Online Appendix

Table A.1: **Other Parameters**

Parameter	Mode	Parameter	Mode	Parameter	Mode	Parameter	Mode
σ	0.0650	ϕ	0.7436	ρ_k	0.9980	scale Baa	0.3998
ϱ_1	0.5372	$\bar{r}$	0.0000	$\lambda_{k,y}$	26.9629	σ_f	23.4733
β	0.7161	γ^T	0.0051	ρ_{lp}	0.8407	σ_i	0.0331
κ_1	0.0036	$e^{\bar{k}}$	0.0507	ϱ_2	0.2338	σ_μ	0.1379
γ	0.0001	σ_p	5.8542	ϱ_3	0.1887	σ_k	6.2614
ρ_g	0.0914	β_p	0.9936	$\lambda_{k,2}$	10.7499	σ_{lp}	0.5699
κ_0	0.0026	$\overline{lp}$	-0.0130	p_s	0.9876	σ_g	1.7200
β_a	0.3905	$\lambda_{\pi,1}$	0.4244	mean beta be1	0.9286		
γ_π	0.0000	$\lambda_{\pi,2}$	0.3139	std beta be1	0.1090		
ρ_f	0.5010	σ_2	0.0383	int Baa	0.0140		

Note: For each realized policy regime, the table reports the posterior mode values of the parameters for the current and alternative policy rules.

Table A.2: FED Announcements

Date	**Day**	**Hour**	**Event**	**Information**
			Top Ten FOMC: 6-month FFF rate	
1/22/2008	Tuesday	8:30 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 75 basis points to 3-1/2 percent. • The Committee took this action in view of a weakening of the economic outlook and increasing downside risks to growth • The Committee expects inflation to moderate in coming quarters, but it will be necessary to continue to monitor inflation developments carefully. Appreciable downside risks to growth remain.
4/18/2001	Wednesday	10:00 AM	Conference Call	• The Federal Open Market Committee decided today to lower its target for the federal funds rate by 50 basis points to 4-1/2 percent. • Capital investment has continued to soften and the persistent erosion in current and expected profitability, in combination with rising uncertainty about the business outlook, seems poised to dampen capital spending going forward.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
10/15/1998	Thursday	3:15 PM	Conference Call	• The Board of Governors approved a reduction in the discount rate by 25 basis points from 5 percent to 4-3/4 percent. • The federal funds rate is expected to fall 25 basis points from around 5-1/4 percent to around 5 percent. • Growing caution by lenders and unsettled conditions in financial markets more generally are likely to be restraining aggregate demand in the future. Against this backdrop, further easing of the stance of monetary policy was judged to be warranted to sustain economic growth in the context of contained inflation.
1/3/2001	Wednesday	1:15 PM	Conference Call	• The Federal Open Market Committee decided today to lower its target for the federal funds rate by 50 basis points to 6 percent. • These actions were taken in light of further weakening of sales and production, and in the context of lower consumer confidence...Moreover, inflation pressures remain contained. Nonetheless, to date there is little evidence to suggest that longer-term advances in technology and associated gains in productivity are abating.
10/2/2001	Tuesday	1:15 PM	FOMC Meeting	• The Federal Open Market Committee decided today to lower its target for the federal funds rate by 50 basis points to 2-1/2 percent • The terrorist attacks have significantly heightened uncertainty in an economy that was already weak...Nonetheless, the long-term prospects for productivity growth and the economy remain favorable and should become evident once the unusual forces restraining demand abate. • The Committee continues to believe that...the risks are weighted mainly toward conditions that may generate economic weakness in the foreseeable future.
9/18/2007	Tuesday	2:15 PM	FOMC Meeting	• The Federal Open Market Committee decided today to lower its target for the federal funds rate 50 basis points to 4-3/4 percent. • Economic growth was moderate during the first half of the year, but the tightening of credit conditions has the potential to intensify the housing correction and to restrain economic growth more generally.
7/6/1995	Thursday	2:15 PM	FOMC Meeting	• Today's action will be reflected in a 25 basis point decline in the federal funds rate from about 6 percent to about 5-3/4 percent. • As a result of the monetary tightening initiated in early 1994, inflationary pressures have receded enough to accommodate a modest adjustment in monetary conditions.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
6/25/2003	Wednesday	1:30 PM	FOMC Meeting	• The Federal Open Market Committee decided today to lower its target for the federal funds rate by 25 basis points to 1 percent. • Recent signs point to a firming in spending, markedly improved financial conditions, and labor and product markets that are stabilizing...With inflationary expectations subdued, the Committee judged that a slightly more expansive monetary policy would add further support for an economy which it expects to improve over time. • The Committee perceives that the upside and downside risks to the attainment of sustainable growth for the next few quarters are roughly equal.
1/30/2008	Wednesday	2:15 PM	FOMC Meeting	• The Federal Open Market Committee decided today to lower its target for the federal funds rate 50 basis points to 3 percent. • TodayŠs policy action, combined with those taken earlier, should help to promote moderate growth over time and to mitigate the risks to economic activity. However, downside risks to growth remain. The Committee will continue to assess the effects of financial and other developments on economic prospects and will act in a timely manner as needed to address those risks.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
Top Ten FOMC: BBG Expected Inflation				
1/22/2008	Tuesday	8:30 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 75 basis points to 3-1/2 percent. • The Committee took this action in view of a weakening of the economic outlook and increasing downside risks to growth • The Committee expects inflation to moderate in coming quarters, but it will be necessary to continue to monitor inflation developments carefully. Appreciable downside risks to growth remain.
12/16/2009	Wednesday	2:15 PM	FOMC Meeting	• The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions...are likely to warrant exceptionally low levels of the federal funds rate for an extended period. • The Federal Reserve is in the process of purchasing $1.25 trillion of agency mortgage-backed securities and about $175 billion of agency debt...the Committee is gradually slowing the pace of these purchases, and it anticipates that these transactions will be executed by the end of the first quarter of 2010. • In light of ongoing improvements in the functioning of financial markets, the Committee and the Board of Governors anticipate that most of the Federal ReserveŠs special liquidity facilities will expire on February 1, 2010, consistent with the Federal ReserveŠs announcement of June 25, 2009.
12/12/2012	Wednesday	12:30 PM	FOMC Meeting	• The Committee will continue purchasing additional agency mortgage-backed securities at a pace of $40 billion per month. The Committee also will purchase longer-term Treasury securities after its program to extend the average maturity of its holdings of Treasury securities is completed at the end of the year, initially at a pace of $45 billion per month. • To support continued progress toward maximum employment and price stability, the Committee expects that a highly accommodative stance of monetary policy will remain appropriate for a considerable time after the asset purchase program ends and the economic recovery strengthens. In particular, the Committee decided to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that this exceptionally low range for the federal funds rate will be appropriate at least as long as the unemployment rate remains above 6-1/2 percent, inflation between one and two years ahead is projected to be no more than a half percentage point above the CommitteeŠs 2 percent longer-run goal, and longer-term inflation expectations continue to be well anchored.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
3/11/2008	Tuesday	8:30 AM	Conference Call	• The Federal Reserve announced today an expansion of its securities lending program. Under this new Term Securities Lending Facility (TSLF), the Federal Reserve will lend up to $200 billion of Treasury securities to primary dealers secured for a term of 28 days. • In addition, the Federal Open Market Committee has authorized increases in its existing temporary reciprocal currency arrangements (swap lines) with the European Central Bank (ECB) and the Swiss National Bank (SNB). The FOMC extended the term of these swap lines through September 30, 2008.
5/9/2007	Wednesday	2:15 PM	FOMC Meeting	• Economic growth slowed in the first part of this year and the adjustment in the housing sector is ongoing. Nevertheless, the economy seems likely to expand at a moderate pace over coming quarters. • Core inflation remains somewhat elevated. Although inflation pressures seem likely to moderate over time, the high level of resource utilization has the potential to sustain those pressures.
3/16/2004	Tuesday	1:15 PM	FOMC Meeting	• The Committee perceives the upside and downside risks to the attainment of sustainable growth for the next few quarters are roughly equal...With inflation quite low and resource use slack, the Committee believes that it can be patient in removing its policy accommodation.
8/10/2010	Tuesday	2:15 PM	FOMC Meeting	• Information received since the Federal Open Market Committee met in June indicates that the pace of recovery in output and employment has slowed in recent months. • The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions...are likely to warrant exceptionally low levels of the federal funds rate for an extended period. • To help support the economic recovery in a context of price stability, the Committee will keep constant the Federal Reserve's holdings of securities at their current level by reinvesting principal payments from agency debt and agency mortgage-backed securities in longer-term Treasury securities. The Committee will continue to roll over the Federal Reserve's holdings of Treasury securities as they mature.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
9/13/2012	Thursday	12:30 PM	FOMC Meeting	• The Committee agreed today to increase policy accommodation by purchasing additional agency mortgage-backed securities at a pace of $40 billion per month. The Committee also will continue through the end of the year its program to extend the average maturity of its holdings of securities...and it is maintaining its existing policy of reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities. • If the outlook for the labor market does not improve substantially, the Committee will continue its purchases of agency mortgage-backed securities, undertake additional asset purchases, and employ its other policy tools as appropriate until such improvement is achieved in a context of price stability. • In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015.
10/8/2008	Wednesday	7:00 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 50 basis points to 1-1/2 percent. The Committee took this action in light of evidence pointing to a weakening of economic activity and a reduction in inflationary pressures. • Inflation expectations are diminishing and remain anchored to price stability. The recent intensification of the financial crisis has augmented the downside risks to growth and thus has diminished further the upside risks to price stability.
3/13/2012	Tuesday	2:15 PM	FOMC Meeting	• The Committee decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that economic conditions...are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014. • The Committee expects moderate economic growth over coming quarters and consequently anticipates that the unemployment rate will decline gradually toward levels that the Committee judges to be consistent with its dual mandate. • The Committee also decided to continue its program to extend the average maturity of its holdings of securities as announced in September. The Committee is maintaining its existing policies of reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities and of rolling over maturing Treasury securities at auction.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
Top Ten FOMC: SP-to-GDP ratio				
1/22/2008	Tuesday	8:30 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 75 basis points to 3-1/2 percent. • The Committee took this action in view of a weakening of the economic outlook and increasing downside risks to growth • The Committee expects inflation to moderate in coming quarters, but it will be necessary to continue to monitor inflation developments carefully. Appreciable downside risks to growth remain.
4/28/2010	Wednesday	2:15 PM	FOMC Meeting	• The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions...are likely to warrant exceptionally low levels of the federal funds rate for an extended period. • The Federal Reserve has closed all but one of the special liquidity facilities...The only remaining such program, the Term Asset-Backed Securities Loan Facility, is scheduled to close on June 30 for loans backed by new-issue commercial mortgage-backed securities; it closed on March 31 for loans backed by all other types of collateral.
3/11/2008	Tuesday	8:30 AM	Conference Call	• The Federal Reserve announced today an expansion of its securities lending program. Under this new Term Securities Lending Facility (TSLF), the Federal Reserve will lend up to $200 billion of Treasury securities to primary dealers secured for a term of 28 days. • In addition, the Federal Open Market Committee has authorized increases in its existing temporary reciprocal currency arrangements (swap lines) with the European Central Bank (ECB) and the Swiss National Bank (SNB). The FOMC extended the term of these swap lines through September 30, 2008.
10/8/2008	Wednesday	7:00 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 50 basis points to 1-1/2 percent. The Committee took this action in light of evidence pointing to a weakening of economic activity and a reduction in inflationary pressures. • Inflation expectations are diminishing and remain anchored to price stability. The recent intensification of the financial crisis has augmented the downside risks to growth and thus has diminished further the upside risks to price stability.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
12/16/2008	Tuesday	2:15 PM	FOMC Meeting	• The Federal Open Market Committee decided today to establish a target range for the federal funds rate of 0 to 1/4 percent...The Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time. • Since the Committee's last meeting, labor market conditions have deteriorated...Financial markets remain quite strained and credit conditions tight. Overall, the outlook for economic activity has weakened further. • The Committee is also evaluating the potential benefits of purchasing longer-term Treasury securities. Early next year, the Federal Reserve will also implement the Term Asset-Backed Securities Loan Facility to facilitate the extension of credit to households and small businesses.
8/7/2007	Tuesday	2:15 PM	FOMC Meeting	• Economic growth was moderate during the first half of the year...The economy seems likely to continue to expand at a moderate pace over coming quarters, supported by solid growth in employment and incomes and a robust global economy. • Although the downside risks to growth have increased somewhat, the Committee's predominant policy concern remains the risk that inflation will fail to moderate as expected.
10/29/2008	Wednesday	2:15 PM	FOMC Meeting	• The Federal Open Market Committee decided today to lower its target for the federal funds rate 50 basis points to 1 percent. • In light of the declines in the prices of energy and other commodities and the weaker prospects for economic activity, the Committee expects inflation to moderate in coming quarters to levels consistent with price stability.
8/9/2011	Tuesday	2:15 PM	FOMC Meeting	• The Committee now expects a somewhat slower pace of recovery over coming quarters than it did at the time of the previous meeting...Moreover, downside risks to the economic outlook have increased. The Committee also anticipates that inflation will settle, over coming quarters, at levels at or below those consistent with the Committee's dual mandate. • The Committee currently anticipates that economic conditions...are likely to warrant exceptionally low levels for the federal funds rate at least through mid-2013. The Committee also will maintain its existing policy of reinvesting principal payments from its securities holdings.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
12/19/2018	Wednesday	2:00 PM	FOMC Meeting	• The Committee judges that some further gradual increases in the target range for the federal funds rate will be consistent with sustained expansion of economic activity, strong labor market conditions, and inflation near the Committee's symmetric 2 percent objective over the medium term. • In view of realized and expected labor market conditions and inflation, the Committee decided to raise the target range for the federal funds rate to 2-1/4 to 2-1/2 percent.
2/5/1997	Wednesday	12:00 PM	FOMC Meeting	• The Committee at this meeting established ranges for growth of M2 and M3 of 1 to 5 percent and 2 to 6 percent respectively, measured from the fourth quarter of 1996 to the fourth quarter of 1997. The monitoring range for growth of total domestic nonfinancial debt was set at 3 to 7 percent for the year. • In the context of the Committee's long-run objectives for price stability and sustainable economic growth...somewhat greater reserve restraint would or slightly lesser reserve restraint might be acceptable in the intermeeting period.
Top Ten FOMC: BBG Expected GDP Growth				
3/11/2008	Tuesday	8:30 AM	Conference Call	• The Federal Reserve announced today an expansion of its securities lending program. Under this new Term Securities Lending Facility (TSLF), the Federal Reserve will lend up to $200 billion of Treasury securities to primary dealers secured for a term of 28 days. • In addition, the Federal Open Market Committee has authorized increases in its existing temporary reciprocal currency arrangements (swap lines) with the European Central Bank (ECB) and the Swiss National Bank (SNB). The FOMC extended the term of these swap lines through September 30, 2008.
1/22/2008	Tuesday	8:30 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 75 basis points to 3-1/2 percent. • The Committee took this action in view of a weakening of the economic outlook and increasing downside risks to growth • The Committee expects inflation to moderate in coming quarters, but it will be necessary to continue to monitor inflation developments carefully. Appreciable downside risks to growth remain.
10/8/2008	Wednesday	7:00 AM	Conference Call	• The Federal Open Market Committee has decided to lower its target for the federal funds rate 50 basis points to 1-1/2 percent. The Committee took this action in light of evidence pointing to a weakening of economic activity and a reduction in inflationary pressures. • Inflation expectations are diminishing and remain anchored to price stability. The recent intensification of the financial crisis has augmented the downside risks to growth and thus has diminished further the upside risks to price stability.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
3/16/2004	Tuesday	1:15 PM	FOMC Meeting	• The Committee perceives the upside and downside risks to the attainment of sustainable growth for the next few quarters are roughly equal...With inflation quite low and resource use slack, the Committee believes that it can be patient in removing its policy accommodation.
12/13/2017	Wednesday	2:00 PM	FOMC Meeting	• In view of realized and expected labor market conditions and inflation, the Committee decided to raise the target range for the federal funds rate to 1-1/4 to 1-1/2 percent. • The Committee expects that economic conditions will evolve in a manner that will warrant gradual increases in the federal funds rate; the federal funds rate is likely to remain, for some time, below levels that are expected to prevail in the longer run. • Information received since the Federal Open Market Committee met in November indicates that the labor market has continued to strengthen and that economic activity has been rising at a solid rate.
12/16/2009	Wednesday	2:15 PM	FOMC Meeting	• The Committee will maintain the target range for the federal funds rate at 0 to 1/4 percent and continues to anticipate that economic conditions...are likely to warrant exceptionally low levels of the federal funds rate for an extended period. • The Federal Reserve is in the process of purchasing $1.25 trillion of agency mortgage-backed securities and about $175 billion of agency debt...the Committee is gradually slowing the pace of these purchases, and it anticipates that these transactions will be executed by the end of the first quarter of 2010. • In light of ongoing improvements in the functioning of financial markets, the Committee and the Board of Governors anticipate that most of the Federal ReserveŠs special liquidity facilities will expire on February 1, 2010, consistent with the Federal ReserveŠs announcement of June 25, 2009.
9/13/2012	Thursday	12:30 PM	FOMC Meeting	• The Committee agreed today to increase policy accommodation by purchasing additional agency mortgage-backed securities at a pace of $40 billion per month. The Committee also will continue through the end of the year its program to extend the average maturity of its holdings of securities...and it is maintaining its existing policy of reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities. • If the outlook for the labor market does not improve substantially, the Committee will continue its purchases of agency mortgage-backed securities, undertake additional asset purchases, and employ its other policy tools as appropriate until such improvement is achieved in a context of price stability. • In particular, the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015.

