

APPENDICES

APPENDIX 1: Household optimization problem

At date 0, the household solves a consumption-saving and portfolio allocation problem, given the financial contracts available to it. Namely, it chooses consumption at each date and in each state $\{\{c_t(s)\}_s\}_t$, and how to allocate its date 0 savings across investment banks i , described by weights the indicator functions f_0^i which take the value of 1 if the household accepts bank i 's contract and 0 otherwise. Given d_0^i , the total amount of funds the household invests in bank i is given by $f_0^i d_0^i$, and aggregate date 0 saving is then $\sum_i f_0^i d_0^i$.¹ We further assume that banks cannot commit at date 0 to investing in particular projects at date 1. Therefore, the household has no information on which projects each bank will invest in at date 1. As a result, the household chooses f_0^i based only on the contract $(d_0^i, \{d_1^i(s), d_2^i(s)\}_s)$ offered by each bank.

At date 1, the household also chooses its date 1 bond holdings to maximize expected utility subject to its budget constraint each period.

$$\max_{\{\{c_t(s)\}_s\}_t, \{f_0^i\}_i, \{B_1(s)\}_s} E[u(c_0) + u(c_1(s)) + u(c_2(s))]$$

$$c_0 + \sum_i f_0^i d_0^i \leq e_0 - T_0 \tag{1}$$

$$c_1(s) + B_1(s) \leq e_1 + \sum_i f_0^i d_1^i(s) - T_1 - q(s)k_1^T(s) \tag{2}$$

$$c_2(s) \leq e_2 + B_1(s) + \sum_i f_0^i d_2^i(s) + \Pi_2(s) - T_2 \tag{3}$$

Here, $\Pi_2(s)$ is the date 2 profits of all banks and traditional firms in state s , and T are lump-sum taxes. Let $e_0 = e_1 = e_2$. Also assume that e is sufficiently large that non-negativity constraints for c_0, c_1 , and c_2 are never binding. The first-order conditions for f_0^i and the date 1 bond holdings are

$$u'(c_0)d_0^i \geq E[u'(c_1(s))d_1^i(s) + u'(c_2(s))d_2^i(s)] \tag{4}$$

$$u'(c_1(s)) = u'(c_2(s)) \tag{5}$$

¹The household's problem is equivalent to a consumption CAPM in which the household simultaneously solves a consumption-savings and portfolio allocation problem, in which it chooses total savings and the share of savings allocated to each bank i .

APPENDIX 2: Contracting environment between the household and banks

Timing of the contracting problems Recall that there are two types of contracting problems: one between households and banks, and another between banks. These contracting problems are solved simultaneously at date 0: at the same time that consumers agree with banks about state contingent payments $\{d_1^i(s), d_2^i(s)\}_s$, the banks make loans ℓ^{ij} to one another in exchange for state contingent payments $\ell^{ij}r^{ij}(s)$ at date 1. Below we describe the contracting environment between households and banks, and how it interacts with the the contracting problem between banks on the interbank market.

Contracting problem between the household and banks At date 0, each bank i may offer the household a contract which specifies an initial loan d_0^i from the household and a set of state-contingent repayments $\{d_1^i(s), d_2^i(s)\}_s$ to the household at dates 1 and 2. We assume that both the household and banks have a limited ability to commit to honoring the contract at dates 1 and 2.

In particular, at dates 1 and 2, the bank chooses whether to honor the contract and make payments $d_1^i(s)$ and $d_2^i(s)$ to the household. If the bank does not pay, it makes the household a take-it-or-leave-it offer regarding the date 1 and 2 payments. If the household refuses the offer, the bank is liquidated. In the event of liquidation, there is a kind of ‘pecking order’ among bank i ’s claimants: the household can seize the bank’s net capital holdings (described below); any of the bank’s interbank obligations must be paid out of the bank’s remaining assets.²

How much of a defaulting bank’s assets can the household seize at date 1? Although the precise assumptions about this are not critical for our results, we assume the following: In the event of liquidation at date 1, the household can seize bank i ’s own capital holdings k_0^i and also the capital holdings that i lent to other banks j in the interbank market at date 0 $\sum_j \ell^{ij}$. However, the household cannot seize the capital that bank i borrowed from other banks j in the interbank market $\sum_j \ell^{ji}$.

In addition, we assume the household can seize a fraction $\Gamma < 1$ of the bank’s profits date 2 profits, where Γ satisfies $\Gamma < \underline{q}$. Any profits not seized by the household is retained by the bank. (While bank profits eventually find their way to to the household in the form of dividends at date 2, this general equilibrium result is not internalized by the atomistic households).

Any assets that the household seizes can be converted to capital and invested in the date 1 project, after incurring the maintenance cost γ . Therefore, the value to the household of a liquidated bank i at date 1 is $(q(s) - \gamma)(k_0^i + \sum_j [\ell^{ij} - \ell^{ji}])$, and at date 2 it is $\Gamma k_1^i(s)$. Then bank i never defaults in equilibrium if and only if the following conditions are met: in each period, the value of repayment does not exceed the liquidation value to the household of bank i .

²That the household is the first claimant on the bank’s assets is not important for the results, but simplifies the model, since the optimal household contract doesn’t depend directly on the solution to the interbank contracting problems.

$$d_1^i(s) + d_2^i(s) \leq (q(s) - \gamma) \left(k_0^i + \sum_j [\ell^{ij} - \ell^{ji}] \right) \quad (6)$$

$$d_2^i(s) \leq \Gamma k_1^i(s) \quad (7)$$

Similarly, the household can always walk away from the contract without consequence. Therefore, the household does not default in equilibrium if and only if two conditions hold.

$$0 \leq d_1^i(s) + d_2^i(s) \quad (8)$$

$$0 \leq d_2^i(s) \quad (9)$$

We can scale the contract by the value of i 's net capital holdings at dates 0 and 1 in units of the numeraire, so that the contract is denoted $(d_0^i, \{b_1^i(s), b_2^i(s)\})$ where $b_1^i(s)$ and $b_2^i(s)$ are given by $b_1^i(s) \equiv \frac{d_1^i(s) + d_2^i(s)}{k_0^i + \sum_j [\ell^{ij} - \ell^{ji}]}$ and $b_2^i(s) \equiv \frac{d_2^i(s)}{k_1^i(s)}$. (Using each bank's binding date 0 budget constraint, $b_1^i(s)$ can equivalently be expressed as $b_1^i(s) \equiv \frac{d_1^i(s) + d_2^i(s)}{n + d_0^i}$.) Then we can rewrite the no-default constraints (26)-(29) as

$$0 \leq b_1^i(s) \leq q(s) - \gamma \quad (10)$$

$$0 \leq b_2^i(s) \leq \Gamma. \quad (11)$$

To entice the household to accept the contract, bank i 's contract must satisfy a participation constraint, which is the household's optimality condition (1).

Interaction between the contracting problems The above discussion implies that the two limited enforcement problems interact in two ways. First, the limited enforcement constraints between households and banks depend on banks' interbank exposures through ℓ^{ij} . (This can be seen from (26) and the definition of $b_1^i(s)$.) Second, interbank contracts must respect the 'pecking order' among a bank's claimants. In other words, in the event that a bank defaults on its obligations to the household at date 1 and its assets are seized, the bank's remaining assets must be sufficient to meet its interbank obligations. As a result, optimal interbank contracts are contingent not only on the state of world, but also on whether the bank defaults on the household at date 1 or not. This ensures that banks always have the resources to meet their interbank obligations in all states and contingencies.

In equilibrium, banks never default on their obligations to households. So for simplicity of

exposition, in the paper we characterize the optimal interbank contracts only for the case in which banks do not default on the household (given in Lemma 3), and we omit the off-equilibrium case in which the banks default on the household and their assets are seized.

Externalities and the household contracting problem Note that both banks and their household creditors fully internalize that by choosing a risky portfolio of interbank claims, the banks are changing the state contingent payoff profile. In this sense, banks act in the best interest of the households that hold their claims. However, while individual, atomistic lenders do not internalize how the bank's investments affect the marginal utility of consumption in general equilibrium. In this sense, the externalities in the paper are all at the general equilibrium level, and do not derive directly from the contracting friction between the household and banks.

