac{f_{-\infty}}{\zeta} + \sum_{s=-\infty}^{\infty} G(t-s)\,\mathbb{E}_t[\Delta f_s], \tag{105}$$

*where*

$$G(\tau) = \begin{cases} \left(\frac{1}{\zeta} + b\right) \frac{1}{(1+\rho)^{\tau}} & \text{if } \tau < 0, \\ \frac{1}{\zeta} + b(1-\Phi)^{\tau} & \text{if } \tau \geq 0. \end{cases} \tag{106}$$

*with $\rho = \zeta/\kappa$ is in* (21)*, and $b$ is in Proposition 11. When there is no inertia, $b = 0$.*

*Proof.* This can be checked by the "plug and verify" method for market clearing, $q_t = 0$. □

**Heterogeneity in inertia across funds** One can generalize this model to the case of heterogeneous inertial funds. Things are particularly tractable when $\phi_i$ is the same across funds $i$ (but $\mu_i, \zeta_i, \kappa_i$ could be different): then (104) holds, with more complex expressions for $b$ and $\Phi$.

## G.5 On the link between the Kyle lambda and the market inelasticity

### G.5.1 Theory: Kyle's lambda versus inelasticity

Suppose that within a certain time window, there is an "order flow" (realized signed trades), with volume $\Delta f_t$ expressed as a fraction of the market capitalization. A typical micro structure regression is, as in Evans and Lyons (2002); Hasbrouck (2007):

$$p_t - p_{t-1} = \lambda\left(\Delta f_t - \mathbb{E}_{t-1}[\Delta f_t]\right) \tag{107}$$

where $\lambda$ is the so-called "Kyle lambda", from Kyle (1985). We analyze what that regression would estimate in our model.

We suppose that our model holds, and that there is completely symmetric information about fundamentals – so, we remove the informational ingredient of Kyle. Still, trades will move prices – because of inelasticity. We clarify this here. As we mentioned above, a very important difference is that in Kyle flows do not change the equity premium on average, whereas in our model, positive inflows lower the equity premium.

To analyze what happens in our model, we suppose some autocorrelation in the order flow (like Madhavan et al. (1997), Lillo et al. (2005) and Bouchaud et al. (2018)):

$$\Delta f_t = (1-\phi_g)\,\Delta f_{t-1} + \varepsilon_t, \tag{108}$$

where $\varepsilon_t$ is i.i.d. So, an innovation $\varepsilon_t$ creates an innovation to the eventual cumulative flow:[95]

$$\lim_{h\to\infty} \mathbb{E}_t \left[f_{t+h} - f_{t-1}\right] = K\varepsilon_t, \qquad K = \frac{1}{\phi_g}.$$

For instance, if a large desired trade ("meta-order") is on average "sliced" into 15 trades, executed slowly over time, then $K = 15$. Likewise, if a fast fund trades, and is followed on average by similar or "copycat" meta-orders by two other funds, then $K = 3$.[96] The two forces combine: if a fund splits its meta-order in five trades, and it is followed by two more similar funds doing a similar trade (also splitting their trade into five chunks), then $K = 5 \times 3 = 15$, the product of the number of "similar" funds (3 in this example), and the number of "chunks" in which they split their trade (5 in this example).

In our model, the total price impact is, in the limit of small time intervals,[97]

$$\Delta p_t = \frac{K}{\zeta}\varepsilon_t = \frac{K}{\zeta}\left(\Delta f_t - \mathbb{E}_{t-1}\left[\Delta f_t\right]\right).$$

Hence, an econometrician estimating (107), will find:

$$\lambda = \frac{K}{\zeta}. \tag{109}$$

This means that, for the aggregate market, the Kyle lambda is the inelasticity $\frac{1}{\zeta}$ times the persistence parameter $K$ associated with the positive autocorrelation of the order flow.

Most empirical work in micro structure is done at the level of one asset, so that the $\lambda$ they estimate is

$$\lambda = \frac{K}{\zeta^{\perp}}, \tag{110}$$

where $\lambda^{\perp}$ is the micro-elasticity of Section G.3.

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[95] Indeed, $\varepsilon_t$ creates an innovation to the cumulative flow $f_{t+h}$ equal to

$$\mathbb{E}_t\left[f_{t+h} - f_{t-1}\right] = \mathbb{E}_t\left[\Delta f_t + \Delta f_{t+1} + \cdots + \Delta f_{t+h}\right] = \varepsilon_t\left(1 + (1-\phi_g) + \cdots + (1-\phi_g)^h\right) = \frac{\varepsilon_t}{\phi_g}\left(1 - (1-\phi_g)^{h+1}\right).$$

[96] More generally (such as in models with multiple time scales, or some form of long memory, see Bouchaud et al. (2018)), $K$ is the "expected value of related orders, given the past". So, the estimation of $K$ is a bit delicate, and not simply the inverse of the speed of mean-reversion of orders. Formally, with $\varepsilon_t \coloneqq \Delta f_t - \mathbb{E}_{t-1}\left[\Delta f_t\right]$, $K = \frac{\partial}{\partial \varepsilon_t} \lim_{h\to\infty} \mathbb{E}_t\left[f_{t+h} - f_{t-1} | \varepsilon_t\right]$. For instance, if we have (39)

$$\Delta f_t = \sum_{l=1}^{k} a_l \Delta f_{t-l} + \varepsilon_t,$$

then the total innovation is $K = \frac{1}{1-\sum_{l=1}^{k} a_l}$. We use this in Section 4.3.

[97] Away from the limit of small time intervals, the calculation is the following. The price is $p_t = Af_{t-1} + B\Delta f_t$ for two coefficients $A$, $B$ to determine. Calculations based on Proposition 5 (plug in that expression in (19) with $q_t = 0$) show: $A = \frac{1}{\zeta}$ and $B = \frac{1+\frac{\kappa}{\zeta}}{\zeta + \kappa\phi_g}$. So, in the limit of small time intervals (as in Section G.12.2), with $\kappa = \boldsymbol{\kappa}/\Delta t \to \infty$, and $\zeta$ constant (as $\delta = \boldsymbol{\delta}\Delta t$, $\delta\kappa$ is constant as $\Delta t \to 0$), we get $B \to \frac{1}{\zeta\phi_g} = \frac{K}{\zeta}$, i.e. $\Delta p_t = \frac{K}{\zeta}\varepsilon_t$.

### G.5.2 Empirical values from the micro structure literature

Frazzini et al. (2018) find that buying 2.5% of the daily volume of a stock creates a permanent price impact $\Delta p = 15$bp (indeed, it creates a total price impact of 18bp, of which 85% is permanent, see their Figures 2 and 6). Using an annual turnover of 100%, and 250 trading days per year, this means that buying a fraction $\Delta q = 2.5\% \times \frac{1}{250} = 1$bp of the stock creates a 15bp price impact. Hence, their Kyle lambda is[98]

$$\lambda = \frac{\Delta p}{\Delta q} = \frac{15\text{bp}}{1\text{bp}} \simeq 15.$$

Hence, the prima facie "micro structure" price impact is $\lambda \simeq 15$.[99] This can be compared with our own $M \simeq 5$. However, in terms of our model, their $\lambda$ reflects the micro-elasticity rather than the macro-elasticity: it is $\lambda = \frac{K}{\zeta^{\perp}}$. As we calibrate $\zeta^{\perp} \simeq 1$, this leads to $K \simeq 15$. This estimate has the interpretation, in inelastic markets with a micro elasticity of 1, that a large market-wide desired trade ("meta-order") is on average split into 15 smaller trades executed over time, by one or several institutions collectively (for example, by three funds pursuing a similar strategy, each splitting their desired position change into five smaller trades).

This factor $K > 10$ may seen surprisingly large, but it is consistent with micro structure data. Bouchaud et al. (2018) report a positive autocorrelation of the decay in the signed of trades $\varepsilon_t = sign\,(\Delta f_t)$, qualitatively consistent with the above model. Importantly, it is also roughly quantitatively consistent too. The empirical correlation between the signs of trades, $c\,(h) = corr\,(\varepsilon_t, \varepsilon_{t+h})$, is approximately $c\,(h) \simeq \frac{0.25}{h^{1/2}}$ for $h \in [1, 10^3]$, which leads to $K = 1 + \sum_{h=1}^{10^3} c\,(h) = 16$.[100] This means that a buy trade today announces 15 more buy trades in the future – a large empirical autocorrelation of market orders. We explain this, in this section, by order splitting and copycat trades (which is also Bouchaud et al. (2018)'s interpretation – here we also relate it to the micro elasticity $\zeta^{\perp}$ of the market, by (110)). This gives, we think, a potentially satisfactory unification of the very high impact measured impact of the micro structure literature, and the more moderate impact measured in inelastic markets.

One lesson is that the market micro structure literature finds price impacts that are larger than the ones we find (with a price impact multiplier over 15), which may help dispel some feeling that our estimates are too large. By estimating things at a low frequency, and using a model taking into account the autocorrelation of the order flow, we can structurally relate their price impact estimates to the market micro-inelasticity (since most of the micro structure literature is about the micro elasticity, not the macro elasticity).

[98] In practice, the measured price impact is not linear, and indeed looks more concave, perhaps like a square root, which may be due to slower trading of large orders (Torre and Ferrari (1998); Gabaix et al. (2003, 2006); Bouchaud et al. (2018)). We think that this elaboration is beyond the scope of this appendix.

[99] Frazzini et al. (2018) also explore the Trades And Quotes (TAQ) data, and find a price impact about 2.5 times bigger (see their Figure 7). This would would then lead to $\lambda \simeq 37$. In our discussion we use their baseline estimate, which is instead constructed using trading data from AQR, a large institutional asset manager.

[100] See their Figure 10.1. This model has a power law decay rather than an exponential decay, because it is a mixture of several exponential decay. Also, a limitation is that Bouchaud et al. (2018) study the sign of flows, whereas our model would like the signed traded, including their size.

## G.6 The model in continuous time

We use the notation $\mathbb{E}_t\left[\frac{dp_t}{dt}\right] = \frac{\mathbb{E}_t[dp_t]}{dt} := \lim_{h\downarrow 0}\mathbb{E}_t\left[\frac{p_{t+h}-p_t}{h}\right]$. So, if $dp_t = \mu_t dt + \sigma_t dZ_t$, then $\mathbb{E}_t\left[\frac{dp_t}{dt}\right] = \mu_t$. Here we record the main expressions in continuous time. The equity premium

$$\pi_t = \mathbb{E}_t\left[\frac{dP_t}{P_t}\right]/dt + \frac{D_t}{P_t} - r_f \tag{111}$$

has the Taylor expansion:

$$\hat{\pi}_t = \mathbb{E}_t\frac{dp_t}{dt} + \delta\left(d_t - p_t\right). \tag{112}$$

We have:

$$q_t = -\zeta p_t + \kappa\delta d_t + \kappa\mathbb{E}_t\left[\frac{dp_t}{dt}\right] + \nu_t + f_t,$$

which in equilibrium (with $q_t = 0$) leads to the stock price equation:

$$\mathbb{E}_t\left[\frac{dp_t}{dt}\right] - \rho p_t + \delta d_t + \frac{\nu_t + f_t}{\kappa} = 0. \tag{113}$$

Integrating forward, the stock price is:

$$p_t = \mathbb{E}_t\int_t^\infty e^{-\rho(\tau-t)}\left(\frac{\rho}{\zeta}\left(f_\tau + \nu_\tau\right) + \delta d_\tau\right)d\tau \tag{114}$$

$$= \frac{\delta}{\rho}d_t + \frac{1}{\zeta}\left(f_t + \nu_t\right) + \mathbb{E}_t\int_t^\infty e^{-\rho(\tau-t)}\left(\frac{\dot{f}_\tau + \dot{\nu}_\tau}{\zeta} + \frac{\delta}{\rho}\dot{d}_\tau\right)d\tau. \tag{115}$$

This allows for easier calculations than the discrete time model. For instance, suppose that flows and dividends follow autoregressive process, e.g. $df_t = -\phi_f f_t dt + \sigma_t dZ_t$ (for $dZ_t$ a mean-zero increment process, e.g. a Brownian motion). Then we have $\mathbb{E}_t\left[f_\tau\right] = e^{-\phi_f(\tau-t)}f_t$ for $\tau \geq t$, so (114) gives:

$$p_t = \frac{\rho}{\rho+\phi_f}\frac{f_t}{\zeta} + \frac{\delta}{\rho+\phi_d}d_t = \frac{1}{\zeta+\kappa\phi_f}f_t + \frac{\delta\kappa}{\zeta+\kappa\phi_d}d_t. \tag{116}$$

which is the continuous time equivalent of (26).

Combining (116) and (22) leads to:

$$\hat{\pi}_t = b_f^\pi f_t + b_d^\pi d_t, \tag{117}$$

with $b_f^\pi < 0$ and $b_d^\pi > 0$. In the random walk case, $b_f^\pi = -\frac{\delta}{\zeta}$ and $b_d^\pi = \frac{\delta(1-\theta)}{\zeta}$, while in the general case, we have $b_f^\pi = -\frac{\left(\delta+\phi_f\right)\rho}{\rho+\phi_f}\frac{1}{\zeta}$ and $b_d^\pi = \frac{\delta(1-\theta)}{\zeta+\kappa\phi_d}$.