Table A.2: FED Announcements (Cont'd)

Date	Day	Hour	Event	Information
5/9/2007	Wednesday	2:15 PM	FOMC Meeting	• Economic growth slowed in the first part of this year and the adjustment in the housing sector is ongoing. Nevertheless, the economy seems likely to expand at a moderate pace over coming quarters. • Core inflation remains somewhat elevated. Although inflation pressures seem likely to moderate over time, the high level of resource utilization has the potential to sustain those pressures.
3/13/2012	Tuesday	2:15 PM	FOMC Meeting	• The Committee decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that economic conditions...are likely to warrant exceptionally low levels for the federal funds rate at least through late 2014. • The Committee expects moderate economic growth over coming quarters and consequently anticipates that the unemployment rate will decline gradually toward levels that the Committee judges to be consistent with its dual mandate. • The Committee also decided to continue its program to extend the average maturity of its holdings of securities as announced in September. The Committee is maintaining its existing policies of reinvesting principal payments from its holdings of agency debt and agency mortgage-backed securities in agency mortgage-backed securities and of rolling over maturing Treasury securities at auction.
12/16/2008	Tuesday	2:15 PM	FOMC Meeting	• The Federal Open Market Committee decided today to establish a target range for the federal funds rate of 0 to 1/4 percent...The Committee anticipates that weak economic conditions are likely to warrant exceptionally low levels of the federal funds rate for some time. • Since the Committee's last meeting, labor market conditions have deteriorated...Financial markets remain quite strained and credit conditions tight. Overall, the outlook for economic activity has weakened further. • The Committee is also evaluating the potential benefits of purchasing longer-term Treasury securities. Early next year, the Federal Reserve will also implement the Term Asset-Backed Securities Loan Facility to facilitate the extension of credit to households and small businesses.

Data

Real GDP

The real Gross Domestic Product is obtained from the US Bureau of Economic Analysis. It is in billions of chained 2012 dollars, quarterly frequency, seasonally adjusted, and at annual rate. The source is from Bureau of Economic Analysis (BEA code: A191RX). The sample spans 1959:Q1 to 2021:Q2. The series was interpolated to monthly frequency using the method in

Stock and Watson (2010). The quarterly series was downloaded on August 20th, 2021.

GDP price deflator

The Gross Domestic Product: implicit price deflator is obtained from the US Bureau of Economic Analysis. Index base is 2012=100, quarterly frequency, and seasonally adjusted. The source is from Bureau of Economic Analysis (BEA code: A191RD). The sample spans 1959:Q1 to 2021:Q2. The series was interpolated to monthly frequency using the method in Stock and Watson (2010). The quarterly series was downloaded on August 20th, 2021.

