

Online Appendix

A Additional Tables Empirical Analysis

Table A1: Relationship of Exchange Rates and Banking Liquidity Ratio Feb. 2001 – July 2020

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat}_t)$	0.220*** (4.422)	0.233*** (3.359)	0.148** (2.557)	-0.141*** (-2.830)	0.277*** (3.947)	0.184*** (2.914)	0.215*** (3.561)	0.140** (2.557)	0.163*** (3.254)
$\pi_t - \pi_t^*$	-0.546*** (-3.750)	-0.408** (-2.136)	-0.653*** (-2.656)	0.050 (0.343)	-0.729*** (-3.815)	-0.118 (-0.807)	-0.504** (-2.528)	-0.649*** (-2.737)	-0.394** (-2.129)
$\Delta(i_t - i_t^*)$	-1.284 (-1.486)	-0.491 (-0.599)	-0.093 (-0.083)	-1.793** (-2.336)	-1.114 (-1.245)	-0.513 (-0.638)	0.249 (0.311)	-1.541* (-1.663)	-0.269 (-0.351)
LiqRat_{t-1}	0.010** (2.141)	0.006 (0.940)	0.002 (0.319)	-0.001 (-0.142)	0.008 (1.279)	0.010 (1.643)	0.006 (1.181)	0.004 (0.775)	0.009* (1.705)
Constant	-0.011*** (-3.239)	-0.004 (-0.950)	-0.007* (-1.912)	0.002 (0.349)	-0.008* (-1.892)	-0.006 (-1.560)	-0.009** (-2.081)	-0.016*** (-2.980)	-0.005 (-1.571)
N	234	232	148	234	232	234	234	234	234
adj. R^2	0.11	0.05	0.05	0.05	0.10	0.03	0.05	0.05	0.04

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A2: Relationship of Exchange Rates and Banking Liquidity Ratio with VIX Feb. 2001 – July 2020

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat}_t)$	0.192*** (3.922)	0.155** (2.592)	0.073 (1.378)	-0.123** (-2.477)	0.208*** (3.230)	0.137** (2.291)	0.174*** (3.012)	0.126** (2.291)	0.138*** (2.769)
$\pi_t - \pi_t^*$	-0.432*** (-2.989)	-0.174 (-1.054)	-0.421* (-1.893)	0.027 (0.191)	-0.530*** (-3.016)	-0.033 (-0.240)	-0.419** (-2.210)	-0.589** (-2.468)	-0.308* (-1.676)
ΔVIX_t	0.001*** (3.794)	0.004*** (9.317)	0.003*** (6.169)	-0.001** (-2.538)	0.003*** (7.018)	0.002*** (5.896)	0.002*** (5.133)	0.001* (1.683)	0.001*** (3.110)
$\Delta(i_t - i_t^*)$	-1.075 (-1.277)	-0.404 (-0.578)	0.456 (0.455)	-1.994*** (-2.615)	-1.404* (-1.725)	-0.289 (-0.385)	0.096 (0.126)	-1.285 (-1.374)	-0.266 (-0.353)
LiqRat_{t-1}	0.010** (2.240)	0.007 (1.299)	0.004 (0.686)	-0.001 (-0.177)	0.008 (1.390)	0.010* (1.808)	0.007 (1.385)	0.004 (0.871)	0.008 (1.587)
_cons	-0.011*** (-3.095)	-0.005 (-1.399)	-0.006* (-1.881)	0.002 (0.291)	-0.008* (-1.941)	-0.006 (-1.648)	-0.009** (-2.095)	-0.015*** (-2.837)	-0.005 (-1.433)
N	234	232	148	234	232	234	234	234	234
adj. R^2	0.16	0.31	0.25	0.07	0.26	0.15	0.15	0.05	0.07

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A3: Relationship of Exchange Rates and Banking Liquidity Ratio Instrumental Variable Regression: $\Delta(\text{VIX})$, $\text{StDev}(\text{Inf})$ and $\text{StDev}(\text{XRate})$ instrument for $\Delta(\text{LiquidityRatio})$ Feb. 2001 – July 2020

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat}_t)$	0.604*** (4.231)	1.099*** (4.663)	0.688*** (4.421)	-0.380*** (-2.904)	0.888*** (4.357)	0.832*** (4.087)	0.710*** (3.961)	0.153 (1.058)	0.799*** (4.249)
$\pi_t - \pi_t^*$	-0.717*** (-4.120)	-0.892*** (-3.245)	-0.732*** (-2.627)	0.094 (0.596)	-0.985*** (-4.176)	-0.450** (-2.234)	-0.778*** (-3.217)	-0.673** (-2.465)	-1.094*** (-3.519)
LiqRat_{t-1}	0.016*** (3.063)	0.018** (2.177)	0.016*** (2.618)	-0.003 (-0.466)	0.017** (2.349)	0.023*** (2.862)	0.012** (1.972)	0.005 (0.951)	0.026*** (3.208)
Constant	-0.018*** (-4.085)	-0.014** (-2.325)	-0.015*** (-3.065)	0.005 (0.723)	-0.017*** (-3.049)	-0.017*** (-2.932)	-0.017*** (-3.068)	-0.017*** (-2.618)	-0.019*** (-3.219)
N	234	232	234	234	232	234	234	234	234

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A4: Relationship of Exchange Rates and Alternative Measure of Banking Liquidity Ratio

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat2}_t)$	0.098*** (3.718)	0.109*** (3.073)	0.069*** (2.675)	-0.012 (-0.450)	0.123*** (3.388)	0.101*** (3.100)	0.088*** (2.789)	0.079*** (2.768)	0.103*** (4.049)
$\pi_t - \pi_t^*$	-0.511*** (-3.480)	-0.406** (-2.135)	-0.421* (-1.944)	-0.082 (-0.540)	-0.673*** (-3.519)	-0.126 (-0.849)	-0.440** (-2.253)	-0.640*** (-2.731)	-0.346** (-2.065)
LiqRat2_{t-1}	0.006** (2.118)	0.005 (1.415)	0.005* (1.833)	0.004 (1.217)	0.005 (1.388)	0.006* (1.796)	0.004 (1.377)	0.003 (1.129)	0.004 (1.280)
Constant	-0.006*** (-2.699)	-0.001 (-0.257)	-0.002 (-1.249)	-0.002 (-0.659)	-0.003 (-1.414)	-0.001 (-0.496)	-0.005* (-1.765)	-0.014*** (-3.175)	-0.001 (-0.311)
N	234	232	234	234	232	234	234	234	234
adj. R^2	0.09	0.04	0.04	-0.00	0.07	0.04	0.04	0.04	0.06

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A5: Relationship of Exchange Rates and Alternative Measure of Banking Liquidity Ratio with VIX

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat2}_t)$	0.090*** (3.501)	0.088*** (2.933)	0.059** (2.544)	-0.008 (-0.283)	0.106*** (3.233)	0.088*** (2.911)	0.078*** (2.625)	0.074*** (2.616)	0.096*** (3.833)
$\pi_t - \pi_t^*$	-0.393*** (-2.721)	-0.188 (-1.163)	-0.299 (-1.543)	-0.112 (-0.742)	-0.480*** (-2.748)	-0.052 (-0.373)	-0.379** (-2.052)	-0.575** (-2.451)	-0.274* (-1.655)
ΔVIX_t	0.001*** (4.215)	0.004*** (9.677)	0.002*** (7.735)	-0.001** (-2.589)	0.003*** (7.238)	0.002*** (6.145)	0.002*** (5.446)	0.001** (2.126)	0.001*** (3.330)
LiqRat2_{t-1}	0.006** (2.224)	0.006* (1.860)	0.005** (2.137)	0.004 (1.259)	0.006* (1.656)	0.007* (1.951)	0.005* (1.663)	0.003 (1.220)	0.003 (1.268)
Constant	-0.005** (-2.372)	-0.001 (-0.660)	-0.002 (-1.235)	-0.003 (-0.829)	-0.003 (-1.466)	-0.001 (-0.502)	-0.004* (-1.698)	-0.013*** (-2.931)	-0.000 (-0.277)
N	234	232	234	234	232	234	234	234	234
adj. R^2	0.15	0.32	0.23	0.02	0.24	0.17	0.15	0.06	0.10

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A6: Relationship of Exchange Rates and Alternative Measure of Banking Liquidity Ratio Instrumental Variable Regression: StDev(Inf) and StDev(XRate) instrument for $\Delta(\text{LiquidityRatio})$

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat2}_t)$	0.362*** (2.758)	0.491*** (2.791)	0.352*** (3.033)	-0.239* (-1.872)	0.411** (2.579)	0.450*** (2.625)	0.373** (2.593)	0.018 (0.146)	0.604*** (2.674)
$\pi_t - \pi_t^*$	-0.607*** (-2.991)	-0.548** (-2.065)	-0.403 (-1.573)	0.100 (0.483)	-0.698*** (-2.996)	-0.324 (-1.496)	-0.552** (-2.347)	-0.490 (-1.630)	-1.149** (-2.436)
LiqRat2_{t-1}	0.008** (2.318)	0.008* (1.966)	0.007** (2.124)	0.000 (0.091)	0.007* (1.832)	0.011** (2.310)	0.007* (1.795)	0.003 (1.087)	0.011** (1.989)
ΔVIX_t	0.001*** (2.617)	0.004*** (6.194)	0.002*** (5.240)	-0.001* (-1.693)	0.003*** (5.402)	0.002*** (4.018)	0.002*** (3.955)	0.001** (2.146)	0.001 (0.859)
Constant	-0.007*** (-2.717)	-0.002 (-0.832)	-0.004 (-1.636)	0.002 (0.484)	-0.005* (-1.839)	-0.003 (-1.137)	-0.007** (-2.146)	-0.011* (-1.945)	-0.004 (-1.228)
N	234	232	234	234	232	234	234	234	234

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A7: Relationship of Exchange Rates and Alternative Measure of Banking Liquidity Ratio Instrumental Variable Regression: $\Delta(\text{VIX})$, $\text{StDev}(\text{Inf})$ and $\text{StDev}(\text{XRate})$ instrument for $\Delta(\text{LiquidityRatio})$

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{LiqRat2}_t)$	0.471*** (3.276)	0.822*** (3.327)	0.484*** (3.420)	-0.296** (-2.251)	0.651*** (3.250)	0.649*** (3.135)	0.507*** (3.076)	0.105 (0.882)	0.686*** (3.036)
$\pi_t - \pi_t^*$	-0.774*** (-3.466)	-1.007*** (-2.679)	-0.552* (-1.734)	0.171 (0.788)	-1.023*** (-3.468)	-0.527** (-1.978)	-0.678** (-2.466)	-0.677** (-2.353)	-1.315*** (-2.760)
LiqRat2_{t-1}	0.008** (2.177)	0.010 (1.597)	0.008* (1.831)	-0.001 (-0.120)	0.008 (1.598)	0.013** (2.219)	0.007 (1.559)	0.003 (1.149)	0.012** (2.074)
Constant	-0.009*** (-2.938)	-0.002 (-0.601)	-0.005 (-1.641)	0.004 (0.780)	-0.007* (-1.841)	-0.004 (-1.240)	-0.008** (-2.235)	-0.015*** (-2.662)	-0.004 (-1.300)
N	234	232	234	234	232	234	234	234	234

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A8: Relationship of Exchange Rates and the Measure of Intraday Fed Funds Rate Volatility Instrumental Variable
 Regression: $\Delta(\text{VIX})$, $\text{StDev}(\text{Inf})$ and $\text{StDev}(\text{XRate})$ instrument for $\Delta(\text{MaxDif}_t)$