## G.7 Infinite horizon model with heterogeneous funds

When there are several funds trading stocks, the setup is very similar to the case of a single mixed fund, but with indices $i$ for each fund. Fund $i$'s mandate says that the fraction invested in equities, $\frac{P_tQ_t}{W_{it}}$, is:

$$\frac{P_tQ_{it}^D}{W_{it}} = \theta_i e^{\kappa_i\hat{\pi}_t + \nu_{it}}, \tag{118}$$

where as before $\hat{\pi}_t := \pi_t - \bar{\pi}$ is the deviation of the equity premium from its average, and we allow for additional demand shocks, $\nu_{it}$. Dividends and bond coupons are passed on to consumer, so that retained dividends would be counted as active flows.

Differentiating the demand for stocks (118) we get:

$$\Delta \ln Q_{it} = \Delta \ln W_{it} - \Delta \ln P_t + \kappa_i \Delta \hat{\pi}_t.$$

Now, from accounting the evolution of wealth of fund $i$ is:

$$\Delta W_{it} = Q_{i,t-1} \Delta P_t + \Delta F_{it},$$

that is,

$$\frac{\Delta W_{it}}{W_{i,t-1}} = \frac{Q_{i,t-1} P_{t-1}}{W_{i,t-1}} \frac{\Delta P_t}{P_{t-1}} + \frac{\Delta F_{it}}{W_{i,t-1}} = \theta_{i,t-1} \frac{\Delta P_t}{P_{t-1}} + \frac{\Delta F_{it}}{W_{i,t-1}},$$

so:

$$\Delta \ln Q_{it} = -\left(1 - \theta_{i,t-1}\right) \Delta \ln P_t + \frac{\Delta F_{it}}{W_{i,t-1}} + \kappa_i \Delta \hat{\pi}_t = -\left(1 - \theta_{i,t-1}\right) \Delta p_t + \kappa_i \Delta \hat{\pi}_t + \frac{\Delta F_{it}}{W_{i,t-1}}.$$

We use the linearization of the risk premium (18), $\hat{\pi}_t = -\delta \left(d_t^e - p_t\right) + \mathbb{E}_t \left[\Delta p_{t+1}\right]$. So, linearizing throughout,

$$\Delta \ln Q_{it} = -\left(1 - \theta_{i,t-1} + \kappa\delta\right) \Delta p_t + \kappa \Delta \mathbb{E}_t \left[\Delta p_{t+1}\right] + \Delta f_{it},$$

with:

$$\Delta f_{it} = \frac{\Delta F_{it}}{W_{it}} - \left(1 - \theta_{i,t-1}\right) \Delta \ln \bar{P}_t. \tag{119}$$

For instance, in the AR(1) model (26), we have $\mathbb{E}_t \left[\Delta p_{t+1}\right] = -\phi p_t$, so $\kappa \Delta \mathbb{E}_t \left[\Delta p_{t+1}\right] = -\kappa\phi\Delta p_t$. This definition of the flow is slightly different from that in (17): definition (119) is more useful for empirical work, and (17) for the equilibrium price. They are obviously very close. They are identical to the leading order when the baseline economy has a growth rate of zero. One can also verify directly that the two expressions are consistent in the case of the mixed fund.

## G.8 Complements to the general equilibrium model

### G.8.1 The model with government bonds

We propose a way to include bonds. We assume that the government issues at the beginning of period $t$ bonds in quantities $B_t$. They are financed via lump-sum taxes: the tax at time $t$ is $T_t = R_{f,t-1} B_{t-1} - B_t$, as taxes pay for the maturing bond value, $R_{f,t-1} B_{t-1}$, minus new bond issuance, $B_t$. We assume that the government issued debt in amount:

$$B_t = \frac{1 - \Theta}{\Theta} \bar{W}_t^{\mathcal{E}} \tag{120}$$

for some $\Theta > 0$, and where $\bar{W}_t^{\mathcal{E}} = \frac{\mathcal{D}_t}{\delta}$ is the aggregate value of equities in the baseline model. This way, in the baseline model, total financial wealth is $\bar{W}_t^{\mathcal{E}} + B_t = \frac{1}{\Theta} \bar{W}_t^{\mathcal{E}}$, and a fraction $\Theta$ of wealth is in equities, while the rest $(1 - \Theta)$ is in government bonds. In short, the government issues enough bonds to maintain a share $1 - \Theta$ of bonds as a fraction of total financial wealth.

With this additional feature of the model, everything remains the same, except that the average equity premium is

$$\bar{\pi} = \gamma \Theta \sigma_r^2. \tag{121}$$

The calibration can be modified in a parsimonious way: we target a average aggregate equity share $\Theta = 0.6$, and increase $\gamma$ by a factor $\frac{1}{\Theta}$ to keep $\bar{\pi}$ constant, changing accordingly $\beta$ to keep the same interest rate $r_f$ in the consumption Euler equation (54). Nothing else needs to change.

### G.8.2 Details of the household's problem in general equilibrium: the consumer's part of the household

The consumer part of the household only decides on consumption, which entails withdrawing money from the pure bond fond. The consumption decision is made under fully rational choice: as discussed in the main text, this ensures that the standard consumption Euler equation for bonds holds, $\mathbb{E}_t \left[ \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} R_{ft} \right] = 1$. Here we detail how this happens. We adopt a formulation based on mental accounts that makes this idea portable to other contexts.

The household has, at the beginning of period $t$, a "consumption account" with wealth $w_t$ (this might be, for example, a checking account), and a "financial wealth account" that holds a quantity $Q_t^B$ of bonds (with a dollar value of 1) and a quantity $Q_t^{\mathcal{E}}$ of equity shares. We let $Q_t = \left( Q_t^B, Q_t^{\mathcal{E}} \right)$. While the aggregate bond holding of the household is $Q_t^B + w_t$, we posit that its two components are not fully interchangeable because of the presence of mental accounts. One can think of $w_t$ as being small (in equilibrium, it will be 0), and $Q_t^B$ as being big. We drop the superscript $h$ in this section for simplicity whenever there is no ambiguity, but all the quantities refer to a given household $h$ (which will be the representative household in equilibrium). Also, we directly refer to the ultimate assets held, as the household is assumed to be smart enough to "see through" the veil of mutual fund intermediation. Recall that $Z_t$ is the macro state vector (see Definition 1), and we call $Z_t^h = \left( Z_t, Q_t^B, Q_t^{\mathcal{E}}, w_t \right)$ the state vector specific to household $h$.

The evolution of wealth in the consumption account is

$$w_{t+1} = R_{ft} \left( w_t + \mathcal{Y}_t^h - C_t \right), \tag{122}$$

where

$$\mathcal{Y}_t^h = Q_t^B r_{f,t-1} + Q_t^{\mathcal{E}} D_t + \Omega_t - T_t =: \mathcal{Y} \left( Z_t^h \right) \tag{123}$$

is the aggregate income to the household, coming from its "financial dividend" stream (from bonds in the financial account, $Q_t^B r_{f,t-1}$, as well as equities, $Q_t^{\mathcal{E}} D_t$), plus residual income $\Omega_t$ (e.g., comprising labor income), minus $T_t$.

The consumer's problem is to maximize lifetime utility, subject to this dynamic budget constraint. Crucially, we assume that when the consumer does that, she takes the income $\mathcal{Y}_t^h$ as exogenous to her consumption decisions: in this sense, the decisions of the consumer and financier sides of the households are decoupled. So, as in any consumption-saving problem, the consumer's Euler equation for bonds holds, (43). But the consumer does not see that as she consumes more, and hence lowers the amount of bonds in the household's holdings, she will induce a (typically small) flow from stocks to bonds in the financier side of the household, so that $Q_{t+1}$ is affected. In equilibrium, $w_t = 0$ and $\mathcal{Y}_t^h = Y_t = C_t$ at all dates, as the representative household consumes aggregate income.

It might be helpful to also state this same argument using dynamic programming. Namely, the consumer side of the household solves:

$$V\left(w_t, Z_t^h\right) = \max_{C_t} u\left(C_t\right) + \beta\mathbb{E}\left[V\left(R_{ft}\left(w_t + \mathcal{Y}\left(Z_t^h\right) - C_t\right), Z_{t+1}^h\right)\right], \tag{124}$$

where the law of motion of $Z_{t+1}^h$ is taken by the agent to be independent of $c_t$. In the case of a strategic consumer wishing to manipulate the financier part of the household, the agent would take into account the (small) dependence on $Z_{t+1}^h$ on $c_t$.

### G.8.3 Details of the household's problem in general equilibrium: mutual fund shares accounting

This section provides some extra details on the household's problem of Section 5.1. We call $t^-$ the beginning of period values, evaluated at the time $t$ price $P_t$. The mixed fund gives a dividend $D_t^M = QD_t + r_{f,t-1}B_{t-1}^M$, so that its cum-dividend value is $W_{t^-}^M = QP_t + B_{t-1}^M + D_t^M$, and the return is $R_t^M = \frac{W_{t^-}^M}{W_{t-1}^M}$.

The mixed fund has issued $N_{t-1}$ shares, of which $N_{t-1}^h$ are owned by household $h$. The value of a share in the mixed fund is $v_t^M = \frac{QP_t + B_{t-1}^M}{N_{t-1}}$. So, the beginning of period wealth of the household is:

$$W_{t^-}^h = \frac{N_{t-1}^h}{N_{t-1}} W_{t^-}^M + B_{t-1}^h R_{f,t-1}. \tag{125}$$

Suppose there are household flows $\Delta F_t^h$ into the mixed fund, while flows from the rest of the economy are $\Delta F_t$ (in equilibrium, the two values are the same). Then, the number of shares owned by the household and in the fund are:[101] $N_t^h = N_{t-1}^h + \frac{\Delta F_t^h}{v_t^M}$ and $N_t = N_{t-1} + \frac{\Delta F_t}{v_t^M}$. The household holds $B_t^h$ in the pure bond fund:

$$B_t^h = B_{t-1}^h R_{f,t-1} + \frac{N_{t-1}^h}{N_{t-1}} D_t^M - C_t - \Delta F_t^h,$$

i.e. the proceeds from the pure bond fund, the dividend of the mixed fund, minus consumption, minus the flow.

The household's problem, in its rational form, is:

$$V\left(W_{t^-}, Z_t\right) = \max_{C_t, B_t^h} u\left(C_t\right) + \beta\mathbb{E}\left[V\left(W_{t+1^-}^h, Z_{t+1}\right)\right].$$

This problem defines a consumption policy, and also desired holdings in the pure bond fund (hence, a flow out of the bond fond).

### G.8.4 On a production economy

If we had a production-based model with capital $K_t$, then investment $I_t$ and labor demand $L_t$ (with $\kappa$ the cost of investment, $w$ the wage) would be characterized by the following problem:

$$\begin{aligned} V\left(K_t, Z_t\right) &= \max_{I_t, L_t}\{F\left(K_t, L_t, Z_t\right) - w\left(Z_t\right)L_t - I_t - \kappa\left(I_t, K_t, Z_t\right) \\ &\quad + \mathbb{E}_t\left[\mathcal{M}_{t+1} V\left(\left(1-\delta\right)K_t + I_t, Z_{t+1}\right)\right]\}, \end{aligned} \tag{126}$$

[101] We also have $v_t^M = \frac{W_t^M}{N_t} = \frac{QP_t + B_{t-1}^M}{N_{t-1}}$, $B_t^M = B_{t-1}^M + \Delta F_t$. Flows change the number of shares issued by the fund, but not (controlling for stock prices) the value of each fund share.

using the SDF $\mathcal{M}_{t+1}$ developed in the paper (see (55)). Hence, we can trace how an inflow into equities increases equity prices, lowers the risk premium, and increases investment. We leave the full, quantitative analysis of this to future research, but hope that this will help economists see more concretely how all fits together.

## G.9 Linking flows to beliefs

### G.9.1 Flows and perceived risk premia

We now develop a model in which flows are determined by households' expectations of excess returns. This simple notion turns out to be quite rich.

We will use the following notations. We call $\Theta_t^h$ the equity share of household $h$, and $\Theta_t$ the aggregate equity share in the economy, and $\Theta$ its baseline value. We call $\hat{\pi}_t^h$ the perception by household $h$ of the excess risk premium $\hat{\pi}_t = \pi_t - \bar{\pi}$, and $\hat{\pi}_t^H$ the perception of this excess risk premium by the representative household. Those perceptions need not be rational. Suppose that household $h$ has an inflow $f_t^h$ into the market, while the average household has an inflow $f_t$. We simply consider a small, hypothetical divergence between household $h$ and the representative household that happened just before $t$. We start with with a bit of equity-share accounting.