Capital Share K_t

The capital share K_t is defined as $1 - LS_t$ where LS_t is the nonfarm business sector labor share. Labor share is measured as labor compensation divided by value added. The labor compensation is defined as Compensation of Employees - Government Wages and Salaries-Compensation of Employees of Nonprofit Institutions - Private Compensation (Households) - Farm Compensation of Employees - Housing Compensation of Employees - Imputed Labor Compensation of Self-Employed. The value added is defined as Compensation of Employees + Corporate Profits + Rental Income + Net Interest Income + Proprietors' Income + Indirect Taxes Less Subsidies + Depreciation. The quarterly, seasonally adjusted data spans from 1959:Q1 to 2021:Q2. The source is from Bureau of Labor Statistics. The labor share index is available at http://research.stlouisfed.org/fred2/series/PRS85006173 and the quarterly LS level can be found from the dataset at https://www.bls.gov/lpc/special_requests/msp_dataset.zip. The series was interpolated to monthly frequency using the method in Stock and Watson (2010). The quarterly series was downloaded on September 21th, 2021.

Federal funds rate (FFR)

The Effective Federal Funds Rate is obtained from the Board of Governors of the Federal Reserve System. It is in percentage points, quarterly frequency, and not seasonally adjusted. The sample spans 1960:02 to 2021:06. The series was downloaded on August 20th, 2021

SP500 and SP500 futures

We use tick-by-tick data on SP500 index obtained from tickdata.com. The series was downloaded on September 22th, 2021 from `https://www.tickdata.com/` .We create the minutely data using the close price within each minute. Our sample spans January 2nd 1986 to September 17th, 2021. Within trading hours, we construct SP500 market capitalization by multiplying the SP500 index by the SP500 Divisor. The SP500 Divisor is available at the URL: `https://ycharts.com/indicators/sp\_500\_divisor`. We supplement SP500 index using SP500 futures for events that occur in off-market hours. We use the current-quarter contract futures.

We purchased the SP500 futures from CME group at URL: https://datamine.cmegroup.com/. The SP500 futures data were downloaded on October 6, 2021.

SP500 Earnings and Market Capitalization

We obtained monthly S&P earnings from multpl.com at URL: `https://www.multpl.com/shiller-pe`. For S&P market cap, we obtain the series from Ycharts.com available at `https://ycharts.com/indicators/sp%\_500\_market\_cap`. Both series were downloaded on December 22nd, 2021.

Baa Spread, 20-yr T-bond, Long-term US government securities

We obtained daily Moody's Baa Corporate Bond Yield from FRED (series ID: DBAA) at URL: `https://fred.stlouisfed.org/series/BAA`, US Treasury securities at 20-year constant maturity from FRED (series ID: DGS20) at URL: `https://fred.stlouisfed.org/series/DGS20`, and long-term US government securities from FRED (series ID: LTGOVTBD) at URL: `https://fred.stlouisfed.org/series/LTGOVTBD`. To construct the long term bond yields, we use LTGOVTBD before 2000 and use DGS20 after 2000. The Baa spread is the difference between the Moody's Corporate bond yield and the 20-year US government yield. The excess bond premium is obtained at URL: `https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/recession-risk-and-the-excess-bond-premium-20160408.html`. All series were downloaded on Feb 21, 2022.

Bloomberg Consensus Inflation and GDP forecasts

We obtain the Bloomberg (BBG) US GDP (id: EXGDUS) and inflation (id: ECPIUS) consensus mean forecast from the Bloomberg Terminal available on a daily basis up to a few days before the release of GDP and inflation data. The Bloomberg (BBG) US consensus forecasts are updated daily (except for weekends and holidays) and reports daily quarter-over-quarter real GDP growth and CPI forecasts from 2003:Q1. These forecasts provide more high-frequency information on the professional outlook for economic indicators. Both forecast series were downloaded on October 21, 2021.

Livingston Survey Inflation Forecast

We obtained the Livingston Survey mean 1-year and 10-year CPI inflation forecast from the Federal Reserve Bank of Philadelphia, URL: https://www.philadelphiafed.org/surveys-and-data/real-time-data-research/livingston-historical-data. The forecast series were downloaded on September 20, 2021.

Michigan Survey of Consumers Inflation Forecasts

We construct MS forecasts of annual inflation of respondents answering at time t. Each month, the SOC contains approximately 50 core questions, and a minimum of 500 interviews are conducted by telephone over the course of the entire month, each month. We use two questions from the monthly survey for which the time series begins in January 1978.

1. Annual CPI inflation: To get a point forecast, we combine the information in the survey responses to questions A12 and A12b.

   - Question A12 asks (emphasis in original): *During the next 12 months, do you think that prices in general will go up, or go down, or stay where they are now?*
   - A12b asks (emphasis in original): *By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?*

2. Long-run CPI inflation: To get a point forecast, we combine the information in the survey responses to questions A13 and A13b.

   - Question A13 asks (emphasis in original): *What about the outlook for prices over the next 5 to 10 years? Do you think prices will be higher, about the same, or lower, 5 to 10 years from now?*
   - A13b asks (emphasis in original): *By about what percent per year do you expect prices to go (up/down) on the average, during the next 5 to 10 years?*

All series were downloaded on September 17th, 2021.

Bluechip Inflation and GDP Forecasts

We obtain Blue Chip expectation data from Blue Chip Financial Forecasts. The surveys are conducted each month by sending out surveys to forecasters in around 50 financial firms such as Bank of America, Goldman Sachs & Co., Swiss Re, Loomis, Sayles & Company, and J.P. Morgan Chase. The participants are surveyed around the 25th of each month and the results published a few days later on the 1st of the following month. The forecasters are asked to forecast the average of the level of U.S. interest rates over a particular calendar quarter, e.g. the federal funds rate and the set of H.15 Constant Maturity Treasuries (CMT) of the following maturities: 3-month, 6-month, 1-year, 2-year, 5-year and 10-year, and the quarter over quarter percentage changes in Real GDP, the GDP Price Index and the Consumer Price Index, beginning with the current quarter and extending 4 to 5 quarters into the future.

In this study, we look at a subset of the forecasted variables. Specifically, we use the Blue Chip micro data on individual forecasts of the quarter-over-quarter (Q/Q) percentage change in the Real GDP, the GDP Price Index and the CPI, and convert to quarterly observations as explained below.

1. CPI inflation: We use quarter-over-quarter percentage change in the consumer price index, which is defined as

   "Forecasts for the quarter-over-quarter percentage change in the CPI (consumer prices for all urban consumers). Seasonally adjusted, annual rate."

   Quarterly and annual CPI inflation are constructed the same way as for PGDP inflation, except CPI replaces PGDP.

2. For real GDP growth, We use quarter-over-quarter percentage change in the Real GDP, which is defined as

   "Forecasts for the quarter-over-quarter percentage change in the level of chain-weighted real GDP. Seasonally adjusted, annual rate. Prior to 1992, Q/Q % change (SAAR) in real GNP."

The surveys are conducted right before the publication of the newsletter. Each issue is always dated the 1st of the month and the actual survey conducted over a two-day period almost always between 24th and 28th of the month. The major exception is the January issue when the survey is conducted a few days earlier to avoid conflict with the Christmas holiday. Therefore, we assume that the end of the last month (equivalently beginning of current month) is when the forecast is made. For example, for the report in 2008 Feb, we assume that the forecast is made on Feb 1, 2008.

Survey of Professional Forecasters (SPF)

The SPF is conducted each quarter by sending out surveys to professional forecasters, defined as forecasters. The number of surveys sent varies over time, but recent waves sent around 50 surveys each quarter according to officials at the Federal Reserve Bank of Philadelphia. Only forecasters with sufficient academic training and experience as macroeconomic forecasters are eligible to participate. Over the course of our sample, the number of respondents ranges from a minimum of 9, to a maximum of 83, and the mean number of respondents is 37. The surveys are sent out at the end of the first month of each quarter, and they are collected in the second or third week of the middle month of each quarter. Each survey asks respondents to provide nowcasts and quarterly forecasts from one to four quarters ahead for a variety of variables. Specifically, we use the SPF micro data on individual forecasts of the price level, long-run inflation, and real GDP.[1] Below we provide the exact definitions of these variables as well as our method for constructing nowcasts and forecasts of quarterly and annual inflation for each respondent.[2]

The following variables are used on either the right- or left-hand-sides of forecasting models:

[1]Individual forecasts for all variables can be downloaded at https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/historical-data/individual-forecasts.

[2]The SPF documentation file can be found at https://www.philadelphiafed.org/-/media/research-and-data/real-time-center/survey-of-professional-forecasters/spf-documentation.pdf?la=en.

1. Quarterly and annual inflation (1968:Q4 - present): We use survey responses for the level of the GDP price index (PGDP), defined as

   "Forecasts for the quarterly and annual level of the chain-weighted GDP price index. Seasonally adjusted, index, base year varies. 1992-1995, GDP implicit deflator. Prior to 1992, GNP implicit deflator. Annual forecasts are for the annual average of the quarterly levels."

   Since advance BEA estimates of these variables for the current quarter are unavailable at the time SPF respondents turn in their forecasts, four quarter-ahead inflation and GDP growth forecasts are constructed by dividing the forecasted level by the survey respondent-type's nowcast. Let $\mathbb{F}_t^{(i)}[P_{t+h}]$ be forecaster i's prediction of PGDP h quarters ahead and $\mathbb{N}_t^{(i)}[P_t]$ be forecaster i's nowcast of PGDP for the current quarter. Annualized inflation forecasts for forecaster i are

   $$\mathbb{F}_t^{(i)}[\pi_{t+h,t}] = (400/h) \times \ln\left(\frac{\mathbb{F}_t^{(i)}[P_{t+h}]}{\mathbb{N}_t^{(i)}[P_t]}\right),$$

   where $h = 1$ for quarterly inflation and $h = 4$ for annual inflation. Similarly, we construct quarterly and annual nowcasts of inflation as

   $$\mathbb{N}_t^{(i)}[\pi_{t,t-h}] = (400/h) \times \ln\left(\frac{\mathbb{N}_t^{(i)}[P_t]}{P_{t-h}}\right),$$

   where $h = 1$ for quarterly inflation and $h = 4$ for annual inflation, and where P_{t-1} is the BEA's advance estimate of PGDP in the previous quarter observed by the respondent in time t, and P_{t-4} is the BEA's most accurate estimate of PGDP four quarters back. After computing inflation for each survey respondent, we calculate the 5th through the 95th percentiles as well as the average, variance, and skewness of inflation forecasts across respondents.

