Asset Prices and Unemployment Fluctuations: A Resolution of the Unemployment Volatility Puzzle
Online Appendix

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Contents

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A Omitted Proofs and Details

We begin with proofs and details omitted from the paper.

A.1 Laws of Motion for Employed and Unemployed

To start, let (z_t, τ_t) denote a consumer's human capital z_t and labor market status $\tau_t \in \{e, u\}$, which denotes whether the consumer is employed or unemployed in period t. To develop the law of motion for (z_t, τ_t), note that such a pair (z_t, τ_t) in t can be reached from various pairs (z_{t-1}, τ_{t-1}) in $t-1$ according to the following probabilities

$$(z_t, \tau_t)|\tau_{t-1} = \begin{cases} ((1+g_e)z_{t-1}, e), & \text{with probability } \phi(1-\sigma) \text{ if } \tau_{t-1} = e \\ ((1+g_e)z_{t-1}, u), & \text{with probability } \phi\sigma \text{ if } \tau_{t-1} = e \\ ((1+g_u)z_{t-1}, e), & \text{with probability } \phi\lambda_{wt}(\theta_t(z_t)) \text{ if } \tau_{t-1} = u \\ ((1+g_u)z_{t-1}, u), & \text{with probability } \phi[1-\lambda_{wt}(\theta_t(z_t))] \text{ if } \tau_{t-1} = u \\ (z, u), & \text{with probability } (1-\phi)\nu(z) \text{ for all } \tau_{t-1} \end{cases}.$$

Thus, for example, the first line in the above refers to a consumer with human capital $z_{t-1} = z_t/(1+g_e)$ who was employed in $t-1$ ($\tau_{t-1} = e$), survives to period t, has human capital z_t in t, and is employed ($\tau_t = e$) in t. The next four lines are similar: note that a consumer's human capital grows at rate $1+g_e$ or $1+g_u$ depending on the consumer's employment status in $t-1$. In the last line, we record that a measure $1-\phi$ of consumers enter the labor market unemployed, drawing their human capital z from a continuous distribution with density $\nu(z)$.

More formally, the state variable describing consumers' human capital z_t and employment status $\tau_t \in \{e, u\}$ is a joint density function that we denote by $f_{z_t,\tau_t}(z_t, \tau_t)$. Given this joint density, we let $e_t(z)$ and $u_t(z)$ denote the induced densities over z_t for the employed and the unemployed, namely,

$$e_t(z_t) \equiv f_{z_t,\tau_t}(z_t, e) \text{ and } u_t(z_t) \equiv f_{z_t,\tau_t}(z_t, u).$$

(Observe that the joint density $f_{z_t,\tau_t}(z_t, \tau_t)$ is a probability density function since it integrates to one, that is,

$$\sum\nolimits_\tau \int_z f_{z_t,\tau_t}(z, \tau)dz = \int_z [e_t(z) + u_t(z)]dz = 1,$$

but the densities over $e(z_t)$ and $u_t(z_t)$ do not. Rather, they simply integrate to the total measure of employed and unemployed consumers in period t, namely, $e_t = \int_z e_t(z)dz$ and $u_t = \int_z u_t(z)dz$.)

We now claim that the densities $e_t(z_t)$ and $u_t(z_t)$ evolve over time according to

$$e_t(z_t) = \frac{\phi(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z_t}{1+g_e}\right) + \lambda_{wt}(\theta_t(z_t))\frac{\phi}{1+g_u} u_{t-1}\left(\frac{z_t}{1+g_u}\right) \tag{1}$$

and

$$u_t(z_t) = \frac{\phi\sigma}{1+g_e} e_{t-1}\left(\frac{z_t}{1+g_e}\right) + [1-\lambda_{wt}(\theta_t(z_t))]\frac{\phi}{1+g_u} u_{t-1}\left(\frac{z_t}{1+g_u}\right) + (1-\phi)\nu(z_t). \tag{2}$$

In contrast to the formulae in the text, we have substituted into both (1) and (2) that

$$u_{bt}(z_t) = \frac{\phi}{1+g_u} u_{t-1}\left(\frac{z_t}{1+g_u}\right). \tag{3}$$

To derive the laws of motion in (1) and (2), we begin by noting that

$$f_{z_t,\tau_t}(z_t,\tau_t) = f_{z_t,\tau_t|\tau_{t-1}}(z_t,\tau_t|e) f_{\tau_{t-1}}(e) + f_{z_t,\tau_t|\tau_{t-1}}(z_t,\tau_t|u) f_{\tau_{t-1}}(u), \tag{4}$$

where $e_{t-1} = f_{\tau_{t-1}}(e) = \int_z f_{z_{t-1},\tau_{t-1}}(z,e)dz$ and $u_{t-1} = f_{\tau_{t-1}}(u) = \int_z f_{z_{t-1},\tau_{t-1}}(z,u)dz$. We then simply apply the general formula in (4) fill in the various probabilities from the model.

Consider the law of motion for $e_t(z)$. Applying (4), we have

$$\begin{aligned}
e_t(z_t) &= f_{z_t,\tau_t|\tau_{t-1}}(z_t,e|e) f_{\tau_{t-1}}(e) + f_{z_t,\tau_t|\tau_{t-1}}(z_t,e|u) f_{\tau_{t-1}}(u) \\
&= f_{z_t|\tau_t,\tau_{t-1}}(z_t|e,e) f_{\tau_t|\tau_{t-1}}(e|e) f_{\tau_{t-1}}(e) + f_{z_t|\tau_t,\tau_{t-1}}(z_t|e,u) f_{\tau_t|\tau_{t-1}}(e|u) f_{\tau_{t-1}}(u) \\
&= f_{z_t|\tau_{t-1}}(z_t|e)\phi(1-\sigma) f_{\tau_{t-1}}(e) + f_{z_t|\tau_{t-1}}(z_t|u)\phi\lambda_{wt}(\theta_t(z_t)) f_{\tau_{t-1}}(u) \\
&= \phi f_{z_{t-1}|\tau_{t-1}}\left(z_{t-1} = \frac{z_t}{1+g_e}\Big|\tau_{t-1}=e\right)\left(\frac{\partial z_{t-1}}{\partial z_t}\Big|\tau_{t-1}=e\right)(1-\sigma) f_{\tau_{t-1}}(e) \qquad (5) \\
&\quad + \phi f_{z_{t-1}|\tau_{t-1}}\left(z_{t-1} = \frac{z_t}{1+g_u}\Big|\tau_{t-1}=u\right)\left(\frac{\partial z_{t-1}}{\partial z_t}\Big|\tau_{t-1}=u\right)\lambda_{wt}(\theta_t(z_t)) f_{\tau_{t-1}}(u) \\
&= \phi\frac{f_{z_{t-1},\tau_{t-1}}\left(z_{t-1}=\frac{z_t}{1+g_e},e\right)}{f_{\tau_{t-1}}(e)}\left(\frac{1-\sigma}{1+g_e}\right) f_{\tau_{t-1}}(e) + \phi\frac{f_{z_{t-1},\tau_{t-1}}\left(z_{t-1}=\frac{z_t}{1+g_u},u\right)}{f_{\tau_{t-1}}(u)}\left[\frac{\lambda_{wt}(\theta_t(z_t))}{1+g_u}\right] f_{\tau_{t-1}}(u) \\
&= \frac{\phi(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z_t}{1+g_e}\right) + \frac{\phi\lambda_{wt}(\theta_t(z_t))}{1+g_u} u_{t-1}\left(\frac{z_t}{1+g_u}\right), \qquad (6)
\end{aligned}$$

where (6) is (1). Now, the third equality follows from the fact that z_t is a deterministic function of τ_{t-1} and z_{t-1} and is therefore independent on τ_t conditional on τ_{t-1}. In the fourth inequality, we have performed a change of variable. Recall that if X is a random variable with probability density function $f_X(x)$ with support $[0,\infty)$ and $Y = g(X)$ is a differentiable strictly increasing function of X on this support with inverse $X = h(Y)$, then

$$F_Y(y) = P(Y = g(X) \le y) = P(X \le g^{-1}(y) \equiv h(y) = \int_{x_1}^{h(y)} f_X(x)dx.$$

Differentiating implies $f_Y(y) \equiv F_Y'(y) = f_X(h(y))h'(y)$ is a random variable with support $[0,\infty)$. Here we let $X = z_{t-1}|\tau_{t-1}$ and $Y = g(X) = (1+g_e)X = z_t|\tau_{t-1}$ with $X = h(y) = Y/(1+g_e)$. Hence,

$$f_Y(y) = f_{z_t|\tau_{t-1}}(z_t|\tau_{t-1}) = f_X(h(y))h'(y) = \frac{1}{1+g_e} f_X\left(\frac{y}{1+g_e}\right) = \frac{1}{1+g_e} f_{z_t|\tau_{t-1}}\left(\frac{z_t}{1+g_e}\Big|\tau_{t-1}\right).$$

Intuitively, this change of variables implies that the density of human capital in the interval, say, $[\underline{z}_{t-1}, \bar{z}_{t-1}]$ in $t-1$ is assigned to the wider interval $[(1+g_e)\underline{z}_{t-1}, (1+g_e)\bar{z}_{t-1}]$ in t. Thus, the density over z_{t-1} in $t-1$ must be scaled *down* by $1/(1+g_e)$ when changed to a density over z_t.

Proceeding analogously with the law of motion for $u_t(z_t)$, we have

$$\begin{aligned}
u_t(z_t) &= f_{z_t,\tau_t|\tau_{t-1}}(z_t,u|e)f_{\tau_{t-1}}(e) + f_{z_t,\tau_t|\tau_{t-1}}(z_t,u|u)f_{\tau_{t-1}}(u) + (1-\phi)\nu(z) \\
&= f_{z_t|\tau_t,\tau_{t-1}}(z_t|u,e)f_{\tau_t|\tau_{t-1}}(u|e)f_{\tau_{t-1}}(e) + f_{z_t|\tau_t,\tau_{t-1}}(z_t|u,u)f_{\tau_t|\tau_{t-1}}(u|u)f_{\tau_{t-1}}(u) + (1-\phi)\nu(z) \\
&= f_{z_t|\tau_{t-1}}(z_t|e)\phi\sigma f_{\tau_{t-1}}(e) + f_{z_t|\tau_{t-1}}(z_t|u)\phi[1-\lambda_{wt}(\theta_t(z_t))]f_{\tau_{t-1}}(u) + (1-\phi)\nu(z) \\
&= \phi f_{z_{t-1}|\tau_{t-1}}\left(z_{t-1} = \frac{z_t}{1+g_e}\middle|\tau_{t-1}=e\right)\left(\frac{\partial z_{t-1}}{\partial z_t}\middle|\tau_{t-1}=e\right)\sigma f_{\tau_{t-1}}(e) \\
&\quad +\phi f_{z_{t-1}|\tau_{t-1}}\left(z_{t-1} = \frac{z_t}{1+g_u}\middle|\tau_{t-1}=u\right)\left(\frac{\partial z_{t-1}}{\partial z_t}\middle|\tau_{t-1}=u\right)[1-\lambda_{wt}(\theta_t(z_t))]f_{\tau_{t-1}}(u)+(1-\phi)\nu(z) \\
&= \phi\frac{f_{z_{t-1},\tau_{t-1}}\left(z_{t-1}=\frac{z_t}{1+g_e},e\right)}{f_{\tau_{t-1}}(e)}\frac{\sigma}{1+g_e}f_{\tau_{t-1}}(e) + \phi\frac{f_{z_{t-1},\tau_{t-1}}\left(z_{t-1}=\frac{z_t}{1+g_u},u\right)}{f_{\tau_{t-1}}(u)}\frac{[1-\lambda_{wt}(\theta_t(z_t))]}{1+g_u}f_{\tau_{t-1}}(u) \\
&\quad +(1-\phi)\nu(z) \\
&= \frac{\phi\sigma}{1+g_e}e_{t-1}\left(\frac{z_t}{1+g_e}\right) + \frac{\phi[1-\lambda_{wt}(\theta_t(z_t))]}{1+g_u}u_{t-1}\left(\frac{z_t}{1+g_u}\right) + (1-\phi)\nu(z), \qquad (7)
\end{aligned}$$

where (7) is (2). Hence, we have derived (1) and (2) from the underlying joint distributions over consumers' human capital and employment status.

A.2 Details of the Competitive Search Equilibrium

In this section, we characterize the competitive search equilibria defined in the paper. We also provide conditions for a symmetric competitive search equilibrium to exist.

Let the matching function be given by $m(u_{bt}(z), v_t(z))$ after suppressing the dependence of $m(\cdot,\cdot)$ on t without loss. Define the (negative of the) elasticity of the job-filling rate with respect to market tightness $\theta_t(z)$ as

$$\eta(\theta_t(z)) \equiv -\frac{d\log(\lambda_f(\theta_t(z)))}{d\log(\theta_t(z))} = -\theta_t(z)\frac{\lambda_f'(\theta_t(z))}{\lambda_f(\theta_t(z))}. \qquad (8)$$

Then, as long as $\theta_t(z) > 0$, we have

$$\frac{\lambda_f'(\theta_t(z))}{\lambda_f(\theta_t(z))} = -\frac{\eta(\theta_t(z))}{\theta_t(z)} \text{ and } \frac{\lambda_w'(\theta_t(z))}{\lambda_w(\theta_t(z))} = \frac{1-\eta(\theta_t(z))}{\theta_t(z)}, \qquad (9)$$

where the first equality in (9) follows directly from (8). The second equality follows from (8) and $\lambda_{wt}(\theta_t(z)) = \theta_t(z)\lambda_{ft}(\theta_t(z))$, which imply

$$\frac{\lambda_w'(\theta_t(z))}{\lambda_w(\theta_t(z))} = \frac{\theta_t(z)\lambda_f'(\theta_t(z))}{\theta_t(z)\lambda_f(\theta_t(z))} + \frac{1}{\theta_t(z)} = -\frac{\eta(\theta_t(z))}{\theta_t(z)} + \frac{1}{\theta_t(z)} = \frac{1-\eta(\theta_t(z))}{\theta_t(z)}, \qquad (10)$$

where the first equality in (10) follows from differentiating $\lambda_{wt}(\theta_t(z)) = \theta_t(z)\lambda_{ft}(\theta_t(z))$ and substituting $\lambda_w'(\theta_t(z)) = \theta_t(z)\lambda_f'(\theta_t(z)) + \lambda_f(\theta_t(z))$ and $\lambda_w(\theta_t(z)) = \theta_t(z)\lambda_f(\theta_t(z))$ into $\lambda_w'(\theta_t(z))/\lambda_w(\theta_t(z))$, whereas the second equality in (10) uses (8). Also, the second equality in (9) and $\lambda_{wt}(\theta_t(z)) = \theta_t(z)\lambda_{ft}(\theta_t(z))$ imply

$$\lambda_w'(\theta_t(z)) = [1-\eta(\theta_t(z))]\lambda_{ft}(\theta_t(z)), \qquad (11)$$

which we will use in what follows. It will be convenient for later results to summarize in a compact form the conditions determining a symmetric competitive search equilibrium. In the next proposition, we do so and provide a condition that implies that if all other firms choose $\theta(z)$ and $W_{mt}(z)$ in the common market, then it is optimal for a firm to choose $\tilde{\theta}_t(z) = \theta_t(z)$ and $\tilde{W}_{mt}(z) = \theta_t(z)$ so that a symmetric competitive search equilibrium exists.

Proposition A.1. *In a competitive search equilibrium, the following properties hold:*
a) The optimality condition for a firm's choice of wage offer and market tightness are

$$-\frac{\lambda_f'(\theta_t(z))}{\lambda_f(\theta_t(z))}[Y_t(z) - W_{mt}(z)] = \frac{\lambda_w'(\theta_t(z))}{\lambda_w(\theta_t(z))}[W_{mt}(z) + W_{pt}(z) - U_t(z)] \tag{12}$$

or, equivalently using (9) when $\theta_t(z) > 0$,

$$\eta(\theta_t(z))[Y_t(z) - W_{mt}(z)] = [1 - \eta(\theta_t(z))][W_{mt}(z) + W_{pt}(z) - U_t(z)]. \tag{13}$$

b) The free-entry condition for any active market can be expressed as

$$\kappa A_t z = \lambda_f(\theta_t(z))[Y_t(z) - W_{mt}(z)]. \tag{14}$$

c) The value of output in a match $Y_t(z)$, the value of unemployment $U_t(z)$, and the post-match value $W_{pt}(z)$ satisfy

$$Y_t(z) = A_t z + \phi(1-\sigma)\mathbb{E}_t[Q_{t,t+1}Y_{t+1}((1+g_e)z)], \tag{15}$$

$$\begin{aligned} U_t(z) = {} & bA_t z + \phi\mathbb{E}_t\{Q_{t,t+1}\lambda_w(\theta_{t+1}(z))\,[W_{mt+1}((1+g_u)z) + W_{pt+1}((1+g_u)z)]\} \\ & + \phi\mathbb{E}_t\{Q_{t,t+1}[1 - \lambda_w(\theta_{t+1}(z))]U_{t+1}((1+g_u)z)\}, \end{aligned} \tag{16}$$

$$W_{pt}(z) = \phi\sigma\mathbb{E}_t[Q_{t,t+1}U_{t+1}((1+g_e)z)] + \phi(1-\sigma)\mathbb{E}_t[Q_{t,t+1}W_{pt+1}((1+g_e)z)], \tag{17}$$

where

$$Q_{t,t+1} = \beta\left(\frac{S_{t+1}C_{t+1}}{S_t C_t}\right)^{-\alpha}. \tag{18}$$

d) Properties a) through c) along with the appropriate limiting conditions completely characterize allocations, prices, and values in a competitive search equilibrium.
e) In any active market, it is optimal for a firm to choose $\tilde{\theta}_t(z) = \theta_t(z)$ and $\tilde{W}_{mt}(z) = W_{mt}(z)$, where $\theta_t(z)$ and $W_{mt}(z)$ are the common market tightness and the common market wage offer so that a symmetric competitive search equilibrium exists. Moreover, there are no asymmetric competitive search equilibria.

Proof. We begin by deriving the optimality condition in (12) for a firm's choice of $\tilde{W}_{mt}(z)$ and $\tilde{\theta}_t(z)$, given that all other firms have chosen the wage offer $W_{mt}(z)$ and market tightness $\theta_t(z)$. Consider then a firm's maximization problem, namely, the problem

$$\begin{aligned} \max_{\tilde{W}_{mt}(z),\tilde{\theta}_t(z)} \quad & (-\kappa A_t z + \lambda_f(\tilde{\theta}_t(z))[Y_t(z) - \tilde{W}_{mt}(z)] + [1 - \lambda_f(\tilde{\theta}_t(z))]\mathbb{E}_t\{\max_{z'}[Q_{t,t+1}V_{t+1}(z')]\}) \\ \text{s.t.} \quad \tilde{\mu}_t(z): \quad & \lambda_w(\tilde{\theta}_t(z))[\tilde{W}_{mt}(z) + W_{pt}(z) - U_t(z)] \geq \lambda_w(\theta_t(z))[W_{mt}(z) + W_{pt}(z) - U_t(z)], \end{aligned}$$

where $\tilde{\mu}_t(z)$ is the multiplier on this participation constraint. Since a firm's value of any offer that

is not a serious one is zero, it is without loss of generality to consider a relaxed version of the firm problem without the constraint discussed in the paper that an offer be serious (in the sense defined there). The first-order conditions for this problem are

$$\tilde{W}_{mt}(z) : \lambda_f(\tilde{\theta}_t(z)) = \tilde{\mu}_t(z)\lambda_w(\tilde{\theta}_t(z)) \tag{19}$$

and

$$\tilde{\theta}_t(z) : -\lambda_f'(\tilde{\theta}_t(z))(Y_t(z) - \tilde{W}_{mt}(z) - \mathbb{E}_t\{\max_{z'}[Q_{t,t+1}V_{t+1}(z')]\}) = \tilde{\mu}_t(z)\lambda_w'(\tilde{\theta}_t(z))[\tilde{W}_{mt}(z) + W_{pt}(z) - U_t(z)].$$

Using the free-entry condition that $V_t(z) = 0$ for any t and z as well as (19) to substitute for $\tilde{\mu}_t(z)$ in this last displayed expression gives

$$-\frac{\lambda_f'(\tilde{\theta}_t(z))}{\lambda_f(\tilde{\theta}_t(z))}[Y_t(z) - \tilde{W}_{mt}(z)] = \frac{\lambda_w'(\tilde{\theta}_t(z))}{\lambda_w(\tilde{\theta}_t(z))}[\tilde{W}_{mt}(z) + W_{pt}(z) - U_t(z)]. \tag{20}$$

By assuming that $\tilde{\theta}_t(z) > 0$, multiplying both sides by $\tilde{\theta}_t(z)$, and substituting from (9), we obtain

$$\tilde{\eta}(\tilde{\theta}_t(z))[Y_t(z) - \tilde{W}_{mt}(z)] = [1 - \tilde{\eta}(\tilde{\theta}_t(z))][\tilde{W}_{mt}(z) + W_{pt}(z) - U_t(z)]. \tag{21}$$

In part e), we show that $\tilde{W}_{mt}(z) = W_{mt}(z)$ and $\tilde{\theta}_t(z) = \theta_t(z)$. Given this result, (20) establishes (12) and (21) establishes (13) for any firm, which prove part a).

For parts b) and c), note that we have derived the free-entry condition in the paper and that once we substitute for $z' = (1 + g_e)z$ when consumers are employed and for $z' = (1 + g_u)z$ when consumers are unemployed, the equations for $Y_t(z)$, $U_t(z)$, and $W_{pt}(z)$ here are just those in the paper.

Part d) simply collects the results from parts a) through c).

We now turn to part e). Before proving that a symmetric equilibrium exists, we note two useful properties. First, in any active market, the surplus $Y_t(z) + W_{pt}(z) - U_t(z)$ is positive. To see why, note that the free-entry condition in (14) implies that $Y_t(z) - W_{mt}(z)$ is positive in the common market. Hence, it follows from (12) that $W_{mt}(z) + W_{pt}(z) - U_t(z)$ in the common market is positive and so is the sum of $Y_t(z) - W_{mt}(z)$ and $W_{mt}(z) + W_{pt}(z) - U_t(z)$, namely, the surplus.