**Lemma 1.** *The equity share of household $h$ is:*

$$\Theta_t^h = \Theta\left(1 + f_t^h - f_t + (1-\Theta)\, p_t\right). \tag{127}$$

*Proof.* Let us call $\Theta_t$ the aggregate equity share (this is also $\theta_{Wt}$ in (15), but here we use a slightly simpler notation), using (120) for the aggregate bond supply:

$$\Theta_t = \frac{P_t Q}{P_t Q + B_t} = \frac{\bar{P}_t Q e^{p_t}}{\bar{P}_t Q e^{p_t} + \frac{1-\Theta}{\Theta}\bar{P}_t Q} = \frac{\Theta e^{p_t}}{\Theta e^{p_t} + 1 - \Theta},$$

so that, taking a first order expansion in small $p_t$,

$$\Theta_t = \Theta\left(1 + (1-\Theta)\, p_t\right). \tag{128}$$

Household $h$ has an equity share $\Theta_t^h = \frac{P_t Q e^{f_t^h - f_t}}{P_t Q + B_t}$, as it has the same aggregate wealth as the representative household (in the denominator), i.e., as a Taylor expansion:

$$\Theta_t^h = \Theta_t\left(1 + f_t^h - f_t\right). \tag{129}$$

Combining those two expressions gives (127). □

Let us now discuss the modeling of the flow $f_t^h$ of a household $h$ that perceives the equity premium to be $\hat{\pi}_t^h$. One natural model is that the financier optimizes over $\Theta_t^h$ (with $V^p(W) = \frac{W^{1-\gamma}-1}{1-\gamma}$) the "narrow framing" value function:

$$U\left(\Theta_t^h, \hat{\pi}_t^h\right) := \mathbb{E}\left[V^p\left(R_{ft} + \Theta_t\left(\bar{\pi} + \hat{\pi}_t^h + \varepsilon_{t+1}^r\right)\right)\right] \simeq 1 + r_{ft} + \Theta_t^h\left(\bar{\pi} + \hat{\pi}_t^h\right) - \frac{1}{2}\gamma\sigma_r^2\left(\Theta_t^h\right)^2, \tag{130}$$

where $\varepsilon_{t+1}^r$ has mean 0 and variance $\sigma_r^2$, and the second expression is in the limit of small time intervals. This implies a targeted equity share (using $\pi_t^h = \bar{\pi} + \hat{\pi}_t^h$) equal to:

$$\Theta_t^h = \frac{\pi_t^h}{\gamma \sigma_r^2} = \frac{\bar{\pi}\left(1 + \kappa^H \hat{\pi}_t^h\right)}{\gamma \sigma_r^2},$$

with $\kappa^H = \frac{1}{\bar{\pi}}$.

This "attentive portfolio choice" formulation, while it may be appealing as it is traditional, leads to three problems. First, in the cross-section, the attentive formulation predicts:

$$f_t^h - f_t = \kappa^H \left(\hat{\pi}_t^h - \hat{\pi}_t^H\right), \tag{131}$$

with a strength of $\kappa^H = \frac{1}{\bar{\pi}} \simeq 22$. This is a very large pass-through from beliefs to actions, and it contradicts both intuition (as an agent seeing a higher equity premium by 2.5% would double his equity share), and the evidence in Giglio et al. (2021a), who instead find a passthrough $\kappa^H \simeq 2$ (using yearly units). The second problem is that, with this formulation, we cannot model a plausible agent who would let his equity share drift passively.[102] The equity share $\Theta_t^h$ is always "actively managed" by the household. The third problem is that the household is so active that it exactly cancels out all the actions of intermediaries, i.e. of the mixed fund. Indeed, plugging this into (128) gives a price equal to $p_t = \frac{\kappa^H \hat{\pi}_t^H}{(1-\Theta)}$, so proportional to the risk premium $\hat{\pi}_t^H$ perceived by the representative household, and independent of $\zeta$ and any other considerations.[103]

We propose a common resolution of those three problems, which relies on "behaviorally inattentive" portfolio choice. We posit that a household $h$ is partially inattentive to some of the normatively relevant determinants of its flow, and chooses its flow according to:

$$\max_{f_t^h} U\left(\Theta^h\left(f_t^h, m_\theta f_t, m_\theta p_t\right), m_\pi \hat{\pi}_t^h\right), \tag{132}$$

where $\Theta^h\left(f_t^h, f_t, p_t\right) = \Theta\left(1 + f_t^h - f_t + (1-\Theta) p_t\right)$ as in (127), $U$ is in (130), $m_\theta \in [0,1]$ is the attention to the rebalancing concerns, and $m_\pi$ is the attention to one's estimate of the risk premium. The traditional case corresponds to $m_\theta$ and $m_\pi$ equal to 1. However, when $m_\theta = 0$, the household pays no attention to rebalancing needs: it lets the portfolio drift. This sort of behavioral modeling is tractable, it applies to a variety of micro and macro setups, and has good empirical support (see the survey in Gabaix (2019)). We derive the resulting flows.

**Proposition 13.** *In the above behavioral formulation, the aggregate flow is linked to the aggregate belief $\hat{\pi}_t^H$ by:*

$$f_t = \frac{\kappa^H \hat{\pi}_t^H - m_\theta (1-\Theta) p_t}{1 - m_\theta}, \tag{133}$$

*where:*

$$\kappa^H = \frac{m_\pi}{\bar{\pi}}. \tag{134}$$

[102] For instance, Giglio et al. (2021b) document a large fraction of investors who do not trade at all during the stock market crash of early 2020.

[103] In a standard model with two types of agents, both agents matter. However, here the household is the principal, and the mixed fund the agent, so that the household in fact owns all equities, and thus (if it is active enough) it can undo the actions of the mixed fund.

*In the cross-section, a household h perceiving the equity premium deviation to be $\hat{\pi}_t^h$ has a flow:*

$$f_t^h = f_t + \kappa^H \left(\hat{\pi}_t^h - \hat{\pi}_t^H\right), \tag{135}$$

*so that the cross-sectional sensitivity of flows to the equity premium is $\kappa^H$, less that then the aggregate sensitivity of flows to the equity premium is, which is $\frac{\kappa^H}{1-m_\theta} > \kappa^H$. Those two relations also imply the following link:*

$$f_t^h = m_\theta \left(f_t - (1-\Theta)\, p_t\right) + \kappa^H \hat{\pi}_t^h. \tag{136}$$

*Proof.* Given (132), at the optimal flow level we should have, with $\kappa^H = \frac{m_\pi}{\bar{\pi}}$:

$$\Theta_t^h = \frac{\pi_t + m_\pi \hat{\pi}_t^h}{\gamma\sigma^2} = \frac{\bar{\pi}\left(1 + \kappa^H \hat{\pi}_t^h\right)}{\gamma\sigma^2} = \Theta\left(1 + \kappa^H \hat{\pi}_t^h\right).$$

We also have

$$\Theta_t^h = \Theta^h\left(f_t^h, m_\theta f_t, m_\theta p_t\right) = \Theta\left(1 + f_t^h - m_\theta f_t + (1-\Theta)\, m_\theta p_t\right).$$

This gives (136). Finally, the flow of the representative agent satisfies $f_t^h = f_t$, i.e. is (133). □

We check that this formulation solves the three problems we outlined. First, the cross-household sensitivity of flows to beliefs is smaller, and can be realistically calibrated. Intuitively, the household does see that high perceived excess return $\hat{\pi}_t^h$ should lead to a high flow $f_t^h$ ($m_\pi > 0$), but doesn't agree that a 2.5% extra risk premium should lead it to increase its equity share from $\Theta = 60\%$ to 94%, as it would in the rational model (with $\kappa^H = \frac{1}{\bar{\pi}} = 22$). Indeed, Giglio et al. (2021a) find $\kappa^H \simeq 2$.[104] To match their evidence, we need to calibrate $m_\pi = \kappa^H \bar{\pi} = 0.08$. This means that people might have "bold forecasts" but "timid choices", very much as in Kahneman and Lovallo (1993). Second, we can model a very inactive household that would let its wealth drift independently of any perception of the risk premium: that would correspond to the case in which $m_\theta = m_\pi = 0$. Third, now the household does not exactly cancel the actions of the mixed fund: in (133), because $m_\theta < 1$, the flow's reaction is finite, and only in the completely reactive case ($m_\theta = 1$) the household would completely undo the actions of the mixed fund (when $m_\theta \to 1$, to avoid an infinite flow the numerator of (133) needs to go to 0, which pins the price to $p_t = \frac{\kappa^H \hat{\pi}_t^H}{1-\Theta}$).

### G.9.2 Application

**Flows and expected risk premia: A simple specification** As a simple calibration of (133), let us take $m_\theta = 0$. Then, we have:

$$f_t = \kappa^H \hat{\pi}_t^H, \tag{137}$$

i.e. the flow is just the perceived risk premium, times the sensitivity $\kappa^H$. If $\kappa^H = 2$, then the volatility of the perceived risk premium is simply the volatility of the flows, divided by $\kappa^H = 2$, so about 1.4% per year in our calibration (see Section 5.4). Recall that we have $f_t = \theta b_t$ (see (47)). So, this is a model where the "behavioral disturbance" is simply a time-variation in the perceived value of equities, $b_t = \frac{\kappa^H}{\theta} \hat{\pi}_t^h$

[104] For instance, column 6 of their Table III gives $\frac{d\Theta_t^h}{d\pi_t^h} = 1.2$. Given a typical $\Theta = 2/3$, that leads to $\kappa^H = \frac{1}{\Theta}\frac{d\Theta_t^h}{d\pi_t^h} = 1.8$.

**Flows and expected growth of fundamentals** Bordalo et al. (2020) propose that stock market fluctuations are linked to subjective expectations of long-term growth in dividends. Let us see how that would be linked to flows and prices in inelastic markets. Call $g_t$ the expectation of the growth rate of dividends (expressed as a deviation from the average growth rate $\bar{g}$, so that the expected growth rate is $\bar{g}+g_t$), and for now suppose that

$$f_t = b_g^f g_t,$$

for some parameter $b_g^f \geq 0$. We assume that $g_t$ mean-reverts with speed $\phi$, so that $\mathbb{E}_t[g_{t+1}] = (1-\phi)\, g_t$. Then, the price deviation from the baseline is:[105]

$$p_t = \frac{\kappa + b_g^f}{\zeta^M} g_t, \tag{138}$$

with $\zeta^M = \zeta + \kappa\phi$. In a frictionless economy where agents hold those beliefs, the price would be $p_t^* = \frac{1}{\delta+\phi} g_t$. So the flow passthrough that replicates that is $b_g^{f,*} = \frac{\zeta_M}{\delta+\phi} - \kappa$. Calibrating, this gives $b_g^{f,*} = \frac{0.2}{0.034+0.04} - 1 \simeq 2$ years. If the growth rate is perceived to be 1% higher, then the flow is 2% higher.

Now, let us see how we link the flows more deeply to explicit beliefs about returns. We find it useful to consider that households have time-varying beliefs about the long-term growth rate of dividends, $m_g g_t$, and the representative fund has beliefs $g_t$. When $m_g > 1$, the general public is more reactive in its beliefs than institutions.

**Proposition 14.** *The solution is as follows. Flows, prices, and the risk premium perceived by institutions ($\hat{\pi}_t$) and households ($\hat{\pi}_t^H$) follow:*

$$f_t = b_g^f g_t, \qquad p_t = b_g^p g_t, \qquad \hat{\pi}_t = b_g^\pi g_t, \qquad \hat{\pi}_t^H = b_g^{\pi,H} g_t,$$

*where, with $\zeta^M = \zeta + \kappa\phi$,*

$$b_g^p = \frac{m_g \kappa^H + (1-m_\theta)\,\kappa}{\kappa^H(\delta+\phi) + (1-m_\theta)\,\zeta^M + m_\theta(1-\Theta)},$$

*and $b_g^f = \zeta^M b_g^p - \kappa$, $b_g^\pi = 1 - (\delta+\phi)\, b_g^p$, $b_g^{\pi,H} = m_g - (\delta+\phi)\, b_g^p$.*

*Proof.* The risk premium perceived by institutions is the belief $\hat{\pi}_t^I = g_t - (\delta+\phi)\, p_t$, so

$$b_g^\pi = 1 - (\delta+\phi)\, b_g^p, \tag{139}$$

while the belief of households is $\hat{\pi}_t^H = m_g g_t - (\delta+\phi)\, p_t$, so that its sensitivity to $g_t$ is $b_g^{\pi,H} = m_g - (\delta+\phi)\, b_g^p$.

[105] Indeed, replacing $g$ by $\bar{g}+g_t$ in (70) we obtain (in the limit of small time intervals):

$$\hat{\pi}_t = g_t + \mathbb{E}_t\left[\Delta p_{t+1} + \delta\,(d_{t+1} - p_t)\right].$$

We seek a solution of the type $p_t = b_g^p g_t$, and we also observe that the deviation of the dividend from the baseline is simply $d_{t+1} = 0$. Plugging this in (61), setting $\nu_t = 0$ for simplicity, gives:

$$0 \;=\; q_t = -(1-\theta)\, p_t + \kappa\hat{\pi}_t + f_t = -(1-\theta)\, b_g^p g_t + \kappa\left(g_t - (\phi+\delta)\, b_g^p g_t\right) + b_g^f g_t,$$

which gives (138).

By market clearing in the equity market, the total demand change by the mixed fund should be 0:

$$0 = q = -\left(1-\theta\right)p_t + f_t + \kappa\hat{\pi}_t^I = -\left(1-\theta\right)p_t + f_t + \kappa\left(g_t - \left(\delta+\phi\right)p_t\right),$$

i.e., with $\zeta^M = 1-\theta+\left(\delta+\phi\right)\kappa = \zeta+\kappa\phi$,

$$0 = -\zeta^M b_g^p + b_g^f + \kappa,$$

hence we recover (138),

$$b_g^f = \zeta^M b_g^p - \kappa \tag{140}$$

Finally, the flow of the representative investor is as in (133), so that:

$$0 = \left(1-m_\theta\right)f_t + \left(1-\Theta\right)m_\theta p_t - \kappa^H\hat{\pi}_t^H,$$

so

$$\begin{aligned} 0 &= \left(1-m_\theta\right)b_g^f + m_\theta\left(1-\Theta\right)b_g^p - \kappa^H b_g^{\pi,H} \\ &= \left(1-m_\theta\right)\left(\zeta^M b_g^p - \kappa\right) + m_\theta\left(1-\Theta\right)b_g^p - \kappa^H\left(m_g - \left(\delta+\phi\right)b_g^p\right), \end{aligned}$$

and

$$b_g^p = \frac{m_g\kappa^H + \left(1-m_\theta\right)\kappa}{\kappa^H\left(\delta+\phi\right) + \left(1-m_\theta\right)\zeta^M + m_\theta\left(1-\Theta\right)}. \tag{141}$$

□

**An illustrative calibration** To match the low passthrough from beliefs to actions in Giglio et al. (2021a), $\kappa^H \simeq 2$ years, we use $m_\pi = \kappa^H\bar{\pi} = 0.08$. We set the attention to rebalancing to $m_\theta = 0.3$, and give equal beliefs to households and institutions. This leads to $b_g^\pi = 0.5$. This means that if agents perceive the growth rate to be 1 percentage point higher than usual, they think that the risk premium is only 0.5 percentage points higher: they believe that the market has incorporated half of those news. This is also in line with Giglio et al. (2021a) (Table IX). This implies that $b_g^p = 6.6$ and $b_g^f = 0.3$. This calibration is illustrative. In future research, it would be highly desirable to quantify those parameters directly, in particular the only partial tendency of households to rebalance ($m_\theta < 1$).