APPENDIX 3: Bank optimization problems

We can now put these elements together to solve each bank's optimization problem. At date 0, each bank i chooses the financial contract $(d_0^i, \{b_1^i(s), b_2^i(s)\})$ with the household, the financial contract $\{\ell^{ji}, r^{ji}(s)\}_s$ with each other bank j , how much to lend to other banks $\{\ell^{ij}\}_j$, investment levels $k_0^i, k_1^i(s)$, and portfolio allocation ω^i across projects, to maximize the value of its investment bank. Here, $m_2(s)$ denotes the stochastic discount factor at date 2 given state s , and reflects the risk-aversion of the household.

$$\max E_0 [m_2(s) (1 - b_2^i(s)) k_1^i(s)] \quad (12)$$

subject to budget constraints

$$k_0^i + \sum_j \ell^{ij} \leq n + d_0^i + \sum_j \ell^{ji} \quad (13)$$

$$q(s)k_1^i(s) \leq \theta_k^i(s, \omega^i, g^i) k_0^i + \sum_j \theta_\ell^i(s, j) \ell^{ij} - \sum_h (r^{hi}(s) - b_1^i(s)) \ell^{hi} + b_2^i(s) k_1^i(s) \quad (14)$$

no-default constraints for the household contract

$$0 \leq b_1^i(s) \leq q(s) - \gamma \quad (15)$$

$$0 \leq b_2^i(s) \leq \Gamma \quad (16)$$

the household participation constraint, where we have combined the household's optimality conditions (1) and (2)

$$u'(c_0)d_0^i \geq E [u'(c_1(s))b_1^i(s)] \left(k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij} \right) \quad (17)$$

and the other banks' participation constraints for each j

$$u^{ji}(\ell^{ji}, \{r^{ji}(s)\}_s) \geq \bar{u}^j \quad (18)$$

and non-negativity constraints on capital holdings and inter-bank loans.

$$k_0^i, k_1^i(s), \ell^{ij} \geq 0 \quad \forall j \quad (19)$$

Let $z_0^i, z_1^i(s), \bar{\lambda}^i(s), \underline{\lambda}^i(s), \bar{\mu}^i(s), \underline{\mu}^i(s)$, and v^{ji} denote Lagrange multipliers on the date 0 budget constraint (33), the date 1 budget constraint (34), the upper and lower bounds on $b_1^i(s)$, the upper and lower bounds on $b_2^i(s)$, and bank j 's participation constraint (38) respectively. Also, let $g^i(s, k_0^i, \omega^i)$ denote the derivative of the government transfer g^i to bank i with respect to ω^i , which represents how a marginal increase in ω^i affects the bailout that i receives conditional on i being bailed out. (Importantly, this may in general depend on not only the state of the world and i 's investment, but also on the investment decisions ω^j of all other banks j .) Because the household has access to a riskless bond at date 1 with gross return 1, and all uncertainty is resolved in date 1, we will have in equilibrium

$$u'(c_2(s)) = u'(c_1(s)). \quad (20)$$

The optimality conditions are then given by

$$\frac{\partial L^i}{\partial k_0^i} \leq 0 \iff z_0^i \left(\frac{1}{u'(c_0)} E [u'(c_1(s))b_1^i(s)] - 1 \right) + E [z_1^i(s)\theta_k^i(s, \omega^i, g^i)] \leq 0 \quad (21)$$

$$\frac{\partial L^i}{\partial k_1^i(s)} \leq 0 \iff m_2(s)(1 - b_2^i(s)) \leq z_1^i(s)(q(s) - b_2^i(s)) \quad (22)$$

$$\frac{\partial L^i}{\partial b_1^i(s)} \leq 0 \iff \left[\frac{u'(c_1(s))}{u'(c_0)} z_0^i - z_1^i(s) \right] \left(k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij} \right) \leq \lambda_1^i(s) - \lambda_0^i(s) \quad (23)$$

$$\frac{\partial L^i}{\partial b_2^i(s)} \leq 0 \iff [z_1^i(s) - m_2(s)] k_1^i(s) \leq \mu_1^i(s) - \mu_0^i(s) \quad (24)$$

$$\frac{\partial L^i}{\partial \omega^i} \leq 0 \iff E \left[z_1^i(s) k_0^i \frac{\partial \theta_k^i(s, \omega^i, g^i)}{\partial \omega^i} \right] \leq 0 \quad (25)$$

$$\frac{\partial L^i}{\partial \ell^{ij}} \leq 0 \iff E \left[z_1^i(s) \theta_\ell^i(s, j) \right] \leq z_0^i \left(1 - E \left[\frac{u'(c_1(s))}{u'(c_0)} b_1^i(s) \right] \right) \quad (26)$$

$$\frac{\partial L^i}{\partial r^{ji}(s)} \leq 0 \iff -v^{ji} \frac{\partial u^{ji}(\ell^{ji}, \{r^{ji}(s)\}_s)}{\partial r^{ji}(s)} \leq \pi(s) z_1^i(s) \ell^{ji} \quad (27)$$

APPENDIX 4: Optimal household contract

Notice from (42) and (44) that when the optimality condition for $k_1^i(s)$ holds, that for $b_2^i(s)$ cannot hold since $b_2^i(s) \leq \Gamma < 1$ and $q(s) \leq 1$. Therefore, given that in equilibrium the optimality condition for $k_1^i(s)$ holds, we have $z_1^i(s) \geq m_2(s)$. Although $b_2^i(s) \in [0, \Gamma]$ when $z_1^i(s) = m_2(s)$, we assume for simplicity it is at its upper bound in this situation. (This does not affect our main results.) Consequently, we always have a corner solution for $b_2^i(s)$ as it is set at its maximum.

$$b_2^i(s) = \Gamma \quad (28)$$

And since $m_2(s) > 0$ by the Inada condition of $u(\cdot)$, it follows that $z_1^i(s) > 0$, so that i 's date 1 budget constraint always binds in equilibrium.

Notice from i 's optimality condition for $b_1^i(s)$, the household's optimality condition for the bond and the definition of the stochastic discount factor $m_2(s) = \frac{u'(c_2(s))}{u'(c_0)}$, we can write (27) the optimality condition for $b_1^i(s)$ as

$$z_0^i m_2(s) \leq z_1^i(s) \quad (29)$$

Then $b_1^i(s)$ is set at its maximum $q(s) - \gamma$ (a corner solution) if and only if $z_0^i > \frac{z_1^i(s)}{m_2(s)} = \frac{1-\Gamma}{q(s)-\Gamma}$, at its minimum 0 (corner solution) if and only if $z_0^i < \frac{1-\Gamma}{q(s)-\Gamma}$, and is indeterminate if and only if $z_0^i = \frac{1-\Gamma}{q(s)-\Gamma}$. Lemma 1 characterizes the individually optimal financial contract in light of these conditions.

APPENDIX 5: Proof of Lemma 3

Proof: The proof relies on two results. First, perfect competition between atomistic banks implies that, in each state, the interbank contract issued from any i to h equates the return on the contract to h to the return on i 's assets in each state, such that $\theta_\ell^h(s, i) = \theta^i(s)$. Second, Lemma 1 showed that each bank is always at a corner solution in its portfolio choice. This implies only one contract

accepted: the contract with highest private valuation $E [z_1(s)\theta_\ell^h(s,i)]$. It follows that $\theta_\ell^h(s,i) = \theta_k^w(s, \omega^w, g^w)$, where $W \equiv \{w \mid E [z_1(s)\theta_k^w(s, \omega^w, g^w)] \geq E [z_1(s)\theta_k^i(s, \omega^i, g^i)] \forall i \in I\}$.

APPENDIX 6: Aggregate investment at date 1

In order to evaluate the date 1 spot market for capital, we first characterize aggregate net investment in capital at date 1. Consider net aggregate investment by all banks in state s at date 1, defined as the difference between aggregate capital holdings at date 1 and aggregate date 0 holdings of capital, $K_1(s) - K_0$, where we have defined $K_0 \equiv \sum_i k_0^i$ and $K_1(s) \equiv \sum_i k_1^i(s)$ to be the aggregate capital holdings of the banking sector at dates 0 and 1, respectively. We can write aggregate net investment in state s as

$$K_1(s) - K_0 = \sum_i \Delta^i(s, \omega^i, g^i) \quad (30)$$

where $\Delta^i(s, \omega^i, g^i) \equiv k_1^i(s) - [n + d_0^i] = k_1^i(s) - [k_0^i + \sum_h \ell^{ih} - \sum_h \ell^{hi}]$ denotes the difference between bank i 's choice of date 1 capital $k_1^i(s)$ and its date 0 funds $n + d_0^i$ available for investment in any asset.³

This object can be derived from each bank i 's date 1 budget constraint in state s , after imposing the partial equilibrium characterization of optimal interbank contracts given in Lemma 3 $\theta_\ell^i(s) = \theta_k^j(s) = \theta_k^w(s, \omega^w, g^w)$ for all i, j in the set of intermediary banks L .

$$K_1(s) - K_0 = \sum_i \Delta^i(s, \omega^w, g^w) = K_0 \left[\frac{\theta_k^w(s, \omega^w, g^w)}{q(s) - \Gamma} - 1 \right] \quad (31)$$

Equation (51) says that aggregate net investment in capital by the banking sector at date 1 is given by the aggregate rate of return on capital holdings at date 1, discounted by the cost of capital at date 1. At date 1, the aggregate rate of return on banks' date 0 capital holdings K_0 is given by the rate of return earned by bank w 's assets $\theta_k^w(s, \omega^w, g^w)$. Since banks do not pay out dividends at date 1, this return is invested in capital at date 1. The cost of capital at date 1 is given by the spot price $q(s)$ net of the date 2 repayment to the household $b_2^i = \Gamma$. Therefore, the aggregate net investment in new capital by the banking sector at date 1 is given by (51).