2. Long-run inflation (1991:Q4 - present): We use survey responses for 10-year-ahead CPI inflation (CPI10), which is defined as

   "Forecasts for the annual average rate of headline CPI inflation over the next 10 years. Seasonally adjusted, annualized percentage points. The "next 10 years" includes the year in which we conducted the survey and the following nine years. Conceptually, the calculation of inflation is one that runs from the fourth quarter of the year before the survey to the fourth quarter of the year that is ten years beyond the survey year, representing a total of 40 quarters or 10 years. The fourth-quarter level is the quarterly average of the underlying monthly levels."

   Only the median response is provided for CPI10, and it is already reported as an inflation rate, so we do not make any adjustments and cannot compute other moments or percentiles.

3. Real GDP growth (1968:Q4 - present): We use the level of real GDP (RGDP), which is defined as

   *"Forecasts for the quarterly and annual level of chain-weighted real GDP. Seasonally adjusted, annual rate, base year varies. 1992-1995, fixed-weighted real GDP. Prior to 1992, fixed-weighted real GNP. Annual forecasts are for the annual average of the quarterly levels. Prior to 1981:Q3, RGDP is computed by using the formula NGDP / PGDP * 100."*

All series were downloaded on September 17th, 2021.

Fed Funds Futures and Eurodollar Futures

We use tick-by-tick data on Fed funds futures (FFF) and Eurodollar futures obtained from the CME Group. Our sample spans January 3, 1995 to June 2, 2020. FFF contracts settle based on the average federal funds rate that prevails over a given calendar month. Fed funds futures are priced at $100 - f_t^{(n)}$, where $f_t^{(n)}$ is the time-t contracted federal funds futures market rate that investors lock in. Contracts are monthly and expire at month-end, with maturities ranging up to 60 months. For the buyer of the futures contract, the amount of $\left(f_t^{(n)} - r_{t+n}\right) \times$ \$D, where r_{t+n} is the ex post realized value of the federal funds rate for month $t+n$ calculated as the average of the daily Fed funds rates in month $t+n$, and \$D is a dollar "deposit", represents the payoff of a zero-cost portfolio.

Eurodollar futures contracts are quarterly, expiring two business days before the third Wednesday in the last month of the quarter. Eurodollar futures are similarly quoted, where $f_t^{(q)}$ is the average 3-month LIBOR in quarter q of contract expiry. Maturities range up to 40 quarters. For both types of contracts, the implied contract rate is recovered by subtracting 100 from the price and multiplying by -1.

Both types of contracts are cleaned following the same procedure following communication with the CME Group. First, trades with zero volume, which indicate a canceled order, are excluded. Floor trades, which do not require a volume on record, are included. Next, trades with a recorded expiry (in YYMM format) of 9900 indicate bad data and are excluded (Only 1390 trades, or less than 0.01% of the raw Fed funds data, have contract delivery dates of 9900). For trades time stamped to the same second, we follow Bianchi, Kind, and Kung (2019) and keep the trade with the lowest sequence number, corresponding to the first trade that second.

Fed funds futures data require additional cleaning. Trade prices were quoted in different units prior to August 2008. To standardize units across our sample, we start by noting that Fed funds futures are priced to the average effective Fed funds rate realized in the contract month. And in our sample, we expect a reasonable effective Fed funds rate to correspond to prices in the 90 to 100 range. As such, we rescale prices to be less than 100 in the pre-August 2008 subsample.[3] After rescaling, a small number of trades still appear to have prices that are far

[3]For trades with prices significantly greater than 100, we repeatedly divide by 10 until prices are in the range

away from the effective Fed funds rates at both trade day and contract expiry, along with trades in the immediate transactions. The CME Group could not explain this data issue, so following Bianchi, Kind, and Kung (2019) and others in the high frequency equity literature(Brownlees and Gallo 2006, Barndorff-Nielsen, Hansen, Lunde, and Shephard 2008, Andersen, Bollerslev, and Meddahi 2005), we apply an additional filter to exclude trades with such non-sensible prices. Specifically, for each maturity contract, we only keep trades where

$$|p_t - \overline{p}_t(k, \delta)| < 3\sigma_t(k, \delta) + \gamma,$$

where p_t denotes the trade price (where t corresponds to a second), and $\overline{p}_t(k, \delta)$ and $\sigma_t(k, \delta)$ denote the average price and standard deviation, respectively, centered with $k/2$ observations on each side of t excluding $\delta k/2$ trades with highest price and excluding $\delta k/2$ trades with lowest price. Finally, γ is a positive constant to account for the cases where prices are constant within the window. Our main specification uses $k = 30$, $\delta = 0.05$ and $\gamma = 0.4$, and alternative parameters produce similar results.

High Frequency Changes Around FOMC Meetings

We follow Guraynak, Sack, and Swanson (2005) and Nakamura and Steinsson (2018) among others in constructing high frequency changes around FOMC meetings. Although we do not use these changes directly in the structural model estimation, we constructed these changes as a cross-check on the construction of our high-frequency FFF data around meetings.

First, we compile dates and times of FOMC meetings from 1994 to 2004 from Guraynak, Sack, and Swanson (2005). The dates of the remaining FOMC meetings are collected from the Federal Reserve Board website. The times of statement releases were coalesced in the following priority: the Federal Reserve Board calendar, the Federal Reserve Board minutes, Bloomberg's FOMC page, and the first news article to appear on Bloomberg. We only include scheduled meetings and unscheduled meetings where a statement was released.

Next, we calculate changes in implied futures rates in a tight window around each FOMC statement release. Our main specification uses an inner window of 30 minutes, from 10 minutes before the FOMC announcement to 20 minutes after it, along with an outer window from 12am to noon the next day. Specifically, on the left side of the window, we use the first trade at 10 minutes before the FOMC announcement, or the nearest trade before 10 minutes if there is no trade at 10 minutes exactly, but not before 12am. Similarly, on the right side of the window, we use the first trade at 20 minutes after the announcement, or nearest trade after 20 minutes otherwise, but not after noon the next day. In other words, we use the nearest trades on or outside the inner window, but inside of the outer window.

For example, suppose the FOMC announcement is at 2:15pm. Then the inner window is from 2:05pm to 2:35pm. On the left side, we take the first trade at 2:05 or earlier, but not

of 90 to 100. We exclude all trades otherwise.

before 12am. On the right side, we take the first trade at 2:35pm or later, but not after noon the next day. Then we subtract the two implied rates.

As a robustness check, we also consider an inner window of 60 minutes (15 minutes before the FOMC announcement and 45 minutes after), along with outer windows of 12am to 1 hour after the statement release, and 12am to 2 hours after the statement release.

In addition to calculating the change in implied rates from Fed funds futures and Eurodollar futures, we also calculate the surprise component of Fed funds futures. We follow Kuttner (2001) in unwinding the average rate into a surprise measure.

To make notation consistent, for a variable X_t^j let the superscript j index the current or future FOMC meetings ($j = 0$ is the current meeting), and let the subscript t index the "real-time" of when the statement is released ($t - \Delta t$ and $t + \Delta t$ are the inner window before and after the statement, respectively). Let d^j be the day of the jth FOMC meeting and m^j denote the number of calendar days in the month of the FOMC meeting. Let r^j denote the target Fed funds rate prevailing after the jth meeting. And let $f^j_{t-\Delta t}$ and $f^j_{t+\Delta t}$ denote the implied rate from the Fed funds futures contract expiring in the month of the jth meeting, before and after the current meeting. Finally, $\mathbb{E}_{t-\Delta t} \equiv \mathbb{E}[\cdot|\mathcal{I}_{t-\Delta t}]$ is the conditional expectation using information up to an inner window before the FOMC meeting at t.

The implied rate from the Fed funds futures in an inner window around the current FOMC can be written as

$$\begin{aligned} f^0_{t-\Delta t} =& \frac{d^0}{m^0} r^{-1} + \frac{m^0 - d^0}{m^0} \mathbb{E}_{t-\Delta t}(r^0) + \mu^0_{t-\Delta t} \\ f^0_{t+\Delta t} =& \frac{d^0}{m^0} r^{-1} + \frac{m^0 - d^0}{m^0} (r^0) + \mu^0_{t+\Delta t}. \end{aligned}$$

Here we make three assumptions. First, the effective Fed funds rate equals the target rate; if not, then replace r^{-1} by the average effective rate realized so far in the month. Second, r^0 only changes from the FOMC meeting, and is constant for the remainder of the month after the FOMC meeting. In other words, $\mathbb{E}_{t+\Delta t}(r^0) = r^0$. Third, high frequency changes around the term premium μ^0 are negligible. Piazzesi and Swanson (2008) argue the narrow daily window largely "differences out" risk premia that are moving primarily at lower, business cycle frequencies.

With these three assumptions, we can then calculate the current FOMC surprise as a scaled change in the current Fed funds implied rates,

$$e^0_{t+\Delta t} \equiv \frac{m^0}{m^0 - d^0} \left[f^0_{t+\Delta t} - f^0_{t-\Delta t} \right], \tag{A.1}$$

where the scaling is proportional to when in the month the FOMC meeting occurs. And the change in implied rate equals the expected component plus the surprise component.

We also calculate longer horizon surprises around the jth meeting, after the current meeting,

as

$$e^j_{t+\Delta t} \equiv \frac{m^j}{m^j - d^j}\left[\left(f^j_{t+\Delta t} - f^j_{t-\Delta t}\right) - \frac{d^j}{m^j} e^{j-1}_t\right]. \tag{A.2}$$

Lastly, the scale factor can get large if the meeting is at the end of the month and Fed funds futures only trade in half a basis point increments. Therefore if a meeting is in the last 3 to 7 days of the month, then we use the current change in next month's Fed funds futures implied rate.