	Euro	Australia	Canada	Japan	New Zealand	Norway	Sweden	Switz	U.K.
$\Delta(\text{MaxDif}_t)$	0.019*** (2.964)	0.030*** (3.376)	0.018*** (3.536)	-0.009** (-2.125)	0.025*** (3.034)	0.020*** (2.951)	0.023*** (2.899)	0.006 (1.195)	0.019*** (3.110)
$\pi_t - \pi_t^*$	-0.710** (-2.364)	-0.822* (-1.947)	-0.168 (-0.547)	0.047 (0.325)	-1.091*** (-2.821)	-0.108 (-0.480)	-1.006** (-2.320)	-0.916** (-2.168)	-0.394 (-1.326)
MaxDif_{t-1}	0.008** (2.409)	0.013*** (2.697)	0.007** (2.550)	-0.006*** (-2.808)	0.013*** (2.786)	0.008** (2.222)	0.011** (2.577)	0.004 (1.291)	0.008** (2.505)
Constant	-0.020** (-2.502)	-0.021** (-2.359)	-0.013** (-2.317)	0.011** (2.099)	-0.026*** (-2.794)	-0.014* (-1.962)	-0.029*** (-2.626)	-0.025** (-2.133)	-0.013** (-2.063)
N	187	187	187	187	187	187	187	187	187

t statistics in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

B Additional Tables Quantitative Analysis

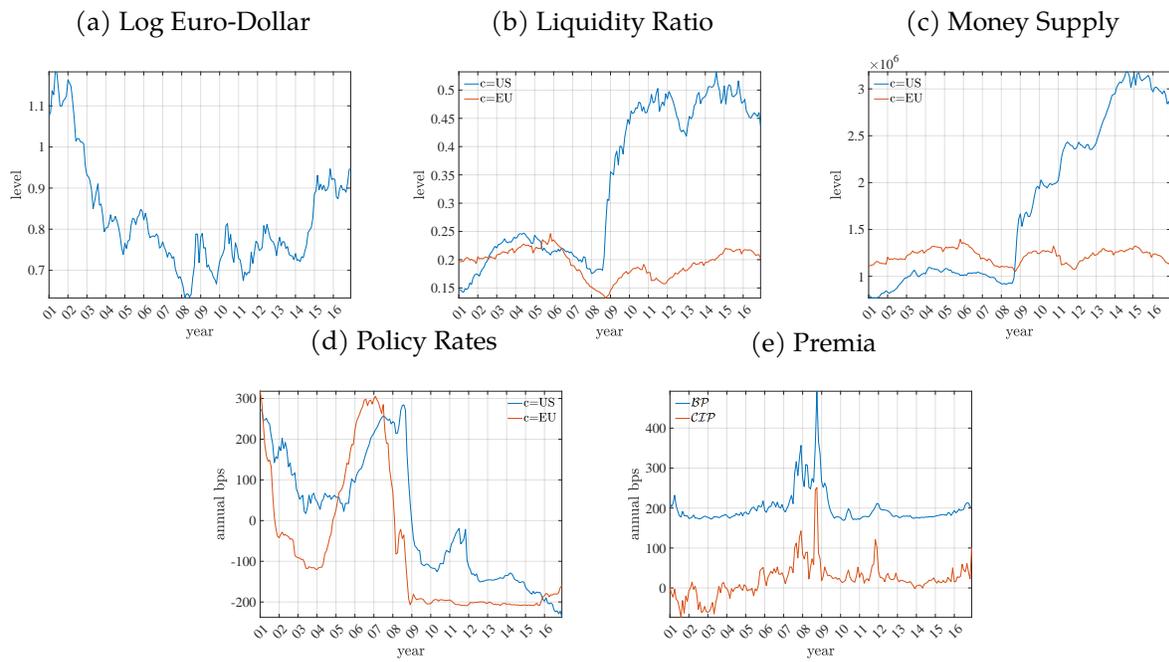


Figure 8: Data series

Table B1: CALIBRATED PARAMETERS

Parameter	Value	Description	Target
External Calibration			
M_{ss}^{us}/M_{ss}^{eu}	0.6841	relative money supply	normalized to exchange rate levels
Θ^b	1	global loan demand scale	normalization
ϵ	-35	loan demand elasticity	Bianchi and Bigio (2021)/symmetry
$\zeta^{us} = \zeta^{eu}$	35	US/EU deposit demand elasticity	Bianchi and Bigio (2021)/symmetry
$\lambda^{us} = \lambda^{eu}$	7.9	US/EU interbank market matching efficiency	Bianchi and Bigio (2021)
$\iota^{us} = \iota^{eu}$	10	US policy corridor spread	Bianchi and Bigio (2021)
$\eta^{us} = \eta^{eu}$	1/2	benchmark/symmetry	
Steady-State Estimation of Financial Variables			
$\Theta_{ss}^{d,*}$	1.0026	US deposit demand scale	To match steady-state moments
Θ_{ss}^d	1.0026	EU deposit demand scale	symmetry
σ_{ss}^{us}	76.8541	average US payment shock	steady-state moment targets
σ_{ss}^{eu}	21.9133	average EU payment shock	steady-state moments targets
ξ_{ss}	-62.5845	average <i>CIP</i> and <i>UIP</i> wedge	steady-state moments targets
Estimates of US and EU policy variables			
$\mathbb{E}(i_t^{m,us})$	0.9957	annualized US interest on reserves	data
$\Sigma(i_t^{m,us})$	0.0026	std annual US policy rate	data
$\rho(i_t^{m,us})$	0.9777	autocorrelation annual US policy rate	data
$\mathbb{E}(i_t^{m,eu})$	1.0027	average annual Euro policy rate	data
$\Sigma(i_t^{m,eu})$	0.0017	std annual Euro policy rate	data
$\rho(i_t^{m,eu})$	0.9894	autocorrelation annual Euro policy rate	data
$\mathbb{E}(\ln M_t^{us})$	6.7255	average monthly US liquid assets stock (\$ Billion)	data
$\Sigma(\ln M_t^{us})$	0.0281	std monthly US liquid assets stock	data
$\rho(\ln M_t^{us})$	0.9894	autocorrelation monthly US liquid assets stock	data
$\mathbb{E}(\ln M_t^{us})$	6.7255	average monthly US liquid assets stock (\$ Billion)	data
$\Sigma(\ln M_t^{us})$	0.0281	std monthly US liquid assets stock	data
$\rho(\ln M_t^{us})$	0.9894	autocorrelation monthly US liquid assets stock	data

Table B2: ESTIMATED PROCESS

Parameter	Prior Fam	Prior Mean	Post. Mean	Post. Interval
$\Sigma^{\sigma^{us}}$	Inv. Gamma	0.150	0.100	0.091-0.108
$\Sigma^{\sigma^{eu}}$	Inv. Gamma	0.150	0.124	0.115-0.134
$\Sigma^{\Theta^{us}}$	Inv. Gamma	0.030	0.017	0.015-0.018
$\Sigma^{\Theta^{eu}}$	Inv. Gamma	0.020	0.015	0.014-0.017
Σ^{ξ}	Inv. Gamma	0.000	0.001	0.000-0.001
$\rho^{\sigma^{us}}$	Beta	0.930	0.989	0.986-0.991
$\rho^{\sigma^{eu}}$	Beta	0.930	0.954	0.931-0.973
$\rho^{\Theta^{us}}$	Beta	0.985	0.991	0.989-0.992
$\rho^{\Theta^{eu}}$	Beta	0.980	0.989	0.988-0.991
ρ^{ξ}	Beta	0.980	0.980	0.979-0.982

Table B3: MODEL AND DATA MOMENTS

Moment	Data	Simulated Moment
Steady-State Targets		
Mean of CIP	13.407	16.030
Mean of \mathcal{BP}	194.605	196.044
Mean of μ^{us}	0.226	0.233
Mean of μ^{eu}	0.222	0.232
Mean of e	1.251	1.281
Untargetted Moments		
Std. of CIP	42.672	121.597
Std. of \mathcal{BP}	42.271	103.433
Std. of μ^{us}	0.133	0.036
Std. of μ^{eu}	0.025	0.041
Std. of e	0.167	0.183
AR(1) Coef. of CIP	0.801	0.947
AR(1) Coef. of \mathcal{BP}	0.879	0.955
AR(1) Coef. of μ^{us}	0.989	0.957
AR(1) Coef. of μ^{eu}	0.986	0.955
AR(1) Coef. of e	0.963	0.953

Figure 9: Euro per Dollar Exchange Rate, March 2020

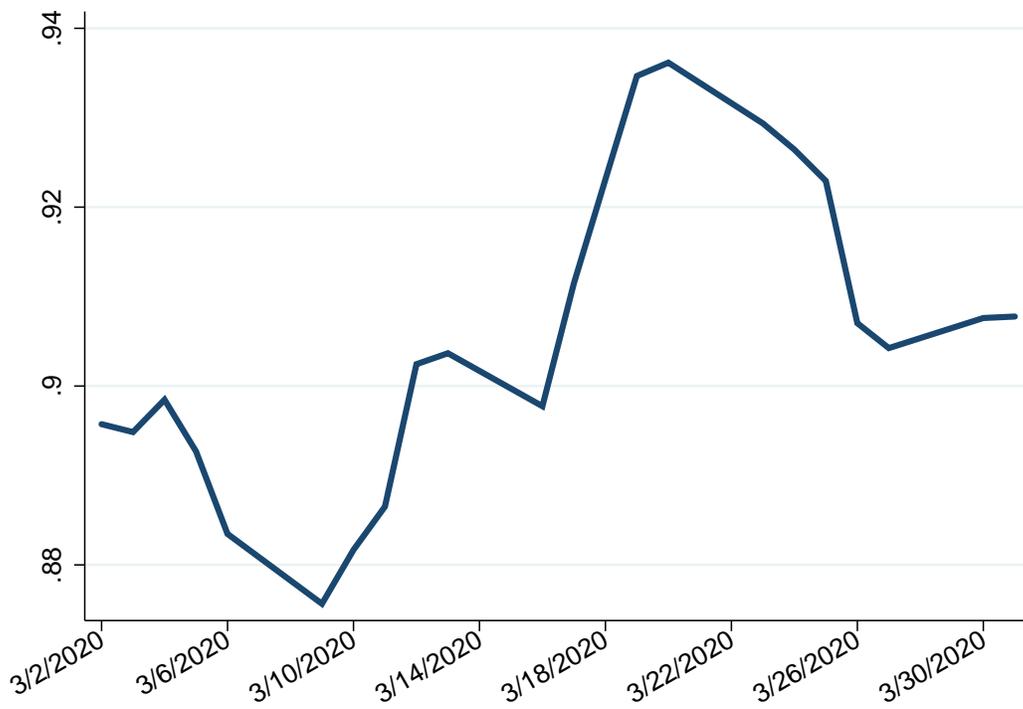


Table B4: U.S. Treasury Yields, March 2020

Date	1 Mo	2 Mo	3 Mo	6 Mo	1 Yr	2 Yr	3 Yr	5 Yr	7 Yr	10 Yr	20 Yr	30 Yr
3/2/2020	1.41	1.27	1.13	0.95	0.89	0.84	0.85	0.88	1.01	1.1	1.46	1.66
3/3/2020	1.11	1.05	0.95	0.83	0.73	0.71	0.72	0.77	0.91	1.02	1.44	1.64
3/4/2020	1	0.87	0.72	0.68	0.59	0.67	0.68	0.75	0.9	1.02	1.45	1.67
3/5/2020	0.92	0.83	0.62	0.53	0.48	0.59	0.61	0.67	0.81	0.92	1.34	1.56
3/6/2020	0.79	0.64	0.45	0.41	0.39	0.49	0.53	0.58	0.69	0.74	1.09	1.25
3/9/2020	0.57	0.52	0.33	0.27	0.31	0.38	0.4	0.46	0.56	0.54	0.87	0.99
3/10/2020	0.57	0.55	0.44	0.43	0.43	0.5	0.58	0.63	0.73	0.76	1.16	1.28
3/11/2020	0.42	0.42	0.42	0.4	0.4	0.5	0.58	0.66	0.78	0.82	1.13	1.3
3/12/2020	0.41	0.33	0.33	0.37	0.39	0.5	0.58	0.66	0.82	0.88	1.27	1.49
3/13/2020	0.33	0.3	0.28	0.38	0.38	0.49	0.58	0.7	0.89	0.94	1.31	1.56
3/16/2020	0.25	0.25	0.24	0.29	0.29	0.36	0.43	0.49	0.67	0.73	1.1	1.34
3/17/2020	0.12	0.18	0.19	0.24	0.3	0.47	0.54	0.66	0.91	1.02	1.45	1.63
3/18/2020	0.04	0.03	0.02	0.08	0.21	0.54	0.66	0.79	1.08	1.18	1.6	1.77
3/19/2020	0.04	0.04	0.04	0.06	0.2	0.44	0.53	0.66	1	1.12	1.56	1.78
3/20/2020	0.04	0.05	0.05	0.05	0.15	0.37	0.41	0.52	0.82	0.92	1.35	1.55
3/23/2020	0.01	0.04	0.02	0.08	0.17	0.28	0.31	0.38	0.63	0.76	1.12	1.33
3/24/2020	0.01	0.01	0.01	0.09	0.25	0.38	0.44	0.52	0.75	0.84	1.19	1.39
3/25/2020	0	0	0	0.07	0.19	0.34	0.41	0.56	0.77	0.88	1.23	1.45
3/26/2020	0.01	0.01	0	0.04	0.13	0.3	0.36	0.51	0.72	0.83	1.2	1.42
3/27/2020	0.01	0.03	0.03	0.02	0.11	0.25	0.3	0.41	0.6	0.72	1.09	1.29
3/30/2020	0.04	0.07	0.12	0.12	0.14	0.23	0.29	0.39	0.57	0.7	1.1	1.31
3/31/2020	0.05	0.12	0.11	0.15	0.17	0.23	0.29	0.37	0.55	0.7	1.15	1.35

Source: U.S. Department of Treasury

Appendix not for Publication

C Expressions for $\{\Psi^+, \Psi^-, \chi^+, \chi^-\}$

Here we reproduce formulas derived from Proposition 1 in [Bianchi and Bigio \(2017\)](#). That proposition gives us the formulas for the liquidity yield function and the matching probabilities as functions of the tightness of the interbank market.