APPENDIX 7: Proof of Lemma 4

Proof of Part (A)

Recall that the assumption that the consumption good can be costlessly converted into the capital good one-for-one, but not vice versa, implies $q(s) \leq 1$. This also implies that aggregate invest-

³To see this, first note that we can re-write aggregate date 0 holdings of capital as $\sum_i k_0^i = \sum_i [n + d_0^i + \sum_j (\ell^{ji} - \ell^{ij})] = \sum_i [n + d_0^i] + \sum_i \sum_j (\ell^{ji} - \ell^{ij}) = \sum_i [n + d_0^i] = \sum_i [k_0^i + \sum_h \ell^{ih} - \sum_h \ell^{hi}]$. Given our definition of $D^i(s, \omega^i, g^i)$, it follows that aggregate net investment can be written as $\sum_i k_1^i(s) - \sum_i k_0^i = \sum_i D^i(s, \omega^i, g^i)$.

ment cannot be negative in equilibrium, i.e. $k_1^T(s) + \sum_i (k_1^i(s) - \chi(s)k_0^i) \geq 0$. If aggregate investment is strictly positive, then $q(s) = 1$ by arbitrage, and so equation (4) implies that $k_1^T(s) = 0$ since $1 = F'(0)$. If, on the other hand, aggregate investment is 0, then we have $k_1^T(s) = \sum_i (k_0^i - k_1^i(s))$. These two cases imply that $q(s) = F'(k_1^T(s))$ and $k_1^T(s) = \max\{0, \sum_i k_0^i - k_1^i(s)\}$. Assumption 1 implies that $\gamma < \underline{q} < q(s)$. Therefore, in equilibrium, we have

$$q(s) = F'(k_1^T(s))$$

$$k_1^T(s) = \max\{0, K_1(s) - K_0\}.$$

Q.E.D.

Proof of Part (B)

Recall that the return to i 's risky project is given by

$$R_A^i(s) = \rho^i R_A(s) - \mu^i \tag{32}$$

where $\mu^i = R_C(\rho^i - 1)$. First we show that we have misallocation if and only if $R^w(s) < b_1^w(s) + \gamma - \Gamma A$. (Recall that we have normalized $A = 1$.) Suppose $b_1^w(L) = 0$. Assumption 2 that $R_C \geq \gamma$ and $R_C + \Gamma A \geq 1$ implies that $R_C > \gamma - \Gamma A$. And for even the smallest ρ^i , we have $R_C - (\gamma - \Gamma A) < \rho^i(R_C - R_A(L))$. Then it follows that $R^w(L) < \gamma - \Gamma A$. It is also easy to see that misallocation does not hold for R_C , i.e. that $R_C \geq \gamma - \Gamma A$. This is true by Assumption 2. Since this holds for $R_A^w(L)$ but not R_C this condition holds for any equilibrium value of $b_1(s)$.

Now we show that there is no misallocation if and only if $R^w(s) \geq q(s) - \Gamma A$. This holds for R_C by Assumption 2 that $R_C + \Gamma A \geq 1$. This holds for $R_A^w(L)$ because of Assumption 2 and the assumption that $\underline{q} > \gamma$, which implies $q(s) \geq \underline{q} > \gamma$. Therefore, these conditions hold for any equilibrium value of $b_1(s)$. So, in equilibrium, $K_1(s) - K_0 < 0$ if and only if $s = L$ and $\omega^w = 0$. Q.E.D.

APPENDIX 8: Deriving the government's optimal bailout policy

Proof of Part (A):

At date 1, the government solves its problem taking date 0 variables as given. First substitute out of the household's date 1 budget constraint lump sum taxes $T_1 = K_0 g^w$ using the governments binding budget constraint.

$$c_1(s) + B_1(s) \leq e_1 + \sum_i f_0^i d_1^i(s) - K_0 g^w - q(s) k_1^T(s) \tag{33}$$

Recall that we ruled out counterfactual situations in which the government bails out banks outside of a crisis. Since the government takes agents' optimizing behavior as given, we impose the conditions for equilibrium at date 1. Below we characterize the bailout per unit of capital g^w , but this is equivalent to characterizing the total bailout $G = g^w K_0$, since the distribution of bailout funds across investing banks is allocatively irrelevant at date 1.

It turns out that, when the conditions for a misallocation of capital at date 1 are satisfied (namely, when $\omega^w = 0$ and $s = L$), we have $\frac{dK_1(L)}{dg^w} > 0$. From the government's date 1 budget constraint, we then have $\frac{d2c_1(s)}{dg^w} = K_1(s) \frac{dq(s)}{dg^w} + A(1 - \Gamma) \frac{dK_1(s)}{dg^w} > 0$. So when $\omega^w = 0$ and $s = L$, household welfare is increasing in g^w when $k_1^T(s) > 0$. Hence, when there is a misallocation of capital at date 1, the government sets g at the minimum to ensure that capital is no misallocated to the traditional sector. This optimal choice of g^w therefore satisfies $k_1^T(s) = 0$ and is given by

$$g^i(s, \omega^w) = \begin{cases} q(s) - \Gamma A - R_A^i(s) & \text{for } i = w, s = L \text{ and } \omega^i = 0 \\ 0 & \text{otherwise} \end{cases}$$

It follows that the total bailout is given by $K_0 g^i(s, \omega^w) = K_0 (q(s) - \Gamma - R_A^w(s))$. This proves part (A) of Lemma 5.

Proof of Part (B):

With regard to part (B), first recall that the government cannot verify the losses that a bank incurs on its interbank claims. As a result, the government does not bail out any intermediaries in equilibrium, and so $g^i(s, \omega^w) = 0$ for all $i \notin W$. How does the government prefer to distribute the bailout across investing banks? First note that any bailout that satisfies the conditions in part (A) will prevent a misallocation of capital ex post, regardless of how it is distributed across investing banks. This is because the aggregate investment of the banking sector, given in (12), is independent of the distribution of funds across banks due to banks' constant returns-to-scale technology. Therefore, any arbitrary distribution of bailout funds across banks which satisfies part (A) is optimal ex post. For ease of exposition, we therefore simply assume that the government bails out investing banks in proportion to their capital holdings.

In principle, however, how the bailout is distributed across banks may affect banks' ex ante incentives. Nevertheless, we show in Appendix 9B that our general equilibrium results are quite robust to alternative assumptions. This relies on the characterization of general equilibrium in Section 3.

APPENDIX 9: Discussion of government problem

9A. Discussion of the frictions faced by the government

An important assumption in the literature on collective moral hazard, and also in our model, is that bailouts cannot be perfectly targeted across banks (e.g. see ?). If bailouts could be perfectly targeted to any bank in the financial system, the government could always design a transfer scheme which punishes SIFIs, thereby getting rid of the moral hazard problem (for example, by bailing out all banks except for the SIFIs). In practice, however, there are frictions which prevent the government from doing this, be it informational frictions, political constraints, etc. In the model, we impose a straightforward assumption which can capture this. While our results do not depend on the precise nature of this assumption, it is an empirically plausible and tractable way to generate imperfect targeting.

Our assumption is that it is difficult for the government to verify the losses that a bank incurs on its holdings of interbank claims. This assumption captures the fact that it is difficult for the government to identify banks' bilateral exposures during a crisis, due to the complexity of inter-bank markets and the fact that these markets are typically over-the-counter. Indeed, the losses that financial institutions incurred in 2008 from their (frequently off-balance-sheet) exposures to other banks on interbank markets were difficult to verify externally, and often these institutions did not themselves know the extent of these exposures in the midst of the crisis.

In the model, this assumption implies that, in general equilibrium, bailouts can be only imperfectly targeted to investing banks. Nevertheless, the results would hold under a broad class of alternative assumptions to the extent that bailouts cannot be perfectly targeted.

9B. Robustness to alternative bailout policies

In this section, we consider alternative policies for Part (B) of Lemma 5 and discuss their implications for our results. All the policies considered are ex post optimal (i.e. they conform with Lemma 5), and differ only in how the bailout is distributed across investing banks at date 1. Ex post, these policies lead to identical outcomes as those in the body of the paper, so below we analyze to what extent they alter banks' ex ante incentives and equilibria. Overall, our results are quite robust to these various alternatives, primarily because interbank risk sharing at date 0 ensures that the benefits of a bailout are widely shared across banks regardless of the government's policy.

1. Transfer of capital from SIFIs to non-SIFIs

One alternative policy would be for the government to simply transfer capital from SIFIs to other banks during a crisis, in a way which keeps production at the first best ex post and eliminates the risk taking incentive of SIFIs ex ante. It is important to note, however, that this would be isomorphic to a bailout of non-SIFI banks. To see why, suppose that, in a crisis, the government obtains the capital of the SIFIs (either through expropriation, or by purchasing the capital at some price) and grants it directly to non-SIFI banks. In a crisis, non-SIFI banks are also, in aggregate, facing losses. Therefore, these non-SIFI banks would be forced to liquidate these capital holdings

to the traditional sector, and we would still end up with a misallocation of capital. This is because, in the bad state of the world, there are losses, incurred from risky investments, that need to be absorbed by some agents in the economy. In order to prevent a misallocation of capital, the government would need to cover losses of other banks via a transfer financed by taxing the household. This is effectively a bailout of non-SIFI banks.

