# ONLINE APPENDIX

*The Flight to Safety and International Risk Sharing*

Rohan Kekre* Moritz Lenel†

## A Equilibrium

In this appendix we provide additional details on the equilibrium excluded from the main text for brevity. We first specify the optimization problems and policy in Foreign. We then outline the market clearing conditions. Finally, we define the equilibrium and characterize the model's equilibrium conditions and solution.

### A.1 Optimization problems and policy in Foreign

**Households** The representative Foreign household seeks to maximize

$$v_t^* = \left( (1-\beta^*) \left( c_t^* \Phi^*(\ell_t^*) \Omega_t^*(B_{Ht,s}^*/(E_t^{-1} P_t^*)) \right)^{1-1/\psi} + \beta^* \mathbb{E}_t \left[ \left( v_{t+1}^* \right)^{1-\gamma^*} \right]^{\frac{1-1/\psi}{1-\gamma^*}} \right)^{\frac{1}{1-1/\psi}},$$

subject to the consumption aggregator

$$c_t^* = \left( \left( \frac{1}{1+\zeta^*} - \frac{\varsigma}{\zeta^*} \right)^{\frac{1}{\sigma}} (c_{Ht}^*)^{\frac{\sigma-1}{\sigma}} + \left( \frac{\zeta^*}{1+\zeta^*} + \frac{\varsigma}{\zeta^*} \right)^{\frac{1}{\sigma}} (c_{Ft}^*)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

the disutility of labor

$$\Phi^*(\ell_t^*) = \left( 1 + (1/\psi - 1) \bar{\nu}^* \frac{(\ell_t^*)^{1+1/\nu}}{1+1/\nu} \right)^{\frac{1/\psi}{1-1/\psi}},$$

the utility from safe dollar bonds

$$\Omega_t^* \left( \frac{B_{Ht,s}^*}{E_t^{-1} P_t^*} \right) = \exp \left( \omega_t^d \frac{B_{Ht,s}^*}{E_t^{-1} P_t^* \bar{c}_t^*} - \frac{1}{2} \frac{1}{\epsilon^d} \left( \frac{B_{Ht,s}^*}{E_t^{-1} P_t^* \bar{c}_t^*} \right)^2 - \right.$$

*Chicago Booth and NBER. Email: rohan.kekre@chicagobooth.edu.
†Princeton and NBER. Email: lenel@princeton.edu.

$$\left[\omega_t^d \frac{\bar{B}^*_{Ht,s}}{E_t^{-1} P_t^* \bar{c}_t^*} - \frac{1}{2}\frac{1}{\epsilon^d}\left(\frac{\bar{B}^*_{Ht,s}}{E_t^{-1} P_t^* \bar{c}_t^*}\right)^2\right]\Bigg),$$

and the resource constraint

$$\begin{aligned} &E_t P_{Ht} c^*_{Ht} + P^*_{Ft} c^*_{Ft} + E_t B^*_{Ht,s} + E_t B^*_{Ht,o} + B^*_{Ft} + E_t Q^k_t k^*_t \leq \\ &E_t(1+i_{t-1})B^*_{Ht-1,s} + E_t(1+\iota_{t-1})B^*_{Ht-1,o} + (1+i^*_{t-1})B^*_{Ft-1} + E_t(\Pi_t + (1-\delta)Q^k_t)k^*_{t-1}\exp(\varphi_t) + \\ &\qquad \int_0^1 W^*_t(j^*)\ell^*_t(j^*)dj^* - \int_0^1 AC^{W*}_t(j^*)dj^* + T^*_t, \end{aligned}$$

where the cost of setting wages is given by

$$AC^{W*}_t(j^*) = \frac{\chi^W}{2} W^*_t \ell^*_t \left(\frac{W^*_t(j^*)}{W^*_{t-1}(j^*)\exp(\varphi_t)} - 1\right)^2.$$

**Labor unions** Foreign union $j^*$ chooses the wage $W^*_t(j^*)$ and labor supply $\ell^*_t(j^*)$ to maximize the utilitarian social welfare of union members.

**Labor packer** A representative Foreign labor packer purchases varieties supplied by each union and combines them to produce a CES aggregate with elasticity of substitution $\epsilon$ and sold at $W^*_t$ to domestic firms. The labor packer thus earns

$$W^*_t \zeta^* \left[\int_0^1 \ell^*_t(j^*)^{(\epsilon-1)/\epsilon}\right]^{\epsilon/(\epsilon-1)} - \int_0^1 W^*_t(j^*)\zeta^* \ell^*_t(j^*)dj^*.$$

**Production** The representative Foreign producer hires $\ell^*_t$ units of labor and rents $\kappa^*_t$ units of capital to maximize

$$P^*_{Ft}\left(z_t z_{Ft} \zeta^* \ell^*_t\right)^{1-\alpha}\left(\kappa^*_t\right)^{\alpha} - W^*_t \zeta^* \ell^*_t - E_t \Pi_t \kappa^*_t.$$

**Policy** Monetary policy is characterized by a Taylor rule

$$1 + i^*_t = (1 + \bar{i}^*)\left(\frac{P^*_t}{P^*_{t-1}}\right)^{\phi},$$

where $P_t^*$ is the ideal price index

$$P_t^* = \left[ \left( \frac{1}{1+\zeta^*} - \frac{\varsigma}{\zeta^*} \right) (E_t P_{Ht})^{1-\sigma} + \left( \frac{\zeta^*}{1+\zeta^*} + \frac{\varsigma}{\zeta^*} \right) P_{Ft}^{*1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

Fiscal policy is characterized by lump-sum transfers

$$T_t^* = \int_0^1 AC_t^{W*}(j^*) dj^*.$$

## A.2 Market clearing

Market clearing in goods each period is

$$c_{Ht} + \zeta^* c_{Ht}^* + \left( \frac{\bar{k}_t}{\bar{k}_{t-1} \exp(\varphi_t)} \right)^{\chi^x} x_{Ht} = (z_t \ell_t)^{1-\alpha} (\kappa_t)^\alpha, \tag{23}$$

$$c_{Ft} + \zeta^* c_{Ft}^* + \left( \frac{\bar{k}_t}{\bar{k}_{t-1} \exp(\varphi_t)} \right)^{\chi^x} x_{Ft} = (z_t z_{Ft} \zeta^* \ell_t^*)^{1-\alpha} (\kappa_t^*)^\alpha, \tag{24}$$

in labor is

$$\left[ \int_0^1 \ell_t(j)^{(\epsilon-1)/\epsilon} dj \right]^{\epsilon/(\epsilon-1)} = \ell_t, \tag{25}$$

$$\left[ \int_0^1 \ell_t^*(j^*)^{(\epsilon-1)/\epsilon} dj^* \right]^{\epsilon/(\epsilon-1)} = \ell_t^*, \tag{26}$$

in the capital rental market is

$$\kappa_t + \kappa_t^* = \bar{k}_{t-1} \exp(\varphi_t), \tag{27}$$

in the capital market is

$$k_{t-1} + \zeta^* k_{t-1}^* = \bar{k}_{t-1}, \tag{28}$$

$$(1-\delta)\bar{k}_{t-1} \exp(\varphi_t) + x_t = \bar{k}_t, \tag{29}$$

and in bonds is

$$B_{Ht,s} + \zeta^* B_{Ht,s}^* + B_{Ht,s}^g = 0, \tag{30}$$

$$B_{Ht,o} + \zeta^* B_{Ht,o}^* = 0, \tag{31}$$

$$B_{Ft} + \zeta^* B^*_{Ft} = 0. \tag{32}$$

## A.3 Definition of equilibrium

**Definition 1.** *An equilibrium is a sequence of prices and policies such that:*

- *each Home representative household chooses* $\{c_{Ht}, c_{Ft}, B_{Ht,s}, B_{Ht,o}, B_{Ft}, k_t\}$ *to maximize (1) subject to (2)-(5) and analogously in Foreign;*
- *each Home union* $j$ *chooses* $\{W_t(j), \ell_t(j)\}$ *to maximize the utilitarian social welfare of its members subject to (5), and analogously in Foreign;*
- *the representative Home labor packer chooses* $\{\ell_t(j)\}$ *to maximize profits (6) and analogously in Foreign;*
- *the representative Home producer chooses* $\{\ell_t, \kappa_t\}$ *to maximize profits (7) and analogously in Foreign;*
- *the representative global capital producer chooses* $\{x_{Ht}, x_{Ft}, x_t\}$ *to maximize profits (9) subject to (8);*
- *the Home government sets* $B^g_{Ht,s}$ *according to (12) and* $\{i_t, \{T_t\}\}$ *according to (10) and (13), and the Foreign government analogously does the latter;*
- *the goods, factor, and asset markets clear according to (23)-(32).*

## A.4 Additional variables

Before turning to the model analysis, defining several additional variables will be helpful. Except for the nominal interest rates $i_t$ and $i^*_t$, we use lower-case variables to denote real variables.

We first define several important relative prices: the real exchange rate

$$q_t \equiv \frac{E_t P_t}{P^*_t},$$

the real interest rates

$$1 + r_{t+1} \equiv (1 + i_t)\frac{P_t}{P_{t+1}},$$

$$1 + r^*_{t+1} \equiv (1 + i^*_t)\frac{P^*_t}{P^*_{t+1}},$$

and the real return to capital (expressed in Home consumption goods)

$$1 + r^k_{t+1} \equiv \frac{(\Pi_{t+1} + (1-\delta)Q^k_{t+1})}{Q^k_t}\frac{P_t}{P_{t+1}}\exp(\varphi_{t+1}).$$

We then define several important quantities: at Home (with analogous definitions in Foreign), output

$$y_t \equiv (z_t \ell_t)^{1-\alpha}(\kappa_t)^{\alpha},$$

the real value of aggregate saving

$$a_t \equiv \frac{1}{P_t}\left(B_{Ht} + E_t^{-1}B_{Ft} + Q^k_t k_t\right),$$

and the real value of net foreign assets

$$nfa_t \equiv a_t - \frac{Q^k_t}{P_t}\kappa_{t+1}\exp(-\varphi_{t+1}),$$

where we define all of these variables at the end of the period, consistent with the way they are measured in the data.[53,54]

## A.5 First-order conditions

### A.5.1 Households

The representative Home household's intratemporal optimality is characterized by

$$\frac{c_{Ht}}{c_{Ft}} = \frac{\frac{1}{1+\zeta^*} + \varsigma}{\frac{\zeta^*}{1+\zeta^*} - \varsigma} s_t^{-\sigma},$$

[53]While $\kappa_{t+1}$ and $\varphi_{t+1}$ are only known after shocks have realized at $t+1$, we still date net foreign assets as of $t$. This is sensible if shocks are realized "just after" a period starts.

[54]By multiplying $\kappa_{t+1}$ by $\exp(-\varphi_{t+1})$ in the definition of net foreign assets, we are undoing the effect of capital destruction at $t+1$ and thus appropriately comparing how much capital is used in production at Home with the capital owned by Home residents.

where we denote the terms of trade

$$s_t \equiv \frac{E_t P_{Ht}}{P_{Ft}^*}.$$

Given the household's pricing kernel

$$m_{t,t+1} = \beta \frac{c_t}{c_{t+1}} \left( \frac{c_{t+1}\Phi(\ell_{t+1})}{c_t \Phi(\ell_t)} \right)^{1-1/\psi} \left( \frac{v_{t+1}}{ce_t} \right)^{1/\psi - \gamma},$$

(where we have used that $\Omega_t(B_{Ht,s}/P_t) = 1$ at all dates and states) as well as the certainty equivalent

$$ce_t = \mathbb{E}_t \left[ (v_{t+1})^{1-\gamma} \right]^{\frac{1}{1-\gamma}},$$

its intertemporal optimality is characterized by

$$\begin{aligned}
1 &= \mathbb{E}_t m_{t,t+1} \left( \frac{1 + r_{t+1}}{1 - \omega_t} \right), \\
1 &= \mathbb{E}_t m_{t,t+1} \frac{q_t}{q_{t+1}} (1 + r_{t+1}^*), \\
1 &= \mathbb{E}_t m_{t,t+1} (1 + r_{t+1}^k),
\end{aligned}$$

where the first equation is implied by optimality in either safe dollar bonds or other dollar bonds given the definition of $\omega_t$ in (15) in the main text. Substituting in government transfers (13) into the resource constraint (4), dividing by $P_t$, and denoting $b_{Ht,s} \equiv \frac{B_{Ht,s}}{P_t}$, $b_{Ht,s}^g \equiv \frac{B_{Ht,s}^g}{P_t}$, $b_{Ht,o} \equiv \frac{B_{Ht,o}}{P_t}$, $b_{Ft} \equiv \frac{B_{Ft}}{P_t^*}$, $q_t^k \equiv \frac{Q_t^k}{P_t}$, $\pi_t \equiv \frac{\Pi_t}{P_t}$, and $w_t \equiv \frac{W_t}{P_t}$, the household's resource constraint becomes

$$\begin{aligned}
c_t + b_{Ht,s} + b_{Ht,s}^g + b_{Ht,o} + q_t^{-1} b_{Ft} + q_t^k k_t = w_t \ell_t + \\
(1 + r_t) \left( b_{Ht-1,s} + b_{Ht-1,s}^g \right) + \left( \frac{1 + r_t}{1 - \omega_{t-1}} \right) b_{Ht-1,o} + q_t^{-1} (1 + r_t^*) b_{Ft-1} + \\
(\pi_t + (1 - \delta) q_t^k) k_{t-1} \exp(\varphi_t).
\end{aligned}$$

Households' first-order conditions in Foreign are analogous. Their resource constraint becomes

$$c_t^* + q_t b_{Ht,s}^* + q_t b_{Ht,o}^* + b_{Ft}^* + q_t q_t^k k_t = q_t w_t^* \ell_t^* +$$

$$q_t(1+r_t)b^*_{Ht-1,s} + q_t\left(\frac{1+r_t}{1-\omega_{t-1}}\right)b^*_{Ht-1,o} + (1+r^*_t)b_{Ft-1} +$$
$$q_t(\pi_t + (1-\delta)q^k_t)k_{t-1}\exp(\varphi_t),$$

where $b^*_{Ht,s} \equiv \frac{B^*_{Ht,s}}{P_t}$, $b^*_{Ht,o} \equiv \frac{B^*_{Ht,o}}{P_t}$, $b^*_{Ft} \equiv \frac{B_{Ft}}{P^*_t}$, and $w^*_t \equiv \frac{W^*_t}{E_t P_t}$.

Now consider households' optimal choice of safe dollar bonds. Given the assumed functional forms of $\Omega_t$ and $\Omega^*_t$, we have by (15) that

$$\omega_t = \omega^d_t - \frac{1}{\epsilon^d}\frac{B_{Ht,s}}{P_t c_t} = \omega^d_t - \frac{1}{\epsilon^d}\frac{B^*_{Ht,s}}{E^{-1}_t P^*_t c^*_t}$$

and thus

$$\frac{B_{Ht,s}}{P_t c_t} = \frac{B^*_{Ht,s}}{E^{-1}_t P^*_t c^*_t}.$$

Combining this with global market clearing in safe dollar bonds, straightforward algebra yields

$$B_{Ht,s} = \frac{P_t c_t}{P_t c_t + \zeta^* E^{-1}_t P^*_t c^*_t}(-B^g_{Ht,s}),$$
$$B^*_{Ht,s} = \frac{E^{-1}_t P^*_t c^*_t}{P_t c_t + \zeta^* E^{-1}_t P^*_t c^*_t}(-B^g_{Ht,s}).$$

It follows that, now in real terms, we can re-write the Home household's resource constraint as

$$c_t + \omega_t \frac{\zeta^* q^{-1}_t c^*_t}{c_t + \zeta^* q^{-1}_t c^*_t} b^g_{Ht,s} + (1-\omega_t)\left(b_{Ht,s} + b^g_{Ht,s}\right) + b_{Ht,o} + q^{-1}_t b_{Ft} + q^k_t k_t = w_t \ell_t +$$
$$(1+r_t)\left(b_{Ht-1,s} + b^g_{Ht-1,s}\right) + \left(\frac{1+r_t}{1-\omega_{t-1}}\right)b_{Ht-1,o} + q^{-1}_t(1+r^*_t)b_{Ft-1} +$$
$$(\pi_t + (1-\delta)q^k_t)k_{t-1}\exp(\varphi_t)$$

and the Foreign household's resource constraint as

$$c^*_t - q_t\omega_t \frac{q^{-1}_t c^*_t}{c_t + \zeta^* q^{-1}_t c^*_t} b^g_{Ht,s} + q_t(1-\omega_t)b^*_{Ht,s} + q_t b^*_{Ht,o} + b^*_{Ft} + q_t q^k_t k_t = q_t w^*_t \ell^*_t +$$
$$q_t(1+r_t)b^*_{Ht-1,s} + q_t\left(\frac{1+r_t}{1-\omega_{t-1}}\right)b^*_{Ht-1,o} + (1+r^*_t)b_{Ft-1} +$$
$$q_t(\pi_t + (1-\delta)q^k_t)k_{t-1}\exp(\varphi_t).$$

Defining households' net positions in dollar-denominated bonds

$$b_{Ht} \equiv (1-\omega_t)\left(b_{Ht,s} + b^g_{Ht,s}\right) + b_{Ht,o},$$
$$b^*_{Ht} \equiv (1-\omega_t) b^*_{Ht,s} + b^*_{Ht,o},$$

their positions in safe dollar bonds are only relevant insofar as they determine the seignorage earned by Home from the safe dollar bonds purchased by Foreign, given by the second term in each resource constraint.

### A.5.2 Unions

The representative union's first-order condition is

$$w_t - \frac{c_t \Phi'(\ell_t)}{\Phi(\ell_t)} + w_t \frac{\chi^W}{\epsilon}\left[\frac{w_t}{w_{t-1}\exp(\varphi_t)}\frac{P_t}{P_{t-1}}\left(\frac{w_t}{w_{t-1}\exp(\varphi_t)}\frac{P_t}{P_{t-1}} - 1\right)\right.$$
$$\left. - \mathbb{E}_t m_{t,t+1}\left(\frac{w_{t+1}}{w_t \exp(\varphi_{t+1})}\right)^2 \frac{P_{t+1}}{P_t}\frac{\ell_{t+1}}{\ell_t}\left(\frac{w_{t+1}}{w_t\exp(\varphi_{t+1})}\frac{P_{t+1}}{P_t} - 1\right)\right] = 0,$$

The representative union's first-order condition in Foreign is analogous.