The average interbank rate is:

$$R^f = (1 - \bar{\eta}(\theta))R^w + \bar{\eta}(\theta)R^m$$

where $\bar{\eta}(\theta)$ is an endogenous bargaining power given by

$$\bar{\eta}(\theta) \equiv \begin{cases} \frac{\theta}{\theta-1} \left(\left(\frac{\bar{\theta}}{\theta} \right)^\eta - 1 \right) (\exp(\lambda) - 1)^{-1} & \text{if } \theta > 1 \\ \eta & \text{if } \theta = 1 \\ \frac{\theta(1-\bar{\theta})-\bar{\theta}}{\theta(1-\theta)} \left(\left(\frac{\bar{\theta}}{\theta} \right)^\eta - 1 \right) (\exp(\lambda) - 1)^{-1} & \text{if } \theta < 1 \end{cases}$$

and η is a parameter associated with the bargaining power of banks with reserve deficits in each trade—a Nash bargaining coefficient. In addition, $\bar{\theta}$ represents the market tightness after the interbank-market trading session is over:

$$\bar{\theta} = \begin{cases} 1 + (\theta - 1) \exp(\lambda) & \text{if } \theta > 1 \\ 1 & \text{if } \theta = 1 \\ (1 + (\theta^{-1} - 1) \exp(\lambda))^{-1} & \text{if } \theta < 1 \end{cases}$$

The parameter λ captures the matching efficiency of the interbank market. Trading probabilities are given by

$$\Psi^+ = \begin{cases} 1 - e^{-\lambda} & \text{if } \theta \geq 1 \\ \theta (1 - e^{-\lambda}) & \text{if } \theta < 1 \end{cases}, \quad \Psi^- = \begin{cases} (1 - e^{-\lambda}) \theta^{-1} & \text{if } \theta > 1 \\ 1 - e^{-\lambda} & \text{if } \theta \leq 1 \end{cases}. \quad (27)$$

Finally, using [7](#) and [6](#), we arrive at the parameters of the liquidity yield function χ :

$$\bar{\chi}^+ = (R^w - R^m) \left(\frac{\bar{\theta}}{\theta} \right)^\eta \left(\frac{\theta^\eta \bar{\theta}^{1-\eta} - \theta}{\bar{\theta} - 1} \right) \text{ and } \bar{\chi}^- = (R^w - R^m) \left(\frac{\bar{\theta}}{\theta} \right)^\eta \left(\frac{\theta^\eta \bar{\theta}^{1-\eta} - 1}{\bar{\theta} - 1} \right). \quad (28)$$

D Proofs

D.1 Preliminaries

Here we provide some intermediate results that we use to prove the propositions.

Recall that the liquidity ratio is denoted by $\mu \equiv m/d$ and $\theta = S^-/S^+$ where $S^- = -\int \min\{s, 0\} d\Phi(\omega)$, $S^+ = \int \max\{s, 0\} d\Phi(\omega)$ and $s = m + \omega d$. Then,

$$\begin{aligned}\theta &= -\frac{\int_{\{s < 0\}} s \cdot d\Phi(\omega; \sigma)}{\int_{\{s > 0\}} s \cdot d\Phi(\omega; \sigma)}, \\ &= -\frac{m\Phi(\{s < 0\}; \sigma) + d \int_{\{s < 0\}} \omega \cdot d\Phi(\omega; \sigma)}{m(1 - \Phi(\{s > 0\}; \sigma)) + d \int_{\{s \geq 0\}} \omega \cdot d\Phi(\omega; \sigma)}.\end{aligned}$$

Note that $s < 0$ occurs when $\omega < -\mu$. Therefore, we express the interbank market tightness as:

$$\theta = -\frac{\int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega; \sigma)}{\int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma)}. \quad (29)$$

With abuse of notation, define $\theta(\mu, \sigma)$ as the function that maps μ and σ into a value of θ (thus, in equilibrium, $\theta = \theta(\mu, \sigma)$). We have the following Lemma:

Lemma 3. *Interbank market tightness is decreasing in the liquidity ratio. That is, $\frac{d\theta}{d\mu} < 0$. Moreover, $\theta \in [0, 1]$.*

Proof. From (29), using Leibniz rule, we obtain

$$\frac{d\theta}{d\mu} = \theta \left(\frac{\Phi(-\mu; \sigma)}{\int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega; \sigma)} - \frac{1 - \Phi(-\mu; \sigma)}{\int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma)} \right). \quad (30)$$

By definition of conditional expectation:

$$\mathbb{E}[\mu + \omega | \omega < -\mu] = \int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega; \sigma) / \Phi(-\mu; \sigma),$$

and

$$\mathbb{E}[\mu + \omega | \omega > -\mu] = \int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma) / (1 - \Phi(-\mu; \sigma)).$$

Replacing these definitions into (30), we obtain:

$$\frac{d\theta}{d\mu} = \theta \cdot \left(\frac{1}{\mathbb{E}[\mu + \omega | \omega < -\mu]} - \frac{1}{\mathbb{E}[\mu + \omega | \omega > -\mu]} \right) < 0,$$

where the inequality follows because $\mathbb{E}[\mu + \omega | \omega < -\mu] < 0$ and $\mathbb{E}[\mu + \omega | \omega > -\mu] > 0$.

Finally, the bounds on θ follow because $\lim_{\mu \rightarrow \infty} \theta = 0$ and $\theta = 1$ if $\mu = 0$. \square

Next, we obtain the derivative of interbank market tightness with respect to σ .

Lemma 4. *Under Assumption 1, we have that $\frac{\partial \theta}{\partial \sigma} > 0$.*

Proof. Passing the differential operator inside the integrals in the numerators, we have that:

$$\begin{aligned} \frac{\partial \theta}{\partial \sigma} &= \theta \cdot \left(\frac{\int_{-\infty}^{-\mu} (\mu + \omega) \phi_{\sigma} d\omega}{\int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega; \sigma)} - \frac{\int_{-\mu}^{\infty} (\mu + \omega) \phi_{\sigma} d\omega}{\int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma)} \right) \\ &= \theta \cdot \left(\frac{\partial}{\partial \sigma} \left[\log \left(\frac{\int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega; \sigma)}{\int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma)} \right) \right] \right). \end{aligned}$$

Since the withdrawal shock is zero mean,

$$\int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega; \sigma) + \int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma) = \mu.$$

Therefore, identity this condition into the derivative just above we obtain:

$$\frac{\partial \theta}{\partial \sigma} = \log \left(\frac{\mu - \int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma)}{\int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega; \sigma)} \right).$$

Therefore, $\frac{\partial \theta}{\partial \sigma} > 0$ holds if and only if:

$$\frac{\partial}{\partial \sigma} \left[\int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega; \sigma) \right] < 0.$$

Using the integration by parts formula:

$$\begin{aligned} \int_{-\infty}^{-\mu} (\mu + \omega) \phi_{\sigma}(\omega; \sigma) d\omega &= (\mu + \omega) \Phi_{\sigma}(\omega; \sigma) \Big|_{-\infty}^{-\mu} - \int_{-\infty}^{-\mu} \Phi_{\sigma}(\omega; \sigma) d\omega \\ &= - \int_{-\infty}^{-\mu} \Phi_{\sigma}(\omega; \sigma) d\omega < 0 \end{aligned}$$

where the last equality follows from $\lim_{\omega \rightarrow -\infty} ((\mu + \omega)) \Phi_{\sigma}(\omega; \sigma) = \frac{\partial}{\partial \sigma} [\lim_{\omega \rightarrow -\infty} ((\mu + \omega)) \Phi(\omega; \sigma)] =$

0 and the strict inequality follows from Assumption 1. We conclude that, $\frac{\partial \theta}{\partial \sigma} > 0$. \square

We will also use the results from the following Lemma.

Lemma 5. *The liquidity coefficients have the following derivatives:*

$$\frac{\partial \chi^+}{\partial \mu} = \frac{\partial \chi^+}{\partial \theta} \frac{\partial \theta}{\partial \mu} < 0 \quad \text{and} \quad \frac{\partial \chi^-}{\partial \mu} = \frac{\partial \chi^-}{\partial \theta} \frac{\partial \theta}{\partial \mu} < 0, \quad (31)$$

$$\frac{\partial \chi^+}{\partial \sigma} = \frac{\partial \chi^+}{\partial \theta} \frac{\partial \theta}{\partial \sigma} > 0 \quad \text{and} \quad \frac{\partial \chi^-}{\partial \mu} = \frac{\partial \chi^-}{\partial \theta} \frac{\partial \theta}{\partial \sigma} > 0, \quad (32)$$

$$\frac{\partial \bar{\chi}^+}{\partial P_t} = \frac{\bar{\chi}^+}{P_t} \quad \text{and} \quad \frac{\partial \bar{\chi}^-(\theta)}{\partial P_t} = \frac{\bar{\chi}^-}{P_t}. \quad (33)$$

Proof. Notice first that $\frac{\partial \chi^+}{\partial \theta} > 0$ and $\frac{\partial \chi^-}{\partial \theta} > 0$ is an immediate result from their definitions in equations (28). Applying Lemmas 3 and 4, we obtain respectively (31) and (32).

In addition, we can express (28) as

$$\bar{\chi}^+ = \frac{P_t}{P_{t+1}} (i^w - i^m) \left(\frac{\bar{\theta}}{\theta} \right)^\eta \left(\frac{\theta^\eta \bar{\theta}^{1-\eta} - \theta}{\bar{\theta} - 1} \right), \quad \bar{\chi}^- = \frac{P_t}{P_{t+1}} (i^w - i^m) \left(\frac{\bar{\theta}}{\theta} \right)^\eta \left(\frac{\theta^\eta \bar{\theta}^{1-\eta} - 1}{\bar{\theta} - 1} \right) \quad (34)$$

Equation (33) follows immediately. \square

It is useful to define $\mathcal{L}(\mu, \sigma, P)$ to be the bond liquidity premium as a function of the liquidity ratio, the index σ and the current price level. That is,

$$\mathcal{L}(\mu, \sigma, P) = (1 - \Phi(-\mu, \sigma)) \cdot \bar{\chi}^+(\theta(\mu, \sigma), P) + \Phi(-\mu, \sigma) \cdot \bar{\chi}^-(\theta(\mu, \sigma), P) \quad (35)$$

In equilibrium $\mathcal{L}(\mu, \sigma, P) = R^b - R^m$. We have the following result.

Lemma 6. *The liquidity bond premium is decreasing in the liquidity ratio and increasing in volatility. That is, $\mathcal{L}_\mu < 0$ and $\mathcal{L}_\sigma > 0$. In addition, $\mathcal{L}_P = -\mathcal{L}/P$.*

Proof. From (35), differentiating \mathcal{L} with respect to μ :

$$\mathcal{L}_\mu = [(1 - \Phi(-\mu, \sigma)) \cdot \chi_\theta^+ + \Phi(-\mu, \sigma) \cdot \chi_\theta^-] - (\bar{\chi}^- - \bar{\chi}^+) \phi(-\mu, \sigma). \quad (36)$$

Using that $\frac{\partial \theta}{\partial \mu} < 0$ from Lemma 3 and that $\bar{\chi}^- > \bar{\chi}^+$, we arrive at $\mathcal{L}_\mu < 0$.

From (35), differentiating \mathcal{L} with respect to σ yields:

$$\mathcal{L}\mathcal{P}_\sigma = \frac{\partial \theta}{\partial \sigma} [(1 - \Phi(-\mu, \sigma)) \cdot \chi_\theta^+ + \Phi(-\mu, \sigma) \cdot \chi_\theta^-] + (\bar{\chi}^- - \bar{\chi}^+) \Phi_\sigma(-\mu, \sigma). \quad (37)$$

Using that $\frac{\partial \theta}{\partial \sigma} > 0$ from Lemma 4 and that $\bar{\chi}^- > \bar{\chi}^+$, we conclude that $\mathcal{L}_\sigma > 0$. Finally, the expression for \mathcal{L}_P follows directly from differentiating \mathcal{L} with respect to P in (33). \square

We now proceed with the proofs and use that these properties apply for both euros and dollars.