## G.10 Pricing kernel consistent with flow-based pricing: Complements

This section gives complements to the flow-based SDF of Section 5.3.

### G.10.1 Basics

Much of asset pricing uses pricing kernels, or stochastic discount factors (SDFs). We show how to express the economics of flows in inelastic markets in the language of pricing kernels. To do so, we outline a simple general method to complete a "default" pricing kernel so that it reflects the impact of flows on asset prices.

**Pricing kernel completion: How to adjust a default pricing kernel to reflect the impact of flows on asset prices** For simplicity, we omit the time subscripts.

*Default pricing kernel.* We allow for a "default pricing kernel", which prices bonds at the equilibrium interest rate $R_f$. The simplest is the "risk-free" default pricing kernel: $\mathcal{M}^d = \frac{1}{R_f}$.[106]

*From the default pricing kernel to the actual pricing kernel.* The default pricing kernel $\mathcal{M}^d$ will not price assets correctly, as it does not react to flows. We propose a method of "pricing kernel completion" that will augment the pricing kernel so that it correctly prices all assets. We posit the existence of a very small mass $\varepsilon$ (which we will take to be infinitesimal, so that it will not impact prices) of "agile optimizers," who start with zero financial wealth and whose objective function is:

$$\max_{Q} \mathbb{E}\left[-\mathcal{M}^d e^{-Q'R}\right], \tag{142}$$

where $R$ is the vector of excess returns at time 1.[107] That is, they maximize (over a vector $Q$ of holdings over all assets) their expected return $R = \frac{P_1+D_1}{P_0} - R^f$, starting from zero wealth, but this is their expected return "under the risk-neutral probability" generated by $\mathcal{M}^d$.[108] Hence we have $\mathbb{E}\left[\mathcal{M}^d e^{-Q'R} R\right] = 0$. So, the following $\mathcal{M}$ is a pricing kernel:

$$\mathcal{M} = \mathcal{M}^d e^{-Q'R+\xi}, \tag{143}$$

where the constant $\xi$ ensures that the risk-free rate is correctly priced ($\mathbb{E}\left[\mathcal{M}\right] = \mathbb{E}\left[\mathcal{M}^d\right]$, so $\xi = \ln \frac{\mathbb{E}\left[\mathcal{M}^d\right]}{\mathbb{E}\left[\mathcal{M}^d e^{-Q'R}\right]}$).

We call this the "completed" pricing kernel. Note that other SDFs could also work (as is generic in incomplete markets), but the one given in (143) is the unique SDF coming from the "pricing kernel completion" procedure. We treated here the simplest case, with just one risky asset, and the simplest default pricing kernel $\mathcal{M}^{d,R_f} = \frac{1}{R_f}$.

**Flow-based SDF for the two-period model** Let us revisit the two-period model of Section 3.1. The excess equity premium is $\hat{\pi} = \delta\left(d - p\right)$ with

$$p = \frac{f + \kappa\delta d}{\zeta}, \tag{144}$$

so that, with $f = (1-\theta)\, d + \tilde{f}$, the total equity premium is: $\pi = \bar{\pi} - \delta\frac{\tilde{f}}{\zeta}$. So, the completed pricing kernel is:

$$\mathcal{M} = \exp\left(-r_f - \pi\frac{\varepsilon^D}{\sigma_d^2} + \xi\right), \qquad \pi = \bar{\pi} - \delta\frac{\tilde{f}}{\zeta}, \tag{145}$$

with $\xi = -\frac{\pi^2}{2\sigma_d^2}$ if $\varepsilon^D$ is Gaussian. This SDF prices correctly stocks and bonds.

This gives the "flow-based" completed pricing kernel, which is an alternative to the consumption-based kernel of Lucas (1978). The core economics is in how flows affect prices, and the pricing kernel (145) just reflects that. If there is a flow $f$, that modifies the price $P$ according to (144), and the pricing kernel $\mathcal{M}$, in such a way that $P = \bar{P}\left(1+p\right) = \mathbb{E}\left[\mathcal{M}D\right]$ holds. The pricing kernel is in a sense a symptom rather than a cause in that market.

[106] In the spirit of maintaining a continuity with the heritage of Lucas (1978), we can also consider a "consumption CAPM" default pricing kernel: $\mathcal{M}^{d,C} = \beta\frac{u'(C_1)}{u'(C_0)}$. We develop this in Section G.10.2.

[107] The implicit risk aversion of 1 is just a normalization.

[108] They start with zero capital at each period, and rebate their profits and losses to the representative household.

**Flow-based SDF for the infinite-horizon model** Section 5.3 developed the SDF for the infinite-horizon model process for flows in (26), something also delivered by our general equilibrium model of Section 5. A justification is that we assume that "dividend strips" are also traded. By the above procedure we obtain the pricing kernel for each date. In the construction, then dividend strips have an equity premium $\pi_t$ independent of maturity. So, the maximum Sharpe ratio is achieved via a one-period dividend strip.

Formally, one obtains the price of any asset, once we have a SDF. However, one can reasonably hope to obtain a correct price only when the novel asset is in very small quantity, as the agile optimizers, which form a very small group, will be able to absorb it. When there are substantially different asset classes, one needs to think about flows in those different classes — they will affect prices, and hence the SDF, along the lines we just saw. We next show how easy it is to generalize the model to several asset classes.

### G.10.2 More general cases to get a pricing kernel

Here we expand Section 5.3 to multiple risky assets and a consumption-based default SDF.

**A Gaussian example** We start with a basic example. We suppose that returns and consumption are lognormal:

$$\frac{C_1}{C_0} = e^{g_c + \sigma_c \varepsilon^c - \frac{1}{2}\sigma_c^2}, \tag{146}$$

with $\varepsilon_t^c$ a standard Gaussian random variable. Consider the consumption pricing kernel, which is:

$$\mathcal{M}^{d,C} \equiv e^{M^{d,C}} = \beta \left( \frac{C_1}{C_0} \right)^{-\gamma} = e^{-r_f - \gamma \sigma_c \varepsilon^c - \frac{1}{2}\gamma^2 \sigma_c^2}$$

for the risk-free rate $r_f = -\ln \beta + \gamma g_c - \frac{1}{2}\gamma (1+\gamma) \sigma_c^2$.

We next consider the agile optimizers' problem, going back to a general default pricing kernel:

$$\mathcal{M}^d = e^{M^d},$$

which might be $\mathcal{M}^{d,R_f}$ or $\mathcal{M}^{d,C}$. We recall that for two jointly Gaussian variables $X, Y$:

$$\frac{\mathbb{E}\left[e^X Y\right]}{\mathbb{E}\left[e^X\right]} = \mathbb{E}\left[Y\right] + cov\left(X, Y\right). \tag{147}$$

For instance, the anomalous excess equity premium is

$$\mathbb{E}^{\mathcal{M}^d}\left[R\right] := \frac{\mathbb{E}\left[\mathcal{M}^d R\right]}{\mathbb{E}\left[\mathcal{M}^d\right]} = \mathbb{E}\left[R\right] + cov\left(M^d, R\right), \tag{148}$$

which is the expected excess return of $R$ that is not explained by the default pricing kernel: indeed, if the pricing kernel $\mathcal{M}^d$ correctly priced $R$, we'd have $\mathbb{E}^{\mathcal{M}^d}\left[R\right] = 0$. Put another way, those are the excess returns above and beyond what is warranted by the default pricing kernel.

The FOC of (142) is $\mathbb{E}\left[\mathcal{M}^d e^{-Q'R} R\right] = 0$, so that using (147), with $V_R = cov\left(R, R\right)$ the variance-covariance matrix of returns,

$$\mathbb{E}\left[R\right] + cov\left(M^d, R\right) - V_R Q = 0,$$

where $-cov\left(M^d, R\right)$ is the equity premium warranted by the default pricing kernel. The optimal portfolio of agile optimizers is $Q = V_R^{-1}\mathbb{E}^{\mathcal{M}^d}[R]$ and their return is a form of "tangency portfolio" return:

$$R^\tau = Q'R = \mathbb{E}^{\mathcal{M}^d}[R]' V_R^{-1} R, \tag{149}$$

which depends on the "anomalous" excess returns $\mathbb{E}^{\mathcal{M}^d}[R]$. Their "excess Sharpe ratio" is:

$$\mathcal{S} = \frac{\mathbb{E}^{\mathcal{M}^d}[R^\tau]}{\sigma_{R^\tau}}, \tag{150}$$

which is the Sharpe ratio they get in excess of the average returns warranted by the default pricing kernel. Given that $\mathbb{E}^{\mathcal{M}^d}[R^\tau] = \mathbb{E}^{\mathcal{M}^d}[R]' V_R^{-1}\mathbb{E}^{\mathcal{M}^d}[R] = \sigma^2_{R^\tau}$, we have $\mathcal{S} = \sigma_{R^\tau}$. Hence, the SDF is their marginal utility (up to a proportional factor that is pinned down by the risk-free rate), which is:

$$\mathcal{M} = \mathcal{M}^d e^{-\mathcal{S}\frac{R^\tau - \mathbb{E}^{\mathcal{M}^d}[R^\tau]}{\sigma_{R^\tau}} - \frac{1}{2}\mathcal{S}^2}. \tag{151}$$

This SDF $\mathcal{M}$ prices all assets correctly: $P_a = \mathbb{E}[\mathcal{M}D_a]$ for all assets.

## G.11 Corporate finance in inelastic markets: Complements

We provide complements to Section 6.2.

### G.11.1 An increase in buyback financed by a decrease in dividends: Infinite horizon

Here we complete the discussion of the main text, with the infinite horizon case.

We provide a simple thought experiment. Suppose that at time 0 there is a permanent change in the share buyback policy: corporations devote a fraction $b$ of their dividend payout to share buybacks, the rest to dividends (see Boudoukh et al. (2007c) for an empirical analysis). So, the aggregate dividend goes from $\mathcal{D}_t$ to $\mathcal{D}'_t = \mathcal{D}_t(1-b)$, and at each date corporations spend $b\mathcal{D}_t$ on share buybacks. To streamline the computations, we use continuous time.

**Buybacks in a frictionless rational model** We first consider the rational model.

**Proposition 15.** *Consider a firm that at time 0 changes its payout policy, and devotes a fraction $b$ of the payout to share buybacks, $1-b$ to dividends (starting from paying only dividends before time 0). Consider a frictionless, rational model. Then, the dividend-price ratio falls by a factor $1-b$:*

$$\delta' = (1-b)\,\delta.$$

*It goes from $\delta = r_f + \pi - g$ to $\delta' = (1-b)\,\delta = r_f + \pi - g'$, where $g' = g + G$ is the new average growth rate of the dividend per share, which is increased by $G = b\delta$. At the same time, the market value of the firm is unchanged, as per Modigliani-Miller.*

The surprise is that it changes the dividend-price ratio by a big amount. The share of dividend as a fraction of the payout has moved from roughly 100% to 50%, so $b = 0.5$. Hence, Proposition 15 implies that the price-dividend ratio went from $\delta$ (empirically, about 4%) to half its value (about 2%). This is simply because the growth rate per share has increased by $G = 2\%$, because the number of shares has decreased by the same $G = 2\%$.

*Proof.* We had $P_t Q_t = \frac{\mathcal{D}_t}{\delta}$. As $b\mathcal{D}_t$ dollars are devoted to purchasing shares each period, the number of shares follows $\dot{Q}_t = -\frac{b\mathcal{D}_t}{P_t} = -bQ_t\delta$, so that the new number of shares is:

$$Q_t' = Q_0 e^{-Gt}, \qquad G = b\delta. \tag{152}$$

Hence, the dividend per share in the new regime is $D_t' = \frac{(1-b)\mathcal{D}_t}{Q_t'} = \frac{(1-b)\mathcal{D}_t}{Q_0 e^{-Gt}}$ i.e. as $D_t = \frac{\mathcal{D}_t}{Q_0}$,

$$D_t' = (1-b)\, e^{Gt} D_t. \tag{153}$$

Because the value of the firm is constant, $P_t' Q_t' = P_t Q_0$, we have

$$P_t' = P_t e^{Gt}. \tag{154}$$

This implies that the new dividend-price ratio is

$$\frac{D_t'}{P_t'} = \frac{D_t}{P_t}(1-b)\,. \tag{155}$$

This is of course consistent with the Gordon formula: as $\delta = r_f + \pi - g$, as under the new regime the dividend per share grows at a rate $g + G$, we have

$$\delta' = r_f + \pi - g - G = \delta - b\delta = (1-b)\,\delta,$$

using (152) in the last equation. □

**Buybacks in an inelastic model** We next study the situation in an inelastic model. We need a stabilizing force, and we assume that the flow has a "reaction to the risk premium" component $\chi\hat{\pi}_t$ as in (158).

