Online Appendix

A Theory of the Global Financial Cycle

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August 2021

In this Online Appendix we first present some additional information and robustness tests related to the empirical section. We then discuss the decomposition of capital flows into portfolio growth, portfolio rebalancing and portfolio reallocation components (discussed in Sections 5 and 6 of the paper).

A Additional Empirical Results

We begin with the results from various robustness tests. We first report the results we using quarterly rather than annual data. We then report the results for a few alternative definitions of the GFC factor, alternative measures of the risk-free rate and different methods to detrend stock prices. After that we present the loadings on the GFC factor for each country in our sample. We finally discuss some data to justify our classification of international debt flows as safe asset flows and international equity flows as risky asset flows.

A.1 Robustness

In Tables 1-2 we present the results when estimating the GFC factor, and running regressions, with quarterly data instead of annual data. The first factor based on quarterly data is shown in Figure 1, together with the MAR factor at the quarterly frequency.

In Tables 3-6 we then repeat all regressions, using both annual and quarterly data, for two alternative measures of the GFC factor. The first alternative GFC measure is the Miranda-Agrippino and Rey (MAR) asset price factor. The second alternative measure is estimated as the first principal component of a larger set of 101 series: the 80 capital flow series we use in the text (4 series per country, 20 countries), the stock price series from each of the 20 countries, and the US real 1yr treasury interest rate.

Finally, in Table 7 we report the results for different measures of the risk-free interest rate, either the real or nominal 3 month or 1 year US treasury rate. We also report regressions when the stock price dependent variable is detrended with log differences instead of a deviation from a linear trend.

Figure 1: First Factor from Capital Flows Factor Model and MAR factor (quarterly data)

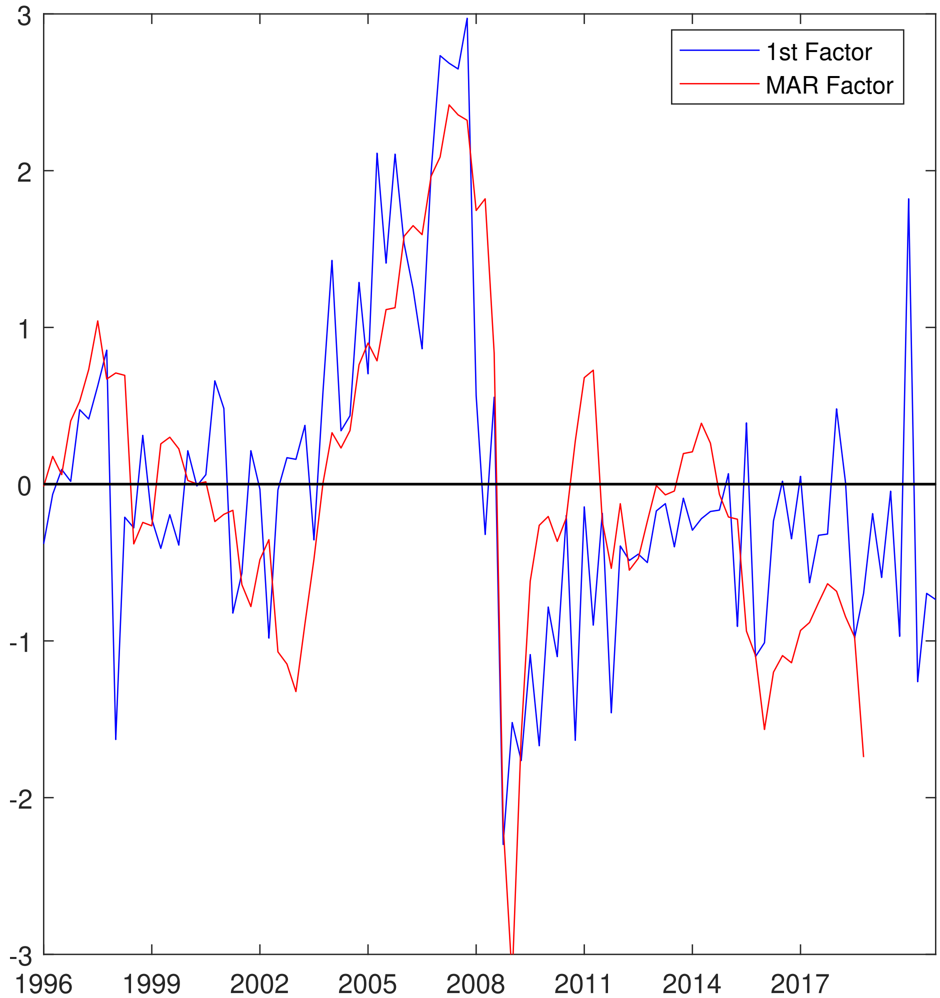

Table 1: Regression Real Interest Rate, Stock Prices, Gross Risky Asset Flows on First Factor (quarterly data)

	r_t	$q_{i,t}$	$of_{i,t}^{risky}$	$if_{i,t}^{risky}$	$of_{i,t}^{risky}+if_{i,t}^{risky}$
F_t	0.492***	9.039***	2.713**	1.772**	4.485**
	(0.118)	(0.664)	(1.220)	(0.835)	(2.004)
R^2	0.153	0.131	0.184	0.217	0.205

Notes: A linear trend is removed from the real interest rate series and each stock price series.

Table 2: Panel Regression Net Capital Flows, Stock Prices on First Factor (quarterly data)

Dep. Var:	nf_t^{safe}	nf_t^{safe}	nf_t^{safe}	nf_t^{risky}	nf_t^{risky}	nf_t^{risky}
F_t	-0.883	-1.029*	-0.735	0.588	0.605**	0.428**
	(0.690)	(0.531)	(0.513)	(0.370)	(0.287)	(0.218)
$nfa_{i,t-1}^{safe} * F_t$	0.030*		0.028**	-0.017**		-0.014***
	(0.016)		(0.012)	(0.007)		(0.004)
$nfa_{i,t-1}^{risky} * F_t$		-0.037	-0.019		0.028	0.018
		(0.039)	(0.022)		(0.023)	(0.015)
R^2	0.240	0.183	0.254	0.196	0.173	0.213
Dep. Var:	nf_t	nf_t	nf_t	q_t	q_t	q_t
F_t	-0.295	-0.424	-0.306	8.962***	9.215***	9.190***
	(0.353)	(0.282)	(0.323)	(0.605)	(0.748)	(0.715)
$nfa_{i,t-1}^{safe} * F_t$	0.013		0.013	-0.018***		-0.016**
	(0.009)		(0.008)	(0.004)		(0.007)
$nfa_{i,t-1}^{risky} * F_t$		-0.009	0.000		-0.055***	-0.058***
		(0.017)	(0.007)		(0.020)	(0.019)
R^2	0.178	0.161	0.178	0.135	0.155	0.160

Notes: A linear trend is removed from each stock price series. nfa^{safe} and nfa^{risky} are a country's net foreign asset positions in safe and risky assets.

Table 3: Regression Real Interest Rate, Stock Prices, Gross Risky Asset Flows on 2 alternative GFC Factors (annual data)

MAR Factor:					
	r_t	$q_{i,t}$	$of_{i,t}^{risky}$	$if_{i,t}^{risky}$	$of_{i,t}^{risky}+if_{i,t}^{risky}$
F_t	0.514**	12.178***	2.449**	1.719**	4.168**
	(0.251)	(0.848)	(1.027)	(0.782)	(1.788)
R^2	0.194	0.251	0.387	0.411	0.415
Factor from Capital Flows and Asset Prices:					
	r_t	$q_{i,t}$	$of_{i,t}^{risky}$	$if_{i,t}^{risky}$	$of_{i,t}^{risky}+if_{i,t}^{risky}$
F_t	0.627***	13.680***	2.994**	1.988***	4.982***
	(0.219)	(0.937)	(1.189)	(0.775)	(1.897)
R^2	0.271	0.331	0.371	0.364	0.382

Notes: A linear trend is removed from the real interest rate series and each stock price series.

Table 4: Panel Regression Net Capital Flows, Stock Prices on 2 alternative GFC Factors (annual data)

MAR Factor:						
Dep. Var:	nf_t^{safe}	nf_t^{safe}	nf_t^{safe}	nf_t^{risky}	nf_t^{risky}	nf_t^{risky}
F_t	-1.140*	-0.982	-0.944**	0.655**	0.484**	0.469**
	(0.634)	(0.632)	(0.428)	(0.323)	(0.231)	(0.221)
$nfa_{i,t-1}^{safe} * F_t$	0.032***		0.029***	-0.005***		-0.003
	(0.009)		(0.004)	(0.001)		(0.005)
$nfa_{i,t-1}^{risky} * F_t$		-0.045	-0.025		0.025	0.024
		(0.041)	(0.025)		(0.021)	(0.023)
R^2	0.522	0.418	0.547	0.215	0.225	0.268
Dep. Var:	nf_t	nf_t	nf_t	q_t	q_t	q_t
F_t	-0.485	-0.497	-0.476	12.195***	12.624***	12.687***
	(0.381)	(0.494)	(0.336)	(0.820)	(0.923)	(0.877)
$nfa_{i,t-1}^{safe} * F_t$	0.026***		0.026***	-0.015**		-0.012
	(0.009)		(0.009)	(0.007)		(0.008)
$nfa_{i,t-1}^{risky} * F_t$		-0.020	-0.001		-0.093***	-0.095***
		(0.021)	(0.007)		(0.029)	(0.029)
R^2	0.657	0.555	0.657	0.253	0.277	0.279
Factor from Capital Flows and Asset Prices:						
Dep. Var:	nf_t^{safe}	nf_t^{safe}	nf_t^{safe}	nf_t^{risky}	nf_t^{risky}	nf_t^{risky}
F_t	-0.742	-0.535	-0.473	0.732*	0.636*	0.578**
	(0.583)	(0.663)	(0.401)	(0.408)	(0.339)	(0.268)
$nfa_{i,t-1}^{safe} * F_t$	0.035***		0.032***	-0.018***		-0.016***
	(0.012)		(0.007)	(0.006)		(0.003)
$nfa_{i,t-1}^{risky} * F_t$		-0.056	-0.025		0.031	0.015
		(0.046)	(0.023)		(0.027)	(0.016)
R^2	0.529	0.407	0.544	0.318	0.246	0.339
Dep. Var:	nf_t	nf_t	nf_t	q_t	q_t	q_t
F_t	-0.010	0.101	0.105	13.709***	13.933***	14.003***
	(0.234)	(0.380)	(0.209)	(0.809)	(0.950)	(0.844)
$nfa_{i,t-1}^{safe} * F_t$	0.017***		0.016***	-0.023***		-0.020***
	(0.006)		(0.005)	(0.006)		(0.006)
$nfa_{i,t-1}^{risky} * F_t$		-0.026	-0.010		-0.057**	-0.059***
		(0.020)	(0.009)		(0.025)	(0.022)
R^2	0.570	0.533	0.572	0.337	0.354	0.359

Notes: A linear trend is removed from each stock price series. nfa^{safe} and nfa^{risky} are a country's net foreign asset positions in safe and risky assets.