Second, let $D(\theta_t(z))$ be defined as $\eta(\theta_t(z))\lambda_w(\theta_t(z))$. It turns out that $D(\theta_t(z))$ equals the derivative of the matching function with respect to unemployment at the beginning of period t, that is,

$$\frac{\partial m(u_{bt}(z), v_t(z))}{\partial u_{bt}(z)} = \eta(\theta_t(z))\lambda_w(\theta_t(z)), \tag{22}$$

and is strictly increasing with $\theta_t(z)$. To establish (22), note that

$$\begin{aligned}\frac{\partial m(u_{bt}(z), v_t(z))}{\partial u_{bt}(z)} &= \frac{\partial\left[v_t(z)\lambda_f(\theta_t(z))\right]}{\partial u_{bt}(z)} = v_t(z)\frac{\partial \lambda_f(\theta_t(z))}{\partial \theta_t(z)}\frac{\partial \theta_t(z)}{\partial u_{bt}(z)} = v_t(z)\lambda_f'(\theta_t(z))\left[-\frac{v_t(z)}{u_{bt}^2(z)}\right] \\ &= -\theta_t(z)^2\lambda_f'(\theta_t(z)) = \left[-\theta_t(z)\frac{\lambda_f'(\theta_t(z))}{\lambda_f(\theta_t(z))}\right]\theta_t(z)\lambda_f(\theta_t(z)) = \eta(\theta_t(z))\lambda_w(\theta_t(z)) = D(\theta_t(z)),\end{aligned}$$

where the sixth equality uses the first equation in (9) and that $\lambda_w(\theta_t(z)) = \theta_t(z)\lambda_f(\theta_t(z))$. That $D(\theta_t(z))$ is strictly increasing with $\theta_t(z)$ follows from the strict concavity of the matching function.

Namely, observe that

$$\frac{\partial^2 m(u_{bt}(z), v_t(z))}{\partial^2 u_{bt}(z)} = \frac{\partial D(\theta_t(z))}{\partial u_{bt}(z)} = \frac{\partial \theta_t(z)}{\partial u_{bt}(z)} \left[\eta'(\theta_t(z))\lambda_w(\theta_t(z)) + \eta(\theta_t(z))\lambda'_w(\theta_t(z))\right] < 0,$$

where the inequality holds by the strict concavity of the matching function. Since $\partial\theta_t(z)/\partial u_{bt}(z) < 0$ in any active market, it then follows $\eta'(\theta_t(z))\lambda_w(\theta_t(z)) + \eta(\theta_t(z))\lambda'_w(\theta_t(z)) > 0$ and so

$$D'(\theta_t(z)) = \eta'(\theta_t(z))\lambda_w(\theta_t(z)) + \eta(\theta_t(z))\lambda'_w(\theta_t(z)) > 0,$$

where the equality follows from the definition of $D(\theta_t(z))$. Thus, $D(\theta_t(z))$ is strictly increasing with $\theta_t(z)$.

We are now ready to show that a symmetric equilibrium exists. To prove that it is optimal for a firm that contemplates deviating from the common market to choose $\tilde{\theta}_t(z) = \theta_t(z)$, we use the condition $Y_t(z) + W_{pt}(z) - U_t(z) \neq 0$ to reduce the firm's optimality condition to

$$D(\theta_t(z)) = D(\tilde{\theta}_t(z)), \tag{23}$$

which has a unique solution $\tilde{\theta}_t(z) = \theta_t(z)$ because, as we have just proved, $D(\cdot)$ is strictly increasing.

To establish (23), we first note that the participation constraint

$$\lambda_w(\tilde{\theta}_t(z))[\tilde{W}_{mt}(z) + W_{pt}(z) - U_t(z)] = \lambda_w(\theta_t(z))[W_{mt}(z) + W_{pt}(z) - U_t(z)] \tag{24}$$

when divided by $\lambda_{wt}(\tilde{\theta}_t(z))$ implies

$$\tilde{W}_{mt}(z) + W_{pt}(z) - U_t(z) = \frac{\lambda_w(\theta_t(z))}{\lambda_w(\tilde{\theta}_t(z))}[W_{mt}(z) + W_{pt}(z) - U_t(z)], \tag{25}$$

which, after rearranging terms and defining $x_t(z) = \lambda_w(\theta_t(z))/\lambda_w(\tilde{\theta}_t(z))$, can be written as

$$\tilde{W}_{mt}(z) = U_t(z) - W_{pt}(z) + x_t(z)[W_{mt}(z) + W_{pt}(z) - U_t(z)] \tag{26}$$

or, equivalently,

$$\tilde{W}_{mt}(z) + W_{pt}(z) - U_t(z) = x_t(z)[W_{mt}(z) + W_{pt}(z) - U_t(z)]. \tag{27}$$

Substituting for $\tilde{W}_{mt}(z)$ on the left side of (21) using (26) and on the right side using (27) gives

$$\begin{aligned}\tilde{\eta}(\tilde{\theta}_t(z))\{Y_t(z) - U_t(z) + W_{pt}(z) - x_t(z)[W_{mt}(z) + W_{pt}(z) - U_t(z)]\} \\ = [1 - \tilde{\eta}(\tilde{\theta}_t(z))]x_t(z)[W_{mt}(z) + W_{pt}(z) - U_t(z)].\end{aligned} \tag{28}$$

We simplify the expression in (28) as follows. From (13) in the common market, namely,

$$\eta(\theta_t(z))[Y_t(z) - W_{mt}(z)] = [1 - \eta(\theta_t(z))][W_{mt}(z) + W_{pt}(z) - U_t(z)], \tag{29}$$

we obtain that

$$W_{mt}(z) + W_{pt}(z) - U_t(z) = \eta(\theta_t(z))[Y_t(z) + W_{pt}(z) - U_t(z)].$$

Using this equation to substitute for $W_{mt}(z) + W_{pt}(z) - U_t(z)$ on both the left and right sides of (28)

yields that

$$\begin{aligned}\tilde{\eta}(\tilde{\theta}_t(z))\{Y_t(z) - U_t(z) + W_{pt}(z) - x_t(z)\eta(\theta_t(z))[Y_t(z) - U_t(z) + W_{pt}(z)]\} \\ = [1 - \tilde{\eta}(\tilde{\theta}_t(z))]x_t(z)\eta(\theta_t(z))[Y_t(z) - U_t(z) + W_{pt}(z)].\end{aligned}$$

Dividing by $Y_t(z) - U_t(z) + W_{pt}(z) \neq 0$ gives

$$\tilde{\eta}(\tilde{\theta}_t(z))[1 - x_t(z)\eta(\theta_t(z))] = [1 - \tilde{\eta}(\tilde{\theta}_t(z))]x_t(z)\eta(\theta_t(z)) \text{ or } x_t(z) = \frac{\tilde{\eta}(\tilde{\theta}_t(z))}{\eta(\theta_t(z))},$$

which when combined with the definition of $x_t(z) = \lambda_w(\theta_t(z))/\lambda_w(\tilde{\theta}_t(z))$ implies that

$$\frac{\tilde{\eta}(\tilde{\theta}_t(z))}{\eta(\theta_t(z))} = \frac{\lambda_w(\theta_t(z))}{\lambda_w(\tilde{\theta}_t(z)} \text{ or } \eta(\theta_t(z))\lambda_w(\theta_t(z)) = \tilde{\eta}(\tilde{\theta}_t(z))\lambda_w(\tilde{\theta}_t(z)).$$

But this result in turn yields that

$$D(\theta_t(z)) = \eta(\theta_t(z))\lambda_w(\theta_t(z)) = \tilde{\eta}(\tilde{\theta}_t(z))\lambda_w(\tilde{\theta}_t(z)) = D(\tilde{\theta}_t(z)), \tag{30}$$

which proves (23). Since $D(\cdot)$ is strictly monotone, we have that $\tilde{\theta}_t(z) = \theta_t(z)$ and thus $\tilde{W}_{mt}(z) = W_{mt}(z)$ by (24).

Next, we show that there are no asymmetric competitive search equilibria. Suppose to the contrary that for some z, there exist (at least) two distinct pairs $(W^A_{mt}(z), \theta^A_t(z))$ and $(W^B_{mt}(z), \theta^B_t(z))$ that solve a firm's problem. We will prove that it must be the case that $W^A_{mt}(z) = W^B_{mt}(z)$ and $\theta^A_t(z) = \theta^B_t(z)$.

To prove this claim, we start by noting that the value of search for a consumer with human capital z is the same across markets $(z, W^A_{mt}(z))$ and $(z, W^B_{mt}(z))$ in that it must be that

$$\lambda_w(\theta^A_t(z))[W^A_{mt}(z) + W_{pt}(z) - U_t(z)] = \mathcal{W}_t(z) = \lambda_w(\theta^B_t(z))[W^B_{mt}(z) + W_{pt}(z) - U_t(z)] \tag{31}$$

for such a consumer to be indifferent between markets $(z, W^A_{mt}(z))$ and $(z, W^B_{mt}(z))$. But if we repeat the above steps starting with (24) and finishing with (30), first for $\mathcal{W}_t(z)$ defined by the first equality in (31) and then for $\mathcal{W}_t(z)$ defined by the second equality in (31), we end up with

$$D(\theta^A_t(z)) = D(\tilde{\theta}_t(z)) \text{ and } D(\theta^B_t(z)) = D(\tilde{\theta}_t(z)).$$

Since $D(\cdot)$ is strictly monotone, we conclude that $\theta^A_t(z) = \theta^B_t(z)$. This result together with (31) further implies that $W^A_{mt}(z) = W^B_{mt}(z)$. Hence, there cannot exist any asymmetric equilibria. □

A.3 Linearity of Competitive Search Equilibrium

In this section we establish that, in a competitive search equilibrium, value functions are linear in z because market production, home production, and the cost of posting job vacancies are linear in z. Thus, market tightness, job-finding rates, and job-filling rates are independent of z. We begin with the statement of Lemma 1 in the paper and then provide its proof.

Lemma 1. *In a competitive search equilibrium, labor market tightness $\theta_t(z)$, the job-finding rate $\lambda_{wt}(\theta_t(z))$, the job-filling rate $\lambda_{ft}(\theta_t(z))$, and the elasticity $\eta_t(\theta_t(z))$ of the job-filling rate with re-*

spect to $\theta_t(z)$ are independent of z, and values are linear in z in that $W_{mt}(z) = W_{mt}z$, $W_{pt}(z) = W_{pt}z$, $U_t(z) = U_t z$, $\mathcal{W}_t(z) = \mathcal{W}_t z$, and $Y_t(z) = Y_t z$.

Proof. By part d) of Proposition A.1, for a given path for the stochastic discount factor, the competitive search equilibrium is characterized by a sequence $\{W_{mt}(z), \theta_t(z), Y_t(z), U_t(z), W_{pt}(z)\}$ satisfying (12), (14), (15), (16), and (17).

We guess and then verify that a solution to this system of equations has labor market tightness independent of z, $\theta_t(z) = \theta_t$, and values linear in z in that $W_{mt}(z) = W_{mt}z$, $Y_t(z) = Y_t z$, $U_t(z) = U_t z$, $W_{pt}(z) = W_{pt}z$, and so $\mathcal{W}_t(z) = \mathcal{W}_t z$. The verification step substitutes this guess in equations (12), (14), (15), (16), and (17) and divides both sides of each equation by z so that we obtain

$$
\begin{aligned}
0 &= \frac{\lambda'_{ft}(\theta_t)}{\lambda_{ft}(\theta_t)}(Y_t - W_{mt}) + \frac{\lambda'_{wt}(\theta_t)}{\lambda_{wt}(\theta_t)}(W_{mt} + W_{pt} - U_t),\\
\kappa A_t &= \lambda_{ft}(\theta_t)(Y_t - W_{mt}),\\
Y_t &= A_t + \phi(1-\sigma)(1+g_e)\mathbb{E}_t(Q_{t,t+1}Y_{t+1}),\\
U_t &= bA_t + \phi(1+g_u)\mathbb{E}_t[Q_{t,t+1}\lambda_{wt}(\theta_t)(W_{mt+1} + W_{pt+1})] + \phi(1+g_u)\mathbb{E}_t\{Q_{t,t+1}[1-\lambda_{wt}(\theta_t)]U_{t+1}\},\\
W_{pt} &= \phi\sigma(1+g_e)\mathbb{E}_t(Q_{t,t+1}U_{t+1}) + \phi(1-\sigma)(1+g_e)\mathbb{E}_t(Q_{t,t+1}W_{pt+1}).
\end{aligned}
$$

This system of equations clearly admits such the posited solution, which is independent of z, thereby verifying the guess. Note that the linearity of flow market production, flow home production, and vacancy costs in z is key to this result. □

A.4 Aggregate Laws of Motion

Here we derive the laws of motion for Z_{et}, Z_{ut}, Z_{vt}, e_t, and u_t. To derive the law of motion for the aggregate human capital of employed consumers $Z_{et} = \int_z z e_t(z)dz$, we multiply the transition law for the density $e_t(z)$ in (1) by z and integrate it with respect to z, using that $Z_{ut} = \int_z z u_t(z)dz$ is the aggregate human capital of unemployed workers. Specifically, using that $\theta_t(z)$ is constant in z by the linearity of the competitive search equilibrium, in that $\theta_t(z) = \theta_t$ for all z, we can express Z_{et} as

$$
\begin{aligned}
Z_{et} &= \phi(1-\sigma)\int_z \frac{z}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right)dz + \phi\lambda_{wt}(\theta_t)\int_z \frac{z}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right)dz\\
&= \phi(1-\sigma)(1+g_e)\int_z \frac{z}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right)d\left(\frac{z}{1+g_e}\right)\\
&\quad + \phi\lambda_{wt}(\theta_t)(1+g_u)\int_z \frac{z}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right)d\left(\frac{z}{1+g_u}\right)\\
&= \phi(1-\sigma)(1+g_e)Z_{et-1} + \phi\lambda_{wt}(\theta_t)(1+g_u)Z_{ut-1}.
\end{aligned}
$$

In the second line, we have changed the variable of integration from $x = z$ to $y = z/(1+g_e)$ in the first integral, and then multiplied the integrand by the Jacobian $dx/dy = 1+g_e$ of this transformation, and have changed the variable of integration from z to $z/(1+g_u)$ in the second integral, and then multiplied the integrand by the Jacobian $dx/dy = 1+g_u$ of this transformation. In the third line, we have used that with the change of integration variable from $z_{t-1} = z_t/(1+g_e)$ to z and from $z_{t-1} = z_t/(1+g_{u,})$

to z in the relevant integrals, we obtain

$$Z_{et-1}=\int_z \frac{z}{1+g_e}e_{t-1}\left(\frac{z}{1+g_e}\right)d\left(\frac{z}{1+g_e}\right) \text{ and } Z_{ut-1}=\int_z \frac{z}{1+g_u}u_{t-1}\left(\frac{z}{1+g_u}\right)d\left(\frac{z}{1+g_u}\right) \tag{32}$$

from $Z_{et-1}=\int_z ze_{t-1}(z)dz$ and $Z_{ut-1}=\int_z zu_{t-1}(z)dz$. Similarly, we can derive the law of motion for the aggregate human capital of unemployed consumers by multiplying the transition law for the density $u_t(z)$ in (2) by z and integrating it with respect to z to obtain

$$\begin{aligned}
Z_{ut} &= \phi\sigma\int_z \frac{z}{1+g_e}e_{t-1}\left(\frac{z}{1+g_e}\right)dz + \phi[1-\lambda_{wt}(\theta_t)]\int_z \frac{z}{1+g_u}u_{t-1}\left(\frac{z}{1+g_u}\right)dz + (1-\phi)\int_z z\nu(z)dz \\
&= \phi\sigma(1+g_e)\int_z \frac{z}{1+g_e}e_{t-1}\left(\frac{z}{1+g_e}\right)d\left(\frac{z}{1+g_e}\right) \\
&\quad + \phi[1-\lambda_{wt}(\theta_t)](1+g_u)\int_z \frac{z}{1+g_u}u_{t-1}\left(\frac{z}{1+g_u}\right)d\left(\frac{z}{1+g_u}\right) + (1-\phi)\int_z z\nu(z)dz \\
&= \phi\sigma(1+g_e)Z_{et-1} + \phi[1-\lambda_{wt}(\theta_t)](1+g_u)Z_{ut-1} + 1 - \phi,
\end{aligned}$$

where, as before, the second equality follows from the change of variable in integration from z to $z/(1+g_e)$ in the first integral and from z to $z/(1+g_u)$ in the second integral. In the third equality, we have used (32) and that the mean human capital of newborns is $\int_z z\nu(z)dz = 1$ by assumption.

We also note that using the definition of market tightness $\theta_t(z) = v_t(z)/u_{bt}(z)$ and its constancy in z, the aggregate efficiency units of vacancies can be written as

$$\begin{aligned}
Z_{vt} &= \int_z zv_t(z)dz = \int_z z\theta_t u_{bt}(z)dz = \theta_t\int_z \frac{z}{1+g_u}\phi u_{t-1}\left(\frac{z}{1+g_u}\right)dz \\
&= \theta_t\phi(1+g_u)\int_z \frac{z}{1+g_u}u_{t-1}\left(\frac{z}{1+g_u}\right)d\left(\frac{z}{1+g_u}\right) = \phi\theta_t(1+g_u)Z_{ut-1},
\end{aligned}$$

where in the third equality, we have used the definition of $u_{bt}(z)$ in (3), in the fourth equality, we have used that $dz = (1+g_u)d(z/(1+g_u))$, and the fifth equality, we have used (32).

Next, to derive the law of motions for aggregate employment $e_t = \int_z e_t(z)dz$ and aggregate unemployment $u_t = \int_z u_t(z)dz$, we integrate the transitions law for $e_t(z)$ and $u_t(z)$, namely, (1) and (2), with respect to z. Proceeding as above, for aggregate employment, it follows that

$$\begin{aligned}
e_t &= \phi(1-\sigma)\int_z \frac{1}{1+g_e}e_{t-1}\left(\frac{z}{1+g_e}\right)dz + \phi\lambda_{wt}(\theta_t)\int_z \frac{1}{1+g_u}u_{t-1}\left(\frac{z}{1+g_u}\right)dz \\
&= \phi(1-\sigma)\int_z e_{t-1}\left(\frac{z}{1+g_e}\right)d\left(\frac{z}{1+g_e}\right) + \phi\lambda_{wt}(\theta_t)\int_z u_{t-1}\left(\frac{z}{1+g_u}\right)d\left(\frac{z}{1+g_u}\right) \\
&= \phi(1-\sigma)e_{t-1} + \phi\lambda_{wt}(\theta_t)u_{t-1},
\end{aligned}$$

where we have used

$$e_{t-1}=\int_z e_{t-1}(z)dz=\int_z e_{t-1}\left(\frac{z}{1+g_e}\right)d\left(\frac{z}{1+g_e}\right) \text{ and } u_{t-1}=\int_z u_{t-1}(z)dz=\int_z u_{t-1}\left(\frac{z}{1+g_u}\right)d\left(\frac{z}{1+g_u}\right). \tag{33}$$

For aggregate unemployment, it follows that

$$\begin{aligned} u_t &= \phi\sigma \int_z \frac{1}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) dz + \phi[1-\lambda_{wt}(\theta_t)] \int_z \frac{1}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right) dz + (1-\phi)\int_z \nu(z)dz \\ &= \phi\sigma \int_z e_{t-1}\left(\frac{z}{1+g_e}\right) d\left(\frac{z}{1+g_e}\right) + \phi[1-\lambda_{wt}(\theta_t)] \int_z u_{t-1}\left(\frac{z}{1+g_u}\right) d\left(\frac{z}{1+g_u}\right) + (1-\phi)\int_z \nu(z)dz \\ &= \phi\sigma e_{t-1} + \phi[1-\lambda_{wt}(\theta_t)]u_{t-1} + 1 - \phi, \end{aligned}$$

where, as before, in the third equality, we have used (33).

We finally note that the aggregate measure of unemployed workers searching for a job at time t is

$$u_{bt} = \int_z u_{bt}(z)dz = \frac{\phi}{1+g_u}\int_z u_{t-1}\left(\frac{z}{1+g_u}\right) dz = \phi \int_z u_{t-1}\left(\frac{z}{1+g_u}\right) d\left(\frac{z}{1+g_u}\right) = \phi u_{t-1}.$$

A.5 Uniqueness, Symmetry, and Efficiency of the Competitive Search Equilibrium

Here we prove Proposition 1 in the paper. For this result, it is convenient to use notation that compactly describes equilibrium allocations and prices. To that end, let (x, p) denote a competitive search equilibrium with allocations $x = \{C_t, e_t(z), u_t(z), v_t(z), \theta_t(z)\}_{t=0}^{\infty}$ and prices (or values) $p = \{Q_{t,t+1}, W_{mt}(z), W_{pt}(z), U_t(z), Y_t(z)\}_{t=0}^{\infty}$—note that $\mathcal{W}_t(z)$ is completely determined by these prices and $V_t(z) = 0$ for each t and z by the free-entry condition. Likewise, let (x, μ) denote a solution to the planning problem with allocations x and associated multipliers $\mu = \{\mu_{et}(z), \mu_{ut}(z)\}_{t=0}^{\infty}$.

Proposition 1. *The competitive search equilibrium is unique, symmetric, and efficient.*

We have established the symmetry of the competitive search equilibrium in Proposition A.1. To complete the proof of Proposition 1, we proceed in four steps by proving four lemmas that together establish the uniqueness and efficiency of the competitive search equilibrium. In Lemma A.1, we characterize a solution to the planning problem. In Lemma A.2, we show that the allocations of a symmetric competitive search equilibrium coincide with those of the planning problem. In Lemma A.3, we show that there exists a unique set of allocations that solve the planning problem. Combining Lemmas A.2 and A.3 implies that there exists a unique set of allocations that are part of a symmetric competitive search equilibrium and they are efficient. In Lemma A.4, we show that given any symmetric competitive search equilibrium allocations, there exists a unique set of prices that decentralize them. Hence, the symmetric competitive search equilibrium is unique in terms of allocations and prices, and is efficient in that its allocations solve the planning problem. We begin with the first lemma.