**Proposition 16.** (Impact of share buybacks in the infinite horizon model) *In the inelastic model, suppose that a change in policy on a scale $b$ is announced, and should last forever: a fraction $b$ of the aggregate payout is devoted to share buybacks. Then the firm value increases by $v_* = \frac{(\mu^D - \mu^G)\theta b}{\chi}$ in the long run, and the risk premium is lower by $\hat{\pi}_* = -\delta v_*$.*

Hence, the economics is similar to the simple model of Proposition 7.

*Proof.* The proof is similar to that of Proposition 7 in Section F.[109] As in the rational case, growth rate in the number of shares is $-G$ with $G = \delta b$, $d_t = -b + Gt$. As each period the aggregate dividends are lower by $b$, and they represented a fraction $\delta\theta$ of the fund's value, and capital gains are increased by $G$, the inflow each period, from the households to the mixed fund, is:

$$\frac{d}{dt} f_t = \left(1 - \mu^D\right)(-b\delta\theta) - \mu^G G\theta + \chi\hat{\pi}_* = \left(-1 + \mu^D - \mu^G\right)\theta G + \chi\hat{\pi}_*$$

We conjecture a long term equilibrium where $\frac{dp_t}{dt} = G$, of the type: $p_t = Gt + \tilde{p}_t$, where $\tilde{p}_t = o(t)$. The demand deviation by the mixed fund is still $q_t = -\zeta p_t + \kappa\delta d_t + f_t$, and it should be equal to the supply deviation, $q_t^S = -Gt$. This gives,

$$0 = \lim_{t\to\infty} \frac{d}{dt}\left(q_t - q_t^S\right) = -\zeta G + \kappa\delta G + \left(-1 + \mu^D - \mu^G\right)\theta G + \chi\hat{\pi} + G = \left(\mu^D - \mu^G\right)\theta G + \chi\hat{\pi}_*.$$

[109]Our deviation from the baseline are in logs, e.g. $p_t = \ln\frac{P_t}{\bar{P}_t}$.

This implies that the long run change in the risk premium is

$$\hat{\pi}_* = -\frac{\left(\mu^D - \mu^G\right)\theta G}{\chi} = -\frac{\left(\mu^D - \mu^G\right)\theta\delta}{\chi}b.$$

The corresponding increase in market value if $v_* = -\frac{\hat{\pi}_*}{\delta}$. □

In the long run, it is very likely that the $\mu^D - \mu^G$ will be more rational, and fall to a value closer to 0—perhaps because the financier will more actively target a constant equity share (as in Section G.9), which would create another corrective inflow. Examining this effect empirically would be interesting.

### G.11.2 An increase in buyback financed by increased debt rather than lower contemporaneous dividends.

Consider an increase in buybacks that is not compensated by a contemporaneous decrease in dividends — so that the total payout is increased, by a factor equal to $b$ times the initial market value.

*Two period model.* The buyback of $B$ dollars decreases the number of shares by $\frac{B}{P_0}$, the future aggregate dividend by $BR$, where $R$ is the gross interest rate. The time-1 dividend is $\mathcal{D}_1' = \mathcal{D}_1 - BR$, the present value of $\mathcal{D}_1'$ falls by a fraction $b$. As the number of shares also falls by $b$, the present value of the time-1 dividend per share remains constant:

$$q^S = -b, \qquad d = 0.$$

In a frictionless model, this buyback does not change the current price per share, and does not change the time-0 return $r$

$$\text{Frictionless model: } p = 0, \qquad v = -b, \qquad r = 0.$$

In an inelastic model, now $q = -\zeta p + f^h = q^S = -b$, so

$$p = \frac{b + f^h}{\zeta}, \qquad v = p - b, \qquad r = p.$$

Hence, the aggregate values of equities increases, and the time-0 return $r$ is positive, unless it's compensated by a flow $f^h = -b$. Using the marginal propensities in Section 6.2, we have $f^h = -\mu^G \theta p$, so that in total:

$$p = \frac{b}{\zeta + \theta\mu^G}. \tag{156}$$

*Infinite horizon.* We suppose that the debt will be repaid very far in the future (at date $T \to \infty$.) Then, the economics is as in the two-period model.[110]

[110] In a rational model, we still have $p_t = 0$, $v_t = -b$. In an inelastic model, as $\rho > \delta$,we have $d_t = 0$, $q_t^S = -b$, so $p_t = \frac{b+f^h}{\zeta}$ (for all dates $t \ll T$) also. Hence the expression is as in the two-period model.

## G.12 When flows respond to risk premia in the long run

### G.12.1 Basic version

The present paper mostly points out how impactful flows are. We provided one microfoundation for flows in the macro model of Section 5.1, via the "behavioral disturbance" $b_t$, which is a stand-in for the forces driving flows. Here, we examine variants of that formulation.

**A necessary trend and cycle decomposition for flows** First, we record that all models of flows should satisfy the following decomposition to keep the price-dividend ratio stationary. For clarity, we use the detrending procedure of Section 3.2, but with the baseline of $\left(\bar{P}_t, \bar{D}_t, \bar{W}_t\right) = \left(\bar{P}_0, \bar{D}_0, \bar{W}_0\right) \mathcal{G}_t$ with $\mathcal{G}_t = 1$. We also use log deviations, e.g., $d_t = \ln \frac{D_t}{\bar{D}_t}$ is the deviation of the dividend from the initial date 0, and is typically non-stationary, for instance we might have $\mathbb{E}_0 \left[d_t\right] = gt$. This allows us to zoom in on the core issue of how households respond to trends in $d_t$. This also means that $\bar{F}_t = 0$, and that $f_t$ is the scaled flow.

**Lemma 2.** *(Trend-cycle decomposition for flows) The price-dividend ratio is stationary if and only if the cumulative flow $f_t$ admits the decomposition*

$$f_t = (1-\theta)\, d_t + \hat{f}_t, \tag{157}$$

*where $d_t$ the realized long term deviation in dividends ($D_t = D_0 e^{d_t}$), which is typically nonstationary, $\theta$ is the equity-weighted equity share, and $\hat{f}_t$ is stationary.*

*Proof.* Recall that $q = f_t - (1-\theta)\, p_t + \kappa \hat{\pi}_t$. As $q = 0$, (157) holds, with $\hat{f}_t = -\kappa \hat{\pi}_t + (1-\theta)\left(p_t - d_t\right)$. □

**How does the market equilibrate in the long run?** One might ask, how does the market discover the trend $(1-\theta)\, d_t$ in (157)? It comes out in the model of Section 5.1, via the assumption of (partial) rational expectations. But what about other models? It turns out that a variety of plausible models of investor behavior also lead to stationarity. We briefly summarize the situation, while Section G.12.2 provides proof and complements. Consider a behavioral rule of the type

$$\Delta f_t = \chi \hat{\pi}_t + \varepsilon_t, \tag{158}$$

with $\chi > 0$: this means that people invest more in equities when they are undervalued, which makes flows stabilizing. Then, one can show that this leads to a stationary $P/D$ ratio, as in Lemma 2, and hence the correct representation (157).[111]

This rule, in turn, generates the following realistic dynamics. We provide the expression in the limit of small time intervals, as the expressions are simpler, and in the case $d_t = 0$ to simplify the analysis.

[111] Rule (158) can have microfoundations of the "behavioral inattention" type:

$$f_t = m f_t^r + (1-m)\, f_{t-1} + \tilde{f}_t, \tag{159}$$

where $f_t^r$ is the rational flow for an investor embedded in this economy, and $f_{t-1}$ is the "default behavioral flow", corresponding to no action, $\tilde{f}_t$ is a stationary "behavioral disturbance", and $m \in [0,1)$ (along with the size of $\tilde{f}_t$) smoothly parametrizes the degree of rationality of the model. This captures that agents are "partially rational," but are also affected by some disturbance $\tilde{f}_t$. Because the rational flow (maximizing $\mathbb{E}_t\left[V^p\left(R_{t+1}\right)\right]$) is $f_t^r = f_t + \frac{\hat{\pi}_t}{\bar{\pi}}$, behavior (159) generates (158) with $\chi = \frac{m}{1-m}\frac{1}{\bar{\pi}} > 0$ and $\varepsilon_t = \frac{\tilde{f}_t}{1-m}$. Formulation (159) extends more easily than (158) to other contexts (Gabaix (2014)).

**Proposition 17.** (Equilibrium when flows respond to the equity premium in a noisy fashion) *In the limit of small time intervals, the specification of flows* (158) *with i.i.d. shocks* $\varepsilon_t$ *generates a deviation of the price from trend equal to:*

$$\mathbb{E}_0 p_t = \frac{1}{\zeta + \kappa\phi}\mathbb{E}_0 f_t, \qquad \mathbb{E}_0 f_t = (1-\phi)^t f_0. \tag{160}$$

*The speed of mean-reversion* $\phi$ *is the positive solution of* $\kappa\phi^2 + (\zeta - \chi)\phi = \chi\delta$. *The speed of mean-reversion* $\phi$ *is increasing in the intensity of the response to the equity premium,* $\chi$, *and decreasing in* $\zeta$ *and* $\kappa$. *It is zero if* $\chi = 0$.

For instance, consider a flow shock $f_0$ at time 0. Then, the dynamics are those in (24). This captures that flows are endogenously "digested" by the market at a rate $\phi$, which is higher when $\chi$ is higher, i.e. when investors chase risk premia more aggressively (see Bouchaud et al. (2009) for a survey, more geared towards shorter time scales).

*Illustrative calibration.* If we assume $\chi = 0.1$, we replicate a slow mean-reversion of the P/D ratio of $\phi \simeq 4\%$ per year.[112]

One could imagine agents with other behavioral rules, or agents optimizing on the parameters $\chi$, $m$, this way providing additional cross-asset predictions.[113] We leave that to future research.

### G.12.2 When flows react to the risk premia: advanced material

Here we derive Proposition 17, and more generally explore the consequences of flows of the type:

$$\Delta f_t = \chi\hat{\pi}_t + \varepsilon_t. \tag{162}$$

We first proceed in continuous time, which is cleanest.

**Continuous time** For simplicity, we assume away dividend surprises. They would be easy to add back. The flows (162) are

$$df_t = \chi\hat{\pi}_t dt + \sigma dz_t. \tag{163}$$

We use the operator $D$,

$$Dx_t := \frac{\mathbb{E}_t[dx_t]}{dt}. \tag{164}$$

So, $\hat{\pi}_t = -\delta p_t + \frac{\mathbb{E}_t[dp_t]}{dt}$ (see Section G.6) becomes:

$$\hat{\pi}_t = (D - \delta)p_t \tag{165}$$

and (163) gives

$$Df_t = \chi\hat{\pi}_t. \tag{166}$$

[112] The parameter $\chi$ is unitless: in continuous time, $df_t = \chi\hat{\pi}_t dt + \sigma dz_t$.

[113] Alternatively, consider a rule like:

$$\Delta f_t = \chi\hat{\pi}_t + \beta(d_t - p_t) + \Delta\tilde{f}_t, \tag{161}$$

where $\chi$ and $\beta$ are weakly positive, one of them is strictly positive, and $\tilde{f}_t$ is an AR(1). The coefficients $\chi$ and $\beta$ are "stabilizing" forces: they make investors buy when expected returns are high. Then, the rule (161) also leads to the correct form shown in Lemma 2. However, a rule like $f_t = \chi\hat{\pi}_t + \beta(d_t - p_t) + \tilde{f}_t$ would not lead to a stationary $P/D$ ratio: while the right-hand side would be stationary, by Lemma 2 the left-hand side should not be stationary.

The basic dynamic pricing equation, (62), becomes:

$$0 = -\zeta p_t + \kappa D p_t + f_t. \tag{167}$$

Differentiating once and taking time-$t$ expectations gives:

$$\begin{aligned} 0 &= -\zeta D p_t + \kappa D^2 p_t + D f_t & (168)\\ &= \left[-\zeta D + \kappa D^2 + \chi \left(D - \delta\right)\right] p_t & (169)\\ &= H\left(D\right) p_t \end{aligned}$$

where

$$H\left(x\right) = \kappa x^2 - \left(\zeta - \chi\right) x - \chi\delta. \tag{170}$$

The fundamental solutions of equation $H\left(D\right) p_t = 0$ are of the form $p_t = Be^{xt}$, with $H\left(x\right) = 0$.

There are two roots to $H\left(x\right) = 0$, of opposite sign: we call them $\rho$ and $-\phi$, with $\rho$ and $\phi$ weakly positive:

$$\rho = \frac{\zeta - \chi + \sqrt{\Delta}}{2\kappa}, \qquad \phi = \frac{-\zeta + \chi + \sqrt{\Delta}}{2\kappa}, \qquad \Delta = \left(\zeta - \chi\right)^2 + 4\chi\kappa\delta. \tag{171}$$

When $\chi = 0$, $\rho = \frac{\zeta}{\kappa}$ (as in Proposition 5) and $\phi = 0$. We record that $\phi$ solves:

$$\left(\zeta + \kappa\phi\right)\phi = \chi\left(\phi + \delta\right) \tag{172}$$

and as the product of the two roots, $-\phi\rho$ is equal to $\frac{-\chi\delta}{\kappa}$ in (170),

$$\phi = \frac{\chi\delta}{\kappa\rho}. \tag{173}$$

This allows us to derive a variety of impulse responses. Calling $y_t$ the process $dy_t = -\phi y_t dt + \sigma dz_t$, let us look for a solution of the form (or "Ansatz"):

$$p_t = Ay_t, \qquad f_t = ay_t.$$

Plugging this Ansatz in (163) and examining the $\sigma z_t$ term gives:

$$a = 1.$$

Next, we have $Dy_t = -\phi y_t$, so plugging this in (167) and matching coefficients gives: $0 = \left[-\left(\zeta + \kappa\phi\right) A + a\right] y_t$, i.e.

$$A = \frac{1}{\zeta + \kappa\phi}.$$

This derived Proposition 17 in continuous time.□

One can derive other things. For instance, here is an expression for the price, expressed in discrete time for convenience.