### A.5.3 Producers

The representative Home producer's first-order conditions are

$$w_t = \frac{P_{Ht}}{P_t}(1-\alpha) z_t^{1-\alpha} \ell_t^{-\alpha} \kappa_t^{\alpha},$$
$$\pi_t = \frac{P_{Ht}}{P_t}\alpha z_t^{1-\alpha} \ell_t^{1-\alpha} \kappa_t^{\alpha-1}.$$

The representative Foreign producer's first-order conditions are

$$w^*_t = q_t^{-1}\frac{P^*_{Ft}}{P^*_t}(1-\alpha)(z_t z_{Ft})^{1-\alpha}(\zeta^* \ell^*_t)^{-\alpha} \kappa_t^{*\alpha},$$
$$\pi_t = q_t^{-1}\frac{P^*_{Ft}}{P^*_t}\alpha (z_t z_{Ft})^{1-\alpha}(\zeta^* \ell^*_t)^{1-\alpha} \kappa_t^{*\alpha-1}.$$

Finally, the representative producer of capital's first-order conditions are

$$\frac{x_{Ht}}{x_{Ft}} = \frac{1}{\zeta^*} s_t^{-\sigma},$$

$$q_t^k = \left(\frac{\bar{k}_t}{\bar{k}_{t-1}\exp(\varphi_t)}\right)^{\chi^x}\left(\frac{1}{1+\zeta^*}\left(\frac{P_{Ht}}{P_t}\right)^{1-\sigma}+\left(\frac{\zeta^*}{1+\zeta^*}\right)\left(\frac{P_{Ft}}{P_t}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}.$$

## A.6 Re-scaled economy

Define the re-scaled variables

$$\tilde{c}_t \equiv \frac{c_t}{z_t},\ \ \tilde{c}_{Ht} \equiv \frac{c_{Ht}}{z_t},\ \ \tilde{c}_{Ft} \equiv \frac{c_{Ft}}{z_t},\ \ \tilde{ce}_t \equiv \frac{ce_t}{z_t},\ \ \tilde{m}_{t,t+1} \equiv m_{t,t+1}\left(\frac{z_{t+1}}{z_t}\right)^{\gamma},$$

$$\tilde{b}_{Ht} \equiv \frac{b_{Ht}}{z_t},\ \ \tilde{b}^g_{Ht,s} \equiv \frac{b^g_{Ht,s}}{z_t},\ \ \tilde{b}_{Ft} \equiv \frac{b_{Ft}}{z_t},\ \ \tilde{\tilde{b}}_{Ht-1} \equiv \frac{\tilde{b}_{Ht-1}}{\exp(\sigma^z\epsilon^z_t+\varphi_t)},\ \ \tilde{\tilde{b}}_{Ft-1} \equiv \frac{\tilde{b}_{Ft-1}}{\exp(\sigma^z\epsilon^z_t+\varphi_t)},$$

$$\tilde{k}_t \equiv \frac{k_t}{z_t},\ \ \tilde{\kappa}_t \equiv \frac{\kappa_t}{z_t},\ \ \tilde{\tilde{k}}_{t-1} \equiv \frac{\tilde{k}_{t-1}}{\exp(\sigma^z\epsilon^z_t)},$$

$$\tilde{w}_t \equiv \frac{w_t}{z_t},\ \ \tilde{\tilde{w}}_{t-1} \equiv \frac{\tilde{w}_{t-1}}{\exp(\sigma^z\epsilon^z_t)},$$

$$\tilde{c}^*_t \equiv \frac{c^*_t}{z_t},\ \ \tilde{c}^*_{Ht} \equiv \frac{c^*_{Ht}}{z_t},\ \ \tilde{c}^*_{Ft} \equiv \frac{c^*_{Ft}}{z_t},\ \ \tilde{ce}^*_t \equiv \frac{ce^*_t}{z_t},\ \ \tilde{m}^*_{t,t+1} \equiv m^*_{t,t+1}\left(\frac{z_{t+1}}{z_t}\right)^{\gamma^*},$$

$$\tilde{b}^*_{Ht} \equiv \frac{b^*_{Ht}}{z_t},\ \ \tilde{b}^*_{Ft} \equiv \frac{b^*_{Ft}}{z_t},\ \ \tilde{\tilde{b}}^*_{Ht-1} \equiv \frac{\tilde{b}^*_{Ht-1}}{\exp(\sigma^z\epsilon^z_t+\varphi_t)},\ \ \tilde{\tilde{b}}^*_{Ft-1} \equiv \frac{\tilde{b}^*_{Ft-1}}{\exp(\sigma^z\epsilon^z_t+\varphi_t)},$$

$$\tilde{k}^*_t \equiv \frac{k^*_t}{z_t},\ \ \tilde{\kappa}^*_t \equiv \frac{\kappa^*_t}{z_t},\ \ \tilde{\tilde{k}}^*_{t-1} \equiv \frac{\tilde{k}^*_{t-1}}{\exp(\sigma^z\epsilon^z_t)},$$

$$\tilde{w}^*_t \equiv \frac{w^*_t}{z_t},\ \ \tilde{\tilde{w}}^*_{t-1} \equiv \frac{\tilde{w}^*_{t-1}}{\exp(\sigma^z\epsilon^z_t)},$$

$$\tilde{x}_{Ht} \equiv \frac{x_{Ht}}{z_t},\ \ \tilde{x}_{Ft} \equiv \frac{x_{Ft}}{z_t},\ \ \tilde{x}_t \equiv \frac{x_t}{z_t},\ \ \tilde{\bar{k}}_t \equiv \frac{\bar{k}_t}{z_t},\ \ \tilde{\tilde{\bar{k}}}_{t-1} \equiv \frac{\tilde{\bar{k}}_{t-1}}{\exp(\sigma^z\epsilon^z_t)}.$$

The re-scaled Home household first-order conditions and constraints are:

$$\tilde{v}_t = \left((1-\beta)\left(\tilde{c}_t\Phi(\ell_t)\right)^{1-1/\psi}+\beta\left(\tilde{ce}_t\right)^{1-1/\psi}\right)^{\frac{1}{1-1/\psi}}, \tag{33}$$

$$\tilde{ce}_t = \mathbb{E}_t\left[\exp\left((1-\gamma)\left[\sigma^z\epsilon^z_{t+1}+\varphi_{t+1}\right]\right)\left(\tilde{v}_{t+1}\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \tag{34}$$

$$\tilde{c}_t = \left(\left(\frac{1}{1+\zeta^*}+\varsigma\right)^{\frac{1}{\sigma}}\left(\tilde{c}_{Ht}\right)^{\frac{\sigma-1}{\sigma}}+\left(\frac{\zeta^*}{1+\zeta^*}-\varsigma\right)^{\frac{1}{\sigma}}\left(\tilde{c}_{Ft}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{35}$$

$$\frac{\tilde{c}_{Ht}}{\tilde{c}_{Ft}} = \frac{\frac{1}{1+\zeta^*}+\varsigma}{\frac{\zeta^*}{1+\zeta^*}-\varsigma}s_t^{-\sigma}, \tag{36}$$

$$\tilde{m}_{t,t+1} = \beta\frac{\tilde{c}_t}{\tilde{c}_{t+1}}\left(\frac{\tilde{c}_{t+1}\Phi(\ell_{t+1})}{\tilde{c}_t\Phi(\ell_t)}\right)^{1-\frac{1}{\psi}}\left(\frac{\tilde{v}_{t+1}}{\tilde{ce}_t}\right)^{1/\psi-\gamma}, \tag{37}$$

$$1 = \mathbb{E}_t \tilde{m}_{t,t+1} \exp\left(-\gamma\left[\sigma^z \epsilon^z_{t+1} + \varphi_{t+1}\right]\right) \frac{(1+r_{t+1})}{(1-\omega_t)}, \tag{38}$$

$$1 = \mathbb{E}_t \tilde{m}_{t,t+1} \exp\left(-\gamma\left[\sigma^z \epsilon^z_{t+1} + \varphi_{t+1}\right]\right) \frac{q_t}{q_{t+1}}(1+r^*_{t+1}), \tag{39}$$

$$1 = \mathbb{E}_t \tilde{m}_{t,t+1} \exp\left(-\gamma\left[\sigma^z \epsilon^z_{t+1} + \varphi_{t+1}\right]\right)(1+r^k_{t+1}), \tag{40}$$

$$\tilde{c}_t + \tilde{b}_{Ht} + q_t^{-1}\tilde{b}_{Ft} + q^k_t \tilde{k}_t = \tilde{w}_t \ell_t + \theta_t(\pi_t + (1-\delta)q^k_t)\tilde{\tilde{\tilde{k}}}_{t-1}. \tag{41}$$

The re-scaled Foreign household first-order conditions and constraints are:

$$\tilde{v}^*_t = \left((1-\beta)\left(\tilde{c}^*_t \Phi^*(\ell^*_t)\right)^{1-1/\psi} + \beta\left(\tilde{ce}^*_t\right)^{1-1/\psi}\right)^{\frac{1}{1-1/\psi}}, \tag{42}$$

$$\tilde{ce}^*_t = \mathbb{E}_t\left[\exp\left((1-\gamma^*)\left[\sigma^z \epsilon^z_{t+1} + \varphi_{t+1}\right]\right)\left(\tilde{v}^*_{t+1}\right)^{1-\gamma^*}\right]^{\frac{1}{1-\gamma^*}}, \tag{43}$$

$$\tilde{c}^*_t = \left(\left(\frac{1}{1+\zeta^*} - \frac{\varsigma}{\zeta^*}\right)^{\frac{1}{\sigma}} (\tilde{c}^*_{Ht})^{\frac{\sigma-1}{\sigma}} + \left(\frac{\zeta^*}{1+\zeta^*} + \frac{\varsigma}{\zeta^*}\right)^{\frac{1}{\sigma}} (\tilde{c}^*_{Ft})^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{44}$$

$$\frac{\tilde{c}^*_{Ht}}{\tilde{c}^*_{Ft}} = \frac{\frac{1}{1+\zeta^*} - \frac{\varsigma}{\zeta^*}}{\frac{\zeta^*}{1+\zeta^*} + \frac{\varsigma}{\zeta^*}} s_t^{-\sigma}, \tag{45}$$

$$\tilde{m}^*_{t,t+1} = \beta \frac{\tilde{c}^*_t}{\tilde{c}^*_{t+1}} \left(\frac{\tilde{c}^*_{t+1}\Phi^*(\ell^*_{t+1})}{\tilde{c}^*_t \Phi^*(\ell^*_t)}\right)^{1-\frac{1}{\psi}} \left(\frac{\tilde{v}^*_{t+1}}{\tilde{ce}^*_t}\right)^{1/\psi-\gamma^*}, \tag{46}$$

$$1 = \mathbb{E}_t \tilde{m}^*_{t,t+1} \exp\left(-\gamma^*\left[\sigma^z \epsilon^z_{t+1} + \varphi_{t+1}\right]\right) \frac{q_{t+1}}{q_t}\frac{(1+r_{t+1})}{(1+\omega_t)}, \tag{47}$$

$$1 = \mathbb{E}_t \tilde{m}^*_{t,t+1} \exp\left(-\gamma^*\left[\sigma^z \epsilon^z_{t+1} + \varphi_{t+1}\right]\right)(1+r^*_{t+1}), \tag{48}$$

$$1 = \mathbb{E}_t \tilde{m}^*_{t,t+1} \exp\left(-\gamma^*\left[\sigma^z \epsilon^z_{t+1} + \varphi_{t+1}\right]\right) \frac{q_{t+1}}{q_t}(1+r^k_{t+1}), \tag{49}$$

$$q_t^{-1}\tilde{c}^*_t + \tilde{b}^*_{Ht} + q_t^{-1}\tilde{b}^*_{Ft} + q^k_t \tilde{k}^*_t = \tilde{w}^*_t \ell^*_t + \frac{1}{\zeta^*}(1-\theta_t)(\pi_t + (1-\delta)q^k_t)\tilde{\tilde{\tilde{k}}}_{t-1}. \tag{50}$$

The global wealth share of Home households, inclusive of seignorage, is

$$\theta_{t+1} = \frac{1}{(\pi_{t+1} + (1-\delta)q^k_{t+1})\tilde{\tilde{\tilde{k}}}_t}\left[\frac{1+r_{t+1}}{1-\omega_t}\tilde{\tilde{b}}_{Ht} + \frac{1}{q_{t+1}}(1+r^*_{t+1})\tilde{\tilde{b}}_{Ft} \right.$$
$$\left. + (\pi_{t+1} + (1-\delta)q^k_{t+1})\tilde{\tilde{k}}_t - \omega_{t+1}\frac{\zeta^* q^{-1}_{t+1}\tilde{c}^*_{t+1}}{\tilde{c}_{t+1} + \zeta^* q^{-1}_{t+1}\tilde{c}^*_{t+1}}\tilde{b}^g_{Ht+1,s}\right]. \tag{51}$$

Supply-side optimality requires:

$$\tilde{w}_t - \frac{\tilde{c}_t \Phi'(\ell_t)}{\Phi(\ell_t)} + \tilde{w}_t \frac{\chi^W}{\epsilon} \Bigg[ \frac{\tilde{w}_t}{\tilde{\tilde{w}}_{t-1}} \frac{P_t}{P_{t-1}} \left( \frac{\tilde{w}_t}{\tilde{\tilde{w}}_{t-1}} \frac{P_t}{P_{t-1}} - 1 \right)$$
$$- \mathbb{E}_t m_{t,t+1} \exp\left(-\gamma\left[\sigma^z \epsilon^z_{t+1} + \varphi_{t+1}\right]\right) \times$$
$$\left( \frac{\tilde{w}_{t+1}}{\tilde{\tilde{w}}_t} \right)^2 \frac{P_{t+1}}{P_t} \frac{\ell_{t+1}}{\ell_t} \left( \frac{\tilde{w}_{t+1}}{\tilde{\tilde{w}}_t} \frac{P_{t+1}}{P_t} - 1 \right) \Bigg] = 0 \tag{52}$$

$$\tilde{w}^*_t - q_t^{-1} \frac{\tilde{c}^*_t \Phi^{*'}(\ell^*_t)}{\Phi^*(\ell^*_t)} + \tilde{w}^*_t \frac{\chi^W}{\epsilon} \Bigg[ \frac{\tilde{w}^*_t}{\tilde{\tilde{w}}^*_{t-1}} \frac{E_t P_t}{E_{t-1} P_{t-1}} \left( \frac{\tilde{w}^*_t}{\tilde{\tilde{w}}^*_{t-1}} \frac{E_t P_t}{E_{t-1} P_{t-1}} - 1 \right)$$
$$- \mathbb{E}_t m^*_{t,t+1} \exp\left(-\gamma^*\left[\sigma^z \epsilon^z_{t+1} + \varphi_{t+1}\right]\right) \times$$
$$\left( \frac{\tilde{w}^*_{t+1}}{\tilde{\tilde{w}}^*_t} \right)^2 \frac{q_{t+1}}{q_t} \frac{E_{t+1} P_{t+1}}{E_t P_t} \frac{\ell^*_{t+1}}{\ell^*_t} \left( \frac{\tilde{w}^*_{t+1}}{\tilde{\tilde{w}}^*_t} \frac{E_{t+1} P_{t+1}}{E_t P_t} - 1 \right) \Bigg] = 0 \tag{53}$$

$$\tilde{w}_t = \frac{P_{Ht}}{P_t}(1-\alpha)\ell_t^{-\alpha}\tilde{\kappa}_t^{\alpha}, \tag{54}$$

$$\tilde{w}^*_t = q_t^{-1}\frac{P^*_{Ft}}{P^*_t}(1-\alpha)z_{Ft}^{1-\alpha}(\zeta^*\ell^*_t)^{-\alpha}\tilde{\kappa}_t^{*\alpha}, \tag{55}$$

$$\pi_t = \frac{P_{Ht}}{P_t}\alpha\ell_t^{1-\alpha}\tilde{\kappa}_t^{\alpha-1}, \tag{56}$$

$$\pi_t = q_t^{-1}\frac{P^*_{Ft}}{P^*_t}\alpha(z_{Ft}\zeta^*\ell^*_t)^{1-\alpha}\tilde{\kappa}_t^{*\alpha-1}, \tag{57}$$

$$\tilde{x}_t = \left( \left(\frac{1}{1+\zeta^*}\right)^{\frac{1}{\sigma}} (\tilde{x}_{Ht})^{\frac{\sigma-1}{\sigma}} + \left(\frac{\zeta^*}{1+\zeta^*}\right)^{\frac{1}{\sigma}} (\tilde{x}_{Ft})^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \tag{58}$$

$$\frac{\tilde{x}_{Ht}}{\tilde{x}_{Ft}} = \frac{1}{\zeta^*} s_t^{-\sigma}, \tag{59}$$

$$q^k_t = \left( \frac{\tilde{\tilde{k}}_t}{\tilde{\tilde{k}}_{t-1}} \right)^{\chi^x} \left( \frac{1}{1+\zeta^*} \left(\frac{P_{Ht}}{P_t}\right)^{1-\sigma} + \left(\frac{\zeta^*}{1+\zeta^*}\right) \left(\frac{P_{Ft}}{P_t}\right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}. \tag{60}$$

Market clearing requires

$$\tilde{c}_{Ht} + \zeta^*\tilde{c}^*_{Ht} + \left( \frac{\tilde{\tilde{k}}_t}{\tilde{\tilde{k}}_{t-1}} \right)^{\chi^x} \tilde{x}_{Ht} = (\ell_t)^{1-\alpha}(\tilde{\kappa}_t)^{\alpha}, \tag{61}$$

$$\tilde{c}_{Ft} + \zeta^*\tilde{c}^*_{Ft} + \left( \frac{\tilde{\tilde{k}}_t}{\tilde{\tilde{k}}_{t-1}} \right)^{\chi^x} \tilde{x}_{Ft} = (z_{Ft}\zeta^*\ell^*_t)^{1-\alpha}(\tilde{\kappa}^*_t)^{\alpha}, \tag{62}$$

$$\tilde{\kappa}_t + \tilde{\kappa}_t^* = \tilde{\tilde{\tilde{k}}}_{t-1}, \tag{63}$$

$$\tilde{k}_t + \zeta^* \tilde{k}_t^* = \tilde{\tilde{k}}_t, \tag{64}$$

$$(1-\delta)\tilde{\tilde{\tilde{k}}}_{t-1} + \tilde{x}_t = \tilde{\tilde{k}}_t, \tag{65}$$

$$\tilde{b}_{Ht} + \zeta^* \tilde{b}_{Ht}^* = 0. \tag{66}$$

The definitions of returns are

$$1 + r_{t+1} = (1+i_t)\frac{P_t}{P_{t+1}}, \tag{67}$$

$$1 + r_{t+1}^* = (1+i_t^*)\frac{P_t^*}{P_{t+1}^*}, \tag{68}$$

$$1 + r_{t+1}^k = \frac{(\pi_{t+1} + (1-\delta)q_{t+1}^k)}{q_t^k}\exp(\varphi_{t+1}). \tag{69}$$

Finally, the definitions of prices imply

$$\frac{P_t}{P_{Ht}} = \left[\left(\frac{1}{1+\zeta^*} + \varsigma\right) + \left(\frac{\zeta^*}{1+\zeta^*} - \varsigma\right) s_t^{\sigma-1}\right]^{\frac{1}{1-\sigma}}, \tag{70}$$

$$\frac{P_t^*}{P_{Ft}^*} = \left[\left(\frac{1}{1+\zeta^*} - \frac{\varsigma}{\zeta^*}\right) s_t^{1-\sigma} + \left(\frac{\zeta^*}{1+\zeta^*} + \frac{\varsigma}{\zeta^*}\right)\right]^{\frac{1}{1-\sigma}}, \tag{71}$$

$$q_t = \frac{E_t P_{Ht}}{P_{Ft}^*}\frac{P_t/P_{Ht}}{P_t^*/P_{Ft}^*} = s_t\left(\frac{\left(\frac{1}{1+\zeta^*} + \varsigma\right) + \left(\frac{\zeta^*}{1+\zeta^*} - \varsigma\right) s_t^{\sigma-1}}{\left(\frac{1}{1+\zeta^*} - \frac{\varsigma}{\zeta^*}\right) s_t^{1-\sigma} + \left(\frac{\zeta^*}{1+\zeta^*} + \frac{\varsigma}{\zeta^*}\right)}\right)^{\frac{1}{1-\sigma}}. \tag{72}$$

Together with the Taylor and fiscal rules and specification of driving forces, (33)-(72) define the equilibrium. Note that by Walras' Law, the Foreign bond market clears as well. As is evident, this environment features 7 state variables:

$$\{p, \omega, z_F, \theta, \tilde{\tilde{\tilde{k}}}_{-1}, \tilde{\tilde{w}}_{-1}, \tilde{\tilde{w}}_{-1}^*\}.$$

## A.7 Solution algorithm

We solve the model globally. We use anistropic, sparse grids as described in Judd, Maliar, Maliar, and Valero (2014). When forming expectations, we use Gauss-Hermite quadrature and interpolate with Chebyshev polynomials for states off the grid. The stochastic equilibrium is determined through backward iteration, while dampening the updating of asset prices and individuals' expectations over the dynamics of the aggregate states. Further details are provided in the document *SolutionAlgorithm.pdf* in our online replication package.

# B Analytical insights

In this appendix we provide supplemental analytical results for the simplified environment described in the main text. We work with the equilibrium conditions (33)-(72) under the parametric conditions in definition 1.

The first subsection characterizes the impulse responses to safety and productivity shocks. The second subsection characterizes agents' pricing kernels and provides a general characterization of equilibrium portfolios and risk premia. The third subsection proves each of Propositions 1-5. The fourth subsection demonstrates these results are robust to asymmetric demand shocks for safe dollar bonds in Foreign and Home. The final subsection outlines an alternative environment without capital mobility and with sticky prices, and presents analogs of our baseline results in this environment.

## B.1 Impulse responses

Without loss of generality, we characterize the impulse responses to shocks in period 1, assuming that the economy was in steady-state in period 0 and there are no other shocks from period 2 onwards. We employ the parametric assumptions in definition 1 except that we allow for a general $\varsigma$ so that the role of home bias is clear.