However, recall from Section 2.9 that the government cannot bail out banks whose losses it cannot verify. Because the government cannot verify exposures from interbank claims, it would then be infeasible for the government to bail out non-SIFI banks, as these banks are facing losses only from their holdings of interbank claims. These frictions prevent the government from perfectly targeting bailouts to non-SIFI banks. Otherwise, the government could simply design a bailout of all banks except for the SIFIs, without ever having to directly reallocate capital across banks. As we discussed above in Part (A), this does not happen in practice for various reasons.

2. Randomized bailouts

We next consider randomized bailouts at date 1, similar to the policies analyzed in ?. We consider two alternatives.

i) *Randomizing the occurrence of a bailout* In ?, the government faces uncertainty about whether a crisis is systemic, and therefore delays intervention to attain more information. This forces banks to internalize the riskiness of their investments to some extent, mitigating the ex ante moral hazard problem. In our setting, there is no such uncertainty; the government knows with certainty whether there is a crisis, and so this mechanism is not at play. Moreover, given the inefficiencies associated with a crisis, it would be suboptimal (and therefore not credible) for the government not to intervene during a crisis with positive probability.

Nevertheless, in practice, a lack of confidence in the government's ability to carry out its optimal bailout policy could mitigate risk taking ex ante. We do not take up this issue in this paper.

ii) *Randomizing the bailout across investing banks* A government could conceivably choose to randomize *which* investing banks it bails out during a crisis. In our setting, however, risk sharing between banks in the interbank market always ensures that the benefits of bailouts are shared perfectly. As a result, randomization does not mitigate the collective moral hazard problem.

To see this, let $\pi^i \in (0, 1]$ denote the probability (chosen by the government) that bank i receives a transfer in the event of a bailout, while $\tilde{g}^i \geq 0$ is the transfer to i per unit of capital, conditional on it receiving one. Consider a bailout policy which satisfies $\sum_{i \in W} \tilde{g}^i k_0^i = G(s, \omega^w)$ for all investing banks which are chosen to receive a bailout, so that the total size of the bailout is consistent with part (A) of Lemma 5 regardless of which banks receive the bailout.

First note that randomness of the bailout effectively adds an additional source of risk to risky assets which is uncorrelated to the aggregate shock: when an investing bank invests in a risky

project, it bears not only the aggregate on the project's return, but also the risk that it is not bailed out during a crisis. Off equilibrium, this means that investing banks are no longer in a corner solution in their portfolio choice, but rather diversify this risk by lending to other investing banks in addition to investing in their own projects. The perfect risk sharing between banks facilitated interbank contracts means that each investing bank can fully diversify away the risk of not receiving bailout funds in a crisis. (Recall that the government will bailout at least one of them with probability 1.)

However, this cannot be sustained as part of an equilibrium. Consider two investing banks i and j such that $\rho^i < \rho^j$. Since perfect interbank risk sharing fully diversifies away the risk of not receiving a bailout during a crisis, all investing banks receive the same return R_C in the bad state. However, in the good state, bank j receives a higher return. Hence, from bank i 's perspective, the risk-adjusted return to lending to bank j is larger than investing in its own project. As a result, each atomistic bank in i prefers to forgo its own investment and lend entirely to bank j . As a result, all other investing banks forgo their own investments in favor of lending to the riskiest investing bank. Hence, the riskiest investing banks become the only SIFIs and are bailed out during a crisis with probability 1. Thus, bailout policies of this type would yield identical equilibria to those analyzed in Section 3 of the paper.

3. Other bailout policies which alter ex ante incentives

We now consider other bailout policies which satisfy Lemma 5, but may lead to different ex ante incentives. Two examples of such policies are a credible commitment by the government to bail out only the least risky investing bank, or only the largest investing bank in a crisis. While these policies may alter the equilibria of the economy, we show that the implications for welfare and policy outlined in Sections 5 and 6 still apply under these alternatives.

Under either of these bailout policies, any equilibrium featuring risk taking features a single type of SIFI. To see why, suppose that the government bails out only the least risky bank, and that at date 0, there are two investing banks i and j such that $\rho^i < \rho^j$. In a crisis, only bank i will be bailed out by the government. Then atomistic banks in j prefer to deviate and forgo their investments in favor of lending to i . So this cannot be an equilibrium. Alternatively, suppose that only the largest investing bank is bailed out, and that i is larger than j . Again, atomistic banks in j prefer to deviate and forgo their investments in favor of lending to i . Hence, a risky equilibrium cannot feature more than one type of SIFI.

When only the least risky investing banks are bailed out, the unique SIFIs are the least risky banks. To see why, suppose that at date 0 bank j is the only investing bank. Bank j will be bailed out in equilibrium. However, atomistic banks in i have incentive to deviate and instead of lending to j , invest in their risky projects: then bank i will be bailed out, and atomistic banks in j will prefer to lend to i rather than invest in their risky projects, since they will not be bailed out. Hence,

this bailout policy will imply that there is a unique risky equilibrium (in addition to the prudent equilibrium described in Section 3 of the paper) in which the unique SIFIs are the least risky banks.

When only the largest investing bank is bailed out, however, the identity of the SIFIs is not pinned down uniquely. To see why, suppose that bank i is the largest investing bank at date 0. Then bank i will be bailed out in a crisis. Consider bank j where $\rho^j \neq \rho^i$. Atomistic banks in j does not have incentive to deviate and start investing in its own asset, because bank i would remain the largest investing bank and therefore the only bank to be bailed out in a crisis. Similarly, atomistic banks in i have no incentive to deviate and start lending to j for the same reason. Therefore, we can have N risky equilibria (one for each bank type in the economy) in addition to the prudent equilibrium which feature a single type of SIFI.

Therefore, with bailout policies of this type, the risky equilibria may differ from those characterized in Section 3 of the paper. However, the welfare implications of each risky equilibrium remain the same. As a result, the scope for ex ante regulation to improve welfare, and the policy implications outlined in Sections 5 and 6 still apply even under these alternative bailout policies.

APPENDIX 10: Proof of Proposition 1

We prove Proposition 1 by backward induction. We have already characterized banks' optimal decisions at dates 1 and 2. Given these, we also characterized each investing bank's best response function for its date 0 portfolio choice. We now prove that, given these best response functions, there exist exactly two subgame perfect Nash equilibria.

Recall that, to complete the characterization of general equilibrium, it remains to determine the investment choices ω^w of investing banks, and to determine which banks are in the set W of investing banks in equilibrium. Once these are determined jointly, the investment choices ω^i of all other banks (i.e. banks in the set $L = I/W$, who simply invest in the liabilities of investing banks) are irrelevant for the allocation.

Proof: The proof is in three parts. In all cases, we make use of the best response functions

$$\omega^i(\{\omega^w\}_{w \in W}) = \begin{cases} 1 & \text{if } g^w(L, \omega^w) = 0 \\ 0 & \text{otherwise} \end{cases}.$$

Claim (i): $\{\omega^w = 1 \quad \forall w \in W\}$ is an equilibrium. This is the 'prudent' equilibrium, as all banks undertake the prudent investment.

Proof: We will show that, when all investing banks in set W choose $\omega^w = 1$, then bank $w \in W$ has no incentive to deviate from $\omega^w = 1$. Suppose that all investing banks choose $\omega^w = 1$. Recall from the government's optimal bailout policy that when all investing banks are exposed to risky projects, then there is never a bailout in the low state at date 1, i.e. $g^i(s, \omega^w) = 0$. The best response function for ω^w then implies that bank w finds it optimal to set $\omega^w = 1$.

Also, recall in that we showed in the partial equilibrium characterization of optimal interbank contracts that the set of investing banks J is given by $J = W \equiv \{w \mid w \equiv \max_{i \in M} E [z_1(s) \theta_k^i(s, \omega^i, g^i)]\}$. In this case when $\omega^w = 1 \quad \forall w \in W$, all banks are invested in only to prudent assets, so that that $E [z_1(s) \theta_k^i(s, \omega^i, g^i)]$ is the same for all banks i . Therefore, the structure of interbank lending in this equilibrium, and therefore the set of investing banks W , is indeterminate – in this prudent equilibrium, we can have any combination of banks investing in the prudent project on their own behalf, with rest of banks investing in their liabilities. W is non-empty, so that at least one bank invests in the prudent project in equilibrium.

Claim (ii): $\{\omega^w = 0 \quad \forall w \in W\}$ is also an equilibrium, where $w \in W \iff \rho^w = \bar{\rho}$. This is the ‘risky’ equilibrium, as all investing banks invest in the riskiest project available.