**Proposition 18.** (Equilibrium price in infinite horizon model, with enriched model of households)*Suppose that $\Delta f_t = \chi\hat{\pi}_t + \Delta\tilde{f}_t$ for some arbitrary $\tilde{f}_t$. Then, the price at time $t$ is:*

$$p_t = \frac{f_{t-1}}{\bar{\zeta}} + \mathbb{E}_t \sum_{\tau=t}^{\infty} \frac{\rho}{\left(1+\rho\right)^{\tau-t+1}} \left(\frac{\tilde{f}_\tau - \tilde{f}_{t-1}}{\bar{\zeta}} + M^D d^e_\tau\right) \tag{174}$$

with $\bar{\zeta} = \zeta + \kappa\phi$ and $M^D = \frac{\kappa\delta}{\bar{\zeta}}$.

This generalizes Proposition 5. The economics is largely the same, except that $\zeta$ is replaced by $\bar{\zeta}$, the expression of $\rho$ changes to (171), and the price impact of an inflow $\Delta\tilde{f}_t$ decays at a rate $\phi$.

**Discrete time** We use the operator

$$\nabla x_t = \mathbb{E}_t \left[ x_{t+1} - x_t \right]$$

so that $\nabla x_t \simeq D x_t \times \Delta t$, where $D$ is the continuous time operator (164). So $\hat{\pi}_t = -\delta p_t + \mathbb{E}_t \left[ \Delta p_{t+1} \right]$ becomes:

$$\hat{\pi}_t = (\nabla - \delta)\, p_t.$$

Likewise, $\Delta f_t = \chi \hat{\pi}_t + \varepsilon_t$ (see (162)) implies:

$$\nabla f_t = \chi \hat{\pi}_{t+1} = \chi (\nabla - \delta)\, p_{t+1} = \chi (\nabla - \delta)(1 + \nabla \omega)\, p_t$$

with $\omega = \Delta t = 1$ in discrete time, and a formal sense that we clarify below, $\omega = 0$ in the continuous time limit.

Then, the basic equation (62) becomes:

$$0 = -(\zeta - \kappa \nabla)\, p_t + f_t.$$

Pre-multiplying by $\nabla$ gives:

$$\begin{aligned} 0 &= -\nabla (\zeta - \kappa \nabla)\, p_t + \nabla f_t \\ &= -\nabla (\zeta - \kappa \nabla)\, p_t + \chi (\nabla - \delta)(1 + \nabla \omega)\, p_t \\ &= \tilde{H}(\nabla)\, p_t \end{aligned}$$

with

$$\tilde{H}(x) = (\kappa + \chi \omega)\, x^2 - (\zeta - \chi (1 - \delta \omega))\, x - \chi \delta. \tag{175}$$

Polynomial $\tilde{H}(x)$ is the discrete-time analogue to the continuous time polynomial $H(x)$ seen above.

Then, we call $\rho$ and $-\phi$ the roots of polynomial $\tilde{H}$.

Now, defining $y_t = (1 - \phi)\, y_{t-1} + \varepsilon_t$, we seek solutions of the type:

$$p_t = A y_t, \qquad f_t = a y_t. \tag{176}$$

This implies

$$\hat{\pi}_t = (\nabla - \delta)\, p_t = -(\phi + \delta)\, A y_t.$$

Plugging this in (162) gives:

$$a = -\chi \omega (\phi + \delta)\, A + 1.$$

Plugging the Ansatz (176) in (62) gives:

$$0 = -(\zeta + \kappa \phi)\, A + a.$$

Hence, we obtain: $A = \frac{a}{\zeta + \kappa \phi}$, with

$$a = \frac{1}{1 + \chi \omega \frac{\phi + \delta}{\zeta + \kappa \phi}}. \tag{177}$$

Again, formally, we obtain the continuous time limit when $\omega \to 0$.

**From discrete to continuous time** We draw from Section H.1, and denote with bolded symbols the continuous-time version of the parameters, and $\Delta t$ the calendar value of a time interval. As they are unitless, $\chi$ and $\zeta$ are the same in discrete and continuous time.

We have $\omega = \Delta t$, and we can write $\mathbb{E}_t [p_{t+1}] = (1 + \omega \nabla) p_t$. Hence, calling $x = \boldsymbol{x} \Delta t$,

$$\begin{aligned} \tilde{H}(x) &= (\kappa + \chi\omega) x^2 - (\zeta - \chi(1 - \delta\omega)) x - \chi\delta\omega \\ \tilde{H}(x)/\Delta t &= (\boldsymbol{\kappa} + \chi\Delta t) \boldsymbol{x}^2 - (\zeta - \chi(1 - \delta\Delta t)) \boldsymbol{x} - \chi\boldsymbol{\delta} \end{aligned}$$

so that indeed, $\lim_{\Delta t \to 0} \frac{\tilde{H}(x)}{\Delta t} = H(\boldsymbol{x})$.

**Impact of a trend on dividends** We prove the following.

**Proposition 19.** *Suppose that $d_t = gt$. Then, if flows follow*

$$\Delta f_t = \chi \hat{\pi}_t + (1 - \theta) g + c$$

*with $f_{-1} = 0$, with $\chi > 0$ and some constant $c$. Then in the long run, the equity premium is higher, $\hat{\pi}_* = -\frac{c}{\chi}$. We have $p_t = d_t + p_*$, $f_t = (1 - \theta) d_t + f_*$, with $p_* = \frac{c}{\chi\delta}$, i.e.*

$$p_* = \frac{c}{\chi\delta} = \frac{c}{\kappa\phi\rho} \tag{178}$$

*and $f = \zeta p_*$. For finite $t$, we have*

$$p_t = d_t + \left(1 - \frac{\zeta}{\zeta + \kappa\phi}(1 - \phi)^t\right) p_* \tag{179}$$

*so that on impact*

$$p_0 = \frac{c}{(\zeta + \kappa\phi)\rho} \tag{180}$$

*where $\rho$,$-\phi$ are the of the characteristic polynomial $H(x)$ in (170). The flows are*

$$f_t = (1 - \theta) d_t + f_* \left(1 - (1 - \phi)^t\right). \tag{181}$$

We write the rule as a deviation $c$ from the rational flow, which is $\Delta f_t = (1 - \theta) g$ by Lemma 2. In the baseline case $\Delta f_t = \chi \hat{\pi}_t + \varepsilon_t$, then $c = -(1 - \theta) g < 0$. Intuitively, if there is a low $\chi$, the "flows don't adjust enough", so that the price is too low, and the equity premium is higher. This is why the intercept $p_*$ is negative.

Also, the long run impact is larger than the short run impact, because the mistakes $c$ "pile up" over time. The speed of convergence is $\phi$, which is about 9%. So, for most purposes, the impact $p_0$ is more important than the long run impact.[114]

*Proof.* First, we derive the long run, which is simpler. Calling $\hat{\pi}_*$ the steady state deviation of the equity premium from $\bar{\pi}$, we have on average $\Delta f_t = \chi \hat{\pi}_* + (1 - \theta) g + c$. But Lemma 2 showed that we need $\Delta f_t = (1 - \theta) g$. So, this implies $\hat{\pi}_* = \frac{-c}{\chi}$. This in turn corresponds to $p_t = p_* + gt$, with $p_* = -\frac{\hat{\pi}_*}{\delta}$.

[114] Note that it could be obtained in the case $\chi \to 0$ from Proposition 5, which gives $p_0 = \frac{c}{\zeta\rho}$ (by plugging in $f_\tau = c\tau$).

Next, we derive the finite-time behavior. For simplicity, we use continuous time, and set $g = 0$ for simplicity (the general case is similar). We have $Df_t = \chi \hat{\pi}_t + c$. Insert this in (169) gives

$$H(D) p_t + c = 0. \tag{182}$$

The solution is $p_t = p_* + Ae^{-\phi t} + Be^{\rho t}$ for constants $A$ and $B$. The large $t$ behavior implies $B = 0$. As time $t = 0$, we must have $f_0 = 0$, so

$$0 = -\zeta p_0 + \kappa \frac{\mathbb{E} dp_t}{dt}_{|t=0} = -\zeta p_* - (\zeta + \kappa \phi) A.$$

This gives $A = -\frac{\zeta}{\zeta + \kappa \phi} p_*$. This implies

$$p_0 = p_* + A = \left(1 - \frac{\zeta}{\zeta + \kappa \phi}\right) p_* = \frac{\kappa \phi}{\zeta + \kappa \phi} \frac{c}{\chi \delta}.$$

We use that $\phi = \frac{\chi \delta}{\kappa \rho}$ from (173), which gives (180).

Finally, as $0 = -\zeta p_t + \kappa D p_t + f_t$, we have

$$f_t = \zeta p_* + (\zeta + \kappa \phi) A e^{-\phi t} = \zeta p_* \left(1 - e^{-\phi t}\right).$$

Using our calibration $g = 2\%$, $\theta = 87.5\%$ and rule (158) with $\chi = 0.1$ we find: $\hat{\pi}_* = \frac{1-\theta}{\chi} g = 2.5\%$.

We next prove a result that synthesizes and expands on our previous results. □

**Proposition 20.** *Suppose an economy with i.i.d. dividend growth $\Delta d_t = g + \varepsilon_t^d$, and consumer flows following the semi-behavioral rule:*

$$\Delta f_t = \chi \hat{\pi}_t + (1 - \theta + \gamma) \Delta d_t + c + \varepsilon_t^f \tag{183}$$

*where disturbances $\varepsilon_t^d$, $\varepsilon_t^f$ have mean 0 and no time correlations. The rational case obtains when $\gamma$, $\chi$, $c$, $var\left(\varepsilon_t^f\right)$ are set to 0. Then, in the steady state, the equity premium is $\bar{\pi} + \hat{\pi}$ with $\hat{\pi} = -\frac{\gamma g + c}{\chi}$, and using $p_* = -\frac{\hat{\pi}}{\delta}$, $f_* = \zeta p_*$, and $\phi$ the mean-reversion of Proposition 17,*

$$f_t = (1 - \theta) d_t + \hat{f}_t + f_*, \qquad p_t = d_t + \frac{\hat{f}_t}{\zeta + \kappa \phi} + p_* \qquad \hat{f}_t = (1 - \phi) \hat{f}_{t-1} + \varepsilon_t^f + \gamma \varepsilon_t^d. \tag{184}$$

Here in (183), $\gamma$ is a "gap", as the rational case would entail $\gamma = 0$. So, the "gap" creates a permanent change in the equity premium (as in Proposition 19). The new part is really the impact of disturbances $\varepsilon_t^d$, $\varepsilon_t^f$: their impact means-reverts at the rate $\phi$. Excess flows $\varepsilon_t^f$ make the price temporarily too high, and high dividends not immediately compensated by a flow ($\gamma \varepsilon_t^d$) make the price temporarily too low, as in the myopia effect of Proposition 5.

*Proof.* The terms corresponding to the non-zero trend $g$ and $c$ are exactly as in Proposition 19, using $c' = \gamma g + c$. So, by linearity, we can set $g = c = 0$ and focus on the stochastic terms. In the case where $\varepsilon_t^d = 0$, which is exactly Proposition 17. Then, the case $\varepsilon_t^d$ is very similar, as the "mistake" in flows is the sum of the shock $\varepsilon_t^f$ and the "excess adjustment" $\gamma \varepsilon_t^d$. □

# H Calibration of the general equilibrium model of Section 5: Details

Here we provide a detailed justification of the calibration in Section 5.4.

## H.1 From discrete to continuous time

Denote with bolded symbols the continuous-time version of the parameters, and $\Delta t$ the calendar value of a time interval. Then, as $\boldsymbol{\phi}, \boldsymbol{\delta}, \boldsymbol{\rho}, \hat{\boldsymbol{\pi}}$ have units of $[\text{Time}]^{-1}$, their discrete time counterparts are:

$$\phi = \boldsymbol{\phi}\Delta t, \qquad \delta = \boldsymbol{\delta}\Delta t, \qquad \rho = \boldsymbol{\rho}\Delta t, \qquad \hat{\pi} = \hat{\boldsymbol{\pi}}\Delta t, \tag{185}$$

but as $\boldsymbol{\kappa}$ has unit of [Time] (indeed, $\boldsymbol{\kappa}\hat{\boldsymbol{\pi}}$ must be unitless in an expression like $\frac{PQ}{W} = \theta e^{\boldsymbol{\kappa}\hat{\boldsymbol{\pi}}}$, see (1)) its discrete time counterpart is:

$$\kappa = \boldsymbol{\kappa} \left(\Delta t\right)^{-1}. \tag{186}$$

As it is unitless, $\zeta$ is the same in discrete and continuous time,

$$\zeta = \boldsymbol{\zeta}.$$

Finally $\boldsymbol{\sigma}_f$ has the units of $[\text{Time}]^{-1/2}$ so

$$\sigma_f = \boldsymbol{\sigma}_f \left(\Delta t\right)^{1/2}$$

and similarly for $\boldsymbol{\sigma}_d$.

## H.2 Calibration steps

The calibration steps are the following.[115] We express things in annualized values, but we use the correspondence in Section H.1 to go between discrete vs continuous time notions.