Structural Breaks as Nonrecurrent Regime-Switching

To capture the phenomenon of nonrecurrent regimes, we suppose that ξ^P_t follows a Markov-switching process in which new regimes can arise but do not repeat exactly as before. This is modeled by specifying the transition matrix over nonrecurrent states, or "structural breaks." If the historical sample has N_P nonrecurrent regimes (implying $N_P - 1$ structural breaks), the transition matrix for the Markov process takes the form

$$\mathbf{H} = \begin{bmatrix} p_{11} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 1-p_{11} & p_{22} & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 1-p_{22} & p_{33} & 0 & \cdots & \cdots & \vdots \\ \vdots & 0 & 1-p_{33} & \ddots & & & \\ \vdots & \vdots & 0 & \vdots & \ddots & & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & p_{N_P,N_P} & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & 1-p_{N_P,N_P} & 1 \end{bmatrix}, \tag{A.3}$$

where $\mathbf{H}_{ij} \equiv p\left(\xi^P_t = i | \xi^P_{t-1} = j\right)$. For example, if there were $N_P = 2$ nonrecurrent regimes in the sample, we would have

$$\mathbf{H} = \begin{bmatrix} p_{11} & 0 \\ 1-p_{11} & 1 \end{bmatrix}.$$

The above process implies that, if you are currently in regime 1, you will remain there next period with probability p_{11} or exit to regime 2 with probability $1 - p_{11}$. Upon exiting to regime 2, since there are only two regimes in the sample and the probability p_{12} of returning exactly to the previous regime 1 is zero, $p_{22} = 1$.

Most Likely Regime Sequence

In this section we explain how to compute the most likely regime sequence. This most likely regime sequence is the particular regime sequence $\xi^{P,T} = \{\hat{\xi}^P_1, ..., \hat{\xi}^P_T\}$ that is most likely to have occurred, given our estimated posterior mode parameter values for $\boldsymbol{\theta}$. This sequence is computed as follows. Let $P\left(\xi^P_t = i | z_{t-1}; \boldsymbol{\theta}_r\right) \equiv \pi^i_{t|t-1}$. First, we run Hamilton's filter to get the vector of filtered regime probabilities $\pi_{t|t}$, $t = 1, 2, ..., T$. The Hamilton filter can be expressed iteratively as

$$\begin{aligned}\pi_{t|t} &= \frac{\pi_{t|t-1} \odot \eta_t}{\mathbf{1}'\left(\pi_{t|t-1} \odot \eta_t\right)} \\ \pi_{t+1|t} &= \mathbf{H}\pi_{t|t}\end{aligned}$$

where η_t is a vector whose *j-th* element contains the conditional density $p(mps_t|\xi_t^P = j; \boldsymbol{\theta}_r)$, the symbol $\odot$ denotes element by element multiplication, and $\mathbf{1}$ is a vector with all elements equal to 1. The final term, $\pi_{T|T}$ is returned with the final step of the filtering algorithm. Then, a recursive algorithm can be implemented to derive the other smoothed probabilities:

$$\pi_{t|T} = \pi_{t|t} \odot \left[\mathbf{H}'\left(\pi_{t+1|T}\left(\div\right)\pi_{t+1|t}\right)\right]$$

where $(\div)$ denotes element by element division. To choose the regime sequence most likely to have occurred given our parameter estimates, consider the recursion in the next to last period $t = T - 1$:

$$\pi_{T-1|T} = \pi_{T-1|T-1} \odot \left[\mathbf{H}'\left(\pi_{T|T}\left(\div\right)\pi_{T|T-1}\right)\right].$$

We first take $\pi_{T|T}$ from the Hamilton filter and choose the regime that is associated with the largest probability, i.e., if $\pi_{T|T} = (.9, .1)$, where the first element corresponds to the probability of regime 1, we select $\hat{\xi}_T^P = 1$, indicating that we are in regime 1 in period T. We now update $\pi_{T|T} = (1, 0)$ and plug into the right-hand-side above along with the estimated filtered probabilities for $\pi_{T-1|T-1}$, $\pi_{T|T-1}$ and estimated transition matrix $\mathbf{H}$ to get $\pi_{T-1|T}$ on the left-hand-side. Now we repeat the same procedure by choosing the regime for $T - 1$ that has the largest probability at $T - 1$, e.g., if $\pi_{T-1|T} = (.2, .8)$ we select $\hat{\xi}_{T-1} = 2$, indicating that we are in regime 2 in period $T - 1$, we then update to $\pi_{T-1|T} = (0, 1)$, which is used again on the right-hand-side now

$$\pi_{T-2|T} = \pi_{T-2|T-2} \odot \left[\mathbf{H}'\left(\pi_{T-1|T}\left(\div\right)\pi_{T-1|T-2}\right)\right].$$

We proceed in this manner until we have a most likely regime sequence $\xi^{P,T}$ for the entire sample $t = 1, 2, ..., T$. Two aspects of this procedure are worth noting. First, it fails if the updated probabilities are exactly $(.5, .5)$. Mathematically this is virtually a zero probability event. Second, note that this procedure allows us to choose the most likely regime sequence by using the recursive formula above to update the filtered probabilities sequentially working backwards from $t = T$ to $t = 1$. This allows us to take into account the time dependence in the regime sequence as dictated by the transition probabilities.

Price-Output Decompositions

Mapping from price to output (measured as GDP_t) is

$$\begin{aligned}\frac{P_t}{GDP_{t-1}} &= \frac{P_t}{D_t}\frac{D_t}{GDP_t}\frac{GDP_t}{GDP_{t-1}} \\ pgdp_t &= pd_t + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}\end{aligned}$$

Below we decompose pd_t to write:

$$\begin{aligned}
pgdp_t &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + y_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} + \underbrace{pdv_t\left(\Delta d\right)}_{\text{earnings}} - \underbrace{pdv_t\left(r^{ex}\right)}_{\text{premia}} - \underbrace{pdv_t\left(rir\right)}_{\text{RIR}} \\
pgdp_{r^{ex},t} &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} - \underbrace{pdv_t\left(r^{ex}\right)}_{\text{premia}} \\
pgdp_{rir,t} &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} - \underbrace{pdv_t\left(rir\right)}_{\text{RIR}} \\
pgdp_{\Delta d,t} &= \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + \widetilde{y}_t + g_t - \widetilde{y}_{t-1}}_{\text{earning share component}} + \underbrace{pdv_t\left(\Delta d\right)}_{\text{earnings}}
\end{aligned}$$

where

$$\begin{aligned}
pd_t &= \kappa_{pd,0} + \mathbb{E}_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right] + \\
&\quad + .5\mathbb{V}_t^b\left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1}\right].
\end{aligned}$$

The solution approximates around the balanced growth path with $\frac{D_{t+1}}{D_t} = G$, where G is the gross growth rate of the economy. The Euler equation under the balanced growth path is

$$\begin{aligned}
1 &= \left[M_{t+1}\left(\frac{P_{t+1}/D_{t+1}+1}{P_t/D_t}\right)\frac{D_{t+1}}{D_t}\right] \\
&= \left[\beta_p\left(\frac{D_{t+1}}{D_t}\right)^{-\sigma_p}\left(\frac{P_{t+1}/D_{t+1}+1}{P_t/D_t}\right)\frac{D_{t+1}}{D_t}\right] \\
&= \left[\underbrace{\beta_p G^{1-\sigma_p}}_{\widetilde{\beta}_p}\left(\frac{P/D+1}{P/D}\right)\right] => \\
\frac{1}{\widetilde{\beta}_p} &= \left(\frac{P/D+1}{P/D}\right) => \\
P/D &= \frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}.
\end{aligned}$$

Denote the log steady state price-payout ratio as $\ln\left(P/D\right) = \overline{pd}$, thus we have

$$\overline{pd} = \ln\left(\frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}\right).$$

$$\begin{aligned}\kappa_{pd,1} &= \exp(\overline{pd})/(1+\exp(\overline{pd})) = \frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}\left[1+\frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p}\right]^{-1} = \widetilde{\beta}_p \\ \kappa_{pd,0} &= \ln(\exp(\overline{pd})+1) - \kappa_{pd,1}pd = \ln\left(\frac{1}{1-\widetilde{\beta}_p}\right) - \widetilde{\beta}_p \ln\frac{\widetilde{\beta}_p}{1-\widetilde{\beta}_p} \\ &= -\widetilde{\beta}_p \ln\widetilde{\beta}_p - \left(1-\widetilde{\beta}_p\right)\ln\left(1-\widetilde{\beta}_p\right)\end{aligned}$$

The log return obeys the following approximate identity (Campbell and Shiller (1989)):

$$r^D_{t+1} = \kappa_{pd,0} + \kappa_{pd,1}pd_{t+1} - pd_t + \Delta d_{t+1},$$

where $\kappa_{pd,1} = \exp(\overline{pd})/(1+\exp(\overline{pd}))$, and $\kappa_{pd,0} = \log\left(\exp(\overline{pd})+1\right) - \kappa_{pd,1}\overline{pd}$. Combining all of the above, the log equity premium is

$$\underbrace{\mathbb{E}^b_t\left[r^D_{t+1}\right] - \left(i_t - \mathbb{E}^b_t\left[\pi_{t+1}\right]\right)}_{\text{Equity Premium}} = \underbrace{\left[\begin{array}{c} -.5\mathbb{V}^b_t\left[r^D_{t+1}\right] - \mathbb{COV}^b_t\left[m_{t+1}, r^D_{t+1}\right] \\ +.5\mathbb{V}^b_t\left[\pi_{t+1}\right] - \mathbb{COV}^b_t\left[m_{t+1}, \pi_{t+1}\right] \end{array}\right]}_{\text{Risk Premium}} + \underbrace{\overline{lp}_t}_{\text{Liquidity Premium}},$$

Then

$$\begin{aligned} pd_t &= \kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+1} - r^D_{t+1} + \kappa_{pd,1}pd_{t+1}\right] \\ pd_t &= \kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+1} - \left(r^{ex}_{t+1} - rir_{t+1}\right) + \kappa_{pd,1}pd_{t+1}\right]\end{aligned}$$

where $\mathbb{E}^b_t\left[r^{ex}_{t+1}\right] = \mathbb{E}^b_t\left[r^D_{t+1}\right] - rir_{t+1}$, where $rir_{t+1} \equiv \left(i_{t+1} - \mathbb{E}^b_t\left[\pi_{t+1}\right]\right)$.