Proof: We will show that, when all banks set $\omega_C^i = 0$, then bank i has no incentive to deviate from $\omega_C^i = 0$. Suppose that all investing banks choose $\omega^w = 0$. Recall from the government’s optimal bailout policy that when all investing banks are exposed to risky projects, then there is a bailout in the low state at date 1 given by $\hat{g}^i(s, \omega^w) = q(s) - \Gamma A - R_A^i(s)$. The best response function for ω^w then implies that bank w finds it optimal to set $\omega^w = 0$.

Again, we showed that interbank contracts in equilibrium are such that the set of investing banks J is given by $J = W \equiv \{w \mid w \equiv \max_{i \in M} E [z_1(s) \theta_k^i(s, \bar{\omega}^i, g^i)]\}$. Since $z_1(s) = m_1(s)$ is proportional to $u'(c_1(s))$ and in this case $\theta_k^i(s, \omega^i, g^i) = R_A^i(s) + g^i(s, \omega^i) = \rho^i R_A(s) - \mu^i + g^i(s, \omega^i)$, it is easy to show that $E [z_1(s) \theta_k^i(s, \omega^i, g^i)]$ is monotonically increasing in ρ^i . This is because: (i) $u(\cdot)$ is strictly concave; (ii) the variance of $R_A^i(s)$ is increasing in ρ^i , while its mean is independent of ρ^i ; and (iii) the government’s optimal $g^i(s, \omega^i)$ bounds $\theta_k^i(s, \omega^i, g^i)$ from below by $1 - \Gamma$. Therefore, $E [z_1(s) \theta_k^i(s, \omega^i, g^i)]$ is highest for the bank with the greatest potential exposure to the aggregate shock, $\rho^i = \bar{\rho}$. Hence, $W = \{w \in W \mid \rho^w = \bar{\rho}\}$, i.e. only banks with access to the riskiest projects invest in equilibrium, while the rest of banks invest in the liabilities of these risky banks.

Claim (iii): There are no other equilibria.

Proof: Suppose for the sake of contradiction that some $\{\omega^w\}_{w \in W}$ is an equilibrium, where $\{\omega^w\}_{w \in W} \neq \{\omega^w = 1 \quad \forall w \in W\}$ and $\{\omega^w\}_{w \in W} \neq \{\omega^w = 0 \quad \forall w \in W\}$. The government’s optimal bailout policy implies that, in any equilibrium, either $g^w(L, \omega^w) = 1 - \Gamma A - R_A^i(L)$ for some $w \in W$ (i.e. a crisis and bailout occurs in the bad state) or $g^w(s) = 0$ for all s (i.e. a crisis and bailout never occur). Take the latter case in which we always have $g^w(s) = 0$. Then all investing bank w ’s best response functions favor investing only in the prudent project by setting $\omega^w = 1$. Moreover, this is consistent with having $g^w(s) = 0$. So we must have $\{\omega^w\}_{w \in W} = \{\omega^w = 1 \quad \forall w \in W\}$, which contradicts the premise that this equality does not hold. So this cannot be an equilibrium.

Now suppose that we have a bailout in the bad state. Then the best response function of each investing bank implies all investing banks invest only in the risky their risky projects by choosing

$\omega^w = 0$, which is consistent with having a bailout in the bad state. So we must have $\{\omega^w\}_{w \in W} = \{\omega^w = 0 \quad \forall w \in W\}$, which contradicts the premise that this equality does not hold. So this cannot be an equilibrium either. Therefore, any equilibrium must be either the prudent equilibrium in which $\{\omega^w = 1 \quad \forall w \in W\}$, or the risky equilibrium in which $\{\omega^w = 0 \quad \forall w \in W\}$. Q.E.D.

Uniqueness of representative SIFI

Although the results above imply that, in the risky equilibrium, the SIFIs are always the riskiest banks (i.e. the banks with the highest ρ^i), it may be instructive to reiterate why this is necessarily the case. Suppose we have an equilibrium with risk taking in which bank j is the only investing bank, where $\rho^j < \rho^h$ for some h (i.e. bank j is not the riskiest bank). Can this be an equilibrium? Given that bank j is the only investing bank, it will be bailed out in the bad state. All other banks have incentive to lend their funds to bank j in order to benefit from the bailout in the bad state. Bank j in turn invests in its risky project. Indeed, other banks may not have incentive to deviate and lend to a different bank (since it may not be bailed out) or invest in its own project. (This would indeed be the case if the government announced in advance that it would bail out the least risky investing bank.) However, ex ante, bank h has incentive to deviate and invest its funds in its own risky project rather than lend to j . The reason for this is that, per the government bailout policy in Lemma 5, bank h will be bailed out in the bad state and therefore receive the same return from lending to j . But in the good state, the return on h 's own project exceeds that paid on j 's liability, since $\rho^j < \rho^h$. Therefore, this cannot be an equilibrium.

Now suppose that we have a situation with risk taking in which two banks j and h are both investing banks, where $\rho^j < \rho^h$ (i.e. bank h 's project is riskier). Can this be an equilibrium? Recall that Lemma 5 implies that bank h will be bailed out in equilibrium, since it is an investing bank. Then ex ante, each atomistic bank in j has incentive to deviate and lend all of its funds to the riskiest bank h . This is because, giving the perfect risk sharing facilitated by interbank contracts, it would benefit from a higher upside in the good state (since $\rho^j < \rho^h$), and still benefit equally from the bailout in the bad state. Therefore, this cannot be an equilibrium.

Therefore, any equilibrium with risk taking features banks $w \in W$ (the riskiest banks) as the only investing banks. Appendix 9B discusses the robustness of this result to alternative assumptions about the government's bailout policy.

APPENDIX 11: Proof of Lemma 1

This follows from the linearity of the firm's portfolio allocation problem. Namely, the optimality conditions for the bank's portfolio allocation decisions for k_0^i, ℓ^{ij} , and ω^i do not depend on size

of the firm's investment. Therefore, it immediately follows that, for each firm i , we have one of two cases. Either we are in case 1, in which there is a firm $j \neq i$ such that $E [z_1^i(s)\theta_\ell^i(s, j)] \geq E [z_1^i(s)\theta_\ell^i(s, h)]$ for all other firms h , and $E [z_1^i(s)\theta_\ell^i(s, j)] \geq E [z_1^i(s)\theta_k^i(s, \omega^i, g^i)]$ for any $\omega^i \in [0, 1]$. In this case, the contract offered by firm j to firm i has a more favorable risk-return tradeoff than that offered to i by any other firm h . In addition, the return to lending to firm j is preferable to investing any amount in either the risky or prudent project on i 's own behalf. In case 1, we have $k_0^i = 0$ and $\ell^{ij} > 0$, meaning the firm forgoes investing in its own projects in favor of lending to firm j .

The other possibility is that we are in case 2, in which there is a $\tilde{\omega}^i \in [0, 1]$ such that $E [z_1^i(s)\theta_k^i(s, \tilde{\omega}^i, g^i)] \geq E [z_1^i(s)\theta_k^i(s, \omega^i, g^i)]$ for all $\omega^i \neq \tilde{\omega}^i$ and $E [z_1^i(s)\theta_k^i(s, \tilde{\omega}^i, g^i)] \geq E [z_1^i(s)\theta_\ell^i(s, h)] \quad \forall h$. This implies that at the optimal ω^i , the return to investing ω^i in the prudent project and $1 - \omega^i$ of its capital has a more favorable risk-return profile than the returns offered by any firm's inter-firm contract. In case 2, we have $k_0^i > 0$ and $\ell^{ij} = 0$ for all j , meaning the firm does not lend to any other firm. Furthermore, since the condition for ω^i does not depend on ω^i , firm i will always be at a corner solution in its choice of ω^i , so that the optimal ω^i satisfies $\tilde{\omega}^i \in \{0, 1\}$. (This is partly due to the fact that, in the government's optimization problem, we will show that g^i will be zero for $\omega^i = 1$.) Q.E.D.

APPENDIX 12: Benchmark 2: Comparative static on degree of risk aversion

How does risk sharing between the SIFIs and non-SIFI banks generate excessive risk taking? In this benchmark variant of the model, we isolate the role of risk sharing *per se* in generating excessive risk taking by all banks by varying the degree of risk aversion of agents in the model.

In general, the interbank market plays two roles in the risky equilibrium. First, it directs funds at date 0 to the projects with the highest expected return. Second, as we showed in Section 3.2.3., the interbank market facilitates risk sharing between SIFIs and other banks by allowing other banks to benefit from the government guarantee indirectly, thereby reducing the variance of their portfolios. This second risk sharing motive of interbank lending arises because the stochastic discount factor reflects the household's risk aversion. To elucidate this point we modify the model in this section so that only the risk sharing role of the interbank market ultimately affects banks' portfolio choices. Then when capture how risk sharing incentivizes risk taking through a comparative static exercise by varying the degree of risk aversion of the household.