1. For Tables 5 and 6, first we set $\gamma = 2, g = 2\%, \sigma_y = 0.8\%, \sigma_D = 5\%, r_f = 1\%, \sigma_f = 2.8\%$. Next we impute $\beta$ from $r_f$ given $\left(\gamma, g, \sigma_y^2\right)$. Then we calculate $(\sigma_r^2, \bar{\pi}, \delta)$ and set $\phi_f = 4\%, \zeta^M = 0.2, \kappa = 1, \theta = 0.875$, which jointly imply $\zeta = \zeta^M - \kappa\phi_f = 0.16$. Last we calculate $\left(\rho, b_f^p, b_f^\pi, \sigma_b\right)$.

2. In Table 6, we use $\hat{r}_t := r_t - \mathbb{E}_{t-1}\left[r_t\right] = \varepsilon_t^D + b_f^p \varepsilon_t^f$: so, the share of the variance of excess stock returns that is due to flows is $\frac{cov(\hat{r}, b_f^p \varepsilon_t^f)}{var(\hat{r})}$ . Likewise, the share of the variance of stock returns due to fundamentals is $\frac{cov(\hat{r}, \varepsilon_t^D)}{var(\hat{r})}$. The two shares add up to 1.

3. For Table 7a, the model implied mean of $P/D$ is

$$\mathbb{E}\left[\frac{P}{D}\right] = \mathbb{E}\left[\frac{e^{p_t}}{\delta}\right] = \frac{1}{\delta}\mathbb{E}\left[\exp\left(b_f^p \tilde{f}_t\right)\right] = \frac{1}{\delta}\exp\left(\frac{1}{2}\left(b_f^p\right)^2 \frac{\sigma_f^2}{1 - \left(1 - \phi_f\right)^2}\right), \tag{187}$$

the log mean of $P/D$ is

$$\exp\left(\mathbb{E}\log P/D\right) = \frac{1}{\delta}, \tag{188}$$

[115] We thank Lingxuan Wu for performing those computations.

and the variance of log $D/P$ is

$$\operatorname{Var}(\log D/P) = \operatorname{Var}(p_t) = \frac{\left(b_f^p\right)^2 \sigma_f^2}{1 - \left(1 - \phi_f\right)^2}. \tag{189}$$

Up to now, everything is in units of continuous time.

4. For Table 7b, we simulate the model with $\Delta t = \frac{1}{12}$ over 72 years with $N = 1000$ simulations. We report the mean and 95% confidence interval for the slopes, the mean 8-lag Newey-West standard errors and the mean $R^2$ across the $N = 1000$ simulations as the model-generated results.

   (a) We generate $T = \frac{72}{\Delta t}$ periods (72 years) of i.i.d. innovations $\varepsilon_t^D \sim \mathcal{N}\left(0, \sigma_D^2 \cdot \Delta t\right), \varepsilon_t^f \sim \mathcal{N}\left(0, \sigma_f^2 \cdot \Delta t\right)$ with $\Delta t = \frac{1}{12}$ and calculate the path of $d_t$, $\tilde{f}_t$ from

$$d_t - d_{t-1} = g \cdot \Delta t + \varepsilon_t^D - \frac{1}{2}\sigma_D^2 \cdot \Delta t, \tag{190}$$

$$\tilde{f}_t - \tilde{f}_{t-1} = -\phi_f \Delta t \cdot \tilde{f}_{t-1} + \varepsilon_t^f, \tag{191}$$

   for a total of $N = 1000$ times.

   (b) We calculate the time series for the log dividend, the deviation of prices from their rational average, the log price, the log dividend price ratio, and the equity premium as follows:

$$\log D_t = d_t, \tag{192}$$

$$p_t = b_f^p \tilde{f}_t, \tag{193}$$

$$\log P_t = \log D_t + p_t - \log \delta, \tag{194}$$

$$\log D_t/P_t = \log D_t - \log P_t, \tag{195}$$

$$\pi_t = \bar{\pi} + b_f^\pi \tilde{f}_t. \tag{196}$$

   (c) For the predictive return regressions, we collapse the data to a yearly frequency by taking the price and the dividend paid in the last month of the year. Then we calculate the 1-year, 4-year and 8-year cumulative returns, and run 1-year, 4-year and 8-year predictive regressions on the collapsed yearly data. (For the 4-year and 8-year horizons, the regressions have overlapping windows when calculating returns.) We report the 8-lag Newey-West standard errors for all three regressions.

      i. In both the data and the simulations, returns are calculated assuming that the investor pockets the monthly dividends without reinvesting them at the risk-free rate. For example, the return at the 1-year horizon is the month-12 post-dividend $P_{12}$ plus an average of $D_1$ to $D_{12}$ (approximating $\int_0^1 D_\tau d\tau$ in continuous time) divided by the month-0 post-dividend price $P_0$.

      ii. We generate the *cumulative* return for the 4-year and 8-year horizons by compounding 1-year return $\left(r_{t,t+4} = \left(1 + r_{t,t+1}\right)\left(1 + r_{t+1,t+2}\right)\left(1 + r_{t+2,t+3}\right)\left(1 + r_{t+3,t+4}\right) - 1\right)$ for the predictive regressions.

Table I.15: Parameters and elasticities in numerical models

| Variable | Lucas I | Lucas II | LRR I | IRR II |
|---|---|---|---|---|
| Risk aversion ($\gamma$) | 4 | 4 | 10 | 10 |
| IES ($\frac{1}{\varphi}$) | 1/4 | 1/4 | 1.5 | 1.5 |
| Time preference ($\beta$) | 0.9 | 0.9 | 0.94 | 0.94 |
| Mean output growth ($g$) | 0.04 | 0.04 | 0.018 | 0.018 |
| Std. dev. of output growth ($\sigma_y$) | 0.1 | 0.1 | 0.046 | 0.046 |
| Mean financialization ratio ($\psi = \frac{D}{Y}$) | 0.2 | 0.2 | 0.082 | 0.082 |
| Persistence of financialization ratio ($\rho_x$) | 1.0 | 0.95 | - | - |
| Std. dev. of financialization shocks ($\sigma_x$) | 0.0 | 0.05 | - | - |
| Growth rate persistence ($\rho$) | - | - | 0.044 | 0.044 |
| Growth risk loading ($\varphi_e$) | - | - | 0.044 | 0.044 |
| Stochastic volatility ($\sigma_w$) | - | - | 0.0 | $0.8 \times 10^{-5}$ |
| Persistence of volatility ($\nu_1$) | - | - | 0.854 | 0.854 |
| Macro elasticity ($\zeta$) | 20.0 | 17.5 | 28.85 | 28.96 |

*Notes.* All parameters in the table are annualized. The parameters are calibrated so that in each model the risk premium is around 4% and the equity market's capitalization is 1.25 times GDP. These parameters are defined throughout our discussion. In *Lucas I*, we reproduce the analytical results of Section F.4 using the numerical methods to confirm their consistency. In *Lucas II*, we compute the macro elasticity in an economy where labor income is partially correlated with financial market returns. *LRR I* follows the notation and parameterization of Case I in Bansal and Yaron (2004) with long run risks and constant volatility. We adjust the discount rate and output volatility in order to hit the calibration targets. *LRR II* follows the parameterization in their Case II with stochastic volatility.

# I Computing elasticities numerically in equilibrium models

## I.1 Summary of the main results

This section computes numerically the macro elasticity in classic asset pricing models.[116] This complements the closed forms that we obtained in Section F.4. We consider two important classes of models with different parameterizations: the Lucas (1978) model with labor income, and long run risk models as in Bansal and Yaron (2004).

Table I.15 summarizes the parameterization and the moments generated by the numerical exercises. These parameters are defined throughout our discussion. We find that the macro elasticity is around 17.5 in a Lucas model with partially correlated labor income, and close to 30 in long run risk models. Section I.2 describes the general procedure that we follow to compute the macro elasticities numerically, and Sections I.3 to I.4 outline the models and the numerical algorithms in detail.

[116] We thank Zhiyu Fu for performing those computations.

## I.2 General procedure

For both models considered, we suppose $\Delta \ln Y_t = g - \frac{\sigma_Y^2}{2} + \varepsilon_{Yt}$ and define $\Omega_t = Y_t - D_t$ to be residual "labor income" (broadly understood). Suppose that stocks are tradable (with dividends $D_t$), but not labor income. The consumer receives a stream of labor income $\Omega_t$ every period. At time 0, he is endowed with all the shares in the "stock market."

Call $\psi_t = \frac{D_t}{Y_t} = 1 - \frac{\Omega_t}{Y_t}$ the relative size of dividends versus aggregate consumption, to which we refer as the degree of "financialization" of the economy. In Bansal and Yaron (2004), we assume that $\psi_t$ is a constant at all dates. In the Lucas model, we allow $\psi_t$ to be time-varying so that labor income is only partially correlated with the stock market.

### I.2.1 General procedure

Define $Z_t$ to be a vector of aggregate state variables and $\bar{Z}$ the unconditional mean of $Z_t$. In a Lucas model with partially-correlated labor income, $Z_t = \psi_t$, and in a long run risk model $Z_t = (x_t, \sigma_t^2)'$. We compute the equilibrium price-dividend ratio and risk-free rate, $\frac{P_t^*}{D_t} = pd^*(Z_t)$ and $r_t^{f*} = r^{f*}(Z_t)$. We define the perturbed price-dividend ratio as:

$$pd(Z_t, p) = pd^*(Z_t)(1+p)$$

for a small constant $p$ close to 0.

Given $\left(pd(Z_t, p), r^{f*}(Z_t)\right)$, we compute the value function, $V(Z_t, w_t; p)$, the optimal portfolio rule $\theta(Z_t, w_t; p)$, and the optimal consumption plan $c(Z_t, w_t; p)$ for households with different $w_t$, where $w_t = W_t^{\mathcal{E}}/Y_t$ is their financial wealth relative to GDP, which equals aggregate consumption. The representative household holds $\bar{w}(Z_t) = \psi(Z_t)(pd^*(Z_t)+1)$ in financial wealth relative to GDP. We calibrate the parameters so that on average the consumption/market capitalization is equal to $\frac{Y}{W_t^{\mathcal{E}}} = \frac{1}{\bar{w}(\bar{Z})} = 0.8$. Define $\theta^*(Z_t, p) \equiv \theta(Z_t, \bar{w}(Z_t), p)$ and $c^*(Z_t, p) = c(Z_t, \bar{w}(Z_t), p)$ as the policy functions for the representative household given the perturbation $p$. We have $\theta^*(Z, 0) = 1$ and $c^*(Z, 0) = 1$ for any $Z$.

The macro elasticity is then give as:

$$\zeta^r = -\frac{dQ^{\mathcal{E}}/Q^{\mathcal{E}}}{dP/P} = -\left.\frac{d\theta^*(\bar{Z}, p)}{dp}\right|_{p=0}$$

The numerical solution procedure is as follows:

1. We solve for the equilibrium objects $pd^*(Z_t)$ and $r^{f*}(Z_t)$ on the grid of aggregate state variables $Z_t$ using standard methods, as discussed below.

2. Taking the unperturbed $pd^*(Z_t)$, $r^{f*}(Z_t)$ as given, we solve for the optimal policy functions via the following dynamic programming problem:

$$V_t(Z_t, w_t) = \max_{(\theta, c)} u(c, Z_t) + \beta \mathbb{E}_t \left[ \left( \frac{Y_{t+1}}{Y_t} \right)^{1-\gamma} V(Z_{t+1}, w_{t+1}) \right],$$

where[117]:

$$w_{t+1} = \frac{Y_t}{Y_{t+1}} \left(w_t + 1 - \psi_t - c_t\right) R_{t+1},$$

with $R_{t+1}$ the gross return on financial assets:

$$R_{t+1} = \theta_t \frac{D_{t+1}}{D_t} \frac{pd\left(Z_{t+1}\right)+1}{pd\left(Z_t\right)} + \left(1-\theta_t\right)\left(1 + r_t^{f*}\left(Z_t\right)\right).$$

For the long run risk model, the wealth evolution is the same, but the Bellman equation uses Epstein-Zin preferences, so it is nonlinear. We exploit the homotheticity of the long run risk model to speed up the computation, as explained below.

3. We check the consistency of the policy functions with the aggregate equilibrium, $\theta^*(Z,0) = 1$ and $c^*(Z,0) = 1$.

4. We solve another dynamic programming with the perturbed price-dividend ratio, taking $p = 0.01$. We then compute:

$$\zeta^r = -\frac{\theta^*(\bar{Z}, 0.01) - \theta^*(\bar{Z}, 0)}{0.01}.$$

## I.3 Lucas models with partially correlated labor income

We allow for shocks to the financialization ratio $\psi$ in the Lucas model. We consider the parameterization $\psi_t = \frac{x_t}{\mu_x + x_t}$, where $\ln x_t = \rho_x \ln x_{t-1} + \sigma_x \epsilon_{x,t}$, $\mu_x > 0$. In the simple Lucas model studied in the main body of the paper, we have $\psi_0 = \psi < 1$, but $\sigma_x = 0$.

### I.3.1 Stationary equilibrium

We first solve for the steady state price-dividend ratio. The asset pricing equation is, with $M_{t+1}$ being the SDF:

$$\begin{aligned}
\mathbb{E}_t\left[\frac{M_{t+1}}{M_t} R_{t+1}\right] &= 1, \\
\mathbb{E}_t\left[\beta \left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma} \frac{D_{t+1}}{D_t} \frac{pd(\psi_{t+1})+1}{pd(\psi_t)}\right] &= 1, \\
\mathbb{E}_t\left[\beta \left(\frac{Y_{t+1}}{Y_t}\right)^{1-\gamma} \frac{\psi_{t+1}}{\psi_t} \frac{pd(\psi_{t+1})+1}{pd(\psi_t)}\right] &= 1. \qquad (197)
\end{aligned}$$

For ease of computation, we decompose the shock to $x$ into $\varepsilon_{Y,t}$ and another component independent from it:

$$\ln x_t = \rho_X \ln x_{t-1} + \sigma_x \epsilon_{xt} + \sigma_{x,Y} \varepsilon_{Y,t} \quad \epsilon_{x,t} \perp\!\!\!\perp \varepsilon_{Y,t}. \qquad (198)$$

The stationary equilibrium is defined as the function $pd(\psi)$ that solves equation (197) subject to the law of motion of the underlying state variable $x$ in (198).