A.5.1 First Step

Lemma A.1. *A solution to the planning problem is characterized by the following conditions:*

$$\kappa A_t z = \left[\lambda_{ft}(z) + \theta_t(z)\lambda'_{ft}(\theta_t(z))\right][\mu_{et}(z) - \mu_{ut}(z)]z = [1-\eta_t(z)]\lambda_{ft}(z)[\mu_{et}(z) - \mu_{ut}(z)]z, \tag{34}$$

$$\mu_{et}(z) = A_t z + \phi\mathbb{E}_t\{Q_{t,t+1}[(1-\sigma)\mu_{et+1}(z'_e) + \sigma\mu_{ut+1}(z'_e)]\}, \tag{35}$$

$$\mu_{ut}(z) = bA_t z + \phi\mathbb{E}_t(Q_{t,t+1}\{[1-\eta_{t+1}(z'_u)\lambda_{wt+1}(z'_u)]\mu_{ut+1}(z'_u) + \eta_{t+1}(z'_u)\lambda_{wt+1}(z'_u)\mu_{et+1}(z'_u)\}), \tag{36}$$

where we used the shorthand notation $\eta_t(z) = \eta_t(\theta_t(z))$ *for any* t, $z'_u = (1+g_u)z$, *and* $z'_e = (1+g_e)z$. *Equations (34) to (36) are three equations that determine* $\theta_t(z)$, $\mu_{et}(z)$, *and* $\mu_{ut}(z)$.

Proof. Recall from the paper that the transition laws for employment and unemployment for consumers with human capital z are

$$e_t(z) = \frac{\phi(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + \lambda_{wt}(\theta_t(z))\frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right), \tag{37}$$

$$u_t(z) = \frac{\phi\sigma}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + [1-\lambda_{wt}(\theta_t(z))]\frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right) + (1-\phi)\nu(z), \tag{38}$$

where $\lambda_{wt}(\theta_t(z)) = m_t(u_{bt}(z), v_t(z))/u_{bt}(z)$ is the job-finding rate of an unemployed consumer with human capital z and $\theta_t(z) = v_t(z)/u_{bt}(z)$ is the tightness of the labor market for consumers with human capital z. To reduce the controls to $\{C_t, e_t(z), u_t(z), v_t(z)\}$, we find it convenient to express the transition laws for $e_t(z)$ and $u_t(z)$ using that

$$m_t\left[\frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right), v_t(z)\right] = \lambda_{wt}(\theta_t(z))\frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right),$$

which follows since $\lambda_{wt}(\theta_t(z)) = m_t(u_{bt}(z), v_t(z))/u_{bt}(z)$.

Consider now the planning problem. Here $C_t - X_t = \tilde{S}_t C_t = S_t^{\frac{\alpha}{\alpha-1}} C_t$, where S_t is an exogenous stochastic process so $(C_t - X_t)^{1-\alpha} = S_t^{-\alpha} C_t^{1-\alpha}$. Hence, given the exogenous processes for aggregate productivity A_t and the surplus consumption ratio S_t, the planning problem can be expressed as the problem of choosing $\{C_t, e_t(z), u_t(z), v_t(z)\}$ to solve

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{S_t^{-\alpha} C_t^{1-\alpha}}{1-\alpha}, \tag{39}$$

subject to the following constraints for all periods t, levels of human capital z, and histories of shocks (for which we suppressed the notation) with multipliers ς_t, $\varsigma_t \mu_{et}(z)$, and $\varsigma_t \mu_{ut}(z)$, namely,

$$\varsigma_t : C_t = A_t \int_z z e_t(z) dz + b A_t \int_z z u_t(z) dz - \kappa A_t \int_z z v_t(z) dz, \tag{40}$$

$$\varsigma_t \mu_{et}(z) : e_t(z) = \frac{\phi(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + m_t\left[\frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right), v_t(z)\right], \tag{41}$$

$$\varsigma_t \mu_{ut}(z) : u_t(z) = \frac{\phi\sigma}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + \frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right) - m_t\left[\frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right), v_t(z)\right] + (1-\phi)\nu(z), \tag{42}$$

and the initial condition $\int_z [e_{-1}(z) + u_{-1}(z)]dz = 1$. The Lagrangian for this problem is

$$\mathbb{E}_0 \Bigg\{ \sum_{t=0}^{\infty} \beta^t \frac{S_t^{-\alpha} C_t^{1-\alpha}}{1-\alpha} + \varsigma_t \left[A_t \int_z z e_t(z) dz + b A_t \int_z z u_t(z) dz - \kappa A_t \int_z z v_t(z) dz - C_t \right]$$
$$+ \varsigma_t \int_z \mu_{et}(z) \left[\frac{\phi(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + m_t(u_{bt}(z), v_t(z)) - e_t(z) \right] dz$$
$$+ \varsigma_t \int_z \mu_{ut}(z) \left[\frac{\phi\sigma}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + \frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right) - m_t(u_{bt}(z), v_t(z)) + (1-\phi)\nu(z) - u_t(z) \right] dz \Bigg\}.$$

Next ,we carefully go through the first-order conditions with respect to $e_t(z)$ and $u_t(z)$. We do so in steps I) through III).

I: Derivative with Respect to Employment. When we take the derivative with respect to $e_t(z)$, we need to trace through where this variable enters the Lagrangian problem in t and $t+1$. Consider the constraints for employment of this problem in t,

$$\varsigma_t \int_z \mu_{et}(z) e_t(z) dz = \varsigma_t \int_z \mu_{et}(z) \left\{ \frac{\phi(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + m_t \left[\frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right), v_t(z) \right] \right\} dz \quad (43)$$

and in $t+1$

$$\varsigma_t \int_z \mu_{et+1}(z) e_{t+1}(z) dz = \varsigma_t \int_z \mu_{et+1}(z) \left\{ \frac{\phi(1-\sigma)}{1+g_e} e_t\left(\frac{z}{1+g_e}\right) + m_{t+1} \left[\frac{\phi}{1+g_u} u_t\left(\frac{z}{1+g_u}\right), v_{t+1}(z) \right] \right\} dz. \quad (44)$$

Notice that the density $e_t(\cdot)$ enters the left side of (43) in the form of $e_t(z)$ and the right side of (44) in the form of $e_t(z/(1+g_e))$. It is then useful to perform a change of variable in (44) so that only $e_t(z)$ appears. Specifically, we divide and multiply (44) by $(1+g_e)$,

$$\varsigma_{t+1} \int_z \mu_{et+1}(z) e_{t+1}(z)(1+g_e) d\left(\frac{z}{1+g_e}\right)$$
$$= \varsigma_{t+1} \int_z \mu_{et+1}(z) \left\{ \frac{\phi(1-\sigma)}{1+g_e} e_t\left(\frac{z}{1+g_e}\right) + m_{t+1} \left[\frac{\phi}{1+g_u} u_t\left(\frac{z}{1+g_u}\right), v_{t+1}(z) \right] \right\} (1+g_e) d\left(\frac{z}{1+g_e}\right)$$

and then rewrite the integrals in this constraint by changing the integration variable from $x = z/(1+g_e)$ to $y = z$ so that

$$\varsigma_{t+1} \int_z \mu_{et+1}((1+g_e)z) e_{t+1}((1+g_e)z)(1+g_e) dz$$
$$= \varsigma_{t+1} \int_z \mu_{et+1}((1+g_e)z) \left\{ \phi(1-\sigma) e_t(z) + (1+g_e) m_{t+1} \left[\frac{\phi}{1+g_u} u_t\left(\frac{(1+g_e)z}{1+g_u}\right), v_{t+1}((1+g_e)z) \right] \right\} dz. \quad (45)$$

Likewise, consider the constraints for unemployment in $t+1$,

$$\begin{aligned}&\varsigma_{t+1}\int_z \mu_{ut+1}(z)u_{t+1}(z)dz\\&=\varsigma_{t+1}\int_z \mu_{ut+1}(z)\left\{\frac{\phi\sigma}{1+g_e}e_t\left(\frac{z}{1+g_e}\right)+\frac{\phi}{1+g_u}u_t\left(\frac{z}{1+g_u}\right)-m_{t+1}\left[\frac{\phi}{1+g_u}u_t\left(\frac{z}{1+g_u}\right),v_{t+1}(z)\right]\right\}dz,\end{aligned}$$

where we have dropped the original term $(1-\phi)\nu(z)$ from the right side since it does not vary with the choice of $e_t(z)$. By changing again the integration variable from $x=z/(1+g_e)$ to $y=z$, we can rewrite this constraint as

$$\begin{aligned}&\varsigma_{t+1}\int_z \mu_{ut+1}((1+g_e)z)u_{t+1}((1+g_e)z)(1+g_e)dz\\&\quad=\varsigma_{t+1}\int_z \mu_{ut+1}((1+g_e)z)\left\{\phi\sigma e_t(z)+\frac{\phi(1+g_e)}{1+g_u}u_t\left(\frac{(1+g_e)z}{1+g_u}\right)\right.\\&\quad\left.-(1+g_e)m_{t+1}\left[\frac{\phi}{1+g_u}u_t\left(\frac{(1+g_e)z}{1+g_u}\right),v_{t+1}((1+g_e)z)\right]\right\}dz,\end{aligned}\tag{46}$$

where we have also dropped the original term $(1-\phi)\nu(z)$ from the right side.

When taking the derivative of the Lagrangian with respect to $e_t(z)$, we note that $e_t(z)$ appears in it only through the constraints (43), (45), and (46). Namely, the relevant terms of the Lagrangian that depend on $e_t(z)$ are the following:

$$\begin{aligned}&\varsigma_t A_t\int_z ze_t(z)dz-\varsigma_t\int_z \mu_{et}(z)e_t(z)dz\\&+\mathbb{E}_t\left[\varsigma_{t+1}\int_z \mu_{et+1}((1+g_e)z)\phi(1-\sigma)e_t(z)dz\right]+\mathbb{E}_t\left[\varsigma_{t+1}\int_z \mu_{ut+1}((1+g_e)z)\phi\sigma e_t(z)dz\right].\end{aligned}$$

The derivative of these terms with respect to $e_t(z)$ is then

$$\varsigma_t A_t z-\varsigma_t\mu_{et}(z)+\mathbb{E}_t[\varsigma_{t+1}\mu_{et+1}((1+g_e)z)\phi(1-\sigma)]+\mathbb{E}_t[\varsigma_{t+1}\mu_{ut+1}((1+g_e)z)\phi\sigma].\tag{47}$$

II: Derivative with Respect to Unemployment. When we take the derivative with respect to $u_t(z)$, we need to trace through where this variable enters in t and $t+1$. We drop the terms $(1-\phi)\nu(z)$ that do not vary with $u_t(z)$. Consider the constraints that enter the Lagrangian for unemployment in t,

$$\begin{aligned}&\varsigma_t\int_z \mu_{ut}(z)u_t(z)dz\\&=\varsigma_t\int_z \mu_{ut}(z)\left\{\frac{\phi\sigma}{1+g_e}e_{t-1}\left(\frac{z}{1+g_e}\right)+\frac{\phi}{1+g_u}u_{t-1}\left(\frac{z}{1+g_u}\right)-m_t\left[\frac{\phi}{1+g_u}u_{t-1}\left(\frac{z}{1+g_u}\right),v_t(z)\right]\right\}dz\end{aligned}\tag{48}$$

and in $t+1$

$$\varsigma_{t+1}\int_z \mu_{ut+1}(z)u_{t+1}(z)dz$$
$$=\varsigma_{t+1}\int \mu_{ut+1}(z)\left\{\frac{\phi\sigma}{1+g_e}e_t\left(\frac{z}{1+g_e}\right)+\frac{\phi}{1+g_u}u_t\left(\frac{z}{1+g_u}\right)-m_{t+1}\left[\frac{\phi}{1+g_u}u_t\left(\frac{z}{1+g_u}\right),v_{t+1}(z)\right]\right\}dz.$$

Proceeding as in the above, divide and multiply this constraint by $(1+g_u)$ to then rewrite the integrals in the constraint by a change of integration variable from $x=z/(1+g_u)$ to $y=z$ as

$$\begin{aligned}\varsigma_{t+1}\int_z&\mu_{ut+1}((1+g_u)z)u_{t+1}((1+g_u)z)(1+g_u)dz\\&=\varsigma_{t+1}\int_z\mu_{ut+1}((1+g_u)z)\left\{\frac{\phi\sigma(1+g_u)}{1+g_e}e_t\left(\frac{1+g_u}{1+g_e}z\right)+\phi u_t(z)\right.\\&\left.-(1+g_u)m_{t+1}\left[\frac{\phi}{1+g_u}u_t(z),v_{t+1}((1+g_u)z)\right]\right\}dz.\end{aligned}\tag{49}$$

Likewise, consider the constraints for employment in $t+1$ that enter the Lagrangian,

$$\begin{aligned}&\varsigma_{t+1}\int_z\mu_{et+1}(z)e_{t+1}(z)dz\\&=\varsigma_{t+1}\int_z\mu_{et+1}(z)\left\{\frac{\phi(1-\sigma)}{1+g_e}e_t\left(\frac{z}{1+g_e}\right)+m_{t+1}\left[\frac{\phi}{1+g_u}u_t\left(\frac{z}{1+g_u}\right),v_{t+1}(z)\right]\right\}dz\end{aligned}$$

and after a change of variable, rewrite the integrals as

$$\begin{aligned}&\varsigma_{t+1}\int_z\mu_{et+1}((1+g_u)z)e_{t+1}((1+g_u)z)(1+g_u)dz\\&=\varsigma_{t+1}\int_z\mu_{et+1}((1+g_u)z)\left\{\phi\left(1-\sigma\right)e_t\left(\frac{(1+g_u)z}{1+g_e}\right)+(1+g_u)m_{t+1}\left[\frac{\phi}{1+g_u}u_t\left(z\right),v_{t+1}((1+g_u)z)\right]\right\}dz.\end{aligned}\tag{50}$$

When taking the derivative with respect to $u_t(z)$, we note that $u_t(z)$ appears in the Lagrangian only through the constraints (48), (49), and (50). Namely, the relevant terms of the Lagrangian that depend on $u_t(z)$ are the following:

$$\begin{aligned}&\varsigma_t bA_t\int_z zu_t(z)dz-\varsigma_t\int_z\mu_{ut}(z)u_t(z)dz\\&+\mathbb{E}_t\left\{\varsigma_{t+1}\int_z\mu_{et+1}((1+g_u)z)(1+g_u)m_{t+1}\left[\frac{\phi}{1+g_u}u_t\left(z\right),v_{t+1}((1+g_u)z)\right]dz\right\}\\&+\mathbb{E}_t\left(\varsigma_{t+1}\int_z\mu_{ut+1}((1+g_u)z)\left\{\phi u_t\left(z\right)-(1+g_u)m_{t+1}\left[\frac{\phi}{1+g_u}u_t\left(z\right),v_{t+1}((1+g_u)z)\right]\right\}dz\right).\end{aligned}$$

The derivative of these terms with respect to $u_t(z)$ is then

$$\varsigma_t b A_t z - \varsigma_t \mu_{ut}(z) + \phi \mathbb{E}_t \left\{ \varsigma_{t+1} \frac{\partial m_{t+1}\left[u_{bt+1}((1+g_u)z), v_{t+1}((1+g_u)z)\right]}{\partial u_{bt+1}((1+g_u)z)} \right\} \tag{51}$$
$$\cdot \left[\mu_{et+1}((1+g_u)z) - \mu_{ut+1}((1+g_u)z)\right] + \phi \mathbb{E}_t \left[\varsigma_{t+1}\mu_{ut+1}((1+g_u)z)\right],$$

where $u_{bt+1}((1+g_u)z) = \phi u_t(z)/(1+g_u)$.

III: First-Order Conditions. With the (47) and (51) in hand, it is immediate to conclude that the optimality conditions of the planning problem are

$$C_t : \varsigma_t = \beta^t \left(S_t C_t\right)^{-\alpha}, \tag{52}$$
$$v_t(z) : \kappa A_t \varsigma_t z = \frac{\partial m_t\left[u_{bt}(z), v_t(z)\right]}{\partial v_t(z)} [\mu_{et}(z) - \mu_{ut}(z)] \varsigma_t z, \tag{53}$$
$$e_t(z) : \varsigma_t \mu_{et}(z) = \varsigma_t A_t z + \phi(1-\sigma)\mathbb{E}_t \varsigma_{t+1}\mu_{et+1}[(1+g_e)z] + \phi\sigma \mathbb{E}_t \varsigma_{t+1}\mu_{ut+1}[(1+g_e)z], \tag{54}$$
$$u_t(z) : \varsigma_t \mu_{ut}(z) = \varsigma_t b A_t z + \phi \mathbb{E}_t \varsigma_{t+1}\mu_{ut+1}[(1+g_u)z], \tag{55}$$
$$+ \phi \mathbb{E}_t \varsigma_{t+1} \frac{\partial m_{t+1}\left[u_{bt+1}((1+g_u)z), v_{t+1}((1+g_u)z)\right]}{\partial u_{bt+1}((1+g_u)z)} \{\mu_{et+1}[(1+g_u)z] - \mu_{ut+1}[(1+g_u)z]\},$$

where (54) and (55) simply set the derivatives (47) and (51) equal to zero.

We now use $\lambda_{ft}(\theta_t(z)) = m_t(u_{bt}(z), v_t(z))/v_t(z)$ with $\theta_t(z) = v_t(z)/u_{bt}(z)$ to rewrite

$$\frac{\partial m_t(u_{bt}(z), v_t(z))}{\partial v_t(z)} = \frac{\partial\{v_t(z)\lambda_{ft}(\theta_t(z))}{\partial v_t(z)} = \lambda_{ft}(\theta_t(z)) + v_t(z)\lambda'_{ft}(\theta_t(z))\frac{\partial \theta_t(z)}{\partial v_t(z)} = \lambda_{ft}(\theta_t(z)) + \theta_t(z)\lambda'_{ft}(\theta_t(z))$$

and

$$\begin{aligned}\frac{\partial m_t(u_{bt}(z), v_t(z))}{\partial u_{bt}(z)} &= \frac{\partial\{v_t(z)\lambda_{ft}(\theta_t(z))\}}{\partial u_{bt}(z)} = v_t(z)\lambda'_{ft}(\theta_t(z))\frac{\partial \theta_t(z)}{\partial u_{bt}(z)} \\ &= -v_t(z)\lambda'_{ft}(\theta_t(z))\frac{v_t(z)}{u_{bt}(z)^2} = -\theta_t^2(z)\lambda'_{ft}(\theta_t(z)).\end{aligned}$$

Therefore, using the definitions of the stochastic discount factor, $Q_{t,t+1} = \varsigma_{t+1}/\varsigma_t$, of the (negative of the) elasticity of the job-filling rate with respect to $\theta_t(z)$, $\eta_t(\theta_t(z)) = -\theta_t(z)\lambda'_{ft}(\theta_t(z))/\lambda_{ft}(\theta_t(z))$, and of the job-finding rate, $\lambda_{wt}(\theta_t(z)) = \theta_t(z)\lambda_{ft}(\theta_t(z))$, we can rewrite (52) to (55) as (34)-(36), that is,

$$\begin{aligned}\kappa A_t z &= \left[\lambda_{ft}(z) + \theta_t(z)\lambda'_{ft}(\theta_t(z))\right][\mu_{et}(z) - \mu_{ut}(z)]z = [1-\eta_t(z)]\lambda_{ft}(z)[\mu_{et}(z) - \mu_{ut}(z)]z, \\ \mu_{et}(z) &= A_t z + \phi \mathbb{E}_t\{Q_{t,t+1}[(1-\sigma)\mu_{et+1}(z'_e) + \sigma\mu_{ut+1}(z'_e)]\}, \\ \mu_{ut}(z) &= bA_t z + \phi \mathbb{E}_t(Q_{t,t+1}\{\left[1 + \theta^2_{t+1}(z'_u)\lambda'_{ft+1}(\theta_{t+1}(z'_u))\right]\mu_{ut+1}(z'_u) - \theta^2_{t+1}(z'_u)\lambda'_{ft+1}(\theta_{t+1}(z'_u))\mu_{et+1}(z'_u)\}) \\ &= bA_t z + \phi \mathbb{E}_t(Q_{t,t+1}\{[1-\eta_{t+1}(z'_u)\lambda_{wt+1}(z'_u)]\mu_{ut+1}(z'_u) + \eta_{t+1}(z'_u)\lambda_{wt+1}(z'_u)\mu_{et+1}(z'_u)\}),\end{aligned}$$

where we used the shorthand notation $\eta_t(z) = \eta_t(\theta_t(z))$ for any t, $z'_u = (1+g_u)z$, and $z'_e = (1+g_e)z$. These are the three equations that determine $\theta_t(z)$, $\mu_{et}(z)$ and $\mu_{ut}(z)$. □

A.5.2 Second Step

Consider now the second lemma.

Lemma A.2. *If (x, p) is a symmetric competitive search equilibrium, then x solves the planning problem. Conversely, if x solves the planning problem, then there exist prices p such that (x, p) is a symmetric competitive search equilibrium. Hence, the set of symmetric competitive search equilibrium allocations coincides with the set of allocations that solve the planning problem.*

Proof. We claim that if we replace $Y_t(z) + W_{pt}(z)$ with $\mu_{et}(z)$ and $U_t(z)$ with $\mu_{ut}(z)$, where $Y_t(z)$, $W_{pt}(z)$ and $U_t(z)$ were defined in Proposition 1, then the equations characterizing a competitive search equilibrium stated in Proposition A.1, namely,

$$\kappa A_t z = \lambda_{ft}(z)[Y_t(z) - W_{mt}(z)], \tag{56}$$
$$\eta_t(z)[Y_t(z) - W_{mt}(z)] = [1 - \eta_t(z)][W_{mt}(z) + W_{pt}(z) - U_t(z)], \tag{57}$$
$$Y_t(z) = A_t z + \phi(1-\sigma)\mathbb{E}_t[Q_{t,t+1}Y_{t+1}(z'_e)], \tag{58}$$
$$U_t(z) = bA_t z + \phi\mathbb{E}_t(Q_{t,t+1}\{\lambda_{wt+1}(z'_u)[W_{mt+1}(z'_u) + W_{pt+1}(z'_u)] + [1 - \lambda_{wt+1}(z'_u)]U_{t+1}(z'_u)\}), \tag{59}$$
$$W_{pt}(z) = \phi\mathbb{E}_t\{Q_{t,t+1}[(1-\sigma)W_{pt+1}(z'_e) + \sigma U_{t+1}(z'_e)]\}, \tag{60}$$

reduce to equations (34) to (36) so the allocations of a competitive search equilibrium solve the planning problem. We conclude that the solution to the planning problem and the allocations of the competitive search equilibrium coincide.