Solving forward:

$$\begin{aligned} pd_t &= \kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+1} - r^{ex}_{t+1} - rir_{t+1}\right] + \\ &\quad + \kappa_{pd,1}\mathbb{E}^b_t\left[\kappa_{pd,0} + \mathbb{E}^b_t\left[\Delta d_{t+2} - r^{ex}_{t+2} - rir_{t+1} + \kappa_{pd,1}pd_{t+2}\right]\right]\end{aligned}$$

Thus:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + \left(1_{\Delta d} - 1_{\mathbb{E}(r^{ex})} - 1_{rir}\right)\sum_{h=0}^{\infty}\kappa^h_{pd,1}\mathbb{E}^b_t\left[S_{t+1+h}\right]$$

where 1_x is a vector of all zeros except for a 1 in the xth position. This can be written as:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + pdv_t\left(\Delta d\right) - pdv_t\left(r^{ex}\right) - pdv_t\left(rir\right)$$

Using the solution:

$$pd_t = \frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + \left(1_{\Delta d} - 1_{\mathbb{E}(r^{ex})} - 1_{r^b}\right)\left(\mathbf{I} - \kappa_{pd,1}T_{\xi_t}\right)^{-1}\left[T_{\xi_t}S_t + \left(\mathbf{I}-\kappa_{pd,1}\right)^{-1}C_{\xi_t}\right].$$

Thus, we can decompose movements in the pd_t into those attributable to expected dividends, equity premia, and expected real interest rates:

$$pgdp_t = \underbrace{\frac{\kappa_{pd,0}}{1-\kappa_{pd,1}} + k_t + y_t + g_t - y_{t-1}}_{\text{earning share component}} + \underbrace{pdv_t\left(\Delta d\right)}_{\text{earnings}} - \underbrace{pdv_t\left(r^{ex}\right)}_{\text{premia}} - \underbrace{pdv_t\left(rir\right)}_{\text{RIR}}.$$

Solution and Estimation Details

Computing Expectations with Regime Switching and Alternative Policy Rules

In what follows, we explain how to use expectations to infer what alternative regimes agents have in mind. Expectations about inflation, FFR, and GDP growth depend on the regime currently in place, the alternative regime, and the probability of moving to such regime. This note is based on "Methods for measuring expectations and uncertainty in Bianchi (2016). That paper explains how to computed expected values in presence of regime changes. In the models described above, for each policy rule in place, agents would have different beliefs about alternative future policy rules. This would lead to changes in expected values for the endogenous variables of the model.

Consider a MS model:

$$S_t = C_{\xi_t} + T_{\xi_t} S_{t-1} + R_{\xi_t} Q \varepsilon_t \tag{A.4}$$

where $\xi_t = \left\{\xi_t^P, \xi_t^b\right\}$ controls the policy regime ξ_t^P controls the policy rule currently in place and the alternative policy rule, while the belief regime ξ_t^b controls agents' beliefs about the possibility of moving to the alternative policy rule.

Let n be the number of variables in S_t. Let $m = B + 1$ be the number of Markov-switching states and define

$$\xi_t = i \equiv \left\{\xi_t^P, \xi_t^b = i\right\}, \quad i = 1, ..., B+1.$$

Define the $mn \times 1$ column vector q_t as:

$$\underset{mn\times 1}{q_t} = \left[q_t^{1\prime}, ..., q_t^{m\prime}\right]'$$

where the individual $n \times 1$ vectors $q_t^i = \mathbb{E}_0\left(S_t 1_{\xi_t=i}\right) \equiv \mathbb{E}\left(S_t 1_{\xi_t=i} | \mathbb{I}_0\right)$ and $1_{\xi_t=i}$ is an indicator variable that is one when belief regime i is in place and zero otherwise. Note that:

$$q_t^i = \mathbb{E}_0\left(S_t 1_{\xi_t=i}\right) = \mathbb{E}_0\left(S_t | \xi_t = i\right) \pi_t^i$$

where $\pi_t^i = P_0\left(\xi_t = i\right) = P\left(\xi_t = i | \mathbb{I}_0\right)$. Therefore we can express $\mu_t = \mathbb{E}_0\left(S_t\right)$ as:

$$\mu_t = \mathbb{E}_0\left(S_t\right) = \textstyle\sum_{i=1}^m q_t^i = w q_t$$

where the matrix $\underset{n\times mn}{w} = [I_n, ..., I_n]$ is obtained placing side by side m n-dimensional identity matrices. Then the following proposition holds:

PROPOSITION 1: *Consider a Markov-switching model whose law of motion can be described by (A.4) and define* $q_t^i = \mathbb{E}_0\left(S_t 1_{\xi_t=i}\right)$ *for* $i = 1...m$. *Then* $q_t^j = C_j \pi_t^j + \sum_{i=1}^m T_j q_{t-1}^i p_{ji}$.

It is then straightforward to compute expectations conditional on the information available at a particular point in time. Suppose we are interested in $\mu_{t+s|t} \equiv \mathbb{E}_t^b\left(S_{t+s}\right)$, i.e. the expected value for the vector S_{t+s} conditional on the information set available at time t. If we define:

$$q_{t+s|t} = \left[q_{t+s|t}^{1\prime}, ..., q_{t+s|t}^{m\prime}\right]'$$

where $q^i_{t+s|t} = \mathbb{E}^b_t\left(S_{t+s}1_{\xi_t=i}\right) = \mathbb{E}^b_t\left(S_{t+s}|\xi_t = i\right)\pi^i_{t+s|t}$, where $\pi^i_{t+s|t} \equiv P\left(\xi_{t+s} = i|\mathbb{I}_t\right)$, we have

$$\mu_{t+s|t} = \mathbb{E}^b_t\left(S_{t+s}\right) = wq_{t+s|t}, \tag{A.5}$$

where for $s \geq 1$, $q_{t+s|t}$ evolves as:

$$\begin{aligned} q_{t+s|t} &= C\pi_{t+s|t} + \Omega q_{t+s-1|t} && \text{(A.6)} \\ \pi_{t+s|t} &= \mathbf{H}^b\pi_{t+s-1|t} && \text{(A.7)} \end{aligned}$$

with $\pi_{t+s|t} = \left[\pi^1_{t+s|t}, ..., \pi^m_{t+s|t}\right]'$, $\Omega = bdiag\left(T_1, ..., T_m\right)\left(\mathbf{H}^b \otimes I_n\right)$, and $\underset{mn\times m}{C} = bdiag\left(C_1, ..., C_m\right)$, where e.g., C_1 is the $n \times 1$ vector of constants in regime 1, $\otimes$ represents the Kronecker product and *bdiag* is a matrix operator that takes a sequence of matrices and use them to construct a block diagonal matrix.

The formulas above are used to compute expectations conditional on each belief regime ξ^b_t and policy rule regime ξ^P_t. For each composite regime $\xi_t = \left\{\xi^P_t, \xi^b_t\right\}$, we can obtain a forecast for each of the variables of the model. For example, conditional on ξ^P_t and $\xi^b_t = j$ in place we have

$$q_{t,\xi_t=j} = e_j \otimes S_t$$

where e_j is a variable that has elements equal to zero except for the one in position ξ^b_t. For example, with $B = 5$ belief regimes and $\xi^b_t = 3$ we have

$$q_{t,\xi_t=3} = \left[\mathbf{0}', \mathbf{0}', S_t', \mathbf{0}', \mathbf{0}', \mathbf{0}'\right]'.$$

where $\mathbf{0}$ and S_t are column vectors with n rows. We have $B + 1$ subvectors in $q_{t,\xi_t=j}$ to take into account the alternative policy mix. The fact that all subvectors are zero except for the one corresponding to the belief regime $b = 3$ reflects the assumption that agents can observe the current state S_t and, by definition, their own beliefs (while the econometrician cannot observe any of the two and she uses macro data and survey expectations to estimate both S_t and agents' beliefs).

Thus, suppose we want to compute the expected value for a variable x over the next year under the assumption that agents' beliefs are $\xi^b_t = j$. With monthly data, we have:

$$\begin{aligned} \mathbb{E}^b_t\left(x_{t,t+s}|\xi_t = j\right) &= \textstyle\sum_{s=1}^{12}\mathbb{E}^b_t\left(x_{t+s}|\xi_t = j\right) \\ &= e_x\textstyle\sum_{s=1}^{12}\mu_{t+s|t,\xi_t=j} \\ &= e_xw\textstyle\sum_{s=1}^{12}q_{t+s|t,\xi_t=j} \end{aligned}$$

where for $s \geq 1$, $q_{t+s|t}$ evolves as:

$$\begin{aligned} q_{t+s|t,\xi_t=j} &= C\pi_{t+s|t} + \Omega q_{t+s-1|t,\xi_t=j} && \text{(A.8)} \\ \pi_{t+s|t,\xi_t=j} &= \mathbf{H}^b\pi_{t+s-1|t,\xi_t=j} && \text{(A.9)} \end{aligned}$$

with $\pi_{t+s|t} = \left[\pi^1_{t+s|t}, ..., \pi^m_{t+s|t}\right]'$, $\Omega = bdiag\left(T_1, ..., T_m\right)\left(\mathbf{H}^b \otimes I_n\right)$, and $\underset{mn\times m}{C} = bdiag\left(C_1, ..., C_m\right)$, where e.g., C_1 is the $n \times 1$ vector of constants in regime 1, $\otimes$ represents the Kronecker product and *bdiag* is a matrix operator that takes a sequence of matrices and use them to construct a block diagonal matrix. The recursive algorithm is initialized with $\pi_{t|t,\xi_t=j} = 1_{\xi_t=j}$ and $q_{t,\xi_t=j} = e_j \otimes S_t$.

The formulas (A.8) and (A.9) can be written in a more compact form. If we define $\widetilde{q}_{t|t} = [q'_{t|t}, \pi'_{t|t}]'$, with $\pi_{t|t}$ a vector with elements $\pi^i_{t|t} \equiv P\left(\xi_t = i|\mathbb{I}_t\right)$ we can compute the conditional expectations in one step:

$$\mu_{t+s|t} = \mathbb{E}^b_t\left(S_{t+s}\right) = \widetilde{w}\widetilde{\Omega}^s\widetilde{q}_{t|t} \tag{A.10}$$

where $\widetilde{w} = [w, 0_{n\times m}]$. The formula above can be used to compute the expected value from the point of view of the agent of the model with beliefs $\xi_t = j$:

$$\mathbb{E}^b_t\left(x_{t+s}|\xi_t = j\right) = e_x\mu_{t+s|t,\xi_t=j} = e_x\widetilde{w}\widetilde{\Omega}^s\widetilde{q}_{t|t,\xi^b_t=j} = \underbrace{e_x w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}}_{Z_{\xi_t,x_{t+s}}}\underbrace{S_t}_{(n\times 1)} + \underbrace{e_x w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,x_{t+s}}} \tag{A.11}$$

where $D_{\xi_t,x_{t+s}}$ is a scalar, $Z_{\xi_t,x_{t+s}}$ is an $(1 \times n)$ vector, $\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}$ is the submatrix obtained taking the first nm rows and the columns from $n(j-1)+1$ to nj of $\widetilde{\Omega}^s$, while $\widetilde{\Omega}^s_{\{1,nm\},nm+j}$ is the submatrix obtained taking the first nm rows and the $nm+j$ column of $\widetilde{\Omega}^s$. Thus, we have that conditional on one belief regime and a policy rule regime, we can map the current state of the economy S_t into the expected value reported in the survey. The matrix algebra in (A.11) returns the same results of the recursion in (A.8) and (A.9).