To do this, we modify the baseline model in three respects. First, for concreteness, we suppose that the representative household's utility features constant relative risk aversion so that, $u(c) = \frac{c^{1-\eta}-1}{1-\eta}$, where $0 \leq \eta \leq 1$. Second, rather than assuming that all risky projects are a mean-

preserving spread of the prudent project, we now assume that $R_C > E[R_A^i(s)]$ for all i .⁴ This implies that the risky projects are not only riskier than the prudent project, but also offer a lower expected return. Moreover, we assume that a stronger condition holds: $\pi(H)R_A^i(H) + \pi(L)(1 - \Gamma) < R_C$. This assumption will ensure that the higher expected return on risky assets afforded by the government guarantee is not sufficient by itself to entice banks to invest in risky assets. For the purpose of this exercise, we also make an assumption to ensure that there is some threshold degree of risk aversion above which banks prefer to lend to SIFIs and below which they prefer hold prudent assets only.

Assumption OA.1: a) $\frac{(1-\pi(L))}{\pi(L)} \left(\frac{c_1(L)}{c_1(H)} \right)^\eta < \frac{(1-\theta_\ell(L,w))}{(\theta_\ell(H,w)-1)}$; b) $\frac{(1-\pi(L))}{\pi(L)} \left(\frac{c_1(L)}{c_1(H)} \right)^{\eta-1} > \frac{(1-\theta_\ell(L,w))}{(\theta_\ell(H,w)-1)}$; and c) $\log \left[\frac{\pi(L)}{(1-\pi(L))} \frac{(R_C - \theta_\ell(L,w))}{(\theta_\ell(H,w) - R_C)} \right] < \log \left(\frac{c_1(L)}{c_1(H)} \right)$ hold.

(Note that (b) and (c) can be assured by setting $\pi(L)$ sufficiently low. While these conditions depend in part on equilibrium variables, these can be solved in closed form. For ease of exposition do not present that here.)

In this modified environment, the characterization of the date 1 spot market for capital and optimal interbank and household contracts all go through. Moreover, the government's optimal bailout policy is still characterized by Lemma 5. Therefore, to characterize the equilibrium in this version of the model, it remains to characterize banks' best response functions for their date 0 portfolio choices and interbank lending decisions. We characterize these best response functions for different degrees of the household's risk aversion η .

How does risk sharing affect portfolio choices, risk taking? Recall from Section 3.5.1. that the value to bank i of an interbank claim issued by a SIFI w promising a return $\theta_\ell^i(s, w)$ is given by the sum of the expected discounted return $E_A \equiv E[m_1(s)]E[\theta_\ell^i(s, w)]$ and a safety premium component given by $SP_A \equiv Cov(m_1(s), \theta_\ell^i(s, w))$, where the total value V_A of the claim is given by the sum of the two. We already showed in Proposition 2 that the implicit guarantee lowers riskiness of SIFI's assets, and that the interbank market facilitates risk sharing between the SIFIs and non-SIFI banks whereby banks can benefit from safety of the SIFIs interbank claims. These results apply in this modified setting as well. We now vary the degree of risk aversion of the household to show how this interbank risk sharing actually exacerbates excessive risk taking, generating collective risk shifting problem.

First suppose that $\eta = 0$, so that the household is risk neutral. In this case, the stochastic discount factor $m_1(s)$ is constant across states, and so the covariance term is 0. Agents do not value risk sharing - the variance of their portfolios is irrelevant for their portfolio choice and they care only about the expected return. Since the bailout policy $g^w(s)$ is given by Lemma 5, our assumption above $\pi(H)R_A^w(H) + \pi(L)(1 - \Gamma) < R_C$ implies that $E[R_A^w(s) + g^w(s)] < R_C$. Therefore, the value

⁴For this to hold, we need to modify our assumption that $R_C \geq 1 - \Gamma$ instead holds with strict inequality.

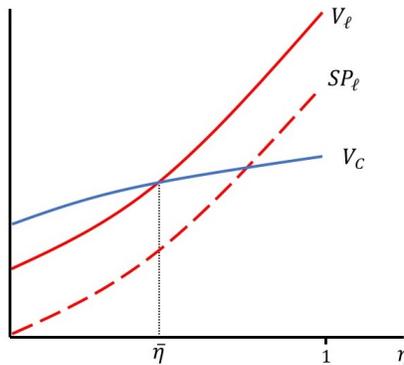
of the investing in the claim issued by the SIFI V_A (which is backed by the SIFI's risky project) exceeds that of investing in the prudent project V_C , i.e. we have $V_A < V_C \equiv E[m_1(s)]R_C = m_1(s)R_C$. Banks never want to invest in interbank claims issued by SIFIs, because the government guarantee does not increase the expected return on these claims sufficiently to entice banks away from prudent assets. As a result, each bank i 's best response function is to always invest in prudent assets. As a result, no bank ever undertakes a risky investment in equilibrium. This is summarized in the corollary below.

Corollary: *No excessive risk taking with risk neutrality*

Under Benchmark economy 2, when the household is risk neutral ($\eta = 0$), there is never excessive risk taking in equilibrium by any bank.

Now suppose that the household is risk averse, so that $\eta > 0$. As the household's risk aversion increases, banks care more about the covariance of their portfolio returns with the stochastic discount factor, and therefore the risk premium on an interbank claim issued by a SIFI w is lower, as captured by a higher safety premium $SP_A \equiv Cov(m_1(s), \theta_\ell^i(s, w))$. In other words, the safety offered by the SIFI's interbank claim is valued more by non-SIFI banks. By Assumption OA.1, there is a threshold risk aversion value $\bar{\eta}$ above which the value of the SIFI's interbank claim exceeds the value of the prudent project. This is summarized in the figure below, which plots the value of investing in the prudent project $V_C \equiv E[m_1(s)]R_C$ together with the total value of the interbank claim issued by SIFI V_A and the safety premium component SP_A of this claim, each as a function of risk aversion parameter η . (The difference between V_A and SP_A is given by $E_A \equiv E[m_1(s)]E[\theta_\ell^i(s, w)]$.)

Figure 3:



How does this affect banks' portfolio choices? Recall that, for sufficiently high risk aversion $\eta > \bar{\eta}$, we have $E[m_1(s)(R_A(s) + g(s))] > E[m_1(s)R_C]$. As a result, non-SIFI banks choose to invest in claims issued by the SIFI for all $\eta > \bar{\eta}$: The insurance value of interbank claims issued by the SIFI (together with expected discounted return) is sufficiently high to entice banks to forgo their prudent projects in favor of buying financial claims issued by the SIFI. (At same time, the

SIFI invests in its risky project.) As a result, the risk sharing facilitated by the interbank market incentivizes excessive risk taking.

Corollary: *Risk sharing generates excessive risk taking by all banks*

When the household is risk averse, the insurance value of interbank claims issued by SIFIs is sufficiently high to entice non-SIFI banks to forgo their prudent investments in favor of buying claims on the SIFIs' portfolio. As a result, in equilibrium, the SIFIs invests in their risky project and non-SIFIs invest in financial claims issued by SIFIs.

Takeaway These comparative static exercises show that, in Benchmark economy 2, risk sharing between the SIFIs and non-SIFI banks in the risky equilibrium is precisely what facilitates excessive risk taking in the first place. When the insurance value of interbank claims on the SIFIs are low, banks do not have incentive to invest in risky assets. Only when the insurance provided by these SIFI claims is sufficiently high do banks undertake excessive risks.

APPENDIX 13: Full planner problem

The planner's problem is to choose $c_t(s)$, f_0^i , $B_1(s)$, d_0^i , $b_1^i(s)$, $b_2^i(s)$, ℓ^{ji} , $r^{ji}(s)$, k_0^i , $k_1^i(s)$, ω^i , $T_1(s)$, and $g^i(s, \omega^i)$ for all banks i, j , all states s and all periods t to solve

$$\max E[u(c_0) + u(c_1(s)) + u(c_2(s))]$$

s.t.

$$c_0 + \sum_i f_0^i d_0^i \leq e_0 - T_0 \quad (34)$$

$$c_1(s) + B_1(s) \leq e_1 + \sum_i f_0^i d_1^i(s) - T_1(s) - q(s)k_1^T(s) \quad (35)$$

$$c_2(s) \leq e_2 + B_1(s) + \sum_i f_0^i d_2^i(s) + \Pi_2(s) \quad (36)$$

Final dividend payout (including dividend from traditional firms)

$$\Pi_2(s) = \sum_i (A - b_2^i(s)) k_1^i(s) + F(k_1^T(s))$$

budget constraints

$$k_0^i + \sum_j \ell^{ij} \leq n + d_0^i + \sum_j \ell^{ji} \quad (37)$$

$$q(s)k_1^i(s) \leq \theta_k^i(s, \omega^i, g^i)k_0^i + \sum_j \theta_\ell^i(s, j)\ell^{ij} - \sum_h \left(r^{hi}(s) - b_1^i(s) \right) \ell^{hi} + b_2^i(s)k_1^i(s)$$

no-default constraints for the household contract

$$0 \leq b_1^i(s) \leq q(s) - \gamma \quad (38)$$

$$0 \leq b_2^i(s) \leq \Gamma A \quad (39)$$

the other firms' participation constraints for each j

$$u^{ji}(\ell^{ji}, \{r^{ji}(s)\}_s) \geq \bar{u}^j \quad (40)$$

and non-negativity constraints on capital holdings and interbank loans.