We first derive (36). To establish this claim, add $[1 - \eta_t(z)][Y_t(z) - W_{mt}(z)]$ to both sides of (57) to obtain

$$Y_t(z) - W_{mt}(z) = [1 - \eta_t(z)][Y_t(z) + W_{pt}(z) - U_t(z)]. \tag{61}$$

Using (61) to substitute for $Y_t - W_{mt}$ in (57) further gives

$$\eta_t(z)[1 - \eta_t(z)][Y_t(z) + W_{pt}(z) - U_t(z)] = [1 - \eta_t(z)][W_{mt}(z) + W_{pt}(z) - U_t(z)]$$

and dividing both sides of this expression by $1 - \eta_t(z)$ yields

$$\eta_t(z)[Y_t(z) + W_{pt}(z) - U_t(z)] = W_{mt}(z) + W_{pt}(z) - U_t(z).$$

Adding $U_t(z)$ to both sides then gives

$$W_{mt}(z) + W_{pt}(z) = \eta_t(z)[Y_t(z) + W_{pt}(z)] + [1 - \eta_t(z)]U_t(z). \tag{62}$$

Now, use this expression for $W_{mt}(z) + W_{pt}(z)$ to substitute for $W_{mt+1}(z'_u) + W_{pt+1}(z'_u)$ in (59) to get

$$\begin{aligned} U_t(z) =& bA_t z + \phi\mathbb{E}_t[Q_{t,t+1}(\lambda_{wt+1}(z'_u)\{\eta_{t+1}(z'_u)[Y_{t+1}(z'_u) + W_{pt+1}(z'_u)] + [1 - \eta_{t+1}(z'_u)]U_{t+1}(z'_u)\} \\ &+ [1 - \lambda_{wt+1}(z'_u)]U_{t+1}(z'_u))] \\ =& bA_t z + \phi\mathbb{E}_t(Q_{t,t+1}\{\lambda_{wt+1}(z'_u)\eta_{t+1}(z'_u)[Y_{t+1}(z'_u) + W_{pt+1}(z'_u)] + [1 - \lambda_{wt+1}(z'_u)\eta_{t+1}(z'_u)]U_{t+1}(z'_u)\}). \end{aligned}$$

Therefore, using the conjecture that $\mu_{et+1}(z) = Y_{t+1}(z) + W_{pt+1}(z)$ and $\mu_{ut+1}(z) = U_{t+1}(z)$ in this last

equation gives (36), namely,

$$\mu_{ut}(z) = bA_t z + \phi \mathbb{E}_t(Q_{t,t+1}\{\lambda_{wt+1}(z'_u)\eta_{t+1}(z'_u)\mu_{et+1}(z'_u) + [1 - \lambda_{wt+1}(z'_u)\eta_{t+1}(z'_u)]\mu_{ut+1}(z'_u)\}).$$

Next, we establish (35). Proceeding similarly, note that by summing $Y_t(z)$ and $W_{pt}(z)$ from (58) and (60), we have

$$Y_t(z) + W_{pt}(z) = A_t z + \phi \mathbb{E}_t(Q_{t,t+1}\{(1-\sigma)[Y_{t+1}(z'_e) + W_{pt+1}(z'_e)] + \sigma U_{t+1}(z'_e)\}) \tag{63}$$

and so using the conjecture that $\mu_{et+1}(z) = Y_{t+1}(z) + W_{pt+1}(z)$ and $\mu_{ut+1}(z) = U_{t+1}(z)$, we obtain (35), that is,

$$\mu_{et}(z) = A_t z + \phi \mathbb{E}_t(Q_{t,t+1}[(1-\sigma)\mu_{et+1}(z'_e) + \sigma\mu_{ut+1}(z'_e)]).$$

Finally, we prove (34). By (61), the free-entry condition (56) under the conjecture implies (34), namely,

$$\kappa A_t z = \lambda_{ft}(\theta_t(z))[1 - \eta(\theta_t(z))][Y_t(z) + W_{pt}(z) - U_t(z)] = [1 - \eta_t(z)]\lambda_{ft}(z)[\mu_{et}(z) - \mu_{ut}(z)].$$

Thus, the equations characterizing the competitive search equilibrium allocations coincide with those characterizing the solution to the planning problem. Hence, the allocations of the competitive search equilibrium solve the planning problem. □

A.5.3 Third Step

Consider now the third lemma.

Lemma A.3. *There is a unique set of allocations that solve the planning problem.*

To set up this proof, we first note that the pair of constraints (41) and (42) are equivalent to the pair of constraints

$$e_t(z) = \frac{\phi(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + m_t\left[\frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right), v_t(z)\right] \tag{64}$$

and

$$u_t(z) + e_t(z) = \frac{\phi}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + \frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right) + (1-\phi)\nu(z). \tag{65}$$

Next, note that it is convenient to consider a relaxed version of the constraints (40), (64), and (65) in which we write all three as inequalities, namely,

$$C_t \leq A_t \int_z z e_t(z) dz + bA_t \int_z z u_t(z) dz - \kappa A_t \int_z z v_t(z) dz, \tag{66}$$

$$e_t(z) \leq \frac{\phi(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + m_t\left[\frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right), v_t(z)\right] \tag{67}$$

$$u_t(z) + e_t(z) \leq \frac{\phi}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + \frac{\phi}{1+g_u} u_{t-1}\left(\frac{z}{1+g_u}\right) + (1-\phi)\nu(z). \tag{68}$$

Proof. We claim that a solution to the relaxed planning problem in which we choose $\{C_t, e_t(z), u_t(z)$,-

$v_t(z)\}_{t=0}^{\infty}$ to solve

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{S_t^{-\alpha} C_t^{1-\alpha}}{1-\alpha} \tag{69}$$

subject to (66)-(68) coincides with the solution to the original planning problem in which we choose $\{C_t, e_t(z), u_t(z), v_t(z)\}_{t=0}^{\infty}$ to maximize this same objective function subject to (40)-(42). Observe that the solution to the relaxed problem coincides with the solution the original problem because the three constraints (66)-(68) hold as equality at the optimum of the relaxed problem.

Next, we claim that the relaxed problem is a strictly concave problem and thus admits a unique solution. Since the objective function is strictly concave, we need only establish that the constraint set is convex in the sequences of allocations $\{C_t, e_t(z), u_t(z), v_t(z)\}_{t=0}^{\infty}$ with initial conditions $e_{-1}(z)$ and $u_{-1}(z)$. Note that allocations are also functions of the history of aggregate productivity shocks that we have suppressed for notational convenience.

To establish convexity of the constraint set, we show that if two sets of allocations, $x^i = \{C_t^i, e_t^i(z),$-$u_t^i(z), v_t^i(z)\}_{t=0}^{\infty}$ for $i = A, B$, satisfy the above infinite sequence of constraints, then so does a convex combination of them, $\hat{x} = \lambda^A x^A + \lambda^B x^B$, for some $\lambda^A + \lambda^B = 1$ with $\lambda^i \in [0, 1]$. Clearly, the first constraint (66) and third constraint (68) are linear in the allocations and hence are satisfied at $\hat{x}$. For the second constraint, we use that the matching function is strictly concave. That is, suppose that x^A and x^B satisfy the constraint for some arbitrary z and t so that

$$e_t^A(z) \leq \frac{\phi(1-\sigma)}{1+g_e} e_{t-1}^A\left(\frac{z}{1+g_e}\right) + m_t\left[\frac{\phi}{1+g_u} u_{t-1}^A\left(\frac{z}{1+g_u}\right), v_t^A(z)\right]$$

and

$$e_t^B(z) \leq \frac{\phi(1-\sigma)}{1+g_e} e_{t-1}^B\left(\frac{z}{1+g_e}\right) + m_t\left[\frac{\phi}{1+g_u} u_{t-1}^B\left(\frac{z}{1+g_u}\right), v_t^B(z)\right].$$

Then, multiplying these constraints by λ^A and λ^B, respectively, and adding them side by side gives

$$\begin{aligned}\lambda^A e_t^A(z) + \lambda^B e_t^B(z) \leq{}& \frac{\phi(1-\sigma)}{1+g_e}\left[\lambda^A e_{t-1}^A\left(\frac{z}{1+g_e}\right) + \lambda^B e_{t-1}^B\left(\frac{z}{1+g_e}\right)\right] \\ &+ \lambda^A m_t\left[\frac{\phi}{1+g_u} u_{t-1}^A\left(\frac{z}{1+g_u}\right), v_t^A(z)\right] + \lambda^B m_t\left[\frac{\phi}{1+g_u} u_{t-1}^B\left(\frac{z}{1+g_u}\right), v_t^B(z)\right]\end{aligned}$$

or, equivalently,

$$\begin{aligned}\hat{e}_t(z) \leq{}& \frac{\phi(1-\sigma)}{1+g_e}\left[\hat{e}_{t-1}\left(\frac{z}{1+g_e}\right)\right] + \lambda^A m_t\left[\frac{\phi}{1+g_u} u_{t-1}^A\left(\frac{z}{1+g_u}\right), v_t^A(z)\right] \\ &+ \lambda^B m_t\left[\frac{\phi}{1+g_u} u_{t-1}^B\left(\frac{z}{1+g_u}\right), v_t^B(z)\right],\end{aligned}$$

where $\hat{e}_t(z) = \lambda^A e_t^A(z) + \lambda^B e_t^B(z)$ for any z and t. But since $m_t(\cdot,\cdot)$ is strictly concave, we know that

$$\begin{aligned}&\lambda^A m_t\left[\frac{\phi}{1+g_u} u_{t-1}^A\left(\frac{z}{1+g_u}\right), v_t^A(z)\right] + \lambda^B m_t\left[\frac{\phi}{1+g_u} u_{t-1}^B\left(\frac{z}{1+g_u}\right), v_t^B(z)\right] \\ &< m_t\left[\frac{\phi}{1+g_u} \hat{u}_{t-1}\left(\frac{z}{1+g_u}\right), \hat{v}_t(z)\right],\end{aligned}$$

where $\hat{u}_{t-1}(z) = \lambda^A u^A_{t-1}(z) + \lambda^B u_{t-1}(z)$ and $\hat{v}_t(z) = \lambda^A v^A_t(z) + \lambda^B v^B_t(z)$ for any z.

Since we have established that the planning problem is characterized by a strictly concave objective function and a convex constraint set, it indeed admits a unique set of allocations as a solution. □

It is immediate that this result implies that the solution to the aggregate planning problem described in the paper is also unique.

Corollary A.1. *There is a unique set of allocations that solve the aggregate planning problem.*

A.5.4 Fourth Step

Consider now the final lemma.

Lemma A.4. *Given any symmetric competitive search equilibrium allocations, there is a unique set of prices that decentralize them.*

Proof. Given the competitive search equilibrium allocations characterized above, we show how to construct a unique set of prices $\{Q_{t,t+1}, W_{mt}(z), W_{pt}(z), U_t(z), Y_t(z)\}_{t=0}^{\infty}$ that decentralize them. First, observe that the one-period pricing kernel $Q_{t,t+1} = \beta[S_{t+1}C_{t+1}/(S_tC_t)]^{-\alpha}$ is clearly unique and so are the pricing kernels $Q_{t,r} = \beta^{r-t}[S_rC_r/(S_tC_t)]^{-\alpha}$ for more periods. Consider next the value of output $Y_t(z)$ defined by

$$Y_t(z) = A_t z + \phi(1-\sigma)\mathbb{E}_t[Q_{t,t+1}Y_{t+1}((1+g_e)z)].$$

Recursing this expression forward gives

$$\begin{aligned} Y_t(z) &= A_t z + \phi(1-\sigma)\mathbb{E}_t\{Q_{t,t+1}A_{t+1}(1+g_e)z + \phi(1-\sigma)\mathbb{E}_{t+1}[Q_{t+1,t+2}Y_{t+2}((1+g_e)^2 z)]\} \\ &= A_t z + \phi(1-\sigma)\mathbb{E}_t[Q_{t,t+1}A_{t+1}(1+g_e)z] + \phi^2(1-\sigma)^2\mathbb{E}_t[Q_{t+1,t+2}Y_{t+2}((1+g_e)^2 z)] \\ &= \sum\nolimits_{s=t}^{\infty} \phi^{s-t}(1-\sigma)^{s-t}\mathbb{E}_t\left\{Q_{t,s}A_s(1+g_e)^{s-t}z\right\}, \end{aligned} \tag{70}$$

where in the second line, we have used the law of iterated expectations and in the third line, we have imposed the limiting condition that

$$\lim_{s\to\infty} \phi^{s-t}(1-\sigma)^{s-t}\mathbb{E}_t\left\{Q_{t,s}Y_s((1+g_e)^{s-t}z)\right\} = 0.$$

Next, note that given $\lambda_{ft}(z)$ calculated from the allocations and $Y_t(z)$ calculated from (70), the free-entry condition (56), namely, $\kappa A_t z = \lambda_{ft}(z)[Y_t(z) - W_{mt}(z)]$, pins down $W_{mt}(z)$.

Finally, to solve for $W_{pt}(z)$ and $U_t(z)$, we can stack up (59) and (60) as a vector linear difference equation and solve it forward to obtain a solution in terms of the known variables $W_{ms}(z)$, $\lambda_{ws}(z)$, and $Q_{t,s}$ for all $s \geq t$. □

A.6 Risk-Free Rate for One-Period Bonds

In this section, we derive approximate formulae for the one-period risk-free rate in our baseline model, which show that the risk-free rate is approximately linear in the surplus consumption ratio s_t. Recall that the risk free rate is defined as $e^{-r_{ft}} = \mathbb{E}_t\left(Q_{t,t+1}\right)$ or, taking logs,

$$r_{ft} = -\log\left[\mathbb{E}_t\left(Q_{t,t+1}\right)\right]. \tag{71}$$

Next, since

$$Q_{t,t+1} = \beta \left(\frac{S_{t+1} C_{t+1}}{S_t C_t} \right)^{-\alpha},$$

then with $c_t = \log(C_t)$ and $s_t = \log(S_t)$, we obtain

$$\log(Q_{t,t+1}) = \log(\beta) - \alpha(s_{t+1} - s_t) - \alpha(c_{t+1} - c_t). \tag{72}$$

Here, we make the assumption that consumption growth is approximately equal to productivity growth, $\Delta c_{t+1} = \Delta a_{t+1}$ with $\Delta c_{t+1} = \log(C_{t+1}) - \log(C_t)$ and $\Delta a_{t+1} = \log(A_{t+1}) - \log(A_t)$, which implies that $\mathbb{E}_t(\Delta c_{t+1}) = \mathbb{E}_t(\Delta a_{t+1})$ and that log consumption innovations are approximately equal to log productivity innovations in that

$$\Delta c_{t+1} - \mathbb{E}_t(\Delta c_{t+1}) = \Delta a_{t+1} - \mathbb{E}_t(\Delta a_{t+1}) = \sigma_a \varepsilon_{at+1} \tag{73}$$

and approximately normal. Given the process for s_t,

$$s_{t+1} = (1 - \rho_s)\, s + \rho_s s_t + \lambda_a(s_t) \sigma_a \varepsilon_{at+1}, \tag{74}$$

the innovations for s_t are also normal. Hence, from (72), $\log(Q_{t,t+1})$ is approximately normal. We then use that if $\log(Q_{t,t+1})$ is normal with conditional mean $\mathbb{E}_t[\log(Q_{t,t+1})]$ and conditional variance $\sigma_t^2[\log(Q_{t,t+1})]$, then $Q_{t,t+1}$ is log-normal with conditional mean

$$\mathbb{E}_t(Q_{t,t+1}) = \exp\{\mathbb{E}_t[\log(Q_{t,t+1})] + \sigma_t^2[\log(Q_{t,t+1})]/2\},$$

so that the risk-free rate (71) can be expressed as

$$r_{ft} = -\log\left[\mathbb{E}_t\left(Q_{t,t+1}\right)\right] = -\mathbb{E}_t[\log(Q_{t,t+1})] - \sigma_t^2[\log(Q_{t,t+1})]/2.$$

Simplifying the first term gives

$$\begin{aligned}
\mathbb{E}_t[\log(Q_{t,t+1})] &= \mathbb{E}_t\left\{ \log\left[\beta \left(\frac{S_{t+1} C_{t+1}}{S_t C_t} \right)^{-\alpha} \right] \right\} = \log(\beta) - \alpha \mathbb{E}_t(\Delta s_{t+1}) - \alpha \mathbb{E}_t(\Delta c_{t+1}) \\
&= \log(\beta) + \alpha(1 - \rho_s)(s_t - s) - \alpha \mathbb{E}_t(\Delta c_{t+1}),
\end{aligned} \tag{75}$$

where in (75) we have used (74) to obtain $\Delta s_{t+1} = s_{t+1} - s_t = -(1 - \rho_s)(s_t - s) + \lambda_a(s_t)\sigma_a \varepsilon_{at+1}$.

Similarly, simplifying the second term gives

$$\begin{aligned}
\sigma_t^2[\log(Q_{t,t+1})] &= \sigma_t^2\left\{ \log\left[\beta \left(\frac{S_{t+1} C_{t+1}}{S_t C_t} \right)^{-\alpha} \right] \right\} = \alpha^2 \sigma_t^2(\Delta s_{t+1} + \Delta c_{t+1}) \\
&= \alpha^2 \sigma_t^2(-(1 - \rho_s)(s_t - s) + \lambda_a(s_t)\sigma_a \varepsilon_{at+1} + \Delta a_{t+1}) && (76) \\
&= \alpha^2 \sigma_t^2([1 + \lambda_a(s_t)]\, \sigma_a \varepsilon_{at+1}) = \alpha^2 \left[1 + \lambda_a(s_t)\right]^2 \sigma_t^2(\sigma_a \varepsilon_{at+1}) && (77) \\
&= \frac{\alpha^2 \left[1 - 2(s_t - s)\right]}{S^2} \sigma_a^2, && (78)
\end{aligned}$$

where in (76) we have used our approximation that $\Delta c_{t+1} \approx \Delta a_{t+1}$ and the law of motion for s_t in

(74), in (77) we have used that the conditional variance in t of random variables known in t such as s_t is zero, and in (78) we have used that the sensitivity function $\lambda_a(s_t)$ is

$$\lambda_a(s_t) = \frac{1}{S}\sqrt{[1 - 2\,(s_t - s)]} - 1.$$

Combining (75) and (78) gives that

$$\begin{aligned} r_{ft} =& -\mathbb{E}_t[\log(Q_{t,t+1})] - \frac{\sigma_t^2[\log(Q_{t,t+1})]}{2} = -\log(\beta) - \alpha(1-\rho_s)(s_t - s) + \alpha\mathbb{E}_t(\Delta c_{t+1}) - \frac{\alpha^2[1-2(s_t-s)]}{2S^2}\sigma_a^2 \\ =& -\log(\beta) + \alpha g_a + \alpha(1-\rho_s)s - \frac{\alpha^2(1+2s)}{2S^2}\sigma_a^2 + \left[\frac{\alpha^2\sigma_a^2}{S^2} - \alpha(1-\rho_s)\right] s_t \\ =& \text{const} + \alpha\left[\frac{\alpha\sigma_a^2}{S^2} - (1-\rho_s)\right] s_t, \end{aligned} \tag{79}$$

where in the second line we used that $\mathbb{E}_t(\Delta c_{t+1}) = \mathbb{E}_t(\Delta a_{t+1}) = g_a$. Hence,

$$\sigma^2(r_{ft}) = \alpha^2\left[\frac{\alpha\sigma_a^2}{S^2} - (1-\rho_s)\right]^2 \sigma^2(s_t). \tag{80}$$

Define the parameter d as satisfying the following relationship

$$S^2 = \frac{\alpha\sigma_a^2}{1 - \rho_s - d/\alpha}. \tag{81}$$

In this case, the risk-free rate in (79) reduces to

$$r_{ft} = \text{const} - d(s_t - s). \tag{82}$$

Therefore, we have

$$\sigma^2(r_{ft}) = d^2\sigma^2(s_t) \tag{83}$$

and by appropriately choosing d in (81), which implies setting S, we can control the variance of the risk-free rate.

B Nominal and Real Bonds

In this section, we provide approximate formulae for the equilibrium prices of nominal and real bonds.

B.1 Pricing Real Zero-Coupon Bonds

The price at time t of the n-period zero-coupon real bond P_{nt}^r is defined as the expected value of one unit of consumptions goods in n periods. Hence, it is given by $P_{nt}^r = \mathbb{E}_t(Q_{t,t+n})$. We can express this formula recursively as $P_{nt}^r = \mathbb{E}_t(Q_{t,t+1}P_{n-1,t+1}^r)$ with $P_{0t}^r = 1$ following steps analogous to those in the proof of Lemma 2 in the appendix of the paper, where we establish that the price of an n-period strip at time t is $P_{nt} = \mathbb{E}_t(Q_{t,t+1}P_{n-1,t+1})$.