### B.1.1 Dynamics from period 2 onwards

Since there are no shocks from period 2 onwards, under the parametric conditions assumed in definition 1 it is straightforward to use the equilibrium conditions from period 2 onwards to show

$$\mathbb{E}_1\hat{\hat{c}}_2 = \tilde{\alpha}\mathbb{E}_1\hat{\theta}_2 + \alpha\hat{\hat{\bar{k}}}_1, \tag{73}$$

$$\mathbb{E}_1\hat{\hat{c}}_2^* = -\frac{1}{\zeta^*}\tilde{\alpha}\mathbb{E}_1\hat{\theta}_2 + \alpha\hat{\hat{\bar{k}}}_1, \tag{74}$$

$$\mathbb{E}_1 \hat{s}_2 = \frac{\varsigma \left(\frac{1+\zeta^*}{\zeta^*}\right)^2}{\frac{\alpha}{1-\alpha} + \sigma \left(1 - \left(\varsigma \frac{1+\zeta^*}{\zeta^*}\right)^2\right)} \tilde{\alpha} \mathbb{E}_1 \hat{\theta}_2, \tag{75}$$

$$\mathbb{E}_1 \hat{q}_2 = \varsigma \frac{1+\zeta^*}{\zeta^*} \mathbb{E}_1 \hat{s}_2, \tag{76}$$

$$\mathbb{E}_1 \hat{\hat{v}}_2 = \mathbb{E}_1 \hat{\hat{c}}_2 = \tilde{\alpha} \mathbb{E}_1 \hat{\theta}_2 + \alpha \hat{\bar{\bar{k}}}_1, \tag{77}$$

$$\mathbb{E}_1 \hat{\hat{v}}_2^* = \mathbb{E}_1 \hat{\hat{c}}_2^* = -\frac{1}{\zeta^*} \tilde{\alpha} \mathbb{E}_1 \hat{\theta}_2 + \alpha \hat{\bar{\bar{k}}}_1, \tag{78}$$

where

$$\tilde{\alpha} \equiv \frac{\alpha}{1 - \left(\frac{\zeta^*}{1+\zeta^*} - \varsigma\right) \frac{\varsigma \left(\frac{1+\zeta^*}{\zeta^*}\right)^2}{\frac{\alpha}{1-\alpha} + \sigma \left(1 - \left(\varsigma \frac{1+\zeta^*}{\zeta^*}\right)^2\right)}}.$$

These are the only conditions we need to solve for the equilibrium in period 1, to which we now turn.

### B.1.2 Log-linearized conditions in period 1

Log-linearizing the definition of the real exchange rate implies

$$\hat{q}_1 = \varsigma \frac{1+\zeta^*}{\zeta^*} \hat{s}_1, \tag{79}$$

a relationship we use repeatedly in what follows.

Log-linearizing the intratemporal allocation of consumption, the equilibrium factor prices, the resource constraints, and goods market clearing yields

$$\frac{1}{1+\zeta^*} \hat{\hat{c}}_1 + \frac{\zeta^*}{1+\zeta^*} \hat{\hat{c}}_1^* = (1-\alpha) \left[\frac{1}{1+\zeta^*} \hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*} \hat{\ell}_1^*\right] + \alpha \hat{\bar{\bar{k}}}_1 \tag{80}$$

and

$$\varsigma \frac{1+\zeta^*}{\zeta^*} \left(\hat{\hat{c}}_1 - \hat{\hat{c}}_1^*\right) = \left[\frac{\alpha}{1-\alpha} + \sigma \left(1 - \left(\varsigma \frac{1+\zeta^*}{\zeta^*}\right)^2\right)\right] \hat{s}_1 + \hat{\ell}_1 - \hat{\ell}_1^*. \tag{81}$$

Log-linearizing the Euler equations yields

$$\Delta \mathbb{E}_1 \hat{\hat{c}}_2 = \Delta \mathbb{E}_1 \hat{\hat{c}}_2^* - \varsigma \frac{1+\zeta^*}{\zeta^*} \Delta \mathbb{E}_1 \hat{s}_2, \tag{82}$$

$$\Delta \mathbb{E}_1 \hat{\hat{c}}_2 = \mathbb{E}_1 \hat{r}_2^k, \tag{83}$$

$$\mathbb{E}_1 \hat{r}_2^k = \mathbb{E}_1 \hat{r}_2 + \hat{\omega}_1, \tag{84}$$

$$\mathbb{E}_1 \hat{r}_2^* = \mathbb{E}_1 \hat{r}_2 + \hat{\omega}_1 + \varsigma \frac{1+\zeta^*}{\zeta^*} \Delta \mathbb{E}_1 \hat{s}_2, \tag{85}$$

where we have used (76).

Log-linearizing the expected evolution of Home's wealth share, using the equilibrium factor prices and Home resource constraint, implies

$$\mathbb{E}_1 \hat{\theta}_2 = \frac{1}{\beta\alpha} \Bigg[ (1-\beta) \left( \frac{\zeta^*}{1+\zeta^*} - \varsigma \right) \hat{s}_1 + (1-\beta)(1-\alpha)\hat{\ell}_1 + (1-\beta)\alpha \hat{\bar{\bar{k}}}_1 + \alpha \hat{\theta}_1 - (1-\beta)\hat{\bar{c}}_1 \Bigg]. \tag{86}$$

Linearizing the definition of Home net foreign assets, using the equilibrium factor prices, Home resource constraint, and capital allocation across countries, implies

$$\widehat{nfa}_1 = a \Bigg[ \frac{1}{\beta\alpha} \Bigg[ (1-\beta) \left( \frac{\zeta^*}{1+\zeta^*} - \varsigma \right) \hat{s}_1 + (1-\beta)(1-\alpha)\hat{\ell}_1 + \alpha \hat{\bar{\bar{k}}}_1 + \alpha \hat{\theta}_1 - (1-\beta)\hat{\bar{c}}_1 \Bigg] - \frac{\zeta^*}{1+\zeta^*} \frac{1}{1-\alpha} \hat{s}_2 - \hat{\bar{\bar{k}}}_1 \Bigg]. \tag{87}$$

Log-linearizing the Fisher equations and Taylor rules yields

$$\mathbb{E}_1 \hat{r}_2 = \hat{i}_1, \tag{88}$$

$$\mathbb{E}_1 \hat{r}_2^* = \hat{i}_1^*, \tag{89}$$

$$\hat{i}_1 = \phi \Delta \hat{P}_1, \tag{90}$$

$$\hat{i}_1^* = \phi \Delta \hat{P}_1^*, \tag{91}$$

where we use that the Taylor rules implement $\Delta \hat{P}_2 = \Delta \hat{P}_2^* = 0$.

Log-linearizing the realized evolution of Home's wealth share implies

$$\hat{\theta}_1 = \left( \frac{q^k k}{a} - 1 \right) \left( \hat{r}_1^k - \hat{r}_1 \right) + \frac{b_F}{a} \left( \hat{r}_1 - \hat{q}_1 - \hat{r}_1 \right) - \beta \frac{\zeta^*}{1+\zeta^*} \frac{b_{H,s}^g}{a} \hat{\omega}_1. \tag{92}$$

Log-linearizing the realized returns on capital, using the equilibrium profits, yields

$$\hat{r}^k_1 = (1-\beta)\left[-\varsigma\hat{s}_1 + (1-\alpha)\left[\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}^*_1\right] - (1-\alpha)\hat{\bar{\bar{k}}}_1\right] + \beta\hat{q}^k_1. \tag{93}$$

Log-linearizing the expected returns on capital implies

$$\hat{q}^k_1 = -\varsigma\mathbb{E}_1\hat{s}_2 - (1-\alpha)\hat{\bar{\bar{k}}}_1 - \mathbb{E}_1\hat{r}^k_2. \tag{94}$$

Log-linearizing the realized returns on dollar and Foreign bonds yields

$$\hat{r}_1 = -\Delta\hat{P}_1, \tag{95}$$

$$\hat{r}^*_1 = -\Delta\hat{P}^*_1. \tag{96}$$

Finally, with flexible wages and an infinite Frisch elasticity ($\nu \to 0$), it is clear from the union's wage-setting condition that

$$\hat{\ell}_1 = \hat{\ell}^*_1 = 0.$$

Alternatively, if wages are set one period in advance, it is straightforward to show that up to first-order

$$\begin{aligned}\hat{\tilde{w}}_1 &= -\Delta\hat{P}_1 - \sigma^z\hat{\epsilon}^z_1,\\ \hat{\tilde{w}}^*_1 + \hat{q}_1 &= -\Delta\hat{P}^*_1 - \sigma^z\hat{\epsilon}^z_1.\end{aligned}$$

Combining these with log-linearized labor demand of firms yields

$$\begin{aligned}&\left[\left(\frac{\zeta^*}{1+\zeta^*} - \varsigma\right) + \frac{\zeta^*}{1+\zeta^*}\frac{\alpha}{1-\alpha}\right]\hat{s}_1 - \alpha\left(\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}^*_1\right)\\ &\qquad - (1-\alpha)\hat{\bar{\bar{k}}}_1 = -\Delta\hat{P}_1 \end{aligned}\tag{97}$$

$$\begin{aligned}-&\left[\left(\frac{1}{1+\zeta^*} - \frac{\varsigma}{\zeta^*}\right) + \frac{1}{1+\zeta^*}\frac{\alpha}{1-\alpha}\right]\hat{s}_1 - \alpha\left(\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}^*_1\right)\\ &\qquad - (1-\alpha)\hat{\bar{\bar{k}}}_1 = -\Delta\hat{P}^*_1. \end{aligned}\tag{98}$$

We now combine these log-linearized conditions to facilitate the proof of the results provided in the main text.

(80) implies

$$\hat{\tilde{c}}_1^* = -\frac{1}{\zeta^*}\hat{\tilde{c}}_1 + \frac{1+\zeta^*}{\zeta^*}\left[(1-\alpha)\left(\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}_1^*\right) + \alpha\hat{\bar{\tilde{k}}}_1\right].$$

Substituting in (81) implies

$$\hat{s}_1 = \frac{1}{\frac{\alpha}{1-\alpha} + \sigma\left(1 - \left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2\right)}\left[\left(\hat{\ell}_1^* - \hat{\ell}_1\right) + \varsigma\left(\frac{1+\zeta^*}{\zeta^*}\right)^2\left(\hat{\tilde{c}}_1 - \left[(1-\alpha)\left(\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}_1^*\right) + \alpha\hat{\bar{\tilde{k}}}_1\right]\right)\right].$$

Combining these with (73), (74), (75), and (82) implies

$$\hat{\tilde{c}}_1 = \tilde{\alpha}\mathbb{E}_1\hat{\theta}_2 + (1-\alpha)\left(\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}_1^*\right) + \alpha\hat{\bar{\tilde{k}}}_1 - \frac{\varsigma}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}_1^* - \hat{\ell}_1\right),$$

which we can substitute into the previous result to give

$$\hat{s}_1 = \frac{1}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}_1^* - \hat{\ell}_1\right) + \frac{\varsigma\left(\frac{1+\zeta^*}{\zeta^*}\right)^2}{\frac{\alpha}{1-\alpha} + \sigma\left(1 - \left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2\right)}\tilde{\alpha}\mathbb{E}_1\hat{\theta}_2.$$

Substituting these into (86) implies

$$\mathbb{E}_1\hat{\theta}_2 \;=\; \hat{\theta}_1 \;+\; (1\;-\;\beta)\frac{\zeta^*}{1+\zeta^*}(\sigma\;-\;1)\frac{1-\alpha}{\alpha}\frac{1-\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}_1 - \hat{\ell}_1^*\right),$$

while substituting these into (87) implies

$$\widehat{nfa}_1 = a\left[\hat{\theta}_1 + (1-\beta)\frac{\zeta^*}{1+\zeta^*}(\sigma-1)\frac{1-\alpha}{\alpha}\frac{1-\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}_1 - \hat{\ell}_1^*\right) - \right.$$

$$\left. \frac{\zeta^*}{1+\zeta^*}\frac{1}{1-\alpha}\hat{s}_2 \right]$$

and substituting these into (83) and making use of (73) implies

$$\mathbb{E}_1\hat{r}_2^k = -(1-\alpha)\left(\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}_1^*\right) + \frac{\varsigma}{\frac{\alpha}{1-\alpha}+\sigma+(1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}_1^* - \hat{\ell}_1\right).$$

Then (88)-(91) imply

$$\Delta\hat{P}_1 = \frac{1}{\phi}\left(\mathbb{E}_1\hat{r}_2^k - \hat{\omega}_1\right),$$

$$\Delta\hat{P}_1^* = \frac{1}{\phi}\left(\mathbb{E}_1\hat{r}_2^k - \frac{1+\zeta^*}{\zeta^*}\frac{\varsigma}{\frac{\alpha}{1-\alpha}+\sigma+(1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}_1^* - \hat{\ell}_1\right)\right),$$

so (95) together with the first implies

$$\hat{r}_1 = -\frac{1}{\phi}\Bigg( -(1-\alpha)\left(\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}_1^*\right) + \frac{\varsigma}{\frac{\alpha}{1-\alpha}+\sigma+(1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}_1^* - \hat{\ell}_1\right) - \hat{\omega}_1\Bigg),$$

while (96) together with the second implies

$$\hat{r}_1^* = -\frac{1}{\phi}\Bigg( -(1-\alpha)\left(\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}_1^*\right) - \frac{1}{\zeta^*}\frac{\varsigma}{\frac{\alpha}{1-\alpha}+\sigma+(1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}_1^* - \hat{\ell}_1\right)\Bigg),$$

Moreover, (93) and (94) imply

$$\hat{r}_1^k = -\frac{\varsigma}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}_1^* - \hat{\ell}_1\right) - \frac{\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}{\frac{\alpha}{1-\alpha} + \sigma\left(1 - \left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2\right)}\tilde{\alpha}\mathbb{E}_1\hat{\theta}_2 + (1-\alpha)\left(\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}_1^*\right) - (1-\alpha)\hat{\hat{\bar{k}}}_1.$$

## B.2 Pricing kernels, portfolios, and risk premia

Now a second order approximation of the optimal portfolio choice conditions in period 0 implies

$$\begin{aligned}
\mathbb{E}_0\left[\hat{r}_1^k - \hat{r}_1\right] &+ \text{Jensen terms} \\
&= \hat{\omega}_0 - \mathbb{E}_0\left[\hat{\hat{m}}_{0,1} - \gamma\sigma^z\hat{\epsilon}_1^z\right]\left[\hat{r}_1^k - \hat{r}_1\right], \\
\mathbb{E}_0\left[\hat{r}_1^* - \Delta\hat{q}_1 - \hat{r}_1\right] &+ \text{Jensen terms} \\
&= \hat{\omega}_0 - \mathbb{E}_0\left[\hat{\hat{m}}_{0,1} - \gamma\sigma^z\hat{\epsilon}_1^z\right]\left[\hat{r}_1^* - \Delta\hat{q}_1 - \hat{r}_1\right],
\end{aligned}$$

and analogously in Foreign, where *Jensen terms* reflect the component of excess returns which do not reflect safety shocks nor the covariance with agents' pricing kernels (and instead reflect the variance of returns).

In period 1, the log deviation in the representative Home household's pricing kernel is given by

$$\hat{\hat{m}}_{0,1} = -\hat{\hat{c}}_1 + (1-\gamma)\hat{\hat{v}}_1.$$

Now,

$$\hat{\hat{v}}_1 = (1-\beta)\hat{\hat{c}}_1 - (1-\beta)(1-\tau)(1-\alpha)\hat{\ell}_1 + \beta\hat{\hat{ce}}_1,$$

where $\tau$ denotes the labor wedge in the deterministic steady-state. By the results of the previous subsection,

$$\hat{\hat{c}}_1 = \tilde{\alpha}\mathbb{E}_1\hat{\theta}_2 + (1-\alpha)\left(\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}_1^*\right) + \alpha\hat{\hat{\bar{k}}}_1 - \frac{\varsigma}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}_1^* - \hat{\ell}_1\right),$$

while the log-linearized certainty equivalent is given by

$$
\begin{aligned}
\hat{\tilde{ce}}_1 &= \mathbb{E}_1 \hat{\tilde{v}}_2, \\
&= \tilde{\alpha}\mathbb{E}_1\hat{\theta}_2 + \alpha\hat{\bar{\tilde{k}}}_1,
\end{aligned}
$$

where the second equality uses (77). Combining these implies

$$
\hat{\tilde{v}}_1 = \tilde{\alpha}\mathbb{E}_1\hat{\theta}_2 - \alpha\sigma^z\hat{\epsilon}^z_1 +
$$

$$
\tau(1-\beta)(1-\alpha)\hat{\ell}_1 + (1-\beta)\left((1-\alpha)\frac{\zeta^*}{1+\zeta^*} - \frac{\varsigma}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\right)\left(\hat{\ell}^*_1 - \hat{\ell}_1\right).
$$

Combining the previous results, we obtain

$$
\begin{aligned}
\hat{\tilde{m}}_{0,1} - \gamma\sigma^z\hat{\epsilon}^z_1 &= -\gamma\left[\tilde{\alpha}\mathbb{E}_1\hat{\theta}_2 + (1-\alpha)\sigma^z\hat{\epsilon}^z_1\right] \\
&\quad - (1-\alpha)\left(\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}^*_1\right) + \frac{\varsigma}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}^*_1 - \hat{\ell}_1\right)
\end{aligned}
$$

$$
+(1-\gamma)(1-\beta)\left[\tau(1-\alpha)\hat{\ell}_1 + \left((1-\alpha)\frac{\zeta^*}{1+\zeta^*} - \frac{\varsigma}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\right)\left(\hat{\ell}^*_1 - \hat{\ell}_1\right)\right].
$$

Analogous steps in Foreign yield

$$
\begin{aligned}
\hat{\tilde{m}}^*_{0,1} - \gamma^*\sigma^z\hat{\epsilon}^z_1 &= -\gamma^*\left[-\frac{1}{\zeta^*}\tilde{\alpha}\mathbb{E}_1\hat{\theta}_2 + (1-\alpha)\sigma^z\hat{\epsilon}^z_1\right] \\
&\quad - (1-\alpha)\left(\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}^*_1\right) - \frac{1}{\zeta^*}\frac{\varsigma}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}^*_1 - \hat{\ell}_1\right)
\end{aligned}
$$

$$
+(1-\gamma^*)(1-\beta)\left[\tau(1-\alpha)\hat{\ell}^*_1 - \left((1-\alpha)\frac{1}{1+\zeta^*} - \frac{1}{\zeta^*}\frac{\varsigma}{\frac{\alpha}{1-\alpha} + \sigma + (1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\right)\left(\hat{\ell}^*_1 - \hat{\ell}_1\right)\right].
$$

Now, the present environment is *locally complete* as defined by Coeurdacier and Gourinchas (2016). It follows that the equilibrium portfolios ensure that

$$
\hat{\tilde{m}}_{0,1} - \gamma\sigma^z\hat{\epsilon}^z_1 = \hat{\tilde{m}}^*_{0,1} - \gamma^*\sigma^z\hat{\epsilon}^z_1 + \Delta\hat{q}_1.
$$

Substituting in using the above results and those of the previous section and collecting terms, we obtain

$$\left(\frac{q^k k}{a}-1\right)\left(\hat{r}_1^k-\hat{r}_1\right)+\frac{b_F}{a}\left(\hat{r}_1^*-\hat{q}_1-\hat{r}_1\right)-\beta\frac{\zeta^*}{1+\zeta^*}\frac{b_{H,s}^g}{a}\hat{\omega}_1 =$$
$$\frac{1}{\Gamma}(\gamma^*-\gamma)(1-\alpha)\sigma^z\hat{\epsilon}_1^z+$$
$$\frac{1}{\Gamma}(\gamma^*-\gamma)\tau(1-\beta)(1-\alpha)\left(\frac{1}{1+\zeta^*}\hat{\ell}_1+\frac{\zeta^*}{1+\zeta^*}\hat{\ell}_1^*\right)+$$
$$(1-\beta)\frac{\zeta^*}{1+\zeta^*}(\sigma-1)\frac{1-\alpha}{\alpha}\frac{1-\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}{\frac{\alpha}{1-\alpha}+\sigma+(1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}_1^*-\hat{\ell}_1\right)-$$
$$\frac{1}{\Gamma}\left(\frac{1}{\zeta^*}(\gamma^*-1)+(\gamma-1)\right)(1-\beta)(1-\tau)(1-\alpha)\frac{\zeta^*}{1+\zeta^*}\left(\hat{\ell}_1^*-\hat{\ell}_1\right)+$$
$$\frac{1}{\Gamma}\left(\frac{1}{\zeta^*}(\gamma^*-1)+(\gamma-1)\right)(1-\beta)\frac{\varsigma}{\frac{\alpha}{1-\alpha}+\sigma+(1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}\left(\hat{\ell}_1^*-\hat{\ell}_1\right), \quad (99)$$

where

$$\Gamma \equiv \left[\gamma+\frac{1}{\zeta^*}\gamma^*+\frac{\varsigma^2\left(\frac{1+\zeta^*}{\zeta^*}\right)^3}{\frac{\alpha}{1-\alpha}+\sigma\left(1-\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2\right)}\right]\tilde{\alpha}.$$

Thus, international risk sharing calls for Home wealth (on the left-hand side) to rise with:

- *productivity*, provided $\gamma^*>\gamma$: since a positive TFP shock raises aggregate production and thus consumption;
- *aggregate employment*, provided $\tau(\gamma^*-\gamma)>0$: since an increase in labor raises welfare;
- *Foreign employment less Home employment*, if:
  - $(\sigma-1)\left(1-\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2\right)>0$: since this implies that Foreign labor income rises relative to Home labor income; or
  - $\left(\frac{1}{\zeta^*}(\gamma^*-1)+(\gamma-1)\right)\left(\frac{\varsigma}{\frac{\alpha}{1-\alpha}+\sigma+(1-\sigma)\left(\varsigma\frac{1+\zeta^*}{\zeta^*}\right)^2}-(1-\tau)(1-\alpha)\frac{\zeta^*}{1+\zeta^*}\right)>0$: since this implies that Home requires more wealth when its real exchange rate

appreciates, despite the larger disutility of labor in Foreign.

Finally, the equilibrium risk premium on Foreign bonds relative to dollar bonds is given by the covariance of (the negative of) the log deviation in any agent's pricing kernel with the excess log return.

## B.3 Proofs

### B.3.1 Propositions 1-3

Now consider the case with identical portfolios (so $q^k k = a$, $b_F = 0$, and $b_H = 0$) and zero safe debt issued by the Home government ($b^g_{H,s} = 0$) assumed in Propositions 1 and 2. Thus $\hat{\theta}_1 = 0$. Further, since $\varsigma \to \frac{\zeta^*}{1+\zeta^*}$, we have that $\mathbb{E}_1 \hat{\theta}_2 = 0$.

Then in the further case absent nominal rigidity and with $\nu \to 0$, the claims follow immediately from the above results given $\hat{\ell}_1 = \hat{\ell}^*_1 = 0$.

Alternatively in the case with wages set one period ahead, we can substitute the above results into (97) and (98) and solve for $\hat{\ell}_1$ and $\hat{\ell}^*_1$, yielding

$$\hat{\ell}_1 = -\frac{1}{\alpha + \frac{1}{\phi}(1-\alpha)} \frac{1}{\phi} \hat{\omega}_1 - \frac{1}{\alpha + \frac{1}{\phi}(1-\alpha)} (1-\alpha) \hat{\bar{\bar{k}}}_1,$$

$$\hat{\ell}^*_1 = -\frac{1}{\alpha + \frac{1}{\phi}(1-\alpha)} (1-\alpha) \hat{\bar{\bar{k}}}_1.$$

Thus, in response to a safety shock, $\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}^*_1 \propto -\hat{\omega}_1$ and $\hat{\ell}^*_1 - \hat{\ell}_1 \propto \hat{\omega}_1$ as claimed. We note that the limit of complete home bias $\varsigma \to \frac{\zeta^*}{1+\zeta^*}$ implies that $\ell^*_1$ is unaffected by a safety shock (up to first order), but for $\varsigma < \frac{\zeta^*}{1+\zeta^*}$ it is straightforward to show that $\hat{\ell}^*_1 \propto -\hat{\omega}_1$ for $\sigma$ sufficiently low and $\hat{\ell}^*_1 \propto \hat{\omega}_1$ for $\sigma$ sufficiently high.

### B.3.2 Propositions 4-5

When $\varsigma \to \frac{\zeta^*}{1+\zeta^*}$, the international portfolios solve

$$\left(\frac{q^k k}{a} - 1\right)\left(\hat{r}^k_1 - \hat{r}_1\right) + \frac{b_F}{a}\left(\hat{r}^*_1 - \hat{q}_1 - \hat{r}_1\right) - \beta \frac{\zeta^*}{1+\zeta^*} \frac{b^g_{H,s}}{a} \hat{\omega}_1 =$$

$$\frac{1}{\Gamma}(\gamma^* - \gamma)(1-\alpha)\sigma^z \hat{\epsilon}^z_1 +$$

$$\frac{1}{\Gamma}(\gamma^* - \gamma)\tau(1-\beta)(1-\alpha)\left(\frac{1}{1+\zeta^*}\hat{\ell}_1 + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}^*_1\right) +$$

$$\frac{1}{\Gamma}\left(\frac{1}{\zeta^*}(\gamma^*-1)+(\gamma-1)\right)(1-\beta)\tau(1-\alpha)\frac{\zeta^*}{1+\zeta^*}\left(\hat{\ell}_1^*-\hat{\ell}_1\right).$$

For arbitrary portfolios, it is straightforward to show that

$$\frac{1}{1+\zeta^*}\hat{\ell}_1+\frac{\zeta^*}{1+\zeta^*}\hat{\ell}_1^*=-\frac{1}{\alpha+\frac{1}{\phi}(1-\alpha)}\frac{1}{\phi}\frac{1}{1+\zeta^*}\hat{\omega}_1+\frac{1}{\alpha+\frac{1}{\phi}(1-\alpha)}(1-\alpha)\sigma^z\hat{\epsilon}_1^z,$$
$$\hat{\ell}_1^*-\hat{\ell}_1=\frac{1}{\alpha+\frac{1}{\phi}(1-\alpha)}\frac{1}{\phi}\hat{\omega}_1-\frac{1}{\alpha+\frac{1}{\phi}(1-\alpha)}\frac{1+\zeta^*}{\zeta^*}\alpha\mathbb{E}_1\hat{\theta}_2,$$

generalizing the results given in the proof of Proposition 1 to arbitrary portfolios. Substituting these into the expression for international portfolios, and using that $\mathbb{E}_1\hat{\theta}_2=\left(\frac{q^k k}{a}-1\right)\left(\hat{r}_1^k-\hat{r}_1\right)+\frac{b_{F-1}}{a}\left(\hat{r}_1^*-\hat{q}_1-\hat{r}_1\right)-\beta\frac{\zeta^*}{1+\zeta^*}\frac{b_{H,s}^g}{a}\hat{\omega}_1$, we obtain

$$\left(\frac{q^k k}{a}-1\right)\left(\hat{r}_1^k-\hat{r}_1\right)+\frac{b_F}{a}\left(\hat{r}_1^*-\hat{q}_1-\hat{r}_1\right)-\beta\frac{\zeta^*}{1+\zeta^*}\frac{b_{H,s}^g}{a}\hat{\omega}_1=$$
$$\frac{1}{\Gamma'}(\gamma^*-\gamma)(1-\alpha)\left[1+\tau(1-\beta)\frac{1}{\alpha+\frac{1}{\phi}(1-\alpha)}(1-\alpha)\right]\sigma^z\hat{\epsilon}_1^z-$$
$$\frac{1}{\Gamma'}(\gamma^*-\gamma)\tau(1-\beta)(1-\alpha)\frac{1}{\alpha+\frac{1}{\phi}(1-\alpha)}\frac{1}{\phi}\frac{1}{1+\zeta^*}\hat{\omega}_1+$$
$$\frac{1}{\Gamma'}\left(\frac{1}{\zeta^*}(\gamma^*-1)+(\gamma-1)\right)(1-\beta)\tau(1-\alpha)\frac{\zeta^*}{1+\zeta^*}\frac{1}{\alpha+\frac{1}{\phi}(1-\alpha)}\frac{1}{\phi}\hat{\omega}_1,$$

where

$$\Gamma'\equiv\Gamma+\left(\frac{1}{\zeta^*}(\gamma^*-1)+(\gamma-1)\right)(1-\beta)\tau(1-\alpha)\frac{1}{\alpha+\frac{1}{\phi}(1-\alpha)}\alpha>0.$$

Thus, in comparative statics with respect to $\frac{\gamma^*}{\gamma}$ holding fixed $\gamma+\frac{1}{\zeta^*}\gamma^*$, on the right-hand side it is clear that only the first two terms vary; the third, capturing the effect of the relative labor response on international portfolios, is constant. When $\varsigma\to\frac{\zeta^*}{1+\zeta^*}$, it is straightforward to show that the earlier results imply

$$\hat{r}_1^*-\hat{q}_1-\hat{r}_1=-\left(\frac{1}{\phi}+\left(1-\frac{1}{\phi}\right)\frac{1-\alpha}{\alpha+\frac{1}{\phi}(1-\alpha)}\frac{1}{\phi}\right)\hat{\omega}_1-\frac{1+\zeta^*}{\zeta^*}\frac{\frac{1}{\phi}(1-\alpha)}{\alpha+\frac{1}{\phi}(1-\alpha)}\mathbb{E}_1\hat{\theta}_2,$$
$$\hat{r}_1^k-\hat{r}_1=-\left(\frac{1}{\phi}+\left(1-\frac{1}{\phi}\right)\frac{\frac{1}{\phi}(1-\alpha)}{\alpha+\frac{1}{\phi}(1-\alpha)}\right)\hat{\omega}_1+$$

$$\left[(1-\alpha)+\left(1-\frac{1}{\phi}\right)(1-\alpha)^2\frac{1}{\alpha+\frac{1}{\phi}(1-\alpha)}\right]\sigma^z\hat{\epsilon}_1^z-$$
$$\frac{\frac{1}{\phi}(1-\alpha)}{\alpha+\frac{1}{\phi}(1-\alpha)}\mathbb{E}_1\hat{\theta}_2,$$
$$\mathbb{E}_1\hat{\theta}_2=\left(\frac{q^k k}{a}-1\right)\left(\hat{r}_1^k-\hat{r}_1\right)+\frac{b_F}{a}\left(\hat{r}_1^*-\hat{q}_1-\hat{r}_1\right)-\beta\frac{\zeta^*}{1+\zeta^*}\frac{b_{H,s}^g}{a}\hat{\omega}_1,$$
$$=\frac{1}{\Delta}\left(\left[\frac{1}{\phi}+\left(1-\frac{1}{\phi}\right)\frac{\frac{1}{\phi}(1-\alpha)}{\alpha+\frac{1}{\phi}(1-\alpha)}\right]\frac{b_H}{a}-\beta\frac{\zeta^*}{1+\zeta^*}\frac{b_{H,s}^g}{a}\right)\hat{\omega}_1+$$
$$\frac{1}{\Delta}\left(\frac{q^k k}{a}-1\right)\left[(1-\alpha)+\left(1-\frac{1}{\phi}\right)(1-\alpha)^2\frac{1}{\alpha+\frac{1}{\phi}(1-\alpha)}\right]\sigma^z\hat{\epsilon}_1^z,$$

where we define $\Delta\equiv 1+\left[\left(\frac{q^k k}{a}-1\right)+\frac{b_F}{a}\frac{\zeta^*}{1+\zeta^*}\right]\frac{\frac{1}{\phi}(1-\alpha)}{\alpha+\frac{1}{\phi}(1-\alpha)}$, which evaluates to one in the case with symmetric portfolios. By the method of undetermined coefficients, it follows that at the point of symmetric portfolios,

$$\frac{dk}{db_{H,s}^g}=0,\ \frac{db_H}{db_{H,s}^g}>0,$$
$$\left.\frac{dk}{d[\gamma^*/\gamma]}\right|_{\gamma+\frac{1}{\zeta^*}\gamma^*}>0,\ \left.\frac{db_H}{d[\gamma^*/\gamma]}\right|_{\gamma+\frac{1}{\zeta^*}\gamma^*}<0,$$

where the second line holds $\gamma+\frac{1}{\zeta^*}\gamma^*$ fixed and assumes $\tau>0$.

Finally, consider

$$-\mathbb{E}_0\left[\hat{m}_{0,1}-\gamma\sigma^z\hat{\epsilon}_1^z\right]\left[\hat{r}_1^*-\hat{q}_1-\hat{r}_1\right].$$

Assuming that productivity and safety are independent, this can be expressed as a linear combination of $(\sigma^z)^2$ and $(\sigma^\omega)^2$. The above results imply that the coefficient on the former takes the sign of $\gamma-\gamma^*$ and the coefficient on the latter is positive, completing the claim.

## B.4 Foreign-only demand for safe dollar bonds

We now demonstrate that our analysis is robust to Foreign-only demand for safe dollar bonds, as discussed in section 3.5.

We augment the model with a non-negativity constraint on households' positions in the safe dollar bond, reflecting the assumption that they cannot create these safe assets (only the Home government can). This is irrelevant when the demand shock for the safe dollar bond is global, since in that case all agents hold the outstanding supply of safe dollar debt issued by the Home government.

The Home representative agent's FOC for safe dollar bonds, other dollar bonds, Foreign bonds, and capital are now

$$
\begin{aligned}
1-\mu_t &= \mathbb{E}_t m_{t,t+1}(1+i_t)\frac{P_t}{P_{t+1}},\\
1 &= \mathbb{E}_t m_{t,t+1}(1+\iota_t)\frac{P_t}{P_{t+1}},\\
1 &= \mathbb{E}_t m_{t,t+1}\frac{q_t}{q_{t+1}}(1+r^*_{t+1}),\\
1 &= \mathbb{E}_t m_{t,t+1}(1+r^k_{t+1}),
\end{aligned}
$$

where $\mu_t \geq 0$ is the (scaled) multiplier on the non-negativity constraint on safe dollar bonds and the last three FOCs are as in the baseline model.

The Foreign representative agent's FOC for safe dollar bonds, other dollar bonds, Foreign bonds, and capital are

$$
\begin{aligned}
1-c^*_t\Omega^{*'}_t(B^*_{Ht,s}/(E^{-1}_t P^*_t))/\Omega^*_t(B^*_{Ht,s}/(E^{-1}_t P^*_t)) &= \mathbb{E}_t m^*_{t,t+1}\frac{q_{t+1}}{q_t}(1+i_t)\frac{P_t}{P_{t+1}},\\
1 &= \mathbb{E}_t m^*_{t,t+1}\frac{q_{t+1}}{q_t}(1+\iota_t)\frac{P_t}{P_{t+1}},\\
1 &= \mathbb{E}_t m^*_{t,t+1}(1+r^*_{t+1}),\\
1 &= \mathbb{E}_t m^*_{t,t+1}\frac{q_{t+1}}{q_t}(1+r^k_{t+1}),
\end{aligned}
$$

as in the baseline model. Given the same functional form for $\Omega^*_t$ as in the baseline model,

$$
c^*_t\Omega^{*'}_t(B^*_{Ht,s}/(E^{-1}_t P^*_t))/\Omega^*_t(B^*_{Ht,s}/(E^{-1}_t P^*_t)) = \omega^{d*}_t - \frac{B^*_{Ht,s}}{E^{-1}_t P^*_t c^*_t},
$$

where $\omega^{d*}_t$ is the Foreign latent demand shock for safe dollar bonds. We assume for expositional simplicity that the non-negativity constraint on safe dollar bonds for Foreign agents does not bind, which will be the case when $\omega^{d*}_t$ is sufficiently high.

Now note that each agent's FOCs for safe and other dollar bonds imply

$$
\frac{1+i_t}{1-\mu_t} = 1+\iota_t = \frac{1+i_t}{1-\left(\omega^{d*}_t - \frac{B^*_{Ht,s}}{E^{-1}_t P^*_t c^*_t}\right)}.
$$

It follows that

$$\mu_t = \omega_t^{d*} - \frac{B^*_{Ht,s}}{E_t^{-1} P_t^* c_t^*} \equiv \omega_t.$$

That is, the Lagrange multiplier on the non-negativity constraint for safe dollar bonds for Home agents must be equated to the convenience yield perceived by Foreign agents for these bonds. The equilibrium system is thus exactly as in the baseline model, except with $B_{Ht,s} = 0$ (zero holdings of safe dollar bonds by Home agents). It follows that the propagation of Foreign demand shocks for safe dollar bonds is exactly like global demand shocks for these assets, up to any differences in seignorage because in this case only Foreign agents hold them.[55]

Note in particular that despite the fact that only foreigners have a special demand for safe dollar bonds, the international risk sharing conditions are unchanged. That is, we still have that

$$\begin{aligned}
\mathbb{E}_t m_{t,t+1} \frac{1 + r_{t+1}}{1 - \omega_t} &= \mathbb{E}_t m^*_{t,t+1} \frac{q_{t+1}}{q_t} \frac{1 + r_{t+1}}{1 - \omega_t}, \\
\mathbb{E}_t m_{t,t+1} \frac{q_t}{q_{t+1}} (1 + r^*_{t+1}) &= \mathbb{E}_t m^*_{t,t+1} (1 + r^*_{t+1}), \\
\mathbb{E}_t m_{t,t+1} (1 + r^k_{t+1}) &= \mathbb{E}_t m^*_{t,t+1} \frac{q_{t+1}}{q_t} (1 + r^k_{t+1}),
\end{aligned}$$

With one more asset than shocks, this simplified environment is locally complete as in the prior subsections.

## B.5 Distinct capital stocks and sticky prices

We finally provide more detail on the sensitivity of our results to the alternative environment with distinct capital stocks and sticky prices discussed in section 3.5.

### B.5.1 Environment

We first outline the changes relative to the baseline environment to accommodate distinct capital stocks and sticky prices.

The representative Home household's budget constraint is now

$$P_{Ht} c_{Ht} + E_t^{-1} P^*_{Ft} c_{Ft} + B_{Ht,s} + B_{Ht,o} + E_t^{-1} B_{Ft} + Q^k_t k_{Ht} + E_t^{-1} Q^{k*}_t k_{Ft} \leq$$

[55] When the Home government issues zero debt ($B^g_{Ht,s} = 0$), there is no seignorage and hence the propagation of Foreign demand shocks is identical to global demand shocks.

$$(1+i_{t-1})B_{Ht-1,s} + (1+\iota_{t-1})B_{Ht-1,o} + E_t^{-1}(1+i_{t-1}^*)B_{Ft-1} + \\ (\Pi_t + Q_t^k)k_{Ht-1} + E_t^{-1}(\Pi_t^* + Q_t^{k*})k_{Ft-1} + W_t\ell_t + T_t,$$

and the representative Foreign household's budget constraint is analogous.[56] In particular, Home agents now choose distinct positions in capital used at Home, $k_{Ht}$, and capital used at Foreign, $k_{Ft}$, which offer distinct payoffs and trade at distinct prices. Consumption of goods produced in each country is now a CES aggregator across varieties with elasticity of substitution $\epsilon$. We no longer need to assume distinct labor varieties.

In each country, there are now intermediate good producers and final goods retailers.[57] The representative Home intermediate good producer earns profits

$$P_t^i\left(z_t\ell_t\right)^{1-\alpha}\left(\bar{k}\right)^{1-\alpha} - W_t\ell_t - \Pi_t^i\bar{k}, \tag{100}$$

where $P_t^i$ is the price of the intermediate good, $\Pi_t^i$ is the rental rate on capital, and $\bar{k}$ now denotes the fixed capital stock at Home. A unit measure of monopolistically competitive retailers at Home (indexed by $j$) purchase the domestic intermediate good and earn a discounted stream of profits

$$J_t(j) = \Pi_t(j) + \mathbb{E}_t M_{t,t+1}^f J_{t+1}(j),$$

where the stochastic discount factor $M_{t,t+1}^f$ is described below. Flow profits are

$$\Pi_t(j) = \left(P_{Ht}(j) - P_t^i\right) y_{Ht}(j)$$

given the retailer's global demand (which it internalizes)

$$y_{Ht}(j) = \left(\frac{P_{Ht}(j)}{P_t}\right)^{-\epsilon}\left(c_{Ht} + \zeta^* c_{Ht}^*\right),$$

and we already make use of the assumption that the retailer engages in producer currency pricing. Retailers either set prices flexibly or one period in advance. The

[56] We ignore disaster risk and depreciation because we are studying this environment analytically in the absence of such features, following the maintained assumptions of section 3. It would be straightforward to add them in a quantitative analysis of this environment.

[57] Again, we abstract from capital good producers because we are studying this environment analytically in the absence of capital accumulation, following the assumptions of section 3.

problems of intermediate good producers and final good retailers in Foreign are analogous given Foreign capital stock $\bar{k}^*$, except that Foreign intermediate good producers are also subject to an additional productivity shock $z_{Ft}$. Relative to the baseline environment studied in section 3, we need one more shock to pin down portfolios given that there is trade in one more asset. We assume this productivity shock is fully transitory ($\rho^F = 0$) analogous to our assumption for safety shocks in section 3.

We assume that households own a share in all retailers from a given country in proportion to their ownership of capital in that country. That is, the aggregate profits earned by owners of Home capital are

$$\Pi_t \bar{k} = \Pi_t^i \bar{k} + \int_0^1 \Pi_t(j) dj,$$

and analogously for owners of Foreign capital. With incomplete markets, there is the usual problem that there is no standard way to describe firms' stochastic discount factor in making dynamic decisions. But because asset markets are *locally* complete in this environment studied analytically, the results which follow will be the same using any owner's stochastic discount factor (or weighted average).

Intermediate good market clearing is now

$$\int_0^1 y_{Ht}(j) dj = (z_t \ell_t)^{1-\alpha} \left(\bar{k}\right)^{1-\alpha},$$
$$\int_0^1 y_{Ft}^*(j^*) dj^* = (z_t z_{Ft} \zeta^* \ell_t^*)^{1-\alpha} \left(\bar{k}^*\right)^{1-\alpha},$$

final good market clearing is now

$$c_{Ht}(j) + \zeta^* c_{Ht}^*(j) = y_{Ht}(j), \ \ \forall j,$$
$$c_{Ft}(j^*) + \zeta^* c_{Ft}^*(j^*) = y_{Ft}^*(j^*), \ \ \forall j^*,$$

and capital market clearing is now

$$k_{Ht} + \zeta^* k_{Ht}^* = \bar{k},$$
$$k_{Ft} + \zeta^* k_{Ft}^* = \bar{k}^*.$$

All other features of the environment are unchanged from that studied in section 3, except we do not impose the limit of an infinite Frisch elasticity $\nu$. We refer to this as the *simplified environment with distinct capital shocks* in what follows.

### B.5.2 Results

We now provide analogs of Propositions 1-5 in this environment. We exclude the proofs for brevity but they are available on request.

Propositions 1 and 2 characterize the dynamics of prices, quantities, and wealth provided that $\mathbb{E}_t\hat{\theta}_{t+1} = 0$ on impact of all shocks at $t$. Proposition 3 demonstrates that $\mathbb{E}_t\hat{\theta}_{t+1} = 0$ is implied by efficient risk sharing in the natural benchmark with $\gamma = \gamma^* = 1$, so that the earlier results apply at least in neighborhood of this benchmark. Finally, Propositions 4 and 5 characterize the comparative statics of portfolios and risk premia around this benchmark.

We first consider the analog of Proposition 1 in this environment:

**Proposition 1.** *Consider the simplified environment with distinct capital stocks and assume $\mathbb{E}_t\hat{\theta}_{t+1} = 0$ on impact of all shocks at $t$. If prices are flexible, then on impact of a positive safety shock:*

- *the Home CPI declines ($\Delta\hat{P}_t = -\frac{1}{\phi}\hat{\omega}_t$); and*
- *the Home real interest rate declines ($\mathbb{E}_t\hat{r}_{t+1} = -\hat{\omega}_t$);*
- *the Home real exchange rate and employment in each country are unchanged ($\hat{q}_t = \hat{\ell}_t = \hat{\ell}_t = 0$).*

*If prices are set one period in advance, then on impact of a positive safety shock:*

- *the Home CPI is unchanged ($\Delta\hat{P}_t = 0$);*
- *the Home real interest rate is unchanged ($\mathbb{E}_t\hat{r}_{t+1} = 0$);*
- *the Home real exchange rate appreciates ($\hat{q}_t = \hat{\omega}_t$); and*
- *global employment falls, disproportionately so in Home ($\frac{1}{1+\zeta^*}\hat{\ell}_t + \frac{\zeta^*}{1+\zeta^*}\hat{\ell}_t^* = -\frac{1}{1-\alpha}\hat{\omega}_t$ and $\hat{\ell}_t - \hat{\ell}_t^* = -\frac{1}{1-\alpha}\hat{\omega}_t$).*

Thus, the transmission of safety shocks to prices and quantities is largely unchanged from Proposition 1.

We next consider the analog of Proposition 2 in this environment, defining the real returns to each capital stock as

$$1 + r_t^k = \frac{\Pi_t + Q_t^k}{Q_{t-1}^k} \frac{P_{t-1}}{P_t},$$
$$1 + r_t^{k*} = \frac{\Pi_t^* + Q_t^{k*}}{Q_{t-1}^{k*}} \frac{P_{t-1}^*}{P_t^*}.$$

We obtain:

**Proposition 2.** *Consider the simplified environment with distinct capital stocks and assume $\mathbb{E}_t \hat{\theta}_{t+1} = 0$ on impact of all shocks at $t$. Then on impact of a positive safety shock:*

- *the real return on dollar bonds rises if prices are flexible ($\hat{r}_t = \frac{1}{\phi}\omega_t$) but is unchanged if prices are set in advance ($\hat{r}_t = 0$);*
- *the real return on Foreign bonds is unaffected if prices are flexible ($\hat{r}_t^* - \hat{q}_t = 0$) but falls if prices are set in advance ($\hat{r}_t^* - \Delta\hat{q}_t = -\hat{\omega}_t$);*
- *the real return on Home capital is unaffected if prices are flexible ($\hat{r}_t^k = 0$) but may rise or fall if prices are set in advance ($\hat{r}_t^k = \left[\frac{(1-\beta)(1-\tau)\left(\frac{1}{\nu}+1\right)}{1-(1-\alpha)(1-\tau)} - 1\right]\hat{\omega}_t$);*
- *the real return on Foreign capital is unaffected if prices are flexible ($\hat{r}_t^{k*} - \hat{q}_t = 0$) but falls if prices are set in advance ($\hat{r}_t^{k*} - \hat{q}_t = -\hat{\omega}_t$).*

The ambiguous response of the real return on Home capital owes to competing effects of a safety shock given sticky prices: it reduces Home output, but raises the Home mark-up $P_{Ht}/P_t^i$. What is definitive, however, is that the real return on Home capital exceeds that on Foreign capital ($\hat{r}_t^k - \hat{r}_t^{k*} + \hat{q}_t \propto \hat{\omega}_t$.).

This plays an important role in the following new result:

**Proposition 3.** *Consider the simplified environment with distinct capital stocks and prices set one period in advance. When $\gamma = \gamma^* = 1$, the equilibrium portfolios are:*

- $\frac{q^k k_H}{a} = 1$, $\frac{q^k k_F}{a} = 0$, $\frac{b_F}{a} = -\beta \frac{\zeta^*}{1+\zeta^*} \frac{b_{H,s}^g}{a}$, *and* $\frac{b_H}{a} = \beta \frac{\zeta^*}{1+\zeta^*} \frac{b_{H,s}^g}{a}$*;*
- *implying that Home's financial wealth share $\theta_t$ rises on impact of a positive safety shock, falls on impact of a positive Foreign productivity shock, and is unchanged on impact of a global productivity shock at $t$;*

- *while its net foreign assets $nfa_t$ and expected future wealth share $\mathbb{E}_t\theta_{t+1}$ are unchanged on impact of all shocks at $t$.*

In this benchmark case efficient risk sharing calls for Home's overall wealth share (inclusive of labor income) to be constant in response to all shocks.[58] When agents fully own their domestic capital stock, this ensures that returns on financial assets offset the changes in labor income in response to safety and Foreign productivity shocks, and wealth does not redistribute across countries in response to global productivity shocks. This builds on Engel and Matsumoto (2009) and Coeurdacier and Gourinchas (2016). And if the Home government earns seignorage revenues from safety shocks, an offsetting position of Home agents long Foreign bonds by shorting dollar bonds can neutralize these revenues.

The next result characterizes portfolios around this benchmark, an analog of Proposition 4 in this environment:

**Proposition 4.** *Consider the simplified environment with distinct capital stocks and prices set one period in advance. At least around the case with $\gamma = \gamma^* = 1$, $b^g_{H,s} < 0$, and the same, positive steady-state labor wedge in each country:*

- *Home's portfolio share in Home capital (Foreign capital, dollar bonds) is unaffected (unaffected, falls) with $-b^g_{H,s}$; and*
- *Home's portfolio share in Home capital (Foreign capital, dollar bonds) rises (rises, falls) with $\frac{\gamma^*}{\gamma}$, holding $\gamma + \frac{1}{\zeta^*}\gamma^*$ fixed.*

That is, at least around the benchmark with $\gamma = \gamma^* = 1$ and $b^g_{H,s} < 0$, efficient risk sharing calls for Home's overall wealth share to fall upon a positive safety shock as it gets more risk tolerant versus Foreign. This is implemented by Home owning a leveraged portfolio of Home capital and Foreign capital financed by dollar bonds.

An interesting implication of the last two results is that even if Home's overall wealth share and thus net foreign assets fall upon a safety shock because it is insuring Foreign, it can still be the case that its *financial* wealth share $\theta_t$ rises on impact because the return on Home capital outperforms Foreign capital.

The final result characterizes the currency risk premium around the $\gamma = \gamma^* = 1$ and $b^g_{H,s} < 0$ benchmark, essentially identical to Proposition 5:

[58]We need to assume not only that risk aversions are the same across countries, but that they are 1, for this result. This is because, consistent with (99) in the baseline simplified environment, unitary risk aversions imply that equilibrium financial portfolios hedge only labor income risk.

**Proposition 5.** *Consider the same environment as in Proposition 4 and suppose safety and productivity shocks are independent. Then at least around the case with* $\gamma = \gamma^* = 1$*:*

- $Cov_t\left(-\hat{m}_{t,t+1}, \hat{r}^*_{t+1} - \Delta\hat{q}_{t+1} - \hat{r}_{t+1}\right) \propto \gamma - \gamma^*$ *if* $\sigma^\omega = 0$*; and*
- $Cov_t\left(-\hat{m}_{t,t+1}, \hat{r}^*_{t+1} - \Delta\hat{q}_{t+1} - \hat{r}_{t+1}\right)$ *is rising in* $\sigma^\omega$*.*

*This result holds as well for the pricing kernel of a Foreign household.*

# C Additional quantitative results

In this appendix we provide supplementary material accompanying the quantitative results in the paper. We first provide the complete set of model impulse responses. We next study how the transmission of safety shocks depends on the parameters of monetary policy rules in each country. We then isolate the effects of individual model parameters on equilibrium portfolios and the risk premium on Foreign bonds. We provide additional detail on our analysis of U.S. external adjustment. Finally, we decompose the role of each driving force in our simulation of the Great Recession.

## C.1 Impulse responses

The responses to an increase in disaster risk are provided at the end of this appendix in Figures 11 and 12. The responses to a disaster realization are provided in Figures 13 and 14. The responses to a negative global productivity shock are provided in Figures 15 and 16. The responses to a negative Foreign productivity shock are provided in Figures 17 and 18. Finally, the responses to a positive safety shock are provided in Figures 19 and 20.

## C.2 Sensitivity to monetary policy rules

Consider the following generalization of the monetary policy rule at Home (10)

$$1 + i_t = (1 + \bar{i})(1 + i_{t-1})^{\rho^i} \left[ \left( \frac{P_t}{P_{t-1}} \right)^{\phi^\pi} \left( \frac{y_t}{z_t} \right)^{\phi^y} \right]^{1-\rho^i},$$

| | Model | $\phi^y = 0.5/4$ | $\phi^y = 0.5/4$, $\rho^i = 0.5$ |
|---|---|---|---|
| $\beta(r^*_{t+1} - \Delta \log q_{t+1} - r_{t+1}, \log y_t - \log y_{t-4})$ | -0.11 | -0.10 | -0.13 |
| $\beta(r^*_{t+1} - \Delta \log q_{t+1} - r_{t+1}, r^e_{t+1})$ | 0.06 | 0.06 | 0.08 |
| $\beta((\Delta nfa_{t+1})/y_t, r^*_{t+1} - \Delta \log q_{t+1} - r_{t+1})$ | 1.45 | 1.30 | 0.86 |
| *Memo*: $(k - \kappa)/(4y)$ | 60% | 52% | 38% |
| $b_H/(4y)$ | -103% | -127% | -73% |
| $b_F/(4y)$ | 20% | 52% | 13% |

Table 9: comovements under alternative monetary policy rules

Notes: moments are computed as described in note to Table 2.

and analogously in Foreign. Now the nominal interest rate can respond to output relative to trend with elasticity $\phi^y$,[59] and there may be inertia in the nominal interest rate as captured by the parameter $\rho^i$.

Figure 5 depicts the response to a safety shock in the baseline model with $\phi^\pi = 1.5$, $\phi^y = 0$, and $\rho^i = 0$ (same as the dark blue line of Figure 3); an alternative specification with $\phi^\pi = 1.5$, $\phi^y = 0.5/4$, and $\rho^i = 0$ (as in Gali (2008)); and an alternative specification with $\phi^\pi = 1.5$, $\phi^y = 0.5/4$, and $\rho^i = 0.5$. In each case the other model parameters are recalibrated to match the same targets in Table 2 (leaving the other parameters unchanged does not change the results which follow). As is evident, when the central bank also responds to output, it slightly dampens the exchange rate and output effects of safety shocks, but quantitatively not by much. Adding interest rate inertia amplifies the exchange rate and output effects of safety shocks. With this moderate amount of inertia, the impact responses are in fact larger than in the baseline model. Taken together, we conclude that the effects of safety shocks are broadly robust to monetary policy rules characterizing the U.S. and G10 economies that have been studied in the literature.

Given this result, the main comovements of interest in Table 9 (the same as Table 3 in the main text) are also robust to these alternative policy rules.

[59]Recall that trend output is proportional to productivity $z_t$ since the latter follows a unit root.

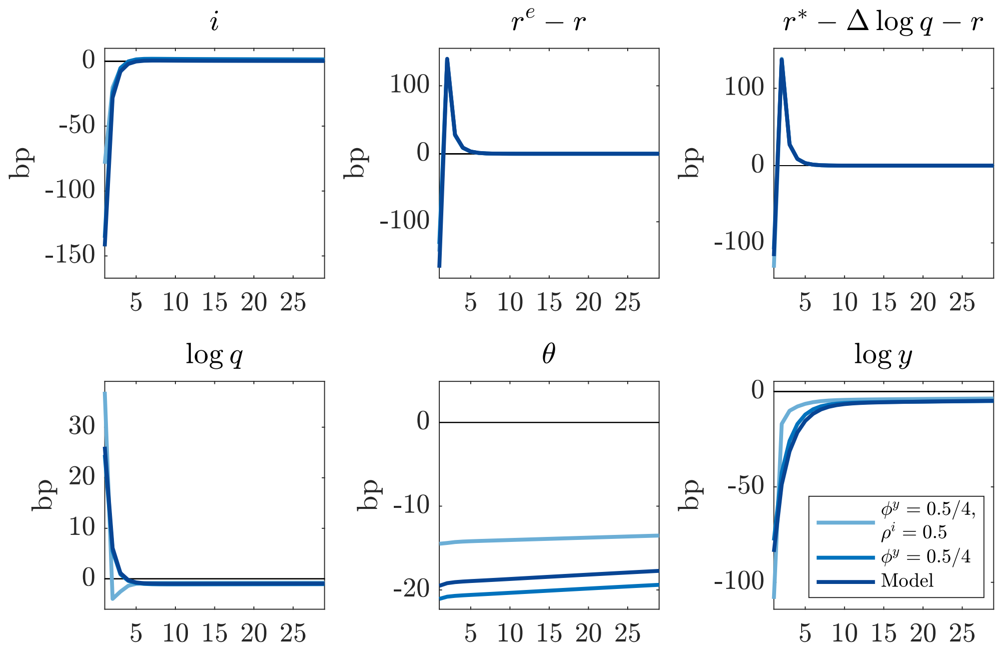


Figure 5: effects of increase in safety under alternative monetary policy rules

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.

## C.3 Determinants of portfolios and currency risk premium

Table 10 examines the sensitivity of Home's portfolio shares and the conditional correlation of excess Foreign bond returns with each country's pricing kernel to model parameters. Holding all other parameters as in the calibrated model, we vary $\{\gamma, \chi^W, \sigma^\omega, \bar{b}^g, \rho^{p\omega}\}$ one at a time. This contains similar insights as, but in a more granular manner than, Table 3 in the main text. The results also illustrate the usefulness of Propositions 4 and 5 in the simplified environment.

Beginning with identical risk tolerance and no safety shocks (column 1), agents hold identical per-capita positions in capital but Home is long dollar bonds financed by Foreign bonds to hedge the effects of relative productivity shocks. While our analytical results did not include this shock, it is consistent with the demands to hedge labor income risk and real exchange rate risk in (99): a negative Foreign productivity shock generates a relative decrease in Foreign labor income and real depreciation of the dollar, so efficient risk sharing calls for Home financial wealth to fall on impact. This is achieved by Home being long dollar bonds, financed by Foreign bonds.

| | | | | | |
|---|---|---|---|---|---|
| $\gamma$ | $= \gamma^*$ | Model | Model | Model | Model |
| $\sigma^{\omega}$ | 0 | 0 | Model | Model | Model |
| $\bar{b}^g$ | n/a | n/a | 0 | Model | Model |
| $\rho^{p\omega}$ | n/a | n/a | 0 | 0 | Model |
| $k/a$ | 100% | 137% | 138% | 138% | 142% |
| $b_H/a$ | 106% | 73% | 5% | 2% | -52% |
| $b_F/a$ | -106% | -110% | -42% | -40% | 10% |
| $\rho_t(r^*_{t+1} - \Delta \log q_{t+1} - r_{t+1}, \log m_{t,t+1})$ | 0.06 | 0.09 | 0.02 | 0.02 | -0.53 |
| $\rho_t(r^*_{t+1} - \Delta \log q_{t+1} - r_{t+1}, \log m^*_{t,t+1} + \Delta \log q_{t+1})$ | 0.07 | 0.08 | 0.02 | 0.02 | -0.49 |

Table 10: portfolios and risk premium

Notes: model moments are computed as described in note to Table 2.

Furthermore, because the relative productivity shock implies that the dollar depreciates in bad times (when Foreign productivity is low), excess Foreign bond returns are high when marginal utility is high. We note that the small magnitude of the correlation coefficients is because most of the volatility in pricing kernels is due to time varying disaster risk, not productivity shocks.

Making Home more risk tolerant than Foreign (column 2) implies that it increases its exposure to capital, financed by less positive / more negative positions in both bonds. Consistent with Proposition 4 in the paper, differences in risk tolerance are essential to rationalize Home's disproportionate exposure to capital. Consistent with Proposition 5 in the paper, differences in risk tolerance alone exacerbate the reserve currency paradox, as they imply that the dollar depreciates in bad times (because Home consumption disproportionately falls). Hence, the correlation between excess Foreign bond returns and marginal utility rises in both countries.

Introducing safety shocks (column 3) implies that Home substantially reduces its position in dollar bonds and raises its position in Foreign bonds, again consistent with Proposition 4 in the paper. However, because of relative productivity shocks the sign of these positions does not yet fully switch in this column. Consistent with Proposition 5 in the paper, the presence of safety shocks mitigates the reserve currency paradox, as it pushes downward the correlation between excess Foreign bond returns and marginal utility. But again, the sign of this correlation does not yet switch.

Introducing Home government issuance of safe dollar bonds (column 4) implies that Home further reduces its position in dollar bonds and raises its position in Foreign bonds, since it can naturally insure against safety shocks due to the seignorage revenue it receives. This is again consistent with Proposition 4 in the paper, but is quantitatively small since the supply of Treasury bills relative to aggregate wealth is small in the data. Relatedly, the effects on asset price comovements are essentially unaffected in the last two rows.

Finally, matching the correlation between safety shocks and disaster risk in the data (column 5) further reduces Home's position in dollar bonds and raises its position in Foreign bonds, since it means that the dollar is more likely to appreciate in bad times when disaster risk is high. Relatedly, this model feature pushes the correlation between excess Foreign bond returns and marginal utility in both countries to be negative. That is, we have resolved the reserve currency paradox, as dollar bonds pay relatively better when marginal utility globally is high. Moreover, this correlation is now large in magnitude, since disaster risk most drives variation in pricing kernels.

## C.4 U.S. external adjustment

It is useful to first review the timing of events within a model period:

1. Exogenous driving forces are realized, including a rare disaster which destroys capital.
2. Production:
    (a) Firms hire domestic labor and import capital in excess of that supplied by domestic households.
    (b) Firms produce, pay workers, pay dividends to capital owners, and export undepreciated capital in excess of that supplied by domestic households.
3. Consumption, savings, and capital production:
    (a) Households close nominal positions from the previous period, consume domestically produced and imported goods, and trade new nominal claims and capital.
    (b) Global capital producers import goods from Home and Foreign and export capital to capital owners.

Net foreign assets dated in period $t$ are measured accounting for capital used in domestic production in step #2(a) of period $t+1$, appropriately undoing the effect of capital destruction that occurs at $t+1$. Hence, Home's net foreign assets dated in period $t$ are

$$nfa_t \equiv b_{Ht} + q_t^{-1} b_{Ft} + q_t^k \left(k_t - \kappa_{t+1} \exp(-\varphi_{t+1})\right),$$

where we use the lower-case notation for real variables introduced in appendix A. We similarly assume exports and imports dated in period $t$ measure all transactions from the beginning of step #2(b) in period $t$ through the end of step #2(a) in period $t+1$, thus obtaining:

$$nx_t \equiv \frac{P_{Ht}}{P_t} \zeta^* c_{Ht}^{h*} + \frac{P_{Ht}}{P_t} \left(\frac{\bar{k}_t}{\bar{k}_{t-1} \exp(\varphi_t)}\right)^{\chi^x} x_{Ht} + \\ q_t^k \left((1-\delta)\kappa_t - \kappa_{t+1} \exp(-\varphi_{t+1})\right) - q_t^{-1} \frac{P_{Ft}^*}{P_t^*} c_{Ft}.$$

It is then straightforward to use the model's resource constraints to obtain the accounting identity

$$\Delta nfa_t = nx_t + r_t^k nfa_{t-1} + val_t,$$

where

$$val_t \equiv -\left(r_t^k - \left(\frac{1+r_t}{1-\omega_{t-1}} - 1\right)\right)(b_{Ht-1} + b_{Ft-1}) + \\ \left(q_t^{-1}(1+r_t^*) - q_{t-1}^{-1} - \left(\frac{1+r_t}{1-\omega_{t-1}} - 1\right)\right) b_{Ft-1} - \omega_t \frac{\zeta^* q_t^{-1} c_t^*}{c_t + \zeta^* q_t^{-1} c_t^*} b_{Ht,s}^g.$$

That is, the change in net foreign assets equals net exports plus interest income at $r_t^k$ and excess returns. The latter are collected in the term $val_t$.

## C.5 Great Recession

Figure 6 decomposes the role of each driving force in our simulation of the Great Recession. It shuts down each driving force (holding it at its mean) and simulates the effects of the other alone. It demonstrates that both safety and disaster risk shocks play important roles in our simulation of the Great Recession. The flight to safety is important in generating a U.S. output decline and valuation loss in late 2008. However, the increase in disaster risk is important in accounting for the persistence

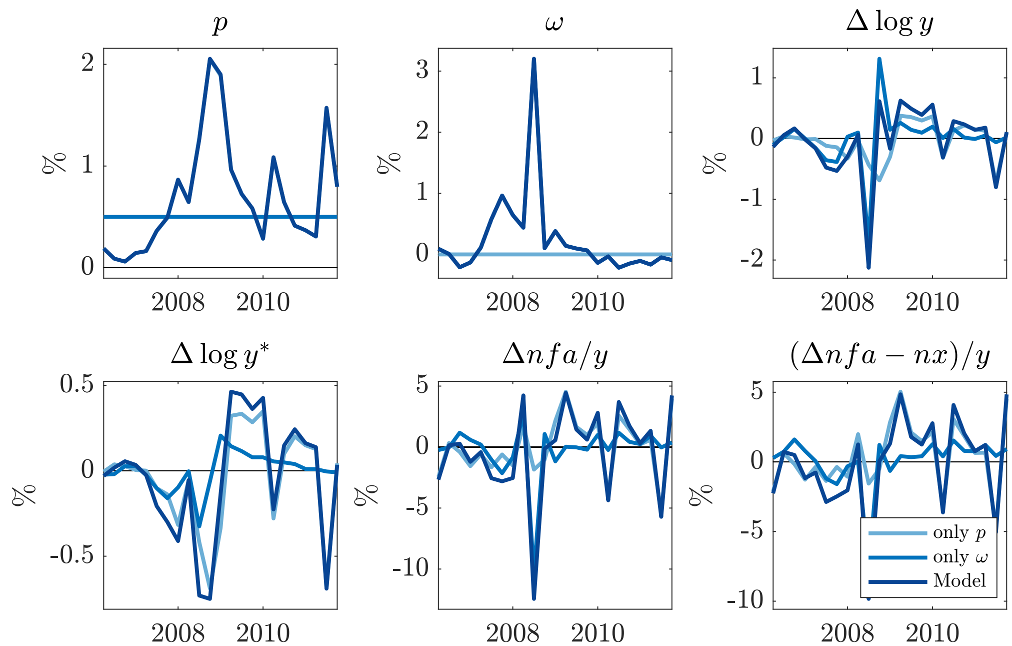


Figure 6: simulation using observed $p$ and $\omega$ series

Notes: see notes to Figure 4.

of the output decline, particularly in Foreign, as well as high excess returns through 2009 on the U.S. external position.

Figure 7 depicts additional variables of interest. The first two panels report nominal interest rates compared to their empirical counterparts (recalling that the latter are three-month government bond yields). Nominal interest rates globally (and especially in the U.S.) fall well below zero in the model, while they were constrained by the zero lower bound in the data. While this is consistent with the decline in "shadow rates" in practice (Wu and Xia (2016)), owing to policies such as quantitative easing which are outside the model, this suggests that the model may understate the effects of disaster risk and safety shocks during this period, if anything.

The third panel of Figure 7 reports the U.S. financial wealth share in the model. While both the increase in disaster risk and flight to safety lower the U.S. wealth share on impact, the elevated disaster risk induces a rise in the wealth share thereafter as the U.S. earns high excess equity returns, while the flight to safety dissipates. Hence, the U.S. wealth share in fact slightly rises from Q1 2008 through Q1 2010 in the model. Dahlquist, Heyerdahl-Larsen, Pavlova, and Penasse (2023) estimate a rise in the U.S. wealth share over this period, while Sauzet (2023) estimates a decline,

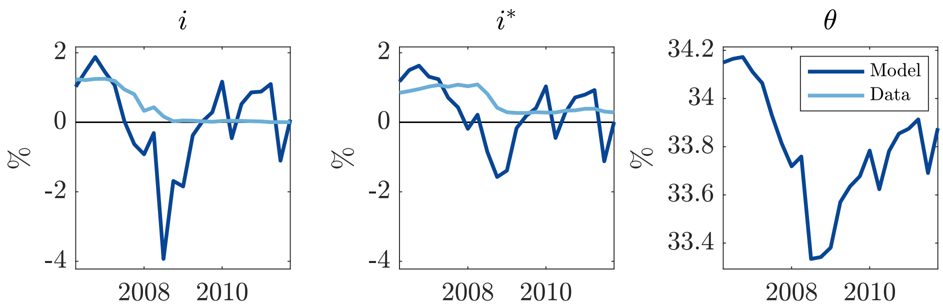


Figure 7: simulation using observed $p$ and $\omega$ series

Notes: see notes to Figure 4.

reflecting the difficulty in measuring market values of wealth in a comprehensive way across countries. The model-implied change in the wealth share over this period is well within the range estimated by these papers. The model further clarifies that it is fully consistent for the U.S. wealth share to rise over the 2008-2009 period even if, on impact, both an increase in disaster risk and flight to safety reduce it.

# D Empirical estimates

In this appendix we provide additional detail on empirical estimates which inform or validate the model. We first estimate the conditional correlation between global equity returns and excess G10 currency bond returns, used to calibrate the magnitude of safety shocks in the model. We then provide evidence on the effects of safety shocks in the data. We finally describe how the evidence on swap line announcement effects can be used to discipline $\epsilon^d$ in the model.

## D.1 Equity returns and excess foreign bond returns

We first estimate the conditional correlation between global equity returns and excess returns on G10 currency bonds versus Treasuries. Our approach builds on that in Maggiori (2013). As we use monthly data, in this subsection we write $t$ to mean a month in time but everywhere use three-month interest rates, as in the model.

We first estimate unexpected return innovations over the next three months by

| | $r_t^e$ | $r_t^F$ |
|---|---|---|
| $dp_{t-3}$ | 2.3 | |
| | (1.7) | |
| $i_{t-3}^* - i_{t-3}$ | | 1.8 |
| | | (1.5) |
| $\log y_{t-3} - \log y_{t-15}$ | | -0.1 |
| | | (0.1) |

Table 11: predicting global equity and excess foreign bond returns

Notes: sample period is 1/1995 - 12/2019. Standard errors are given in parenthesis and follow Hansen and Hodrick (1980) with 4 lags to correct for overlapping observations.

running the regressions

$$r_t^e = \alpha_0^e + \alpha_1^e dp_{t-3} + \varepsilon_t^e, \tag{101}$$

$$r_t^F = \alpha_0^F + \alpha_1^F (i_{t-3}^* - i_{t-3}) + \alpha_2^F (\log y_{t-3} - \log y_{t-15}) + \varepsilon_t^F. \tag{102}$$

Here, $r_t^e$ is the real return on global equities from month $t-3$ to $t$ and $r_t^F \equiv i_{t-3}^* - (\log E_t - \log E_{t-3}) - i_{t-3}$ is the return on a position short 3-month U.S. Treasury bills and long 3-month G10 currency bonds from month $t-3$ to $t$. The variables known at $t-3$ used to predict returns are the dividend-price ratio on the global equity index $dp_{t-3}$, the interest rate differential $i_{t-3}^* - i_{t-3}$, and the year-over-year change in U.S. industrial production $\log y_{t-3} - \log y_{t-15}$. The first regression is a standard predictability regression for equity returns. The second regression is consistent with Lustig, Roussanov, and Verdelhan (2014). The estimated coefficients are provided in Table 11.

The resulting estimated return innovations are given by the estimated residuals $\hat{\varepsilon}_t^e$ and $\hat{\varepsilon}_t^F$. A time-series of their product is given in Figure 8. As argued in Maggiori (2013), the disproportionately positive values imply in a wide class of environments a positive risk premium on foreign bonds relative to U.S. bonds. Consistent with the "exchange rate reconnect" emphasized by Lilley, Maggiori, Neiman, and Schreger (2020), the values are more consistently positive after 2008. We use as our calibration target in the model the correlation of $\hat{\varepsilon}_t^e$ and $\hat{\varepsilon}_t^F$ over the entire period, 0.5. We obtain quantitatively similar results if we include additional conditioning variables in the predictability regressions (101) and (102) such as lagged returns or the VIX.

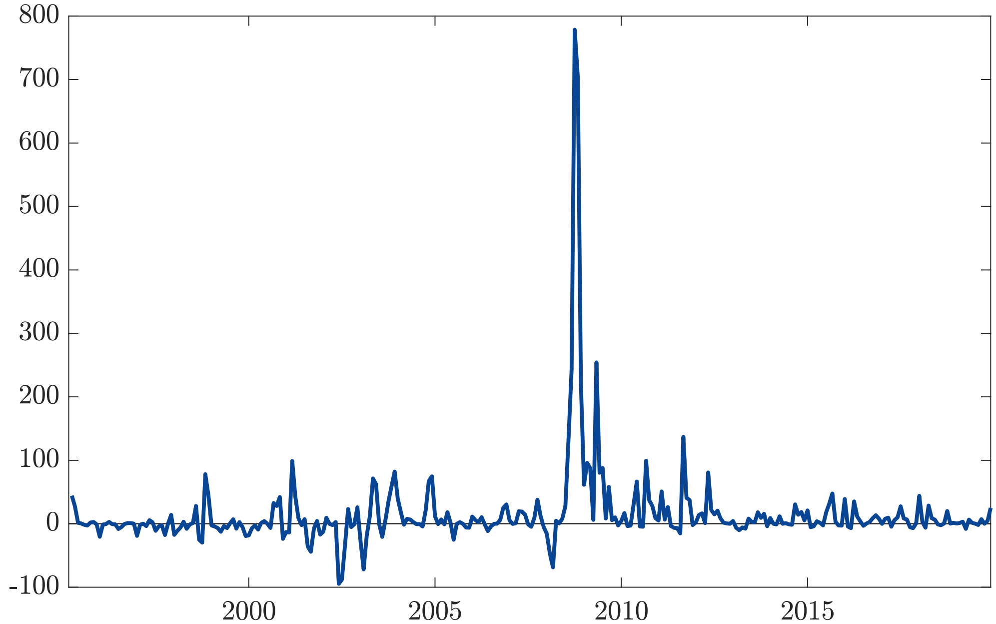


Figure 8: $\hat{\varepsilon}_t^e \hat{\varepsilon}_t^F$

Notes: $\hat{\varepsilon}_t^e$ and $\hat{\varepsilon}_t^F$ are residuals from the specifications estimated in Table 11. Each is expressed in percentage points.

## D.2 Estimated effects of safety shocks

We now provide direct evidence on the effects of safety shocks in the data.

We compute the simple average of the log real exchange rate, the three-month interest rate differential, and the difference in log industrial production between the U.S. and each of the G10 countries. Over January 1995 through December 2019, we then run a six-variable, four-lag recursive VAR with the swapped G10/T-bill spread (from Du, Im, and Schreger (2018)), log real exchange rate, interest rate differential, global equity returns, log U.S. industrial production, and difference in log industrial production. We identify the effects of a safety shock by ordering the swapped G10/T-bill spread first in the VAR, so other variables can respond contemporaneously to it. This is consistent with our assumption that safety shocks are an exogenous driving force.

Figure 9 summarizes the results. As in Jiang, Krishnamurthy, and Lustig (2021) as well as our model, a positive innovation to the yield on swapped G10 bonds relative to T-bills leads to a dollar appreciation and increase in the foreign interest rate relative to U.S. interest rate. More novel, we find that a positive innovation leads to an initial

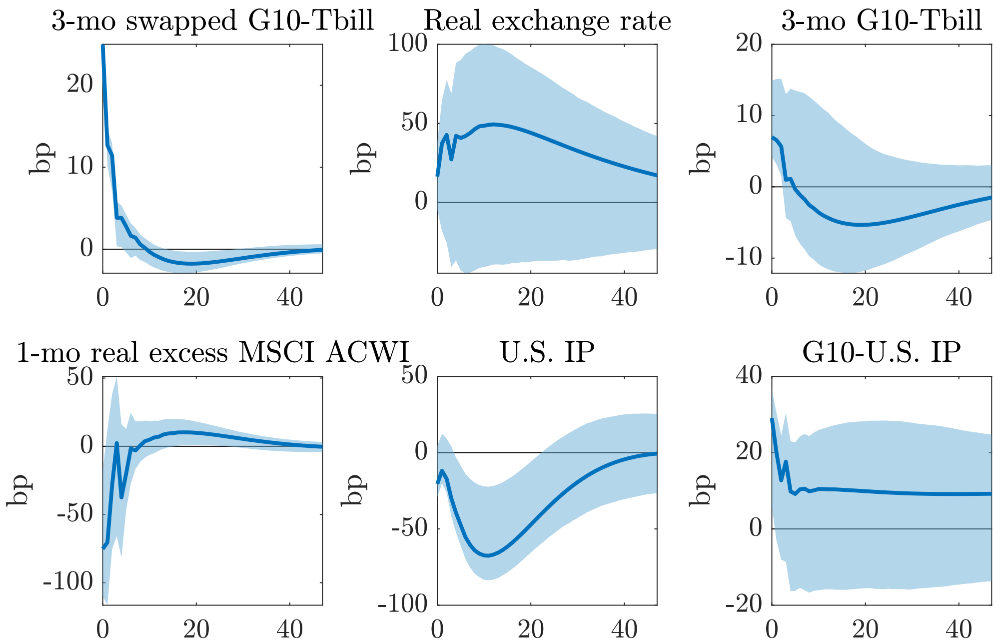


Figure 9: effects of innovation to swapped G10/T-bill spread

Notes: VAR is estimated with four lags in each variable over January 1995 – December 2019. Swapped G10/T-bill spread is ordered first. Bootstrapped 90% confidence intervals at each horizon are computed using 10,000 iterations.

decline followed by sustained increase in excess returns on the MSCI ACWI; a decline in U.S. industrial production; and an increase in foreign production relative to U.S. production. All of these are consistent with our model.

Figure 10 quantitatively compares the empirical and model impulse responses. We first re-estimate the effects of safety shocks using quarterly data, since our model is solved at a quarterly frequency. We run a one-lag recursive VAR over Q1 1995 through Q4 2019 with the same variables as above.[60] We then simulate a safety shock in the model, setting the initial innovation to equal the estimated innovation in the swapped G10/Tbill spread, multiplied by the ratio of the unconditional volatilities of $\omega_t$ to the swapped G10/Tbill spread.[61] The top right panel reflects that U.S. monetary policy is too responsive to safety shocks in the model. This is consistent with the model undershooting the effects on other asset prices and quantities in all other panels. We

[60]The only exception is that we replace the one-month excess equity return with the three-month excess equity return.

[61]Recall that the swapped G10/Tbill spread understates the volatility of $\omega_t$ if swapped G10 bonds are also partially valued for their liquidity or safety, so we calibrate $\sigma^\omega$ to match the conditional correlation between equity returns and excess foreign bond returns in the data.

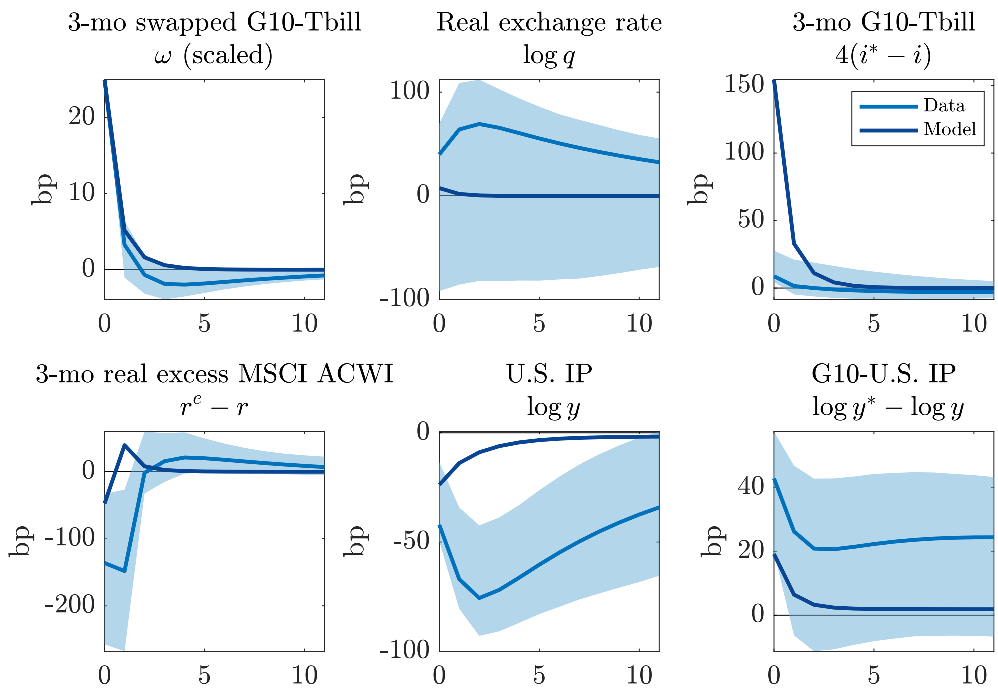


Figure 10: effects of safety shock in data and model

Notes: in data, impulse responses estimated as in Figure 9 except using quarterly data over Q1 1995 – Q4 2019. In model, innovation to $\omega_t$ equals estimated innovation in swapped G10/Tbill spread, multiplied by ratio of unconditional volatilities of $\omega_t$ in model to swapped G10/Tbill spread in data.

thus conclude that the model may be conservative, if anything, in quantifying the importance of safety shocks for asset prices and real fluctuations.

## D.3 Estimating $\epsilon^d$ from swap line announcements

We finally describe how the estimated announcement effects of swap lines can be used to discipline $\epsilon^d$ in our model.

Section 6.4 describes how we translate the announcement effects estimated in Kekre and Lenel (2023) into a $14bp$ decline in $\omega_t$. Given this decline in $\omega_t$, we can estimate the elasticity of safe asset demand $\epsilon^d$ in (17) given an assumption on the news regarding the expanded supply of safe dollar assets contained in these announcements. A plausible range is $\$50bn - \$300bn$. The lower end corresponds to the assumption that only the March 19 announcement contained news about incremental swap line usage (of $\$50bn$ in the subsequent weeks, by the central banks granted temporary swap lines),[62] since the March 20 announcement only pertained to the frequency of swap line operations. The upper end corresponds to the assumption that all \$300bn in swap line usage in the subsequent weeks was communicated to the public in the March 19-20 announcements.[63] Since a range of $\$50bn - \$300bn$ corresponds to roughly $0.25\% - 1.5\%$ of annual U.S. GDP, and annual Home GDP in the model is roughly 2 times quarterly global consumption, this implies $\bar{b}_t^g \equiv -B_{Ht,s}^g / \left(P_t c_t + \zeta^* E_t^{-1} P_t^* c_t^*\right)$ rose by $50 - 300bp$. With a $14bp$ resulting decline in $\omega_t$, equation (17) then implies that $\epsilon^d$ is between 4 and 21.

By construction, the scenarios of a $\$50bn$ increase in $-B_{Ht,s}^g$ and $\epsilon^d = 4$, and a $\$300bn$ increase in $-B_{Ht,s}^g$ and $\epsilon^d = 21$, induce the same increase in $\omega_t$. They only differ in the implications for seignorage earned by the U.S. However, the latter is small relative to the general equilibrium effects of the change in $\omega_t$. For instance, even in the case of a $\$300bn$ increase in $-B_{Ht,s}^g$ fully absorbed by foreigners (1.5% of annual U.S. GDP), given an initial value of $\omega_t$ of say 1%, the seignorage earned by the U.S. in the first period would be $1.5bp$ of annual U.S. GDP.[64] This compares to the roughly $440bp$ response in U.S. NFA from the decline in $\omega_t$ reported in section 6.4. This is why in the main text we simply simulate a shock to $\omega_t$ of $-14bp$ in the first period, with the understanding that this corresponds to a shock to $B_{Ht,s}^g$ in the background and appropriate value of $\epsilon^d$.

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[62] See Chart 3 of Choi, Goldberg, Lerman, and Ravazzolo (2022) depicting peak usage by country.

[63] See Figure 1 of Kekre and Lenel (2023) depicting total usage over time.

[64] An initial value of $\omega_t = 1\%$ is consistent with the annualized swapped G10/Tbill spread of 0.74% as of March 19, 2020, multiplied by the ratio of the volatility of $\omega_t$ relative to the swapped G10/Tbill spread in the data.

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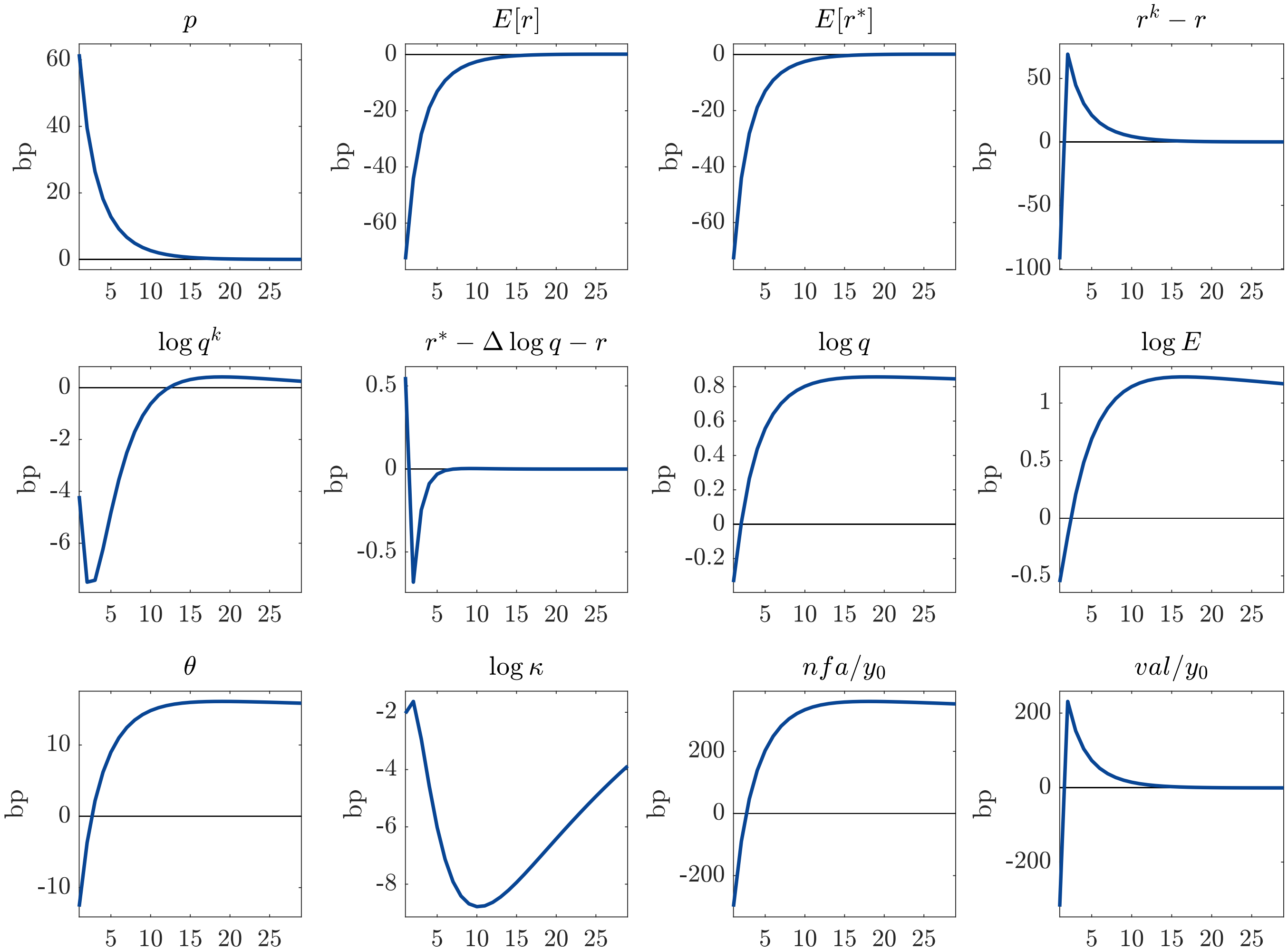


Figure 11: effects of disaster risk (1/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.

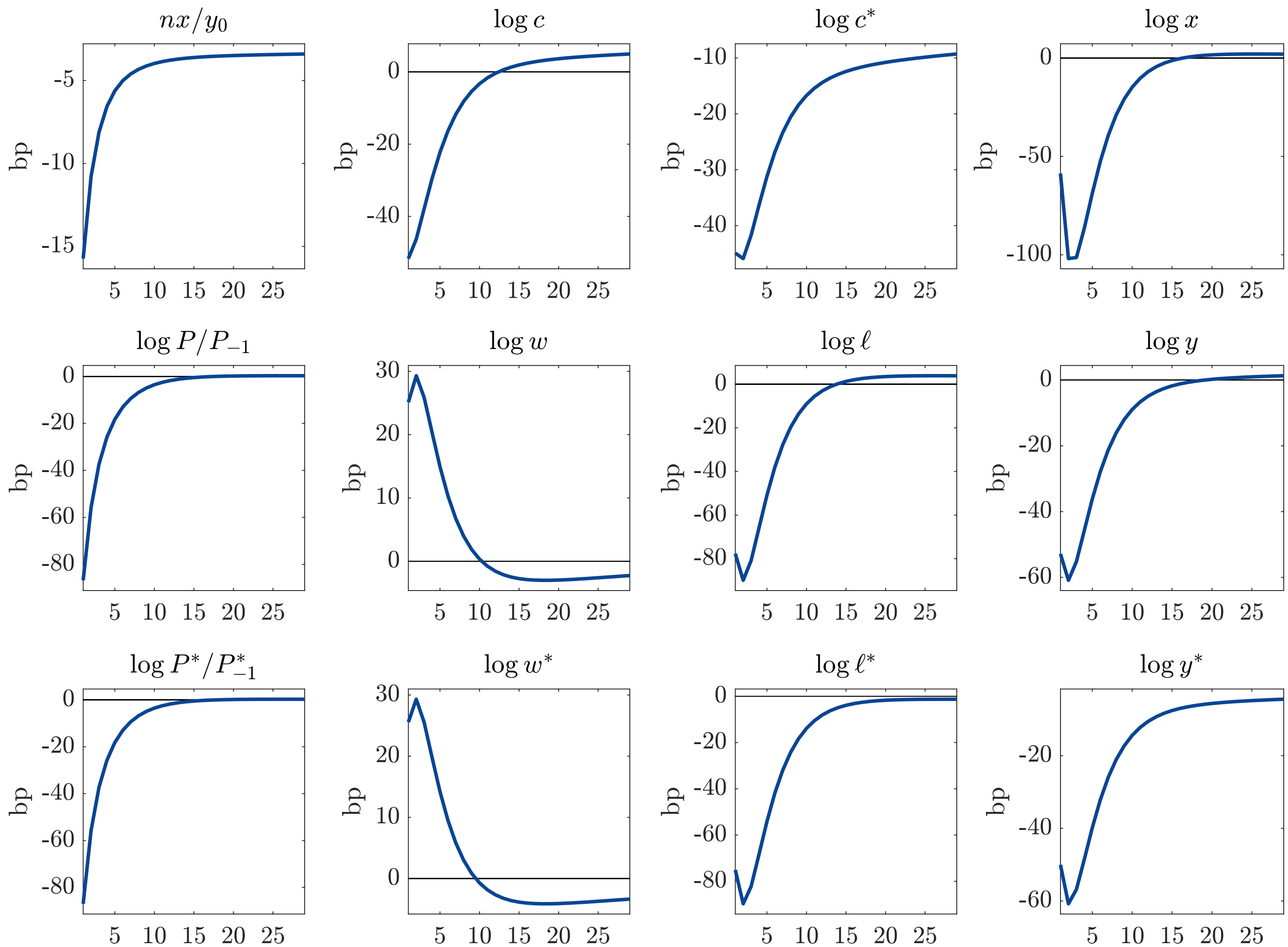


Figure 12: effects of disaster risk (2/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.

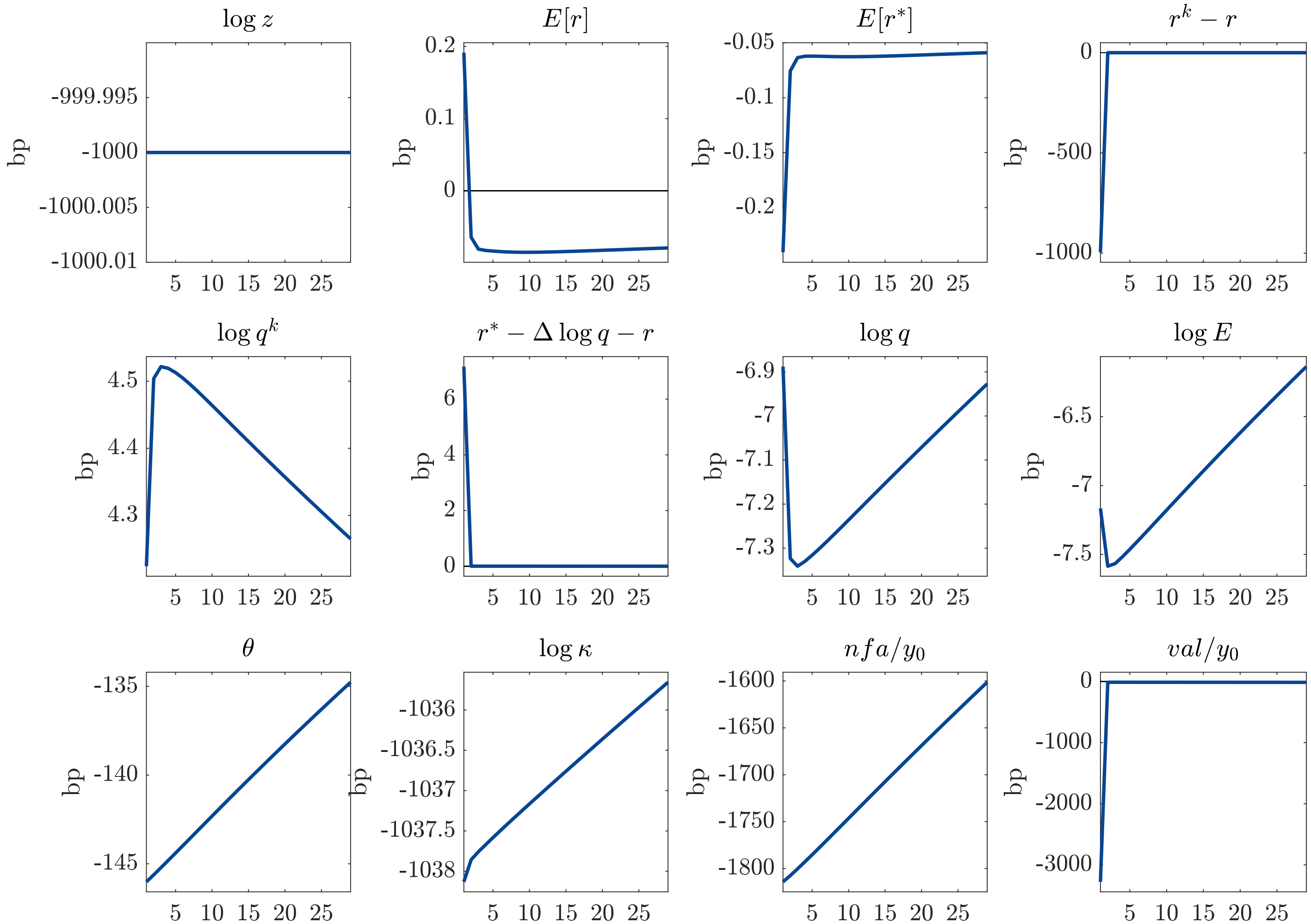


Figure 13: effects of disaster realization (1/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.

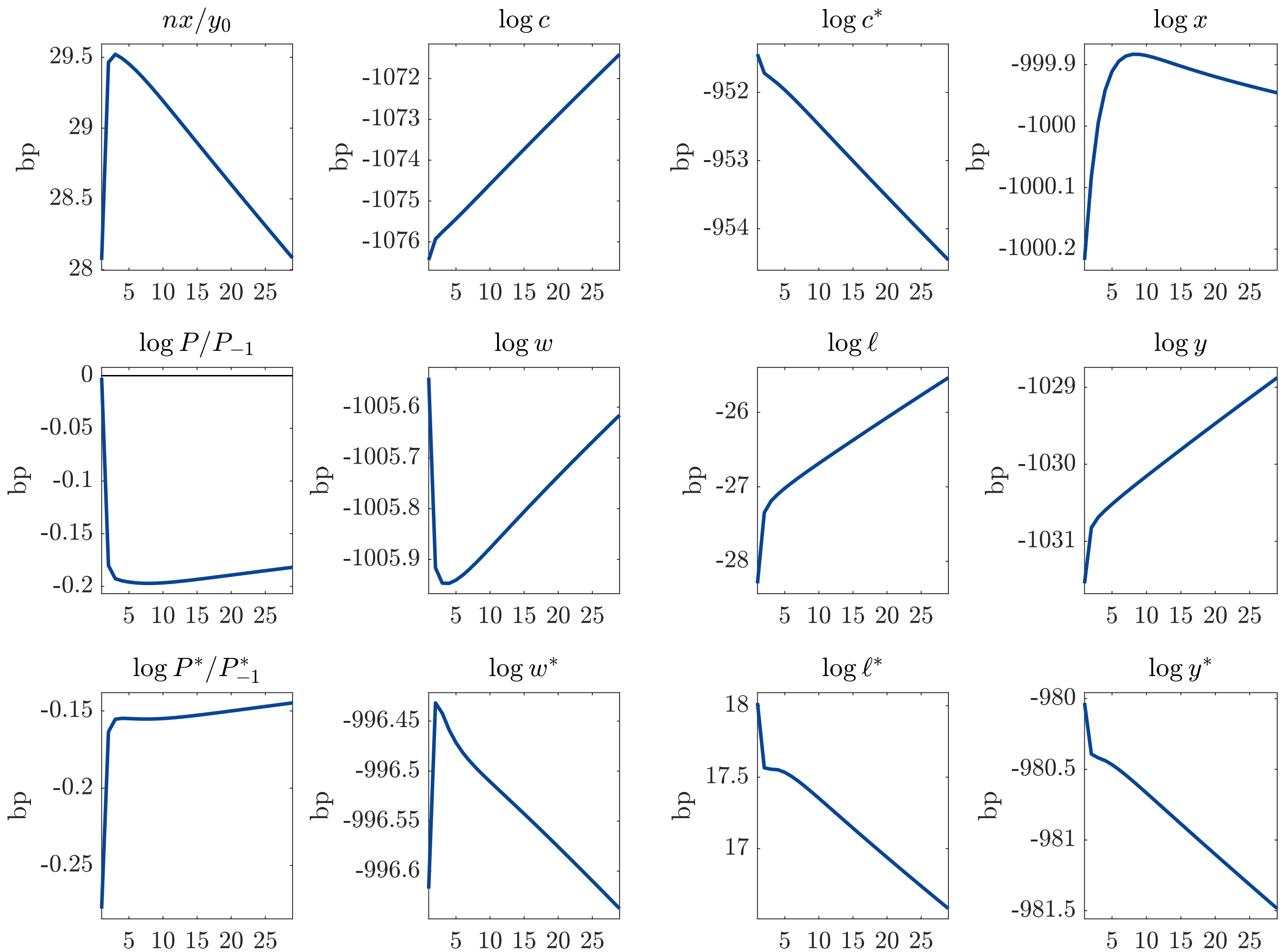


Figure 14: effects of disaster realization (2/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.

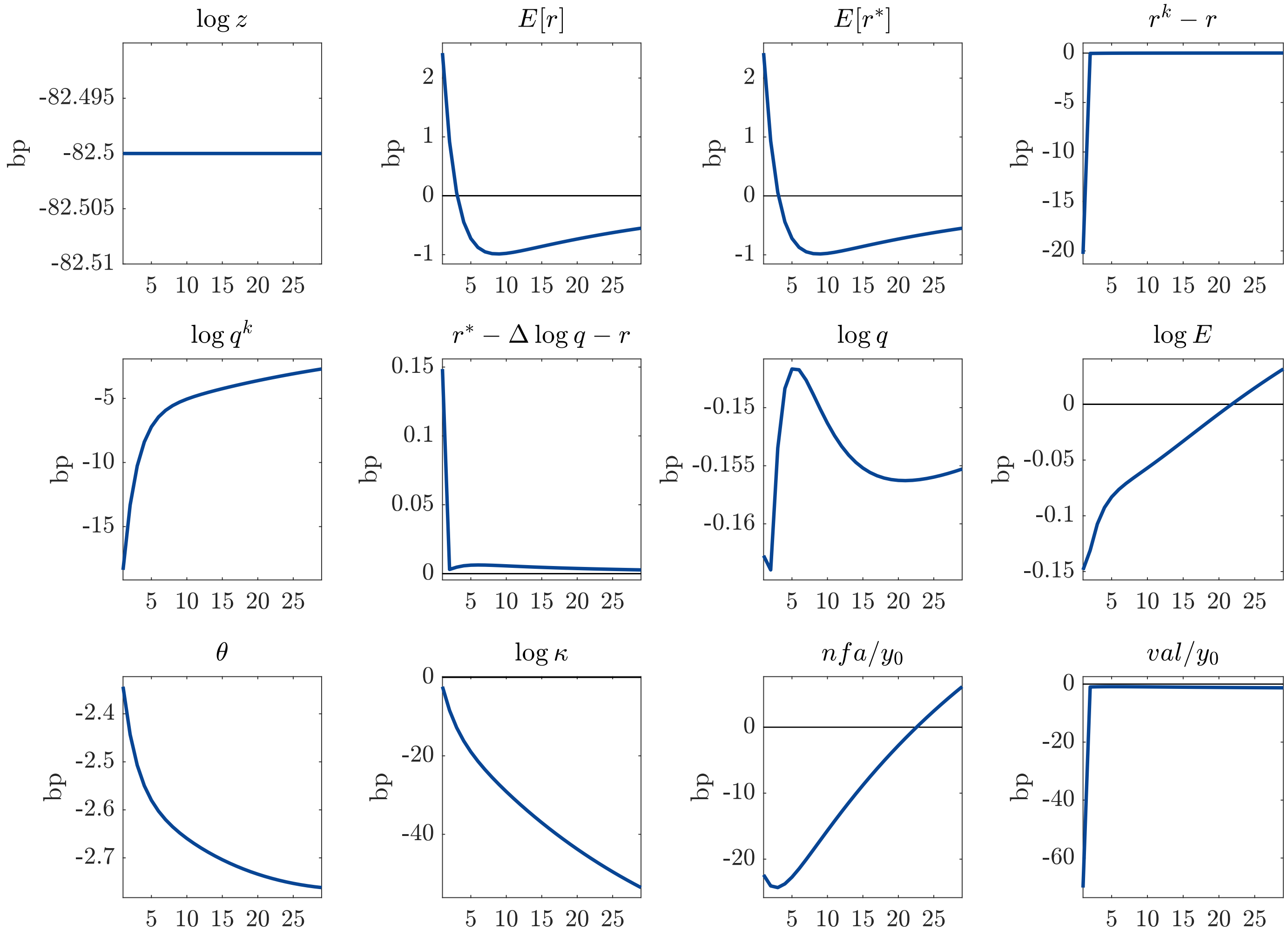


Figure 15: effects of global productivity shock (1/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.