D.2 Proof of Proposition 1

Proof. Part i). By definition, the liquidity ratio μ^* is given by

$$\mu^*(P^*, D^*) = \frac{M^*/P^*}{D^*} \quad (38)$$

where we made explicit the dependence of μ^* on (P^*, D^*) . Using that M^* is exogenously given, totally differentiating (38) yields

$$d\mu^* = -\mu^* \left(\frac{dP^*}{P^*} + \frac{dD^*}{D^*} \right). \quad (39)$$

The dollar liquidity premium is

$$R^b - (1 + i^{m,*}) \frac{P^*}{\mathbb{E}[P^*(X')]} = \mathcal{L}^*(\mu^*(P^*, D^*), P^*). \quad (40)$$

Totally differentiating (40) with respect to P^* and D^* , and using (39), we obtain:

$$-R^{m,*} \left(\frac{dP^*}{P^*} \right) = -\mathcal{L}_{\mu^*}^* \left[\mu \left(\frac{dP^*}{P^*} + \frac{dD^*}{D^*} \right) \right] + \mathcal{L}_P^* dP^* \quad (41)$$

where $\mathbb{E}[P^*(X')]$ remains constant because the shock is i.i.d. and the loan rate is constant at $R^b = 1/\beta$.

Using $\mathcal{L}_{P^*}^* = \frac{\mathcal{L}^*}{P^*}$ from Lemma 6, $R^b = R^{m,*} + \mathcal{L}^*$ and replacing in (41), we arrive to

$$\frac{d \log P^*}{d \log D^*} = \frac{\mathcal{L}_{\mu^*}^* \mu^*}{R^b - \mathcal{L}_{\mu^*}^* \mu^*} \in (-1, 0). \quad (42)$$

The bounds follows immediately because $\mathcal{L}_{\mu^*}^* < 0$ as established in Lemma 6 and from $R^b > 0$.

Notice also that the euro bond premium remains constant. To see this, we can replace $\mu = (M/P)/D$ in (17) and use (29) to obtain

$$R^b - (1 + i^m) \frac{P}{\mathbb{E}[P(X')]} = \left(1 - \Phi\left(-\frac{M/P}{D}\right)\right) \bar{\chi}^+(\theta((M/P)/D, \sigma)) + \Phi\left(-\frac{M/P}{D}\right) \bar{\chi}^-(\theta((M/P)/D, \sigma)). \quad (43)$$

From (43), it follows that P must be constant and thus μ and \mathcal{L} are also constant. As a result, $d\mathcal{L}^* = d\mathcal{D}\mathcal{L}\mathcal{P}$, $d\mathcal{L}_{\mu^*}^* = d\mathcal{D}\mathcal{L}\mathcal{P}_{\mu^*}$.

By the law of one price and using that P remains constant, we then have $\frac{d \log e}{d \log D^*} = -\frac{\mathcal{L}_{\mu^*}^*}{R^b - \mathcal{L}_{\mu^*}^*}$ which implies an appreciation of the dollar. Finally, we can rewrite (41) as $\bar{R}^{m,*} (d \log e) = d\mathcal{L}^* = d\mathcal{D}\mathcal{L}\mathcal{P}$.

Part ii). When the shock is permanent, expected inflation remains constant. Moreover, given that nominal policy rates and expected inflation are constant, we have from (18) that \mathcal{L}^* is constant. Hence, $\mathcal{D}\mathcal{L}\mathcal{P}$ is constant. Furthermore, the fact that \mathcal{L}^* is constant, implies that μ must also be constant. Thus, using that (39) and that M^* is constant, we have from the law of one price that:

$$\frac{d \log e}{d \log D^*} = -\frac{d \log P^*}{d \log D^*} = 1.$$

□

D.3 Proof of Proposition 2

Proof. Part i). Totally differentiating (38) with respect to P^* yields

$$d\mu^* = -\mu^* \left(\frac{dP^*}{P^*} \right). \quad (44)$$

The dollar liquidity premium is

$$R^b - (1 + i^{m,*}) \frac{P^*}{\mathbb{E}[P^*(X')]} = \mathcal{L}^*(\mu^*(P^*, \sigma^*), P^*). \quad (45)$$

Totally differentiating (45) with respect to P^* and σ^* and using (44) yields:

$$-R^{m,*} \left(\frac{dP^*}{P^*} \right) = -\mathcal{L}_{\mu}^* \left[\mu \left(\frac{dP^*}{P^*} \right) \right] + \mathcal{L}_{\sigma^*}^* d\sigma^* + \mathcal{L}_P^* dP^* \quad (46)$$

where we used that $\mathbb{E}[P^*(X')]$ is constant because the shock is i.i.d. and $R^b = 1/\beta$.

Using $\mathcal{L}_{P^*}^* = \frac{\mathcal{L}^*}{P^*}$ from Lemma 6, $R^b = R^{m,*} + \mathcal{L}^*$, and replacing in (44), we obtain

$$\frac{d \log P^*}{d \log \sigma^*} = -\frac{\mathcal{L}_{\sigma^*}^* \sigma^*}{R^b - \mathcal{L}_{\mu^*}^* \mu^*} < 0 \quad (47)$$

where the sign follows from Lemma 6. Notice also that the euro bond premium remains constant, and so do P , μ and \mathcal{L} , as demonstrated in the proof of Proposition 1.

By the law of one price, and using that P remains constant, we then have $\frac{d \log e}{d \log D^*} = \frac{\mathcal{L}_{\sigma^*}^* \sigma^*}{R^b - \mathcal{L}_{\mu^*}^* \mu^*}$ which implies an appreciation of the dollar. Finally, we can rewrite (46) as $\bar{R}^{m,*} (d \log e) = d\mathcal{L}^* = d\mathcal{DLP}$.

Part ii). When the shock is permanent, expected inflation is constant. Given that nominal policy rates are constant, \mathcal{L}^* and \mathcal{DLP} are constant. Thus,

$$\mathcal{L}_{\mu^*}^* d\mu^* + \mathcal{L}_{\sigma^*}^* d\sigma^* = 0 \quad (48)$$

and so

$$\frac{d \log \mu^*}{d \log \sigma^*} = -\frac{\mathcal{L}_{\sigma^*}^* \sigma^*}{\mathcal{L}_{\mu^*}^* \mu^*} > 0 \quad (49)$$

where the sign follows from $\mathcal{L}_{\mu^*}^* < 0$ and $\mathcal{L}_{\sigma^*}^* > 0$ from Lemma 6. Using that $d \log \mu^* = -d \log P^*$, from the law of one price, $\frac{d \log e^*}{d \log \sigma^*} = \frac{d \log \mu^*}{d \log \sigma^*}$. \square

D.4 Approximation to Mean Reverting Shocks

Proof. We now derive approximate analogues to propositions 1 and 2 for cases where shocks are mean reverting. In particular, shocks follow a log AR(1) process:

$$\log(x_t) = (1 - \rho^x) \log(x_{ss}) + \rho^x \cdot \log(x_{t-1}) + \Sigma^x \varepsilon_t^x. \quad (50)$$

We have the following result. We use x_{ss} to refer to the deterministic steady-state value of any variable x . The proof extends the results in Propositions 1 and 2. We first show this intermediate result. In the model, prices are a function of the aggregate state, X . Thus, an equilibrium will feature a function $P^*(X_t)$ such that $P_t^* = P^*(X_t)$. Then, near the steady state, using a Taylor expansion of first-order with respect to the variable x . We have that:

$$\log P_t^* \approx \log P_{ss}^* + \frac{P_x^*(x_{ss}) x_{ss}}{P_{ss}^*} \frac{x_t - x_{ss}}{x_{ss}}.$$

Thus, we have that for small deviations around the steady state:

$$d \log P_t^* \approx \frac{P_x^*(x_{ss}) x_{ss}}{P_{ss}^*} d \log x_t. \quad (51)$$

Shifting this condition forward:

$$d \log P_{t+1}^* \approx \frac{P_x^*(x_{ss}) x_{ss}}{P_{ss}^*} d \log x_{t+1}$$

Taking expectations:

$$\mathbb{E} [d \log P_{t+1}^*] \approx \frac{P_x^*(x_{ss}) x_{ss}}{P_{ss}^*} \rho^x d \log x_t. \quad (52)$$

Dividing the left-hand side of (52) by (51),

$$\frac{\mathbb{E} [d \log P_{t+1}^*]}{d \log P_t^*} = \rho^x d \log x_t. \quad (53)$$

Next, we proof the main items of the propositions. The proof uses that for either currency:

$$\frac{\partial \bar{\chi}^+}{\partial P_{t+1}} = -\frac{\bar{\chi}^+}{\mathbb{E}[P_{t+1}]}, \text{ and } \frac{\partial \bar{\chi}^-(\theta)}{\partial P_{t+1}} = -\frac{\bar{\chi}^-}{\mathbb{E}[P_{t+1}]}. \quad (54)$$

Hence:

$$\mathcal{L}_{P_{t+1}^*}^* = -\frac{\mathcal{L}^*}{P_{t+1}^*}$$

Recall that the dollar liquidity premium can be expressed as

$$R^b - (1 + i^{m,*}) \frac{P_t^*}{\mathbb{E}[P_{t+1}^*]} = \mathcal{L}^*(\mu^*(P^*, D^*), P_t^*, P_{t+1}^*), \quad (55)$$

where we now make explicit that \mathcal{L}^* depends on both P_t and P_{t+1} .

Part (i). We present here the proof for item (i). Totally differentiating (55) with respect to P_t , P_{t+1} , and D^* and using (39) near the steady state, we obtain

$$-R^{m,*} \left(\frac{dP_t^*}{P_t^*} \right) + R^{m,*} \frac{\mathbb{E}[dP_{t+1}^*]}{\mathbb{E}[P_{t+1}^*]} = -\mathcal{L}_{\mu^*}^* \mu^* \left(\frac{dP_t^*}{P_t^*} + \frac{dD_t^*}{D_t^*} \right) + \mathcal{L}_{P_t^*}^* dP_t^* - \mathcal{L}_{P_{t+1}^*}^* \mathbb{E}[dP_{t+1}^*]. \quad (56)$$

Then, collecting terms:

$$-(R^{m,*} + \mathcal{L}^*) \left(1 - \frac{\mathbb{E}[d \log P_{t+1}^*]}{d \log P_t^*} \right) d \log P_t^* = -\mathcal{L}_{\mu^*}^* \mu^* \left(\frac{dP_t^*}{P_t^*} + \frac{dD_t^*}{D_t^*} \right). \quad (57)$$

Substituting $R^b = R^{m,*} + \mathcal{L}^*$ and (53), we obtain:

$$R^b (1 - \rho^{D^*}) d \log P_t^* \approx \mathcal{L}_{\mu^*}^* \mu^* \left(\frac{dP_t^*}{P_t^*} + \frac{dD_t^*}{D_t^*} \right).$$

Thus, we obtain

$$\frac{d \log P^*}{d \log D^*} \approx \frac{\mathcal{L}_{\mu^*}^* \mu^*}{(1 - \rho^{D^*}) R^b - \mathcal{L}_{\mu^*}^* \mu^*} < 0.$$

Then, it follows from the law of one price and the differential form of μ that

$$\epsilon_{D^*}^e \equiv \frac{d \log e}{d \log D^*} \approx -\frac{\mathcal{L}_{\mu^*}^* \mu^*}{(1 - \rho^{D^*}) R^b - \mathcal{L}_{\mu^*}^* \mu^*} \in (0, 1),$$

and

$$\epsilon_{\mu^*}^e \equiv \frac{d \log \mu}{d \log D^*} \approx -\frac{(1 - \rho^{D^*}) R^b}{(1 - \rho^{D^*}) R^b - \mathcal{L}_{\mu^*}^* \mu^*} \in (-1, 0).$$

Part (ii). We present here the proof for item (ii). It follows the same steps as in Part (i): We totally differentiate (55) with respect to P_t , P_{t+1} , and σ^* and using (39) for the case where $dD^* = 0$. We obtain:

$$-R^{m,*} \left(\frac{dP_t^*}{P_t^*} \right) + R^{m,*} \frac{\mathbb{E} [dP_{t+1}^*]}{\mathbb{E} [P_{t+1}^*]} = \mathcal{L}_{\sigma^*}^* d\sigma^* - \mathcal{L}_{\mu^*}^* \mu^* \left(\frac{dP_t^*}{P_t^*} \right) + \mathcal{L}_{P_t^*}^* dP_t^* - \mathcal{L}_{P_{t+1}^*}^* \mathbb{E} [dP_{t+1}^*]. \quad (58)$$

Collecting terms and using the same identities that we use to derive 57, we arrive at:

$$(R^b (1 - \rho^{\sigma^*}) - \mathcal{L}_{\mu^*}^* \mu^*) d \log P_t^* \approx -\mathcal{L}_{\sigma^*}^* d\sigma^*.$$

Therefore, we obtain:

$$\frac{d \log P^*}{d \log \sigma^*} \approx \frac{-\mathcal{L}_{\sigma^*}^* d\sigma^*}{(1 - \rho^{\sigma^*}) R^b - \mathcal{L}_{\mu^*}^* \mu^*} < 0.$$

Then using that $\mu^* = M^*/(P^* D^*)$ and that $e = P/P^*$ and that P, M^* and D^* are constant, we arrive at:

$$\frac{d \log \mu^*}{d \log \sigma^*} \approx \frac{d \log e}{d \log \sigma^*} = -\frac{d \log P^*}{d \log \sigma^*} = \frac{\mathcal{L}_{\sigma^*}^* d\sigma^*}{(1 - \rho^{\sigma^*}) R^b - \mathcal{L}_{\mu^*}^* \mu^*} > 0.$$

□

D.5 Proof of Proposition 3

We again consider that any shock x follows a log AR(1) process:

$$\log(x_t) = (1 - \rho^x) \log(x_{ss}) + \rho^x \cdot \log(x_{t-1}) + \Sigma^x \varepsilon_t^x.$$

We consider only shocks to dollar funding risk and the dollar funding scale and that $Var(\varepsilon_t^x) = 1$ for all shocks. Thus

$$Var(x_t) = \frac{(\Sigma^x)^2}{(1 - (\rho^x)^2)}. \quad (59)$$

Consider a univariate linear regression of $\Delta \log e^*$ against $\Delta \log \mu^*$ where $\Delta x_t = x_t - x_{t-1}$. The regression coefficient is a function of two moments:

$$\gamma_{\mu^*}^{e^*} = \frac{CoV(\Delta \log e^*, \Delta \log \mu^*)}{Var(\Delta \log \mu^*)}. \quad (60)$$

Consider an endogenous variable Y_t in the model. An equilibrium will feature a function $Y(X_t)$ such that $Y_t = Y(X_t)$, where X_t is the exogenous state. Then, using a first-order Taylor expansion:

$$\log Y_t \approx \log Y_{ss} + \sum_{x \in X} \frac{Y_x(x_{ss}) \cdot x_{ss}}{Y_{ss}} \frac{x_t - x_{ss}}{x_{ss}} \text{ for } x \in X.$$

Therefore, we have that:

$$\Delta \log Y_t \approx \sum_{x \in X} \frac{Y_x(x_{ss}) \cdot x_{ss}}{Y_{ss}} \left(\frac{x_t - x_{ss}}{x_{ss}} - \frac{x_{t-1} - x_{ss}}{x_{ss}} \right).$$

Near a steady state:

$$\frac{x_t - x_{ss}}{x_{ss}} - \frac{x_{t-1} - x_{ss}}{x_{ss}} \approx \Delta \log(x_t) = \rho^x \cdot (\log(x_{t-1}) - \log(x_{ss})) + \Sigma^x \varepsilon_t^x.$$

Using this identity,

$$\Delta \log Y_t \approx \sum_{x \in X} \frac{Y_x(x_{ss}) \cdot x_{ss}}{Y_{ss}} (\rho^x \cdot (\log(x_{t-1}) - \log(x_{ss})) + \Sigma^x \varepsilon_t^x).$$

Then, for small shocks the log-deviation from steady-state is approximately the elasticity near steady state.

$$\frac{Y_x(x) \cdot x}{Y_{ss}} = \epsilon_x^Y.$$

Hence, we have that $\Delta \log e_t^*$ and $\Delta \log \mu_t^*$ follow:

$$\begin{aligned} \Delta \log e_t^* &= \epsilon_{\sigma^*}^{e^*} (\rho^{\sigma^*} \cdot (\log(\sigma_{t-1}^*) - \log(\sigma_{ss}^*)) + \Sigma^{\sigma^*} \varepsilon_t^{\sigma^*}) \dots \\ &\quad + \epsilon_{D^*}^{e^*} (\rho^{D^*} \cdot (\log(D_{t-1}^*) - \log(D_{ss}^*)) + \Sigma^{D^*} \varepsilon_t^{D^*}). \end{aligned} \quad (61)$$

Likewise, for the dollar liquidity ratio:

$$\begin{aligned}\Delta \log \mu_t^* &= \epsilon_{\sigma^*}^{\mu^*} (\rho^{\sigma^*} \cdot (\log(\sigma_{t-1}^*) - \log(\sigma_{ss}^*)) + \Sigma^{\sigma^*} \epsilon_t^{\sigma^*}) \dots \\ &\quad + \epsilon_{D^*}^{\mu^*} (\rho^{D^*} \cdot (\log(D_{t-1}^*) - \log(D_{ss}^*)) + \Sigma^{D^*} \epsilon_t^{D^*}).\end{aligned}\quad (62)$$

From, (62) variance of the change in the liquidity ratio is:

$$Var(\Delta \log \mu^*) = (\epsilon_{\sigma^*}^{\mu^*})^2 \left((\rho^{\sigma^*})^2 Var(\sigma^*) + (\Sigma^{\sigma^*})^2 \right) + (\epsilon_{D^*}^{\mu^*})^2 \left((\rho^{D^*})^2 Var(D^*) + (\Sigma^{D^*})^2 \right).$$

Substituting (59) into the equation above:

$$\begin{aligned}Var(\Delta \log \mu^*) &= (\epsilon_{\sigma^*}^{\mu^*})^2 \left((\rho^{\sigma^*})^2 \frac{(\Sigma^{\sigma^*})^2}{(1 - (\rho^{\sigma^*})^2)} + (\Sigma^{\sigma^*})^2 \right) + (\epsilon_{D^*}^{\mu^*})^2 \left((\rho^{D^*})^2 \frac{(\Sigma^{D^*})^2}{(1 - (\rho^{D^*})^2)} + (\Sigma^{D^*})^2 \right) \\ &= (\epsilon_{\sigma^*}^{\mu^*})^2 \frac{(\Sigma^{\sigma^*})^2}{(1 - (\rho^{\sigma^*})^2)} + (\epsilon_{D^*}^{\mu^*})^2 \frac{(\Sigma^{D^*})^2}{(1 - (\rho^{D^*})^2)}.\end{aligned}$$

Provided that the shocks to σ^* and D^* are orthogonal, from (61) and (62), we have that following covariance between the change in the exchange rate and the change in the dollar liquidity ratio:

$$\begin{aligned}Cov(\Delta \log e^*, \Delta \log \mu^*) &\approx \epsilon_{\sigma^*}^{e^*} \cdot \epsilon_{\sigma^*}^{\mu^*} \left((\rho^{\sigma^*})^2 Var(\sigma^*) + (\Sigma^{\sigma^*})^2 \right) + \epsilon_{D^*}^{e^*} \cdot \epsilon_{D^*}^{\mu^*} \left((\rho^{D^*})^2 Var(D^*) + \Sigma^{D^*} \right)^2 \\ &= \epsilon_{\sigma^*}^{e^*} \cdot \epsilon_{\sigma^*}^{\mu^*} \left((\rho^{\sigma^*})^2 Var(\sigma^*) + (\Sigma^{\sigma^*})^2 \right) + \epsilon_{D^*}^{e^*} \cdot \epsilon_{D^*}^{\mu^*} \left((\rho^{D^*})^2 Var(D^*) + \Sigma^{D^*} \right)^2 \\ &= \epsilon_{\sigma^*}^{e^*} \cdot \epsilon_{\sigma^*}^{\mu^*} \frac{(\Sigma^{\sigma^*})^2}{(1 - (\rho^{\sigma^*})^2)} + \epsilon_{D^*}^{e^*} \cdot \epsilon_{D^*}^{\mu^*} \frac{(\Sigma^{D^*})^2}{(1 - (\rho^{D^*})^2)}.\end{aligned}$$

Thus, substituting the approximations to $Var(\Delta \log \mu^*)$ and $Cov(\Delta \log e^*, \Delta \log \mu^*)$ back into (60), we obtain that the univariate regression coefficient is approximately:

$$\begin{aligned}\beta_{\mu^*}^{e^*} &\approx \frac{\epsilon_{\sigma^*}^{e^*} \cdot \epsilon_{\sigma^*}^{\mu^*} \cdot Var(\sigma^*) + \epsilon_{D^*}^{e^*} \cdot \epsilon_{D^*}^{\mu^*} Var(D^*)}{(\epsilon_{\sigma^*}^{\mu^*})^2 Var(\sigma^*) + (\epsilon_{D^*}^{\mu^*})^2 Var(D^*)} \\ &= \frac{\epsilon_{\sigma^*}^{e^*}}{\epsilon_{\sigma^*}^{\mu^*}} \cdot \mathbf{w}^{\sigma^*} + \frac{\epsilon_{D^*}^{e^*}}{\epsilon_{D^*}^{\mu^*}} \mathbf{w}^{D^*}.\end{aligned}$$

where:

$$w^{\sigma^*} = \frac{\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2 \frac{(\Sigma^{\sigma^*})^2}{1-(\rho^{\sigma^*})^2}}{\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2 \frac{(\Sigma^{\sigma^*})^2}{1-(\rho^{\sigma^*})^2} + \left(\epsilon_{D^*}^{\mu^*}\right)^2 \frac{(\Sigma^{D^*})^2}{1-(\rho^{D^*})^2}} = \frac{\left(\epsilon_{\sigma^*}^{\mu^*} \Sigma^{\sigma^*}\right)^2 \left(1 - (\rho^{D^*})^2\right)}{\left(\epsilon_{\sigma^*}^{\mu^*} \Sigma^{\sigma^*}\right)^2 \left(1 - (\rho^{D^*})^2\right) + \left(\epsilon_{D^*}^{\mu^*} \Sigma^{D^*}\right)^2 \left(1 - (\rho^{\sigma^*})^2\right)}.$$

and

$$w^{D^*} = \frac{\left(\epsilon_{D^*}^{\mu^*}\right)^2 \frac{(\Sigma^{D^*})^2}{1-(\rho^{D^*})^2}}{\left(\epsilon_{\sigma^*}^{\mu^*}\right)^2 \frac{(\Sigma^{\sigma^*})^2}{1-(\rho^{\sigma^*})^2} + \left(\epsilon_{D^*}^{\mu^*}\right)^2 \frac{(\Sigma^{D^*})^2}{1-(\rho^{D^*})^2}} = \frac{\left(\epsilon_{\sigma^*}^{\mu^*} \Sigma^{\sigma^*}\right)^2 \left(1 - (\rho^{\sigma^*})^2\right)}{\left(\epsilon_{\sigma^*}^{\mu^*} \Sigma^{\sigma^*}\right)^2 \left(1 - (\rho^{D^*})^2\right) + \left(\epsilon_{D^*}^{\mu^*} \Sigma^{D^*}\right)^2 \left(1 - (\rho^{\sigma^*})^2\right)}.$$

D.6 Proof of Proposition (4)

Proof. Part i) Totally differentiating (38) with respect to P^* yields

$$d\mu^* = -\mu^* \left(\frac{dP^*}{P^*} \right). \quad (63)$$

The dollar liquidity premium is

$$R^b - (1 + i^{m,*}) \frac{P^*}{\mathbb{E}[P^*(X')]} = \mathcal{L}^*(\mu^*(P^*), P^*) \quad (64)$$

Totally differentiating (64) with respect to P^* and $(1 + i^{m,*})$, and using (63), we obtain

$$-R^{m,*} \left(\frac{dP^*}{P^*} \right) - \frac{P^*}{\mathbb{E}[P^*(X')]} d(1 + i^{m,*}) = -\mathcal{L}_{\mu^*}^* \mu^* \left(\frac{dP^*}{P^*} \right) + \mathcal{L}_{P^*}^* dP^* \quad (65)$$

where notice that $\mathbb{E}[P^*(X')]$ is constant because the shock is i.i.d. and $R^b = 1/\beta$.

Using $\mathcal{L}_{P^*}^* = \frac{\mathcal{L}^*}{P^*}$ from Lemma 6, $R^b = R^{m,*} + \mathcal{L}^*$, and $\bar{R}^m = P^*(1 + i^{m,*})/\mathbb{E}[P^*(X')]$, and replacing these equalities in (65), we obtain:

$$\frac{d \log P^*}{d \log (1 + i^{m,*})} = -\frac{\bar{R}^m}{R^b - \mathcal{L}_{\mu^*}^* \mu^*} \in (-1, 0) \quad (66)$$

where the sign follows from Lemma 6. The upper bound follows because $R^b > \bar{R}^m$.

Notice also that the euro bond premium remains constant, and so do P , μ and \mathcal{L} , as demonstrated in the proof of Proposition 1. This implies that $d\mathcal{L}^* = d\mathcal{D}\mathcal{L}\mathcal{P}$, $d\mathcal{L}_{\mu^*}^* = d\mathcal{D}\mathcal{L}\mathcal{P}_{\mu^*}$.

By the law of one price, we then have $\frac{d \log e^*}{d \log(1+i^{m,*})} = \frac{\bar{R}^m}{R^b - \mathcal{L}_\mu^* \mu^*}$ which implies an appreciation of the dollar.

Finally, we can rewrite (65) as

$$R^{m,*} (d \log e - d \log(1 + i^{m,*})) = d\mathcal{L}^* = d\mathcal{DLP} < 0 \quad (67)$$

where the sign follows from the bounds on (66).

Part ii). When the shock is permanent, expected inflation is constant. From (18), it follows that the increase in $1 + i^{m,*}$ leads to a decrease in \mathcal{L}^* and a reduction in \mathcal{DLP} . Total differentiation of (64) with respect to $1 + i^{m,*}$ and μ^* yields

$$-\bar{R}^{m,*} d \log(1 + i^{m,*}) = \mathcal{L}_\mu^* \mu^* d \log \mu^*, \quad (68)$$

and thus

$$\frac{d \log \mu^*}{d \log(1 + i^{m,*})} = -\frac{\bar{R}^{m,*}}{\mathcal{L}_\mu^* \mu^*} > 0. \quad (69)$$

where the sign follows from Lemma 6. Using that $d \log \mu^* = -d \log P^*$ when M^* and D^* are constant, we have from the law of one price that $\frac{d \log e^*}{d \log 1+i^{m,*}} = \frac{\bar{R}^{m,*}}{-\mathcal{L}_\mu^* \mu^*}$. Finally

$$d\mathcal{DLP} = -\bar{R}^{m,*} d \log(1 + i^{m,*})$$

□

Proofs of Proposition 5 (Open-Market Operations)

Preliminary Observations. We make two assumptions: first, deposits and securities are perfect substitutes, but the demand for the sum of deposits and securities is perfectly inelastic. Second, the supply of securities is fixed. Let $S^{H,*}$ indicate the household holding of dollar securities and $S^{G,*}$ the central bank's holdings of dollar securities. Thus, we have $S^{H,*} + S^{G,*} = S^*$ where S^* is a fixed supply of securities.

Consider a purchase of securities with reserves. The central banks' budget constraint in this case is modified to:

$$M_t^* + T_t^* + W_{t+1}^* + \left(1 + i_t^{d,*}\right) \cdot P_{t-1}^* S_{t-1}^{G,*} = P_t^* \cdot S_t^{G,*} + M_{t-1}^* (1 + i_t^{m,*}) + W_t^* (1 + i_t^{w,*}).$$

As in earlier proofs, we avoid time subscripts. Consider a small change in the holdings of

central bank securities purchased with reserves. We obtain:

$$dM^* = S^{G,*} dP^* + P^* dS^{G,*} = P^* S^{G,*} \frac{dP^*}{P^*} + P^* S^{G,*} \frac{dS^{G,*}}{S^{G,*}}. \quad (70)$$

Assuming that the central bank has a balance sheet such that Υ of its liabilities are backed with securities,

$$\Upsilon^* = \frac{P^* S^{G,*}}{M^*},$$

we modify (70) to obtain:

$$\frac{dM^*}{M^*} = \frac{P^* S^{G,*}}{M^*} \left(\frac{dP^*}{P^*} + \frac{dS^{G,*}}{S^{G,*}} \right) = \Upsilon^* \left(\frac{dP^*}{P^*} + \frac{dS^{G,*}}{S^{G,*}} \right).$$

Thus, expressed in logs, this condition is:

$$d \log M^* = \Upsilon^* (d \log P^* + d \log S^{G,*}). \quad (71)$$

The equation accounts for the fact that the growth in the money supply needed to finance the open-market operation has consider the change in the price level.

Next, since households are inelastic regarding the some of securities and deposits, it must be that $dD^* = -dS^{H,*}$. Since the supply of the security is fixed $-dS^{H,*} = dS^{G,*}$. Hence,

$$dD^* = dS^{G,*}.$$

Next, we express the change in the liquidity ratio in its differential form:

$$\begin{aligned} d\mu^* &= \mu^* \left(\frac{dM^*}{M^*} - \left(\frac{dP^*}{P^*} + \frac{dD^*}{D^*} \right) \right), \\ &= \mu^* \left(\frac{dM^*}{M^*} - \left(\frac{dP^*}{P^*} + \mu^* \Upsilon^* \frac{dS^{G,*}}{S^{G,*}} \right) \right), \end{aligned}$$

where the second line applies the definitions of μ^* and Υ^* .

In log terms, the last equation is:

$$d \log \mu^* = (d \log M^* - (d \log P^* + \Upsilon^* \mu^* \cdot d \log S^{G,*})).$$

Substituting (71) we obtain:

$$\begin{aligned} d \log \mu^* &= (\Upsilon^* (d \log P^* + d \log S^{G,*}) - d \log P^* - \Upsilon^* \mu^* \cdot d \log S^{G,*}) \\ &= -(1 - \Upsilon^*) d \log P^* + \Upsilon^* (1 - \mu^*) \cdot d \log S^{G,*}. \end{aligned} \quad (72)$$

Item (i). We now derive the main results. We follow the earlier proofs. Totally differentiating the liquidity premium with respect to μ^* and P^* , we obtain:

$$\bar{R}^{m,*} d \log P^* + \mathcal{L}^* d \log P^* + \mathcal{L}_{\mu^*}^* \mu^* d \log \mu^* = 0. \quad (73)$$

Substituting (72) and collecting terms we obtain:

$$(\bar{R}^{m,*} + \mathcal{L}^* - (1 - \Upsilon^*) \mathcal{L} \mathcal{P}_{\mu^*}^* \mu^*) d \log P^* + \mathcal{L}_{\mu^*}^* \mu^* \Upsilon^* (1 - \mu^*) \cdot d \log S^{G,*} = 0. \quad (74)$$

Thus, we obtain:

$$\frac{d \log P^*}{d \log S^{G,*}} = \frac{-\mathcal{L} \mathcal{P}_{\mu^*}^* \mu^* (1 - \mu^*) \Upsilon^*}{R^b - (1 - \Upsilon^*) \mathcal{L} \mathcal{P}_{\mu^*}^* \mu^*} > 0.$$

If we substitute this expression in the left back into (72) we obtain:

$$\frac{d \log \mu^*}{d \log S^{G,*}} = \frac{R^b \Upsilon^* (1 - \mu^*)}{R^b - (1 - \Upsilon^*) \mathcal{L} \mathcal{P}_{\mu^*}^* \mu^*} > 0.$$

Finally, by the law of one price:

$$d \log e = -d \log P^* = \frac{\mathcal{L}_{\mu^*}^* \mu^* (1 - \mu^*) \Upsilon^*}{R^b - (1 - \Upsilon^*) \mathcal{L}_{\mu^*}^* \mu^*} < 0.$$

Finally, the excess-bond premium and the dollar liquidity premium is:

$$d\mathcal{L}^* = d\mathcal{D}\mathcal{L}\mathcal{P}^* = -R^{*,m} d \log P^* < 0.$$

Item (ii). If the shock is permanent expected inflation does not change. Since nominal rates are fixed, we have that the dollar liquidity ratio must remain constant:

$$d \log \mu^* = 0. \quad (75)$$

Moreover, $d\mathcal{B}\mathcal{P}^* = d\mathcal{D}\mathcal{L}\mathcal{P}^* = 0$. From (72)

$$\frac{d \log P^*}{d \log S^{G,*}} = \frac{\Upsilon^*}{(1 - \Upsilon^*)} (1 - \mu^*) > 0.$$

By the law of one price then:

$$\frac{d \log e}{d \log S^{G,*}} = -\frac{d \log P^*}{d \log S^{G,*}} = -\frac{\Upsilon^*}{(1 - \Upsilon^*)} (1 - \mu^*).$$

E Microfoundations for Deposit Supplies and Loan Demands

Here we provide the micro-foundations for the loan demand and deposit supply schedules in a deterministic version of the model. We consider a representative global household. The household saves in dollar and euro deposits, supplies labor to an international firm, holds shares of this firm and owns a diversified portfolio of banks.

E.1 The Non-Financial Sector

Global household problem. The household enters the periods with a portfolio of dollar and euro deposits, denoted by $\{D_t, D_t^*\}$, holds shares of a global firm, Σ_t , and shares in a perfectly diversified portfolio of global banks, ϑ_t . These shares entitle the household to the firm's and bank's profits. The financial wealth available to the household (expressed in euros) is given by:

$$P_t n_t^h \equiv (1 + i_t^d) D_t + T_t + e_t \left((1 + i_t^{*,d}) D_t^* + T_t^* \right) + P_t (q_t + r_t^h) \Sigma_t + P_t (Q_t + div_t) \vartheta_t \quad (76)$$

where T_t and T_t^* represent euro and dollar central bank transfers, q_t is the price of the firm (in terms of goods), r_t^h is the profit of the international firm, and Q_t is the price of the bank portfolio and div_t the dividend payout of banks.

In addition, the household supplies h_t hours that are remunerated at z_t euros per hour. The household uses its wealth to purchase deposits, to buy shares, and to consume. There are three types of consumption goods: dollar goods, denoted by c_t^* , euro goods, denoted by c_t , and a linear good, denoted by c_t^h . The household's budget constraint is:

$$e_t P_t^* c_t^* + P_t c_t + P_t c_t^h + D_{t+1} + e_t D_{t+1}^* + P_t q_t \Sigma_{t+1} + P_t Q_t \vartheta_{t+1} = P_t n_t^h + z_t h. \quad (77)$$

Both dollar and euro consumption are subject to deposit-in-advance (DIA) constraints:

$$c_t \leq (1 + i_t^d) \frac{D_t}{P_t}, \quad (78)$$

and

$$c_t^* \leq (1 + i_t^d) \frac{D_t}{P_t}. \quad (79)$$

The period utility is

$$U^*(c_t^*) + U(c_t) + c_t^h - \frac{h_t^{1+\nu}}{1+\nu},$$

where U^* and U are concave utility functions over both goods and $h_t^{1+\nu}/(1+\nu)$ is a labor dis-utility. To simplify the algebra of this section, we assume that $U_{c^*}^*(1) = U_c(1) = 1$.

The household's problem is:

$$V_t^h(D_t, D_t^*, \Sigma_t, \vartheta_t) = \max_{\{c_t, c_t^*, c_t^h, h_t, D_t, D_{t+1}^*, \Sigma_{t+1}, \vartheta_{t+1}\}} U^*(c_t^*) + U(c_t) + c_t^h - \frac{h_t^{1+\nu}}{1+\nu} \dots + \beta V_{t+1}^h(D_{t+1}, D_{t+1}^*, \Sigma_{t+1}, \vartheta_{t+1}) \quad (80)$$

subject to the budget constraint (77 and 76) and the two DIA constraints (78-79).

Firm Problem. The firm produces all goods in the economy using the same production function

$$y_t = A_{t+1} h_t^\alpha.$$

The firm's output is divided into:

$$c_t^* + c_t + c_t^h = y_t. \quad (81)$$

The firm revenues come from selling goods in the dollar, euro, and linear good markets:

$$e_t P_t^* c_t^* + P_t c_t + P_t c_t^h = P_t y_t.$$

To produce positive amounts of all goods, the firm must be indifferent between selling in either market. Hence, the law of one price will hold in an equilibrium with positive consumption of all goods—the Inada conditions guarantee this is the case.

To maximize profits, the firm chooses borrowed funds B_{t+1}^d and labor h_t . The demand for loansemerges from a working capital constraint: $z_t h_t \leq B_{t+1}^d$. The firm saves in deposits whatever borrowings it doesn't spend in wages.

The firm's problem is given by:

$$\begin{aligned} P_{t+1} r_{t+1}^h &= \max_{B_{t+1}^d \geq 0, h_t \geq 0} P_{t+1} A_{t+1} h_t^\alpha - (1 + i_{t+1}^b) B_{t+1}^d + (1 + i_{t+1}^d) (B_{t+1}^d - z_t h_t) \\ &= \max_{B_{t+1}^d \geq 0, h_t \geq 0} P_{t+1} A_{t+1} h_t^\alpha - (1 + i_{t+1}^b) z_t h_t - (i_{t+1}^b - i_{t+1}^d) (B_{t+1}^d - z_t h_t). \end{aligned} \quad (82)$$

Equilibrium. In the body of the paper we characterized the equilibrium in loan and deposit markets, taking as given the loan demand and deposit supply schedules, and the transfers rules. In addition to these financial markets, the non-financial sector features a labor market, firm shares market, bank shares market, and the three goods markets. Next, we derive the

loan demand and deposit supply schedules and comment on how once these asset markets clear, all other markets clear.

E.2 Derivation of Deposit Supply and Loan Demand

Step 1 - deposit demand. We clear the linear good, c_t^h , from the household's budget constraint:

$$\begin{aligned} c_t^h &= \frac{P_t n^h + z_t h - (e_t P_t^* c^* + P_t c + D_{t+1} + e_t D_{t+1}^* + P_t (r_t + q_t) \Sigma_t + P_t (Q_t + div_t) \vartheta_t)}{P_t} \\ &= n^h + \frac{z_t}{P_t} h_t - \left(c_t^* + c_t + (r_t + q_t) \Sigma_t + (Q_t + div_t) \vartheta_t + \frac{D_{t+1}}{P_t} + \frac{D_{t+1}^*}{P_t^*} \right). \end{aligned} \quad (83)$$

where the second line uses the law of one price.

Substituting (83) into the objective of the household's problem (80) we obtain:

$$\begin{aligned} V_t^h(D_t, D_t^*, \Sigma_t, \vartheta_t) &= n_t^h + \max_{\{c_t, c_t^*, h_t, D_t, D_{t+1}^*, \Sigma_{t+1}, \vartheta_{t+1}\}} U^*(c_t^*) + U(c_t) - \frac{h_t^{1+\nu}}{1+\nu} \dots \\ &\quad + \frac{z_t}{P_t} h_t - \left(c_t^* + c_t + (r_t + q_t) \Sigma_t + (Q_t + div_t) \vartheta_t + \frac{D_{t+1}}{P_t} + \frac{D_{t+1}^*}{P_t^*} \right) \dots \\ &\quad + \beta V_{t+1}^h(D_{t+1}, D_{t+1}^*, \Sigma_{t+1}, \vartheta_{t+1}) \end{aligned}$$

subject to the two DIA constraints (78-79).

We proceed to obtain the deposit supply.

Since $\{D_{t+1}, D_{t+1}^*\}$ enter symmetrically, we derive the deposit supply only for one currency. We take the first-order condition with respect to c_t and notice that if the DIA constraint does not bind, $U_c(c) = 1$. In turn, if the deposit in advance constraint indeed binds, then:

$$c = (1 + i_t^d) \frac{D_t}{P_t} = \frac{(1 + i_t^d)}{P_t/P_{t-1}} \frac{D_t}{P_{t-1}} = R_t^d \frac{D_t}{P_{t-1}}.$$

Thus, we can combine both cases, with and without the binding DIA constraint, to write down the optimal consumption rule:

$$c = \min \left\{ (U_c)^{-1}(1), R_t^d \cdot \frac{D_t}{P_{t-1}} \right\}. \quad (84)$$

By analogy:

$$c^* = \min \left\{ (U_{c^*})^{-1}(1), R_t^{d,*} \cdot \frac{D_t^*}{P_{t-1}^*} \right\}.$$

It is convenient to treat c and c^* directly as functions of $\frac{D}{P_{t-1}}$ and $\frac{D^*}{P_{t-1}^*}$ in the next step.

Step 2 - deposit supply schedules. We replace the optimal euro and dollar consumption rules into the objective. We have:

$$\begin{aligned}
V_t^h(D_t, D_t^*, \Sigma_t, \vartheta_t) &= n^h + \max_{\{c_t, c_t^*, h_t, D_t, D_{t+1}^*, \Sigma_{t+1}, \vartheta_{t+1}\}} U^* \left(\min \left\{ (U_{c^*}^*)^{-1}(1), R_t^{d,*} \cdot \frac{D^*}{P_{t-1}^*} \right\} \right) \dots \\
&+ U \left(\min \left\{ (U_c)^{-1}(1), R_t^d \cdot \frac{D_t}{P_{t-1}} \right\} \right) - \frac{h^{1+\nu}}{1+\nu} \dots \\
&+ n^h + \frac{z_t}{P_t} h - \min \left\{ (U_{c^*}^*)^{-1}(1), R_t^{d,*} \cdot \frac{D_t^*}{P_{t-1}^*} \right\} \dots \\
&- \min \left\{ (U_c)^{-1}(1), R_t^d \cdot \frac{D_t}{P_{t-1}} \right\} \dots \tag{85}
\end{aligned}$$

$$\begin{aligned}
&- \left((r_t + q_t) \Sigma_t + (Q_t + div_t) \vartheta_t + \frac{D_{t+1}}{P_t} + \frac{D_{t+1}^*}{P_t^*} \right) \dots \\
&+ \beta V_{t+1}^h(D_{t+1}, D_{t+1}^*, \Sigma_{t+1}, \vartheta_{t+1}). \tag{86}
\end{aligned}$$

Next, we derive deposit demand: We take the first-order conditions with respect to D_{t+1}/P_t to obtain:

$$1 = \beta \frac{\partial V_{t+1}^h}{\partial (D_{t+1}/P_t)}. \tag{87}$$

Next, we derive the envelope condition. We have two cases.

Case 1: binding DIA constraint the following period. For the case where

$$R_t^d \cdot D_t/P_{t-1} < 1$$

we have from (86) that:

$$\frac{\partial V_t^h}{\partial (D_t/P_{t-1})} = U_c R_t^d - R_t^d + R_t^d = U_c R_t^d.$$

Case 1: non-binding DIA constraint the following period. For the case where $R_{t-1}^d \frac{D}{P_{t-1}} \geq 1$, we have from (86) that

$$\frac{\partial V_t^h}{\partial (D_t/P_{t-1})} = R_t^d.$$

Thus, combining the two envelope conditions we obtain:

$$\frac{\partial V_t^h}{\partial (D_t/P_{t-1})} = \begin{cases} U_c (R_t^d \cdot D_t/P_{t-1}) R_t^d & \text{for } R_t^d \cdot D_t/P_{t-1} < 1 \\ R_t^d & \text{otherwise.} \end{cases} \quad (88)$$

We shift (88) one period forward and substitute in (87) in the left-hand side to obtain:

$$\frac{1}{\beta} = \begin{cases} U_c (R_{t+1}^d \cdot D_{t+1}/P_t) R_{t+1}^d & R_{t+1}^d \cdot D_t/P_{t-1} < 1 \\ R_{t+1}^d & \text{otherwise.} \end{cases}$$

In the body of the paper, using the banks' problem, we show that $R_{t+1}^d < R^b = 1/\beta$. Thus, the only relevant portion to determine the demand condition is the one where there's no satiation of deposits. We hence, will only use this portion.

We now adopt power utility. Assume that $U = c^{1-\gamma}/(1-\gamma)$ and $U^* = (c^*)^{1-\gamma^*}/(1-\gamma^*)$. Then,

$$\frac{1}{\beta} = (R_{t+1}^d \cdot D_{t+1}/P_t)^{-\gamma} R_{t+1}^d.$$

We clear D/P_t to obtain the euro deposit supply schedule:

$$D_{t+1}/P_t = \beta^{1/\gamma} (R_{t+1}^d)^{\frac{1-\gamma}{\gamma}}.$$

By analogy, we have the dollar supply schedule:

$$D_{t+1}^*/P_t^* = \beta^{1/\gamma^*} (R_{t+1}^{*,d})^{\frac{1-\gamma^*}{\gamma^*}}.$$

More generically, following the same states, if we introduce preference shocks to the utility specifications, as follows:

$$U_t = (c/x_t)^{1-\gamma}/(1-\gamma) \text{ and } U_t^* = (c^*/x_t^*)^{1-\gamma^*}/(1-\gamma^*),$$

the demand schedules generalize to:

$$D_{t+1}/P_t = x_t \beta^{1/\gamma} (R_t^d)^{\frac{1-\gamma^x}{\gamma^x}}.$$

By analogy, we have that

$$D_{t+1}^*/P_t^* = x_t^* \beta^{1/\gamma} (R_t^d)^{\frac{1-\gamma^x}{\gamma^x}}.$$

We obtain this conditions following exactly the same steps.

All in all, the demand schedules are akin to those in the body of the paper, where the reduced form coefficients are given by:

$$\Theta_t^d = x_t \beta^{1/\gamma^d} \text{ and } \epsilon^d = \frac{1}{\gamma^d} - 1,$$

and

$$\Theta_t^{*,d} = x_t^* \beta^{1/\gamma^{*,d}} \text{ and } \epsilon^{*,d} = \frac{1}{\gamma^{*,d}} - 1.$$

Next, we describe the labor supply schedule.

Step 3 - labor supply. The first-order condition with respect to h in the household's problem yields a labor supply that only depends on the real wage:

$$h_t^\nu = z_t / P_t. \quad (89)$$

Next, we move to the firm's problem to obtain the labor demand.

Step 4 - labor demand. Since from the bank's problem, it will be the case that $i_{t+1}^b > i_{t+1}^d$, then the working capital constraint in (82) is binding, $z_t h_t = B_{t+1}^d$. Thus, the firm's objective is to

$$\max_{h_t \geq 0} P_{t+1} A_{t+1} h_t^\alpha - (1 + i_{t+1}^b) z_t h_t.$$

The first-order condition for labor h_t yields:

$$P_{t+1} \alpha A_{t+1} h_t^{\alpha-1} = (1 + i_{t+1}^b) z_t.$$

Dividing both sides by P_t , we obtain

$$\frac{P_{t+1}}{P_t} \alpha A_{t+1} h_t^{\alpha-1} = (1 + i_{t+1}^b) \frac{z_t}{P_t}. \quad (90)$$

Step 5 - loan demand. Next, we use the labor supply (89) and labor demand (90), to solve for labor as a function of the loans rate:

$$\frac{P_{t+1}}{P_t} \alpha A_{t+1} h_t^{\alpha-1} = (1 + i_{t+1}^b) h_t^{\nu+1} \rightarrow R_t^b = \frac{\alpha A_{t+1} h_t^\alpha}{h_t^{\nu+1}}. \quad (91)$$

Since the working capital constraint binds:

$$\frac{B_{t+1}^d}{P_t} = h_t \frac{z_t h_t}{P_t} = h_t^{\nu+1} \rightarrow h_t = \left(\frac{B_{t+1}^d}{P_t} \right)^{\frac{1}{\nu+1}}. \quad (92)$$

Thus, we can combine (91) and (92) to obtain the loans demand:

$$R_t^b = \alpha A_{t+1} \left(\frac{B_{t+1}^d}{P_t} \right)^{-1} \left(\frac{B_{t+1}^d}{P_t} \right)^{\frac{\alpha}{\nu+1}} \rightarrow \frac{B_{t+1}^d}{P_t} = \Theta_t (R_{t+1}^b)^{\epsilon^b}, \quad (93)$$

where the reduced form coefficients of the loans demand are:

$$\Theta_t^b = (\alpha A_{t+1})^{-\epsilon^b} \text{ and } \epsilon^b = \left(\frac{\nu + 1}{\alpha - (\nu + 1)} \right).$$

Step 6 - output, firm value and bank values. We replace the loans demand (93) into (92) to obtain the equilibrium labor as a function of the equilibrium loans rate:

$$h_t = \left(\frac{1}{\alpha A_{t+1}} \right)^{\frac{1}{\alpha - (\nu + 1)}} (R_{t+1}^b)^{\frac{1}{\alpha - (\nu + 1)}}.$$

We replace (92) into the production function to obtain:

$$y_{t+1} = A_{t+1} \left(\frac{1}{\alpha A_{t+1}} \right)^{\frac{\alpha}{\alpha - (\nu + 1)}} (R_{t+1}^b)^{\frac{\alpha}{\alpha - (\nu + 1)}} \rightarrow y_{t+1} = \left(\frac{1}{\alpha} \right)^{\frac{\alpha}{\alpha - (\nu + 1)}} A_{t+1}^{\frac{\nu + 1}{\alpha - (\nu + 1)}} (R_{t+1}^b)^{\frac{\alpha}{\alpha - (\nu + 1)}}.$$

The profit of the international firm is given by:

$$r_{t+1}^h = y_{t+1} - R_{t+1}^b B_{t+1} \rightarrow r_{t+1}^h = A_{t+1}^{\frac{\nu + 1}{\alpha - (\nu + 1)}} \left(\alpha^{-\frac{\alpha}{\alpha - (\nu + 1)}} - \alpha^{-\frac{\nu + 1}{\alpha - (\nu + 1)}} \right) \cdot (R_{t+1}^b)^{\frac{\alpha}{\alpha - (\nu + 1)}}.$$

The price of the firm is given by the first-order condition with respect to Σ . In that case, q_t must satisfy:

$$q_t = \beta (r_{t+1}^h + q_{t+1}) \rightarrow q_t = \sum_{\tau \geq 1} \beta^\tau (r_{t+\tau}^h).$$

With this, we conclude that output, hours, and the firm price are decreasing in current (and future) loans rate.

