Online Appendix

Trade Policy

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A Appendix: Shift-share Equations

In this appendix, we present the derivation of the the shift-share equations under different economic environments.

Baseline Model

Consider an economy with R regions indexed by n and J sectors indexed by j. Assume that labor is freely mobile across sectors inside a region and costly mobile across regions. Firms produce with a constant return to scale technology that uses local factors of production; labor (L) and a fixed factor (H). Labor and the fixed factor are aggregated with a Cobb-Douglas technology. Denote by Y_{nj} the output in sector j and region n, given by

$$Y_{nj} = A_{nj} L_{nj}^{\beta_{nj}} H_{nj}^{1-\beta_{nj}},$$

where A_{nj} is the TFP in sector j and region n, β_{nj} is the share of labor in output, and $1-\beta_{nj}$ is the share of the fixed factor in output. The demand for labor and the fixed factor in sector j and region n are given by L_{nj} and H_{nj}, respectively. From the firm's cost-minimization problem we obtain that the optimal labor demand in nj is given by

$$L_{nj} = \frac{\beta_{nj} P_{nj} Y_{nj}}{w_n},$$

where P_{nj} is the price of output in sector j and region n.

The regional labor-market clearing condition is given by

$$L_n = \sum_j L_{nj}.$$

Using the optimal labor demand, we obtain that

$$L_n = \sum_j \frac{\beta_{nj} P_{nj} Y_{nj}}{w_n} \text{ for all } n.$$

Totally differentiating the labor market clearing condition

$$d\ln L_n = \frac{1}{L_n} \sum_j \frac{\beta_{nj} P_{nj} Y_{nj}}{w_n} \left[d\ln P_{nj} + d\ln Y_{nj} - d\ln w_n \right],$$

which can also be expressed as

$$d\ln L_n = \sum_j \frac{L_{nj}}{L_n} \left[d\ln P_{nj} + d\ln Y_{nj} - d\ln w_n \right].$$

The total differential of output is given by $d\ln Y_{nj} = d\ln A_{nj} + \beta_{nj} d\ln L_{nj}$. Using the first-order condition of the firm's cost-minimization problem, we obtain

$$d\ln Y_{nj} = \frac{d\ln A_{nj}}{1-\beta_{nj}} + \frac{\beta_{nj}\left[d\ln P_{nj} - d\ln w_n\right]}{1-\beta_{nj}},$$

and plugging this equation in the labor market clearing we obtain

$$d\ln L_n = \sum_j \frac{L_{nj}}{L_n}\left[\frac{d\ln A_{nj} + d\ln P_{nj} - d\ln w_n}{1-\beta_{nj}}\right],$$

and solving for the change in wages, we get

$$d\ln w_n = -\delta_n d\ln L_n + \sum_j \omega_{nj} d\ln P_{nj} + \sum_j \omega_{nj} d\ln A_{nj}, \tag{A.1}$$

where $\delta_n \equiv \left[\sum_j \frac{L_{nj}}{L_n}\frac{1}{1-\beta_{nj}}\right]^{-1}$ and $\omega_{nj} \equiv \delta_n \frac{L_{nj}}{L_n}\frac{1}{1-\beta_{nj}}$.

We now discuss the regional supply labor. We assume frictional labor mobility, where moving to location n entails a multiplicative cost ε_n that is an *i.i.d.* draw from an extreme value Frechet distribution with shape parameter ν (the cost draw can also be interpreted as an amenity shock). Using the properties of the Frechet distribution, we have that the mass of workers that locate in n is given by

$$L_n = \frac{[w_n]^{\nu}}{\sum_i [w_i]^{\nu}} L,$$

where L is the country total endowment of labor. Total differentiating this expression we obtain that

$$d\ln L_n = \nu d\ln w_n - d\ln \Phi,$$

where we denote $\Phi \equiv \sum_i [w_i]^{\nu}$, and where $d\ln\Phi = \nu \sum_i L_i d\ln w_i$. Using this equilibrium relation, we substitute into (A.1) and obtain that

$$d\ln w_n = -\delta_n\left[\nu d\ln w_n - d\ln\Phi\right] + \sum_j \omega_{nj} d\ln P_{nj} + \sum_j \omega_{nj} d\ln A_{nj},$$

and solving for wages, one obtains that

$$d\ln w_n = \frac{\delta_n}{1+\delta_n\nu} d\ln\Phi + \sum_j \frac{\omega_{nj}}{1+\delta_n\nu} d\ln P_{nj} + \sum_j \frac{\omega_{nj}}{1+\delta_n\nu} d\ln A_{nj}.$$

The last important assumption is that each region is a small open economy, which allows to express price changes as functions of tariff changes, namely $d\ln P_{nj} = d\ln\tau_j$. It follows then that

$$d\ln w_n = \frac{\delta_n}{1+\delta_n\nu} d\ln\Phi + \frac{1}{1+\delta_n\nu}\sum_j \omega_{nj} d\ln\tau_j + \sum_j \frac{\omega_{nj}}{1+\delta_n\nu} d\ln A_{nj}. \tag{A.2}$$

We finish this section analyzing the implication of two extreme cases, namely labor immobility across regions and free labor mobility across regions.

Two Special Cases

First, in the case of labor immobility across regions we have that $d\ln L_n = 0$, which is equivalent to say that $\nu d\ln w_n = d\ln \Phi$. Imposing this restriction in (A.2) we get

$$d\ln w_n = \sum_j \omega_{nj} d\ln \tau_j + \sum_j \omega_{nj} d\ln A_{nj}.$$

An implication of the regional labor immobility assumption is that the predicted coefficient in the shift-share regression is one.

Second, we assume free mobility across regions. In this case, the labor market clearing is given by

$$L = \sum_n L_n = \sum_n \sum_j L_{nj}.$$

The total differential of the labor market clearing condition is given by

$$d\ln L = \frac{1}{L} \sum_n \sum_j \frac{\beta_{nj} P_{nj} Y_{nj}}{w_n} \left[d\ln P_{nj} + d\ln Y_{nj} - d\ln w_n \right].$$

Hence, we get

$$d\ln L = \sum_n \sum_j \frac{L_{nj}}{L} \left[d\ln P_{nj} + d\ln Y_{nj} - d\ln w_n \right].$$

Using the total differential of output $d\ln Y_{nj} = \frac{d\ln A_{nj}}{1-\beta_{nj}} + \frac{\beta_{nj}[d\ln P_{nj} - d\ln w_n]}{1-\beta_{nj}}$, we obtain

$$d\ln L = \sum_n \sum_j \frac{L_{nj}}{L} \left[\frac{d\ln A_{nj}}{1-\beta_{nj}} + \frac{d\ln P_{nj} - d\ln w_n}{1-\beta_{nj}} \right].$$

and solving for the change in earnings we obtain

$$d\ln w_n = -\delta_n^{mov} d\ln L + \sum_n \sum_j \omega_{nj}^{mov} d\ln P_{nj} + \sum_n \sum_j \omega_{nj}^{mov} d\ln A_{nj},$$

where $\delta_n^{mov} = \left[\sum_n \sum_j \frac{L_{nj}}{L_n} \frac{1}{1-\beta_{nj}} \right]^{-1}$ and $\omega_{nj}^{mov} = \delta_n^{mov} \frac{L_{nj}}{L_n} \frac{1}{1-\beta_{nj}}$.

Finally, we assume that each region is a small open economy that face world's prices as given, namely $d\ln P_{nj} = d\ln \tau_j$. Therefore,

$$d\ln w_n = -\delta_n^{mov} d\ln L + \sum_n \sum_j \omega_{nj}^{mov} d\ln \tau_j + \sum_n \sum_j \omega_{nj}^{mov} d\ln A_{nj}.$$

As the shift-share equation shows, the predicted coefficient of a shift-share regression would be zero with free regional mobility since there is no variation on the shift-share term across regions.

Model with Non-traded Goods

As in the previous section of this appendix, consider an economy with R regions indexed by n and J sectors indexed by j. Assume that labor is freely mobile across sectors inside a region and not mobile across regions. Firms produce with a constant return to scale technology using labor (L) and a fixed factor (H). We assume that labor and the fixed factor are aggregated with a Cobb-Douglas. We denote by Y_{nj} the output in sector

j and region n, given by

$$Y_{nj} = A_{nj} L_{nj}^{\beta_{nj}} H_{nj}^{1-\beta_{nj}},$$

where A_{nj} is the TFP in sector j and region n, β_{nj} is the share of labor in output, and $1-\beta_{nj}$ is the share of the fixed factor in output. The demand for labor and the fixed factor in sector j and region n are given by L_{nj} and H_{nj}, respectively.

From (A.1), we have that the shift-share equation for earnings is given by

$$d\text{ln}w_n = -\delta_n d\text{ln}L_n + \sum_j \omega_{nj} d\text{ln}P_{nj} + \sum_j \omega_{nj} d\text{ln}A_{nj},$$

where $\delta_n \equiv \left[\sum_j \frac{L_{nj}}{L_n} \frac{1}{1-\beta_{nj}}\right]^{-1}$ and $\omega_{nj} \equiv \delta_n \frac{L_{nj}}{L_n} \frac{1}{1-\beta_{nj}}$.

Denote by NT a non-tradable sector, hence with a non-tradable sector we have that

$$d\text{ln}w_n = -\delta_n d\text{ln}L_n + \sum_{j \neq NT} \omega_{nj} d\text{ln}P_{nj} + \omega_{nNT} d\text{ln}P_{nNT} + \sum_j \omega_{nj} d\text{ln}A_{nj}.$$

As we can see from this equation, we now need to solve for the endogenous change in the price of non-tradable. To do so, we need to specify the demand side of the economy. We assume that consumers have Cobb-Douglas preferences of goods produced in each industry, namely

$$U_n = \prod_j C_{nj}^{\alpha_j},$$

with $\sum_j \alpha_j = 1$. The demand for goods from industry j is the given by

$$C_{nj} = \frac{\alpha_j I_n}{P_{nj}},$$

where I_n is the income of a representative consumer as defined below.

Total differentiating this equation we get

$$d\text{ln}C_{nj} = d\text{ln}I_n - d\text{ln}P_{nj}.$$

We assume that consumers are the owners of factors of production. Hence

$$I_n = \sum_j P_{nj} Y_{nj},$$

and total differentiating this equation we get

$$dlnI_n = \sum_j \varrho_{nj} \left[d\text{ln}Y_{nj} + d\text{ln}P_{nj}\right],$$

where $\varrho_{nj} = \frac{Y_{nj} P_{nj}}{\sum_j P_{nj} Y_{nj}}$ is the share of industry j in total output value.

The total differential of output is given by $d\text{ln}Y_{nj} = d\text{ln}A_{nj} + \beta_{nj} d\text{ln}L_{nj}$, and plugging this equation into the income equation we obtain

$$dlnI_n = \sum_j \left[\varrho_{nj} d\text{ln}A_{nj} + \varrho_{nj}\beta_{nj} d\text{ln}L_{nj} + \varrho_{nj} d\text{ln}P_{nj}\right].$$

Notice that we can re-express the second term on the right hand side as

$$\sum_j \varrho_{nj}\beta_{nj} d\text{ln}L_{nj} = \frac{w_n L_n}{\sum_j P_{nj} Y_{nj}} \sum_j \frac{L_{nj}}{L_n} d\text{ln}L_{nj} = \zeta_n d\text{ln}L_n,$$

where $\zeta_n = \frac{w_n L_n}{\sum_j P_{nj} Y_{nj}}$ is the share of labor payment in total output value.

Hence, we have

$$dlnI_n = \sum_j \left[\varrho_{nj} d\text{ln}A_{nj} + \varrho_{nj} d\text{ln}P_{nj}\right] + \zeta_n d\text{ln}L_n.$$

Plugging the income equation in the demand for non-tradable goods we obtain

$$dlnC_{nj} = \sum_j \varrho_{nj} d\text{ln}A_{nj} + \sum_j \varrho_{nj} d\text{ln}P_{nj} + \zeta_n d\text{ln}L_n - d\text{ln}P_{nj},$$

and solving for the price of non-tradable we get

$$dlnC_{nT} = \sum_j \varrho_{nj} d\text{ln}A_{nj} + \sum_{j \neq NT} \varrho_{nj} d\text{ln}P_{nj} + \zeta_n d\text{ln}L_n + \left[\varrho_{nNT} - 1\right] d\text{ln}P_{nT},$$

or

$$dlnP_{nT} = \sum_j \frac{\varrho_{nj}}{1 - \varrho_{nNT}} d\text{ln}A_{nj} + \sum_{j \neq NT} \frac{\varrho_{nj}}{1 - \varrho_{nNT}} d\text{ln}P_{nj} + \frac{\zeta_n}{1 - \varrho_{nNT}} d\text{ln}L_n - \frac{d\text{ln}C_{nT}}{1 - \varrho_{nNT}}.$$

Notice that $d\text{ln}C_{nNT} = d\text{ln}Y_{nNT}$, hence

$$d\text{ln}P_{nT} = \sum_j \frac{\varrho_{nj}}{1 - \varrho_{nNT}} d\text{ln}A_{nj} + \sum_{j \neq NT} \frac{\varrho_{nj}}{1 - \varrho_{nNT}} d\text{ln}P_{nj} + \frac{\zeta_n}{1 - \varrho_{nNT}} d\text{ln}L_n - \frac{d\text{ln}Y_{nT}}{1 - \varrho_{nNT}}.$$

Now we use that

$$d\text{ln}Y_{nNT} = \frac{d\text{ln}A_{nNT}}{1 - \beta_{nNT}} + \frac{\beta_{nNT}}{1 - \beta_{nNT}} \left[d\text{ln}P_{nNT} - d\text{ln}w_n\right],$$

to obtain and expression for non-tradable price

$$\begin{aligned} d\text{ln}P_{nNT} = {} & \frac{\left[1 - \beta_{nNT}\right]}{1 - \left[1 - \beta_{nNT}\right]\varrho_{nNT}} \sum_j \varrho_{nj} d\text{ln}A_{nj} + \frac{\left[1 - \beta_{nNT}\right]}{1 - \left[1 - \beta_{nNT}\right]\varrho_{nNT}} \sum_{j \neq NT} \varrho_{nj} d\text{ln}P_{nj} \\ & + \frac{\left[1 - \beta_{nNT}\right]\zeta_n}{1 - \left[1 - \beta_{nNT}\right]\varrho_{nNT}} d\text{ln}L_n - \frac{d\text{ln}A_{nNT}}{1 - \left[1 - \beta_{nNT}\right]\varrho_{nNT}} + \frac{\beta_{nNT}}{1 - \left[1 - \beta_{nNT}\right]\varrho_{nNT}} d\text{ln}w_n. \end{aligned}$$

Turning to the earning equation, we have

$$d\text{ln}w_n = -\delta_n d\text{ln}L_n + \sum_{j \neq NT} \omega_{nj} d\text{ln}P_{nj} + \omega_{nNT} d\text{ln}P_{nNT} + \sum_j \omega_{nj} d\text{ln}A_{nj},$$

here $\delta_n \equiv \left[\sum_j \frac{L_{nj}}{L_n} \frac{1}{1-\beta_{nj}}\right]^{-1}$ and $\omega_{nj} \equiv \delta_n \frac{L_{nj}}{L_n} \frac{1}{1-\beta_{nj}}$.

Using the equilibrium earnings, we solve again for the price of non-tradable, and obtain

$$[1-[1-\beta_{nNT}]\,\varrho_{nNT}-\beta_{nNT}\omega_{nNT}]\,d\ln P_{nNT}=\sum_j\left[[1-\beta_{nNT}]\,\varrho_{nj}+\beta_{nNT}\omega_{nj}\right]d\ln A_{nj}-d\ln A_{nNT}$$
$$+\sum_{j\neq NT}\left[[1-\beta_{nNT}]\,\varrho_{nj}+\beta_{nNT}\omega_{nj}\right]d\ln P_{nj}+\left[[1-\beta_{nNT}]\,\zeta_n-\beta_{nNT}\delta_n\right]d\ln L_n.$$

Let's denote $\vartheta_{nNT}=[1-\beta_{nNT}]\,\varrho_{nNT}+\beta_{nNT}\omega_{nNT}$ and $\vartheta_{nj}=[1-\beta_{nNT}]\,\varrho_{nj}+\beta_{nNT}\omega_{nj}$. Solving for the price of non-tradable we get

$$d\ln P_{nNT}=\sum_j\frac{\vartheta_{nj}}{1-\vartheta_{nNT}}d\ln A_{nj}-\frac{d\ln A_{nNT}}{1-\vartheta_{nNT}}+\sum_{j\neq NT}\frac{\vartheta_{nj}}{1-\vartheta_{nNT}}d\ln P_{nj}+\frac{[1-\beta_{nNT}]\,\zeta_n-\beta_{nNT}\delta_n}{1-\vartheta_{nNT}}dlnL_n.$$

Hence, the earning equation can be written as

$$d\ln w_n=\left[\frac{\omega_{nNT}\left[[1-\beta_{nNT}]\,\zeta_n-\beta_{nNT}\delta_n\right]}{1-\vartheta_{nNT}}-\delta_n\right]d\ln L_n+\sum_j\left[\frac{\omega_{nNT}\vartheta_{nj}}{1-\vartheta_{nNT}}+\omega_{nj}\right]d\ln A_{nj}$$
$$-\frac{\omega_{nNT}d\ln A_{nNT}}{1-\vartheta_{nNT}}+\sum_{j\neq NT}\left[\frac{\omega_{nNT}\vartheta_{nj}}{1-\vartheta_{nNT}}+\omega_{nj}\right]d\ln P_{nj},$$

Finally, we assume that each region is a small open economy that face world's prices as given, namely $d\ln P_{nj}=d\ln\tau_j$. Therefore,

$$d\ln w_n=\left[\frac{\omega_{nNT}\left[[1-\beta_{nNT}]\,\zeta_n-\beta_{nNT}\delta_n\right]}{1-\vartheta_{nNT}}-\delta_n\right]d\ln L_n+\sum_j\left[\frac{\omega_{nNT}\vartheta_{nj}}{1-\vartheta_{nNT}}+\omega_{nj}\right]d\ln A_{nj}$$
$$-\frac{\omega_{nNT}}{1-\vartheta_{nNT}}d\ln A_{nNT}+\sum_{j\neq NT}\left[\frac{\omega_{nNT}\vartheta_{nj}}{1-\vartheta_{nNT}}+\omega_{nj}\right]d\ln\tau_j,$$

Notice that the the shares sum up to one, specifically

$$\sum_{j\neq NT}\left[\frac{\omega_{nNT}\vartheta_{nj}}{1-\vartheta_{nNT}}+\omega_{nj}\right]=\sum_{j\neq NT}\left[\frac{\omega_{nNT}\vartheta_{nj}}{1-\vartheta_{nNT}}+\omega_{nj}\right]$$
$$=\frac{\omega_{nNT}\sum_{j\neq NT}\vartheta_{nj}}{1-\vartheta_{nNT}}+1-\omega_{nNT}=1,$$

where we use the fact that $\sum_{j\neq NT}\vartheta_{nj}=1-\vartheta_{nNT}$.

Model with Intermediate Inputs

Similar to the previous sections, we consider an economy with R regions indexed by n and J sectors indexed by j. We also assume that labor is freely mobile across sectors inside a region and not mobile across regions. Firms produce with a constant return to scale technology using labor (L) and a fixed factor (H), and materials (M). We assume that labor, the fixed factor and materials are aggregated with a Cobb-Douglas technology. We denote by Y_{nj} the output in sector j and region n, given by

$$Y_{nj} = A_{nj}\left[L_{nj}^{\beta_{nj}} H_{nj}^{1-\beta_{nj}}\right]^{1-\gamma_{nj}} [M_{nj}]^{\gamma_{nj}},$$

where A_{nj} is the TFP in sector j and region n, β_{nj} is the share of labor in value added, $1-\beta_{nj}$ is the share of the fixed factor in value added, and γ_{nj} is the share of intermediate inputs in output. The demand for labor and the fixed factor are L_{nj} and H_{nj}, respectively, and M_{nj} is the demand for materials in sector j and region n. The regional labor-market clearing condition is given by

$$L_n = \sum_j L_{nj}.$$

From the first-order condition of the firm's cost-minimization problem we have that the demand for labor in nj is given by

$$L_{nj} = \frac{\beta_{nj}[1-\gamma_{nj}] P_{nj}}{w_n} Y_{nj},$$

where P_{nj} is the price of output in sector j and region n.

Plugging this first-order condition into the labor market clearing condition we get

$$L_n = \sum_j \frac{\beta_{nj}[1-\gamma_{nj}] P_{nj}}{w_n} Y_{nj}.$$

Total differentiating the labor market clearing condition we obtain

$$d\text{ln}L_n = \frac{1}{L_n}\sum_j \frac{\beta_{nj}[1-\gamma_{nj}] P_{nj}Y_{nj}}{w_n}\left[d\text{ln}P_{nj} + d\text{ln}Y_{nj} - d\text{ln}w_n\right],$$

which can also be expressed as

$$d\text{ln}L_n = \sum_j \frac{L_{nj}}{L_n}\left[d\text{ln}P_{nj} + d\text{ln}Y_{nj} - d\text{ln}w_n\right].$$

Using the total differential of output $d\text{ln}Y_{nj} = d\text{ln}A_{nj} + \beta_{nj}[1-\gamma_{nj}]\, d\text{ln}L_{nj} + \gamma_{nj} d\text{ln}M_{nj}$ and the total differential of the first-order conditions of the firm's cost-minimization problem with respect to labor $L_{nj} = \frac{\beta_{nj}[1-\gamma_{nj}]P_{nj}}{w_n} Y_{nj}$, and materials, $M_{nj} = \frac{\gamma_{nj}P_{nj}}{P_{nj}} Y_{nj}$, we have that

$$d\text{ln}Y_{nj} = d\text{ln}A_{nj} + \beta_{nj}[1-\gamma_{nj}]\left[d\text{ln}P_{nj} + d\text{ln}Y_{nj} - d\text{ln}w_n\right] + \gamma_{nj} d\text{ln}Y_{nj},$$

or

$$d\text{ln}Y_{nj} = \frac{d\text{ln}A_{nj}}{[1-\gamma_{nj}][1-\beta_{nj}]} + \frac{\beta_{nj}}{1-\beta_{nj}}\left[d\text{ln}P_{nj} - d\text{ln}w_n\right],$$

and plugging this equation in the labor market clearing we obtain

$$d\text{ln}L_n = \sum_j \frac{L_{nj}}{L_n}\left[\frac{d\text{ln}A_{nj}}{[1-\gamma_{nj}][1-\beta_{nj}]} + \frac{1}{1-\beta_{nj}}\left[d\text{ln}P_{nj} - d\text{ln}w_n\right]\right].$$

Solving for wages we get

$$d\ln w_n = -\delta_n d\ln L_n + \sum_j \omega_{nj} d\ln P_{nj} + \sum_j \frac{\omega_{nj}}{1-\gamma_{nj}} d\ln A_{nj},$$

where $\delta_n \equiv \left[\sum_j \frac{L_{nj}}{L_n}\frac{1}{1-\beta_{nj}}\right]^{-1}$ and $\omega_{nj} \equiv \delta_n \frac{L_{nj}}{L_n}\frac{1}{1-\beta_{nj}}$.

Finally, we assume that each region is a small open economy, namely $d\ln P_{nj} = d\ln \tau_j$. Therefore,

$$d\ln w_n = -\delta_n d\ln L_n + \sum_j \omega_{nj} d\ln \tau_j + \sum_j \frac{\omega_{nj}}{1-\gamma_{nj}} d\ln A_{nj}.$$

Model with Input-Output Linkages

We now derive a shift-share equation with intermediate inputs and input-output linkages. Similar to the previous sections of this appendix, we consider an economy with R regions indexed by n and J sectors indexed by j. We also assume that labor is freely mobile across sectors inside a region and not mobile across regions. Firms produce with a constant return to scale technology using labor (L) and a fixed factor (H), and materials (M) from all sectors according to the input-output structure of the economy. We assume that labor, the fixed factor, and materials are aggregated with a Cobb-Douglas technology. We denote by Y_{nj} the output in sector j and region n, given by

$$Y_{nj} = A_{nj}\left[L_{nj}^{\beta_{nj}} H_{nj}^{1-\beta_{nj}}\right]^{1-\gamma_{nj}} \left[\prod_k M_{jk,n}^{\gamma_{jk,n}}\right]^{\gamma_{nj}},$$

where A_{nj} is the TFP in sector j and region n, β_{nj} is the share of labor in value added, $1-\beta_{nj}$ is the share of the fixed factor in value added, and γ_{nj} is the share of intermediate inputs in output. The term $\gamma_{jk,n}$ is the share of sector k in intermediate consumption in sector j and region n, with $\sum_k \gamma_{jk,n} = 1$. The demand for labor and the fixed factor are L_{nj} and H_{nj}, respectively, and $M_{jk,n}$ is the demand for materials from sector k to produce in sector j and region n. The regional labor-market clearing condition is given by

$$L_n = \sum_j L_{nj}.$$

From the first-order condition of the firm's cost-minimization problem we have that the demand for labor in nj is given by $L_{nj} = \frac{\beta_{nj}[1-\gamma_{nj}]P_{nj}}{w_n} Y_{nj}$, where P_{nj} is the price of output in sector j and region n.

Plugging this first-order condition into the labor market clearing condition we get

$$L_n = \sum_j \frac{\beta_{nj}\left[1-\gamma_{nj}\right] P_{nj}}{w_n} Y_{nj}.$$

Total differentiating the labor market clearing condition we obtain

$$d\ln L_n = \frac{1}{L_n}\sum_j \frac{\beta_{nj}\left[1-\gamma_{nj}\right] P_{nj} Y_{nj}}{w_n}\left[d\ln P_{nj} + d\ln Y_{nj} - d\ln w_n\right],$$

which can also be expressed as

$$d\ln L_n = \sum_j \frac{L_{nj}}{L_n} \left[d\ln P_{nj} + d\ln Y_{nj} - d\ln w_n \right].$$

The total differential of output is

$$d\ln Y_{nj} = d\ln A_{nj} + \beta_{nj} \left[1 - \gamma_{nj}\right] d\ln L_{nj} + \gamma_{nj} \sum_k \gamma_{jk,n} d\ln M_{jk,n},$$

and differentiating the first-order conditions of the firm's cost-minimization problem with respect to labor $L_{nj} = \frac{\beta_{nj}[1-\gamma_{nj}]P_{nj}}{w_n} Y_{nj}$, and materials, $M_{jk,n} = \frac{\gamma_{nj}\gamma_{jk,n}P_{nj}}{P_{nk}} Y_{nj}$, we have that

$$d\ln L_{nj} = d\ln P_{nj} + d\ln Y_{nj} - d\ln w_n,$$

$$d\ln M_{jk,n} = d\ln P_{nj} + d\ln Y_{nj} - d\ln P_{nk}.$$

Putting together these equations, we obtain the total differential of output as a function of factor and goods prices, namely

$$\begin{aligned} d\ln Y_{nj} = & \frac{d\ln A_{nj}}{\left[1-\beta_{nj}\right]\left[1-\gamma_{nj}\right]} + \frac{\beta_{nj}\left[1-\gamma_{nj}\right] + \gamma_{nj}\left[1-\gamma_{jj,n}\right]}{\left[1-\beta_{nj}\right]\left[1-\gamma_{nj}\right]} d\ln P_{nj} \\ & - \frac{\beta_{nj}}{1-\beta_{nj}} d\ln w_n - \frac{\gamma_{nj}}{\left[1-\beta_{nj}\right]\left[1-\gamma_{nj}\right]} \sum_{k \neq j} \gamma_{jk,n} d\ln P_{nk}, \end{aligned}$$

and plugging this equation in the labor market clearing we obtain

$$d\ln L_n = \sum_j \frac{L_{nj}}{L_n} \left[\frac{1-\gamma_{nj}\gamma_{jj,n}}{\left[1-\beta_{nj}\right]\left[1-\gamma_{nj}\right]} d\ln P_{nj} + \frac{d\ln A_{nj}}{\left[1-\beta_{nj}\right]\left[1-\gamma_{nj}\right]} - \frac{1}{1-\beta_{nj}} d\ln w_n - \frac{\gamma_{nj}}{\left[1-\beta_{nj}\right]\left[1-\gamma_{nj}\right]} \sum_{k \neq j} \gamma_{jk,n} d\ln P_{nk} \right].$$

Solving for wages we get

$$d\ln w_n = -\delta_n d\ln L_n + \sum_j \frac{\omega_{nj} d\ln A_{nj}}{1-\gamma_{nj}} + \sum_j \omega_{nj} \frac{1-\gamma_{nj}\gamma_{jj,n}}{1-\gamma_{nj}} d\ln P_{nj} - \sum_j \frac{\omega_{nj}\gamma_{nj}}{1-\gamma_{nj}} \sum_{k \neq j} \gamma_{jk,n} d\ln P_{nk}.$$

where $\delta_n = \left[\sum_j \frac{L_{nj}}{L_n} \frac{1}{1-\beta_{nj}}\right]^{-1}$ and $\omega_{nj} = \delta_n \frac{L_{nj}}{L_n} \frac{1}{1-\beta_{nj}}$.

Finally, as in the previous sections, we assume that each region is a small open economy, namely $dlnP_{nj} = dln\tau_j$. Therefore,

$$d\ln w_n = -\delta_n d\ln L_n + \sum_j \frac{\omega_{nj} d\ln A_{nj}}{1-\gamma_{nj}} + \sum_j \omega_{nj} \frac{1-\gamma_{nj}\gamma_{jj,n}}{1-\gamma_{nj}} d\ln \tau_j - \sum_j \frac{\omega_{nj}\gamma_{nj}}{1-\gamma_{nj}} \sum_{k \neq j} \gamma_{jk,n} d\ln \tau_k.$$

Denote $\omega_{nj}^{IO} = \omega_{nj} \frac{1-\gamma_{nj}\gamma_{jj,n}}{1-\gamma_{nj}}$ and $\tilde{\omega}_{nj}^{IO} = \frac{\omega_{nj}\gamma_{nj}}{1-\gamma_{nj}} \sum_{k \neq j} \gamma_{jk,n}$, then notice that with $\gamma_{jk,n} = 0$ and $\gamma_{jj,n} = 1$, then $\omega_{nj}^{IO} = \omega_{nj}$ and $\tilde{\omega}_{nj}^{IO} = 0$.

We now analyze two extreme cases where this shift-share term is isomorphic to the the one derived in the baseline model.

Symmetric changes in tariffs

First, assume that the changes in tariffs are the same across all sectors, namely $d\ln\tau_j = d\ln\tau$. The shift-share equation can be expressed as

$$d\ln w_n = -\delta_n d\ln L_n + \sum_j \frac{\omega_{nj} d\ln A_{nj}}{1-\gamma_{nj}} + \sum_j \left[\omega_{nj}\frac{1-\gamma_{nj}\gamma_{jj,n}}{1-\gamma_{nj}} - \frac{\omega_{nj}\gamma_{nj}}{1-\gamma_{nj}}\sum_{k\neq j}\gamma_{jk,n}\right] d\ln\tau,$$

or

$$d\ln w_n = -\delta_n d\ln L_n + \sum_j \frac{\omega_{nj} d\ln A_{nj}}{1-\gamma_{nj}} + \sum_j \omega_{nj} d\ln\tau.$$

Symmetric sectors

Second, assume that the production function is symmetric across sector, namely $\gamma_{jk,n} = \frac{1}{J}$, $\gamma_{nj} = \gamma_n$, $\omega_{nj} = \omega_n$. Starting from the shift-share equation,

$$d\ln w_n = -\delta_n d\ln L_n + \sum_j \frac{\omega_{nj} d\ln A_{nj}}{1-\gamma_{nj}} + \sum_j \omega_{nj}\frac{1-\gamma_{nj}\gamma_{jj,n}}{1-\gamma_{nj}} d\ln\tau_j - \sum_j \frac{\omega_{nj}\gamma_{nj}}{1-\gamma_{nj}}\sum_{k\neq j}\gamma_{jk,n} d\ln\tau_k,$$

we impose $\gamma_{jk,n} = \frac{1}{J}$, $\gamma_{nj} = \gamma_n$, $\omega_{nj} = \omega_n$, then

$$d\ln w_n = -\delta_n d\ln L_n + \sum_j \frac{\omega_n d\ln A_{nj}}{1-\gamma_n} + \sum_j \omega_n \frac{\left[1-\gamma_n\frac{1}{J}\right]}{1-\gamma_n} d\ln\tau_j - \sum_j \frac{\omega_n\gamma_n\frac{1}{J}}{1-\gamma_n}\sum_{k\neq j} d\ln\tau_k,$$

or

$$d\ln w_n = -\delta_n d\ln L_n + \sum_j \frac{\omega_n d\ln A_{nj}}{1-\gamma_n} + \sum_j \omega_n d\ln\tau_j + \frac{\omega_n\gamma_n}{1-\gamma_n}\sum_j \left[d\ln\tau_j - \frac{1}{J}\sum_j d\ln\tau_k\right],$$

since $\frac{\omega_n\gamma_n}{1-\gamma_n}\sum_j \left[d\ln\tau_j - \frac{1}{J}\sum_j d\ln\tau_k\right] = 0$, we get

$$d\ln w_n = -\delta_n d\ln L_n + \sum_j \frac{\omega_n d\ln A_{nj}}{1-\gamma_n} + \sum_j \omega_n d\ln\tau_j.$$

Model with CES Production Function

We now derive the shift-share equation with a CES production function. Similar to the previous sections in this appendix, we consider an economy with R regions indexed by n and J sectors indexed by j. We also assume that labor is freely mobile across sectors inside a region and not mobile across regions. Firms produce with a constant return to scale technology using labor (L) and a fixed factor (H). We assume that labor and the fixed factor are aggregated with and elasticity of substitution ρ_j. We denote by Y_{nj} the output in sector j and region n, given by

$$Y_{nj} = A_{nj}\left[[\varpi]^{\frac{1}{\rho_j}}\left[L_{nj}\right]^{\frac{\rho_j-1}{\rho_j}} + [1-\varpi]^{\frac{1}{\rho_j}}\left[H_{nj}\right]^{\frac{\rho_j-1}{\rho_j}}\right]^{\frac{\rho_j}{\rho_j-1}},$$

where A_{nj} is the TFP in sector j and region n, and L_{nj} and H_{nj} are the demand for labor and the fixed factor, respectively, in sector j and region n. The term ρ_j is the elasticity of substitution between labor and the fixed-factor in the CES production function. The regional labor-market clearing condition is given by

$$L_n = \sum_j L_{nj}.$$

From the first-order condition of the firm's cost-minimization problem we have that the demand for labor in satisfies the optimality condition

$$w_n = \frac{P_{nj} Y_{nj} \left[\varpi\right]^{\frac{1}{\rho_j}} \left[L_{nj}\right]^{-\frac{1}{\rho_j}}}{\left[\varpi\right]^{\frac{1}{\rho_j}} \left[L_{nj}\right]^{\frac{\rho_j - 1}{\rho_j}} + \left[1-\varpi\right]^{\frac{1}{\rho_j}} \left[H_{nj}\right]^{\frac{\rho_j - 1}{\rho_j}}}.$$

Taking the total differential of this first-order condition, we get

$$d\ln w_n = d\ln P_{nj} + d\ln Y_{nj} - \frac{1}{\rho_j} d\ln L_{nj} - \frac{\rho_j - 1}{\rho_j} \frac{\left[\varpi\right]^{1/\rho_j} \left[L_{nj}\right]^{\frac{\rho_j - 1}{\rho_j}}}{\left[\varpi\right]^{1/\rho_j} \left[L_{nj}\right]^{\frac{\rho_j - 1}{\rho_j}} + \left[1-\varpi\right]^{1/\rho_j} \left[H_{nj}\right]^{\frac{\rho_j - 1}{\rho_j}}} dlnL_{nj}.$$

Using the first-order condition for the demand for labor, we re-express this equation as

$$d\ln w_n = d\ln P_{nj} + d\ln Y_{nj} - \frac{\left[\beta_{nj}\left[\rho_j - 1\right] + 1\right]}{\rho_j} d\ln L_{nj}$$

where $\beta_{nj} = \frac{w_n L_n}{P_{nj} Y_{nj}}$ is the cost-share of labor in output. We now total differentiate the production function, and obtain

$$d\ln Y_{nj} = d\ln A_{nj} + \frac{\left[\varpi\right]^{\frac{1}{\rho_j}} \left[L_{nj}\right]^{\frac{\rho_j - 1}{\rho_j}}}{\left[\varpi\right]^{\frac{1}{\rho_j}} \left[L_{nj}\right]^{\frac{\rho_j - 1}{\rho_j}} + \left[1-\varpi\right]^{\frac{1}{\rho_j}} \left[H_{nj}\right]^{\frac{\rho_j - 1}{\rho_j}}} d\ln L_{nj},$$

and therefore

$$d\ln Y_{nj} = d\ln A_{nj} + \beta_{nj} d\ln L_{nj}.$$

Using this equation and the first-order condition we obtain and expression for output as a function of prices. In particular, we have that

$$d\ln L_{nj} = \frac{\rho_j}{\beta_{nj}\rho_j + \left[1 - \beta_{nj}\right]} \left[d\ln P_{nj} + d\ln Y_{nj} - d\ln w_n\right],$$

and therefore,

$$d\ln Y_{nj} = d\ln A_{nj} + \frac{\beta_{nj}\rho_j}{\beta_{nj}\rho_j + \left[1 - \beta_{nj}\right]} \left[d\ln P_{nj} + d\ln Y_{nj} - d\ln w_n\right],$$

or

$$d\ln Y_{nj} = \frac{\beta_{nj}\rho_j + \left[1 - \beta_{nj}\right]}{1 - \beta_{nj}} \ln A_{nj} + \frac{\beta_{nj}\rho_j}{1 - \beta_{nj}} \left[d\ln P_{nj} - d\ln w_n\right].$$

The labor market clearing condition is given by

$$d\ln L_n = \sum_j \frac{L_{nj}}{L_n} \frac{\rho_j}{\beta_{nj}\rho_j + [1-\beta_{nj}]} \left[d\ln P_{nj} + d\ln Y_{nj} - d\ln w_n\right].$$

Therefore, using the equation for the change output we get

$$d\ln L_n = \sum_j \frac{L_{nj}}{L_n} \frac{\rho_j}{1-\beta_{nj}} \left[\ln A_{nj} + d\ln P_{nj} - d\ln w_n\right].$$

Finally, solving for wages, we obtain

$$d\ln w_n = -\delta_n^{CES} d\ln L_n + \sum_j \omega_{nj}^{CES} \ln A_{nj} + \sum_j \omega_{nj}^{CES} d\ln P_{nj},$$

where $\delta_n^{CES} = \left[\sum_j \frac{L_{nj}}{L_n} \frac{\rho_j}{1-\beta_{nj}}\right]^{-1}$ and $\omega_{nj}^{CES} = \delta_n^{CES} \frac{L_{nj}}{L_n} \frac{\rho_j}{1-\beta_{nj}}$.

Finally, as in the previous sections we assume that each region is a small open economy, namely $d\ln P_{nj} = d\ln \tau_j$. Therefore,

$$d\ln w_n = -\delta_n^{CES} d\ln L_n + \sum_j \omega_{nj}^{CES} \ln A_{nj} + \sum_j \omega_{nj}^{CES} d\ln \tau_j.$$

B Appendix: Countries and Sectors

This appendix list the sample of countries and industries used in the quantitative analysis throughout the chapter.

B.1 List of Sectors:

The sectors in the 2014 WIOD database, were mapped into our 22 sectors as follows: Food Products, Beverage, and Tobacco Products (c10–c12); Textile, Textile Product Mills, Apparel, Leather, and Allied Products (c13–c15); Wood Products, Paper, Printing, and Related Support Activities (c16–c18); Petroleum and Coal Products (c19); Chemical (c20–c21); Plastics and Rubber Products (c22); Nonmetallic Mineral Products (c23); Primary Metal and Fabricated Metal Products (c24–c25); Machinery (c28); Computer and Electronic Products, and Electrical Equipment and Appliances (c26–c27 and c33); Transportation Equipment (c29–c30); Furniture and Related Products, and Miscellaneous Manufacturing (c31–c32); Construction (F); Wholesale and Retail Trade (G45–G47); Transport Services (H49–H52); Information Services (J58–J63); Finance and Insurance (K64–K66); Real Estate (L68); Education (P85); Health Care (Q); Accommodation and Food Services (I); and Other Services (M69–M75, N, R_S, and H52–H53).

B.2 List of Countries:

The set of countries included in the quantitative analysis is: Australia, Austria, Belgium, Brazil, Bulgaria, Canada, China, Croatia, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India, Indonesia, Ireland, Italy, Japan, Korea, Latvia, Lithuania, Luxembourg, Malta, Mexico, Netherlands, Norway, Poland, Portugal, Romania, Rest of the World, Russia, Slovakia, Slovenia, Spain, Sweden, Switzerland, Taiwan, Turkey, United Kingdom, United States.

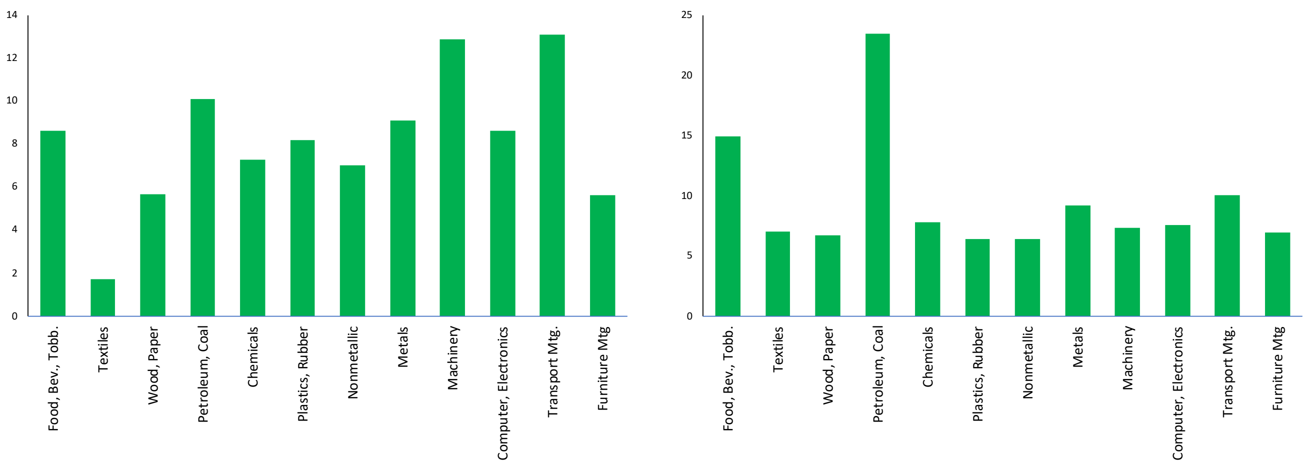
B.3 Sectoral Tariff Changes:

Figure B.1 shows the changes in tariffs applied the United States to China at our level of aggregation on the left panel, and the retaliatory changes in tariffs applied by China to the United States on the right panel. Table B.1 presents the trade-weight averages changes in tariffs applied to the United States to China and the rest of the world, as well as the retaliatory changes in tariffs applied to the United States by different trading partners.

Figure B.1: Changes in tariffs applied between the U.S. and China due to the 2018 trade war

a) Changes in tariffs applied by the U.S. to China (percentage points)

b) Changes in tariffs applied by China to the U.S. (percentage points)



Note: This figure shows the changes in tariffs applied between the United States and China due to the 2018 trade war.

Table B.1: Changes in tariffs applied to the manufacturing sector during the 2018 trade war

Changes in tariffs applied by the United States to:	
China	7.68%
Rest of the World	0.67%
Changes in retaliatory tariffs applied to the United States by:	
China	9.17%
European Union	0.68%
Canada	1.13%
Mexico	0.74%
Turkey	2.31%

Note: This table shows trade-weighted changes in tariffs applied between the United States and other countries to the manufacturing sector as a consequence of the 2018 trade war.

C Appendix: Algorithm to Compute Counterfactuals

In this section of the online appendix we describe the algorithms implemented to compute the static and dynamic frameworks.

C.1 Static Trade Equilibrium

Conditional on the observed initial variables π_{in}^{j} , $w_i L_i$, γ_n^j, γ_i^{jk} α_n^j, the trade elasticities θ^j, and the counterfactual tariff structure $\left\{\tau_{in}^{'j}\right\}_{i=1;n=1;j=1}^{N,N,J}$, an algorithm used to solve the model describe above builds on Alvarez and Lucas (2007) and the extension to multiple sectors with input-output linkages developed in Caliendo and Parro (2015).