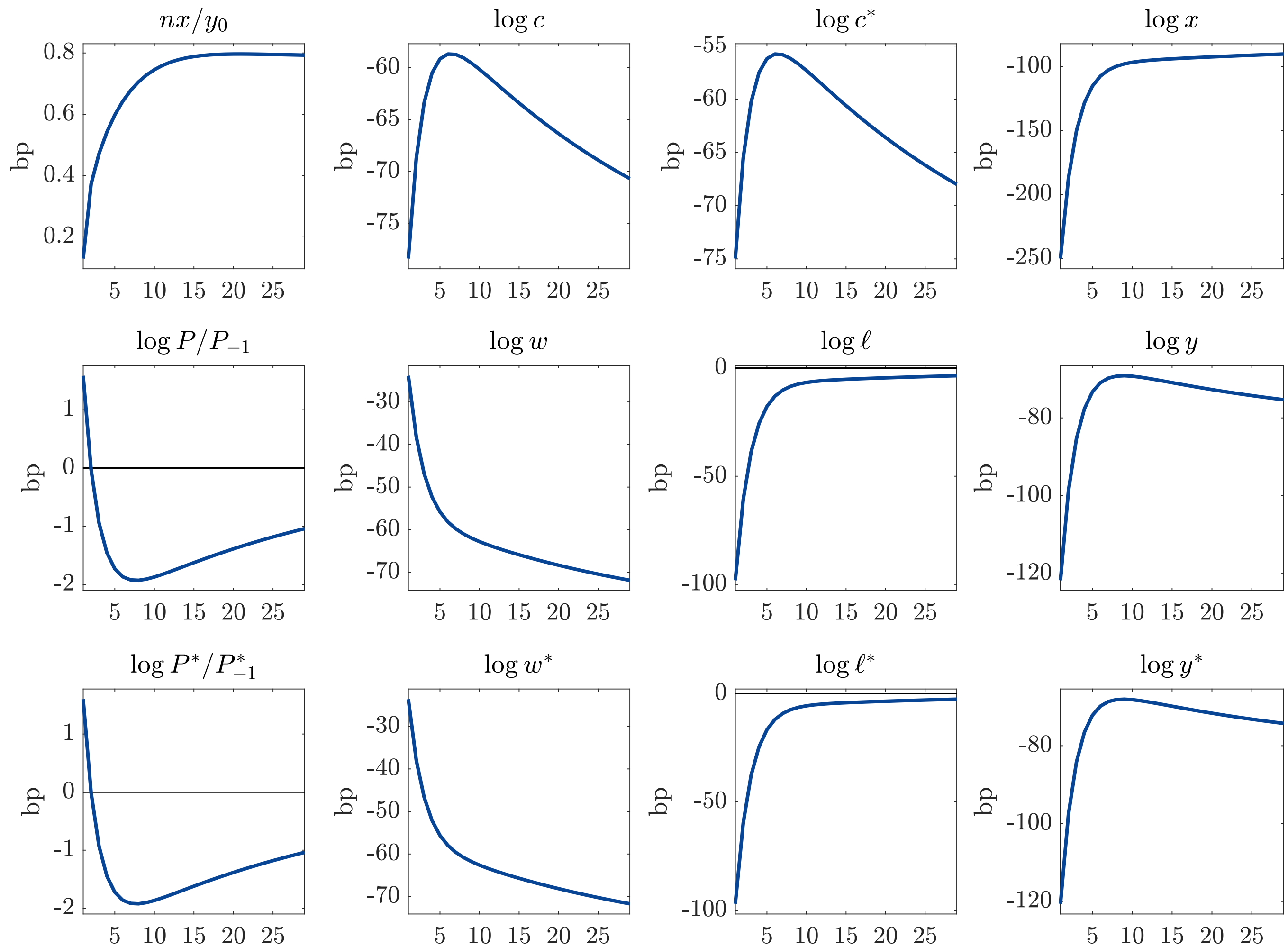


Figure 16: effects of global productivity shock (2/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.

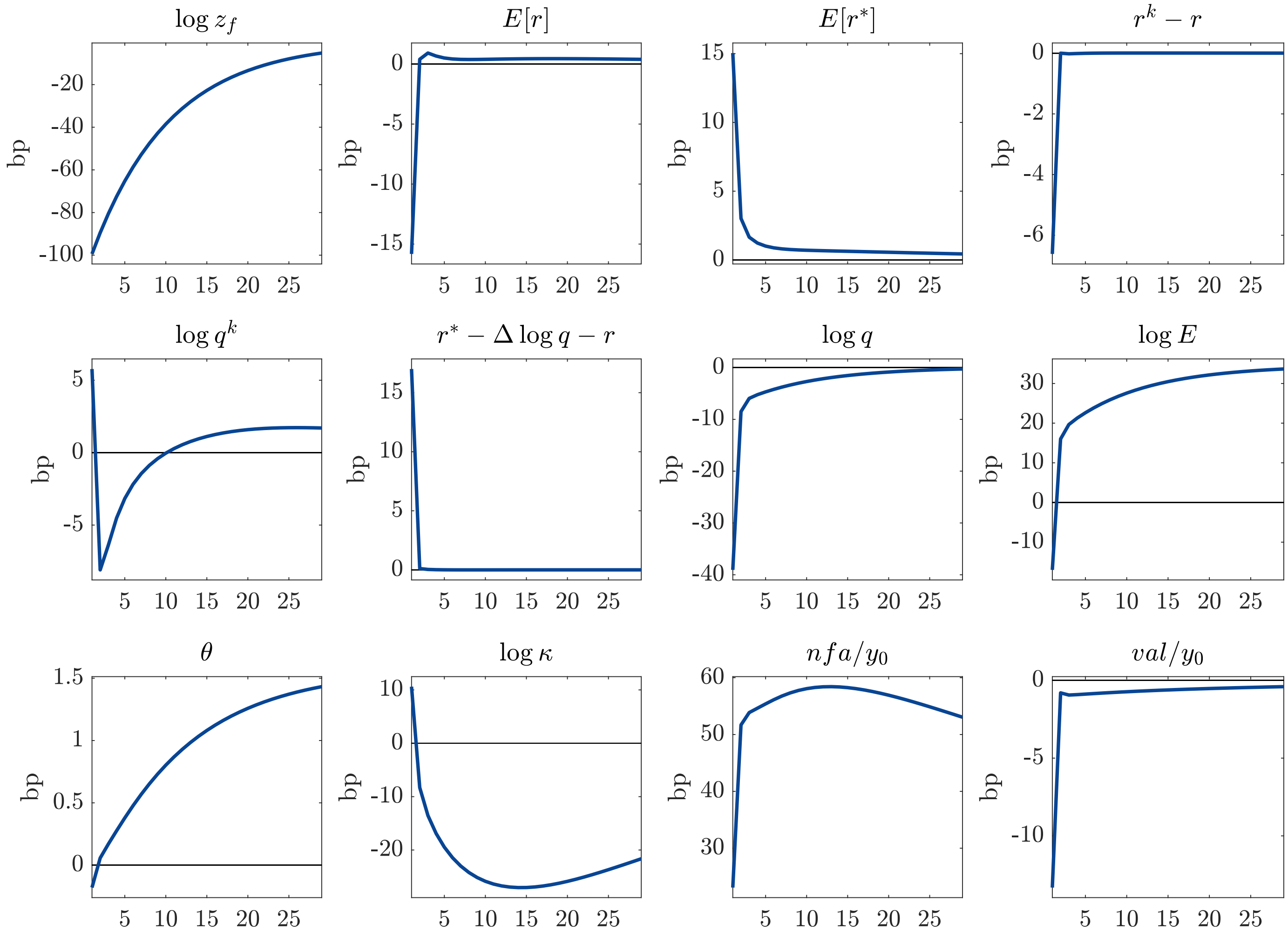


Figure 17: effects of relative productivity shock (1/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.

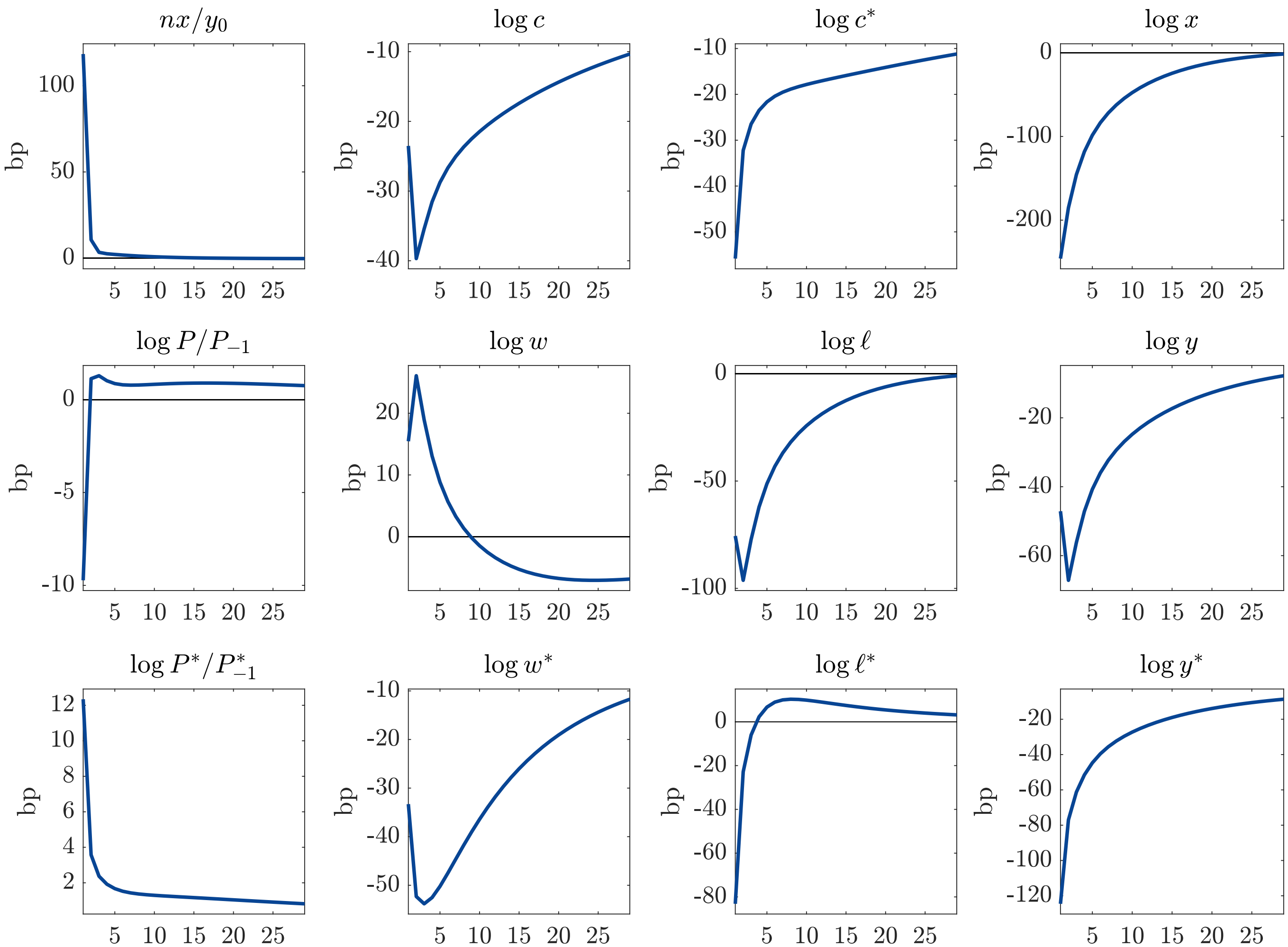


Figure 18: effects of relative productivity shock (2/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.

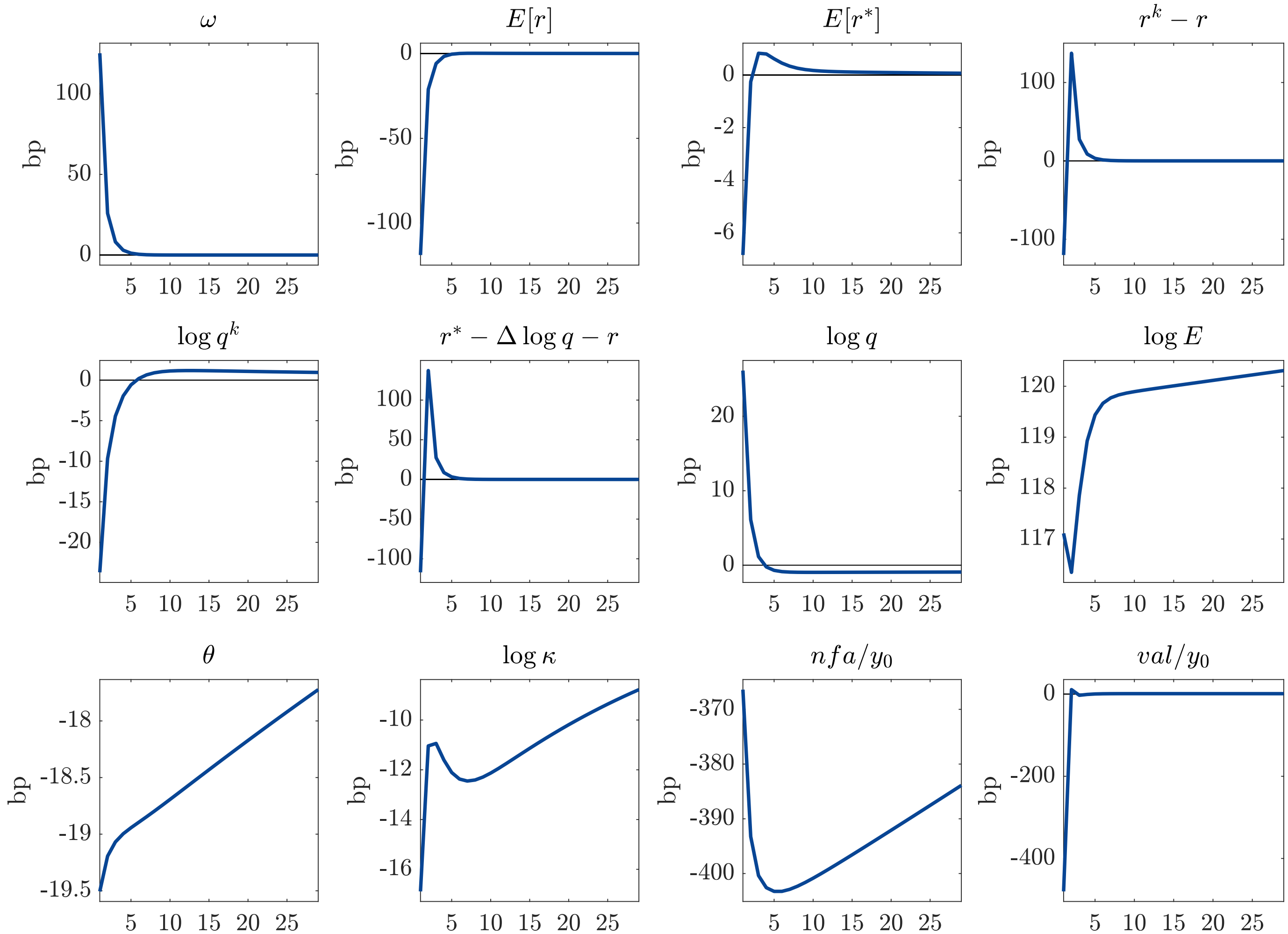


Figure 19: effects of safety shock (1/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.

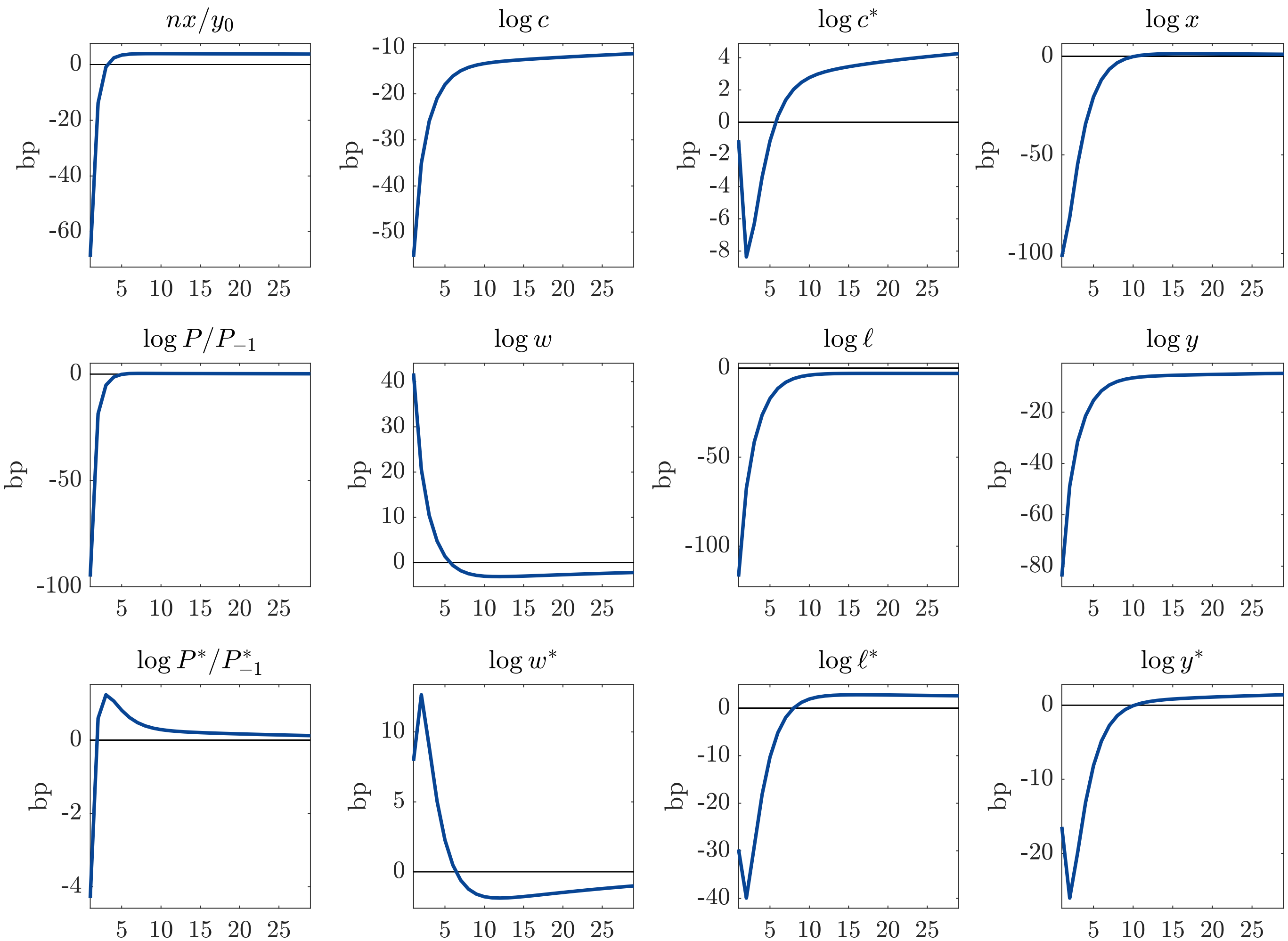


Figure 20: effects of safety shock (2/2)

Notes: impulse responses are average responses starting from 100 points drawn from ergodic distribution as described in note to Table 2.