We can derive approximate formulae for our baseline model along the same lines of the proof of

Lemma 2 in the appendix of the paper. We apply the risk-adjusted affine approximation around the risky steady state described by Lopez, Lopez-Salido, and Vazquez-Grande (2017) to the pricing equation $P^r_{nt} = \mathbb{E}_t(Q_{t,t+1}P^r_{n-1,t+1})$ with $P^r_{0t} = 1$ or, in log form,

$$\log(P^r_{nt}) = \log\left\{\mathbb{E}_t\left[\exp\left(q_{t,t+1} + \log(P^r_{n-1,t+1})\right)\right]\right\}, \tag{84}$$

where $q_{t,t+1} = \log(Q_{t,t+1})$. Now, defining $\hat{s}_t = s_t - s$ and using the approximation

$$\log(P^r_{nt}) = a^r_n + b^r_n \hat{s}_t, \tag{85}$$

we can rewrite both sides of (84) to obtain

$$a^r_n + b^r_n \hat{s}_t = \log\left\{\mathbb{E}_t\left[\exp\left(q_{t,t+1} + a^r_{n-1} + b^r_{n-1}\hat{s}_{t+1}\right)\right]\right\} \tag{86}$$

where a^r_n satisfies

$$a^r_n = \log(\beta) - \alpha g_a + a^r_{n-1} + \left(-b^r_{n-1} - \frac{\alpha - b^r_{n-1}}{S}\right)^2 \frac{\sigma_a^2}{2}, \tag{87}$$

and b^r_n satisfies

$$b^r_n = (1-\rho_s)\alpha + \rho_s b^r_{n-1} + \left(-b^r_{n-1} - \frac{\alpha - b^r_{n-1}}{S}\right)\left(\frac{\alpha - b^r_{n-1}}{S}\right)\sigma_a^2. \tag{88}$$

To prove formulae (87) and (88), we use that the pricing kernel for our baseline preferences in log form is

$$q_{t,t+1} = \log(\beta) - \alpha\Delta s_{t+1} - \alpha\Delta c_{t+1}, \tag{89}$$

with $\Delta s_{t+1} = s_{t+1} - s_t$, $\Delta c_{t+1} = c_{t+1} - c_t$, and the law of motion for $\hat{s}_t$ is

$$\hat{s}_{t+1} = \rho_s \hat{s}_t + \lambda_a(s_t)\sigma_a \varepsilon_{at+1} \tag{90}$$

by (74) so that

$$\Delta\hat{s}_{t+1} = \hat{s}_{t+1} - \hat{s}_t = (\rho_s - 1)\hat{s}_t + \lambda_a(s_t)\sigma_a\varepsilon_{at+1}. \tag{91}$$

Using (89), that $\Delta s_{t+1} = \Delta\hat{s}_{t+1}$, the approximation that $\Delta c_{t+1} = \Delta a_{t+1}$ where $\Delta a_{t+1} = g_a + \sigma_a\varepsilon_{at+1}$, and (90), we can write the argument inside the expectation in (86) as

$$\begin{aligned}
q_{t,t+1} + a^r_{n-1} + b^r_{n-1}\hat{s}_{t+1} &= [\log(\beta) - \alpha\Delta s_{t+1} - \alpha\Delta c_{t+1}] + a^r_{n-1} + b^r_{n-1}\hat{s}_{t+1} \\
&= \{\log(\beta) - \alpha[(\rho_s - 1)\hat{s}_t + \lambda_a(s_t)\sigma_a\varepsilon_{at+1}] - \alpha\Delta a_{t+1}\} + a^r_{n-1} + b^r_{n-1}[\rho_s\hat{s}_t + \lambda_a(s_t)\sigma_a\varepsilon_{at+1}] \\
&= \log(\beta) - \alpha(g_a + \sigma_a\varepsilon_{at+1}) + \left[b^r_{n-1}\rho_s - \alpha(\rho_s - 1)\right]\hat{s}_t + a^r_{n-1} + (b^r_{n-1} - \alpha)\lambda_a(s_t)\sigma_a\varepsilon_{at+1} \\
&= \log(\beta) - \alpha g_a + [b^r_{n-1}\rho_s - \alpha(\rho_s - 1)]\hat{s}_t + a^r_{n-1} + \{-\alpha[1 + \lambda_a(s_t)] + b^r_{n-1}\lambda_a(s_t)\}\sigma_a\varepsilon_{at+1}.
\end{aligned} \tag{92}$$

Next, we evaluate the right side of (86) using (92). Note first that, except for the last term, all of the variables in (92) are known in t so that

$$\begin{aligned}
a^r_n + b^r_n\hat{s}_t = \log\left\{\mathbb{E}_t\left[\exp\left(q_{t,t+1} + a^r_{n-1} + b^r_{n-1}\hat{s}_{t+1}\right)\right]\right\} &= \log(\beta) - \alpha g_a + a^r_{n-1} + [b^r_{n-1}\rho_s - \alpha(\rho_s - 1)]\hat{s}_t \\
&+ \log\left\{\mathbb{E}_t\left[\exp\left(\{-\alpha[1 + \lambda_a(s_t)] + b^r_{n-1}\lambda_a(s_t)\}\sigma_a\varepsilon_{at+1}\right)\right]\right\}.
\end{aligned} \tag{93}$$

For the last term in (93), we use that the conditional expectation of a log-normal random variable with mean 0 and variance σ^2 is $\exp(\sigma^2/2)$ so that

$$\begin{aligned}
\log\left\{\mathbb{E}_t\left[\exp\left(\{-\alpha[1+\lambda_a(s_t)]+b^r_{n-1}\lambda_a(s_t)\}\sigma_a\varepsilon_{at+1}\right)\right]\right\} &= \{-\alpha[1+\lambda_a(s_t)]+b^r_{n-1}\lambda_a(s_t)\}^2\frac{\sigma_a^2}{2} \\
&\approx \{-\alpha[1+\lambda_a(s)]+b^r_{n-1}\lambda_a(s)\}^2\frac{\sigma_a^2}{2}+\left(\sigma_a^2\left(b^r_{n-1}-\alpha\right)\lambda_a'(s)\{-\alpha[1+\lambda_a(s)]+b^r_{n-1}\lambda_a(s)\}\right)\hat{s}_t \\
&= \left[-\frac{\alpha}{S}+b^r_{n-1}\left(\frac{1}{S}-1\right)\right]^2\frac{\sigma_a^2}{2}+\sigma_a^2\frac{\left(\alpha-b^r_{n-1}\right)}{S}\left[-\frac{\alpha}{S}+b^r_{n-1}\left(\frac{1}{S}-1\right)\right]\hat{s}_t \\
&= \left(-b^r_{n-1}-\frac{\alpha-b^r_{n-1}}{S}\right)^2\frac{\sigma_a^2}{2}+\left(-b^r_{n-1}-\frac{\alpha-b^r_{n-1}}{S}\right)\left(\frac{\alpha-b^r_{n-1}}{S}\right)\sigma_a^2\hat{s}_t, \qquad (94)
\end{aligned}$$

where in the second line we have performed a first-order Taylor expansion around $s_t = s$ and in the third line we have used that $\lambda_a(s_t) = \frac{1}{S}\sqrt{[1-2(s_t-s)]} - 1$ evaluated at a steady state implies that

$$1+\lambda_a(s) = 1/S \text{ and } \lambda_a'(s) = -1/S. \qquad (95)$$

Using (93) and (94) and grouping together the constant terms and those in $\hat{s}_t$, we have that

$$\begin{aligned}
\log\left\{\mathbb{E}_t\left[\exp\left(q_{t,t+1}+a^r_{n-1}+b^r_{n-1}\hat{s}_{t+1}\right)\right]\right\} &= \log(\beta)-\alpha g_a+a^r_{n-1}+\left(-b^r_{n-1}-\frac{\alpha-b^r_{n-1}}{S}\right)^2\frac{\sigma_a^2}{2} \\
&+\left[(1-\rho_s)\alpha+\rho_s b^r_{n-1}+\left(-b^r_{n-1}-\frac{\alpha-b^r_{n-1}}{S}\right)\left(\frac{\alpha-b^r_{n-1}}{S}\right)\sigma_a^2\right]\hat{s}_t. \qquad (96)
\end{aligned}$$

Matching coefficients, we obtain (87) and (88).

We can isolate the role of intertemporal substitution motives for the elasticity of real bond prices with respect to the state s_t by dropping the terms in (88) proportional to σ_a^2 that arise because of risk. The components of b^r_n due to intertemporal substitution are denoted by $b^{r,IS}_n$ and can be expressed as $b^{r,IS}_n = (1-\rho_s)\alpha+\rho_s b^{r,IS}_{n-1} = \alpha(1-\rho_s^n)$ and coincide with b^{IS}_n, the corresponding elasticity of the price of productivity claims with respect to s_t.

Proceeding similarly, we can isolate the portion of the elasticity of productivity claims with respect to s_t, b_n, that is due to precautionary saving motives and the risk premium effect discussed in the paper. Recall from the paper that

$$b_n = (1-\rho_s)\alpha+\rho_s b_{n-1}+\left(1-b_{n-1}-\frac{\alpha-b_{n-1}}{S}\right)\left(\frac{\alpha-b_{n-1}}{S}\right)\sigma_a^2$$

and that

$$b^r_n = (1-\rho_s)\alpha+\rho_s b^r_{n-1}+\left(-b^r_{n-1}-\frac{\alpha-b^r_{n-1}}{S}\right)\left(\frac{\alpha-b^r_{n-1}}{S}\right)\sigma_a^2.$$

Here $(1-\rho_s)\alpha$ can be interpreted as an intertemporal substitution motive whereas the last term of b^r_n can be interpreted as a precautionary savings term, which is negative under our conditions for the monotonicity of b^r_n between 0 to α, namely,

$$1-\rho_s-\frac{\alpha\sigma_a^2}{S^2} > 0,$$

which is analogous to the one in the paper that we maintain when characterizing the elasticity of the prices of claims to future productivity with respect to the state s_t. We prove this claim at the end of this subsection.

We can then define the risk premium component or compensation for risk motive, which is > 0 under our conditions for monotonicity of b_n^r between 0 to α, as

$$\begin{aligned}
b_n - b_n^r &= \rho_s(b_{n-1} - b_{n-1}^r) + \left[\left(1 - b_{n-1} - \frac{\alpha - b_{n-1}}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right) - \left(-b_{n-1}^r - \frac{\alpha - b_{n-1}^r}{S}\right)\left(\frac{\alpha - b_{n-1}^r}{S}\right)\right]\sigma_a^2 \\
&= \rho_s(b_{n-1} - b_{n-1}^r) + \left[\left(1 - b_{n-1} - \frac{\alpha - b_{n-1}}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right) - \left(-b_{n-1}^r - \frac{\alpha - b_{n-1}^r}{S}\right)\left(\frac{\alpha - b_{n-1}^r}{S}\right)\right. \\
&\quad \left. - \left(-b_{n-1}^r - \frac{\alpha - b_{n-1}^r}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right) + \left(-b_{n-1}^r - \frac{\alpha - b_{n-1}^r}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right)\right]\sigma_a^2 \qquad (97) \\
&= \rho_s(b_{n-1} - b_{n-1}^r) + \left[\left(1 - b_{n-1} - \frac{\alpha - b_{n-1}}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right) - \left(-b_{n-1}^r - \frac{\alpha - b_{n-1}^r}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right)\right. \qquad (98) \\
&\quad \left. - \left(-b_{n-1}^r - \frac{\alpha - b_{n-1}^r}{S}\right)\left(\frac{\alpha - b_{n-1}^r}{S}\right) + \left(-b_{n-1}^r - \frac{\alpha - b_{n-1}^r}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right)\right]\sigma_a^2 \\
&= \rho_s(b_{n-1} - b_{n-1}^r) + \left[\left(1 - b_{n-1} - \frac{\alpha - b_{n-1}}{S} + b_{n-1}^r + \frac{\alpha - b_{n-1}^r}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right)\right. \\
&\quad \left. + \left(b_{n-1}^r + \frac{\alpha - b_{n-1}^r}{S}\right)\left(\frac{b_{n-1} - b_{n-1}^r}{S}\right)\right]\sigma_a^2 \\
&= \rho_s(b_{n-1} - b_{n-1}^r) + \left\{\left[1 + \frac{(1-S)}{S}(b_{n-1} - b_{n-1}^r)\right]\left(\frac{\alpha - b_{n-1}}{S}\right) + \left(b_{n-1}^r + \frac{\alpha - b_{n-1}^r}{S}\right)\left(\frac{b_{n-1} - b_{n-1}^r}{S}\right)\right\}\sigma_a^2, \qquad (99)
\end{aligned}$$

where in the second equality, we have subtracted and added

$$\left(-b_{n-1}^r - \frac{\alpha - b_{n-1}^r}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right)\sigma_a^2$$

and in the last equality, we have used

$$\begin{aligned}
&\left(1 - b_{n-1} - \frac{\alpha - b_{n-1}}{S} + b_{n-1}^r + \frac{\alpha - b_{n-1}^r}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right) \\
&\quad = \left(1 - \frac{Sb_{n-1}}{S} + \frac{b_{n-1}}{S} + \frac{Sb_{n-1}^r}{S} - \frac{b_{n-1}^r}{S}\right)\left(\frac{\alpha - b_{n-1}}{S}\right) = \left[1 + \frac{(1-S)}{S}(b_{n-1} - b_{n-1}^r)\right]\left(\frac{\alpha - b_{n-1}}{S}\right).
\end{aligned}$$

It is immediate from (99) that if $b_{n-1} \geq b_{n-1}^r$, then $b_n \geq b_n^r$, since $\rho_s > 0$, $S \leq 1$, and both b_{n-1} and b_{n-1}^r range between 0 and α, as we have argued. Note that $b_n - b_n^r$ is indeed the elasticity of the risk premium on an n-period productivity strip or, more precisely, the elasticity of the hold-to-maturity expected excess return on a n-period productivity strip with respect to the state s_t, given that $\log[\mathbb{E}_t(Q_{t,t+n}A_{t,t+n})/\mathbb{E}_t Q_{t,t+n}] \approx a_n - a_n^r + (b_n - b_n^r)s_t$.

As for the monotonicity of b_n^r between 0 to α, we next show that the assumptions that

$$\alpha > 1 \qquad \text{and} \qquad 1 - \rho_s - \frac{\alpha\sigma_a^2}{S^2} > 0 \qquad (100)$$

imply that the coefficients $\{b_n^r\}$ grow monotonically from 0 to α. To this end, it is useful to rearrange the formula for b_n^r as

$$b_n^r - b_{n-1}^r = (\alpha - b_{n-1}^r)\phi^r(b_{n-1}^r), \tag{101}$$

where

$$\phi^r(b_{n-1}^r) = 1 - \rho_s - \frac{\alpha\sigma_a^2}{S^2} + b_{n-1}^r\left(\frac{1-S}{S}\right)\frac{\sigma_a^2}{S} \qquad \text{and} \qquad S = [(C-X)/C]^{(\alpha-1)/\alpha} \leq 1 \tag{102}$$

using that $S = \tilde{S}^{(\alpha-1)/\alpha}$.

First, we claim that if $b_{n-1}^r \in [0, \alpha]$, then $\phi^r(b_{n-1}^r) \geq 0$ and $b_n^r \geq b_{n-1}^r$. To prove this claim, we note that under our assumptions in (100) and (102), $\phi^r(b_{n-1}^r) \geq 0$ if $b_{n-1}^r \geq 0$. Therefore, $b_n^r \geq b_{n-1}^r$ for any $b_{n-1}^r \in [0, \alpha]$ by (101).

Second, we claim that if $b_{n-1}^r \leq \alpha$, then $b_n^r \leq \alpha$. To establish this claim, we observe that the term $[(1-S)/S]\sigma_a^2/S$ is nonnegative since $S \leq 1$, which implies that the function $\phi^r(\cdot)$ is weakly increasing. Hence, if $b_{n-1}^r \leq \alpha$, then $\phi^r(b_{n-1}^r) \leq \phi^r(\alpha)$. Therefore, $b_n^r \leq \alpha$ since

$$b_n^r = b_{n-1}^r + (\alpha - b_{n-1}^r)\phi^r(b_{n-1}^r) \leq b_{n-1}^r + (\alpha - b_{n-1}^r)\phi^r(\alpha) = [1-\phi^r(\alpha)]b_{n-1}^r + \phi^r(\alpha)\alpha \leq \alpha, \tag{103}$$

where in the first inequality, we have used that $\phi^r(b_{n-1}^r) \leq \phi^r(\alpha)$ and $\alpha - b_{n-1}^r \geq 0$. The last inequality follows because $[1-\phi^r(\alpha)]b_{n-1}^r + \phi^r(\alpha)\alpha$ is a convex combination of b_{n-1}^r and α, since $\phi^r(\alpha) = 1 - \rho_s - \alpha\sigma_a^2/S$ is in $(0,1)$, and both b_{n-1}^r and α are less than or equal to α. To see that $\phi^r(\alpha) \in (0,1)$, note that $\phi^r(\alpha) < 1$ follows since ρ_s and α are positive. That $\phi^r(\alpha) > 0$ follows because $S \leq 1$ and we have assumed that $1 - \rho_s - \alpha\sigma_a^2/S^2 > 0$. These facts imply that

$$\phi^r(\alpha) = 1 - \rho_s - \frac{\alpha\sigma_a^2}{S} \geq 1 - \rho_s - \frac{\alpha\sigma_a^2}{S^2} > 0.$$

Finally, we argue that b_n^r monotonically converges to α. To begin, recall that the sequence $\{b_n^r\}$ starts with $b_0^r = 0 \in [0, \alpha]$. Recursively applying our first two claims implies that $b_n^r \in [b_{n-1}^r, \alpha]$ for all n. Since the sequence $\{b_n^r\}$ is monotone and bounded, it converges (monotonically) to some limit point b^{r*}. To prove that this limit point is $b^{r*} = \alpha$, note that b^{r*} solves the quadratic equation $(\alpha - b^{r*})\phi^r(b^{r*}) = 0$ by (101). This equation admits two roots, a positive one with $b^{r*} = \alpha$ and, by (100) and (102), a negative one. The negative one is not relevant because, as argued, b_n^r is a sequence that monotonically increases from $b_0^r = 0$ to b^{r*} so its limit point is positive. Hence, this limit point must be the positive root α. □

B.2 Pricing Nominal Zero-Coupon Bonds

Here we describe how we price nominal bonds in our model, show the inflation process we posit, and then briefly discuss why bonds carry an inflation-risk premium. Note that the price at time t of an n-period zero-coupon nominal bond is defined as the expected value of a claim to one unit of consumption goods in n periods in period-t goods, $P_{nt}^b = \mathbb{E}_t(Q_{t,t+n}/\Pi_{t,t+n})$, where $\Pi_{t,t+n}$ is the gross inflation rate between t and $t+n$. Proceeding as before, we can write this formula recursively as

$$P_{nt}^b = \mathbb{E}_t\left(\frac{Q_{t,t+1}}{\Pi_{t,t+1}}P_{n-1,t+1}^b\right) \qquad \text{with } P_{0t}^b = 1. \tag{104}$$

We follow Wachter (2006) and posit an exogenous monthly process for inflation of the form

$$\pi_{t+1} = \bar{\pi} + x_t + w_{t+1} \text{ with } x_{t+1} = \phi x_t + \psi w_{t+1}, \tag{105}$$

which we estimate below. Note for later that we allow the nominal shock w_t to be correlated with the aggregate productivity shock ε_{at}. Observe also that the gross inflation rate between t and $t+1$, $\Pi_{t,t+1}$, and the net inflation rate between t and $t+1$, π_{t+1}, are related by

$$\Pi_{t,t+1} = \exp(\pi_{t+1}). \tag{106}$$

We first discuss how we solve for the price of a n-period nominal bond. Recall that the aggregate state of our baseline (real) model is (s_t, Z_{et}, Z_{ut}), which we refer to as the *real state*. The nominal model augments the equations of the baseline model with the exogenous inflation process (105). The nominal model thus adds the state x_t so that the aggregate state of the nominal model is $(s_t, Z_{et}, Z_{ut}, x_t)$. We focus on a stationary equilibrium of this economy, in which the price of the n-period nominal bond is a stationary function of this state and so has the form $P^b_{nt} = P^b_n(s_t, Z_{et}, Z_{ut}, x_t)$. To solve for such an equilibrium, we posit (and then subsequently verify) a solution in which the log of this price is the sum of a nominal part that is affine in the nominal state x_t and a real part that depends nonlinearly on the real state (s_t, Z_{et}, Z_{ut}), that is,

$$\log(P^b_n(s_t, Z_{et}, Z_{ut}, x_t)) = a^x_n + b^x_n x_t + \log(F^b_n(s_t, Z_{et}, Z_{ut}))$$

or

$$P^b_n(s_t, Z_{et}, Z_{ut}, x_t) = \exp(a^x_n + b^x_n x_t) F^b_n(s_t, Z_{et}, Z_{ut}). \tag{107}$$

Now, starting with (104), namely,

$$P^b_{nt} = \mathbb{E}_t(Q_{t,t+1}\Pi^{-1}_{t,t+1}P^b_{n-1,t+1}), \tag{108}$$

we substitute on the left side of (108) using (107), where we let $F^b_{n,t}$ denote $F^b_n(s_t, Z_{et}, Z_{ut})$ for any t, and on the right side using $\Pi_{t,t+1} = \exp(\pi_{t+1})$ to obtain

$$\begin{aligned}
\exp(a^x_n + b^x_n x_t) F^b_{nt} &= \mathbb{E}_t\left[Q_{t,t+1}\exp(-\pi_{t+1})P^b_{n-1,t+1}\right] \\
&= \mathbb{E}_t\left[Q_{t,t+1}\exp(-\pi_{t+1})\exp(a^x_{n-1} + b^x_{n-1}x_{t+1})F^b_{n-1,t+1}\right] \\
&= \mathbb{E}_t\left[Q_{t,t+1}\exp(-\bar{\pi} - x_t - w_{t+1} + a^x_{n-1} + b^x_{n-1}(\phi x_t + \psi w_{t+1}))F^b_{n-1,t+1}\right] \\
&= \exp(-\bar{\pi} + a^x_{n-1} - (1 - \phi b^x_{n-1})x_t)\mathbb{E}_t\left[Q_{t,t+1}\exp((b^x_{n-1}\psi - 1)w_{t+1})F^b_{n-1,t+1}\right],
\end{aligned} \tag{109}$$

where in the second line we have used a version (107) for a $n-1$-period bond in period $t+1$ and let $F^b_{n-1,t+1}$ denote $F^b_{n-1}(s_{t+1}, Z_{et+1}, Z_{ut+1})$, in the third line we have substituted for π_{t+1} using the first equation in (105) and for x_{t+1} using the second equation in (105) and collected terms, and in the fourth line we have rearranged terms. Hence, matching the constant terms and the terms in x_t on both sides of the last line in (109) gives

$$a^x_n = -\bar{\pi} + a^x_{n-1} \text{ and } b^x_n = -\left(1 - \phi b^x_{n-1}\right) \tag{110}$$

with $a_0^x = b_0^x = 0$. By solving (110) recursively, we obtain

$$a_n^x = -n\bar{\pi} \text{ and } b_n^x = -\frac{1-\phi^n}{1-\phi}.$$

Now using from (110) that

$$\exp(a_n^x + b_n^x x_t) = \exp(-\bar{\pi} + a_{n-1}^x + (\phi b_{n-1}^x - 1)x_t), \tag{111}$$

we can divide both sides of the third line of (109) by (111) to obtain

$$F_{nt}^b = \mathbb{E}_t\left[Q_{t,t+1}\exp((b_{n-1}^x\psi - 1)w_{t+1})F_{n-1,t+1}^b\right] \text{ with } F_{0t}^b = P_{0t}^b e^{-a_0^x - b_0^x x_t} = 1, \tag{112}$$

where we used that $P_{0t}^b = 1$ and $a_0^x = b_0^x = 0$. We then solve (112) nonlinearly.

Finally, notice that the term $\mathbb{E}_t\left[Q_{t,t+1}\exp((b_{n-1}^x\psi - 1)w_{t+1})F_{n-1,t+1}^b\right]$ in the third line of (109) can be expressed as

$$\begin{aligned}\mathbb{E}_t\left[Q_{t,t+1}\exp((b_{n-1}^x\psi - 1)w_{t+1})F_{n-1,t+1}^b\right] &= \mathbb{E}_t\left[\exp((b_{n-1}^x\psi - 1)w_{t+1})\right]\mathbb{E}_t(Q_{t,t+1}F_{n-1,t+1}^b)\\ &+ \mathrm{Cov}_t(\exp((b_{n-1}^x\psi - 1)w_{t+1}, Q_{t,t+1}F_{n-1,t+1}^b),\end{aligned}$$

where the last term would be zero if the nominal shock w_{t+1} was uncorrelated with the real shock ε_{at+1}, in which case the real interest rate would reduce to the nominal interest rate minus the (log of the) expected inflation rate. Therefore, the inflation-risk premium would be zero. Given our estimates, w_{t+1} and ε_{at+1} are negatively correlated, so that a nominal bond tends to pay off fewer goods when the marginal utility of these goods tends to be high. Hence, nominal bonds carry an inflation-risk premium.

B.3 Estimation of Exogenous Inflation Process

As discussed in the paper, when we assess the implications of our model for nominal interest rates, one approach we follow is to augment our model by appending to it a purely exogenous process for inflation with shocks that are correlated with those to the real side of the economy. In this approach, this correlation is the key way the process for inflation is connected to the real economy. We start by estimating an ARMA(1,1) monthly process for inflation. Since measures of aggregate productivity are only available on a quarterly basis whereas our model is monthly, we proceed as follows. We append to our real model a monthly process for inflation of the same form as the quarterly process in Wachter (2006), namely,

$$\pi_{t+1} = \bar{\pi} + x_t + w_{t+1} \text{ and } x_{t+1} = \phi x_t + \psi w_{t+1},$$

where w_t is an i.i.d. random variable that is distributed $N(0, \sigma_w^2)$ and correlated with productivity innovations with $corr(\varepsilon_{at}, w_t) = \rho_{\pi a}$. This state-space representation implies an ARMA(1,1) univariate representation for monthly inflation. We estimate the model by maximum likelihood and recover the parameters $(\bar{\pi}, \phi, \psi, \sigma_w)$.

We now define quarterly inflation and productivity growth as

$$\pi_{q,t+1} \equiv \pi_{t+1} + \pi_{t+2} + \pi_{t+3} \text{ and } \Delta a_{q,t+1} \equiv \Delta a_{t+1} + \Delta a_{t+2} + \Delta a_{t+3}.$$

These quarterly rates can be written in terms of the monthly parameters and processes as

$$\begin{aligned}\pi_{t+3}+\pi_{t+2}+\pi_{t+1} &= \bar{\pi}+x_{t+2}+w_{t+3}+\bar{\pi}+x_{t+1}+w_{t+2}+\bar{\pi}+x_t+w_{t+1}\\ &= 3\bar{\pi}+[\phi(\phi x_t+\psi w_{t+1})+\psi w_{t+2}]+w_{t+3}+\phi x_t+\psi w_{t+1}+w_{t+2}+x_t+w_{t+1}\\ &= 3\bar{\pi}+(1+\phi+\phi^2)x_t+w_{t+3}+(1+\psi)w_{t+2}+[1+\psi(1+\phi)]w_{t+1}\end{aligned}$$

and

$$\Delta a_{t+3}+\Delta a_{t+2}+\Delta a_{t+1}=3g_a+\varepsilon_{at+3}+\varepsilon_{at+2}+\varepsilon_{at+1}.$$

Thus, after estimating the monthly inflation process, we can construct

$$w_{q,t+1}\equiv w_{t+3}+(1+\psi)w_{t+2}+[1+\psi(1+\phi)]w_{t+1},$$

and hence we can compute

$$\begin{aligned}\text{Cov}(w_{q,t+1},\Delta a_{q,t+1}) &= \text{Cov}(w_{t+3}+(1+\psi)w_{t+2}+[1+\psi(1+\phi)]w_{t+1},\varepsilon_{at+3}+\varepsilon_{at+2}+\varepsilon_{at+1})\\ &= \{1+(1+\psi)+[1+\psi(1+\phi)]\}\rho_{\pi a}\sigma_w\sigma_a.\end{aligned}$$

Rearranging terms, we can recover the monthly correlation between inflation and productivity innovations as

$$\rho_{\pi a}=\frac{\text{Cov}(w_{q,t+1},\Delta a_{q,t+1})}{\{1+(1+\psi)+[1+\psi(1+\phi)]\}\sigma_w\sigma_a}.$$

In Table A.8, we display the results of this estimation.

C Model with Physical Capital

In this section, we state and characterize the detailed planning problem for the model with physical capital. Recall that we assume that physical capital augments the production of goods in the market so that if a measure $e_t(z)$ of consumers with human capital z are paired with $K_t(z)$ units of physical capital, a firm produces $K_t^{\vartheta}(z)[A_t z e_t(z)]^{1-\vartheta}$, where $ze_t(z)$ are effective units of labor. We allow for costs of adjustment of the aggregate capital stock and maintain the same specifications of the technology for producing goods at home and job vacancies as in our baseline model.

We characterize the competitive search equilibrium allocations by solving the planning problem. This problem consists of choosing the processes for aggregate consumption, $\{C_t\}$, individual physical capital for consumers with human capital z, $\{K_t(z)\}$, vacancies $\{v_t(z)\}$, employment $\{e_t(z)\}$, and unemployment $\{u_t(z)\}$ for such consumers in order to solve

$$\max\ \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\frac{S_t^{-\alpha}C_t^{1-\alpha}}{1-\alpha},$$

subject to the resource constraint

$$C_t+I_t\leq\int_z K_t^{\vartheta}(z)[A_t z e_t(z)]^{1-\vartheta}dz+b_h A_t\int_z z u_t(z)dz-\kappa A_t\int_z z v_t(z)dz,$$

the law of motion of employment for consumers with human capital z,

$$e_t(z) = \frac{\phi\,(1-\sigma)}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + \lambda_{wt}(\theta_t(z))u_{bt}(z), \tag{113}$$

the law of motion of unemployment for consumers with human capital z,

$$u_t(z) = \frac{\phi\sigma}{1+g_e} e_{t-1}\left(\frac{z}{1+g_e}\right) + [1-\lambda_{wt}(\theta_t(z))]u_{bt}(z) + (1-\phi)\nu(z), \tag{114}$$

and the law of motion of aggregate physical capital $K_t = \int_z K_t(z)dz$, specified as in Jermann (1998),

$$K_{t+1} = (1-\delta)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t, \text{ where } \Phi\,(X) = \delta + e^{g_a} - 1 + \frac{\delta}{1-\frac{1}{\xi}}\left[\left(\frac{X}{\delta}\right)^{1-\frac{1}{\xi}} - 1\right], \tag{115}$$

with $\Phi(\delta) = \delta + e^{g_a} - 1$ and $\Phi'(\delta) = 1$. Note that here, for convenience, we have expressed the transition laws for employment and unemployment, (113) and (114), in the same form as in the paper, namely, (1) and (2).

Also note that investing I_t units of consumption goods yields $\Phi(I_t/K_t)K_t$ units of new capital, rather than I_t units of new capital as it would in the no adjustment cost case. In this sense, of the total amount I_t of consumption goods invested, the capital stock only increases by $\Phi(I_t/K_t)K_t \leq I_t$ and the rest of the consumption goods, namely, $I_t - \Phi(I_t/K_t)K_t \geq 0$ are lost to adjustment costs. Written as a fraction of the capital stock, these adjustment costs are then

$$\frac{I_t}{K_t} - \Phi\left(\frac{I_t}{K_t}\right). \tag{116}$$

In Jermann (1988), the function $\Phi\,(\cdot)$ is specified as $\Phi\,(I/K) = \frac{b}{1-a}\,(I/K)^{1-a} + c$. We let $a = 1/\xi$ and impose two standard conditions. First, we assume that $\Phi\,(\delta) = e^{g_a} - (1-\delta)$. This condition implies that $I_t/K_t = \delta$ is consistent with a balanced growth path in which $K_{t+1} = e^{g_a}K_t$, since in this this case

$$\frac{K_{t+1}}{K_t} = e^{g_a} = 1 - \delta + \Phi\left(\frac{I_t}{K_t}\right).$$

Hence, along such a balanced growth path, capital adjustment costs are such that

$$\frac{I_t}{K_t} - \Phi\left(\frac{I_t}{K_t}\right) = \delta - \Phi(\delta) = 1 - e^{g_a} \cong -g_a.$$

Note that these costs are zero when productivity growth is zero ($g_a = 0$) and are approximately $-g_a$ when productivity growth is positive.

Second, we assume that $\Phi'(\delta) = 1$, which implies that along such a balanced growth path, adjustment costs are (locally) constant. That is, letting $i_t \equiv I_t/K_t$ denote the investment to capital ratio, this condition implies that the derivative of the capital adjustment costs in (116) with respect to i_t is zero on a balanced growth path, that is,

$$\frac{\partial\,[i_t - \Phi(i_t)]}{\partial i_t}|_{i_t=\delta} = 1 - \Phi'(\delta) = 0.$$

Hence, along a balanced growth, the adjustment costs are constant on this path and locally around it. (Note that without productivity growth, these two conditions imply that adjustment costs are zero both in steady state and locally around it.) These restrictions imply that $\Phi'(\delta) = b\delta^{-1/\xi} = 1$, so that $b = \delta^{1/\xi}$, and that

$$\Phi(\delta) = \frac{\delta^{\frac{1}{\xi}}}{1-\frac{1}{\xi}}\delta^{1-\frac{1}{\xi}} + c = \frac{\delta}{1-\frac{1}{\xi}} + c.$$

Substituting the condition that $\Phi(\delta) = e^{g_a} - (1-\delta)$ in this last displayed expression, we obtain $c = e^{g_a} - (1-\delta) - \frac{\delta}{1-1/\xi}$. Hence, $\Phi(I/K)$ in Jermann (1988) reduces to

$$\Phi\left(\frac{I}{K}\right) = \frac{b}{1-a}\left(\frac{I}{K}\right)^{1-a} + c = \delta + e^{g_a} - 1 + \frac{\delta}{1-\frac{1}{\xi}}\left[\left(\frac{I}{\delta K}\right)^{1-\frac{1}{\xi}} - 1\right].$$

We solve separately the static and the dynamic parts of the planner's problem.

C.1 Static Part of the Planner's Problem with Physical Capital

Consider period t in which there is a measure $\{e_t(z)\}$ of employed consumers over z and an aggregate amount of capital K_t. The static part of the planning problem consists of allocating capital $K_t(z)$ for each z to employed consumers $e_t(z)$ subject to the constraint $\int_z K_t(z)dz \leq K_t$ in order to maximize aggregate output. That is, the static problem at each date t is to solve

$$\max_{K_t(z)} \int_z K_t^{\vartheta}(z)[A_t z e_t(z)]^{1-\vartheta} dz,$$

subject to

$$\lambda_{kt} : \int_z K_t(z)dz \leq K_t.$$

The first-order condition for capital for consumers with human capital z is

$$\vartheta\left[\frac{A_t z e_t(z)}{K_t(z)}\right]^{1-\vartheta} = \lambda_{kt} \quad \text{for all} \quad z$$

or

$$\frac{K_t(z)}{z e_t(z)} = A_t\left(\frac{\vartheta}{\lambda_{kt}}\right)^{\frac{1}{1-\vartheta}} \quad \text{for all} \quad z. \tag{117}$$

Since the right side of (117) does not vary with z neither does the left side. Hence, for all z and z',

$$\frac{K_t(z)}{z e_t(z)} = \frac{K_t(z')}{z' e_t(z')}.$$

It follows that

$$K_t(z) = z e_t(z)\varphi_t, \quad \text{where} \quad \varphi_t \equiv \frac{K_t(1)}{e_t(1)}, \tag{118}$$

and that the ratio of aggregate capital to aggregate effective labor equals the same ratio, since

$$\frac{K_t}{Z_{et}} = \frac{\int_z K_t(z)dz}{\int_z ze_t(z)dz} = \frac{\int_z \varphi_t ze_t(z)dz}{\int_z ze_t(z)dz} = \varphi_t. \tag{119}$$

Therefore, the economy aggregates in a similar fashion as does the economy of our baseline model. In particular, aggregate output is given by

$$\int_z K_t^{\vartheta}(z)[A_t ze_t(z)]^{1-\vartheta}dz = A_t^{1-\vartheta}\int_z \left[\frac{K_t(z)}{ze_t(z)}\right]^{\vartheta} ze_t(z)dz = \varphi_t^{\vartheta}A_t^{1-\vartheta}Z_{et} = K_t^{\vartheta}(A_tZ_{et})^{1-\vartheta},$$

where we have used (118) in the second equality and (119) in the third equality. The aggregate resource constraint can be expressed as

$$C_t + I_t = K_t^{\vartheta}(A_tZ_{et})^{1-\vartheta} + b_hA_tZ_{ut} - \kappa A_tZ_{vt}.$$

C.2 Dynamic Part of the Aggregate Planner's Problem with Physical Capital

The aggregate planning problem for this model with physical capital is set up analogously to the aggregate planning problem for our baseline model without physical capital. Because the economy of this model with physical capital aggregates in the same way as our baseline model does, the dynamic part of the planner's problem similarly consists of choosing processes for aggregate consumption $\{C_t\}$, market tightness $\{\theta_t\}$, the aggregate human capital of employed consumers $\{Z_{et}\}$, the aggregate human capital of unemployed consumers $\{Z_{ut}\}$, aggregate investment $\{I_t\}$, and aggregate physical capital $\{K_{t+1}\}$ to solve

$$\max \ \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\frac{S_t^{-\alpha}C_t^{1-\alpha}}{1-\alpha},$$

where $S_t = [(C_t - X_t)/C_t]^{(\alpha-1)/\alpha}$ is the exogenous scaled surplus consumption ratio, subject to the resource constraint, the law of motion of aggregate physical capital and the laws of motion of the aggregate human capital of employed and unemployed workers,

$$C_t + I_t = K_t^{\vartheta}(A_tZ_{et})^{1-\vartheta} + b_hA_tZ_{ut} - \kappa A_t\phi\theta_t(1+g_u)Z_{ut-1}, \tag{120}$$

$$K_{t+1} = (1-\delta)K_t + \Phi\left(\frac{I_t}{K_t}\right)K_t, \tag{121}$$

$$Z_{et} = \phi(1-\sigma)(1+g_e)Z_{et-1} + \phi\lambda_w(\theta_t)(1+g_u)Z_{ut-1}, \tag{122}$$

$$Z_{ut} = \phi\sigma(1+g_e)Z_{et-1} + \phi[1-\lambda_w(\theta_t)](1+g_u)Z_{ut-1} + 1 - \phi, \tag{123}$$

with multipliers ς_t, $\varsigma_t\mathcal{Q}_t$, $\varsigma_t\mu_{et}$, and $\varsigma_t\mu_{ut}$. The Lagrange multiplier $\mathcal{Q}_t$ denotes the shadow value of an additional unit of capital or the Tobin's Q, whereas the remaining Lagrange multipliers retain their interpretation in the planning problem without physical capital.

The first-order conditions for this problem are

$$C_t: \quad \varsigma_t = \beta^t (S_t C_t)^{-\alpha}, \tag{124}$$

$$\theta_t: \quad \kappa A_t = (1-\eta_t)\lambda_f(\theta_t)(\mu_{et} - \mu_{ut}), \tag{125}$$

$$Z_{et}: \quad \mu_{et} = (1-\vartheta) A_t \left(\frac{K_t}{A_t Z_{et}}\right)^{\vartheta} + \phi(1+g_e)\mathbb{E}_t\{Q_{t,t+1}[(1-\sigma)\mu_{et+1} + \sigma\mu_{ut+1}]\}, \tag{126}$$

$$Z_{ut}: \quad \mu_{ut} = b_h A_t + \phi(1+g_u)\mathbb{E}_t\{Q_{t,t+1}[\eta_{t+1}\lambda_{wt+1}\mu_{et+1} + (1-\eta_{t+1}\lambda_{wt+1})\mu_{ut+1}]\}, \tag{127}$$

$$I_t: \quad 1 = \mathcal{Q}_t \Phi'\left(\frac{I_t}{K_t}\right), \tag{128}$$

$$K_{t+1}: \quad \mathcal{Q}_t = \mathbb{E}_t Q_{t,t+1}\left\{\vartheta\left(\frac{K_{t+1}}{A_{t+1}Z_{et+1}}\right)^{\vartheta-1} + \mathcal{Q}_{t+1}\left[1-\delta+\Phi\left(\frac{I_{t+1}}{K_{t+1}}\right) - \Phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\right]\right\}, \tag{129}$$

with the stochastic discount factor defined as usual as $Q_{t,t+1} = \varsigma_{t+1}/\varsigma_t$.

Equations (124) to (129) along with the constraints (120) to (123) of the aggregate planning problem describe the solution to the planner's problem.

D Numerical Solution of the Model

Here we describe the global algorithm that we use to solve the model. We adopt a global numerical strategy because asset prices are highly nonlinear in the state for the asset-pricing preferences we consider. (Petrosky-Nadeau, Zhang, and Kuehn, 2018 highlight the importance of a model's nonlinearities when search frictions are present, even in the absence of risk-sensitive preferences, at least under parametrizations as that in Hagedorn and Manovskii, 2008.) Specifically, we solve our model by projecting the global solution of our model onto the space spanned by a basis of high-order Chebyshev polynomials and evaluating expectations by a multidimensional Gauss-Hermite quadrature with a sufficiently large number of nodes so that results are not sensitive to small increases or decreases in their number. We turn to providing a few details for each of our models.

Baseline Preferences. Recall that the sensitivity function is given by

$$\lambda_a(s_t) = \frac{1}{S}\sqrt{[1-2(s_t-s)]} - 1. \tag{130}$$

Rather than using the transformation $s_t - s$ of the log surplus consumption ratio s_t as the state, it is convenient to use the term $g(s_t) \equiv \sqrt{[1-2(s_t-s)]}$ from (130) as the state. Intuitively, the function $g(s_t)$, which affects the volatility of the process for the surplus consumption ration, is hard to approximate with a polynomial in $s_t - s$ due to its kink at $s_t - s = 1/2$. It is therefore more appropriate to use $g(s_t)$ as the state, from which we can recover $s_t - s$. We use Chebyshev polynomials of degree twenty for the transformation $g(s_t)$ of the surplus consumption state $s_t - s$ and of degree four for the human capital states Z_{et} and Z_{ut}.

As shown by Wachter (2005), the best practice in solving models with preferences as in Campbell and Cochrane (1999) is to consider a large and fine grid over the surplus consumption ratio space that importantly places many grid points close to zero. Accordingly, we construct a grid for the state $g(s_t)$

that ranges from 0 (the minimum value possible) to 8.5, which reflects a minimum value of S of 10^{-16} and so is very close to zero. We chose the maximum value by progressively widening the grid until results no longer change. For the human capital states, we adopt instead an adaptive grid, namely, a grid that covers the minimum and maximum values of long simulations of the variables of interest from the solved model.

After having set the grid $\mathbb{S}_t$ and the basis functions, our algorithm proceeds as follows. Let $\mathbb{S}_{ht}$, $h = 1, \ldots, H$, denote the h-th element of the grid, which spans the space of the state $(g(s_t), Z_{et}, Z_{ut})$. We start from a constant function as the initial guess for the functional $\theta_t = \theta^{(0)}(\mathbb{S}_t)$ that maps elements of the state space into values for market tightness. We then use such a guess and the J nodes for tomorrow's aggregate productivity shocks given by the Gauss-Hermite quadrature formulae to construct tomorrow's values of the state $\mathbb{S}_{h,t+1}(j)$, for $j = 1, \ldots, J$ and $h = 1, \ldots, H$, using the laws of motion of the state variables. We can then evaluate tomorrow's values for θ_{t+1} and hence compute μ_{et} and μ_{ut} using

$$\mu_{et} = A_t + \phi(1 + g_e)\mathbb{E}_t\{Q_{t,t+1}[(1 - \sigma)\mu_{et+1} + \sigma\mu_{ut+1}]\}, \tag{131}$$

and

$$\mu_{ut} = bA_t + \phi(1 + g_u)\mathbb{E}_t\{Q_{t,t+1}[\eta_{t+1}\lambda_{wt+1}\mu_{et+1} + (1 - \eta_{t+1}\lambda_{wt+1})\mu_{ut+1}]\}. \tag{132}$$

With those values in hand, we can use

$$\kappa A_t = (1 - \eta_t)\lambda_{ft}(\mu_{et} - \mu_{ut})$$

to compute a new value for the functional $\theta^{(1)}(\mathbb{S}_t)$ for each grid point. We repeat this procedure to calculate the n-th functional $\theta^{(n)}(\mathbb{S}_t)$ for all grid points. The algorithm stops when the maximum difference in the value of the functional across successive iterations is small, technically, when $\|\theta^{(n)}(\mathbb{S}_t) - \theta^{(n-1)}(\mathbb{S}_t)\|_\infty < 10^{-6}$, where $\|\cdot\|_\infty$ denotes the sup norm.

Campbell and Cochrane with External Habit. The global solution method we use for preferences with an external habit in consumption is the same as that we use for our baseline preferences with one modification. As before, we approximate market tightness θ_t by Chebyshev polynomials in the state $(g(s_t), Z_{et}, Z_{ut})$. However, for the original preferences in Campbell and Cochrane (1999) with external habit, s_t is an endogenous variable in that the innovations to it are endogenous, since they depend on the process for consumption. In particular, to compute s_{t+1}, we need to determine $\varepsilon_{ct+1} \equiv \Delta c_{t+1} - \mathbb{E}_t(\Delta c_{t+1})$, which is a different function of $(g(s_t), Z_{et}, Z_{ut})$ for each realization of ε_{at+1}. We allow for this dependence by specifying a different Chebyshev polynomial of a fixed order in the state $(g(s_t), Z_{et}, Z_{ut})$ for each realization of ε_{at+1}. Specifically, we set a 100-point grid for aggregate productivity shocks ε_{at+1}, and for each value of ε_{at+1} on this grid, we approximate $s_{t+1}(\varepsilon_{at+1})$ by Chebyshev polynomials in the state $(g(s_t), Z_{et}, Z_{ut})$ of order 20 in $g(s_t)$ and order 4 in Z_{et} and Z_{ut}. Letting j denote a grid point for ε_{at+1} and $\mathbb{S}_t$ denote the grid for $(g(s_t), Z_{et}, Z_{ut})$, our algorithm starts from guesses for the functionals $\theta_t = \theta^{(0)}(\mathbb{S}_t)$ and $s_{t+1} = s^{(0)}(\mathbb{S}_t, j)$ for each grid point $j = 1, \ldots, 100$ for ε_{at+1} and constructs $\theta^{(n)}(\mathbb{S}_t)$ and $s^{(n)}(\mathbb{S}_t, j)$. The algorithm stops when both $\theta^{(n)}$ and $s^{(n)}$ converge—that is, when under the sup norm, $\|(\theta^{(n)}(\mathbb{S}_t) - \theta^{(n-1)}(\mathbb{S}_t), s^{(n)}(\mathbb{S}_t, j) - s^{(n-1)}(\mathbb{S}_t, j))\|_\infty < 10^{-6}$.

Baseline Preferences and Physical Capital. In solving this model, we proceed analogously to the baseline model without physical capital. Specifically, the order of the Chebyshev polynomial for surplus consumption and the human capital states are the same as for the baseline model. We span the additional dimension of the state space, K_t/A_t, by a Chebyshev-polynomial of order four and an adaptive grid. In solving the model, we also iterate over the functional for the investment-capital ratio $I_t/K_t = \Upsilon(\mathbb{S}_t)$ using the optimality condition for investment.

Epstein-Zin Preferences with Long-Run Risk. To solve this model, we use Chebyshev polynomials of degree four and an adaptive grid in all four dimensions of the state space $(x_t, s_t, Z_{et}, Z_{ut})$, which consists of the persistence component of the productivity process x_t, the discount-rate shock s_t, and the human capital stocks of employed and unemployed consumers, Z_{et} and Z_{ut}. After having set the grid $\mathbb{S}_t$ and the basis functions, our algorithm proceeds as follows. Let $\mathbb{S}_{ht}$, $h = 1, \ldots, H$ denote the h-th element of the grid, which spans the space of the state $(x_t, s_t, Z_{et}, Z_{ut})$. We start from a constant guess for the functionals for market tightness $\theta_t = \theta^{(0)}(\mathbb{S}_t)$ and the detrended value function $w_t = w^{(0)}(\mathbb{S}_t)$, where $w_t = \log(W_t) \equiv \log[V_t/(S_t^{1/(1-\rho)} C_t)]$ on the grid $\mathbb{S}_t$. We then use that guess and the J nodes for tomorrow's shocks given by the Gauss-Hermite quadrature formulae to construct tomorrow's values of the state $\mathbb{S}_{h,t+1}(j)$ for $j = 1, \ldots, J$ and $h = 1, \ldots, H$ exploiting the laws of motion of the state variables. We can then evaluate tomorrow's values for θ_{t+1} and w_{t+1} and hence compute μ_{et} and μ_{ut} using (131) and (132). Once we have those values, we can use the free-entry condition to compute a new value for the functional $\theta^{(1)}(\mathbb{S}_t)$ for each grid point. We can likewise evaluate tomorrow's values for w_{t+1} and use the definition of utility to compute a new value for the functional $w^{(1)}(\mathbb{S}_t)$ for each grid point. We repeat this procedure to calculate the n-th functionals $\theta^{(n)}(\mathbb{S}_t)$ and $w^{(n)}(\mathbb{S}_t)$ for all grid points. The algorithm stops when $\|(\theta^{(n)}(\mathbb{S}_t) - \theta^{(n-1)}(\mathbb{S}_t), w^{(n)}(\mathbb{S}_t) - w^{(n-1)}(\mathbb{S}_t))\|_\infty < 10^{-6}$.

Epstein-Zin Preferences with Variable Disaster Risk. We solve this model by using Chebyshev polynomials of degree four and an adaptive grid in all three dimensions of the state space, which consists of the time-varying disaster intensity s_t and the stocks of human capital of employed and unemployed consumers, Z_{et} and Z_{ut}. The algorithm is analogous to the one for the model with Epstein-Zin preferences with long-run risk, except that the state space is now three-dimensional.

Affine Discount Factor Model. To solve this model, we use Chebyshev polynomials of degree twenty in the s dimension and of degree four for the stocks of human capital of employed and unemployed consumers, Z_{et} and Z_{ut}, over an adaptive grid. The algorithm is analogous to the one for the model with Campbell-Cochrane preferences with an exogenously driven habit.

E Extensions

Here, we first examine the implications for stock and bond returns of versions of our model without production, that is, for pure exchange economies. We then examine the robustness of our results to alternative protocols to determine flow wages.

Implications of Pure-Exchange Versions of the Economy We turn to comparing the asset-pricing implications of our search model with endogenous production to those of the original versions of

these models, which were developed for pure exchange economies. Table A.2 shows the implications of the pure-exchange (or endowment) versions of our different economies for stock and bond prices, once we approximate consumption growth by the exogenous process for aggregated productivity growth parametrized as we did for our baseline model. The asset-pricing implications of our search model with endogenous production under each of these preference structures are essentially identical to those of the original pure exchange economies.

Alternative Wage Protocols. We now consider how the parameters g_e and g_u of the human capital process that we infer through our moment-matching exercise vary with the chosen protocol for flow wages. We do so by examining two alternative and very different wage protocols and show that, despite their opposite assumptions on the degree to which wages can be renegotiated over the course of a match, imply similar values for g_e. This way, we demonstrate that the parametrization of the human capital process is fairly insensitive to the details of the wage setting mechanism.

The first protocol, which we refer to as *constant wage protocol*, leads to the most rigid wages. That is, given a present value of wages in t for a worker with human capital z, $W_t(z)$, we compute the constant wage series $w_{t,t+n}(z) = w_{t,t}(z)$ for all n that leads to the present value of $W_t(z)$. In the second protocol, which we refer to as *recontracting Nash bargaining protocol*, wages are instead renegotiated every period by Nash bargaining under the assumption that the Hosios condition holds. Such a protocol gives rise to the most flexible wages. In this second case, we compute the sequence of resulting wages that again leads to the same present value of $W_t(z)$.

For simplicity, we consider a steady state with no movement in aggregate productivity. We simulate 50,000 workers for 40 years, and discard the initial 10-year period. For each individual, risk arises from the possibility of idiosyncratic transition from employment to unemployment with probability σ and from unemployment to employment with probability λ_{wt}. To mimic the way the statistics on wage growth are computed in the data, for each worker we calculate the growth rate of annual wages over each two-year period of consecutive employment, provided the individual was employed full time, that is, worked at least 10 months in each of the two years.

When we do so, we find that our model implies a very similar growth rate of wages across these two extreme protocols. In particular, the average growth rate of wages inferred from the constant wage protocol equals 3.4%, whereas the one inferred from the recontracting Nash bargaining protocol equals 3.6%. These two values bracket the value of 3.5% in our baseline parametrization.

Finally, if we look at the isovolatility lines for the job-finding rate in Figure 3, we see that when g_e equals 3.4 or 3.6, the volatility of the job-finding rate is very similar to its value in our baseline. So in this sense too, our results seem robust across very different protocols.

F Sensitivity to Habit Persistence

The degree of habit persistence in consumption, captured by the parameter ρ_s, is key for our model's ability to generate time-varying risk. In this section, we conduct sensitivity exercises to assess the importance of the degree of habit persistence, as measured ρ_s, for our findings. Specifically, we discuss how precisely the data on asset prices pin down this parameter and how our results on the volatility of the job-finding rate vary for empirically relevant values of this parameter.

As Table 1 in the paper shows, in the baseline we set $\rho_s^{12} = 0.935$, which we pin down by targeting

the autocorrelation of the log price-dividend ratio in our moment-matching exercise. We begin our discussion by first quantifying our confidence in the estimate of this autocorrelation. Next, we show how varying this autocorrelation in our exercise affects the implied value of the habit persistence parameter and, in turn, the volatility of the job-finding rate. Finally, we investigate how the variability of other moments in the data, which we can alternatively use to discipline the habit persistence parameter, affects the value of this parameter and, correspondingly, the volatility of the job-finding rate. We do so in three steps described next.

In our moment-matching exercise, the habit persistence parameter is primarily pinned down by the annualized autocorrelation of the log price-dividend ratio, denoted by ρ_{pd}. In the first step, we quantify our confidence in ρ_{pd} by constructing a lower bound on the estimate of the annual autocorrelation of the log price-dividend ratio ρ_{pd}, which, through the lens of our model, provides a lower bound on ρ_s^{12} and so implies a lower bound on the job-finding rate.

We choose as our lower bound on ρ_{pd} the 5th percentile of the bootstrap distribution of the sample estimator of the annual autocorrelation of the log price-dividend ratio, denoted by $\underline{\rho}_{pd} = 0.794$. To do so, we construct 50,000 bootstrap samples and, because of the persistence of the time series of the log price-dividend ratio, we resample blocks of data to preserve time-series dependencies. Based on the autocorrelogram of the series, we select blocks of 12 observations, as the serial correlation of the series is close to zero after 12 lags.

In the second step, we keep the six assigned parameters, $\{B, b, \sigma, \eta, \phi, g_e\}$, at their baseline values and recompute the remaining seven endogenously chosen parameters, $\{g_a, \sigma_a, \kappa, \beta, S, \alpha, \rho_s\}$, to match the original six moments except for ρ_{pd}. For the seventh moment, we replace our baseline point estimate of $\rho_{pd} = 0.954$ with the 5th percentile of its bootstrap distribution, $\underline{\rho}_{pd} = 0.794$. The resulting value of ρ_s^{12} is 0.701. To help visualize the second step, on the vertical axis of panel a) of Figure A.1, we report both the baseline point estimate of the annualized autocorrelation of the price-dividend ratio ρ_{pd} and the 5th percentile of its bootstrap distribution, $\underline{\rho}_{pd}$. On the horizontal axis, we highlight the corresponding values of the annualized habit persistence parameter implied by our moment-matching exercise.

Clearly, through the lens of the model, a lower estimate of ρ_{pd} implies a lower estimate of the degree of habit persistence, as the price-dividend ratio and the surplus consumption ratio positively comove. Hence, a higher value of ρ_{pd} is associated with a higher value of ρ_s. Intuitively, in a recession, the surplus consumption ratio is low, which implies that the marginal utility of consumption is high but is expected to revert back to its mean. Therefore, consumers are less willing to invest in risky assets and, in turn, the price-dividend ratio is low. So, the price-dividend ratio comoves with the surplus consumption ratio and inherits its persistence.

Given these parameters, in the third step, we calculate a conservative lower bound on the volatility of the job-finding rate implied by our model, which is given by $\sigma(\lambda_{wt}) = 5.11$ or 77% (5.11%/6.60%) of its baseline value. To help understand this third step, on the vertical axis of panel d) of Figure A.1, we plot this implied volatility against the value of the annualized habit persistence parameter.

So far, we have relied on one moment from the data—namely, the annual autocorrelation of the log price-dividend ratio—to assess our confidence in the value of the habit persistence parameter ρ_s. We now show that other asset-pricing moments imply even higher lower bounds for this moment. The other asset-pricing moments we use are two of our untargeted moments: the standard deviation of the excess return on stocks and the mean of the 20-year real bond yield.

In panel b) of Figure A.1, we show that our model implies that a lower value of the volatility of excess returns $\sigma(r_{emt})$ is associated with a higher value of habit persistence. Intuitively, a higher degree of habit persistence has two offsetting effects. First, it increases the volatility of the surplus consumption ratio directly because of the autoregressive form of this ratio, thereby increasing the range of movements of price-dividend ratio and excess returns and correspondingly increasing $\sigma(r_{emt})$. Second, a higher degree of habit persistence implies that in a recession, the price-dividend ratio will remain low for longer so that asset returns do not react as much to the state, which tends to lower the volatility of excess returns. Under our parameter values, the second effect dominates the first one and the model implies that higher habit persistence leads to a lower volatility of excess returns.

Since a higher volatility $\sigma(r_{emt})$ is associated with a lower degree of habit persistence (and a lower volatility of the job-finding rate), a natural lower bound for the habit persistence parameter ρ_s^{12} is provided by the estimated 95th percentile of the distribution of the sample estimator of $\sigma(r_{emt})$. In panel d) of Figure A.1, we see that using this percentile as targeted moment, the implied lower bound on the volatility of the job-finding rate is $\sigma(\lambda_{wt}) = 5.74$ or 87% (5.74%/6.60%) of its baseline value.

Panel c) of Figure A.1 further shows that according to our model, a lower expected return on 20-year bonds, $\mathbb{E}(r_{20t})$, is associated with a higher degree of habit persistence. Intuitively, in a recession, a higher degree of habit persistence implies that a consumer's sensitivity to risk $\lambda_a(s_t)$ remains high for longer. Hence, due to precautionary saving motives, consumers are willing to accept a lower yield on long term risk-free bonds. As a result, our model implies that a higher degree of habit persistence is associated with lower long-term bond yields. As clear from panel c) of Figure A.1, an obvious lower bound on the degree of habit persistence, as measured by ρ_s^{12}, is then given by the estimated 95th percentile of the distribution of 20-year bond yields. From panel d) of Figure A.1, it is immediate that if we use this percentile as targeted moment, then the implied lower bound on the volatility of the job-finding rate is $\sigma(\lambda_{wt}) = 6.18$ or 94% (6.18%/6.60%) of its baseline value.

We summarize all these results in Table A.9 and panel d) of Figure A.1. As this table and figure show, for several alternative ways of calculating a lower bound on the degree of habit persistence in consumption, our model still reproduces the bulk of the measured volatility of the job-finding rate.[1] In this sense, the finding that our model generates a degree of volatility of the job-finding rate similar to that in the data holds for a range of values of the habit persistence parameter consistent with the uncertainty in both the key targeted moment that pins down this parameter—namely, the annual persistence of the log-price dividend ratio—and in the alternative moments that can be used to discipline its value.

G Data

This appendix contains details about the data omitted from the paper.

[1] We have also considered more conservative bounds provided by the 1st and 99th percentiles, instead of the 5th and 95th percentiles, of the bootstrap distributions of the sample estimators of the moments considered. When we set the value of ρ_{pd} equal to the 1st percentile of the distribution of its sample estimator, $\sigma(\lambda_{wt})$ is equal to 67% of the volatility in our baseline. Likewise, when we set the value of $\sigma(r_{emt})$ equal to the 99th percentile of the distribution of its sample estimator, $\sigma(\lambda_{wt})$ is equal to 83% of the volatility in our baseline. Similarly, when we set the value of $\mathbb{E}(r_{20t})$ equal to the 99th percentile of the distribution of its sample estimator, $\sigma(\lambda_{wt})$ is equal to 90% of the volatility in our baseline.

G.1 Wage Growth on the Job and Wage Loss off the Job

Here we describe how we infer the rate of human capital accumulation on the job and depreciation off the job, respectively, from measured wage growth during employment and wage changes associated with spells of nonemployment.

Evidence on Wage Growth. In our baseline model, we measure the rate of human capital accumulation on the job based on the estimates of average annual wage growth by Rubinstein and Weiss (2006) from a sample of workers from the 1979-2000 waves of the NLSY, who are between 14 and 21 years of age in 1979 and are surveyed annually since the initial wave of the survey in 1979. These authors exclude from their sample the military sub-sample and the non-black, non-Hispanic disadvantaged samples, workers with a reported hourly wage lower than \$4 or higher than \$2,000 (adjusted for the 2000 CPI), and workers who work less than 35 weeks or less than 1,000 annual hours.[2] To be able to estimate wage growth, the authors restrict their sample according to these criteria in two consecutive years.

As reported in their Table 2b, Rubinstein and Weiss (2006) estimate that the average annual growth rate of real hourly wages is 7.7% for workers with up to 10 years of labor market experience, 3.3% for workers with 11 to 15 years of labor market experience, and 4.9% for workers with 16 to 25 years of labor market experience, gross of the annual growth rate of labor productivity. Given the difficulty of estimating growth rates at higher levels of experience than 25 years, we proceed by first interpolating these estimated growth rates by nonlinear least squares according to the following specification,

$$\Delta w_t \equiv f(x_t) = \log\left\{\exp(g_w) + \exp[\beta_0 + \beta_1 \exp(-\beta_2 x_t)]\right\} + \varepsilon_t, \tag{133}$$

where Δw_t denotes the growth rates of wages estimated by Rubinstein and Weiss (2006) and x_t denotes labor market experience. We then extrapolate growth rates for the missing experience years based on their predicted values from (133). To understand (133), note that the expression on the right-hand side of (133) is the so-called soft maximum between g_w, the growth rate estimated by Rubinstein and Weiss (2006) for the 16-25 years experience group, and the function $\beta_0 + \beta_1 \exp(-\beta_2 x_t)$. To see how the parameters $(\beta_0, \beta_1, \beta_2)$ are identified, observe that

$$m(x_t) \equiv \frac{\exp\{\mathbb{E}[f(x_t+1)]\} - \exp(g_w)}{\exp\{\mathbb{E}[f(x_t)]\} - \exp(g_w)} = \frac{\exp\left[\beta_0 + \beta_1 \exp(-\beta_2 x_t - \beta_2)\right]}{\exp\left[\beta_0 + \beta_1 \exp(-\beta_2 x_t)\right]} = e^{\beta_1[\exp(-\beta_2 x_t - \beta_2) - \exp(-\beta_2 x_t)]}.$$

Thus, the ratio $\log(m(x_t'))/\log(m(x_t))$ for any two x_t and x_t',

$$\frac{\log(m(x_t'))}{\log(m(x_t))} = \frac{\beta_1[\exp(-\beta_2) - 1]\exp(-\beta_2 x_t')}{\beta_1[\exp(-\beta_2) - 1]\exp(-\beta_2 x_t)} = \exp(-\beta_2(x_t' - x_t)),$$

identifies β_2. Once β_2 is identified, $m(x_t)$ or $m(x_t')$ identifies β_1 and $\exp\{\mathbb{E}[f(x_t)]\}$ evaluated at any other x_t identifies β_0. Based on this procedure, we estimate that the average annual growth rate of real hourly wages is 4.22% up to the first 17 years of labor market experience, and is 2.75% for the remaining years (up to 60). We use these statistics to discipline the growth rate of human capital on the job for young and mature workers in the life-cycle version of our model.

[2] They also omit observations on workers with missing data on own or parents' education, Armed Forces Qualification Test score, or labor market outcomes, and on individuals enrolled in schooling in a given year.

Evidence on Wage Losses. We discipline the rate of human capital depreciation off the job for the life-cycle version of our model based on the wage changes that workers experience upon spells of nonemployment, defined as episodes of either unemployment or nonparticipation. We measure the change in wages for workers who transit from employment to nonemployment and back to employment—so-called E-N-E transitions—as the percentage difference between the first wage in the first employment spell *after* a nonemployment spell and the last wage in the last employment spell *before* a nonemployment spell.

To this purpose, given the typical short duration of nonemployment spells in U.S. labor markets, we use monthly data from the Panel Study of Income Dynamics (PSID) family and individual (merged) files—we follow the same strategy as in Krolikowski (2017). To elaborate, the PSID starts in 1968 with an interview of approximately 5,000 families, and follows any new families formed from this original group. In the survey years prior to 1988, though, the PSID did not collect workers' monthly information on their employment status at different employers so it is not possible to calculate monthly employment spells for these years. Moreover, although interviews were conducted annually between 1968 and 1997, since 1997 interviews have been biennial. For these reasons, we only use data from the 1988-1997 waves. In order to obtain results that are comparable to alternative data sources and to focus, in accord with our model, on workers more likely to be attached to the labor market, we restrict the sample to working-age males aged 18 through 65. We omit observations on individuals who are self-employed, and use individual weights to account for the PSID's poverty over-sample and nonrandom attrition.[3]

An important feature of the data for our analysis is that for the years between 1988 and 1997, respondents were asked to report their employment status in each month of the previous calendar year, as well as monthly employment information for up to two main employers. From this monthly information, it is then possible to determine the actual length of workers' employment and nonemployment spells.[4] The data also include detailed information on wages. Specifically, for employed individuals, the data provide the starting wage at a worker's current employer as well as the ending wage at the worker's former employer. Similarly, for nonemployed individuals, the data record the starting wage at a worker's former employer and the ending wage at the worker's employer before the former one. Combined with information on workers' labor force status, this information allows us to calculate wage changes associated with the E-N-E transitions of interest.