Table 5: Regression Real Interest Rate, Stock Prices, Gross Risky Asset Flows on 2 alternative GFC Factors (quarterly data)

MAR Factor:					
	r_t	$q_{i,t}$	$of^{risky}_{i,t}$	$if^{risky}_{i,t}$	$of^{risky}_{i,t}+if^{risky}_{i,t}$
F_t	0.462***	15.545***	2.572**	1.798**	4.370**
	(0.123)	(0.879)	(1.023)	(0.819)	(1.821)
R^2	0.157	0.379	0.223	0.261	0.253
Factor from Capital Flows and Asset Prices:					
	r_t	$q_{i,t}$	$of^{risky}_{i,t}$	$if^{risky}_{i,t}$	$of^{risky}_{i,t}+if^{risky}_{i,t}$
F_t	0.607***	13.576***	3.157**	2.085**	5.243**
	(0.109)	(0.990)	(1.255)	(0.883)	(2.088)
R^2	0.241	0.295	0.198	0.226	0.218

Notes: A linear trend is removed from the real interest rate series and each stock price series.

Table 6: Panel Regression Net Capital Flows, Stock Prices on 2 alternative GFC Factors (quarterly data)

MAR Factor:						
Dep. Var:	nf_t^{safe}	nf_t^{safe}	nf_t^{safe}	nf_t^{risky}	nf_t^{risky}	nf_t^{risky}
F_t	-1.046*	-1.141**	-0.892**	0.628**	0.530**	0.473**
	(0.571)	(0.518)	(0.408)	(0.274)	(0.224)	(0.195)
$nfa_{i,t-1}^{safe} * F_t$	0.025***		0.022***	-0.008***		-0.005*
	(0.009)		(0.004)	(0.003)		(0.002)
$nfa_{i,t-1}^{risky} * F_t$		-0.038	-0.027		0.026	0.025
		(0.041)	(0.032)		(0.021)	(0.021)
R^2	0.227	0.192	0.249	0.153	0.159	0.181
Dep. Var:	nf_t	nf_t	nf_t	q_t	q_t	q_t
F_t	-0.418	-0.611*	-0.419	15.481***	15.845***	15.822***
	(0.349)	(0.365)	(0.302)	(0.894)	(0.907)	(0.897)
$nfa_{i,t-1}^{safe} * F_t$	0.018***		0.017***	-0.016*		-0.016
	(0.006)		(0.005)	(0.008)		(0.010)
$nfa_{i,t-1}^{risky} * F_t$		-0.012	-0.001		-0.051*	-0.056*
		(0.020)	(0.011)		(0.028)	(0.029)
R^2	0.208	0.178	0.209	0.383	0.386	0.391
Factor from Capital Flows and Asset Prices:						
Dep. Var:	nf_t^{safe}	nf_t^{safe}	nf_t^{safe}	nf_t^{risky}	nf_t^{risky}	nf_t^{risky}
F_t	-0.989	-1.091*	-0.764	0.712**	0.631**	0.477**
	(0.699)	(0.643)	(0.507)	(0.341)	(0.309)	(0.208)
$nfa_{i,t-1}^{safe} * F_t$	0.035**		0.031***	-0.016***		-0.012***
	(0.016)		(0.011)	(0.005)		(0.001)
$nfa_{i,t-1}^{risky} * F_t$		-0.053	-0.026		0.037	0.026
		(0.044)	(0.023)		(0.024)	(0.018)
R^2	0.275	0.204	0.291	0.195	0.184	0.216
Dep. Var:	nf_t	nf_t	nf_t	q_t	q_t	q_t
F_t	-0.277	-0.460	-0.286	13.459***	13.735***	13.675***
	(0.404)	(0.368)	(0.360)	(0.876)	(1.040)	(0.936)
$nfa_{i,t-1}^{safe} * F_t$	0.019*		0.019*	-0.024***		-0.022***
	(0.011)		(0.011)	(0.005)		(0.006)
$nfa_{i,t-1}^{risky} * F_t$		-0.017	0.000		-0.053**	-0.056**
		(0.021)	(0.006)		(0.027)	(0.025)
R^2	0.202	0.165	0.202	0.301	0.316	0.323

Notes: A linear trend is removed from each stock price series. nfa^{safe} and nfa^{risky} are a country's net foreign asset positions in safe and risky assets.

Table 7: Robustness Analysis: Real Interest Rates, Stock Price Detrending

Dep. Variable: US Interest Rate:				
	3M	Real 3M	1y	Real 1y
F_t	0.726***	0.625***	0.704***	0.603***
	(0.267)	(0.238)	(0.261)	(0.231)
R^2	0.252	0.239	0.249	0.237
Dep. Variable: Log Difference in Stock Price				
F_t	3.564***	3.490***	3.622***	
	(0.549)	(0.515)	(0.573)	
$nfa^{safe}_{i,t-1} * F_t$		-0.007	-0.005	
		(0.006)	(0.005)	
$nfa^{risky}_{i,t-1} * F_t$			-0.038	
			(0.029)	
R^2	0.020	0.023	0.030	

Notes: A linear trend is removed from each interest rate series. nfa^{safe} and nfa^{risky} are a country's net foreign asset positions in safe and risky assets.

A.2 Factor Loadings

The factor loadings for individual countries are reported in Table 8 for annual data and Table 9 for quarterly data. The tables show the loadings on the first factor for risky and safe outflows and inflows ($\lambda^{out,risky}, \lambda^{out,safe}, \lambda^{in,risky}, \lambda^{in,safe}$). It also reports the net flow loadings $\lambda^{net,safe} = \lambda^{out,safe} - \lambda^{in,safe}$, $\lambda^{net,risky} = \lambda^{out,risky} - \lambda^{in,risky}$, $\lambda^{net} = \lambda^{net,safe} + \lambda^{net,risky}$.

Table 8: Debt and Equity Capital Flow Loadings on the First Factor (annual data)

	λ^{net}	$\lambda^{net,risky}$	$\lambda^{net,safe}$	$\lambda^{out,risky}$	$\lambda^{in,risky}$	$\lambda^{out,safe}$	$\lambda^{in,safe}$
CHE	2.29	1.78	0.51	2.80	1.01	11.95	11.45
DEU	-0.12	-0.18	0.07	0.23	0.42	3.40	3.33
DNK	-0.30	1.20	-1.50	1.52	0.32	3.76	5.25
ESP	-2.09	1.55	-3.64	1.90	0.34	2.01	5.65
FIN	1.30	-0.54	1.84	1.33	1.88	-0.94	-2.78
FRA	0.27	0.39	-0.12	1.30	0.91	4.79	4.91
GBR	-0.15	2.69	-2.84	3.96	1.27	15.64	18.48
ISL	-9.23	11.57	-20.80	20.58	9.01	44.44	65.24
ITA	-0.24	0.13	-0.36	0.70	0.58	1.96	2.32
NLD	-1.53	0.15	-1.68	12.99	12.84	9.27	10.95
NOR	0.27	-0.65	0.92	0.74	1.40	8.61	7.69
PRT	-1.38	0.60	-1.97	1.16	0.57	2.35	4.32
SWE	0.66	-0.04	0.69	1.48	1.51	3.26	2.57
SGP	2.42	-1.11	3.53	3.34	4.45	23.36	19.83
AUS	-0.79	0.68	-1.46	0.33	-0.35	-0.28	1.18
JPN	0.12	-0.80	0.93	-0.37	0.43	1.08	0.15
KOR	-0.45	0.79	-1.24	0.30	-0.49	0.17	1.41
USA	-0.91	0.08	-0.99	0.36	0.28	1.68	2.68
CAN	1.36	-0.01	1.37	0.50	0.51	0.54	-0.83
ISR	0.30	-0.47	0.77	0.87	1.34	0.83	0.05

Table 9: Debt and Equity Capital Flow Loadings on the First Factor (quarterly data)

	λ^{net}	$\lambda^{net,risky}$	$\lambda^{net,safe}$	$\lambda^{out,risky}$	$\lambda^{in,risky}$	$\lambda^{out,safe}$	$\lambda^{in,safe}$
DEU	-0.08	0.08	-0.16	0.24	0.15	4.73	4.90
DNK	-0.84	1.27	-2.12	1.46	0.18	4.07	6.19
ESP	-2.25	1.33	-3.58	1.66	0.33	2.18	5.76
FIN	1.41	0.28	1.13	1.79	1.51	2.80	1.67
FRA	0.44	0.47	-0.02	1.04	0.57	8.80	8.83
GBR	-0.11	1.50	-1.62	2.35	0.84	21.78	23.40
ISL	-7.28	11.56	-18.84	20.23	8.67	40.21	59.05
ITA	-0.27	0.09	-0.36	0.68	0.60	2.30	2.65
NLD	-0.84	0.54	-1.38	15.27	14.74	14.37	15.74
NOR	0.82	0.14	0.68	0.86	0.72	7.80	7.12
PRT	-1.39	0.74	-2.13	0.93	0.19	2.93	5.06
SWE	0.60	0.14	0.45	1.96	1.82	5.32	4.86
SGP	-0.16	-0.77	0.61	1.33	2.10	40.68	40.08
AUS	-0.90	0.56	-1.46	0.18	-0.38	0.55	2.00
JPN	0.28	-0.50	0.78	-0.39	0.11	2.97	2.19
KOR	-0.59	0.84	-1.43	0.35	-0.50	0.58	2.01
USA	-0.88	0.06	-0.94	0.52	0.46	2.18	3.12
CAN	1.15	-0.15	1.30	0.32	0.47	0.17	-1.13
ISR	0.73	-0.32	1.05	0.77	1.09	1.19	0.14

A.3 Classification of Debt as Safe and Equity as Risky

Assets of course exist on a continuum of riskiness, from a 3M T-bill at one end of the spectrum to equity of start-up companies on the other. For the empirical analysis we have to decide where to draw the line dividing safe and risky. Because of the way the inflows and outflows data is arranged into the broad categories of FDI, portfolio equity, portfolio debt, and “other” (where other includes currency and deposits, bank lending, trade credit, and other account receivable/payable), it makes it natural to draw the line between risky and safe as the line between debt and equity.

The fact that we classify equity (FDI and portfolio equity) as risky is not controversial. However debt, which we are calling safe, includes not only government bonds but also high-yield corporate debt. So with this image of a continuum from T-bills on the left and risky equity on the right, should the line dividing safe and risky in the data be moved a little to the left? What share of the variance of international debt flows is due to safe debt flows (government bonds, central banks, and high grade corporate) and what share is due to risky debt flows (high yield corporate)?