$$k_0^i, k_1^i(s), \ell^{ij} \geq 0 \quad \forall j \quad (41)$$

asset prices

$$q(s) = F'(k_1^T(s))$$

$$k_1^T(s) = \max \{0, K_1(s) - K_0\}.$$

the government's optimal bailout policy

$$k_0^w g^w(s, \omega^w) = \begin{cases} (q(s) - b_2^w(s)) \sum_i k_0^i - \sum_i \frac{q(s) - b_2^w(s)}{q(s) - b_2^i(s)} X & \text{for } s = L \\ 0 & \text{otherwise} \end{cases}$$

where

$$X \equiv \left(q(s) + \omega^i \mathbf{R}^i(s) - \gamma - b_1^i(s) \right) k_0^i + \sum_j \theta_\ell^i(s, j)\ell^{ij} - \sum_h \left(r^{hi}(s) - b_1^i(s) \right) \ell^{hi}$$

and the government budget constraint

$$\sum_j k_0^j g^j(s, \omega^j) + D(k_1^T(s)) = T_1(s). \quad (42)$$

Since the planner has the same limited commitment that the government does, the planner solves its problem recursively. The planner first solves the date 1 problem taking as given date 0 variables.

The optimal government transfers at date 1 in the planner's solution will, by construction, coincide with the government's optimal bailout policy. Given this date 1 solution, the planner then solves the date 0 problem. The recursive nature of this problem is captured by including the optimal bailout policy as a constraint in the planner's date 0 problem above. Note, however, that this optimal bailout policy is a generalized version of that which appears in the competitive equilibrium, because we do not impose equilibrium conditions, such as full interbank risk sharing, in the planner's problem. Recall that the government's optimal bailout policy implies capital is never misallocated at date 1. Therefore, we have $q(s) = 1, k_1^T(s) = 0$. Imposing that the government budget constraint binds, replace date 1 taxes $T_1(s)$. We also replace $d_1^i(s)$ and $d_2^i(s)$ using the definitions of $b_1^i(s)$ and $b_2^i(s)$.

Notice that the planner takes the constraints of all banks i as constraints simultaneously in the Lagrangian. Hence, unlike in the competitive economy, the planner's first order conditions for ℓ^{ij} and $r^{ji}(s)$ will also capture how they affect the budget constraints of other banks j (i.e. k_0^j and $k_1^j(s)$). The planner's first order conditions are

$$\frac{\partial L'}{\partial f_0^i} \leq 0 \iff E [u'(c_2(s))] b_2^i(s) k_1^i(s) + \dots \quad (43)$$

$$\dots + E \left[u'(c_1(s)) \left(\left[b_1^i(s) \left(k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij} \right) - b_2^i(s) k_1^i(s) \right] \right) \right] - E [u'(c_0)] d_0^i \leq 0$$

$$\frac{\partial L'}{\partial B_1(s)} \leq 0 \iff E [u'(c_2(s))] - E [u'(c_1(s))] \leq 0 \quad (44)$$

$$\frac{\partial L'}{\partial d_0^i} \leq 0 \iff -u'(c_0) f_0^i + z_0^i \leq 0 \quad (45)$$

$$\frac{\partial L'}{\partial k_0^i} \leq 0 \iff E [u'(c_1(s)) f_0^i b_1^i(s)] - z_0^i + E [z_1^i(s) \theta_k^i(s, \omega^i, g^i)] - E \left[u'(c_1(s)) \frac{\partial T_1(s)}{\partial k_0^i} \right] \leq 0 \quad (46)$$

$$\frac{\partial L'}{\partial k_1^i(s)} \leq 0 \iff -u'(c_1(s)) f_0^i b_2^i(s) + u'(c_2(s)) f_0^i b_2^i(s) + \dots \quad (47)$$

$$+ u'(c_2(s)) (A - b_2^i(s)) - z_1^i(s) (1 - b_2^i(s)) \leq 0$$

$$\frac{\partial L'}{\partial b_1^i(s)} \leq 0 \iff (u'(c_1(s)) f_0^i - z_1^i(s)) \left(k_0^i - \sum_h \ell^{hi} + \sum_j \ell^{ij} \right) - u'(c_1(s)) \frac{\partial T_1(s)}{\partial b_1^i(s)} \leq \bar{\lambda}_1^i(s) - \bar{\lambda}_0^i(s) \quad (48)$$

$$\begin{aligned} \frac{\partial L'}{\partial b_2^i(s)} \leq 0 &\iff -u'(c_1(s))f_0^i k_1^i(s) + u'(c_2(s))f_0^i k_1^i(s) - u'(c_2(s))k_1^i(s) + z_1^i(s)k_1^i(s) - \dots \\ &\dots - u'(c_1(s))\frac{\partial T_1(s)}{\partial b_2^i(s)} \leq \mu_1^i(s) - \mu_0^i(s) \end{aligned} \quad (49)$$

$$\frac{\partial L'}{\partial \omega^i} \leq 0 \iff E \left[z_1^i(s)k_0^i \frac{\partial \theta_k^i(s, \omega^i, g^i)}{\partial \omega^i} \right] - E \left[u'(c_1(s)) \frac{\partial T_1(s)}{\partial \omega^i} \right] \leq 0 \quad (50)$$

$$\frac{\partial L'}{\partial \ell^{ij}} \leq 0 \iff E \left[u'(c_1(s))f_0^i b_1^i(s) \right] - z_0^i + z_0^j + E \left[z_1^i(s)\theta_\ell^i(s, j) \right] - E \left[u'(c_1(s)) \frac{\partial T_1(s)}{\partial \ell^{ij}} \right] \leq 0 \quad (51)$$

$$\frac{\partial L'}{\partial r^{ji}(s)} \leq 0 \iff -z_1^i(s)\ell^{ji} + z_1^j(s)\ell^{ji} - \hat{v}^{ji} \frac{\partial u^{ji}(\ell^{ji}, \{r^{ji}(s)\}_s)}{\partial r^{ji}(s)} - u'(c_1(s)) \frac{\partial T_1(s)}{\partial r^{ji}(s)} \leq 0 \quad (52)$$

APPENDIX 14: Motivating empirical evidence

Here, we present a brief review of the empirical evidence on the structure of interbank markets, with a focus on three ‘stylized facts’. The overall picture painted by these facts is one of a highly concentrated financial system in which a small number of large and interconnected institutions hold riskier assets, and benefit from an implicit government guarantee which lowers the cost of their liabilities.

The first stylized fact is that interbank financial markets typically exhibit a strong core-periphery structure, in which a few highly interconnected institutions at the core interact with the many sparsely connected institutions in the periphery. This has been shown for a wide range of markets including inter-dealer markets for corporate bonds, over-the-counter derivatives markets, interbank markets, and fed funds markets.⁵

The second fact is that these large and interconnected financial institutions often benefit from an implicit government guarantee of their assets or liabilities. Moreover, this guarantee lowers their costs of funding on deposit or wholesale funding markets, and lowers their cost of insurance via credit default swaps or put options on equity prices.⁶

⁵For example, see ? for evidence of a core-periphery structure in the inter-dealer market for corporate bonds, ? and ? for over-the-counter derivatives markets, ?, ?, ?, and ? for interbank markets, and ?, ?, ? for the fed funds market.

⁶See ? for evidence of the size of implicit government guarantees from out of the money put options, and ? from data on credit default swaps for the largest firms from 2008 Paulson plan, and ? and ? for the size of guarantees government-sponsored enterprises. See also ?, ?, and ?.

The third fact is that these large and interconnected institutions often make riskier investments than those in the periphery. ², and several papers cited therein, show that the anticipation of government support is associated with increased risk taking. Moreover, ² provide evidence that banks who are more interconnected also undertake more correlated risks.

Consistent with these three features of the data, our model will endogenously feature a core-periphery structure in the interbank market in which large, interconnected banks at the core benefit from an implicit government subsidy and undertake riskier investments. In addition, the liabilities of these SIFIs will command a lower risk premium, reflecting the insurance value provided by the implicit government guarantee.

APPENDIX 15: Proof that selective bailouts can eliminate a TCTF equilibrium

We show that the Pigouvian taxes outlined in Lemma 6 are sufficient to eliminate all risk taking in equilibrium. By design, the Pigouvian taxes in Lemma 6 prevents SIFIs from arising in equilibrium by ensuring that no bank wants to lend to any bank with risky portfolios. However, it could still be the case that each bank invests in its own risky project, since these are perfectly correlated (a too-correlated-to-fail equilibrium). Therefore, it remains to show that there is a credible bailout policy which eliminates the too-correlated-to-fail equilibrium. This equivalent to considering an economy with the interbank market shut down, and show that there is a credible selective bailout policy which can eliminate all risk taking in equilibrium. A sufficient condition for the proof below is strengthening Assumption 2(a) to $R_C > 1 - \Gamma$ instead of $R_C \geq 1 - \Gamma$. (This amounts to assuming that the returns from prudent investments are high enough that banks can afford buying capital on net at $q(s) = 1$.)