To see what the formulas above do, consider a simple example with $B = 2$ and we are currently in belief regime $b = 2$:

$$\begin{aligned}
\mathbb{E}^b_t\left(x_{t+s}|\xi_t = 2\right) &= e_x\widetilde{w}\widetilde{\Omega}^s\widetilde{q}_{t|t,\xi_t=2} = e_x\widetilde{w}\widetilde{\Omega}^s\begin{bmatrix}\mathbf{0}_{n\times 1}\\ S_t\\ n\times 1\\ \mathbf{0}_{n\times 1}\\ 0\\ 1\\ 0\end{bmatrix} = e_x\widetilde{w}\begin{bmatrix}\widetilde{\Omega}^s_{11} & \widetilde{\Omega}^s_{12} & \widetilde{\Omega}^s_{13} & \widetilde{\Omega}^s_{14} & \widetilde{\Omega}^s_{15} & \widetilde{\Omega}^s_{16}\\ \widetilde{\Omega}^s_{21} & \widetilde{\Omega}^s_{22} & \widetilde{\Omega}^s_{23} & \widetilde{\Omega}^s_{24} & \widetilde{\Omega}^s_{25} & \widetilde{\Omega}^s_{26}\\ \widetilde{\Omega}^s_{31} & \widetilde{\Omega}^s_{32} & \widetilde{\Omega}^s_{33} & \widetilde{\Omega}^s_{34} & \widetilde{\Omega}^s_{35} & \widetilde{\Omega}^s_{36}\\ & & & \widetilde{\Omega}^s_{44} & \widetilde{\Omega}^s_{45} & \widetilde{\Omega}^s_{46}\\ & & & \widetilde{\Omega}^s_{54} & \widetilde{\Omega}^s_{55} & \widetilde{\Omega}^s_{56}\\ & & & \widetilde{\Omega}^s_{64} & \widetilde{\Omega}^s_{65} & \widetilde{\Omega}^s_{66}\end{bmatrix}\begin{bmatrix}\mathbf{0}_{n\times 1}\\ S_t\\ n\times 1\\ \mathbf{0}_{n\times 1}\\ 0\\ 1\\ 0\end{bmatrix}\\
&= e_x\widetilde{w}\begin{bmatrix}\widetilde{\Omega}^s_{12}S_t + \widetilde{\Omega}^s_{15}\\ \widetilde{\Omega}^s_{22}S_t + \widetilde{\Omega}^s_{25}\\ \widetilde{\Omega}^s_{32}S_t + \widetilde{\Omega}^s_{35}\\ \widetilde{\Omega}^s_{44}\\ \widetilde{\Omega}^s_{54}\\ \widetilde{\Omega}^s_{64}\end{bmatrix} = e_x\left(\widetilde{\Omega}^s_{12} + \widetilde{\Omega}^s_{22} + \widetilde{\Omega}^s_{32}\right)S_t + e_x\left(\widetilde{\Omega}^s_{15} + \widetilde{\Omega}^s_{25} + \widetilde{\Omega}^s_{35}\right)
\end{aligned}$$

Finally, suppose we are interested in the forecast $\mathbb{E}^b_t\left(x_{t,t+s}|\xi^b_t = j, \xi^p_t\right)$:

$$\mathbb{E}^b_t\left(x_{t,t+s}|\xi_t = j\right) = \underbrace{\left[e_x\sum_{s=1}^{12} w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}\right]}_{Z_{\xi_t,x_{t,t+s}}}\underbrace{S_t}_{(n\times 1)} + \underbrace{e_x\sum_{s=1}^{12} w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,x_{t,t+s}}} \tag{A.12}$$

Thus, we can include $Z_{\xi_t,x_{t,t+s}}$ as a row in Z_{ξ_t} and $D_{\xi_t,x_{t,t+s}}$ as a row in D_{ξ_t} in the mapping from the model to the observables described in (A.13). Note that the matrix Z and vector D are now regime dependent.

For GDP growth, we are interested in the average growth over a certain horizon. Our state vector contains $\widetilde{y}_t$. Thus, we can use the following approach:

$$\begin{aligned}\mathbb{E}_t^b\left[\left(gdp_{t+h}-gdp_t\right)h^{-1}|\xi_t=j\right] &= \mathbb{E}_t^b\left[\left(\widetilde{y}_{t+h}-\widetilde{y}_t+h\mu\right)h^{-1}|\xi_t=j\right]\\ &= h^{-1}\mathbb{E}_t^b\left[\widetilde{y}_{t+h}|\xi_t=j\right]-h^{-1}\widetilde{y}_t+\mu\end{aligned}$$

where μ is the average growth rate in the economy and $\widetilde{y}_t$ is GDP in deviations from the trend. With deterministic growth we have $gdp_{t+h}-gdp_t-h\mu\equiv\widetilde{y}_{t+h}-\widetilde{y}_t$. We then have

$$\begin{aligned}\mathbb{E}_t^b\left[\left(gdp_{t+h}-gdp_t\right)h^{-1}|\xi_t=j\right] &= h^{-1}\mathbb{E}_t^b\left[\widetilde{y}_{t+h}|\xi_t=j\right]-h^{-1}\widetilde{y}_t+\mu\\ &= h^{-1}\left[\underbrace{e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}}_{Z_{\xi_t,\widetilde{y}_{t+s}}}\underbrace{S_t}_{(n\times1)}+\underbrace{e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,\widetilde{y}_{t+s}}}-e_{\widetilde{y}}S_t\right]+\mu\\ &= h^{-1}\underbrace{\left[e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},\{n(j-1)+1,nj\}}-e_{\widetilde{y}}\right]}_{Z_{\xi_t,\widetilde{y}_{t+s}-\widetilde{y}_t}}\underbrace{S_t}_{(n\times1)}+h^{-1}\underbrace{e_{\widetilde{y}}w\widetilde{\Omega}^s_{\{1,nm\},nm+j}}_{D_{\xi_t,\widetilde{y}_{t+s}}}+\mu\end{aligned}$$

The expected values for the endogenous variables depend on the perceived transition matrix $\mathbf{H}^b$ and the properties of the alternative regime. The latter can be seen by recalling that the regime $\xi_t=B+1$ applies to the perceived alternative regime. Thus, data on expectations provide information about the perceived probability of moving across belief regimes as well as the parameters of the alternative regime.

Estimation

The solution of the model takes the form of a Makrov-switching vector autoregression (MS-VAR) in the state vector $S_t=\left[S_t^M,m_t,pd_t,k_t,z_t,lp_t,\mathbb{E}_t^b\left(m_{t+1}\right),\mathbb{E}_t^b\left(pd_{t+1}\right)\right]$. Here, S_t^M is a vector of macro block state variables given by $S_t^M\equiv\left[\widetilde{y}_t,g_t,\pi_t,i_t,\overline{\pi}_t,f_t\right]'$. The asset pricing block of equations involves conditional subjective variance terms that are affected by Markov-switching random variables in the model. The subsection "Risk Adjustment with Lognormal Approximation," below, explains the approximation used to preserve lognormality of the entire system.

The model solution in state space form is

$$
\begin{aligned}
X_t &= D_{\xi_t,t} + Z_{\xi_t,t}\left[S_t', \widetilde{y}_{t-1}\right]' + U_t v_t \\
S_t &= C\left(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b\right) + T(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b) S_{t-1} + R(\theta_{\xi_t^P}, \xi_t^b, \mathbf{H}^b) Q \varepsilon_t \\
Q &= diag\left(\sigma_{\varepsilon_1}, ..., \sigma_{\varepsilon_G}\right), \ \varepsilon_t \sim N(0, I) \\
U &= diag\left(\sigma_1, ..., \sigma_X\right), \ v_t \sim N(0, I) \\
\xi_t^P &= 1...N_P, \ \xi_t^b = 1, ...B+1, H_{i,j} = p\left(\xi_t^b = i | \xi_{t-1}^b = j\right).
\end{aligned}
$$

where X_t is a $N_X \times 1$ vector of data, v_t are a vector of observation errors, U_t is a diagonal matrix with the standard deviations of the observation errors on the main diagonal, and $D_{\xi_t,t}$, and $Z_{\xi_t,t}$ are parameters mapping the model counterparts of X_t into the latent discrete- and continuous-valued state variables ξ_t and S_t, respectively, in the model. The vector X_t of observables is explained below. Note that the parameters $D_{\xi_t,t}$, $Z_{\xi_t,t}$, and U_t vary over time in part because not all variables are observed at each data sampling period. In addition, some of the parameters in the system are dependent on the current policy rule and the associated Alternative rule, ξ_t^P, and the unobserved, discrete-valued $(B+1)$-state Markov-switching variable ξ_t^b $\left(\xi_t^b = 1, 2, ..., B+1\right)$ with perceived transition probabilities

$$
\mathbf{H}^b = \begin{bmatrix}
p_{11} & \cdots & p_{1B} & 0 \\
\vdots & \ddots & \vdots & \vdots \\
p_{B1} & \cdots & p_{BB} & 0 \\
1 - \sum_{i=1}^{B} p_{i1} & \cdots & 1 - \sum_{i=1}^{B} p_{iB} & p_{B+1,B+1} = 1
\end{bmatrix},
$$

where $\mathbf{H}_{ij}^b \equiv p\left(\xi_t^b = i | \xi_{t-1}^b = j\right)$. We use the following notation:

$$
\begin{aligned}
C_{\xi_t^P,j} &= C\left(\theta_{\xi_t^P}, \xi_t^b = j\right), \ T_{\xi_t^P,j} = T\left(\theta_{\xi_t^P}, \xi_t^b = j\right), \ R_{\xi_t^P,j} = R\left(\theta_{\xi_t^P}, \xi_t^b = j\right) \\
D_{j,t} &= D_{\xi_t,t|\xi_t^b=j}, \quad Z_{j,t} = Z_{\xi_t,t|\xi_t^b=j}.
\end{aligned}
$$

Kim's Approximation to the Likelihood and Filtering We use Kim's (Kim (1994)) basic filter and approximation to the likelihood.