The problem is that we can't observe this with the available data. The only further disaggregation that we have is the domestic counterparty to an inflow or outflow transaction, as in Avdjiev, Hardy, Kalemli-Ozcan, and Severn (2017). We can observe the party that is selling the asset in the case of a capital inflow or the party that is buying the asset in the case of a capital outflow. As far as determining the riskiness of the security being bought or sold, the outflow data is not much use to us (when we see that a US bank buys a foreign bond, we don't know if that is a safe German bund or risky high yield Italian corporate debt). But the inflows data can be useful. We can observe the sector that sold the asset, where the sectors are Central bank, Deposit Corporations, General Government, and Other. Other includes "other financial corporations", non-financial corporates, and households and non-profits. If we choose to draw the risky/safe line within the debt category, then one option is to group debt issued by the Central Bank, Deposit Corporations and the General Government sector into the safe category and debt of the Other sector into the risky category.

Since we can only do this for inflows, and not outflows, we can't get an idea of net flows. Moreover, the sector level data coverage is poor. But we can at least get some idea of relative magnitudes. If the "other" sector debt is classified as a risky asset, what share of the variance of total debt inflows does it represent? Of course the corporate debt from the "other" sector includes everything from AAA rated nonfinancial sector debt to high yield junk bonds, and the household debt in this category includes both prime and subprime mortgages, but this exercise will establish an upper bound on the share of the variance of debt flows that is due to this risky debt.

We observe Portfolio debt and Other inflows broken into these 4 sectors, government (g), central bank (c), deposit corporations (b), and other (o) for our 20 countries over the 1996-2020 period. Write total portfolio debt or other inflows as

Pd_t and Ot_t, where $Pd_t = Pd_t^g + Pd_t^c + Pd_t^b + Pd_t^o$ and $Ot_t = Ot_t^g + Ot_t^c + Ot_t^b + Ot_t^o$. Then the share of the variance of Pd_t or Ot_t explained by each of these subcomponents is $S_{Pd}^i = \frac{cov\left(Pd_t, Pd_t^i\right)}{var(Pd_t)}$ and $S_{Ot}^i = \frac{cov\left(Ot_t, Ot_t^i\right)}{var(Ot_t)}$ for $i = g, c, b, o$. These results are presented in Table 10. There are some holes in the data, but we do see that on average, inflows to the "Other" sector only account for about 12% of the variance of portfolio debt inflows and about 20% of the variance of Other Debt inflows. A less risky sector like Deposit Corporations accounts for a larger share of the variance of both portfolio debt and other debt inflows, while Government also accounts for a larger share of the variance of portfolio debt inflows.

Table 10: Sectoral Variance Decomposition of Debt Capital Inflows.

	S^o_{Pd}	S^g_{Pd}	S^c_{Pd}	S^b_{Pd}	S^o_{Ot}	S^g_{Ot}	S^c_{Ot}	S^b_{Ot}
CHE	0.18	0.82	0.00	0.00	0.09	0.00	-0.02	0.93
DEU	0.01	0.25	-0.14	0.88	0.06	0.04	0.18	0.72
DNK	0.02	0.20	0.00	0.78	0.20	-0.01	0.08	0.74
ESP	0.12	0.56	0.00	0.32	0.02	0.01	0.69	0.28
FIN	0.16	0.45	0.00	0.40	0.05	-0.01	0.00	0.96
FRA	0.23	0.55	0.00	0.21	0.05	-0.01	0.02	0.94
GBR	NA	NA	NA	NA	0.44	-0.01	0.00	0.57
ISL	0.02	0.03	0.00	0.95	0.16	0.00	0.01	0.82
ITA	0.10	0.72	0.00	0.18	0.20	0.02	0.78	-0.01
NLD	0.21	0.29	0.00	0.50	0.20	-0.01	0.05	0.76
NOR	0.01	0.16	0.00	0.82	0.07	0.64	0.14	0.14
PRT	0.11	0.49	0.00	0.40	0.05	0.14	0.04	0.77
SWE	0.11	0.38	0.00	0.52	0.12	0.00	0.21	0.68
SGP	NA	NA	NA	NA	NA	NA	NA	NA
AUS	0.19	0.13	0.00	0.68	0.13	0.00	0.27	0.61
JPN	0.06	0.89	0.00	0.04	0.48	0.05	0.05	0.42
KOR	0.21	0.30	0.25	0.24	0.06	0.04	0.19	0.71
USA	0.14	0.82	0.00	0.04	0.59	0.00	-0.05	0.46
CAN	NA	NA	NA	NA	0.39	0.00	0.00	0.61
ISR	0.10	0.30	0.60	0.00	0.45	0.05	0.00	0.51
Average	0.12	0.43	0.04	0.41	0.20	0.05	0.14	0.61
Median	0.11	0.38	0.00	0.40	0.13	0.00	0.05	0.68

B Capital Flow Decomposition

This Online Appendix derives expressions for the decomposition of capital flows into portfolio growth, portfolio rebalancing and portfolio reallocation components for the case where there is only cross-country heterogeneity. After that we take second-order derivatives of this portfolio decomposition, first for the case of risk-aversion heterogeneity across countries and then for the case of expected dividend heterogeneity. These results are used to provide intuition for Theorem 4 in the paper.

B.1 Capital Flow Decomposition

Recall that $z_{n,m}$ is the fraction that country n invests in the risky asset of country m in period 1. Define the overall portfolio share invested in risky assets as

$$z_n = \sum_{m=1}^{N+1} z_{n,m} \tag{B.1}$$

The same portfolio share during time 0 is denoted $z_{n,0}$.

Further denote

$$z_{n,F} = \sum_{m \neq n} z_{n,m} \tag{B.2}$$

$$z_{n,m|risky} = \frac{z_{n,m}}{z_n} \tag{B.3}$$

These are respectively the risky asset portfolio share in all foreign countries and the share of the risky asset portfolio invested in the country m risky asset.

It is also useful to define the change in the overall risky asset price index from the perspective of investors in country n:

$$\frac{Q^n - a}{a} = \sum_{m=1}^{N+1} z_{n,m,0|risky} \frac{Q_m - a}{a} \tag{B.4}$$

B.2 Decomposition of Risky Asset Capital Flows

We have

$$OF_n^{risky} = \sum_{m \neq n} z_{n,m} A_n - \sum_{m \neq n} Q_m z_{n,m,0} \tag{B.5}$$

where $A_n = \beta W_n$ is total financial wealth of investors in country n in period 1.

In period 1 we have

$$A_n = A + S_n + A \sum_{m=1}^{N+1} z_{n,m,0} \frac{Q_m - a}{a} \tag{B.6}$$

where time zero financial wealth is $A = a$ for all countries by Assumption 1, and S_n is country n saving by investors.

We can also write (B.6) as

$$A_n = A + S_n + Az_{n,0}\frac{Q^n - a}{a} \tag{B.7}$$

If we substitute this into (B.5), we get

$$\begin{aligned} OF_n^{risky} &= z_{n,F}S_n + z_{n,F}A + Az_{n,F}z_{n,0}\frac{Q^n - a}{a} \\ &\quad -A\sum_{m \neq n}\frac{Q_m - a}{a}z_{n,m,0} - Az_{n,F,0} \end{aligned}$$

This is equal to

$$\begin{aligned} OF_n^{risky} &= z_{n,F}S_n + \Delta z_{n,F}A + Az_{n,F}z_{n,0}\frac{Q^n - a}{a} \\ &\quad -Az_{n,0}\sum_{m=1}^{N+1} z_{n,m,0|risky}\frac{Q_m - a}{a} + A\frac{Q_n - a}{a}z_{n,n,0} \end{aligned}$$

where $\Delta z_{n,F} = z_{n,F} - z_{n,F,0}$. We can also write this as

$$\begin{aligned} OF_n^{risky} &= z_{n,F}S_n + \Delta z_{n,F}A + Az_{n,F}z_{n,0}\frac{Q^n - a}{a} \\ &\quad -Az_{n,0}\frac{Q^n - a}{a} + A\frac{Q_n - a}{a}z_{n,n,0} \end{aligned}$$

We have

$$z_{n,F} = z_n z_{n,F|risky} \tag{B.8}$$

so that

$$\Delta z_{n,F} = z_{n,F|risky}\Delta z_n + z_{n,0}\Delta z_{n,F|risky} \tag{B.9}$$

Then

$$\begin{aligned} OF_n^{risky} &= z_{n,F}S_n \\ &\quad +(z_{n,F}z_{n,0} - z_{n,F,0})A\frac{Q^n - a}{a} + Az_{n,n,0}\left(\frac{Q_n - a}{a} - \frac{Q^n - a}{a}\right) \\ &\quad +z_{n,F|risky}A\Delta z_n + z_{n,0}A\Delta z_{n,F|risky} \end{aligned} \tag{B.10}$$

The first term is a portfolio growth term. New saving is invested with a fraction $z_{n,F}$ in foreign risky assets. The second line captures portfolio rebalancing, both between safe and risky assets and among risky assets. It is associated with changes in asset prices. The last line captures portfolio reallocation. It is associated with changes in portfolio shares, both the share allocated to risky assets and the share of the risky assets allocated to foreign risky assets.

Next consider capital inflows of risky assets, which for country n are

$$IF_n^{risky} = \sum_{m \neq n} z_{m,n} A_m - Q_n \sum_{m \neq n} z_{m,n,0} \tag{B.11}$$

Again use

$$A_m = A + S_m + A z_{m,0} \frac{Q^m - a}{a} \tag{B.12}$$

If we substitute this into (B.11), we get

$$\begin{aligned} IF_n^{risky} &= \sum_{m \neq n} z_{m,n} S_m + A \sum_{m \neq n} \Delta z_{m,n} + \\ &\quad A \sum_{m \neq n} z_{m,n} z_{m,0} \frac{Q^m - a}{a} - A \frac{Q_n - a}{a} \sum_{m \neq n} z_{m,n,0} \end{aligned}$$

We can write

$$z_{m,n} = z_m z_{m,n|risky} \tag{B.13}$$

Therefore

$$\Delta z_{m,n} = z_{m,n|risky} \Delta z_m + z_{m,0} \Delta z_{m,n|risky} \tag{B.14}$$

Then

$$\begin{aligned} IF_n^{risky} &= \sum_{m \neq n} z_{m,n} S_m \\ &\quad + A \sum_{m \neq n} \left(z_{m,n} z_{m,0} - z_{m,n,0} \right) \frac{Q^m - a}{a} - A \sum_{m \neq n} z_{m,n,0} \left(\frac{Q_n - a}{a} - \frac{Q^m - a}{a} \right) \\ &\quad + A \sum_{m \neq n} z_{m,n|risky} \Delta z_m + A \sum_{m \neq n} z_{m,0} \Delta z_{m,n|risky} \end{aligned} \tag{B.15}$$

The first line again captures portfolio growth, which depends on saving of foreign countries times the fraction that they allocate to country n. The second line captures portfolio rebalancing and is associated with risky asset price changes. The last line captures portfolio reallocation. It involves changes in the share that foreign countries allocate to risky assets and the share of the risky assets that they allocate to country n.