Suppose that, in the event of a fire sale, the government is anticipated to bail out only some number $N - x \in (0, N)$ of banks which are suffering losses on their risky projects, and is anticipated to force the remainder $x \in (0, N)$ to absorb losses without a government transfer. Let S denote the set of banks who are anticipated to receive bailout funds in the event of a bailout ('selected' banks), and S^C the set of investing banks who are not ('non-selected' banks). (Note that S and S^C are both subsets of the set W of investing banks, since we've shut down the interbank market.) As we argued in section 2.9, the government can credibly commit to this policy as long as it ensures that $K_1(s) \geq K_0$ so that all capital remains within the banking sector (and $q(s) = 1$). (Recall from section 2.8 that a fire sale occurs if and only if $K_1(s) < K_0$.)

The best response functions characterized in section 3.1 imply that each non-selected bank $i \in S^C$ invests only in prudent assets since it internalizes the riskiness of its projects, so that $\omega^i = 1$. Therefore, the portfolio return of these banks in the bad state is given by $\theta_k^i(s_L, \omega^i) = R_C + 1 - \gamma - b_1^i(s_L, \omega^i)$. Each selected bank $w \in S$ invests in the risky project, if and only if there is expected to be a bailout in the bad state, and their portfolio returns in the bad state are given by $\theta_k^w(s_L, \omega^w) =$

$$R_A^w(s_L) + 1 - \gamma - b_1^w(s_L, \omega^w).$$

Our goal is to show that the government can credibly choose a number $x \in (0, N)$ of selected banks such that, given banks' best response functions, there is never a fire-sale in the bad state even in the absence of a bailout. (This amounts to that the aggregate resources available to banking sector as a whole are sufficient to the bear losses in bad state without liquidating capital to the traditional sector, so that $K_1(s_L) \geq K_0$ even without government bailout.) If that is the case, even selected banks $w \in S$ would choose to invest in prudent assets only.

Under the government's selective bailout policy, a fire sale does not occur in the bad state if and only if the aggregate investment in capital at date 1 is weakly positive. Note that, while the portfolio returns of each non-selected bank are the same since they invest in the prudent project only, the portfolio returns of selected banks may differ since the returns on each of their risky projects may differ. Therefore, aggregate net investment in capital is comprised of the portfolio returns $\frac{\theta_k^i(s_L, \omega^i)}{1-\Gamma} - 1$ of non-selected banks times the amount of capital $\frac{x}{N}K_0$ owned by these banks, plus the sum of the portfolio returns $\frac{\theta_k^w(s, \omega^w, s^w)}{1-\Gamma} - 1$ of each selected bank w times the amount of capital owned by each of these banks $\sum_{w \in S} k_0^w$. This is a version of equation (12) in the bad state, where we have imposed $q(s_L) = 1$ and $g^w(s) = 0$.

$$\frac{x}{N}K_0 \left[\frac{\theta_k^i(s_L, \omega^i)}{1-\Gamma} - 1 \right] + \sum_{w \in S} k_0^w \left[\frac{\theta_k^w(s_L, \omega^w)}{1-\Gamma} - 1 \right] \geq 0$$

Let \tilde{x} be defined by the above condition holding with equality. Then \tilde{x} is the threshold number of selected banks below which there is never a fire sale. Essentially, we want to show that $\tilde{x} \in (0, N)$, or $\frac{\tilde{x}}{N} \in (0, 1)$. (This amounts to showing that there is a selective bailout policy which ensures that there is never a fire sale in equilibrium.) As we show below, it will be trivially true that $\frac{\tilde{x}}{N} > 0$ (since, if all banks are bailed out in a crisis and they anticipate this, then all banks will make risky investments, so that there is a fire sale in the absence of a bailout). It is less obvious that $\frac{\tilde{x}}{N} < 1$.

Because net aggregate investment is increasing in the returns $\theta_k^w(s_L, \omega^w)$ on each selected bank's portfolio in the bad state, it suffices to show that $\frac{\tilde{x}}{N} < 1$ even when all selected banks receive the lowest return among them $\theta_k^v(s_L, \omega^v)$, where $v \in S$ and $R_A^v(s_L) \leq R_A^w(s_L)$ for all $w \in S/v$. (Put differently, in the absence of a bailout, if the resources of non-selected banks in bad state are sufficient to buy all capital sold by failing banks $w \in S$ even when all failing banks get the worst return, then these resources will also be sufficient if each failing bank gets the return on its risky project.) Suppose that all selected banks receive the lowest return $\theta_k^v(s_L, \omega^v)$ in the bad state. Then we could express the total capital holdings of selected banks as $\sum_{w \in S} k_0^w = (1 - \frac{x}{N})K_0$, and so net aggregate investment would be positive if and only if

$$\frac{x}{N} K_0 \left[\frac{\theta_k^i(s_L, \omega^i)}{1 - \Gamma} - 1 \right] + \left(1 - \frac{x}{N} \right) K_0 \left[\frac{\theta_k^v(s_L, \omega^v)}{1 - \Gamma} - 1 \right] \geq 0.$$

So it suffices to show that $\frac{\hat{x}}{N} \in (0, 1)$, where \hat{x} is defined by the above condition holding with equality. Solving for $\frac{\hat{x}}{N}$ yields

$$\frac{\hat{x}}{N} = \frac{1 - \Gamma - \theta_k^v(s_L, \omega^v)}{\theta_k^i(s_L, \omega^i) - \theta_k^v(s_L, \omega^v)} \quad (53)$$

(So there is no fire sale as long as x exceeds this threshold \hat{x} .) We want to show that $\frac{\hat{x}}{N} \in (0, 1)$. Note that from section 2.5, we have that, in the absence of bailouts, $\theta_k^v(s_L, \omega^v) = 1 + R_A^v(s_L) - \gamma - b_1^v(s)$ and $\theta_k^i(s_L, \omega^i) = 1 + R_C - \gamma - b_1^i(s)$.

First we show that $\frac{\hat{x}}{N} > 0$. Note that the denominator is always strictly positive. To see this, note that $\theta_k^i(s_L, \omega^i) > \theta_k^v(s_L, \omega^v)$ if and only if $R_C - R_A^v(s_L) > b_1^i(s, \omega^i) - b_1^v(s, \omega^v)$. Note also that, since $b_1^i(s) \in [0, 1 - \gamma]$, we have $b_1^i(s, \omega^i) - b_1^v(s, \omega^v) \leq 1 - \gamma$. So it suffices to show that $R_C - R_A^v(s_L) > 1 - \gamma$. To see this, recall assumption 2a that $R_C - R_A(L) > R_C - \gamma + \Gamma$. Then it suffices to show that $R_C - \gamma + \Gamma > 1 - \gamma$, which holds based on our assumption above that $R_C > 1 - \Gamma$.

Turn to the numerator. If $b_1^v(s, \omega^v) = 0$, then we have $1 - \Gamma > \theta_k^v(s_L, \omega^v) = 1 - \gamma + R_A^v(s_L)$, by Assumption 2(a) that $\gamma - \Gamma - R_A^L > 0$. So the numerator is strictly positive. If, on the other hand $b_1^v(s, \omega^v) = 1 - \gamma$, then we have $1 - \Gamma > \theta_k^v(s_L, \omega^v) = 1 - \gamma - (1 - \gamma) + R_A^v(s_L)$ which also holds by Assumption 2(a) and the assumption that $\gamma < 1$. So the numerator is strictly positive in any case. Thus, we have $\frac{\hat{x}}{N} > 0$.

Now we show that $\frac{\hat{x}}{N} < 1$. This holds if and only if $1 - \Gamma - \theta_k^v(s_L, \omega^v) < \theta_k^i(s_L, \omega^i) - \theta_k^v(s_L, \omega^v)$, i.e. if and only if $1 - \Gamma < \theta_k^i(s_L, \omega^i)$. If $b_1^i(s, \omega^i) = 0$, then this condition holds since $\gamma - \Gamma < R_C$, which holds because $R_C \geq \gamma$ and $\Gamma > 0$. If, on the other hand, $b_1^i(s, \omega^i) = 1 - \gamma$, then this condition still holds since $1 - \Gamma < R_C$ (which holds based on our assumption above). Thus, we always have that $\frac{\hat{x}}{N} < 1$.

Thus, the government can design a selective bailout policy in which it concentrates a bailout on $N - x$ and leaves the remaining x investing banks to bear losses in the event of a crisis. As long as $x > \hat{x}$, this policy will ensure that there are no fire sales or bailouts in the bad state. As a result, even banks who would be selectively bailed out in the event of a fire sale choose to invest in prudent projects only, since they would never be bailed out in the bad state, and hence $\omega^w = 1$ for all $w \in S$ as well. Therefore, any selective bailout policy in which $x > \hat{x}$, is optimal and credible ex ante. Q.E.D.