Net of an annual growth rate of labor productivity of 2.2%, we estimate the average change in hourly wages between the *first* wage after a complete spell of nonemployment and the *last* wage before such a spell to be -0.99% for workers with less than 17 years of labor market experience and -18.20% for workers with more than 17 years of labor market experience.

G.2 Job-Finding Rate and Unemployment Rate

We rely on the method in Shimer (2012) to compute the job-finding rate and the unemployment rate overall and by experience groups. Specifically, we use aggregate monthly data constructed by the Bureau of Labor Statistics (BLS) based on data from the Current Population Survey (CPS) as well

[3] We include the supplementary low income sample and the 1997 immigrant sample in the analysis, but exclude the Latino sample introduced in 1990. We are grateful to Pawel Krolikowski for his help in the use of these data.

[4] Transition probabilities based on such a sample are broadly consistent with those obtained from the Survey of Income and Program Participation (SIPP) and the Current Population Survey (CPS).

as CPS micro data between 1948 and 2007. We calculate quarterly averages of monthly job-finding rates as in Shimer (2012) and corresponding constant-separation unemployment rates, as described in the paper. For the experience-specific counterparts of these statistics, we use BLS aggregate monthly data between 1976 and 2007, since information on short-term unemployment, which we use to compute job-finding rates, is unavailable before 1976. Given that the BLS provides the necessary data only for selected age ranges, we consider those age groups below and above 35 years of age that map into the experience groups of interest, as discussed in the paper. Moreover, since the age-specific short-term unemployment data from the BLS are available only non-deseasonalized, we apply to these data the same deseasonalization procedure that the BLS applies using the 13ARIMA-SEATS (X-13) seasonal adjustment program. We adjust the short-term unemployment series for the 1994 redesign of the CPS using the CPS microdata following the approach in Elsby, Michaels, and Solon (2009).

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Table A.1: Alternative parametrization and results for model with exogenously-driven habit preferences

Panel A: Parameters		Panel B: Moments		
Endogenously chosen		*Targeted*	Data	Model
g_a, mean productivity growth (% p.a.)	2.28	Mean productivity growth (% p.a.)	2.28	2.28
σ_a, s.d. productivity growth (% p.a.)	1.84	S.d. productivity growth (% p.a.)	1.84	1.84
κ, hiring cost	1.93	Mean unemployment rate (%)	5.9	5.9
β, time preference factor	0.999	Mean risk-free rate (% p.a.)	0.74	0.74
$\bar{S}$, mean of state S_t	0.20	S.d. risk-free rate (% p.a.)	2.35	2.35
α, inverse EIS	5.25	Sharpe ratio of market excess return (p.a.)	0.45	0.45
Assigned		*Labor market results*		
B, efficiency of matching technology	0.46	Mean job-finding rate (%)	45.35	48.69
b, home production parameter	0.6	S.d. job-finding rate (%)	6.67	6.60
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate (%)	0.79	0.77
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.96	0.99
ρ_s, persistence of state	0.9944	Correlation unemployment and job-finding rate	-0.96	-0.98
g_e, human capital growth on job (% p.a.)	1.5			
g_u, human capital loss off job (% p.a.)	-7.9			
		Asset market results		
		Mean excess return (% p.a.)	7.05	5.50
		S.d. excess return (% p.a.)	15.6	12.4
		Mean log price-dividend ratio	3.51	3.55
		S.d. log price-dividend ratio	0.44	0.37

Table A.2: Implications of search economies vs. endowment economies for stock and bond prices

	Data	Baseline	CC	EZ w/ LRR	EZ w/ disasters	Affine SDF
Search Economies						
Mean excess return (% p.a.)	7.05	6.15	6.32	4.26	4.77	6.92
S.d. excess return (% p.a.)	15.6	13.7	14.1	9.50	10.7	15.6
Mean log price-dividend ratio	3.51	3.42	3.41	3.85	3.76	3.29
S.d. log price-dividend ratio	0.44	0.38	0.39	0.31	0.36	0.38
Mean 20-year real yield (% p.a.)	4.74	3.55	3.70	2.79	-1.62	4.10
S.d. 20-year real yield (% p.a.)	1.95	2.24	2.40	1.26	2.15	2.07
Mean 20-year nominal yield (% p.a.)	7.71	7.53	7.67	6.48	2.04	8.19
S.d. 20-year nominal yield (% p.a.)	2.41	2.30	2.47	1.28	2.16	2.20
Endowment Economies						
Mean excess return (% p.a.)	7.05	6.74	6.74	4.47	4.80	6.92
S.d. excess return (% p.a.)	15.6	15.0	15.0	9.84	10.6	15.5
Mean log price-dividend ratio	3.51	3.30	3.30	3.78	3.75	3.29
S.d. log price-dividend ratio	0.44	0.36	0.36	0.31	0.35	0.37
Mean 20-year real yield (% p.a.)	4.74	4.25	4.25	2.79	-1.67	4.10
S.d. 20-year real yield (% p.a.)	1.95	2.42	2.42	1.26	2.15	2.07
Mean 20-year nominal yield (% p.a.)	7.71	8.22	8.22	6.54	1.99	8.19
S.d. 20-year nominal yield (% p.a.)	2.41	2.49	2.49	1.28	2.16	2.20

Table A.3: Parametrization and results for model with Campbell-Cochrane preferences with external habit

Panel A: Parameters		Panel B: Moments		
Endogenously chosen		*Targeted*	Data	Model
g_a, mean productivity growth (% p.a.)	2.28	Mean productivity growth (% p.a.)	2.28	2.28
σ_a, s.d. productivity growth (% p.a.)	1.84	S.d. productivity growth (% p.a.)	1.84	1.84
κ, hiring cost	1.01	Mean unemployment rate (%)	5.9	5.9
β, time preference factor	0.999	Mean risk-free rate (% p.a.)	0.74	0.74
$\bar{S}$, mean of state S_t	0.14	S.d. risk-free rate (% p.a.)	2.35	2.35
α, inverse EIS	4.75	Sharpe ratio of market excess return (p.a.)	0.45	0.45
Assigned		*Labor market results*		
B, efficiency of matching technology	0.46	Mean job-finding rate (%)	45.35	49.87
b, home production parameter	0.6	S.d. job-finding rate (%)	6.67	6.66
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate (%)	0.79	0.77
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.96	0.99
ρ_s, persistence of state	0.9944	Correlation unemployment and job-finding rate	-0.96	-0.98
g_e, human capital growth on job (% p.a.)	3.5			
		Asset market results		
		Mean excess return (% p.a.)	7.05	6.32
		S.d. excess return (% p.a.)	15.6	14.1
		Mean log price-dividend ratio	3.51	3.41
		S.d. log price-dividend ratio	0.44	0.39

Table A.4: Parametrization and results for model with Epstein-Zin preferences with long-run risk

Panel A: Parameters		Panel B: Moments		
Endogenously chosen		*Targeted*	Data	Model
g_a, mean productivity growth (% p.a.)	2.28	Mean productivity growth (% p.a.)	2.28	2.28
σ_a, s.d. productivity growth (% p.a.)	1.80	S.d. productivity growth (% p.a.)	1.84	1.84
κ, hiring cost	1.31	Mean unemployment rate (%)	5.9	5.9
β, time preference factor	0.998	Mean risk-free rate (% p.a.)	0.74	0.74
ϕ_s, relative standard deviation of s_t	0.0379	S.d. risk-free rate (% p.a.)	2.35	2.35
α, risk aversion coefficient	4.35	Sharpe ratio of market excess return (p.a.)	0.45	0.45
Assigned		*Labor market results*		
B, efficiency of matching technology	0.46	Mean job-finding rate (%)	45.35	49.92
b, home production parameter	0.6	S.d. job-finding rate (%)	6.67	6.60
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate (%)	0.79	0.70
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.96	0.99
ρ_x, persistence of x_t	0.9977	Correlation unemployment and job-finding rate	-0.96	-0.98
ρ_s, persistence of s_t	0.9944			
g_e, human capital growth on job (% p.a.)	3.5	*Asset market results*		
ρ, inverse EIS	0.1			
Relative size of x_t component of productivity	0.0445	Mean excess return (% p.a.)	7.05	4.26
		S.d. excess return (% p.a.)	15.6	9.50
		Mean log price-dividend ratio	3.51	3.85
		S.d. log price-dividend ratio	0.44	0.31

Table A.5: Parametrization and results for model with Epstein-Zin preferences with variable disaster risk

Panel A: Parameters		Panel B: Moments		
Endogenously chosen		*Targeted*	Data	Model
g_a, mean productivity growth (% p.a.)	2.28	Mean productivity growth (% p.a.)	2.28	2.28
σ_a, s.d. productivity growth (% p.a.)	1.84	S.d. productivity growth (% p.a.)	1.84	1.84
κ, hiring cost	1.27	Mean unemployment rate (%)	5.9	5.9
β, time preference factor	0.999	Mean risk-free rate (% p.a.)	0.74	0.74
σ_s, disaster intensity volatility parameter	0.0083	S.d. risk-free rate (% p.a.)	2.35	2.35
α, risk aversion coefficient	2.65	Sharpe ratio of market excess return (p.a.)	0.45	0.45
Assigned		*Labor market results*		
B, efficiency of matching technology	0.46	Mean job-finding rate (%)	45.35	50.03
b, home production parameter	0.6	S.d. job-finding rate (%)	6.67	5.66
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate (%)	0.79	0.78
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.96	0.99
ρ_s, persistence of disaster intensity	0.9966	Correlation unemployment and job-finding rate	-0.96	-0.98
g_e, human capital growth on job (% p.a.)	3.5			
ρ, inverse EIS	0.1	*Asset market results*		
s, mean disaster intensity (% p.a.)	3.55			
θ, disaster impact	0.26	Mean excess return (% p.a.)	7.05	4.77
		S.d. excess return (% p.a.)	15.6	10.7
		Mean log price-dividend ratio	3.51	3.76
		S.d. log price-dividend ratio	0.44	0.36

Table A.6: Parametrization and results for model with affine stochastic discount factor

Panel A: Parameters		Panel B: Moments		
Endogenously chosen		*Targeted*	Data	Model
g_a, mean productivity growth (% p.a.)	2.28	Mean productivity growth (% p.a.)	2.28	2.28
σ_a, s.d. productivity growth (% p.a.)	1.84	S.d. productivity growth (% p.a.)	1.84	1.84
κ, hiring cost	0.93	Mean unemployment rate (%)	5.9	5.9
μ_0, level mean parameter	0.00062	Mean risk-free rate (% p.a.)	0.74	0.74
μ_1, slope mean parameter	-0.039	S.d. risk-free rate (% p.a.)	2.35	2.35
γ_0, level variance parameter	25.8	Sharpe ratio of market excess return (p.a.)	0.45	0.45
γ_1, slope variance parameter	0.85	S.d. excess return	15.6	15.6
Assigned		*Labor market results*		
B, efficiency of matching technology	0.46	Mean job-finding rate (%)	45.35	49.56
b, home production parameter	0.6	S.d. job-finding rate (%)	6.67	7.08
σ, probability of separation	0.028	Autocorrelation job-finding rate	0.94	0.99
η, matching function elasticity	0.5	S.d. unemployment rate (%)	0.79	0.72
ϕ, survival probability	0.9972	Autocorrelation unemployment rate	0.96	0.99
ρ_s, persistence of state	0.9944	Correlation unemployment and job-finding rate	-0.96	-0.98
g_e, human capital growth on job (% p.a.)	3.5			
		Asset market results		
		Mean excess return (% p.a.)	7.05	6.92
		S.d. excess return (% p.a.)	15.6	15.6
		Mean log price-dividend ratio	3.51	3.29
		S.d. log price-dividend ratio	0.44	0.38

Table A.7: Accuracy of approximations for job-finding rate in the baseline model

Approximation	Linear around RSS	Linear around RSS and $\lambda_{wt+n} \approx \lambda_w$
$\sigma(\lambda_{wt}^{approx})/\sigma(\lambda_{wt}^{global})$	1.10	2.18
$corr(\lambda_{wt}^{approx}, \lambda_{wt}^{global})$	0.99	0.99

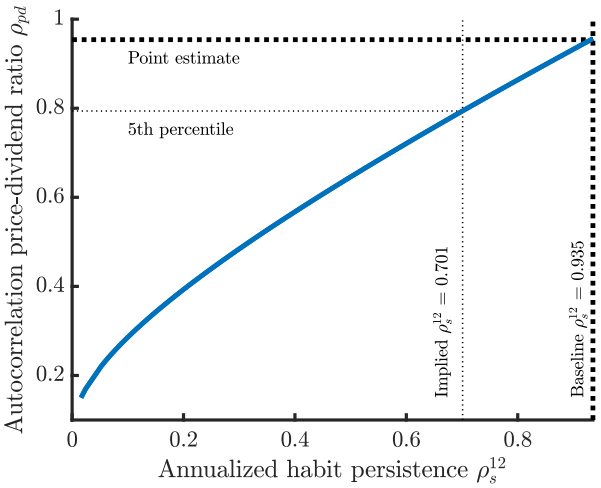
Table A.8: Estimated parameters for inflation process

Parameters	Estimates
$\bar{\pi}$, mean inflation rate	0.0031
ϕ, autocorrelation coefficient	0.9479
ψ, relative volatility of latent variable	0.2499
σ_w, volatility of inflation shocks	0.0027
$\rho_{\pi a}$, correlation inflation shocks and productivity shocks	−0.1620

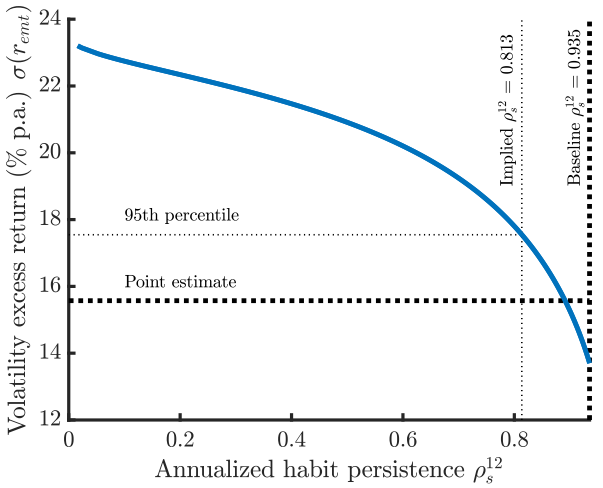
Table A.9: Sensitivity analysis of volatility of job-finding rate

	5th or 95th percentile of estimated moment	Implied 5th percentile of ρ_s^{12}	Implied $\sigma(\lambda_{wt})$	Fraction of baseline $\sigma(\lambda_{wt})$
ρ_{pd}, autocorrelation log price-dividend ratio	0.794 (5th)	0.701	5.11	77%
$\sigma(r_{emt})$, std. dev. market excess return (%)	17.55 (95th)	0.826	5.74	87%
$E(r_{20t})$, mean 20-year real yield (% p.a.)	5.93 (95th)	0.877	6.18	94%

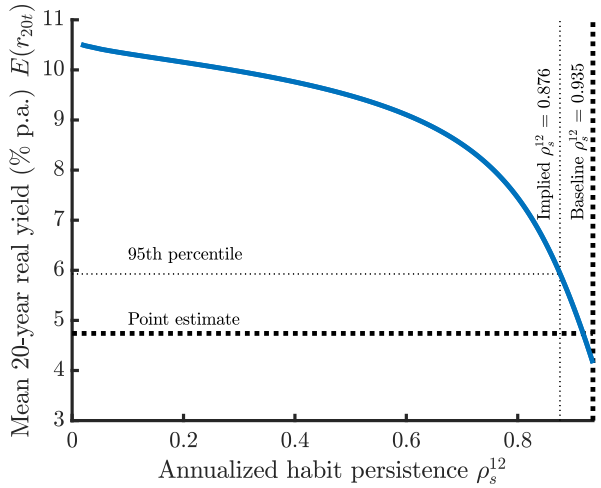
Figure A.1: Lower bounds on habit persistence based on asset-pricing moments



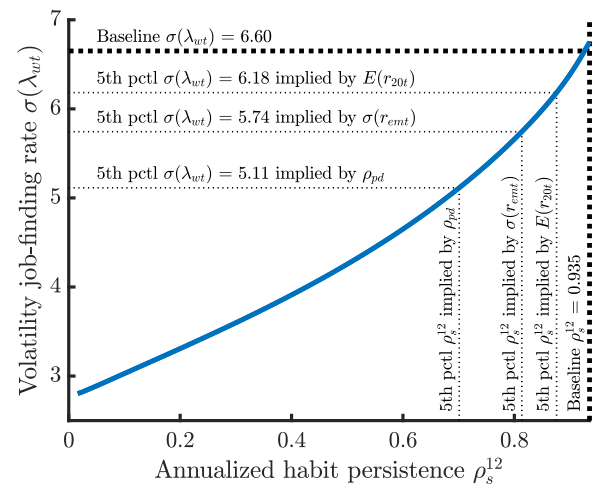
(a) Lower bound based on autocorrelation of price-dividend ratio



(b) Lower bound based on volatility of excess market return



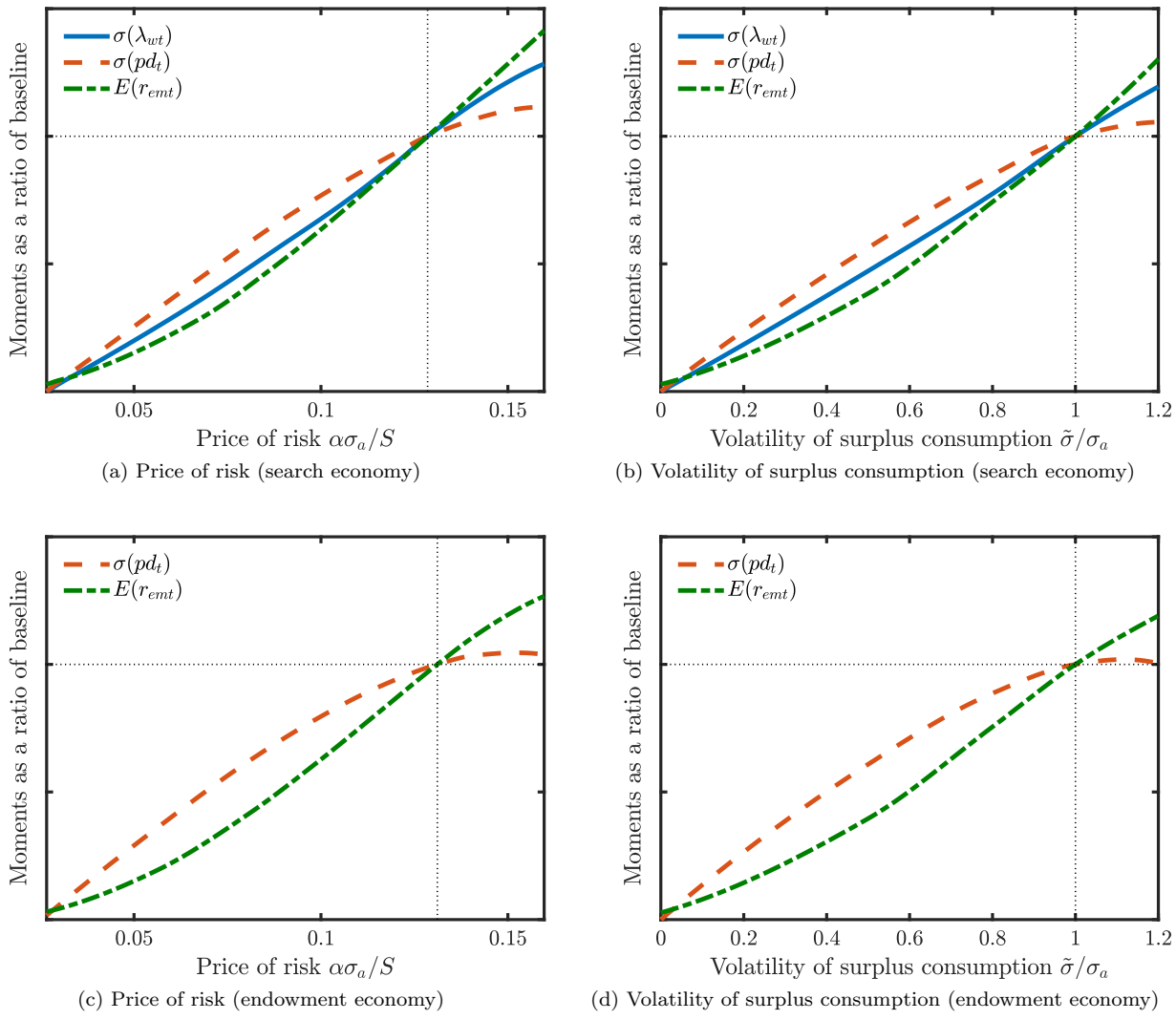
(c) Lower bound based on mean 20-year real yield



(d) Summary of lower bounds from asset-pricing statistics

Note: We use asset-pricing moments to construct lower bounds on the volatility of the job-finding rate. Dotted lines show percentiles of the bootstrap distribution of the estimator of ρ_{pd}, $\sigma(r_{emt})$, and $E(r_{20t})$ and the associated values of habit persistence and the volatility of the job-finding rate. The solid line shows the value of the volatility of the job-finding rate as habit persistence increases and the other endogenous parameters are chosen so as to reproduce the other targeted moments.

Figure A.2: Sensitivity of key moments to preference parameters in baseline and endowment economies



(a) Price of risk (search economy)

(b) Volatility of surplus consumption (search economy)

(c) Price of risk (endowment economy)

(d) Volatility of surplus consumption (endowment economy)

Note: λ_{wt} denotes the job-finding rate, pd_t denotes the log price-dividend ratio of the consumption claim, and r_{emt} denotes the market excess return, measured in the model as the excess return on a consumption claim. As the relevant parameter varies, the vacancy posting cost κ is adjusted so as to keep the mean unemployment rate constant at its baseline value.