Finally consider net flows of safe assets, which are

$$NF_n^{safe} = S_n^h + (1 - z_n) A_n - (1 - z_{n,0}) A \tag{B.16}$$

Substitute

$$A_n = A + S_n + Az_{n,0}\frac{Q^n - a}{a} \tag{B.17}$$

This gives

$$NF_n^{safe} = S_n^h + (1 - z_n)S_n + A(1 - z_n)z_{n,0}\frac{Q^n - a}{a} - A\Delta z_n \tag{B.18}$$

The first two terms capture portfolio growth, by respectively households and investors. The third term captures portfolio rebalancing between safe and risky assets, while the last term reflects portfolio reallocation between safe and risky assets.

B.3 Risk-Aversion Heterogeneity

We start with risk-aversion heterogeneity across countries, where $\epsilon_n^G = g_n\epsilon$, with g_n averaging to zero across countries. Before taking second-order derivatives of the capital flow decomposition with respect to ϵ and G, it is useful to first establish some results with regards to first-order derivatives. We refer back to results that are already derived in Appendix D of the paper.

B.3.1 First-Order Derivatives with Respect to ϵ

Since the risky asset prices in the initial equilibrium are all equal to a, independent of ϵ, the first-order derivatives of all risky asset prices with respect to ϵ are zero. The same is the case for the price index Q^n. Therefore the same is also the case for investor wealth and saving. The latter is $S_n = 1 - W_n/(1 + a)$.

The first-order derivatives of portfolio shares with respect to ϵ are generally not zero. These portfolio shares correspond to the ones in the initial equilibrium in (24)-(25) in the paper, which are both the time zero and time 1 portfolio shares. Using this, the derivative of $z_{n,m}$ and $z_{n,m,0}$ ($m \neq n$) with respect to ϵ is $\kappa g_n/(1 + N\kappa)$. The derivative of $z_{n,F}$ and $z_{n,F,0}$ with respect to ϵ is $N\kappa g_n/(1 + N\kappa)$. The derivative of z_n and $z_{n,0}$ with respect to ϵ is g_n, and the derivative of $z_{n,n,0}$ with respect to ϵ is $g_n/(1 + N\kappa)$. Finally, the derivatives of $z_{n,m|risky}$ and $z_{n,F|risky}$ with respect to ϵ are zero.

B.3.2 First-Order Derivatives with Respect to G

We have already shown that the first-order derivatives of all risky asset prices with respect to G are equal and positive. This is also equal to the first-order derivative of Q^n and W_n with respect to G. Using that $S_n = 1 - W_n/(1+a)$, the first-order derivative of investor saving with respect to G is $-(\partial Q/\partial G)/(1+a)$.

In order to determine the first-order derivatives of portfolio shares with respect to G, we need to use the portfolio share expressions (17)-(18) in the paper:

$$z_{n,n} = \Gamma\left(1+\epsilon g_n\right) G \frac{1}{\sigma^2} Q_n \left(1 + a + \frac{\sigma^2}{a\Gamma(1+N\kappa)} - RQ_n\right) \tag{B.19}$$

$$z_{n,m} = \Gamma\kappa\left(1+\epsilon g_n\right) G \frac{1}{\sigma^2} Q_m \left(1 + a + \frac{\sigma^2}{a\Gamma(1+N\kappa)} - RQ_m\right) \tag{B.20}$$

We then have

$$\begin{aligned}\frac{\partial z_{n,n}}{\partial G} &= \Gamma\left(1+\epsilon g_n\right) \frac{1}{\sigma^2} Q_n \left(1 + a + \frac{\sigma^2}{a\Gamma(1+N\kappa)} - RQ_n\right) \\ &+\Gamma\left(1+\epsilon g_n\right) G \frac{1}{\sigma^2} \frac{\partial Q_n}{\partial G} \left(1 + a + \frac{\sigma^2}{a\Gamma(1+N\kappa)} - RQ_n\right) \\ &-\Gamma\left(1+\epsilon g_n\right) G \frac{1}{\sigma^2} Q_n \left(R\frac{\partial Q_n}{\partial G} + Q_n \frac{\partial R}{\partial G}\right)\end{aligned} \tag{B.21}$$

The derivative $\partial R/\partial G$ from (D.6) in Appendix D of the paper, evaluated at $\epsilon = 0$ and $G = 1$, is

$$\frac{\partial R}{\partial G} = \frac{1}{(1+a)\lambda} \frac{\partial Q}{\partial G} \tag{B.22}$$

Then evaluated at $\epsilon = 0$ and $G = 1$, we then have

$$\frac{\partial z_{n,n}}{\partial G} = \frac{1}{1+N\kappa} + \frac{\partial Q}{\partial G} \frac{1}{a(1+N\kappa)} - \Gamma \frac{1}{\sigma^2} a \left(\frac{1+a}{a} + \frac{a}{(1+a)\lambda}\right) \frac{\partial Q}{\partial G} \tag{B.23}$$

Substituting $\partial Q/\partial G$ from equation (28) in the paper, we have

$$\frac{\partial z_{n,n}}{\partial G} = \frac{\sigma^2}{a\Gamma(1+N\kappa)^2} \frac{1}{(1+a)^2 - \frac{\sigma^2}{\Gamma(1+N\kappa)} + \frac{a^2}{\lambda}} \tag{B.24}$$

The denominator of the last ratio is positive as a result of Assumption 2, making this partial derivative positive. The same is the case for the derivative of $z_{n,m}$ with respect to G, which is κ times the derivative of $z_{n,n}$ with respect to G. Then the same is the case also for the derivatives of z_n and $z_{n,F}$ with respect to G. The derivatives of all time 0 portfolio shares with respect to G are naturally zero. The derivatives of $z_{n,m|risky}$ and $z_{n,F|risky}$ with respect to G are zero also as all changes in risky asset portfolio shares are proportional to each other.

B.3.3 Second-Order Derivatives Capital Flow Decomposition

Using the results above with regards to first-order derivatives, taking the second-order derivative of (B.10) with respect to ϵ and G, evaluated at $\epsilon = 0$ and $G = 1$ gives

$$\begin{aligned} \frac{\partial^2 OF_n^{risky}}{\partial G \partial \epsilon} &= \frac{N\kappa}{1+N\kappa}\frac{\partial^2 S_n}{\partial G \partial \epsilon} + \frac{N\kappa}{1+N\kappa} g_n \frac{\partial S_n}{\partial G} \\ &+ \frac{N\kappa}{1+N\kappa} g_n \frac{\partial Q}{\partial G} + \frac{1}{1+N\kappa}\frac{\partial^2 (Q_n - Q^n)}{\partial G \partial \epsilon} \\ &+ \frac{N\kappa}{1+N\kappa} a \frac{\partial^2 z_n}{\partial G \partial \epsilon} + a \frac{\partial^2 z_{n,F|risky}}{\partial G \partial \epsilon} \end{aligned} \tag{B.25}$$

Taking the second-order derivative of (B.15) with respect to ϵ and G, evaluated at $\epsilon = 0$ and $G = 1$ gives

$$\begin{aligned} \frac{\partial^2 IF_n^{risky}}{\partial G \partial \epsilon} &= \frac{\kappa}{1+N\kappa}\sum_{m\neq n}\frac{\partial^2 S_m}{\partial G \partial \epsilon} + \frac{\kappa}{1+N\kappa}\sum_{m\neq n} g_m \frac{\partial S_m}{\partial G} \\ &- \frac{\kappa}{1+N\kappa} g_n \frac{\partial Q}{\partial G} - \frac{\kappa}{1+N\kappa}\sum_{m\neq n}\frac{\partial^2 (Q_n - Q^m)}{\partial G \partial \epsilon} \\ &+ \frac{\kappa}{1+N\kappa} a \sum_{m\neq n}\frac{\partial^2 z_m}{\partial G \partial \epsilon} + a\sum_{m\neq n}\frac{\partial^2 z_{m,n|risky}}{\partial G \partial \epsilon} \end{aligned} \tag{B.26}$$

The first term of the second line uses that $\sum_{m\neq n} g_m = -g_n$.

Finally, taking the second-order derivative of (B.18) with respect to ϵ and G, evaluated at $\epsilon = 0$ and $G = 1$ gives

$$\frac{\partial^2 NF_n^{safe}}{\partial G \partial \epsilon} = \frac{\partial^2 S^h}{\partial G \partial \epsilon} - g_n \frac{\partial S_n}{\partial G} - g_n \frac{\partial Q}{\partial G} - a\frac{\partial^2 z_n}{\partial G \partial \epsilon} \tag{B.27}$$

In (B.25) and (B.26) the first line relates to portfolio growth, the second line to portfolio rebalancing and the third line to portfolio reallocation. In (B.27) the first two terms relate to portfolio growth, the third to portfolio rebalancing and the last to portfolio reallocation. We now compute these various derivatives in order to determine their sign and provide intuition about net capital flows of risky and safe assets in response to a global risk-aversion shock. We analyze terms related to portfolio growth, portfolio rebalancing and portfolio reallocation in that order.

B.3.4 Portfolio Growth

To understand their sign of the portfolio growth terms, start from the fact that

$$\sum_{n=1}^{N+1} \frac{\partial^2 Q_n}{\partial G \partial \epsilon} = 0$$

This is shown in Appendix D of the paper. Taking the derivative of (D.6) in Appendix D of the paper with respect to ϵ, it then follows that

$$\frac{\partial^2 R}{\partial G \partial \epsilon} = 0 \tag{B.28}$$

Since household saving only depends on the interest rate, it is then also the case that

$$\frac{\partial^2 S^h}{\partial G \partial \epsilon} = 0 \tag{B.29}$$

Since we established in Appendix D that

$$\frac{\partial^2 CA_n}{\partial G \partial \epsilon} \tag{B.30}$$

is a negative linear function of g_n, and the current account is the sum of household and investor saving, it is then also the case that

$$\frac{\partial^2 S_n}{\partial G \partial \epsilon} \tag{B.31}$$

is a negative linear function of g_n. It then also follows that

$$\sum_{m \neq n} \frac{\partial^2 S_m}{\partial G \partial \epsilon} \tag{B.32}$$

is a negative linear function of $\sum_{m=n} g_m = -g_n$ and therefore a positive linear function of g_n.

Using that $S_n = 1 - W_n/(1+a)$ and $\partial W_n / \partial G = \partial Q / \partial G$, we also have that

$$\frac{\partial S_n}{\partial G} = -\frac{1}{1+a} \frac{\partial Q}{\partial G} \tag{B.33}$$

which is negative as $\partial Q / \partial G$ is positive. We also have

$$\sum_{m \neq n} g_m \frac{\partial S_m}{\partial G} = -\frac{1}{1+a} \frac{\partial Q}{\partial G} \sum_{m \neq n} g_m = g_n \frac{1}{1+a} \frac{\partial Q}{\partial G} \tag{B.34}$$

which is a positive linear function of g_n.

Altogether these results imply that the portfolio growth term in the expression (B.25) for capital outflow of risky assets is a negative linear function of g_n, the portfolio growth term in the expression (B.26) for capital inflows of risky assets is a positive linear function of g_n, while the portfolio growth term in the expression (B.27) for net capital flows of safe assets is a positive linear function of g_n. These results imply that, as a result of portfolio growth, countries that are less risk-averse than average will experience positive net capital outflows of risky assets and negative net capital outflows of safe assets in response to the global risk-aversion shock. The opposite is the case for countries that are more risk-averse than average. These effects are stronger the further a country deviates from the average.