First note that, from the econometricians viewpoint, only the first B regimes can actually be realized, since the true alternative that arises after one regime ends is never exactly as previously conceived by the investor. Thus, the estimation algorithm involves estimation just the upper $B \times B$ submatrix of $\mathbf{H}^b$, rescaled so that the elements sum to unity. The filtering described here therefore loops over just B states, rather than $B+1$.

The sample is divided into N_P policy regime subperiods indexed by ξ_t^P. Denote the last observation of each regime subperiod of the sample $T_1, ..., T_{N_P}$. The algorithm for the basic filter is described as follows.

Initiate values $\widetilde{S}_{0|0}$, $P_{0|0}$, for the Kalman filter and $\Pr\left(\xi_0^b\right) = \pi_0$ for the Hamilton filter and initialize $\mathcal{L}(\theta) = 0$. Denote $X^{t-1} \equiv \{X_1, ..., X_{t-1}\}$ and $\xi^{PT} = \left\{\xi_1^P, ..., \xi_T^P\right\}$.

In the mixed-frequency estimation, we use intra-month data to provide "early" estimates of the state space, while "final" estimates are obtained using a more complete set of data available at the end of each month. Let t denote a month. Let d_i denote the number of time units that have passed *within* a month when we have reached a particular point in time, and let nd denote the total number of time units in the month. Then $0 \leq d_i/nd \leq 1$, and the end of month t is denoted $t-1+d_i/nd$ with $d_i = nd$. For example, d_{100}/nd could denote the point within the month that is exactly 10 minutes before an FOMC meeting during the month, while d_{130}/nd could denote point in the month 20 minutes after the same meeting. Intra-month observations used just prior to an FOMC meeting will typically include the daily BBG consensus forecasts from the day before the meeting, and the 10-minutes before FFF, ED and stock market data. Intermonth observations for the point of the month right after the FOMC meeting will typically include the daily BBG consensus forecasts from the day after the meeting, and the 20-minutes after FFF, ED and stock market data.

- For $t = 1$ to T_1 and $\theta_{\xi_t^P}$ relevant when $\xi_t^P = 1$:

1. Suppose we have information up through month $t-1$ and new information arrives at $t-1+d_i/nd$. Conditional on $\xi_{t-1}^b = i$ and $\xi_t^b = j$ run the Kalman filter given below for $i, j = 1, 2, ..., B$ to update estimates of the latent state:

$$\begin{aligned}
S_{t|t-1}^{(i,j)} &= C_{\xi_t^P,j} + T_{\xi_t^P,j} S_{t-1|t-1}^i \\
P_{t|t-1}^{(i,j)} &= T_{\xi_t^P,j} P_{t-1|t-1}^i T_{\xi_t^P,j}' + R_{\xi_t^P,j} Q^2 R_{\xi_t^P,j}' \text{ with } Q^2 \equiv QQ' \\
e_{t|t-1+d_i/nd,t-1}^{(i,j)} &= X_{t-1+d_i/nd} - D_{j,t-1+d_i/nd} - Z_{j,t-1+d_i/nd} \left[S_{t|t-1}^{(i,j)\prime}, \widetilde{y}_{t-1} \right] \\
f_{t|t-1+d_i/nd,t-1}^{(i,j)} &= Z_{j,t-1+d_1/nd} P_{t|t-1}^{(i,j)} Z_{j,t-1+d_1/nd}' + U_{t-1+d_1/nd}^2 \quad \text{with } U_t^2 \equiv U_t U_t' \\
S_{t|t-1+d_i/nd}^{(i,j)} &= S_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)} Z_{j,t-1+d_i/nd}' \left(f_{t|t-1+d_i/nd,t-1}^{(i,j)} \right)^{-1} e_{t|t-1+d_i/nd,t-1}^{(i,j)} \\
P_{t|t-1+d_i/nd}^{(i,j)} &= P_{t|t-1}^{(i,j)} - P_{t|t-1}^{(i,j)} Z_{j,t-1+d_i/nd}' \left(f_{t|t-1+d_i/nd,t-1}^{(i,j)} \right)^{-1} Z_{j,t-1+d_i/nd} P_{t|t-1}^{(i,j)}
\end{aligned}$$

2. Run the Hamilton filter to calculate new regime probabilities $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+d_i/nd}, X^{t-1}\right)$ and $\Pr\left(\xi_t^b | X_{t-1+d_i/nd}, X^{t-1}\right)$, for $i, j = 1, 2, ..., B$

$$\begin{aligned}
\Pr\left(\xi_t^b, \xi_{t-1}^b | X^{t-1}\right) &= \Pr\left(\xi_t^b | \xi_{t-1}^b\right) \Pr\left(\xi_{t-1}^b | X^{t-1}\right) \\
\ell\left(X_{t-1+d_i/nd} | X^{t-1}\right) &= \textstyle\sum_{j=1}^B \sum_{i=1}^B f\left(X_{t-1+d_i/nd} | \xi_{t-1}^b = i, \xi_t^b = j, X^{t-1}\right) \Pr\left[\xi_{t-1}^b = i, \xi_t^b = j | X^{t-1}\right] \\
f\left(X_{t-1+d_i/nd} | \xi_{t-1}^b = i, \xi_t^b = j, X^{t-1}\right) &= (2\pi)^{-N_X/2} |f_{t|t-1+d_i/nd}^{(i,j)}|^{-1/2} \exp\left\{ -\frac{1}{2} e_{t|t-1+d_i/nd,t-1}^{(i,j)\prime} f_{t|t-1+d_i/nd,t-1}^{(i,j)} e_{t|t-1+d_i/nd,t-1}^{(i,j)} \right\} \\
\mathcal{L}(\theta) &= \mathcal{L}(\theta) + \ln\left(\ell\left(X_{t-1+d_i/nd} | X^{t-1}\right)\right) \\
\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+d_i/nd}, X^{t-1}\right) &= \frac{f\left(X_{t-1+d_i/nd} | \xi_t^b, \xi_{t-1}^b, X^{t-1}\right) \Pr\left(\xi_t^b, \xi_{t-1}^b | X^{t-1}\right)}{\ell\left(X_{t-1+d_i/nd} | X^{t-1}\right)} \\
\Pr\left(\xi_t^b | X_{t-1+d_i/nd}, X^{t-1}\right) &= \sum_{i=1}^B \Pr\left(\xi_t^b, \xi_{t-1}^b = i | X_{t-1+d_i/nd}, X^{t-1}\right)
\end{aligned}$$

3. Using $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+d_i/nd}, X^{t-1}\right)$ and $\Pr\left(\xi_t^b | X_{t-1+d_i/nd}, X^{t-1}\right)$, collapse the $B \times B$ values of $S_{t|t-1+d_i/nd}^{(i,j)}$ and $P_{t|t-1+d_i/nd}^{(i,j)}$ into B values represented by $S_{t|t-1+d_i/nd}^{j}$ and $P_{t|t-1+d_i/nd}^{j}$:

$$S_{t|t-1+d_i/nd}^{j} = \frac{\sum_{i=1}^{B} \Pr\left[\xi_{t-1}^b = i, \xi_t^b = j | X_{t-1+d_i/nd}, X^{t-1}\right] S_{t|t-1+d_i/nd}^{(i,j)}}{\Pr\left[\xi_t^b = j | X_{t-1+d_i/nd}, X^{t-1}\right]}$$

$$P_{t|t-1+d_i/nd}^{j} = \frac{\sum_{i=1}^{B} \Pr\left[\xi_{t-1}^b = i, \xi_t^b = j | X_{t-1+d_i/nd}, X^{t-1}\right] \left(P_{t|t-1+d_i/nd}^{(i,j)} + \left(S_{t|t-1+d_i/nd}^{j} - S_{t|t-1+d_i/nd}^{(i,j)}\right)\left(S_{t|t-1+d_i/nd}^{j} - S_{t|t-1+d_i/nd}^{(i,j)}\right)'\right)}{\Pr\left[\xi_t^b = j | X_{t-1+d_i/nd}, X^{t-1}\right]}$$

4. If $t - 1 + d_i/nd = t$, move to the next period by setting $t - 1 = t$ and returning to step 1

5. else, store the updated $S_{t|t-1+d_i/nd}^{j}$, $P_{t|t-1+d_i/nd}^{j}$, $\Pr\left(\xi_t^b, \xi_{t-1}^b | X_{t-1+d_i/nd}, X^{t-1}\right)$, and $\Pr\left(\xi_t^b | X_{t-1+d_i/nd}, X^{t-1}\right)$, and repeat steps 1-5 keeping $t - 1$ fixed.

- At $t = T_1 + 1$ use $\theta_{\xi_t^P}$ relevant when $\xi_t^P = 2$, set $t - 1 = t$, and repeat steps 1-5
- At $t = T_2 + 1$ use $\theta_{\xi_t^P}$ relevant when $\xi_t^P = 3$, set $t - 1 = t$, and repeat steps 1-5
- $\vdots$
- At $t = T_{N_P - 1} + 1$ use $\theta_{\xi_t^P}$ relevant when $\xi_t^P = N_P$, set $t - 1 = t$ and repeat steps 1-5
- At $t = T_N = T$ stop. Obtain $\mathcal{L}(\theta) = \sum_{t=1}^{T} \ln\left(\ell\left(X_t | X^{t-1}\right)\right)$.

The algorithm above is described in general terms; in principle the intermonth loop could be repeated at every instant within a month for which we have new data. In application, we repeat steps 1-5 only at certain minutes or days pre- and post-FOMC meeting.

Observation Equation The mapping from the variables of the model to the observables in the data can be written using matrix algebra to obtain the observation equation $X_t = D_{\xi_t,t} + Z_{\xi_t,t}\left[S_t', \widetilde{y}_{t-1}\right]' + U_t v_t$. