On-Line Appendix for "Charter School Practices and Student Selection: An Equilibrium Analysis"

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A. Section 3 Derivations and Proofs

Sufficient requirements for A2: $\frac{\partial^2 a}{\partial e^2} \leq 0$ and $\left(\frac{\partial a}{\partial e} - \frac{\partial c}{\partial e}\right)\Big|_{e=0} > 0$, along with the increasing marginal cost assumption made in the text.

Proof that $\hat{m}_S(b)$ and $\hat{m}_0(b)$ are downward sloping with $\hat{m}_S(b) > \hat{m}_0(b)$: Regarding the $\hat{m}_s(b)$ locus, one can also define it as: $a(q(b), b, e^*(q, \hat{m}_s, b)) = S$. Note that we write q(b), this from (1). q also depends on (B,Q) but we will *generally* suppress arguments that are fixed. Differentiating:

$$(A.1) \quad \left[\frac{\partial a}{\partial q}\frac{\partial q}{\partial b} + \frac{\partial a}{\partial b}\right] + \frac{\partial a}{\partial e}\frac{\partial e^*}{\partial b} = -\frac{\partial a}{\partial e}\frac{\partial e^*}{\partial m}\frac{d\hat{m}_s}{db}$$

Using the FOC on e^* one obtains:

$$(A.2) \quad \frac{\partial e^*}{\partial b} = \frac{\left[\frac{\partial^2 a}{\partial e \partial b}\right]}{\frac{\partial^2 c}{\partial e^2} - \frac{\partial^2 a}{\partial e^2}} > 0,$$

the inequality using the SOC on e^* and that the cross-derivative in the numerator is positive (see (2)). One also obtains:

$$(A.3) \quad \frac{\partial e^*}{\partial m} = -\frac{\partial^2 c / \partial e \partial m}{\partial^2 c / \partial e^2} > 0,$$

the inequality from the cost function assumptions. Using (A.2) and (A.3) and Assumption 1 in the text (that implies the bracketed term in (A.1) is positive), one can see by inspection of (A.1) that $\frac{d\hat{m}_s}{db} < 0$.

Next consider $\hat{m}_0(b)$, defined by $e_s = e_0$. Now, e_0 satisfies

$$(A.4) \quad a(q(b), b, e_0) = c(e_0, m),$$

and e_s satisfies:

$$(A.5) \quad a(q(b), b, e_s) = S.$$

Since $e_s = e_0$, a on the LHS of (A.4), and (A.5) are equal, implying $c(e_0, \hat{m}_0) = S$, where $e_0 = \hat{e}_0(q(b), b, m)$ satisfies (A.4). Differentiating $c(\hat{e}_0(q(b), b, \hat{m}_0), \hat{m}_0) = S$, the former:

(A.6) $$\left[\frac{\partial a}{\partial q}\frac{\partial q}{\partial b}+\frac{\partial a}{\partial b}\right]+\frac{\partial a}{\partial e}\underbrace{\left\{\frac{\partial \hat{e}_0}{\partial b}+\frac{\partial \hat{e}_0}{\partial q}\frac{\partial q}{\partial b}\right\}}_{d\hat{e}_0/db}=-\frac{\partial a}{\partial e}\frac{\partial \hat{e}_0}{\partial m}\frac{d\hat{m}_0}{db}.$$

Differentiating (A.4) with respect to (e_0,b) and then with respect to (e_0,m), one obtains respectively:

(A.7) $$\frac{d\hat{e}_0}{db}=-\frac{\frac{\partial a}{\partial q}\frac{\partial q}{\partial b}+\frac{\partial a}{\partial b}}{\left(\frac{\partial a}{\partial e_0}-\frac{\partial c}{\partial e_0}\right)}>0, \quad \text{and}$$

(A.8) $$\frac{\partial \hat{e}_0}{\partial m}=\frac{\partial c/\partial m}{\left(\frac{\partial a}{\partial e_0}-\frac{\partial c}{\partial e_0}\right)}>0.$$

The inequalities in both (A.7) and (A.8) use that the term in parenthesis in the numerator are negative, this because $e_0 > e^*$ (recall that e_0 is the e such that a – e is 0 while e^* maximizes a – e). Using these inequalities and again Assumption 1, one can see by inspection of (A.6) that $\frac{d\hat{m}_0}{db}<0$.

It is "obvious" that $\hat{m}_S(b) > \hat{m}_0(b)$. Students on the former have $e = e^*$ and positive utility while those on the latter have $e = e_0$ and 0 utility. Since effort cost declines with m, a not lower m on $\hat{m}_S(b)$ implies a contradiction.

Proof of Lemma 1: We know that $e_0 > e^*$ by (A2) so this element of each case need not be proved.

(i) Since $e^* = e_s$ on $\hat{m}_s(b)$ and achievement increases in effort, the results here follow from (A.3).

(ii) If $e^* > e_s$, a student would optimally choose e^* and have achievement exceeding S. Students for whom $e_s \le e_0$ are at least as well off choosing e_s and meeting the standard than dropping out (strictly except on the bound). Combining these, students with effort values in the interval $e_0 \ge e_S \ge e^*$ optimally choose $e = e_s$ and meet the standard. Using (A.3) and the definitions of $\hat{m}_S(b)$ and $\hat{m}_0(b)$, it follows that students with $m \in \left[\hat{m}_0(b), \hat{m}_S(b)\right]$ choose $e = e_s$.

(iii) A student on $\hat{m}_0(b)$ has $e_s = e_0$ by definition. Since e_s is invariant to m, $e_0 < e_s$ for a student with $m < \hat{m}_s(b)$. Moreover, such a student prefers to drop out, since utility would be negative choosing e high enough to graduate (using U = 0 for those with $m = \hat{m}_s(b)$).

B. Section 4 Derivations and Proofs

Results Presented in Section 4.1: We first show that the $\hat{m}_S^j(b)$ and $\hat{m}_0^j(b)$, $j \in \{P, T\}$, loci shift with B^j as shown in Figure 2, staying with the example of $B^T < B^P$ in the figure. The loci are as shown provided $\frac{\partial \hat{m}_s}{\partial q}<0$ and $\frac{\partial \hat{m}_0}{\partial q}<0$ since students with b less (greater) than $\tilde{b}$ will have higher quality at the charter

(public) school and will (not) apply to it. As above, $\hat{m}_s$ can be defined in $a(q, b, e^*(q, \hat{m}_s, b)) = S$, though we can suppress that q depends on b. Differentiating with respect to q gives:

$$\text{(B.1)} \qquad \frac{\partial \hat{m}_s}{\partial q} = -\frac{\frac{\partial a}{\partial q} + \frac{\partial a}{\partial e}\frac{\partial e^*}{\partial q}}{\frac{\partial e^*}{\partial m}}$$

We have shown $\partial e^* / \partial m$ is positive; see (A.3) above. Then, $\frac{\partial \hat{m}_s}{\partial q} < 0$ if $\frac{\partial e^*}{\partial q} > 0$, which we now show. Using the FOC identifying e^*, one obtains:

$$\text{(B.2)} \qquad \frac{\partial e^*}{\partial q} = -\frac{\frac{\partial^2 a}{\partial e \partial q}}{\frac{\partial^2 a}{\partial e^2} - \frac{\partial^2 c}{\partial e^2}} > 0,$$

the inequality by the SOC and (2) in the text.

Regarding $\hat{m}_0$, as above, it satisfies $a(q, b, e_0) = c(e_0, \hat{m}_0)$, where $a(q, b, e_0) = S$. (To write the latter, we have substituted $e_0 = e_s$ into (A.4) and, again, we suppress that q has b as an argument.) Differentiating the second equation with respect to q gives:

$$\text{(B.3)} \qquad \frac{\partial \hat{m}_0}{\partial q} = -\frac{\frac{\partial a}{\partial q} + \frac{\partial a}{\partial e}\frac{\partial e_0}{\partial q}}{\frac{\partial a}{\partial e}\frac{\partial e_0}{\partial m}}$$

We showed above the denominator of the latter expression is positive (see (A.8)). From $a(q, b, e_0) = c(e_0, \hat{m}_0)$, we get:

$$\text{(B.4)} \qquad \frac{\partial e_0}{\partial q} = \frac{\frac{\partial c}{\partial m}}{\left[\frac{\partial a}{\partial e} - \frac{\partial c}{\partial e}\right]} > 0,$$

the inequality because, as above, the bracketed term is negative at $e = e_0$. Combining these facts, (B.3) implies $\frac{\partial \hat{m}_0}{\partial q} < 0$.

Now we collect the results regarding achievement gains illustrated in Figure 2 in Lemma B.1.

Lemma B.1. Assume $B^T < B^P$. Those that gain from attending the charter (assuming they matriculate) have $b < \tilde{b} = \frac{B^T + B^P}{2}$ and $m > \hat{m}_0^T(b)$. Among them, those with:

(i) $m > \hat{m}_s^P(b)$ would choose e^* at either school and then gain $a_T^* - a_P^*$ by attending the charter school.

(ii) $m \in (\hat{m}_s^T(b), \hat{m}_s^P(b)]$ would choose e^* at the charter school and e_s at the public school and then gain $a_T^* - S$ by attending the charter school.
(iii) $m \in (\hat{m}_0^P(b), \hat{m}_s^T(b)]$ would choose e_s at both schools at then gain zero in achievement, but students have higher utility (since e_s is lower at the charter school).
(iv) $m \in (\hat{m}_0^T(b), \hat{m}_s^P(b)]$ would choose e_s at the charter school and would drop out of the public school and then gain S at by attending the charter school.

Proof of Lemma B.1. Using the implications of the loci for choice of effort, the rotation in the loci that have been shown, and that quality is higher for students with $b < \tilde{b}$ if attending the charter school, the results are all immediate.

A set of analogous results hold for $B^T > B^P$, here with students having $b > \tilde{b} = \dfrac{B^T + B^P}{2}$ and again $m > \hat{m}_0^T(b)$ those that gain from attending the charter school.

Proof of Proposition 1. We focus on the curriculum choice problem with $B^T \le B^P$, i.e., with low end entry. The case with $B^T \ge B^P$ is analogous. The problem is (7) in the text, restated here for convenience:

$$\text{(B.5)} \quad \underset{B_M^T \le B^T \le B^P}{\text{Max}} \ \bar{\Delta} = \left[\begin{array}{l} \int_{y_m}^{y_x} \int_{b_m}^{\tilde{b}(B^T)} \int_{\hat{m}_0^T(b,B^T)}^{\hat{m}_0^P(b)} S f dbdmdy + \int_{y_m}^{y_x} \int_{b_m}^{\tilde{b}(B^T)} \int_{\hat{m}_S^T(b,B^T)}^{\hat{m}_S^P(b)} \left[a^*(B^T) - S \right] f dmdbdy + \\ \\ \int_{y_m}^{y_x} \int_{b_m}^{\tilde{b}(B^T)} \int_{\hat{m}_S^P(b)}^{m_x} \left[a^*(B^T) - a_P^* \right] f dmdbdy \end{array} \right] / N^T,$$

$$\text{s.t. } N^T = \int_{y_m}^{y_x} \int_{b_m}^{\tilde{b}(B^T)} \int_{\hat{m}_0^T(B^T)}^{m_x} f dbdmdy.$$

The objective in (B.5) is the average achievement gain of the *applicant set*, while the actual objective is maximization of aggregate achievement gains of matriculants. Because of random admission if over-subscribed and random matriculation of those admitted, along with the size constraint, aggregate achievement of matriculants is given by $\kappa\bar{\Delta}$. Then maximization of $\bar{\Delta}$ is optimal so long as at least κ students apply. This condition is met, since B^T is constrained to be at least B_M^T, using (A3). Formally, choice of $B^T = B^P$ is infeasible since this would lead no students to gain by attending the charter and none would apply. However, $\bar{\Delta}(B^T)$ converges toward 0 as $B^T \uparrow B^P$, so this can be ignored. $\bar{\Delta}(B^T)$ is a continuously differentiable function of B^T defined on $[B_M^T, B^P]$. It reaches its global minimum at $B^T = B^P$ where the two schools are identical so that the no students would gain in achievement $\bar{\Delta}(B^P) = 0$. For any other value of $B^T \in [B_M^T, B^P] \neq B^P$, the set of applicants is non-empty and only contains applicants who would experience positive achievement gains or no achievement loss but effort reduction. Lemma B.1 implies that some of those students experience strictly positive gains – e.g. those with motivation $m \in (m_0^T(B^T), m_0^P(B^P))$. By the extreme value theorem, then, problem (B.5) has (at least) one local maximum. The proposition assumes it is unique and denoted by B^{T*}. Using D to denote the numerator of (B.5):

$$\frac{\partial\bar{\Delta}}{\partial B^T}=\frac{N^T\frac{\partial D}{\partial B^T}-N^T\cdot\bar{\Delta}\cdot\frac{\partial N^T}{\partial B^T}}{(N^T)^2},$$

and so

$$\text{(B.6)}\qquad \operatorname{sign}\frac{\partial\bar{\Delta}}{\partial B^T}=\operatorname{sign}\left[\frac{\partial D}{\partial B^T}-\bar{\Delta}\cdot\frac{\partial N^T}{\partial B^T}\right].$$

Apply Leibniz's rule to obtain $\partial D/\partial B^T$:

$$\frac{\partial D}{\partial B^T}=\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\left[-S\frac{\partial\hat{m}_0^T(b,B^T)}{\partial B^T}f(b,\hat{m}_0^c(b,B^C),y)-[a^*(\hat{m}_S^T(b,B^T))-S]\frac{\partial\hat{m}_S^T(b,B^T)}{\partial B^T}f(b,\hat{m}_S^T(b,B^T),y)dbdy\right]$$

$$+\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\int_{\hat{m}_S^c(b,B^T)}^{\hat{m}_S^p(b,B^P)}\frac{\partial a^*(B^T)}{\partial B^T}f(b,m,y)dbdmdy+\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\int_{\hat{m}_S^p(b,B^P)}^{m_x}\frac{\partial a^*(B^T)}{\partial B^T}f(b,m,y)dbdmdy;$$

where all the terms multiplied by $\partial\tilde{b}/\partial B^T$ vanish, since $a^*(\tilde{b},m,B^T)=a^*(\tilde{b},m,B^P)$, $\hat{m}_0^T(\tilde{b},B^T)=\hat{m}_0^P(\tilde{b},B^P)$ and $\hat{m}_S^T(\tilde{b},B^T)=\hat{m}_S^P(\tilde{b},B^P)$. Because $a^*(b,\hat{m}_S^T(b,B^T))=S$, the derivative simplifies further to:

$$\text{(B.7)}\quad \frac{\partial D}{\partial B^T}=\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\left[-S\frac{\partial\hat{m}_0^T(b,B^T)}{\partial B^T}f(b,\hat{m}_0^T(b,B^T),y)dbdy\right]+\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\int_{\hat{m}_S^T(b,B^T)}^{m_x}\frac{\partial a^*(b,B^T)}{\partial B^T}f(b,m,y)dbdmdy$$

Next, from the definition of N^T (see the constraint in (B.5)):

$$\text{(B.8)}\qquad \frac{\partial N^T}{\partial B^T}=\frac{\partial\tilde{b}}{\partial B^T}\int_{y_m}^{y_x}\int_{\hat{m}_0^T(\tilde{b},B^T)}^{m_x}f(\tilde{b},m,y)dmdy-\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\left[\frac{\partial\hat{m}_0^T(b,B^T)}{\partial B^T}f(b,\hat{m}_0^T(b,B^T),y)dbdy\right].$$

Combining (B.8), (B.7) and (B.6) yields:

$$\operatorname{Sign}\left[\frac{\partial\bar{\Delta}}{\partial B^T}\right]=$$

$$\text{(B.9)}\qquad \operatorname{Sign}\left[\begin{array}{l}\displaystyle\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\left\{(\bar{\Delta}-S)\frac{\partial\hat{m}_0^T(b,B^T)}{\partial B^T}f(b,\hat{m}_0^T(b,B^T),y)dbdy\right\}\\ \displaystyle+\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\int_{\hat{m}_S^T(b,B^T)}^{m_x}\frac{\partial a^*(b,B^T)}{\partial B^T}f(b,m,y)dbdmdy-\bar{\Delta}\cdot\frac{\partial\tilde{b}}{\partial B^T}\int_{y_m}^{y_x}\int_{\hat{m}_S^T(\tilde{b},B^T)}^{m_x}f(\tilde{b},m,y)dmdy\end{array}\right].$$

In the vicinity of $B^T=B^P$, $\bar{\Delta}\approx 0$, and (B.9) simplifies to:

$$\text{Sign}\left[\frac{\partial\bar{\Delta}}{\partial B^T}\right] =$$

$$\text{(B.10)}\quad \text{Sign}\left[\begin{array}{l}\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\left\{-S\frac{\partial\hat{m}_0^T(b,B^T)}{\partial B^T}f(b,\hat{m}_s^T(b,B^T),y)dbdy\right\}\\ +\int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(B^T)}\int_{\hat{m}_1^c(b,B^T)}^{m_x}\frac{\partial a^*(b,B^T)}{\partial B^T}f(b,m,y)dbdmdy\end{array}\right].$$

For $B^T < B^P$ but in the latter's vicinity, the derivative in the first integral is positive for virtually all students in the charter school (point B^T is almost on the vertical line at $\tilde{b}$) and the derivative in the second integral is negative for virtually all students (virtually all students in the charter have $b < B^T \approx \tilde{b} = \frac{B^T + B^p}{2}$, the approximate equality because B^T is in the vicinity of B^P). Thus, (B.10) is negative, implying gains from decreasing B^T. This proves $B^{T*} < B^P$.

Given uniqueness of the local maximum for $B^T < B^P$ and that the viability constraint is just satisfied at $B^T = B_m^T$, the corner solution case has:

$\text{sign}\left[\left.\frac{\partial\bar{\Delta}}{\partial B^T}\right|_{B^T=B_m^T}\right] \le 0 \leftrightarrow B^{T*} = B_m^T$, which, using (B.10) is the boundary case of (8). Otherwise:

$B^{T*} = B^T \in (B_m^T, B^p) \leftrightarrow \text{sign}\left[\left.\frac{\partial\bar{\Delta}}{\partial B^T}\right|_{B^T=B^{T*}}\right] = 0$, which is the equality case of (8).

The cases of high-end entry ($B^T > B^P$) are analogous. ■

Results Presented in Section 4.3. Here we confirm the results reported in the second paragraph of Section 4.3 regarding curriculum choices of the public school in the Stackelberg game that would lead to high- or low-end entry by the charter school. Let $\underline{B}^{T*}(B^P)$ denote the optimal curriculum of the charter school conditional on entry with $B^T < B^P$ assuming the charter school can be viable with such entry. Let $\overline{B}^{T*}(B^P)$ denote the optimal curriculum of the charter school conditional on entry with $B^T > B^P$ assuming the charter school can be viable with such entry. To proceed, we define the following functions:

(B.11)

$$\tilde{\underline{\Delta}}(\underline{B}^{T*}(B^P), B^P) =$$

$$\int_{y_m}^{y_x}\left[\int_{b_m}^{\tilde{b}(\underline{B}^{T*}(B^P),B^P)}\left\{\int_{\hat{m}_0^T(\underline{B}^{T*}(B^P))}^{\hat{m}_0^P(B^P)} S\,fdm + \int_{\hat{m}_S^T(\underline{B}^{T*}(B^P))}^{\hat{m}_S^P(B^P)}[a^{*T}(\underline{B}^{T*}(B^P)) - S]fdm + \int_{\hat{m}_S^P(B^P)}^{m_x}[a^{*T}(\underline{B}^{T*}(B^P)) - a^{*P}(B^P)]fdm\right\}db\right]dy$$

(B.12) $$\underline{N}^T(\underline{B}^{T*}(B^P), B^P) = \int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}(\underline{B}^{T*}(B^P),B^P)}\int_{\hat{m}_0^T(\underline{B}^{T*}(B^P))}^{m_x} fdmdbdy;$$

(B.13)

$$\tilde{\bar{\Delta}}(\bar{B}^{T*}(B^P),B^P)=$$

$$\int_{y_m}^{y_x}\left[\int_{\tilde{b}(\underline{B}^{T*}(B^P),B^P)}^{b_x}\left\{\int_{\hat{m}_0^T(\bar{B}^{T*}(B^P))}^{\hat{m}_0^P(B^P)} S\,fdm+\int_{\hat{m}_S^T(\bar{B}^{T*}(B^P))}^{\hat{m}_S^P(B^P)}[a^{*T}(\bar{B}^{T*}(B^P))-S]fdm+\int_{\hat{m}_S^P(B^P)}^{m_x}[a^{*T}(\bar{B}^{T*}(B^P))-a^{*P}(B^P)]fdm\right\}db\right]dy$$

and

$$\text{(B.14)}\quad \bar{N}^T(\bar{B}^{T*}(B^P),B^P)=\int_{y_m}^{y_x}\int_{\tilde{b}(\bar{B}^{T*}(B^P),B^P)}^{b_x}\int_{\hat{m}_0^T(\bar{B}^{T*}(B^P))}^{m_x} fdmdbdy.$$

In these expressions, we have suppressed all arguments except for (B^T, B^P), the values most relevant to the present analysis. The expression in (B.11) equals the aggregate achievement gain to charter school students if all applicants were to attend given optimal low-end entry by the charter school, the same as the numerator of the objective function of (B.5), but now evaluated at the conditionally optimal B^T. The expression in (B.12) is the mass of applicants (the same as the constraint in (B.5)). Given the charter school size constraint, random admissions if over-subscribed, and a constant matriculation probability, aggregate achievement of matriculants with low-end entry equals $\kappa\frac{\tilde{\underline{\Delta}}}{\underline{N}^T}$, the number of matriculants times their average achievement gain. The expressions in (B.13) and (B.14) are analogous for the case of optimal high-end entry by the charter school.

Now we find the bounds of B^P that determine when entry at the low- and high-ends are feasible for the charter school. Define B_M^P and B_X^P respectively in:

$$\text{(B.15)}\quad \underline{N}^T(B_M^P,B_M^P)=\kappa/\mu;$$

and

$$\text{(B.16)}\quad \bar{N}^T(B_X^P,B_X^P)=\kappa/\mu.$$

If $B^P=B_X^P$, by entering with B^T slightly below, the charter school would attract almost κ/μ applicants, would almost be viable. For any higher B^P, viable low-end entry would be feasible. Thus, B_M^P is the infimum of the B^P range permitting low end entry. Analogously, B_X^P is the supremum of the range of B^P that permits entry at the high end. Recall that we have assumed κ is small enough that (B_M^P,B_X^P) is non-empty.

Now define the difference in payoff to the charter school from low- versus high-end entry:

$$\text{(B.17)}\quad DF(B^P)=\kappa\frac{\tilde{\underline{\Delta}}}{\underline{N}^T}-\kappa\frac{\tilde{\bar{\Delta}}}{\bar{N}^T},\ B^P\in(B_M^P,B_X^P).$$

One can see by inspection of the terms that make up DF (suing equations (B.11)-(B.14)) that it is continuous in B^P and almost everywhere differentiable (non-differentiabilities possibly occurring at B^P

that would just make a corner solution for B^T optimal). Observe that $DF(B^P) \to -\kappa\frac{\bar{\tilde{\Delta}}}{\bar{N}^T} < 0$ as $B^P \downarrow B_M^P$. The latter holds since, as B^P declines toward B_M^P, viable entry at the low end requires $B^T \approx B_M^P$ to attract enough students, but then vanishingly small achievement gains. On the other hand, entering at the high end is feasible with curriculum differentiation and thus strict achievement gains to some students. Similarly, $DF(B^P) \to \kappa\frac{\underline{\tilde{\Delta}}}{\underline{N}^T} > 0$ as $B^P \uparrow B_X^P$. Then $DF(B^P)$ has at least one fixed point on (B_M^P, B_X^P); and, if $\tilde{B}_1^P$ is the minimum among them, $DF(B^P) < 0$ for all $B^P < B_M^P$ implying high end entry is optimal for the charter school. If $\tilde{B}_2^P$ is the maximum fixed point, then $DF(B^P) > 0$ for all $B^P > B_X^P$, implying low-end entry is optimal for the charter school.

Now we provide sufficient conditions under which the fixed point is unique ($\tilde{B}_1^P = \tilde{B}_2^P$). Using that $DF(B^P)$ is negative near the lower bound of (B_M^P, B_X^P) and positive near the upper bound, the condition for uniqueness is that $DF'(B^P) > 0$ at any fixed point. Let $\hat{\Delta}$ denote aggregate achievement gains of matriculants at a fixed point, equal to either of the first or second term of (B.17). We assume that at any fixed point, i.e., for B^P such that the charter school is indifferent to entering at the low- or high-end, both $\underline{B}^{T*}(B^P)$ and $\bar{B}^{T*}(B^P)$ are not corner optima. Then, differentiating (B.17) and evaluating at a fixed point gives:

$$(B.18)\quad DF'(B^P) = \hat{\Delta}\left[\frac{\partial\underline{\tilde{\Delta}}/\partial B^P}{\underline{\tilde{\Delta}}} - \frac{\partial\bar{\tilde{\Delta}}/\partial B^P}{\bar{\tilde{\Delta}}} - \frac{\partial\underline{N}^T/\partial B^P}{\underline{N}^T} + \frac{\partial\bar{N}^T/\partial B^P}{\bar{N}^T}\right]$$

In (B.18), we use the Envelope Theorem that implies the derivative with respect to $\underline{B}^{T*}$ and $\bar{B}^{T*}$ can be ignored, this since DF is the difference of conditional maxima over B^T. Computing the derivative in the first term in the brackets of (B.11), one obtains:

$$(B.19)\quad \frac{\partial\underline{\tilde{\Delta}}}{\partial B^P} = \int_{y_m}^{y_x}\int_{b_m}^{\tilde{b}}\left\{S\frac{\partial\hat{m}_0^P}{\partial B^P}f(b,\hat{m}_0^P,y) - \int_{\hat{m}_S^P}^{m_x}\frac{\partial a^{*P}}{\partial B^P}fdm\right\}dbdy > 0.$$

In computing the derivative, all the terms associated with the derivative $\frac{\partial\tilde{b}}{\partial B^P}$ vanish. This is because $\hat{m}_0^P = \hat{m}_0^T$ and $\hat{m}_S^P = \hat{m}_S^T$ at $b = \tilde{b}$ and $a^{*T} = a^{*P}$ at $b = \tilde{b}$ in the last integrand in (B.11). Regarding the remaining (non-zero terms) of (B.19), why is the expression positive? Mathematically, $\frac{\partial m_0^P}{\partial B^P} > 0$. For students with $b < \tilde{b}$ who benefit from the charter school's closer curriculum, the public alternative worsens if B^P increases, implying more of these students would drop out. The achievement gains in the charter school are increased by S for those students with m near $\hat{m}_0^P$. Also, for students with $b < \tilde{b}$, $\frac{\partial a^{*P}}{\partial B^P} < 0$. These students would be high achievers if attending the public school, but their achievement would decline with the increase in B^P. They are the other source of higher achievement gains for charter school students if B^P is increased.

Differentiating $\tilde{\bar{\Delta}}$ to sign the second term in (B.18), using again that the terms having $\frac{\partial \tilde{b}}{\partial B^P}$ vanish, one obtains:

$$\text{(B.20)} \quad \frac{\partial \tilde{\bar{\Delta}}}{\partial B^P} = \int_{y_m}^{y_x} \int_{\tilde{b}}^{b_x} \left\{ S \frac{\partial \hat{m}_0^P}{\partial B^P} f(b, \hat{m}_0^P, y) - \int_{\hat{m}_S^P}^{m_x} \frac{\partial a^{*P}}{\partial B^P} f dm \right\} dbdy < 0.$$

Here, $B^P < \tilde{b} < B^T$ and so fewer of the students in charter school (who have $b > \tilde{b}$) would drop out of the public school if B^P increases, corresponding to $\frac{\partial \hat{m}_0^P}{\partial B^P} < 0$. Likewise, $\frac{\partial a^{*P}}{\partial B^P} > 0$ for the charter school students. Thus, the negative sign on the derivative. Intuitively, the charter school that has entered at the high end produces lower achievement gains as if B^P increases. Thus, the second term with sign of (B.18) is also positive. While the first two terms of (B.18) are positive, the second two terms (with sign) are negative. For example, $\frac{\partial \underline{N}^T}{\partial B^P} = -\int_{y_m}^{y_x} \int_{\hat{m}_0^T}^{m_x} f(\tilde{b}, m, y) \underbrace{(\partial \tilde{b} / \partial B^P)}_{+} dmdbdy < 0$. This is a dissolution effect of achievement gains. Increasing B^P and thus $\tilde{b}$ draws into the charter school some students who barely gain in achievement (i.e., those with $b = \tilde{b}$). A positive sign of (B.18) requires the effects on achievement gains of students of a public-school curriculum change that would attend prior to the change outweigh the effect on achievement gains of students that change attendance. To summarize, the sufficient conditions for uniqueness of the fixed point are: (i) non-corner solutions for $\underline{B}^{T*}$ and $\overline{B}^{T*}$ at any fixed point; and (ii) positivity of the expression in (B.18), for which we have identified the forces.

C. Calibration Appendix

Calibration of Distribution of Ability and Motivation.

Here, we detail the derivation of ρ_{bm} =.2165 and $\sigma_m / \sigma_b = \sqrt{.75}$ from Cunha, Heckman, and Schennach (2010) and its associated appendix.

Let θ_C and θ_N denote cognitive and non-cognitive skills in Cunha, Heckman, and Schennah (CHS). We take these as corresponding, respectively, to b and m in our model, but continue using CHS's notation here for ease of comparison to their paper. CHS model the logarithms of these skills as being normally distributed. They report the following decomposition of variance on page 921, with details of their derivation provided on page 40 of their supplementary appendix.

Together, the logarithms of cognitive and non-cognitive skills explain 34% of the variation in educational attainment. CHS report that logarithm of cognitive skills accounts for 16% and the logarithm of non-cognitive skills accounts for 12%. Educational attainment is measured by years of schooling completed by age 19. (Footnote of Appendix Table A10-1.)

Let σ_α^2 denote the variance of educational attainment. Then the CHS decomposition implies:

$\text{Var}(\ln(\theta_C)) = .16\sigma_\alpha^2$.

$\text{Var}(\ln(\theta_N)) = .12\sigma_\alpha^2$.

$\text{Var}(\ln(\theta_C))+\text{Var}(\ln(\theta_N))+2*\text{Cov}(\ln(\theta_C),\ln(\theta_N)) = .34\sigma_\alpha^2$.

Hence,

$\text{Cov}(\ln(\theta_C),\ln(\theta_N)) = (.34\sigma_\alpha^2 - .16\sigma_\alpha^2 - .12\sigma_\alpha^2)/2 = .03\sigma_\alpha^2$.

The correlation of the logs of the cognitive skills is then:

$\rho = \dfrac{\mathrm{Cov}(\ln(\theta_C), \ln(\theta_N))}{[\mathrm{Var}(\ln(\theta_C)) * \mathrm{Var}(\ln(\theta_N))]^{.5}} = \dfrac{.03}{(.16 * .12)^{.5}} = .2165,$ corresponding to our calibration of ρ_{bm}.

The ratio of the standard deviation of non-cognitive to the standard deviation of cognitive skills is:

$\dfrac{\mathrm{Var}(\ln(\theta_N))^{.5}}{\mathrm{Var}(\ln(\theta_C))^{.5}} = \dfrac{\sigma_\alpha}{\sigma_\alpha}\sqrt{12/16} = \sqrt{.75},$ corresponding to our calibration of $\dfrac{\sigma_m}{\sigma_b}$.

Calibration of Achievement Distribution.

As observed in the text, we use NAEP data on the distribution of 12th grade scores to calibrate the mean (μ_a) and standard deviation (σ_a), of the logarithm of achievement, which we then target using equilibrium predictions to calibrate other values. Score are on mathematics, reading, and writing. Mathematics, reading, and writing have median scores of 152, 289, and 148 respectively with standard deviations of 34, 35, and 41 respectively. NAEP scores are approximately normally distributed.[1] Hence, we use the reported medians as means. We rescale all three variables to have a common standard deviation, 34, yielding mean scores of 152, 240, and 148 respectively for mathematics, reading, and writing. We then create a weighted average with .5 weight for mathematics and .25 each for reading and writing. The weighted mean is 180. Hence, the composite score is distributed normally with mean 180 and standard deviation 34. To provide convenient magnitudes for reporting simulation results, and without loss of generality, we divide NAEP achievement by 10, giving mean achievement equal to 18. Hence, we set μ_a, such that $\exp(\mu_a) = 18,$ implying $\mu_a = \log(18) \approx 2.89.$ Letting σ_x denote the standard deviation of the composite NAEP score, allowing for measurement error, we obtain $\sigma_x / \mu_a = 34/180 = .189.$ This and $\sigma_a^2 = .75\sigma_x^2$ imply $\sigma_a / \mu_a = (.189\sqrt{.75}) = .164 \approx 1/6.$ This and $\mu_a = \log(18) \approx 2.89.$ imply $\sigma_a = \log(18)/6 \approx .48.$ Thus, the logarithm of achievement in our model has mean 2.89 and standard deviation .48. Note that this derivation of μ_a and σ_a does not require that the logarithm of achievement be normally distributed.

With the above parameters, we did a search for the curriculum target that maximizes aggregate public school achievement. This was done by enumeration. With the curriculum target equal to the 40th percentile of ability, we calculated equilibrium in our model to obtain the graduation standard and associated dropout that maximized student achievement. We then did the same incrementing the curriculum target by one percentile for percentiles up to the 60th. The maximum public school achievement was reached at the 49th percentile. The maximum value of achievement at the 49th percentile is very little different from the value at the 50th percentile. Hence, we chose the median as the public school curriculum target for the computational results reported in the paper.

At the above curriculum target, we did a comparison of the distribution of public school achievement in the model to the distribution of the NAEP in our reference year. As noted above, the NAEP is approximately normally distributed with mean 180 and standard deviation 34. Expressed as standard deviations from the mean calculated the 10th, 25th, 50th, 75th, and 90th percentiles of student achievement for the NAEP are (-1.282, -0.674, 0, 0.674, 1.282). The corresponding quantiles from our model are (-1.288, -0.986, 0, 0.650, 1.241). As evident from the comparison of these two sets of quantiles, our model matches closely except for the 25th quantile. Our achievement distribution is not normally distributed, but it is reassuring that, overall, our distribution for achievement matches quite well the NAEP distribution.

Calibration of τ.

[1] See https://nces.ed.gov/nationsreportcard/tdw/analysis/trans_compare.aspx. For mathematics and reading, see respectively https://nces.ed.gov/nationsreportcard/tdw/analysis/2013/trans_compare_math2013g12.aspx and https://nces.ed.gov/nationsreportcard/tdw/analysis/2013/trans_compare_read2013g12.aspx.

This parameter, along with α, determines the magnitude of the effect of curriculum targeting on achievement. We have calibrated α so that the baseline model predicts the empirical magnitude of dropouts. We have little additional to go on to calibrate τ. As such, we set it equal to 1 and perform sensitivity analysis of its effect on equilibrium. This sensitivity analysis regarding τ is discussed in the text with much more detail in the next section of the appendix. To provide perspective on the implications of alternative τ values, including the baseline value, here we compute the effects of changes in the curriculum on achievement of students. Specifically, we compute the (negative of the) average effect on the logarithm of achievement of varying the curriculum from students' ideal value (B = b) to one or two standard deviations in b away from that value. The symmetry of $\Gamma(b,B)$ implies the same value for deviations from B = b to higher or lower values. For our calibrated model,

$$\text{(C.1)} \qquad \ln(a(B)) = \ln(b) + \ln(Q\Gamma_m) + \ln(e) + \ln\left\{\alpha + (1-\alpha)\exp\left[-\frac{(b-B)^2}{\tau}\right]\right\}.$$

The gain to a student of ability b of B being adjusted from z standard deviations in log of b away from B = b to the student's ideal (B = b), measured in standard deviations of the log of achievement, is given by:

$$\text{(C.2)} \qquad G(b, z\sigma_{\ln b}, \tau) \equiv [\ln(a(b)) - \ln(a(b + z\sigma_{\ln b}))] / \sigma_{\ln a}.$$

Holding effort constant, using (C.1), this is given by:

$$\text{(C.3)} \qquad G(b, z\sigma_{\ln b}, \tau) = \left[-\ln\left(\alpha + (1-\alpha)\exp\left\{-\frac{(-z\sigma_{\ln b})^2}{\tau}\right\}\right)\right] / \sigma_{\ln a}$$

$$= \left[-\ln\left(\alpha + (1-\alpha)\exp\left\{-\frac{(z\sigma_{\ln b})^2}{\tau}\right\}\right)\right] / \sigma_{\ln a}$$

The latter shows that the measure is the same whether the deviation from the student's ideal is an increase of decrease in the curriculum. Note, too, the measure is independent of effort, holding effort fixed. We measure the average of this over the student population. In doing this, we use the national population (to which we calibrated) for which $\sigma_{\ln b} = .268$ and the standard deviation of the log of achievement for the national population given just a public school ($\sigma_{\ln a} = .747$).

For one standard deviation curriculum changes given τ = 1, 5, and 11.25, we compute:

$$\text{(C.4)} \qquad E_b[G(b, \sigma_{\ln b}, 1)] = .014;\ E_b[G(b, \sigma_{\ln b}, 5)] = .000578;\ E_b[G(b, \sigma_{\ln b}, 11.25)] = .000114.$$

For two standard deviation curriculum changes:

$$\text{(C.5)} \qquad E_b[G(b, 2\sigma_{\ln b}, 1)] = .051;\ E_b[G(b, 2\sigma_{\ln b}, 5)] = .00230;\ E_b[G(b, 2\sigma_{\ln b}, 11.25)] = .000456.$$

As we observed in the text, for the baseline case of τ = 1, the two standard deviation change to a students' ideal curricula has a modest average achievement effect of about 5% of a standard deviation in the population achievement. We see that higher values of τ would reduce this further. (The reason we examined τ = 11.25 is because we used this value in an earlier version of the paper.)

If students are allowed to adjust their effort optimally, the effects are of higher magnitude. We also computed these average effects assuming every student chooses $e = e^*$ for each curriculum, this ignoring the requirement to meet the standard (some students would fall short of the standard at $e = e^*$). Using that $e^* = mbQ\Gamma$ in our model, one obtains the adjusted measure:

(C.6)

$$G(b, z\sigma_{\ln b}, e^*(B), \tau) = \left[\ln(e^*(B = b)) - \ln(e^*(B = b + z\sigma_{\ln b})) - \ln\left(\alpha + (1-\alpha)\exp\left\{ -\frac{(z\sigma_{\ln b})^2}{\tau} \right\} \right) \right] / \sigma_{\ln a}$$

$$= \left[-2\ln\left(\alpha + (1-\alpha)\exp\left\{ -\frac{(z\sigma_{\ln b})^2}{\tau} \right\} \right) \right] / \sigma_{\ln a}$$

The measure exactly doubles for all values. Thus, for example, in the baseline with $\tau = 1$, a two standard deviation change to students' ideal curricula has an average achievement increase of 10.2% standard deviations in achievement if every student can choose their ideal effort before and after their change.

A noted, in the sensitivity analysis next we explore the effects on equilibrium of varying τ.

D. Sensitivity Analysis

Here we investigate sensitivity of findings with respect to τ, the weight that the charter school places on dropouts in their objective function, the first moments of the student-skill distribution, and the correlation between b and m in the student skills distribution.

Sensitivity with Regard to τ.

First, we examine variation in τ. Table A1 summarizes findings, which contains the same results in the main computational tables in the text as well as a few additional values. This analysis is for the poverty population, which is the most empirically relevant population for study of charter schools. We report equilibrium values with just a public school, with a strategic (Stackelberg-leader) public school and TCS school and with a strategic public school and NEC school, assuming the latter has no enforcement costs of their effort minimum. All parameters other than τ are maintained at the baseline values. The fourth-sixth columns have the baseline value of $\tau = 1$ for comparison. We consider one lower value of τ (and have examined others), and two higher values on the second page of the table.

The pattern of the relative values of the schools' choices of their curriculum targets do not vary with τ, and the magnitudes hardly vary. In particular, in the TCS case the public school induces entry by the TCS with low curriculum target (always a corner solution) with then the TCS serving low ability students. The NEC continues to target higher ability students.

Achievement gains for the TCS continue to come largely from reducing dropouts, with small variation but these gains account for over 98% in all cases (see the second row from the bottom). We have added a row "Prop Condit. Drops" that reports the proportion that apply to the charter that would dropout if the charter school were unavailable, all else constant. This is always 0 for the NEC.[2] Taking examples for the TCS

[2] Effects on equilibrium dropouts in the NEC equilibrium relative to the case of just a public school are because the public school changes its curriculum target. The NEC does not effect dropouts holding constant the curriculum of the public school.

cases, 31.4% of those that apply to the charter school would drop out of the public school if $\tau = .5$, while this percentage is 26.5 in the baseline with $\tau = 1$ as we reported in the text. As τ rises this percentage gets very small because the curriculum difference has little effect.

Perhaps of most interest are the effects on achievement gains of the charter school. For the NEC equilibrium, achievement gains as a function of τ are reverse-U shaped, highest for $\tau = 1$ among the values we report, but always high. The case of $\tau = 11.25$ (from an earlier version of the paper) has the best match to the empirical estimates. Achievement gains for charter students in the case of the TCS decline significantly with τ, only .7% of a standard deviation for $\tau = 11.25$. The difference between the NEC effects and the TCS effects are largely because TCS achievement gains are explained exclusively from curriculum targeting while the effort margin for the NEC is most relevant. One interesting observation is that for the lowest value of τ examined (.5), with then the highest curriculum effect, achievement gains from the TCS are higher than from an NEC. Given that the public school has higher achievement as well in the TCS case, this implies the NEC would not arise in equilibrium; i.e., having the power to enforce an effort minimum none would arise in equilibrium. Relatedly the "NEC equilibrium" that is reported in column three is not actually an equilibrium.

	Table A1: Sensitivity Analysis I Impact of Curriculum (τ) (Poverty Population)					
	Public Only **τ = 0.5**	**Strategic Pub/TCS** **τ = 0.5**	**Strategic Pub/NEC** **τ = 0.5**	**Public Only** **τ = 1**	**Strategic Pub/TCS** **τ = 1**	**Strategic Pub/NEC** **τ = 1**
B^{P}_{Per}*	10.7	49.6	10.7	12.5	49.9	12.5
B^{N}_{Per}*	n/a	9.4	77.3	n/a	7.9	78.5
Prop Apply Charter	n/a	.2	.2	n/a	.2	.20
Prop Attend Charter	n/a	.1	.1	n/a	.1	.10
Med. Charter Inc./Med Pop. Inc.	n/a	.943	1.06	n/a	.935	1.059
Total Achiev	42,102.6	41,875.8	41,746.2	42,964	43,435.1	44,308.8
Achiev Public School	42,102.6	37,746.6	34,951.2	42,964	39,297.9	37,362.4
Achiev Gain Charter/SD**	n/a	.87	.783	n/a	.738	.9
Prop Charter Exceed Standard	n/a	0.047	1	n/a	.077	1
Effort Min NEC/ Avg.	n/a	n/a	2.44	n/a	n/a	2.54
Prop Effort Const. NEC	n/a	n/a	.996	n/a	n/a	.991
Prop Condit. Drops	n/a	.314	0	n/a	.265	0
Prop Gains from Drops	n/a	.995	0	n/a	.991	0
Dropout rate	7.88%	10.73%	13.86%	6.43%	8.8%	6.43%

*Entries equal the percentile targeted ability using poverty population ability distribution.
**Entries equal the expected average log achievement gain of charter applicants reported as the ratio of the achievement gain to the standard deviation of log achievement of students calculated for the national population with the public school only.

	Table A1: Sensitivity Analysis I Impact of Curriculum (τ) (Poverty Population)					
	Public Only **τ = 5**	**Strategic Pub/TCS** **τ = 5**	**Strategic Pub/NEC** **τ = 5**	**Public Only** **τ=11.25**	**Strategic Pub/TCS** **τ = 11.25**	**Strategic Pub/NEC** **τ = 11.25**
B^{P}_{Per}*	50.5	55.4	37.3	47.1	58.3	41.7
B^{N}_{Per}*	n/a	5.08	84.1	n/a	3.95	85.8
Prop Apply Charter	n/a	.2	.2	n/a	.2	.2
Prop Attend Charter	n/a	.1	.1	n/a	.1	.1
Med. Charter Inc./Med Pop. Inc.	n/a	.936	1.06	n/a	.936	1.06
Total Achiev	46,159	46,269	47,201	46,531	46,557	47,520
Achiev Public School	46,159	42,118	40,110	46,531	42,405	40,305
Achiev Gain Charter/SD**	n/a	.076	.708	n/a	.018	.660
Prop Charter Exceed Standard	n/a	.122	1	n/a	.127	1
Effort Min NEC/ Avg.	n/a	n.a	2.69	n/a	n.a	2.71
Prop Effort Const. NEC	n/a	n.a	.987	n/a	n.a	.985
Prop Condit. Drops	n/a	.027	0	n/a	.007	0
Prop Gains from Drops	n/a	.982	0	n/a	.982	0
Dropout rate	6.16%	5.97%	5.94%	5.76%	5.74%	5.73%

*Entries equal the percentile targeted ability using poverty population ability distribution.

**Entries equal the expected average log achievement gain of charter applicants reported as the ratio of the achievement gain to the standard deviation of log achievement of students calculated for the national population with the public school only.

Sensitivity with Regard to Charter Weight on Dropouts.

Before performing sensitivity analysis, we clarify the normalization assumption in the baseline model that assumes that achievement of dropouts is equal to 0. Consider this generalization. Let a student's achievement be given by $a(q^j,b,e) + a^F$, where a^F is a constant corresponding to achievement of those that do not graduate. For the same objective functions of the public school (aggregate achievement maximization including all dropouts), both types of charter schools (both maximize achievement gains), and for students all our results go through. This is because a^F is minimal achievement and does not impact maximization of the objectives. Thus, the assumption is not that achievement of non-graduates is zero, but that it is constant and thus independent of any student effort and student ability.

Three key issues are related to this assumption. First, one might consider a model where achievement of those that do not graduate varies to some degree with school quality, student effort and/or with ability. We have not pursued this alternative. Our assumption can be viewed as the success of those that do not graduate is independent of any schooling, as though they take menial jobs.

Second and separately, one might alternatively weight dropouts in schools' objectives. This is what the sensitivity analysis conducted below does.

The third issue is how this assumption impacts our calibration. We believe that our calibration handles dropouts well. For one, we calibrate to scores on the NAEP, with the lowest score used the 10^{th} percentile. Since the national average dropout rate is 5.4%, such students' scores (if included in the data) are likely to be lower than the 10^{th} percentile. Second, we do calibrate to the dropout rate, and we are able to well match it with our specification.

Turning to the sensitivity analysis, we have seen that dropouts dominate the achievement gains of TCS's, while irrelevant to NEC's. Now we examine alternative weighting of dropout effects in the charter school objective. To explain how we vary the charter school's assessment of dropouts, let a_{ci}, a_{pi}, and a_{Fi} denote achievement of student i in the charter school, in the public school (assuming the student would not drop out of it), and achievement from dropping out ('F' for failure). The objective function for our charter school can be written as:

$$\text{Max } 2\cdot\left[\theta\cdot\sum_{i\in ND}[\ln a_{ci}-\ln a_{pi}]+(1-\theta)\cdot\sum_{i\in D}[\ln a_{ci}-\ln a_{Fi}]\right];$$

where ND is the set of charter students that would not drop out of the public school, D is the set that would drop out or the public school, $\ln a_{Fi} = 0$ (or $a_{Fi} = 1$), and $\theta = ½$. The latter value of θ conforms to that in the main model and means, of course, that the charter school weighs would-be dropouts and would-be nondropouts the same. Here we simply vary θ in the charter's objective, while we continue to assume the public school and households assess dropping out as in the main model. (An alternative would be to vary a_{Fi}, which we have investigated to some extent and with the similar implications.)

The first page of Table A2 shows equilibrium values for six values of θ assuming the poverty population and a strategic public school competing with a TCS. The third column is the baseline. As θ rises up to 2/3, thus reducing the weight on dropouts, equilibrium choices of the curricula are unchanged, with still the corner solution having the TCS target low-ability students. But the charter's measure of achievement gain declines. Interestingly, when θ is increased to 5/6 and beyond, the nature of equilibrium changes. The

charter school has sufficiently weaker incentives to target low ability students and the public school finds it optimal to reduce its curriculum target and permit entry by the charter school with higher curriculum. Charter student gains drop further in going from $\theta = 2/3$ to $\theta = 5/6$, but then increase if $\theta = 1$ due to higher weighting on non-dropouts.

The second page of Table A2 reports equilibrium with instead an NEC as compared to the case of a TCS for three of the values of θ. (The paper, of course, contains the cases with $\theta = 1/2$.) Thus, Columns 1, 3, and 5 for the TCS/public-school equilibrium are the same as columns on the first page. For the NEC cases (Columns 2, 4, and 6), the equilibrium curricula and minimum effort in the NEC do not change with θ because the NEC targets strong students that would not dropout of the public school, then in each case maximizing a multiple of the same objective. In turn, the public school choices are the same across these cases. However, comparing the TCS to the NEC equilibrium, for the case with the highest charter weight on dropouts ($\theta = ¼$; Columns 1 and 2 on second page), the TCS implies higher charter gains. In fact, the "NEC equilibrium" in Column 2 is not really an equilibrium. Rather, the public school can and prefers to induce the outcome in Column 1. Here the public school achievement is higher and the charter school optimally responds as in Column 1, preferring not to impose an effort minimum (as we have confirmed), of course because it would increase dropouts. These parameters then provide another example of an "endogenous TCS."

	Table A2: Sensitivity Analysis II Impact of Dropouts Weighting (θ) (Poverty Population - Strategic Pub/TCS)					
	θ = 1/4	**θ = 1/3**	**θ = 1/2**	**θ = 2/3**	**θ = 5/6**	**θ = 1**
B^{P}_{Per}*	49.9	49.9	49.9	49.9	12.1	12.1
B^{N}_{Per}*	7.9	7.9	7.9	7.9	78.43	78.43
Prop Apply Charter	.2	.2	.2	.2	.524	.524
Prop Attend Charter	.1	.1	.1	.1	.1	.1
Med. Charter Inc./Med Pop. Inc.	.935	.935	.935	.935	1.039	1.039
Total Achiev	43,435	43,435	43,435	43,435	43,295.8	43,295.8
Achiev Public School	39,297.9	39,297.9	39,297.9	39,297.9	38,057.5	38,057.5
Adjusted Achiev Gain Charter/SD**	1.1	.980	.738	.496	.371	.445
Prop Charter Exceed Standard	.077	.077	.077	.077	.911	.911
Effort Min NEC/ Avg.	n/a	n/a	n/a	n/a	n/a	n/a
Prop Effort Const. NEC	n/a	n/a	n/a	n/a	n/a	n/a
Prop Condit. Drops	.265	.265	.265	.265	.0003	.0003
Prop (of unweighted) Gains from Cond Drops	.991	.991	.991	.991	.004	.004
Dropout rate	8.8%	8.8%	8.8%	8.8%	6.39%	6.39%

	Table A2: Sensitivity Analysis II Impact of Dropouts Weighting (θ) (Poverty Population)					
	Strategic PUB/TCS θ = 1/4	**Strategic PUB/NEC θ = 1/4**	**Strategic PUB/TCS θ = 2/3**	**Strategic PUB/NEC θ = 2/3**	**Strategic PUB/TCS θ = 5/6**	**Strategic PUB/NEC θ = 5/6**
B^{P}_{Per}*	49.9	12.5	49.9	12.5	12.1	12.5
B^{N}_{Per}*	7.9	78.45	7.9	78.45	78.43	78.45
Prop Apply Charter	.2	.2	.2	.2	.524	.2
Prop Attend Charter	.1	.1	.1	.1	.1	.1
Med. Charter Inc./Med Pop. Inc.	.935	1.059	.935	1.059	1.039	1.059
Total Achiev	43,435	44,308.7	43,435	44,308.7	43,295.8	44,308.7
Achiev Public School	39,297.9	37,362.4	39,297.9	37,362.4	38,057.5	37,362.4
Adjusted Achiev Gain Charter/SD**	1.1	.45	.496	1.2	.371	1.5
Prop Charter Exceed Standard	.077	1	.077	1	.911	1
Effort Min NEC/ Avg.	n/a	2.54	n/a	2.54	n/a	2.54
Prop Effort Const. NEC	n/a	.991	n/a	.991	n/a	.991
Prop Condit. Drops	.265	0	.265	0	.0003	0
Prop Gains from Cond Drops	.991	0	.991	0	.004	0
Dropout rate	8.8%	6.43%	8.8%	6.43%	6.39%	6.43%

*Entries equal the percentile targeted ability using poverty population ability distribution.

**Entries equal the expected weight-adjusted average log achievement gain of charter applicants reported as the ratio of the adjusted achievement gain to the standard deviation of log achievement of students calculated for the national population with the public school only.

Sensitivity Analysis with Regard to First Moments of Student-Skills Distribution

We used results from Cunha, Heckman, and Schennach (2010) and Borgans, Golsteyn, Heckman, and Humphries (2016) along with model predictions (especially regarding the achievement distribution) to calibrate the student-skills distribution as detailed in the text and above. The calibration was based on the national population and equilibrium with just a public school. Here we investigate sensitivity of the model to moments of the student-skills distribution, specifically the means of ability and motivation (μ_b,μ_m). We examine the effects on the standalone public school and the strategic TCS (Stackelberg) equilibrium of a one-tenth of one standard deviation increase and decrease of each of these means individually, holding constant all other parameters, using the calibrated values of (σ_b,σ_m). Table A3 reports the results, with the first page examining the changes in μ_b. Columns 5 and 6 are the baseline values for comparison, from Table 2 in the text. Columns 1 and 2 for the increase in μ_b are then comparable respectively to Columns 5 and 6. The second page of the Table reports the results from changing μ_m, again comparable to Columns 5 and 6 on the first page.

The equilibrium curriculum choices respond moderately to all the changes. Since the type distributions change, we report the curriculum values in ability percentiles both using the baseline distribution and the new distributions. While the curriculum values change, note that the structure of the Stackelberg equilibrium is unchanged for all the variations, meaning the public school increases its curriculum in the face of entry and the charter school is induced to enter targeting low ability students. For all the variations, the drop out rate is predicted to decline from entry. There is some effect on the achievement gains that are predicted from charter school entry, most notably somewhat higher achievement gains for both a decrease in the mean ability and the mean of motivation. To summarize, equilibrium values are moderately affected but without a change in the structure of the equilibria.

	Table A3: Sensitivity Analysis III (National Population)					
	Public School Only μ_b**=1.827**	**Strategic PUB/TCS** μ_b**=1.827**	**Public School Only** μ_b**=1.773**	**Strategic PUB/TCS** μ_b**=1.773**	**Public School Only (Base)** μ_b**=1.80**	**Strategic PUB/TCS (Base)** μ_b**=1.80**
B^P_{Per}*	51.3 (47.87)	55.2 (51.76)	40.8 (45.34)	47.6 (51.99)	49.9	52.8
B^N_{Per}*	n/a	4.27 (3.21)	n/a	3.26 (3.87)	n/a	3.39
Prop Apply Charter	n/a	.2	n/a	.2	n/a	.20
Prop Attend Charter	n/a	.1	n/a	.1	n/a	.10
Med. Charter Inc./Med Pop. Inc.	n/a	.663	n/a	.67	n/a	.670
Total Achiev	50,852.9	51,517.4	48,638.6	49,433.6	49,830	50,537
Achiev Public School	50,852.9	47,325.6	48,638.6	45,268.7	49,830	
Achiev Gain Charter/SD**	n/a	.449	n/a	.555	n/a	.481
Prop Charter Exceed Standard	n/a	.233	n/a	.165	n/a	19.1
Dropout rate	4.77%	3.3%	6.25%	4.62%	5.51%	3.92%

*Percentiles correspond to the national original (new) populations.

**Entries equal the expected average log achievement gain of charter applicants reported as the ratio of the achievement gain to the standard deviation of log achievement of students calculated for the national population with the public school only.

	Table A3: Sensitivity Analysis III (National Population)			
	Public School Only μ_m=.0464	**Strategic PUB/TCS** μ_m=.0464	**Public School Only** μ_m=−.0004	**Strategic PUB/TCS** μ_m=−.0004
B^P_{Per}*	46.8 (47.29)	50.9 (51.18)	48.9 (49.37)	53.3 (53.61)
B^N_{Per}*	n/a	3.76 (3.53)	n/a	3.32 (3.11)
Prop Apply Charter	n/a	.2	n/a	.2
Prop Attend Charter	n/a	.1	n/a	.1
Med. Charter Inc./Med Pop. Inc.	n/a	.663	n/a	.665
Total Achiev	50,263.2	50,972.4	49,238.9	50,004.4
Achiev Public School	50,263.2	46,787.5	49,238.9	45,839
Achiev Gain Charter/SD**	n/a	.484	n/a	.523
Prop Charter Exceed Standard	n/a	.218	n/a	.165
Dropout rate	5.15%	3.59%	5.97%	4.24%

*Percentiles correspond to the national original (new) populations.

**Entries equal the expected average log achievement gain of charter applicants reported as the ratio of the achievement gain to the standard deviation of log achievement of students calculated for the national population with the public school only.

Sensitivity with Regard to ρ_{bm}.

We set $\rho_{bm} = 0$ in the main analysis, given the scant and mixed evidence on this parameter. Here we examine the same effects as in Table A3 of ρ_{bm} equal -.2 and .2, including the baseline in the middle columns for comparison. One can see that the effects of this variation are small both for the standalone public school and the case with a charter school, without affecting the structure of the Stackelberg equilibrium.

	Table A4: Sensitivity Analysis IV (National Population; Vary ρ_{bm})					
	Public School Only $\rho_{ym} = -.2$	**Strategic PUB/TCS** $\rho_{ym} = -.2$	**Public School Only** $\rho_{ym} = 0$	**Strategic PUB/TCS** $\rho_{ym} = 0$	**Public School Only** $\rho_{ym} = .2$	**Strategic PUB/TCS** $\rho_{ym} = .2$
B^{P}_{Per}*	50.1 (50.66)	50.7 (51.22)	49.9	49.9	49.9 (49.93)	51.1 (51.22)
B^{N}_{Per}*	n/a	4.18 (4.21)	n/a	4.41	n/a	4.05 (3.72)
Prop Apply Charter	n/a	.2	n/a	.2	n/a	.2
Prop Attend Charter	n/a	.1	n/a	.1	n/a	.1
Med. Charter Inc./Med Pop. Inc.	n/a	.674	n/a	.671	n/a	.691
Total Achiev	49,671.3	50,418.5	49,830.5	50,529.6	50,009.7	50,744.7
Achiev Public School	49,671.3	46,233.2	49,830.5	50,529.6	50,009.7	46,566.6
Achiev Gain Charter/SD**	n/a	.500	n/a	.468	n/a	.498
Prop Charter Exceed Standard	n/a	.224	n/a	.227	n/a	.199
Dropout rate	5.85%	4.13%	5.51%	3.91%	5.36%	3.67%

*Percentiles correspond to the national original (new) populations.

**Entries equal the expected average log achievement gain of charter applicants reported as the ratio of the achievement gain to the standard deviation of log achievement of students calculated for the national population with the public school only.

E. Efficiency Analysis

Proof of Proposition 2. For reference, here is Proposition 2 from the text.

Proposition 2.

1. Assuming non-strategic behavior by the public school. Entry of a TCS implies a Pareto Improvement including both parents (achievement) and students, thus also ignoring student utility. Entry of an NEC implies a Pareto Improvement ignoring student utility, but not so counting their utility.

2. In any cases of equilibrium with excess demand for the charter school, aggregate achievement would be increased if the charter school (TCS or NEC) were able to directly admit students on their type (b,m).

Proof: (1) This is immediate since students attend the charter school if their achievement increases or does not decline but student utility increases; and, for the TCS, an achievement increase mean higher quality which increases student utility. Below we provide an example where student utility declines for some attending the NEC.

(2) We show that the achievement gains the charter school can generate with direct admissions are higher, implying aggregate achievement is higher among all students. We proceed with a general analysis that can be applied to both the TCS and NEC. This entails some different but simple notation. Let g(t,P) denote the achievement gain to a student of type t that attends a charter school with policy P. Thus, P would be the curriculum for the TCS and both the curriculum and effort minimum for the NEC. Let f(t) denote the density of student types. Let I(t,P) be an indicator function:

$$\text{(E.1)} \qquad I(t,P) = \begin{cases} 1 \text{ if } g > 0 \text{ or } g = 0 \text{ but e is lower in the charter school} \\ 0 \text{ otherwise} \end{cases}$$

Let p_r denote the probability an applicant wins the lottery and so is admitted to the charter school, then attending with probability μp_r given the fixed matriculation probability. Letting G denote the objective of the charter school, the achievement gains it generates, its problem can be written:[3]

$$\begin{aligned} &\text{Max}_P\, G = \int_t g(t,P) I(t,p) p_r \mu f(t) dt \\ \text{(E.2)} \qquad &\text{s.t.} \int_t I(t,P) \mu f(t) dt \geq \kappa \\ &p_r = \kappa / \int_t I(t,P) \mu f(t) dt \end{aligned}$$

The first constraint is that the applicant set must be at least as large as the charter school capacity (κ), taking account of the matriculation probability, and the second determines the constant probability of winning the lottery that is necessary to meet exactly the capacity constraint as we have assumed.

Proposition 2 applies when there is excess demand, i.e., the first constraint is not binding. We will discuss the case when the constraint binds, but assume it does not at the optimum now and can be ignored. The result that direct admissions could increase aggregate achievement gains can be seen by inspection of (E.2). Allowing direct admissions corresponds to permitting p_r to vary with t (including allowing it to be 1

[3] The proof assumes a continuous objective function in P. An exception to this is discussed in the Remarks below.

and 0), so that the second constraint becomes: $\kappa = \int_t p_r(t)I(t,P)\mu f(t)dt$ with choice of $p_r(t)$. In general some applicants will have higher gains than others, in fact some will have vanishingly small gains. By increasing p_r to one for some with higher gains and lowering p_r to zero for some with lower gains, the constraint could be satisfied (as well as maintaining sufficient demand) with an obvious increase in the total charter gains. As well, this would obviously increase aggregate achievement. This is enough to complete the proof, but we go on to show another margin for gains from varying the policy.

The latter argument holds constant P. We show another source of gains from direct admissions would entail adjusting P, this at once showing the selection effect. This is more difficult because of the lack of continuity of I(t,P). We continue to assume the first constraint in (E.2) is not binding. Consider a small change in the charter policy from the optimum P^* (i.e., the solution to (E.2)) to $P^*+\delta$. Define three more indicators:

$$
\begin{aligned}
&I_1(t,P^*,\delta) = 1 \text{ for t such that } g(t,P^*+\delta) \text{ and } g(t,P^*) > 0, \text{ and } = 0 \text{ otherwise} \\
&I_2(t,P^*,\delta) = 1 \text{ for t such that } g(t,P^*+\delta) > 0 \text{ and } g(t,P^*) \le 0, \text{ and } = 0 \text{ otherwise} \\
&I_3(t,P^*,\delta) = 1 \text{ for t such that } g(t,P^*) > 0 \text{ and } g(t,P^*+\delta) \le 0, \text{ and } = 0 \text{ otherwise}
\end{aligned}
\tag{E.3}
$$

Let $p_r(P)$ satisfy the second constraint in (E.2). Now:

$$
\begin{aligned}
G(P^*+\delta) - G(P^*) = &\int_t [g(t,P^*+\delta)p_r(P^*+\delta) - g(t,P^*)p_r(P^*)]I_1(t,P^*,\delta)\mu f(t)dt \\
&+ \int_t g(t,P^*+\delta)p_r(P^*+\delta)I_2(t,P^*,\delta)\mu f(t)dt - \int_t g(t,P^*)p_r(P^*)I_3(t,P^*,\delta)\mu f(t)dt
\end{aligned}
\tag{E.4}
$$

The optimal P satisfies:

$$
\lim_{\delta\to 0} \frac{G(P^*+\delta) - G(P^*)}{\delta} = 0. \tag{E.5}
$$

The values of the g's and the measure of the I's in the last two terms of (E.4) both vanish in the limit so these terms converge to 0^2 as $\delta \to 0$ (the g's because the students that exit the charter or enter the charter as δ varies in the vicinity of 0 have vanishingly small gains), implying the optimal P satisfies:

$$
\lim_{\delta\to 0} \frac{G(P^*+\delta) - G(P^*)}{\delta} = \lim_{\delta\to 0} \frac{\int_t [g(t,P^*+\delta)p_r(P^*+\delta) - g(t,P^*)p_r(P^*)]I_1(t,P^*,\delta)f(t)dt}{\delta} = 0 \tag{E.6}
$$

Rewrite the numerator:

(E.7)

$$
\begin{aligned}
&\int_t [g(t,P^*+\delta)p_r(P^*+\delta) - g(t,P^*)p_r(P^*)]I_1(t,P^*,\delta)f(t)dt \\
&= \underbrace{\int_t [g(t,P^*+\delta) - g(t,P^*)]p_r(P^*)I_1(t,P^*,\delta)f(t)dt}_{\text{Policy Change Effect on Matriculates}} + \underbrace{\int_t g(t,P^*+\delta)[p_r(P^*+\delta) - p_r(P^*)]I_1(t,P^*,\delta)f(t)dt}_{\text{Selection Effect of Policy Change}}
\end{aligned}
$$

Before continuing with the proof per se, it is useful to understand the source of the two terms. The first integral captures the "pure policy effect" of vary P, i.e., on the students that attend both before and after the policy change. The second effect is the net effect from students exiting and entering as a consequence of the policy change along with the implied change in the probability of winning the lottery. The students that exit or enter for a small policy change are those with vanishingly small gains, this why the last two terms in (E.4) are second-order small and can be ignored. But these students take space in the school and their selection in and out effects aggregate charter gains because they take slots from or are replaced by the randomly selected average student. This is what the "selection effect" measures.

Now we argue that, whether the selection effect is positive or negative, aggregate charter achievement could be increased if the charter school could directly admit (or reject) applicants that would gain and want to attend. Note that $g \geq 0$ in the selection term for all students and positive for some students and the probability difference is constant. If the latter term is positive, this would mean the δ policy change would *decrease* demand, implying $p_r(P^*+\delta) > p_r(P^*)$, i.e., the probability of winning the lottery would need to be increased to satisfy the size constraint. In turn the positivity of the selection effect term implies that the first term in (E.7) is negative to satisfy (E.6), i.e., varying δ marginally in the right direction would increase achievement for the average student. But with direct admissions, the charter school could so vary δ, apparently then increasing demand (since δ had been adjusted to decrease demand), but then not admit the marginal students that would want to attend but who would take up slots. Alternatively, if the selection effect is negative, this means there are selection gains from changing δ to drive out students, but at the expense of the average student in I_1. With direct admissions, δ could be changed in the desired direction while throwing out students instead of adjusting δ to accomplish this. Such gains from direct admissions, here with adjustment to policy, are feasible generically unless it just so happens that the selection effect vanishes at the optimum (marginal students that exit from a small policy change in the vicinity of P^* equal marginal students that enter from the change).

Remarks

1. If the solution to (E.2) is at the boundary, then $p_r(P^*) = 1$, and gains from varying p_r for the given policy are infeasible. That is, the first source of gains discussed in the proof are not feasible. Gains associated with the selection effect may or may not be feasible. If the direction of change in policy that would increase the first term in (E.7) would also increase demand, this implying the selection effect is positive, then gains are feasible combining policy change and direct admissions. Here is an example for the case of a TCS at corner optimum (without direct admissions) with $B^T < B^P$. It could be that increasing B^T would increase demand and achievement of students in the charter school, but this is not optimal because of marginal students that are drawn in on net (some would exit too). If some entering students could be prevented from doing so, then gains from increasing B^T could be realized. On the other hand, if the charter school would like to decrease B^T further but cannot not do so because demand declines, no such gains are feasible with direct admissions.

2. Assuming excess demand, gains from direct admissions for the TCS only require the charter school can observe ability. This is because students in the vicinity of $\tilde{b}$ that experience higher quality from the charter school gain marginally regardless of their motivation. We have not shown this generally for the NEC, which then might need to observe (b,m) to engineer gains. This is because the effort minimum implies both (b,m) need to be observed to identify marginal students.

3. For the NEC, $B^N = B^p$ is a local maximum for the charter school (and can be the global maximum), with all charter students gaining because they are constrained by the effort minimum. Here the objective function in (E.2) is not continuous in B^N because a small deviation would induce a finite set of marginal gainers and finitely increased demand. For this case, the NEC could increase aggregate charter gains whether at a corner solution or not with direct admissions by varying B^N but not admitting (almost all of) the marginal gainers. This argument is made formally in an earlier version of the paper available from the authors.

Achievement and Student Utility Winners and Losers from Charter School Entry

In this part of the appendix, we calculate and characterize both achievement and student-utility winners and losers for two key examples from the text. We first discuss the example of a TCS entrant, and then the NEC entrant example. Keep in mind that achievement gains are what parents value (except that they also support the child's preference for less effort if achievement is the same in the charter school).

TCS Entry (Example): Achievement Winners and Losers

The TCS example compares the equilibria in Columns 4 and 6 of Table 2 in the text. Overall, 60.3% have higher achievement, 6.0% have lower achievement and 33.1% have the same achievement. To explain which students comprise these groups, we must separately consider charter applicants who matriculate and do not, as well as public school applicants (including dropouts). Figures A1 to A3 show in the (b,m)-plane, b and m measured in percentiles, those that gain, lose, and have the same achievement as a result of entry of the charter school.for each group. Figure A1 plots charter-school applicants who matriculate, Figure A2 those that do not matriculate, and Figure A3 students that do not apply to the charter school and remain in the public sector (including dropouts). In the example, there are three relevant curricula: the public school curriculum in equilibria before and after charter school entry (denoted, respectively, B_0^P and B_1^P), and the charter school curriculum (denoted B^T). There also are three relevant indifference ability cut-offs: the one between the initial public school curriculum and the one offered by the charter school, denoted $\tilde{b}'(=10.05)$, another between the final public school curriculum and the one offered by the charter school: $\tilde{b}''(=26.1)$, and the one between the two public school curricula, denoted with $\tilde{b}'''(=30.15)$.

TCS applicants

Consider first *TCS applicants who matriculate* and refer to Figure A1. Only a few of these students gain in achievement because most of them continue to meet the standard but with less effort. The winners (light-blue dots) belong to two groups: (i) A set of highly-motivated students with ability $b < \tilde{b}'$. These students benefit from a better curriculum match and are not effort-constrained (by the standard) in the charter school so that they gain in achievement. (ii) Students at risk of dropping out (the "curve" of light-blue dots on the western edge of the graph), who benefit from a better curriculum match and do not drop out of the charter school, while they would in the initial public school. Losers (the red dots) are from a single group of highly motivated students with ability $b > \tilde{b}'$, who apply to the TCS because it offers them a better match than the public school does after re-adjusting its curriculum, but still end up with a worse match than without entry. The achievement of the rest (the darker blue dots) does not change: they graduate with achievement equal to the standard in each equilibrium. Among the TCS applicants that

matriculate, 87.1% are indifferent (8.71% of the total student population), 2.2% are winners (0.22% of the total student population) and the remaining 10.7% lose in achievement (1.07% of the total student population).

Consider next what happens to the achievement of *TCS applicants that do not matriculate*, which are half of the applicants in the model. Refer to Figure A2 now. First, observe that, since the strategic public school, after entry, chooses to cater its curriculum towards the median of the student ability distribution, there are none among TCS applicants that win in achievement if they do not matriculate. Among the losers, we find two groups: (i) Those in the red triangle-shaped area are highly-motivated students that would be effort-unconstrained by the standard in the original public school and have ability $b < \tilde{b}''$. These students lose achievement because they end up with a worse curriculum match in the "new" public school. (ii) The remaining achievement losers are found within the red area to the left of the figure (which coincides with the red area of Figure 2 in the text: students that would drop out of the -new- public school but not of the charter school). These students would not drop out without entry, but do so if they do not matriculate in the charter school after entry. There are two groups of TCS matriculates whose achievement does not change: (i) the middle band of blue dots, who are students that just graduate in any case; and (ii) the approximate locus of blue dots on the western edge (corresponding to the same set of light-blue dots in Figure A1). These are students who would drop out without entry, who apply to the TCS and graduate if they matriculate, but who still drop out after entry if they were to attend the new public school. Among the TCS applicants who do not matriculate, 35.7% are achievement losers (3.57% of the total student population), while the remaining 64.3% (6.43% of total) are indifferent.

Figure A1. TCS Entry. Achievement Winners and Losers
(TCS Applicants Who Matriculate)

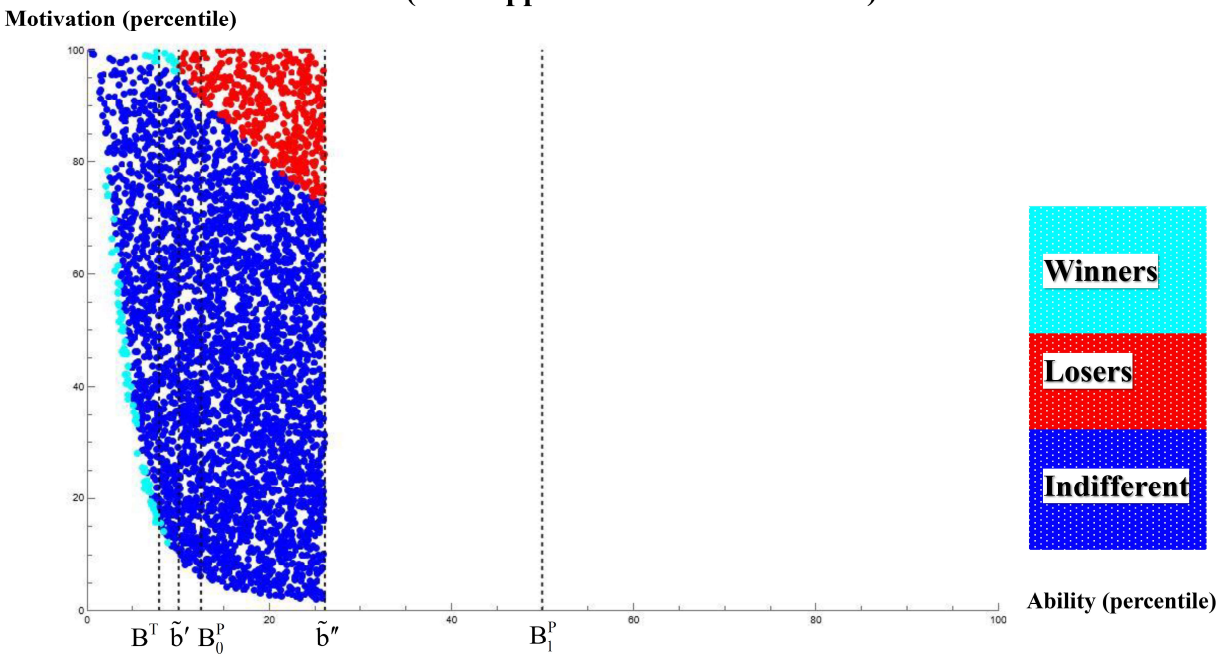
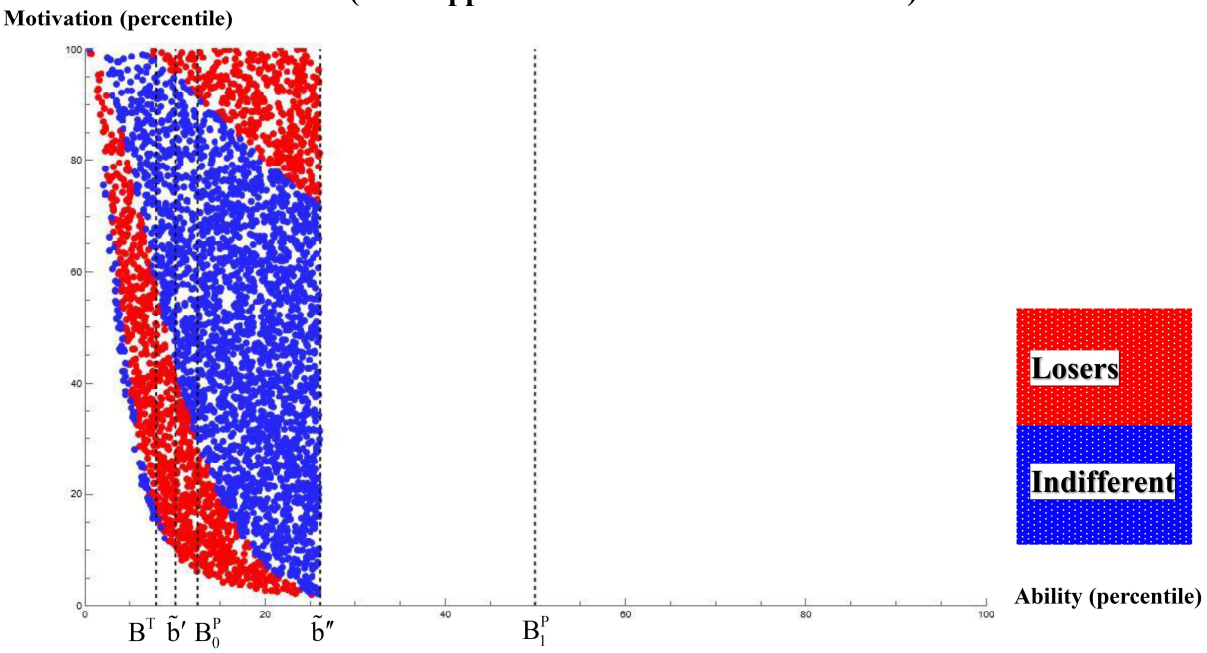


Figure A2. TCS Entry. Achievement Winners and Losers
(TCS Applicants Who Do Not Matriculate)

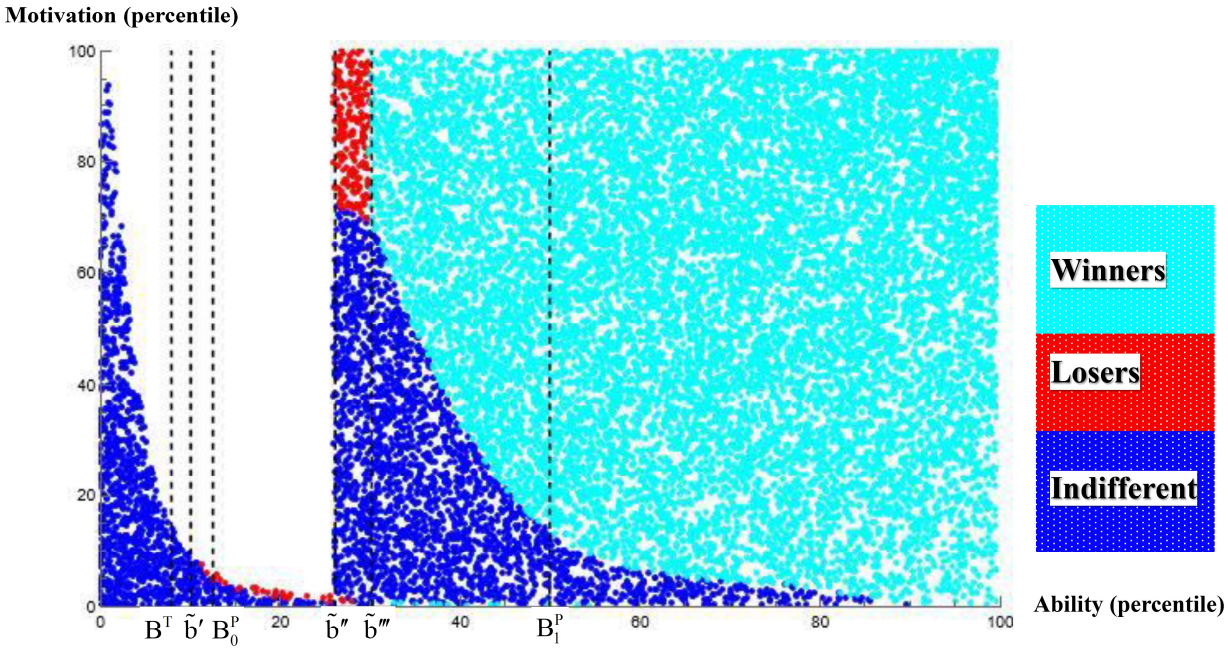


PS applicants

In the example at hand, only 20% of students apply to the charter school. Among the remaining 80% of students, there are two groups of achievement winners: (i) The vast majority of them are in the large light-blue northeast area of Figure A3. They attend the public school prior to and after charter entry, and gain because they are a better match to the public school's adjusted curriculum. (ii) A few more students (the ""curve" of light-blue dots with ability greater than $\tilde{b}'''$ near bottom) who, without TCS entry, would drop out of the public school but who attend it and graduate when the charter enters because of the higher, better-matched public-school curriculum.

We find two small groups of losers: (i) A group of sufficiently motivated students so as to not be effort-constrained in the original public school, and with ability between $\tilde{b}''$ and $\tilde{b}'''$. They attend the public school before and after entry and lose in achievement because, on the one hand, they are not effort-constrained without entry and, on the other hand, because the public school final curriculum is a worse match to their abilities. (ii) A few more of these students have lower achievement, making up the "curve" of red dots with ability between $\tilde{b}'$ and $\tilde{b}'''$ near the bottom. They would not drop out of the public school without TCS entry, but do so after entry due to the increase in the public school curriculum and the resulting weaker match.

Figure A3. TCS Entry. Achievement Winners and Losers (PS Applicants)

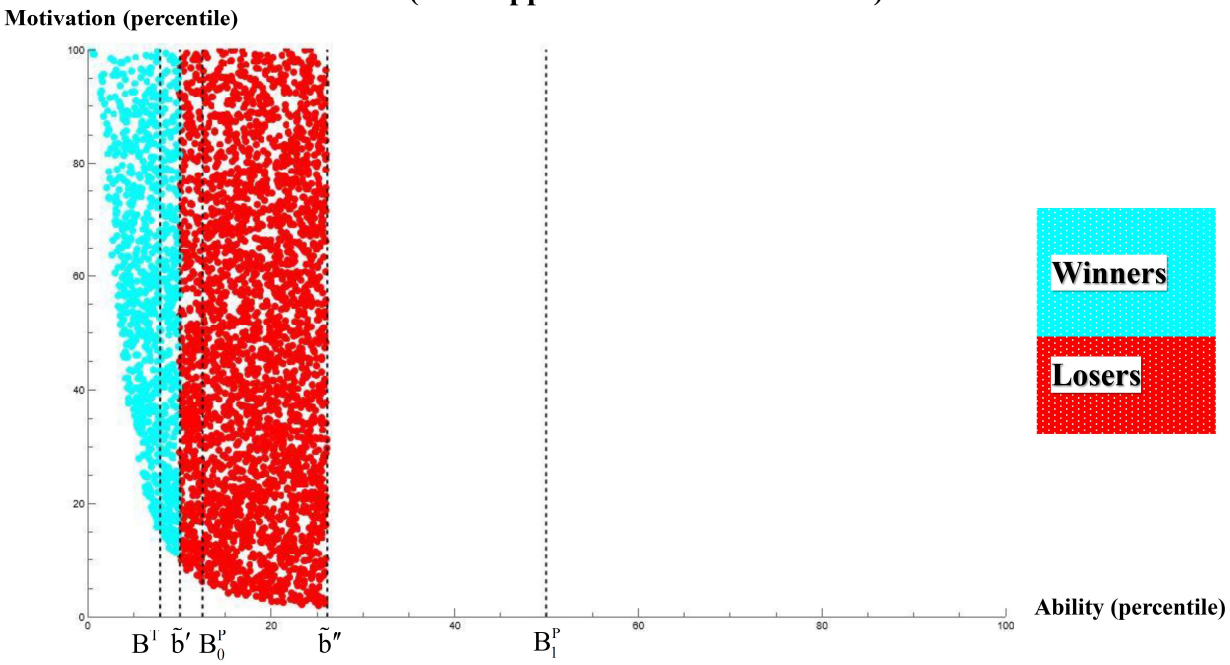


Finally, we have two groups of PS applicants whose achievement does not change (the dark-blue dots in Figure A3): (i) Those that graduate anyway, who have ability greater than $\tilde{b}''$ and are sufficiently motivated to not dropout, but not motivated enough to be effort-unconstrained in the "new" public school. And (ii), those that dropout anyway, most of which are quite low ability (the dark-blue dots with ability below $\tilde{b}''$) with a few exceptions who have ability above $\tilde{b}''$ but are at the bottom of the motivation distribution. The percentages of the total student population of these groups of winners, losers and indifferent students are, respectively, 60.5%, 1.2%, and 18.3%.

TCS Entry (Example): Student Utility Winners and Losers

We compare, again, equilibria in Columns 4 and 6 of Table 2. We present, as before, three graphs, A4 to A6, analogous to A1-A3 but for students who gain, lose and have the same *utility* after entry. Keep in mind that, in the TCS case, achievement effects correspond qualitatively to student-utility effects, except for students who have the same achievement before and after entry. The latter may gain (or lose) in utility because of having to put forth less (or more) effort to meet the standard. Overall, the percent of students that have higher utility is 77.5, the percent with lower utility is 16.36, and the percent with the same utility is 6.18. The detail is provided below, but utility winners consist of most of the matriculates to the charter school and more so those in the public who get higher quality as a result of the increased curriculum. Most of the losers are students that would gain from the charter school but do not matriculate. All those indifferent are students that drop out before and after charter school entry.

Figure A4. TCS Entry. Utility Winners and Losers
(TCS Applicants Who Matriculate)



TCS applicants

Figure A4 identifies utility winners and losers among charter-school matriculates. The public-school curriculum increase worsens the public alternative for all matriculates, but those with sufficiently low ability benefit from the lower charter target. To be precise, matriculates with $b < (>) \tilde{b}'$ experience utility gains (losses), with those losing utility obtaining higher quality in the charter school than in the adjusted public alternative, but lower than in the public-only equilibrium. The percentages of winners are 23.7% (or 2.37% of the total student population) and the losers make up the rest, 76.3% (or 7.63% of the total student population).

Consider now Figure A5, which identifies student utility winners, losers, and unaffected among charter school applicants that do not matriculate and thus remain in the public sector. Note that, as with achievement, there are no utility winners among charter applicants that do not matriculate. On the contrary, most of them experience utility losses because they are left, after entry, with a worse public school curriculum match. The exceptions are the dark-blue dots on the western edge of the figure, a few low ability students who are indifferent because they drop out of the public school with and without the charter school entering. Of these non-matriculates, 98.2% have lower utility (or 9.82% of all students) and the remaining 1.18% (or .118% of all students) have the same utility.

Figure A5. TCS Entry. Utility Winners and Losers
(TCS Applicants Who Do Not Matriculate)

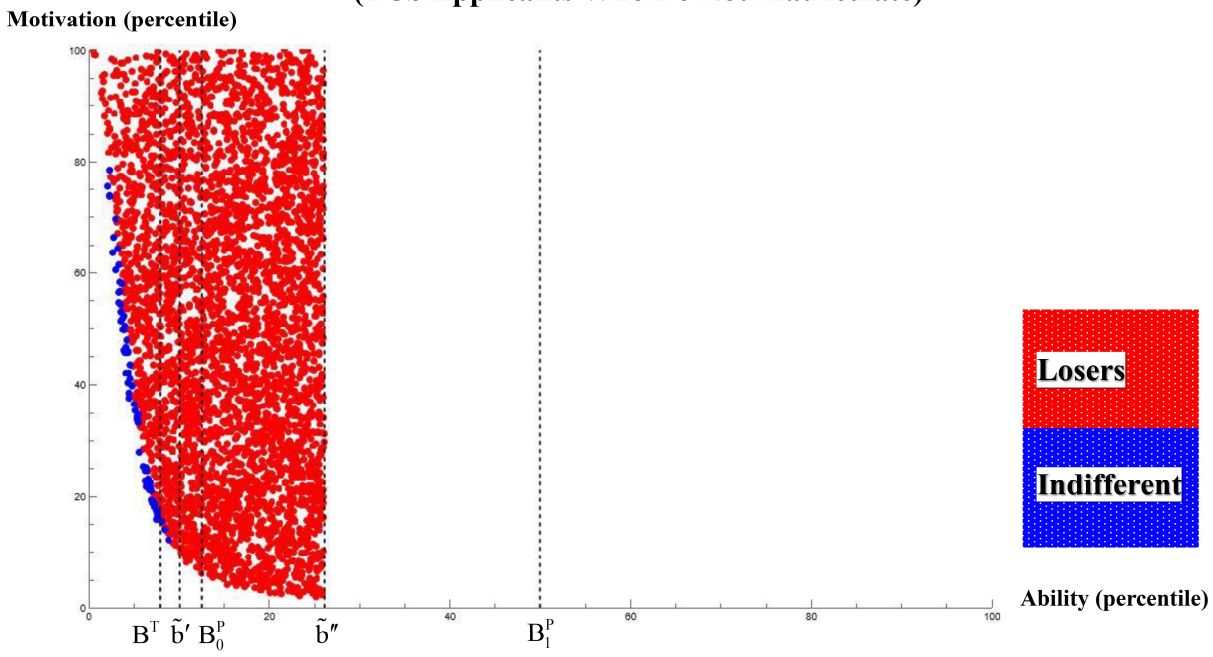
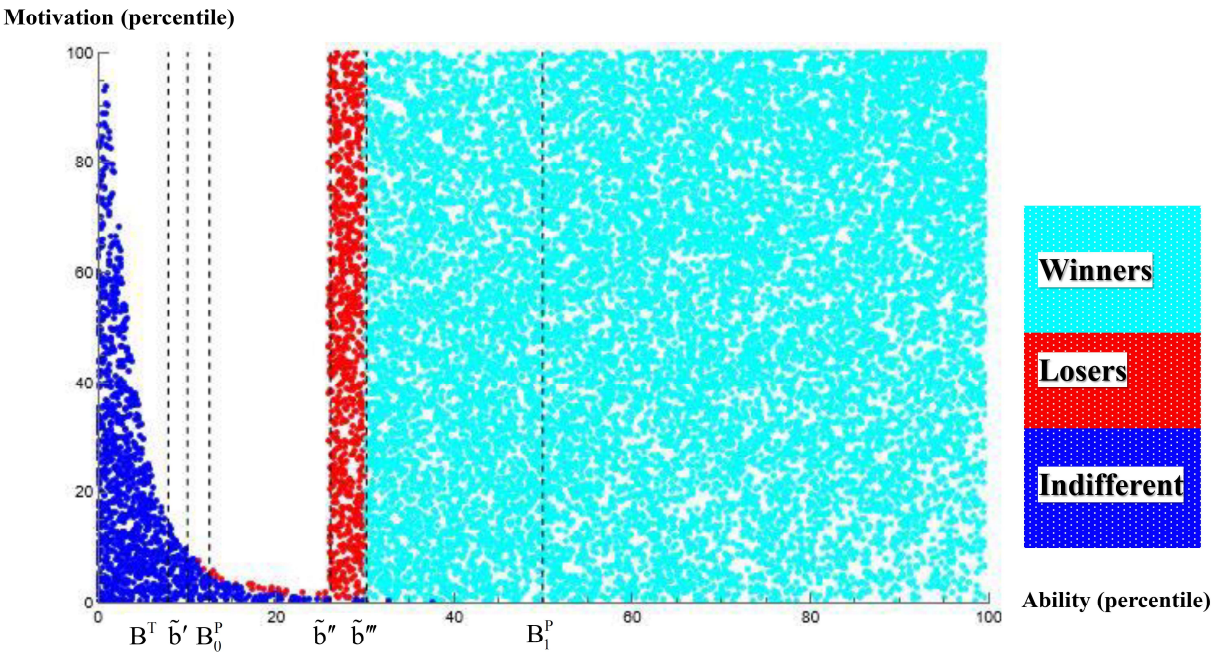


Figure A6. TCS Entry. Utility Winners and Losers (PS Applicants)



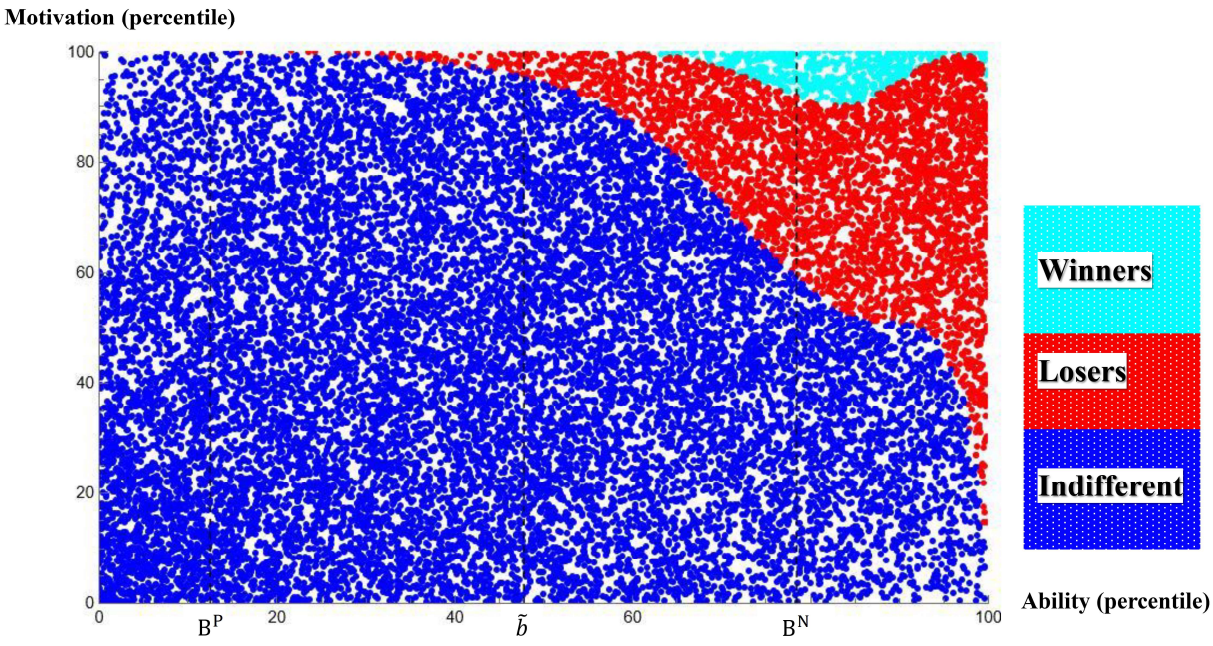
PS applicants

Refer next to Figure A6. Among the students that cannot benefit from the charter school, and so do not apply, the winners (large light-blue right area) are those that do not drop out of the public school in the equilibrium with the charter school and who have sufficient ability to benefit from the higher public school curriculum target: $b > \tilde{b}'''$. The latter are 77.5% of public school applicants, which is 69.8% of all students. Utility losers comprise two groups: (i) Those in the red vertical band of Figure A6, who have ability b, satisfying $\tilde{b}'' < b < \tilde{b}'''$, and experience lower utility because of the worse curriculum match. Most of them have the same achievement following charter entry, i.e., the standard, but must put forth more effort to meet it. (ii) A few more students (the same "line" of red dots with ability between $\tilde{b}'''$ and $\tilde{b}''$ near bottom that we find in figure A3) who would not drop out of the original public school, but do so when the charter enters because of its worse-matched, higher curriculum. The utility losers among public school applicants are 5.2% or 4.2% of all students. Finally, the "indifferent set" are those that drop out in both equilibria, 7.5% of students that do not apply to the charter school (6% of all students).

NEC Entry (Example): Achievement Winners and Losers.

For the NEC example, consider the case with no enforcement costs ($\phi = 0$) and the poverty population, then comparing achievement in Columns 1 and 2 of Table 4. Because the public curriculum does not change, the analysis here is much simpler. We can use Figure 5a in the text to identify achievement effects. All those that attend the NEC gain strictly, and those that do not are unaffected. Gainers are then those that matriculate among the three applicant sets identified in 5a. This is 10% of the population, with then 90% indifferent or unaffected.

Figure A7. NEC Entry. Utility Winners and Losers



NEC Entry (Example): Utility Winners and Losers:

For the same case of the NEC, Figure A7 shows the students that gain (light-blue area), lose (red area), and have the same utility (dark blue area) with the caveat that non-matriculates to the NEC are indifferent. Since the public option is the same, all students that remain in the public school experience no utility change, this making up the dark blue area (and those in the other areas that do not matriculate, i.e., one-half of them). The remaining areas are those that apply to the charter school. Among those that matriculate, 87% percent of them (8.7% of all students) lose in utility, because of having to put in high effort. The remaining charter school applicants that matriculate gain in utility, some of them very highly motivated and unconstrained by the effort minimum but with a better curriculum match. Some of them must increase their effort a bit above what they prefer, but are compensated by the better match. Since 10% ultimately attend the charter school, 90% of students continue in the public school and are unaffected. Overall then, 90% have the same utility, 8.7% have lower utility, and 1.3% have higher utility.