Intuitively, two effects are at work. First, all countries experience a rise in saving by investors due to the drop in risky asset prices. This will lead to an increase in outflows and inflows of risky assets due to portfolio growth. But outflows of risky assets rise more than inflows of risky assets when a country is less risk-averse than average and therefore holds more risky assets. Such a country then experiences net capital outflows of risky assets. Since investors in such a country are leveraged, they invests a negative fraction in safe assets. Portfolio growth will then lead to even more negative holdings of safe assets and therefore negative net capital ouflows of safe assets. Second, countries that are less risk-averse experience a larger drop in their own risky asset price, leading them to reduce consumption and raise saving more than other countries. This leads to even larger net capital outflows of risky assets.

B.3.5 Portfolio Rebalancing

The portfolio rebalancing terms in the expression (B.25) for capital outflows of risky assets are equal to

$$\frac{N\kappa}{1+N\kappa}g_n\frac{\partial Q}{\partial G}+\frac{1}{1+N\kappa}\frac{\partial^2(Q_n-Q^n)}{\partial G\partial\epsilon} \tag{B.35}$$

Using (B.4), we can write

$$Q_n-Q^n=\frac{(N+1)\kappa}{1+N\kappa}Q_n-\frac{\kappa}{1+N\kappa}\sum_{m=1}^{N+1}Q_m \tag{B.36}$$

Since the second derivative of the last term with respect to ϵ and G is zero, we have

$$\frac{\partial^2(Q_n - Q^n)}{\partial G \partial \epsilon} = \frac{(N+1)\kappa}{1+N\kappa}\frac{\partial^2 Q_n}{\partial G \partial \epsilon} \tag{B.37}$$

We know from Appendix D of the paper that this is a positive linear function of g_n. Since $\partial Q/\partial G$ is positive, it then follows that (B.35) is a positive linear function of g_n.

The portfolio rebalancing terms in the expression (B.26) for capital inflows of risky assets are equal to

$$-\frac{\kappa}{1+N\kappa} g_n \frac{\partial Q}{\partial G} - \frac{\kappa}{1+N\kappa}\sum_{m \neq n}\frac{\partial^2(Q_n - Q^m)}{\partial G \partial \epsilon} \tag{B.38}$$

Using (B.4),

$$Q_n - Q^m = Q_n - \frac{1-\kappa}{1+N\kappa}Q_m - \frac{\kappa}{1+N\kappa}\sum_{k=1}^{N+1} Q_k \tag{B.39}$$

Therefore

$$\sum_{m \neq n}(Q_n - Q^m) = \frac{(N+1)(1+N\kappa-\kappa)}{1+N\kappa}Q_n - \frac{1-\kappa+\kappa N}{1+N\kappa}\sum_{k=1}^{N+1} Q_k \tag{B.40}$$

The second-order derivative of the last term with respect to ϵ and G is again zero. Therefore

$$\sum_{m \neq n}\frac{\partial^2(Q_n - Q^m)}{\partial G \partial \epsilon} = \frac{(N+1)(1+N\kappa-\kappa)}{1+N\kappa}\frac{\partial^2 Q_n}{\partial G \partial \epsilon} \tag{B.41}$$

This is a positive linear function of g_n. It then follows that (B.38) is a negative linear function of g_n. The portfolio rebalancing terms associated with net capital outflows of risky assets then depend positively on g_n. The portfolio rebalancing term in the expression (B.27) for net capital outflows of safe assets clearly depends negatively on g_n.

From these results it follows that a country with lower than average risk-aversion experiences negative net capital outflows of risky assets and positive net capital outflows of safe assets as a result of portfolio rebalancing. The intuition for this is straightforward. A country with lower than average risk-aversion will be leveraged. The drop in risky asset prices makes the country even more leveraged. Rebalancing then takes the form of selling risky assets and reducing the debt in safe assets. This reduces outflows of risky assets and increases net outflows of safe assets. In addition, this country experiences larger inflows of risky assets. This is because other countries on average have a portfolio share less than 1 in risky assets. When risky asset prices drop, they rebalance by buying risky assets, including those of our leveraged country.

An additional portfolio rebalancing effect is the result of a drop in the risky asset price of the leveraged country relative to the average risky asset price. This leads to rebalancing among risky assets, towards the risky asset of the leveraged country and away from the other risky assets. This again leads to negative net capital outflows of risky assets, reinforcing the first rebalancing channel.

B.3.6 Portfolio Reallocation

The portfolio reallocation terms in the expression (B.25) for capital outflows of risky assets are equal to

$$\frac{N\kappa}{1+N\kappa} a \frac{\partial^2 z_n}{\partial G \partial \epsilon} + a \frac{\partial^2 z_{n,F|risky}}{\partial G \partial \epsilon} \tag{B.42}$$

The first term depends positively on the second-order derivative of z_n with respect to G and ϵ. We can write

$$\begin{aligned} z_n &= Q_n \Gamma(1-\kappa)\left(1+\epsilon g_n\right) G \frac{1}{\sigma^2}\left(1+a+\frac{\sigma^2}{a\Gamma(1+N\kappa)} - RQ_n\right) \\ &\quad + \sum_{m=1}^{N+1} Q_m \Gamma\kappa \left(1+\epsilon g_n\right) G \frac{1}{\sigma^2}\left(1+a+\frac{\sigma^2}{a\Gamma(1+N\kappa)} - RQ_m\right) \end{aligned} \tag{B.43}$$

Therefore

$$\begin{aligned} \frac{\partial z_n}{\partial \epsilon} &= \frac{g_n}{1+\epsilon g_n} z_n + \frac{\partial Q_n}{\partial \epsilon}\Gamma(1-\kappa)\left(1+\epsilon g_n\right) G \frac{1}{\sigma^2}\left(1+a+\frac{\sigma^2}{a\Gamma(1+N\kappa)} - RQ_n\right) \\ &\quad - Q_n \Gamma(1-\kappa)\left(1+\epsilon g_n\right) G \frac{1}{\sigma^2}\left(R\frac{\partial Q_n}{\partial \epsilon} + Q_n \frac{\partial R}{\partial \epsilon}\right) \\ &\quad + \sum_{m=1}^{N+1} \frac{\partial Q_m}{\partial \epsilon}\Gamma\kappa \left(1+\epsilon g_n\right) G \frac{1}{\sigma^2}\left(1+a+\frac{\sigma^2}{a\Gamma(1+N\kappa)} - RQ_m\right) \\ &\quad - \sum_{m=1}^{N+1} Q_m \Gamma\kappa \left(1+\epsilon g_n\right) G \frac{1}{\sigma^2}\left(R\frac{\partial Q_m}{\partial \epsilon} + Q_m \frac{\partial R}{\partial \epsilon}\right) \end{aligned} \tag{B.44}$$

Taking the derivative with respect to G, we have

$$\frac{\partial^2 z_n}{\partial G \partial \epsilon} = g_n \frac{\partial z_n}{\partial G} + (1-\kappa)\left(\frac{1}{a(1+N\kappa)} - \frac{\Gamma(1+a)}{\sigma^2}\right)\frac{\partial^2 Q_n}{\partial G \partial \epsilon} \tag{B.45}$$

The second term of (B.42) depends positively on the second-order derivative of $z_{n,F|risky}$ with respect to ϵ and G. We have

$$z_{n,F|risky} = 1 - z_{n,n|risky}$$

and

$$z_{n,n|risky} = \frac{Q_n(\bar{D}_n - RQ_n)}{(1-\kappa)Q_n(\bar{D}_n - RQ_n) + \kappa\sum_{m=1}^{N+1} Q_m(\bar{D}_m - RQ_m)} \tag{B.46}$$

Write

$$z_{n,n|risky} = \frac{F_n}{(1-\kappa)F_n + \kappa H}$$

where $F_n = Q_n(\bar{D}_n - RQ_n)$ and $H = \sum_{m=1}^{N+1} Q_m(\bar{D}_m - RQ_m)$. In the initial equilibrium

$$F_n = \frac{\sigma^2}{\Gamma(1+N\kappa)} \equiv F$$

and $H = (N+1)F$.

Using that the first-order derivatives of F_n and H with respect to ϵ are zero, and the second-order derivative of H with respect to ϵ and G is also zero, we find

$$\frac{\partial^2 z_{n,n|risky}}{\partial G \partial \epsilon} = \frac{\kappa(N+1)}{(1+N\kappa)^2 F}\frac{\partial^2 F_n}{\partial G \partial \epsilon} \tag{B.47}$$

Taking the derivative of F_n with respect to ϵ and G, and evaluating at $\epsilon = 0$ and $G = 1$, we have

$$\frac{\partial^2 F_n}{\partial G \partial \epsilon} = \frac{\partial^2 Q_n}{\partial \epsilon \partial G}\left(\frac{\sigma^2}{a\Gamma(1+N\kappa)} - (1+a)\right)$$

Using the slightly tighter condition $\sigma^2 < a(1+a)\Gamma(1+N\kappa)$ than Assumption 2, the term in brackets is negative.

Using all of these results, (B.42) becomes

$$-a\kappa\left(\frac{1}{a(1+N\kappa)} - \frac{\Gamma(1+a)}{\sigma^2}\right)\frac{\partial^2 Q_n}{\partial G \partial \epsilon} + \frac{N\kappa}{1+N\kappa} a g_n \frac{\partial z_n}{\partial G} \tag{B.48}$$

Since the term in brackets is negative, the second-order derivative of Q_n is a positive linear function of g_n, and $\partial z_n / \partial G > 0$, this is a positive linear function of g_n.

The portfolio reallocation terms in the expression (B.26) for capital inflows of risky assets are

$$\frac{\kappa}{1+N\kappa} a \sum_{m \neq n} \frac{\partial^2 z_m}{\partial G \partial \epsilon} + a \sum_{m \neq n} \frac{\partial^2 z_{m,n|risky}}{\partial G \partial \epsilon} \tag{B.49}$$

We have seen that the second-order derivative of z_n is a linear function of g_n. The second-order derivative of $\sum_{m\neq n} z_m$ is then the same linear function of $\sum_{m\neq n} g_m = -g_n$. Therefore the second-order derivative of $\sum_{m\neq n} z_m$ is equal to (B.45), with the sign reversed.

The second term of (B.49) depends on the second-order derivative of $\sum_{m\neq n} z_{m,n|risky}$. We have

$$\sum_{m\neq n} z_{m,n|risky} = \sum_{m\neq n} \frac{\kappa F_n}{(1-\kappa)F_m + \kappa H}$$

Again use that the first-order derivatives of F_n and H with respect to ϵ are zero, and the second-order derivative of H with respect to ϵ and G is also zero. Then

$$\sum_{m\neq n} \frac{\partial^2 z_{m,n|risky}}{\partial G \partial \epsilon} = \frac{\kappa N}{(1+N\kappa)F} \frac{\partial^2 F_n}{\partial G \partial \epsilon} - \frac{\kappa(1-\kappa)}{(1+N\kappa)^2 F} \sum_{m\neq n} \frac{\partial^2 F_m}{\partial G \partial \epsilon} \tag{B.50}$$

Using that $\sum_{m\neq n} F_m = H - F_n$ and that the second-order derivative of H is zero, this becomes

$$\sum_{m\neq n} \frac{\partial^2 z_{m,n|risky}}{\partial G \partial \epsilon} = \frac{\kappa(N+1)(1+(N-1)\kappa)}{(1+N\kappa)^2 F} \frac{\partial^2 F_n}{\partial G \partial \epsilon} \tag{B.51}$$

Using all of these results, (B.49) becomes

$$a\kappa N \left(\frac{1}{a(1+N\kappa)} - \frac{\Gamma(1+a)}{\sigma^2} \right) \frac{\partial^2 Q_n}{\partial G \partial \epsilon} - \frac{\kappa}{1+N\kappa} a g_n \frac{\partial z_n}{\partial G} \tag{B.52}$$

This is a negative linear function of g_n. Net capital outflows of risky assets are then a positive linear function of g_n.

The portfolio reallocation term of net outflows of safe assets is

$$-a \frac{\partial^2 z_n}{\partial G \partial \epsilon} \tag{B.53}$$

Using (B.45), this is

$$-a g_n \frac{\partial z_n}{\partial G} - a(1-\kappa) \left(\frac{1}{a(1+N\kappa)} - \frac{\Gamma(1+a)}{\sigma^2} \right) \frac{\partial^2 Q_n}{\partial G \partial \epsilon} \tag{B.54}$$

The first term is negative linear function of g_n, while the second term is a positive linear function of g_n, leading to an ambiguity.

Overall we can conclude that a country that is less risk-averse than average will experience negative net capital outflows of risky assets due to portfolio reallocation, while the effect on net capital flows of safe assets is ambiguous. The intuition is as follows. The rise in global risk aversion leads to a shift by investors away from risky assets and towards safe assets. In equilibrium this indeed has to be the case as household will save less when the interest rate drops, so that investors must hold more safe assets in equilibrium. This shift away from risky assets towards safe assets is larger for countries that are less risk averse and therefore hold more risky assets to start with. These countries therefore experience a larger drop in capital outflows than inflows of risky assets, leading to negative net ouflows of risky assets. In addition there is reallocation towards the risky asset of a country that is more leveraged than average. The reason is that the risky asset price drops more than average, raising its expected return relative to other risky assets. This reallocation among risky assets further reduces net capital outflows of risky assets for our leveraged country.

Finally consider net capital flows of safe assets. There are two opposing forces at work. First, as discussed, the low risk-averse country that is leveraged experiences a larger shift than average from risky to safe assets as a result of the rise in risk-aversion. This by itself raises its net outflows of safe assets due to portfolio reallocation. But a factor that operates in the other direction is that the relative price of the risky asset of this country drops more than the average price of risky assets. As a result of home bias, this risky asset matters more than other risky assets to the investors in this country. Since it leads to a higher expected return on the domestic risky asset, it generates a reallocation by domestic investors away from all other assets (including safe assets) towards the domestic risky asset. This by itself implies negative net outflows of safe assets.

B.4 Expected Dividend Heterogeneity

We now consider expected dividend heterogeneity, where

$$\bar{D}_n = 1 + a + \frac{\sigma^2}{a\Gamma(1+N\kappa)}\left(1+\epsilon_{D,n}\right) \tag{B.55}$$

$$\epsilon_{D,n} = d_n \epsilon \tag{B.56}$$

where $\sum_{n=1}^{N+1} d_n = 0$. It remains the case that the first-order derivatives of Q_n, Q^n, W_n and investor saving S_n with respect to ϵ are all zero as asset prices are equal to a in the initial equilibrium, irrespective of ϵ. All first-order derivatives with respect to G remain the same as well. What is different are the first-order derivatives of portfolio shares with respect to ϵ, to which we turn next.

B.4.1 Derivatives Portfolio Shares with Respect to ϵ

We have

$$\frac{\partial z_{n,n}}{\partial \epsilon} = \frac{d_n}{1+N\kappa}$$

$$\frac{\partial z_{n,m}}{\partial \epsilon} = \frac{\kappa d_m}{1+N\kappa}$$

$$\frac{\partial z_{n,F}}{\partial \epsilon} = -\frac{\kappa d_n}{1+N\kappa}$$

$$\frac{\partial z_n}{\partial \epsilon} = \frac{(1-\kappa) d_n}{1+N\kappa}$$

The same applies to period 0 portfolio shares.

We have

$$z_{n,m|risky} = \frac{\kappa Q_m F_m}{(1-\kappa)Q_n F_n + \kappa \sum_{k=1}^{N+1} Q_k F_k} \tag{B.57}$$

where $F_n = \bar{D}_n - RQ_n$. Define

$$E_n = \frac{\partial Q_n}{\partial \epsilon} F_n + Q_n \frac{\partial F_n}{\partial \epsilon}$$

Therefore

$$\frac{\partial z_{n,m|risky}}{\partial \epsilon} = \frac{\kappa E_m \left((1-\kappa)Q_n F_n + \kappa \sum_{k=1}^{N+1} Q_k F_k \right) - \kappa Q_m F_m \left((1-\kappa) E_n + \kappa \sum_{k=1}^{N+1} E_k \right)}{\left((1-\kappa)Q_n F_n + \kappa \sum_{k=1}^{N+1} Q_k F_k \right)^2}$$

We have

$$\frac{\partial F_n}{\partial \epsilon} = \frac{\sigma^2 d_n}{a\Gamma(1+N\kappa)} - R\frac{\partial Q_n}{\partial \epsilon} - Q_n \frac{\partial R}{\partial \epsilon} \tag{B.58}$$

Evaluated at $\epsilon = 0$ and $G = 1$, we have

$$F_n = \frac{\sigma^2}{a\Gamma(1+N\kappa)}$$

$$E_n = \frac{\sigma^2}{\Gamma(1+N\kappa)} d_n$$

and therefore

$$\frac{\partial z_{n,m|risky}}{\partial \epsilon} = \frac{\kappa(1+N\kappa)d_m - \kappa(1-\kappa)d_n}{(1+N\kappa)^2} \tag{B.59}$$

Using that $\sum_{m \neq n} d_m = -d_n$, we then also have

$$\frac{\partial z_{n,F|risky}}{\partial \epsilon} = -\frac{\kappa(1+N)d_n}{(1+N\kappa)^2} \tag{B.60}$$

These same derivatives also apply to time zero.

B.4.2 Second-Order Derivatives Capital Flow Decomposition

Using the results with regards to first-order derivatives, taking the second-order derivative of (B.10) with respect to ϵ and G, evaluated at $\epsilon = 0$ and $G = 1$ gives

$$\begin{aligned}\frac{\partial^2 OF_n^{risky}}{\partial G \partial \epsilon} = & \frac{N\kappa}{1+N\kappa}\frac{\partial^2 S_n}{\partial G \partial \epsilon} - \frac{\kappa}{1+N\kappa} d_n \frac{\partial S_n}{\partial G} \\ & + \frac{N\kappa(1-\kappa)}{(1+N\kappa)^2} d_n \frac{\partial Q}{\partial G} + \frac{1}{1+N\kappa}\frac{\partial^2 (Q_n - Q^n)}{\partial G \partial \epsilon} \\ & - \frac{a\kappa(1+N)}{(1+N\kappa)^2} d_n \frac{\partial z_n}{\partial G} + \frac{N\kappa}{1+N\kappa} a \frac{\partial^2 z_n}{\partial G \partial \epsilon} + a \frac{\partial^2 z_{n,F|risky}}{\partial G \partial \epsilon}\end{aligned} \tag{B.61}$$

Taking the second-order derivative of (B.15) with respect to ϵ and G, evaluated at $\epsilon = 0$ and $G = 1$ gives

$$\begin{aligned}\frac{\partial^2 IF_n^{risky}}{\partial G \partial \epsilon} = & \frac{\kappa}{1+N\kappa}\sum_{m \neq n}\frac{\partial^2 S_m}{\partial G \partial \epsilon} + \frac{\kappa N}{1+N\kappa} d_n \frac{\partial S_m}{\partial G} \\ & - \frac{\kappa(1-\kappa)}{(1+N\kappa)^2} d_n \frac{\partial Q}{\partial G} - \frac{\kappa}{1+N\kappa}\sum_{m \neq n}\frac{\partial^2 (Q_n - Q^m)}{\partial G \partial \epsilon} \\ & + \frac{a\kappa(1+N)(1+N\kappa-\kappa)}{(1+N\kappa)^2} d_n \frac{\partial z_m}{\partial G} + \frac{\kappa}{1+N\kappa} a \sum_{m \neq n}\frac{\partial^2 z_m}{\partial G \partial \epsilon} + a \sum_{m \neq n}\frac{\partial^2 z_{m,n|risky}}{\partial G \partial \epsilon}\end{aligned} \tag{B.62}$$

Finally, taking the second-order derivative of (B.18) with respect to ϵ and G, evaluated at $\epsilon = 0$ and $G = 1$ gives

$$\frac{\partial^2 NF_n^{safe}}{\partial G \partial \epsilon} = \frac{\partial^2 S^h}{\partial G \partial \epsilon} - \frac{1-\kappa}{1+N\kappa} d_n \frac{\partial S_n}{\partial G} - \frac{1-\kappa}{1+N\kappa} d_n \frac{\partial Q}{\partial G} - a \frac{\partial^2 z_n}{\partial G \partial \epsilon} \tag{B.63}$$

In (B.25) and (B.26) the first line relates to portfolio growth, the second line to portfolio rebalancing and the third line to portfolio reallocation. In (B.27) the first two terms relate to portfolio growth, the third to portfolio rebalancing and the last to portfolio reallocation. We now compute these various derivatives in order to determine their sign and provide intuition about net capital flows of risky and safe assets in response to a global risk-aversion shock. We analyze terms related to portfolio growth, portfolio rebalancing and portfolio reallocation in that order.

B.4.3 Portfolio Growth

From Appendix E we have

$$\sum_{n=1}^{N+1} \frac{\partial^2 Q_n}{\partial G \partial \epsilon} = 0$$

Taking the derivative of (E.3) in Appendix E of the paper with respect to ϵ, it then follows that

$$\frac{\partial^2 R}{\partial G \partial \epsilon} = 0 \tag{B.64}$$

Since household saving only depends on the interest rate, it is then also the case that

$$\frac{\partial^2 S^h}{\partial G \partial \epsilon} = 0 \tag{B.65}$$

Therefore the second derivative of S_n is equal to the second derivative of CA_n, which from (E.8) in Appendix E of the paper is

$$\frac{\partial^2 S_n}{\partial G \partial \epsilon} = -\frac{1}{1+a}\frac{1-\kappa}{1+N\kappa}\left(d_n \frac{\partial Q}{\partial G} + \frac{\partial^2 Q_n}{\partial G \partial \epsilon}\right) \tag{B.66}$$

This is a negative linear function of d_n. It then also follows that

$$\sum_{m \neq n} \frac{\partial^2 S_m}{\partial G \partial \epsilon} = \frac{1}{1+a}\frac{1-\kappa}{1+N\kappa}\left(d_n \frac{\partial Q}{\partial G} + \frac{\partial^2 Q_n}{\partial G \partial \epsilon}\right) \tag{B.67}$$

which is a positive linear function of d_n.

We also have

$$\frac{\partial S_n}{\partial G} = -\frac{1}{1+a}\frac{\partial Q}{\partial G} \tag{B.68}$$

which is negative as $\partial Q / \partial G$ is positive. Using the first line of (B.61) and (B.62), the portfolio growth component of net outflows of risky assets is then

$$\frac{(N+1)\kappa}{1+N\kappa}\left(\frac{\partial^2 S_n}{\partial G \partial \epsilon} - d_n \frac{\partial S_n}{\partial G}\right) \tag{B.69}$$

Substituting (B.66) and (B.68), this becomes

$$\frac{(N+1)\kappa}{1+N\kappa}\left(-\frac{1}{1+a}\frac{1-\kappa}{1+N\kappa}\frac{\partial^2 Q_n}{\partial G \partial \epsilon} + \frac{\kappa(N+1)}{1+N\kappa}\frac{1}{1+a} d_n \frac{\partial Q}{\partial G}\right) \tag{B.70}$$

The first term in brackets is a negative linear function of d_n, while the second term in brackets is a positive linear function of d_n. There is an ambiguity in the sign as the first term dominates when κ is close to zero, while the second term dominates when κ is close to 1. By contrast, there is no ambiguity for net outflows of safe assets, where the portfolio growth term in (B.63) is clearly a positive linear function of d_n, so that a country with a higher than average expected dividend will have negative net outflows of safe assets due to portfolio growth.

Intuitively, two effects are at work. First, all countries experience a rise in saving by investors due to the drop in risky asset prices. This will lead to an increase in outflows and inflows of risky assets due to portfolio growth. But saving rises more in a country that has higher expected dividends than average, leading to net outflows of risky assets. This is a result of the larger drop in wealth, which has two reasons. The general decline in risky asset prices reduces wealth more because the country is more leveraged. In addition, the drop in the domestic risky asset price is larger than the average risky asset price. Together with home bias, this also reduces wealth more.

The second effect at work applies to a given level of saving. The country with higher expected dividends in its own country invests a smaller fraction in foreign risky assets. This lowers portfolio growth for outflows of risky assets, while raising portfolio growth for inflows of risky assets (as foreign countries invest more in foreign assets). This operates in the opposite direction and leads to negative net outflows of risky assets. These two factors therefore operate in opposite directions and in general the sign of net capital flows of risky assets due to portfolio growth is ambiguous.

The country with the higher than average expected dividend invests a negative fraction in the safe asset, so that portfolio growth unambiguously leads to negative net outflows of safe assets.

B.4.4 Portfolio Rebalancing

Combining the second line of (B.61) and (B.62), together with equations (B.37) and (B.41), the component of net capital flows of risky assets associated with portfolio rebalancing is

$$\frac{(N+1)\kappa(1-\kappa)}{(1+N\kappa)^2} d_n \frac{\partial Q}{\partial G} + \frac{\kappa(N+1)(2+(N-1)\kappa)}{(1+N\kappa)^2} \frac{\partial^2 Q_n}{\partial G \partial \epsilon} \tag{B.71}$$

This is a positive linear function of d_n. The third term on the right hand side of (B.63) is the portfolio rebalancing term associated with net capital flows of safe assets. It therefore follows that as a result of portfolio rebalancing a country with a higher than average expected dividend has negative net outflows of risky assets and positive net outflows of safe assets.

The intuition for this is straightforward. A country with higher than average expected dividends will be leveraged. The drop in risky asset prices makes the country even more leveraged. Rebalancing then takes the form of selling risky assets and reducing the debt in safe assets. This reduces outflows of risky assets and increases net outflows of safe assets. In addition, this country experiences larger inflows of risky assets. This is because other countries on average have a portfolio share less than 1 in risky assets. When risky asset prices drop, they rebalance by buying risky assets, including those of our leveraged country.

An additional portfolio rebalancing effect is the result of a drop in the risky asset price of the leveraged country relative to the average risky asset price. This leads to rebalancing among risky assets, towards the risky asset of the leveraged country and away from the other risky assets. This again leads to negative net capital outflows of risky assets, reinforcing the first rebalancing channel.

B.4.5 Portfolio Reallocation

We have

$$z_n = \frac{\Gamma(1-\kappa)G}{\sigma^2} Q_n F_n + \frac{\Gamma\kappa G}{\sigma^2} \sum_{m=1}^{N+1} Q_m F_m \tag{B.72}$$

Therefore

$$\frac{\partial z_n}{\partial \epsilon} = \frac{\Gamma(1-\kappa)G}{\sigma^2} E_n + \frac{\Gamma\kappa G}{\sigma^2} \sum_{m=1}^{N+1} E_m \tag{B.73}$$

We have

$$\frac{\partial E_n}{\partial G} = \frac{\sigma^2}{a\Gamma(1+N\kappa)} \frac{\partial^2 Q_n}{\partial G \partial \epsilon} + \frac{\sigma^2}{a\Gamma(1+N\kappa)} d_n \frac{\partial Q}{\partial G} + a \frac{\partial^2 F_n}{\partial G \partial \epsilon} \tag{B.74}$$

Substituting

$$\frac{\partial^2 F_n}{\partial G \partial \epsilon} = -\frac{1+a}{a} \frac{\partial^2 Q_n}{\partial G \partial \epsilon}$$

we have

$$\frac{\partial E_n}{\partial G} = \left(\frac{\sigma^2}{a\Gamma(1+N\kappa)} - (1+a) \right) \frac{\partial^2 Q_n}{\partial G \partial \epsilon} + \frac{\sigma^2}{a\Gamma(1+N\kappa)} d_n \frac{\partial Q}{\partial G} \tag{B.75}$$

We then have

$$\frac{\partial^2 z_n}{\partial G \partial \epsilon} = \frac{\Gamma(1-\kappa)}{\sigma^2}\left(E_n + \frac{\partial E_n}{\partial G}\right) = \tag{B.76}$$

$$\Gamma(1-\kappa)\left(\frac{1}{\Gamma(1+N\kappa)}d_n + \frac{1}{a\Gamma(1+N\kappa)}d_n\frac{\partial Q}{\partial G} + \left(\frac{1}{a\Gamma(1+N\kappa)} - \frac{(1+a)}{\sigma^2}\right)\frac{\partial^2 Q_n}{\partial G \partial \epsilon}\right)$$

The sign of this expression, which determines the sign of net capital flows of safe assets due to portfolio reallocation, is not easy to determine. We can also write it as

$$\frac{\partial^2 z_n}{\partial G \partial \epsilon} = \frac{1}{a}\frac{1-\kappa}{1+N\kappa}\left(ad_n + d_n\frac{\partial Q}{\partial G} + \left(1 - \frac{(1+a)a\Gamma(1+N\kappa)}{\sigma^2}\right)\frac{\partial^2 Q_n}{\partial G \partial \epsilon}\right) \tag{B.77}$$

We have from equation (34) of the paper and equation (E.7) in Appendix E of the paper that

$$\frac{\partial Q}{\partial G} = \frac{\frac{(1+a)\sigma^2}{\Gamma(1+N\kappa)}}{(1+a)^2 - \frac{\sigma^2}{\Gamma(1+N\kappa)} + \frac{a^2}{\lambda}} \tag{B.78}$$

$$\frac{\partial^2 Q_n}{\partial G \partial \epsilon} = d_n\frac{\frac{(1-\kappa)^2}{1+N\kappa} + \frac{1}{\sigma^2}\Gamma(1+N\kappa)^2\left((1+a)^2 + \frac{a^2}{\lambda}\right)}{(1+a)^2\Gamma(1+N\kappa)^2\frac{1}{\sigma^2} - \frac{(1-\kappa)^2}{1+N\kappa}}\frac{\partial Q}{\partial G} \tag{B.79}$$

We need to determine the sign of X, which is equal to the term in brackets in (B.77) times the product of the denominators of the last two derivatives. We then have

$$\frac{X}{d_n} = \left(a(1+a)^2 - a\frac{\sigma^2}{\Gamma(1+N\kappa)} + \frac{a^3}{\lambda}\right)\left((1+a)^2\Gamma(1+N\kappa)^2\frac{1}{\sigma^2} - \frac{(1-\kappa)^2}{1+N\kappa}\right)$$
$$+(1+a)^3(1+N\kappa) - (1+a)\sigma^2\frac{(1-\kappa)^2}{\Gamma(1+N\kappa)^2}$$
$$+\left((1+a)\sigma^2 - (1+a)^2 a\Gamma(1+N\kappa)\right)\left(\frac{(1-\kappa)^2}{\Gamma(1+N\kappa)^2} + \frac{1}{\sigma^2}(1+N\kappa)\left((1+a)^2 + \frac{a^2}{\lambda}\right)\right)$$

Collecting terms, we have

$$\frac{X}{d_n} = \left(a(1+a)^2 - a\frac{\sigma^2}{\Gamma(1+N\kappa)} + \frac{a^3}{\lambda}\right)(1+a)^2\Gamma(1+N\kappa)^2\frac{1}{\sigma^2} + (1+a)^3(1+N\kappa)$$
$$+\left((1+a)\sigma^2 - (1+a)^2 a\Gamma(1+N\kappa)\right)\frac{1}{\sigma^2}(1+N\kappa)\left((1+a)^2 + \frac{a^2}{\lambda}\right)$$
$$-\frac{(1-\kappa)^2}{1+N\kappa}a(1+a)^2 + \frac{(1-\kappa)^2\sigma^2}{\Gamma(1+N\kappa)^2}a - \frac{(1-\kappa)^2}{1+N\kappa}\frac{a^3}{\lambda}$$
$$-(1+a)\sigma^2\frac{(1-\kappa)^2}{\Gamma(1+N\kappa)^2} + \frac{(1-\kappa)^2}{\Gamma(1+N\kappa)^2}(1+a)\sigma^2 - \frac{(1-\kappa)^2}{(1+N\kappa)}(1+a)^2 a$$

This is equal to

$$\frac{X}{d_n} = -a(1+a)^2(1+N\kappa) + (1+a)^3(1+N\kappa) + \left((1+a)^2 + \frac{a^2}{\lambda}\right)(1+a)(1+N\kappa)$$
$$+\frac{(1-\kappa)^2\sigma^2}{\Gamma(1+N\kappa)^2}a - \frac{(1-\kappa)^2}{1+N\kappa}\frac{a^3}{\lambda} - 2\frac{(1-\kappa)^2}{1+N\kappa}a(1+a)^2$$

which can be written as

$$\begin{aligned}\frac{X}{d_n} &= \frac{a^2}{\lambda}\left((1+N\kappa)(1+a) - \frac{a(1-\kappa)^2}{1+N\kappa}\right) \\ &+(1+a)^2\left((2+a)(1+N\kappa) - 2a\frac{(1-\kappa)^2}{1+N\kappa}\right) + \frac{a(1-\kappa)^2\sigma^2}{\Gamma(1+N\kappa)^2} \quad \text{(B.80)}\end{aligned}$$

It is clear that the sign is ambiguous. If we set λ very large and σ very small, we can ignore the first and last terms. The middle term, however, is positive when κ is close to 1 and negative when κ is close to 0 and $a > 2$.