Finally, consider the price of the bank's shares. By the same token,

$$Q_t = \beta (div_t + Q_{t+1}),$$

Multiply both sides by $1/\beta$ and recall that $\vartheta_t = 1$. Thus, we have:

$$\frac{1}{\beta} Q_t = \left(div_t + \beta \frac{1}{\beta} Q_{t+1} \right).$$

By change of variables let $v_t \equiv \frac{1}{\beta} Q_t$. Therefore, the value of the firm is given by

$$v_t = div_t + \beta v_{t+1}.$$

Solving this condition from time zero implies that:

$$v_0 = \sum_{t \geq 0} \beta^t div_t.$$

Thus, the bank's objective in the body of the paper is consistent with maximizing the bank's value.

Remark. We priced the firms and banks so that they are held in equilibrium by households. Thus, the shares markets clear. Note that throughout the proof we use the labor market-clearing condition, (91). Hence, the labor market clears. Since in the body of the paper we deal with clearing in the loans and deposit markets, by Walras's law, this implies clearing in the three goods markets.

All in all, the equilibrium in the banking block is an autonomous system. As long as the loan and deposit markets clear, we have clearing in the non-financial sector: Once we compute equilibria taking the schedules as exogenous in the bank's problem, we obtain output and household consumption from the equilibrium loan and deposit rates.

Finally, we should note that in presence of aggregate risk (inflation risk in particular), the deposit demand schedules will feature a risk premium that we are not considering in the derivation. We ignore this terms.

F Computational Algorithms

Steady State: equilibrium conditions. We solve the steady-state of the model where $n = 0$ every period.²⁹ Solving for steady-state equilibrium requires to solve for 11 variables, three interest rates, $\{R^{*,d}, R^d, R^b\}$, three prices $\{P, P^*, e\}$ and five quantities $\{m^*, m, d, d^*, b^*\}$. We summarize these conditions below and show that the system .

Prices given quantities are given by:

$$d^* = \Theta^{*,d} (R^{*,d})^{\epsilon^{d^*}} \quad (94)$$

$$d = \Theta^d (R^d)^{\epsilon^d} \quad (95)$$

and

$$b^* = \Theta^b (R^b)^{\epsilon^b} . \quad (96)$$

The two prices are given by the equilibrium in the market for real dollar reserves,

$$m^* = \frac{M^*}{P^*} . \quad (97)$$

and by the equilibrium in the market for real euro reserves is for euro reserves:

$$m = \frac{M}{P} . \quad (98)$$

In turn, the exchange rate is obtained via the law of one price:

$$e = P/P^* .$$

Finally, we have four first-order conditions and the budget constraint to pin down the quantities:

a) the dollar liquidity premium:

$$R^{m,*} + \mathbb{E}[\chi_{m^*}^*] = R^m + \mathbb{E}[\chi_m]$$

b) the bond premium:

$$R^b = R^{m,*} + \mathbb{E}[\chi_{m^*}^*] \quad (99)$$

²⁹When $n > 0$, $R^b = 1/\beta$ so we drop one variable and the budget constraint.

c-d) the two deposit premia

$$R^{d,*} + \mathbb{E}[\chi_{d^*}^*] = R^{m,*} + \mathbb{E}[\chi_{m^*}^*] \quad (100)$$

and

$$R^{d,*} + \mathbb{E}[\chi_{d^*}^*] = R^{m,*} + \mathbb{E}[\chi_{m^*}^*] \quad (101)$$

Finally, the budget constraint for $n = 0$ is:

$$b + m + m^* = d + d^*.$$

This is a system of 11 equations and 11 unknowns. Next, show how to solve the model in ratios.

Steady State: solving the model in ratios. Recall that liquidity ratios are given by:

$$\mu \equiv \frac{m}{d} \text{ and } \mu^* \equiv \frac{m^*}{d^*}.$$

We define the ratio of real euro to dollar funding as:

$$v \equiv \frac{d}{d^*}.$$

Once we obtain $\{d^*, \nu, \mu, \mu^*\}$, we obtain $\{m, m^*, d\}$ from these three definitions.

We have shown that the interbank market tightness in euros and dollars are given by:

$$\theta(\mu) = \max \left\{ -\frac{\int_{-\infty}^{-\mu} (\mu + \omega) \cdot d\Phi(\omega)}{\int_{-\mu}^{\infty} (\mu + \omega) \cdot d\Phi(\omega)}, 0 \right\}$$

and

$$\theta^*(\mu^*) = \left\{ -\frac{\int_{-\infty}^{-\mu^*} (\mu^* + \omega) \cdot d\Phi^*(\omega^*)}{\int_{-\mu^*}^{\infty} (\mu^* + \omega) \cdot d\Phi^*(\omega^*)}, 0 \right\}.$$

Thus we have introduced three ratios. Notice that once we obtain $\{\mu, \mu^*\}$ we obtain $\{d, d^*\}$.

Once we obtain v , we have $\{e^{-1}\}$.

Furthermore, the budget constraint written in ratios is:

$$b^* = (v(1 - \mu) + (1 - \mu^*))d^*. \quad (102)$$

We substitute out $\{R^b, R^d, R^{*,d}\}$ and work directly with the market clearing conditions.

We replace b^* from the budget constraint. If we substitute the ratios $\{\mu^*, \mu, \nu, d^*\}$ into the equilibrium conditions and, thus only have one quantity variable d^* , and the rest of the system is expressed in ratios.

Steady State: autonomous sub-system. We solve for $\{\mu^*, \mu, \nu, d^*\}$ using:

1) Bond premium:

$$\Theta^b ((\nu(1 - \mu) + (1 - \mu^*)) d^*)^{\epsilon^b} = R^{*,m} + \mathbb{E} [\chi_m (\mu^*)]. \quad (103)$$

2) The dollar liquidity premium:

$$R^m + \mathbb{E} [\chi_m (\mu)] = R^{*,m} + \mathbb{E} [\chi_m (\mu^*)].$$

3) The euro funding premium:

$$\Theta^d (\nu d^*)^{-\epsilon^d} + \mathbb{E} [\chi_d (\mu)] = R^{*,m} + \mathbb{E} [\chi_m (\mu^*)] \quad (104)$$

4) dollar funding premium:

$$\Theta^{*,d} (d^*)^{-\epsilon^{d^*}} + \mathbb{E} [\chi_{d^*} (\mu^*)] = R^{*,m} + \mathbb{E} [\chi_{m^*} (\mu^*)]. \quad (105)$$

These four equations provide us with a solution to $\{d^*, \nu, \mu, \mu^*\}$.

Steady State: Solving the rest of the model. With the solution to $\{\mu^*, \mu, \nu, d^*\}$ we obtain $\{m^*, m, d\}$ using:

$$m = \mu d, \quad m^* = \mu^* d^*, \quad \text{and} \quad d = \nu d^*.$$

Then, we obtain the euro price from

$$P = \frac{M}{\mu \nu d^*},$$

the dollar price from

$$P^* = \frac{M^*}{\mu^* d^*},$$

and the exchange rate from

$$e = \frac{P}{P^*}.$$

F.1 Algorithm to obtain a Global Solution

Define $X \in \mathcal{X} = \{1, 2, 3, \dots, N^s\}$ to be a finite set of states. We let X follow a Markov process with transition matrix Q . Thus, $X' \sim Q(X)$. That is, at each period, $X = \{\sigma^*, \sigma, i^{*,m}, i^m, M, M^*, \Theta^d, \Theta^{*,d}\}$ are all, potentially, functions of the state X .

The algorithm proceeds as follows. We define a “greed” parameter Δ^{greed} and a tolerance parameter ε^{tol} , and construct a grid for \mathcal{X} . We conjecture a price-level functions $\{p_{(0)}(X), p_{(0)}^*(X)\}$ which produces a price levels in both currencies as a function of the state. As an initial guess, we use $p_{(0)}(X) = p_{ss}^*$, and $p_{(0)}^*(X) = p_{ss}^*$ setting the exchange rate to its steady state level in all periods. We proceed by iterations, setting a tolerance count tol to $tol > 2 \cdot \varepsilon^{tol}$.

Outerloop 1: Iteration of price functions. We iterate price functions until they converge.

Let n be the $n - th$ step of a given iteration. Given a $p_{(n)}(X), p_{(n)}^*(X)$, we produce a new price level functions $p_{(n+1)}(X), p_{(n+1)}^*(X)$ if $tol > \varepsilon^{tol}$.

Innerloop 1: Solve for real policy rates. For each X in the grid for \mathcal{X} , we solve for

$$\{\bar{R}^m(X), \bar{R}^{*,m}(X), \bar{R}^w(X), \bar{R}^{*,w}(X)\}.$$

Let j be the $j - th$ step of a given iteration. Conjecture values

$$\{\bar{R}_{(0)}^m(X), \bar{R}_{(0)}^{*,m}(X), \bar{R}_{(0)}^w(X), \bar{R}_{(0)}^{*,w}(X)\}$$

We use $\{\bar{R}_{ss}^m, \bar{R}_{ss}^{*,m}, \bar{R}_{ss}^w, \bar{R}_{ss}^{*,w}\}$ as an initial guess. We then update

$$\{\bar{R}_{(j)}^m(X), \bar{R}_{(j)}^{*,m}(X), \bar{R}_{(j)}^w(X), \bar{R}_{(j)}^{*,w}(X)\}$$

until we obtain convergence:

2.a Given this guess, we solve for the liquidity ratios in Dollars and Euro $\{\mu, \mu^*, \bar{R}^d, \bar{R}^{*,d}\}$ as a function of the state using:

$$\bar{R}^d + \frac{1}{2} (\chi^+(\mu) - \chi^-(\mu)) = \bar{R}^{*,d} + \frac{1}{2} (\chi^{*,+}(\mu^*) - \chi^{*,-}(\mu^*))$$

$$\bar{R}^m + \frac{1}{2} (\chi^+(\mu) + \chi^-(\mu)) = \bar{R}^{*,m} + \frac{1}{2} (\chi^{*,+}(\mu^*) + \chi^{*,-}(\mu^*))$$

$$\Theta^b ((v(1 - \mu) + (1 - \mu^*)) d^*)^{\epsilon^b} = \bar{R}^{*,d} + \frac{1}{2} (\chi^+(\mu) - \chi^-(\mu))$$

$$\bar{R}^m = \bar{R}^{*,d} + \frac{1}{2} (\chi^+(\mu) - \chi^-(\mu)) - \frac{1}{2} (\chi^+(\mu) + \chi^-(\mu)).$$

This step yields an update for $\{R^d(X), R^{*,d}(X)\}$.

2.b Given the solutions to $\{R^d(X), R^{*,d}(X)\}$, we solve $\{d^*, v\}$ using:

$$d^* = \left[\frac{\bar{R}^{*,d}}{\Theta^{*,d}} \right]^{1/\epsilon^{d^*}}$$

$$v = \left[\frac{\bar{R}^d}{\Theta^d} \right]^{1/\epsilon^d} \left[\frac{\bar{R}^{*,d}}{\Theta^{*,d}} \right]^{1/\epsilon^{d^*}}.$$

This step yields an update for $\{d^*(X), v(X)\}$.

2.c Given $\{d^*(X), v(X)\}$ we solve for prices $\{p, p^*, e\}$ using:

$$\mu v d^* = \frac{M}{p}$$

$$\mu^* d^* = \frac{e}{p} M^*$$

$$p^* = e^{-1} p.$$

2.d Finally, we update the real policy rates. For that we construct the expected inflation in each currency:

$$\mathbb{E}[\pi^*] = \frac{\sum_{s' \in S} Q(s'|s) p_{(n)}^*(s)}{p^*(s)}$$

and

$$\mathbb{E}[\pi] = \frac{\sum_{s' \in S} Q(s'|s) p_{(n)}(s)}{p(s)}.$$

We then update the policy rates by:

$$R_{(j+1)}^{*,a} = \frac{(1 + i^{*,a})}{(1 + \pi^*)} \text{ for } a \in \{m, w\}$$

and

$$R_{(j+1)}^a = \frac{(1 + i^a)}{(1 + \pi)} \text{ for } a \in \{m, w\}.$$

2.e Repeat steps 2.a-2.d, unless

$$\left\{ R_{(j)}^m(X), R_{(j)}^{*,m}(X), R_{(j)}^w(X), R_{(j)}^{*,w}(X) \right\}$$

is close to

$$\left\{ R_{(j+1)}^m(X), R_{(j+1)}^{*,m}(X), R_{(j+1)}^w(X), R_{(j+1)}^{*,w}(X) \right\}.$$

If the real policy rates have converged, update prices according to

$$p_{(n+1)}^*(X) = \Delta^{greed} p^* + (1 - \Delta^{greed}) p_{(n)}^*(X)$$

and

$$p_{(n+1)}(X) = \Delta^{greed} p + (1 - \Delta^{greed}) p_{(n)}^*(X)$$

and proceed back to the outer-loop.