[117] Indeed, the law of motion for financial wealth is $W_{t+1}^{\mathcal{E}} = \left(W_t^{\mathcal{E}} + \Omega_t - C_t\right) R_{t+1}$, which gives

$$w_{t+1} = \frac{W_{t+1}^{\mathcal{E}}}{Y_{t+1}} = \frac{Y_t}{Y_{t+1}} \left(\frac{W_t^{\mathcal{E}} + \Omega_t - C_t}{Y_t}\right) R_{t+1} = \frac{Y_t}{Y_{t+1}} \left(w_t + 1 - \psi_t - c_t\right) R_{t+1}.$$

**Discretization** The function $pd(\psi)$ is computed on a discrete grid over $\psi$. The upper and lower boundaries of the grid are chosen to cover at least 99.9% of the stationary distribution of $\psi$. Off-grid values are computed using linear interpolation or extrapolation. Expectations are computed using Gauss–Hermite quadrature with 10 nodes. Increasing the number of nodes does not change the results in a noticeable way. After discretization, the solution to (197) is essentially a vector corresponding to the P/D ratio on the grid for $\psi$. This equation is then solved non-linearly using Newton's method.

The risk-free rate is the same as in the simple Lucas model, as the state variable $\psi$ does not enter the SDF. The expected risky return can be computed as:

$$\mathbb{E}R_{t+1} = \mathbb{E}_t \left[ \frac{Y_{t+1}}{Y_t} \frac{\psi_{t+1}}{\psi_t} \frac{pd(\psi_{t+1}) + 1}{pd(\psi_t)} \right].$$

### I.3.2 Dynamic programming: Outline of the algorithm

The dynamic programming problem is solved using policy function iteration and the endogenous grid point method. For notational ease, for each variable $x$ we denote its value in the next period as $x'$.

The dynamic programming problem can be solved in two steps. First, given post-consumption wealth $\tilde{w} = w + 1 - \psi - c$, the households solve a portfolio choice problem that maximizes:

$$\begin{aligned} &\max_{\theta} \mathbb{E} \left[ \left( \frac{Y'}{Y} \right)^{1-\gamma} V \left( \psi', \frac{Y}{Y'} \tilde{w} R^*(\theta) \right) \right] \\ &\text{s.t.} \\ &R^* = \theta R + (1-\theta)(1 + r_f). \end{aligned}$$

The maximization problem yields the first-order condition:

$$\mathbb{E} \left[ (\frac{Y'}{Y})^{-\gamma} V_w \left( \psi', \frac{Y}{Y'} \tilde{w} R^*(\theta^*) \right) (R - r_f) \right] = 0.$$

With knowledge of $V_w$ (either from an initial guess or from the previous iteration), we can solve for $\theta^*$ non-linearly given each $\tilde{w}$. This way, we obtain the functions $\theta^*(\tilde{w})$ and $R^*(\tilde{w})$, which map post-consumption wealth $\tilde{w}$ into the optimal portfolio choice $\tilde{\theta}$ and into the asset returns $R^*$, independently of the consumption policy.

Second, the household choose optimal consumption by maximizing:

$$\max_{c} u(c) + \beta \mathbb{E} \left[ \left( \frac{Y'}{Y} \right)^{1-\gamma} V \left( \psi', \frac{Y}{Y'} \tilde{w} R^*(\tilde{w}) \right) \right] \tag{199}$$

$$\text{s.t.}$$

$$\tilde{w} = w + 1 - \psi - c, \tag{200}$$

which gives the first-order condition:

$$c^{-\gamma} = \beta \mathbb{E} \left[ \left( \frac{Y'}{Y} \right)^{-\gamma} V_w \left( \psi', \frac{Y}{Y'} \tilde{w} R^*(\tilde{w}) \right) R^*(\tilde{w}) \right].$$

Following the endogenous grid point method, we can solve for $c$ explicitly as a function of the post-consumption wealth $\tilde{w}$ on the grid, that is,

$$c^*(\tilde{w}) = \left\{\beta\mathbb{E}\left[\left(\frac{Y'}{Y}\right)^{-\gamma} V_w\left(\psi', \frac{Y}{Y'}\tilde{w}R^*(\tilde{w})\right) R^*(\tilde{w})\right]\right\}^{-\frac{1}{\gamma}}.$$

then we can back out pre-consumption wealth $w$ (off grid) as a function of $\tilde{w}$ from the budget constraint (200). Finally, using interpolation, we can get the desired policy function $c^*(w)$ on the grid.

By the envelope theorem, we have:

$$V_w = u'(c^*(w)) = (c^*)^{-\gamma}.$$

Using the solved policy function $c^*(w)$ we obtain $V_w$ from the envelope theorem and proceed to the next iteration until we achieve the desired accuracy.

**Verification** Now, we verify that the policy functions are consistent with the aggregate equilibrium. For representative households, that is, households with financial wealth $w^{\mathcal{E}} = (pd(\psi) + 1)\psi$, their consumption is exactly equal to GDP, $c = 1$, and their risky share is also equal to one, $\theta = 1$.

**Initial guess** We make the initial guess by assuming the aggregate state $\psi$ is constant. In this case, the solutions for each value of $\psi$ are independent of one another. We guess and verify that the value function is of the form $V(w) = \frac{A^{-\gamma}w^{1-\gamma}}{1-\gamma}$, where $w$ includes capitalized labor income.

## I.4 Long run risk model

### I.4.1 Stationary equilibrium

As shown in Bansal and Yaron (2004), the SDF for a recursive utility function is:

$$M_{t+1} = \beta^{\vartheta}\left(\frac{Y_{t+1}}{Y_t}\right)^{-\frac{\vartheta}{\varphi}} R_w^{\theta-1},$$

where:

$$R_w = \frac{pd(x', \sigma') + 1}{pd(x, \sigma)}\frac{Y_{t+1}}{Y_t},$$
$$\vartheta = \frac{1-\gamma}{1-\frac{1}{\varphi}}.$$

Following Bansal and Yaron (2004), the dynamics of the economy are:

$$\begin{aligned}
x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1}, \\
g_{t+1} &= \mu + x_t + \sigma_t \eta_{t+1}, \\
\sigma_{t+1}^2 &= \sigma^2 + v_1\left(\sigma_t^2 - \sigma^2\right) + \sigma_w w_{t+1}, \\
&e_{t+1}, u_{t+1}, \eta_{t+1}, w_{t+1} \sim N.i.i.d.(0,1).
\end{aligned}$$

The price-dividend ratio can be solved using:

$$\mathbb{E}\left[\beta^{\vartheta}\left(\frac{Y_{t+1}}{Y_t}\right)^{-\frac{\vartheta}{\varphi}} R_{t+1}^{\vartheta}\right] = 1,$$

$$\mathbb{E}\left[\beta^{\vartheta} \exp\left((1-\gamma)g\right)\left(\frac{pd(x_{t+1})+1}{pd(x_t)}\right)^{\vartheta}\right] = 1.$$

Moving the current $pd$ outside of the expectation operator:

$$pd(x,\sigma^2) = \left[\mathbb{E}\beta^{\theta}\exp\left((1-\gamma)g\right)\left(pd(x',\sigma^2)+1\right)^{\vartheta}\right]^{\frac{1}{\vartheta}},$$

which gives a fixed point problem. We solve for $pd(x,\sigma^2)$ by iterating upon this equation.

### I.4.2 A shortcut for the dynamic programming problem

Labor income can also be modeled as the the dividend from human capital, $W^L$. In our setup of long run risk models, labor income is co-integrated with aggregate output, so human capital $W^L$ and $W^{\mathcal{E}}$ have the same return process. Therefore, we could first solve the bellman equation with total wealth $W = W^L + W^{\mathcal{E}}$ as the state variable, and then take care of the difference between $W^L$ and $W^{\mathcal{E}}$ when calculating the elasticity.

The Bellman equation with Epstein-Zin preferences is:

$$V(x,w) = \max_{c,\vartheta}\left((1-\beta)c^{1-\frac{1}{\varphi}} + \beta\mathbb{E}\left[V^{1-\gamma}(x',w')\right]^{\frac{1-\frac{1}{\varphi}}{1-\gamma}}\right)^{\frac{1}{1-\frac{1}{\varphi}}}$$

$$\begin{aligned}
&\text{s.t.}\\
&w' = (w-c)R^*(x,x'),\\
&R^* = \vartheta R(x,x') + (1-\vartheta)R_f(x),\\
&x' = lom(x),
\end{aligned}$$

where here we use $x$ as a shorthand for both state variables, and $w$ total wealth normalized by GDP, with a slight abuse of notation.

Exploiting the homotheticity in the Bellman equation, we can eliminate $w$ and therefore reduce the dimensionality of the state space by one. We define $v(x)$ as $V(x,w) = v(x)w$. The Bellman equation is then given by:

$$\begin{aligned}
v(x)^{1-\frac{1}{\varphi}}\frac{w^{1-\frac{1}{\varphi}}}{1-\frac{1}{\varphi}} &= \max_{c,\theta}(1-\beta)\frac{c^{1-\frac{1}{\varphi}}}{1-\frac{1}{\varphi}} + \beta\frac{\mathbb{E}\left[v(x')^{1-\gamma}w'^{1-\gamma}\right]^{\frac{1-\frac{1}{\varphi}}{1-\gamma}}}{1-\frac{1}{\varphi}}\\
&= \max_{c,\theta}(1-\beta)\frac{c^{1-\frac{1}{\varphi}}}{1-\frac{1}{\varphi}} + \beta\frac{(w-c)^{1-\frac{1}{\varphi}}}{1-\frac{1}{\varphi}}\mathbb{E}\left[v(x')^{1-\gamma}R^{*1-\gamma}\right]^{\frac{1-\frac{1}{\varphi}}{1-\gamma}}.
\end{aligned}$$

The maximization problem is solved in two steps. First, we take the first order condition with respect to $\theta$:

$$\mathbb{E}\left[v(x')^{1-\gamma}\left(\theta^* R + (1-\theta^*)R_f\right)^{-\gamma}(R-R_f)\right] = 0,$$

which gives $\theta^*$, independent of $w$. Defining $A(x) = \mathbb{E}\left[v(x')^{1-\gamma} R^{*1-\gamma}\right]^{\frac{1-\frac{1}{\varphi}}{1-\gamma}}$, then optimal consumption solves:

$$v(x)^{1-\frac{1}{\varphi}} \frac{w^{1-\frac{1}{\varphi}}}{1-\frac{1}{\varphi}} = \max_c (1-\beta) \frac{c^{1-\frac{1}{\varphi}}}{1-\frac{1}{\varphi}} + \beta A(x) \frac{(w-c)^{1-\frac{1}{\varphi}}}{1-\frac{1}{\varphi}}.$$

The first-order condition is given by:

$$(1-\beta)\left(c^*\right)^{-\frac{1}{\varphi}} = \beta A(x)(w-c^*)^{-\frac{1}{\varphi}},$$

which yields:

$$\frac{c^*(w)}{w} = \frac{1}{(\frac{\beta}{1-\beta})^{\varphi} A^{\varphi} + 1} = \varsigma(x).$$

Again, the consumption-wealth ratio is only a function of $x$ but not $w$. Plugging it into the Bellman equation:

$$\begin{aligned} v(x)^{1-\frac{1}{\varphi}} &= (1-\beta)\varsigma^{1-\frac{1}{\varphi}}(x) + \beta A(x)(1-\varsigma(x))^{1-\frac{1}{\varphi}}, \\ v(x) &\equiv \mathbb{F}(v)(x). \end{aligned}$$

Since it is derived from a Bellman equation, the operator $\mathbb{F}$ is also a contraction mapping. Therefore, we can solve it by iteration.

### I.4.3 Calculating the elasticity with labor income

Now we proceed to calculate the macro elasticity, recognizing that only a fraction $\psi$ of total wealth is capitalized.

Recall the definition of macro elasticity:

$$\zeta^r = -\frac{dQ^{\mathcal{E}}/Q^{\mathcal{E}}}{dP/P},$$

where $\psi = \frac{Q^{\mathcal{E}}}{Q}$. Therefore, the elasticity can be calculated as:

$$\zeta^r = -\frac{d\theta}{dp}\frac{Q}{Q^{\mathcal{E}}} = \frac{d\theta}{dp}\frac{1}{\psi}.$$

The share $\psi$ is calibrated to match $\frac{Y}{W^{\varepsilon}} = \frac{Y}{\psi W} = \frac{1}{\psi(pd+1)} = 0.8$, and $\frac{d\theta}{dp}$ can be computed by perturbing $pd$ as outlined above.

### I.4.4 Calibration

The calibration closely follows Bansal and Yaron (2004), with two exceptions. In the original paper, Bansal and Yaron (2004) obtain a high risk premium from leveraged dividends. In our model, as we do not have an explicit dividend process, we calibrate $\bar{\sigma}$ to match a 4% risk premium on the aggregate market. We also reduce $\beta$ so that we have a reasonable risk-free rate and P/D ratio in the stationary equilibrium.

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