We now turn to other second-order derivatives in the portfolio reallocation terms of capital outflows and inflows of risky assets. We have

$$\sum_{m \neq n} \frac{\partial^2 z_m}{\partial G \partial \epsilon} = -\frac{\partial^2 z_n}{\partial G \partial \epsilon} \quad \text{(B.81)}$$

Next consider the second-order derivative of $z_{n,F|risky}$. We can write

$$z_{n,F|risky} = \frac{z_{n,F}}{z_n}$$

Therefore

$$\frac{\partial z_{n,F|risky}}{\partial \epsilon} = \frac{1}{z_n}\frac{\partial z_{n,F}}{\partial \epsilon} - \frac{z_{n,F}}{z_n^2}\frac{\partial z_n}{\partial \epsilon}$$

Therefore, evaluated at $\epsilon = 0$, $G = 1$ we have

$$\begin{aligned}\frac{\partial^2 z_{n,F|risky}}{\partial G \partial \epsilon} &= \frac{\partial^2 z_{n,F}}{\partial G \partial \epsilon} - \frac{\partial z_n}{\partial G}\frac{\partial z_{n,F}}{\partial \epsilon} \quad \text{(B.82)} \\ &-\frac{\partial z_{n,F}}{\partial G}\frac{\partial z_n}{\partial \epsilon} + 2\frac{N\kappa}{1+N\kappa}\frac{\partial z_n}{\partial G}\frac{\partial z_n}{\partial \epsilon} - \frac{N\kappa}{1+N\kappa}\frac{\partial^2 z_n}{\partial G \partial \epsilon}\end{aligned}$$

Using the same approach as in the start of this subsection, it is easily seen that

$$\frac{\partial^2 z_{n,F}}{\partial G \partial \epsilon} = -\frac{\kappa}{1-\kappa}\frac{\partial^2 z_n}{\partial G \partial \epsilon}$$
$$\frac{\partial z_{n,F}}{\partial \epsilon} = -\frac{\kappa}{1+N\kappa}d_n$$
$$\frac{\partial z_n}{\partial \epsilon} = \frac{1-\kappa}{1+N\kappa}d_n$$

From earlier results

$$\frac{\partial z_{n,F}}{\partial G} = \frac{N\kappa}{1+N\kappa}\frac{\partial z_n}{\partial G}$$

Using these, we have

$$\frac{\partial^2 z_{n,F|risky}}{\partial G \partial \epsilon} = \frac{\partial z_n}{\partial G}\frac{(N+1)\kappa}{(1+N\kappa)^2}d_n - \frac{(1+N)\kappa}{(1-\kappa)(1+N\kappa)}\frac{\partial^2 z_n}{\partial G \partial \epsilon} \tag{B.83}$$

Using the same approach, we can also compute the second-order derivative of $\sum_{m\neq n} z_{m,n|risky}$. We can write

$$z_{m,n|risky} = \frac{z_{m,n}}{z_m}$$

Therefore

$$\frac{\partial z_{m,n|risky}}{\partial \epsilon} = \frac{1}{z_m}\frac{\partial z_{m,n}}{\partial \epsilon} - \frac{z_{m,n}}{z_m^2}\frac{\partial z_m}{\partial \epsilon}$$

Therefore, evaluated at $\epsilon = 0$, $G = 1$ we have

$$\begin{aligned}\frac{\partial^2 z_{m,n|risky}}{\partial G \partial \epsilon} &= \frac{\partial^2 z_{m,n}}{\partial G \partial \epsilon} - \frac{\partial z_m}{\partial G}\frac{\partial z_{m,n}}{\partial \epsilon} \\ &\quad - \frac{\partial z_{m,n}}{\partial G}\frac{\partial z_m}{\partial \epsilon} + 2\frac{\kappa}{1+N\kappa}\frac{\partial z_m}{\partial G}\frac{\partial z_m}{\partial \epsilon} - \frac{\kappa}{1+N\kappa}\frac{\partial^2 z_m}{\partial G \partial \epsilon}\end{aligned} \tag{B.84}$$

Using the same approach as at the start of this subsection, it is easily seen that

$$\begin{aligned}\frac{\partial^2 z_{m,n}}{\partial G \partial \epsilon} &= \frac{\kappa}{1-\kappa}\frac{\partial^2 z_n}{\partial G \partial \epsilon} \\ \frac{\partial z_{m,n}}{\partial \epsilon} &= \frac{\kappa}{1+N\kappa}d_n \\ \frac{\partial z_m}{\partial \epsilon} &= \frac{1-\kappa}{1+N\kappa}d_m\end{aligned}$$

From earlier results

$$\frac{\partial z_{m,n}}{\partial G} = \frac{\kappa}{1+N\kappa}\frac{\partial z_n}{\partial G}$$

Using these, we have

$$\begin{aligned}\frac{\partial^2 z_{m,n|risky}}{\partial G \partial \epsilon} &= \frac{\kappa}{1-\kappa}\frac{\partial^2 z_n}{\partial G \partial \epsilon} - \frac{\partial z_m}{\partial G}\frac{\kappa}{1+N\kappa}d_n \\ &\quad - \frac{\kappa(1-\kappa)}{(1+N\kappa)^2}d_m\frac{\partial z_n}{\partial G} + 2\frac{\kappa(1-\kappa)}{(1+N\kappa)^2}\frac{\partial z_m}{\partial G}d_m - \frac{\kappa}{1+N\kappa}\frac{\partial^2 z_m}{\partial G \partial \epsilon}\end{aligned} \tag{B.85}$$

Use that the first-order derivative of z_n and z_m with respect to G are the same. Also use that

$$\sum_{m\neq n}\frac{\partial^2 z_m}{\partial G \partial \epsilon} = -\frac{\partial^2 z_n}{\partial G \partial \epsilon}$$

Then we have

$$\sum_{m\neq n}\frac{\partial^2 z_{m,n|risky}}{\partial G\partial\epsilon} = \frac{\kappa(N+1)(1+\kappa(N-1))}{(1-\kappa)(1+N\kappa)}\frac{\partial^2 z_n}{\partial G\partial\epsilon} - \frac{\kappa(N+1)(1+\kappa(N-1))}{(1+N\kappa)^2}d_n\frac{\partial z_n}{\partial G} \quad \text{(B.86)}$$

Using the results so far, combining the last lines of (B.61) and (B.62), net capital outflows of risky assets due to portfolio reallocation is

$$-\frac{a\kappa(1+N)(2+N\kappa-\kappa)}{(1+N\kappa)^2}d_n\frac{\partial z_n}{\partial G}+\frac{(N+1)\kappa}{1+N\kappa}a\frac{\partial^2 z_n}{\partial G\partial\epsilon}$$
$$+a\frac{\partial z_n}{\partial G}\frac{\kappa(1+N)}{(1+N\kappa)^2}d_n - a\frac{(1+N)\kappa}{(1-\kappa)(1+N\kappa)}\frac{\partial^2 z_n}{\partial G\partial\epsilon}$$
$$-a\frac{\kappa(N+1)(1+\kappa(N-1))}{(1-\kappa)(1+N\kappa)}\frac{\partial^2 z_n}{\partial G\partial\epsilon}+a\frac{\kappa(N+1)(1+\kappa(N-1))}{(1+N\kappa)^2}d_n\frac{\partial z_n}{\partial G}$$

This is equal to

$$-\frac{(N+1)\kappa}{1-\kappa}a\frac{\partial^2 z_n}{\partial G\partial\epsilon} \quad \text{(B.87)}$$

Both net capital outflows of risky and safe assets due to portfolio reallocation are therefore proportional to

$$-\frac{\partial^2 z_n}{\partial G\partial\epsilon} \quad \text{(B.88)}$$

We have seen that the second-derivative of z_n with respect to G and ϵ is proportional to d_n, with a proportionality factor that is ambiguous. It is generally positive, but can get negative when κ is very close to zero (strong home bias) and σ^2 is very small. Therefore, net outflows of safe and risky assets are a negative function of d_n, unless there is very strong home bias and there is little risk. This means that a country with a higher than average expected dividend will have positive net outflows of safe and risky assets due to portfolio reallocation, unless there is strong home bias and little risk.

The intuition is as follows. The rise in global risk aversion leads to a shift by investors away from risky assets and towards safe assets. Let the Home country have a higher than average expected dividend on its domestic assets. The shift away from risky assets towards safe assets is larger then average for the Home country as it holds more risky assets than average and is leveraged. This leads to positive net outflows of safe assets. The Home country is leveraged because of large holdings of domestic risky assets. The country actually holds less than average in foreign risky assets, while other countries hold a larger than average fraction in the Home country. The reallocation away from risky assets is then larger for foreign investors holding Home risky assets than for the Home country holding foreign risky assets. Capital inflows of risky assets then fall more than capital outflows of risky assets due to portfolio reallocation, leading to positive net outflows of risky assets.

In addition there is reallocation towards the risky asset of the Home country as the relative price of its risky asset drops more than average, raising the expected return on its risky asset more than in other countries. This implies negative net outflows of risky assets. It also leads to negative net outflows of safe assets as Home investors are biased towards the domestic risky asset that has a relatively high return, leading to reallocation away from the safe asset as well. This channel only dominates when there is strong home bias and very low risk.

B.5 Summary Results

Table 11 summarizes all results. It reports the sign of net capital flows of risky and safe assets due to a rise in global risk-aversion, as well as the three components of net capital flows. These results apply to a leveraged country. This is either a country with low risk-aversion ($\epsilon_n^G > 0$) or a high expected dividend ($\epsilon_n^D > 0$). Table 11 shows that the sign of net capital flows of risky and safe assets due to portfolio rebalancing is equal to the overall sign of net capital flows of risky and safe assets. However, there is significant ambiguity when it comes to the sign associated with portfolio growth and portfolio reallocation, where the sign in general depends both on model parameters and on whether the country is leveraged due to low risk-aversion or high expected dividends.

Table 11: Direction Net Capital Flows in Leveraged Country

	Low Risk-Aversion		High Expected Dividend	
	NF^{risky}	NF^{safe}	NF^{risky}	NF^{safe}
Overall	-	+	-	+
Due to Portfolio Growth	+	-	+/-	-
Due to Portfolio Rebalancing	-	+	-	+
Due to Portfolio Reallocation	-	+/-	+/-	+/-

*N*otes: The table shows the sign of net capital flows of risky and safe assets, and its components, due to a rise in global risk-aversion. The results are shown for a leveraged country, both for the case where the leverage is due to low risk-aversion and high expected dividends.