Suppose guessing the changes in wages across all locations, for instance $\mathbf{w} = \{\hat{w}_n\}_{n=1}^N = 1$. Using (14) and (15) we can solve for changes in prices as a function of the initial guess of wage changes, namely $\hat{P}_i^j(\mathbf{w})$, and using the solution for prices we can solve for the changes in expenditure shares as a function of the guess in wage changes, namely $\hat{\pi}_{in}^j(\mathbf{w})$, using (16). With the counterfactual bilateral trade shares we can solve for total expenditure as a function of the wage changes, $X_i^{'j}(\mathbf{w})$, using (17), and then check if under the guess of wage changes, trade balance (18) holds. If not, we adjust the initial guess of wages with an updating rule and repeat the process until reaching convergence. Alvarez and Lucas (2007) show that this algorithm converges to a unique solution.

C.2 Dynamic Model

Conditional on the allocations at the initial period and the values of the the trade and mobility elasticities, the algorithm proposed by Caliendo, Dvorkin, and Parro (2019) to computed the model can be sketched as follows. The system is initiated with a guess of the entire path of changes in values, for instance $\dot{u}_{i,t} = 1$ for all i, t. Use this guess to solve for the evolution of gross flows across locations and time $\mu_{in,t+1}$ using (29). Given the initial allocation of households $L_{i,0}$ and the path of gross flows $\mu_{in,t+1}$, solve for the evolution across locations and time $L_{i,t+1}$. The path of $L_{i,t+1}$ allows us to compute its time changes $\dot{L}_{i,t+1}$ for each t. Conditional on $\dot{L}_{i,t+1}$, solve for a sequence of static trade equilibrium at each time t using the Alvarez and Lucas (2007) and Caliendo and Parro (2015) type algorithms described in previous subsection of this appendix. The sequential static trade equilibrium at each time t results in an equilibrium path of changes in real wages $\dot{\omega}_{i,t+1}$ for each i,t. Use this path of changes in real wages to construct a new path of changes in values $\dot{u}_{i,t}$. Finally, if the new path for changes in values is different from the initial guess, use an updating rule to compute again the system with a new path for the changes in values.

D Appendix: Welfare Effects of the 2018 Trade War

Table D.1 displays the welfare effects of the 2018 trade war across countries.

Table D.1: Welfare effects of the 2018 trade war (percent)

	2018 Trade war		No retaliation	
	Real wages	Real income	Real wages	Real income
United States	**-0.128**	**-0.010**	**-0.097**	**0.024**
China	**-0.112**	**-0.093**	**-0.068**	**-0.090**
Australia	-0.004	-0.005	-0.005	-0.005
Austria	-0.010	-0.005	-0.009	-0.008
Belgium	-0.022	-0.004	-0.012	-0.013
Brazil	-0.010	-0.006	-0.001	-0.001
Bulgaria	0.002	0.002	0.002	0.002
Canada	-0.183	-0.038	-0.064	-0.061
Croatia	-0.010	-0.008	-0.008	-0.008
Cyprus	-0.003	0.001	0.005	0.006
Czech Republic	-0.006	0.000	-0.005	-0.003
Denmark	-0.004	0.001	-0.001	-0.001
Estonia	-0.001	0.005	0.007	0.008
Finland	-0.007	-0.002	-0.005	-0.004
France	-0.007	-0.002	-0.003	-0.004
Germany	-0.011	-0.002	-0.007	-0.007
Greece	0.002	0.005	0.003	0.004
Hungary	-0.001	0.005	0.002	0.003
India	-0.001	-0.001	0.001	0.001
Indonesia	0.002	0.001	0.003	0.002
Ireland	-0.026	-0.008	-0.042	-0.042
Italy	-0.006	-0.001	-0.004	-0.004
Japan	-0.003	-0.003	-0.003	-0.004
Korea	-0.005	-0.004	-0.012	-0.010
Latvia	-0.004	0.003	0.002	0.003
Lithuania	-0.003	0.004	0.008	0.010
Luxembourg	-0.037	-0.032	-0.061	-0.061
Malta	-0.001	0.003	-0.001	0.000
Mexico	-0.091	0.005	-0.032	-0.028
Netherlands	-0.019	0.000	-0.006	-0.006
Norway	-0.006	-0.006	-0.003	-0.003
Poland	-0.001	0.003	0.002	0.003
Portugal	-0.002	-0.001	0.001	0.001
Romania	-0.002	0.000	0.002	0.002
RoW	-0.006	-0.006	-0.011	-0.011
Russia	-0.001	0.005	0.012	0.013
Slovakia	0.001	0.004	0.002	0.003
Slovenia	-0.001	0.003	0.000	0.001
Spain	-0.002	0.002	0.002	0.002
Sweden	-0.008	-0.004	-0.007	-0.007
Switzerland	-0.003	-0.003	-0.003	-0.003
Taiwan	-0.030	-0.031	-0.041	-0.043
Turkey	-0.020	-0.002	-0.008	-0.008
United Kingdom	-0.014	-0.004	-0.005	-0.004

Note: This table shows the real wages and real income effects of the 2018 trade war, and the effects without retaliation.

E Appendix: Optimal Tariffs in Neoclassical Environments

In this appendix we derive the optimal tariff in a single sector neoclassical environment with and without intermediate goods.

E.1 Optimal Tariff in Large Economy

We denote by λ_{ni} the bilateral trade share of goods imported by country n from country i,

$$\lambda_{ni} = \frac{X_{ni}}{X_n} = 1 - \lambda_{nn}.$$

We denote by X_n total expenditure in country n,by β the share of labor in gross output, and by $1-\beta$) the share of intermediate goods in gross output. The term τ_{ni} denote one plus the ad-valorem import tariff applied by country n to country i. Therefore,

$$X_n = w_n L_n + \frac{[\tau_{ni}-1][1-\lambda_{nn}]}{\tau_{ni}} X_n + [1-\beta]\left[\lambda_{nn} X_n + \frac{\lambda_{in}}{\tau_{in}} X_i\right],$$

and using the trade balance condition we get

$$X_n = w_n L_n + \frac{[\tau_{ni}-1][1-\lambda_{nn}]}{\tau_{ni}} X_n + [1-\beta]\left[\frac{\lambda_{nn}\tau_{ni} + 1 - \lambda_{nn}}{\tau_{ni}}\right] X_n,$$

and solving for total expenditure we obtain

$$X_n = \frac{\tau_{ni}}{\tau_{ni}\lambda_{nn} + 1 - \lambda_{nn}} \frac{w_n L_n}{\beta}.$$

Denote by W_n real per-capita income in country n, given by

$$\begin{aligned} W_n &= \frac{w_n L_n + \frac{[\tau_{ni}-1][1-\lambda_{nn}]}{\tau_{ni}} X_n}{L_n P_n} \\ &= \frac{w_n}{\beta P_n}\left[\frac{\tau_{ni}}{\tau_{ni}\lambda_{nn} + 1 - \lambda_{nn}} - [1-\beta]\right]. \end{aligned}$$

Taking the total differential, we obtain

$$d\ln W_n = d\ln(w_n/P_n) + \frac{1}{\frac{\tau_{ni}}{\tau_{ni}\lambda_{nn}+1-\lambda_{nn}} - [1-\beta]}\left[\frac{d\tau_{ni}}{\lambda_{nn}[\tau_{ni}-1]+1} - \frac{\tau_{ni}[d\lambda_{nn}[\tau_{ni}-1] + \lambda_{nn} d\tau_{ni}]}{[\lambda_{nn}[\tau_{ni}-1]+1]^2}\right]$$

$$= d\ln(w_n/P_n) + \frac{\tau_{ni}}{\tau_{ni} - [1-\beta][\tau_{ni}\lambda_{nn} + 1 - \lambda_{nn}]}\left[\frac{1-\lambda_{nn}}{\lambda_{nn}[\tau_{ni}-1]+1} dln\tau_{ni} - \frac{\lambda_{nn}[\tau_{ni}-1] dln\lambda_{nn}}{\lambda_{nn}[\tau_{ni}-1]+1}\right]. \quad \text{(E.1)}$$

We now use the gravity structure in Eaton and Kortum (2002), namely

$$\lambda_{nn} = T_n\left[\frac{w_n^{\beta} P_n^{1-\beta}}{P_n}\right]^{-\theta},$$

where the trade elasticity is given by θ. The gravity structure implies that

$$-\frac{1}{\theta\beta}d\ln\lambda_{nn} = d\ln\left(w_n/P_n\right).$$

Using balanced trade $\frac{\lambda_{in}}{\tau_{in}}X_i = \frac{\lambda_{ni}}{\tau_{ni}}X_n$, we have that

$$\frac{\lambda_{in}}{\tau_{in}}\frac{\tau_{in}}{\tau_{in}\lambda_{ii}+1-\lambda_{ii}}w_iL_i = \frac{\lambda_{ni}}{\tau_{ni}}\frac{\tau_{ni}}{\tau_{ni}\lambda_{nn}+1-\lambda_{nn}}w_nL_n.$$

We assume that $\tau_{in}=1$, then

$$\lambda_{in}w_iL_i = \frac{\lambda_{ni}}{\tau_{ni}\lambda_{nn}+1-\lambda_{nn}}w_nL_n,$$

or

$$\lambda_{in}w_iL_i = \frac{1}{\tau_{ni}\frac{\lambda_{nn}}{\lambda_{ni}}+1}w_nL_n,$$

and using gravity we get

$$\left[\frac{T_i\left[x_i\right]^{-\theta}+T_n\left[x_n\right]^{-\theta}}{T_n\left[x_n\right]^{-\theta}}\right]^{-1}w_iL_i = \frac{1}{\left[\frac{T_n[x_n]^{-\theta}\tau_{ni}}{T_i[x_i\tau_{ni}]^{-\theta}}\right]+1}w_nL_n,$$

where $x_n \equiv w_n^{\beta}P_n^{1-\beta}$
or

$$\left[\frac{T_i\left[x_i\right]^{-\theta}+T_n\left[x_n\right]^{-\theta}}{T_n\left[x_n\right]^{-\theta}}\right]^{-1}w_iL_i = \frac{T_i\left[x_i\tau_{ni}\right]^{-\theta}}{T_n\left[x_n\right]^{-\theta}\tau_{ni}+T_i\left[x_i\tau_{ni}\right]^{-\theta}}w_nL_n,$$

or

$$\frac{T_i\left[x_i\right]^{-\theta}+T_n\left[x_n\right]^{-\theta}}{T_n\left[x_n\right]^{-\theta}}w_nL_n = \frac{T_n\left[x_n\right]^{-\theta}\tau_{ni}+T_i\left[x_i\tau_{ni}\right]^{-\theta}}{T_i\left[x_i\tau_{ni}\right]^{-\theta}}w_iL_i,$$

Let $\varpi \equiv w_n/w_i$, and the normalization $w_i=1$, $x_i=w_i$, so that the foreign country does not use intermediate goods.

$$\left[\frac{T_i}{T_n\left(x_n\right)^{-\theta}}+1\right]w_nL_n = L_i\left[\frac{T_n\left[x_n\right]^{-\theta}\tau_{ni}}{T_i\left[\tau_{ni}\right]^{-\theta}}+1\right].$$

Taking the total differential, we obtain

$$d\ln w_n + \tilde{\lambda}_{ii}\theta d\ln x_n = -\theta\frac{\lambda_{nn}\tau_{ni}}{\lambda_{nn}\tau_{ni}+1-\lambda_{nn}}d\ln x_n + \frac{\lambda_{nn}\tau_{ni}}{\lambda_{nn}\tau_{ni}+1-\lambda_{nn}}\left[1+\theta\right]d\ln\tau_{ni},$$

or

$$d\ln w_n + \left[\tilde{\lambda}_{nn}+\tilde{\lambda}_{ii}\right]\theta d\ln x_n = \left[1+\theta\right]\tilde{\lambda}_{nn}d\ln\tau_{ni}, \tag{E.2}$$

where $\tilde{\lambda}_{nn} = \frac{\tau_{ni}\lambda_{nn}}{1-\lambda_{nn}[1-\tau_{ni}]}$. Taking total derivative of price index

$$\left[P_n\right]^{-\theta} = T_n\left[x_n\right]^{-\theta} + T_i\left[x_i\tau_{ni}\right]^{-\theta},$$

we get

$$d\ln P_n = \lambda_{nn} d\ln x_n + [1-\lambda_{nn}]\, d\ln\tau_{ni}, \tag{E.3}$$

and solving for the input costs, we obtain

$$d\ln x_n = \frac{\beta}{1-\lambda_{nn}[1-\beta]} d\ln w_n + \frac{[1-\beta][1-\lambda_{nn}]}{1-\lambda_{nn}[1-\beta]} d\ln\tau_{ni}. \tag{E.4}$$

We use now into (E.2) and replace the expression for the input bundle costs

$$d\ln w_n = \frac{[1+1/\theta]\,\tilde{\lambda}_{nn} - \left[\tilde{\lambda}_{nn} + \tilde{\lambda}_{ii}\right] \frac{[1-\beta][1-\lambda_{nn}]}{1-\lambda_{nn}[1-\beta]}}{1/\theta + \left[\tilde{\lambda}_{nn} + \tilde{\lambda}_{ii}\right] \frac{\beta}{1-\lambda_{nn}[1-\beta]}} d\ln\tau_{ni}. \tag{E.5}$$

Using

$$\lambda_{nn} = \frac{T_n [x_n]^{-\theta}}{T_n [x_n]^{-\theta} + T_i [\tau_{ni}]^{-\theta}},$$

we express

$$d\ln\lambda_{nn} = -[1-\lambda_{nn}]\,\theta d\ln x_n + [1-\lambda_{nn}]\,\theta d\ln\tau_{ni},$$

and using the expressing for the input-bundle cost, we get

$$d\ln\lambda_{nn} = -[1-\lambda_{nn}]\,\theta \left[\frac{\beta}{1-\lambda_{nn}[1-\beta]} d\ln w_n + \frac{[1-\beta][1-\lambda_{nn}]}{1-\lambda_{nn}[1-\beta]} d\ln\tau_{ni}\right] + [1-\lambda_{nn}]\,\theta d\ln\tau_{ni},$$

or

$$d\ln\lambda_{nn} = \theta \frac{\beta[1-\lambda_{nn}]}{1-\lambda_{nn}[1-\beta]} [d\ln\tau_{ni} - d\ln w_n].$$

Now use (*E*.5) to obtain

$$d\ln\lambda_{nn} = \theta \frac{\beta[1-\lambda_{nn}]}{1-\lambda_{nn}[1-\beta]} \left[\frac{[1/\theta]\left[1-\tilde{\lambda}_{nn}\right] + \tilde{\lambda}_{ii}}{1/\theta + \left[\tilde{\lambda}_{nn} + \tilde{\lambda}_{ii}\right] \frac{\beta}{1-\lambda_{nn}[1-\beta]}}\right] d\ln\tau_{ni},$$

or

$$d\ln\tau_{ni} = \frac{1-\lambda_{nn}[1-\beta]}{\theta\beta[1-\lambda_{nn}]} \left[\frac{1/\theta + \left[\tilde{\lambda}_{nn} + \tilde{\lambda}_{ii}\right] \frac{\beta}{1-\lambda_{nn}[1-\beta]}}{[1/\theta]\left[1-\tilde{\lambda}_{nn}\right] + \tilde{\lambda}_{ii}}\right] d\ln\lambda_{nn}.$$

Plugging this expression into welfare

$$d\ln W_n = d\ln(w_n/P_n) + \frac{\tau_{ni}}{\tau_{ni} - [1-\beta][\tau_{ni}\lambda_{nn} + 1 - \lambda_{nn}]} \left[\frac{1-\lambda_{nn}}{[\lambda_{nn}[\tau_{ni}-1]+1]} d\ln\tau_{ni} - \frac{\lambda_{nn}[\tau_{ni}-1]\, d\ln\lambda_{nn}}{[\lambda_{nn}[\tau_{ni}-1]+1]}\right].$$

Now, the optimal tariff solves $d\ln W_n = 0$, then

$$\frac{1}{\theta\beta} = \frac{\tau_{ni}}{\tau_{ni} - [1-\beta][\tau_{ni}\lambda_{nn} + 1 - \lambda_{nn}]} \left[\begin{array}{c} \frac{1-\lambda_{nn}}{[\lambda_{nn}[\tau_{ni}-1]+1]} \frac{1-\lambda_{nn}[1-\beta]}{\theta\beta[1-\lambda_{nn}]} \left[\frac{1/\theta + \left[\tilde{\lambda}_{nn}+\tilde{\lambda}_{ii}\right]\frac{\beta}{1-\lambda_{nn}[1-\beta]}}{[1/\theta]\left[1-\tilde{\lambda}_{nn}\right]+\tilde{\lambda}_{ii}}\right] \\ -\frac{\lambda_{nn}[\tau_{ni}-1]}{[\lambda_{nn}[\tau_{ni}-1]+1]} \end{array}\right],$$

or

$$\frac{1}{\theta\beta}\frac{\tau_{ni}-[1-\beta]\left[\tau_{ni}\lambda_{nn}+1-\lambda_{nn}\right]}{\tau_{ni}} = \frac{1-\lambda_{nn}[1-\beta]}{\left[\lambda_{nn}[\tau_{ni}-1]+1\right]}\frac{1}{\theta\beta}\left[\frac{1/\theta+\left[\tilde{\lambda}_{nn}+\tilde{\lambda}_{ii}\right]\frac{\beta}{1-\lambda_{nn}[1-\beta]}}{1/\theta-[1+1/\theta]\tilde{\lambda}_{nn}+\left[\tilde{\lambda}_{nn}+\tilde{\lambda}_{ii}\right]}\right] - \frac{\lambda_{nn}[\tau_{ni}-1]}{\left[\lambda_{nn}[\tau_{ni}-1]+1\right]}.$$

Take the denominator for the right-hand side and bring it to left, and put the first element multiplying the first term on the right-hand side inside the parenthesis

$$\frac{1}{\theta\beta}\frac{\tau_{ni}\left[\lambda_{nn}[\tau_{ni}-1]+1\right]-[1-\beta]\left[\tau_{ni}\lambda_{nn}+1-\lambda_{nn}\right]^2}{\tau_{ni}} = \frac{1}{\theta\beta}\left[\frac{[1/\theta]\left[1-\lambda_{nn}[1-\beta]\right]+\left[\tilde{\lambda}_{nn}+\tilde{\lambda}_{ii}\right]\beta}{[1/\theta]\left[1-\tilde{\lambda}_{nn}\right]+\tilde{\lambda}_{ii}}\right]-\lambda_{nn}[\tau_{ni}-1],$$

work on the left-hand side to obtain

$$\frac{1}{\theta\beta}\lambda_{nn}[\tau_{ni}-1]+\frac{1}{\theta\beta}-\frac{1}{\theta\beta}[1-\beta]\frac{\left[\tau_{ni}\lambda_{nn}+1-\lambda_{nn}\right]^2}{\tau_{ni}} = \frac{1}{\theta\beta}\left[\frac{[1/\theta]\left[1-\lambda_{nn}[1-\beta]\right]+\left[\tilde{\lambda}_{nn}+\tilde{\lambda}_{ii}\right]\beta}{[1/\theta]\left[1-\tilde{\lambda}_{nn}\right]+\tilde{\lambda}_{ii}}\right]-\lambda_{nn}[\tau_{ni}-1],$$

bring part of left-hand side to the right-hand side,

$$\frac{1/\theta+\beta}{\beta}\lambda_{nn}[\tau_{ni}-1]+\frac{1}{\theta\beta}-\frac{1}{\theta\beta}[1-\beta]\frac{\left[\tau_{ni}\lambda_{nn}+1-\lambda_{nn}\right]^2}{\tau_{ni}} = \frac{1}{\theta\beta}\left[\frac{[1/\theta]\left[1-\lambda_{nn}[1-\beta]\right]+\left[\tilde{\lambda}_{nn}+\tilde{\lambda}_{ii}\right]\beta}{[1/\theta]\left[1-\tilde{\lambda}_{nn}\right]+\tilde{\lambda}_{ii}}\right],$$

multiply by β and obtain

$$[1/\theta+\beta]\lambda_{nn}[\tau_{ni}-1] = [1/\theta]\left[\frac{[1+1/\theta]\tilde{\lambda}_{nn}-\left[[1/\theta]\lambda_{nn}+\left[\tilde{\lambda}_{nn}+\tilde{\lambda}_{ii}\right]\right][1-\beta]}{[1/\theta]\left[1-\tilde{\lambda}_{nn}\right]+\tilde{\lambda}_{ii}}\right] + [1/\theta][1-\beta]\frac{\left[\tau_{ni}\lambda_{nn}+1-\lambda_{nn}\right]^2}{\tau_{ni}}.$$

Using

$$\tilde{\lambda}_{nn} = \frac{\lambda_{nn}\tau_{ni}}{1-\lambda_{nn}[1-\tau_{ni}]},$$

we get

$$[1/\theta+\beta]\lambda_{nn}[\tau_{ni}-1] = [1/\theta]\left[\frac{[1+1/\theta]\tilde{\lambda}_{nn}-\left[[1/\theta]\lambda_{nn}+\left[\tilde{\lambda}_{nn}+\tilde{\lambda}_{ii}\right]\right][1-\beta]}{[1/\theta]\left[1-\tilde{\lambda}_{nn}\right]+\tilde{\lambda}_{ii}}\right] + [1/\theta][1-\beta]\frac{\left[\frac{\tau_{ni}}{\tau_{ni}+\tilde{\lambda}_{nn}[1-\tau_{ni}]}\right]^2}{\tau_{ni}}.$$

Multiply all the expression by $\left[\tau_{ni}+\tilde{\lambda}_{nn}[1-\tau_{ni}]\right]$, and use this expression again

$$\lambda_{nn}\tau_{ni}+1-\lambda_{nn}=\frac{\tau_{ni}}{\tau_{ni}+\tilde{\lambda}_{nn}\left[1-\tau_{ni}\right]},$$

and working the algebra, we obtain

$$0=\left[\frac{\left[1/\theta+\beta\right]\lambda_{nn}\left[1/\theta-\left[\tau_{ni}-1\right]\tilde{\lambda}_{ii}\right]-\left[1/\theta\right]\left[1-\beta\right]\tilde{\lambda}_{ii}}{\left[1/\theta\right]\left[1-\tilde{\lambda}_{nn}\right]+\tilde{\lambda}_{ii}}\right]\left[\tau_{ni}+\tilde{\lambda}_{nn}\left[1-\tau_{ni}\right]\right]$$
$$+\left[1/\theta\right]\left[1-\beta\right]\frac{\tilde{\lambda}_{ii}\left[\tau_{ni}-1\right]-1/\theta}{\left[1/\theta\right]\left[1-\tilde{\lambda}_{nn}\right]+\tilde{\lambda}_{ii}}\lambda_{nn}+\left[1/\theta\right]\left[1-\beta\right].$$

Use again

$$\tau_{ni}+\tilde{\lambda}_{nn}\left[1-\tau_{ni}\right]=\frac{\tilde{\lambda}_{nn}}{\lambda_{nn}},$$

to obtain

$$0=\left[\frac{\left[1/\theta-\left[\tau_{ni}-1\right]\tilde{\lambda}_{ii}\right]\left[\left[\left[1/\theta\right]+\beta\right]\tilde{\lambda}_{nn}-\left[1/\theta\right]\left[1-\beta\right]\lambda_{nn}\right]-\left[1/\theta\right]\left[1-\beta\right]\tilde{\lambda}_{ii}\frac{\tilde{\lambda}_{nn}}{\lambda_{nn}}}{\left[1/\theta\right]\left[1-\tilde{\lambda}_{nn}\right]+\tilde{\lambda}_{ii}}\right]+\left[1/\theta\right]\left[1-\beta\right],$$

and after some algebra, obtain

$$\left[\tau_{ni}-1\right]=\frac{1}{\theta\tilde{\lambda}_{ii}}\left[\frac{1+\frac{\left[1-\beta\right]}{\left[1+\theta\beta\right]\tilde{\lambda}_{nn}-\left[1-\beta\right]\lambda_{nn}}\left[1-\tilde{\lambda}_{nn}\right]}{1+\frac{\left[1-\beta\right]}{\left[1+\theta\beta\right]\tilde{\lambda}_{nn}-\left[1-\beta\right]\lambda_{nn}}\left[\frac{1+\lambda_{nn}}{1+\lambda_{nn}\left[\tau_{ni}-1\right]}\right]}\right].$$

Now use $\left[1-\tilde{\lambda}_{nn}\right]=\left[\frac{1-\lambda_{nn}}{1-\lambda_{nn}\left[1-\tau_{ni}\right]}\right]$ to arrive to the optimal tariff formula

$$\tau_{ni}^{*}-1=\frac{1}{\theta\tilde{\lambda}_{ii}}\left[\frac{1+\frac{\left[1-\beta\right]}{\left[1+\theta\beta\right]\tilde{\lambda}_{nn}-\left[1-\beta\right]\lambda_{nn}}\left[\frac{1-\lambda_{nn}}{1+\lambda_{nn}\left[\tau_{ni}^{*}-1\right]}\right]}{1+\frac{\left[1-\beta\right]}{\left[1+\theta\beta\right]\tilde{\lambda}_{nn}-\left[1-\beta\right]\lambda_{nn}}\left[\frac{1+\lambda_{nn}}{1+\lambda_{nn}\left[\tau_{ni}^{*}-1\right]}\right]}\right].$$

Optimal tariff with no intermediate goods

Without intermediate goods we have that $\beta=1$, hence the tariff formula becomes

$$\tau_{ni}^{*}-1=\frac{1}{\theta\tilde{\lambda}_{ii}}.$$

E.2 Optimal Tariff in a Small Open Economy

Starting from equation (E.1) and imposing $\lambda_{nn}=0$ and $\lambda_{ii}=1$ we obtain

$$d\ln W_{n}=d\ln\left(w_{n}/P_{n}\right)+\frac{\tau_{ni}}{\tau_{ni}-\left[1-\beta\right]}d\ln\tau_{ni}.$$

Similarly, imposing the same restrictions in equations (E.3) and (E.5), we get

$$d\ln w_{n}-d\ln P_{n}=-\frac{\left[1-\beta\right]}{1/\theta+\beta}d\ln\tau_{ni}-d\ln\tau_{ni}.$$

Hence, we have that

$$d\ln W_n = \left[-\frac{[1-\beta]}{1/\theta+\beta} - 1 + \frac{\tau_{ni}}{\tau_{ni}-[1-\beta]}\right] d\ln\tau_{ni},$$

and therefore, the optimal tariff satisfies

$$-\frac{[1-\beta]}{1/\theta+\beta} - 1 + \frac{\tau_{ni}^*}{\tau_{ni}^*-[1-\beta]} = 0,$$

Solving for the optimal tariff we obtain

$$\tau_{ni}^* - 1 = 1/\theta.$$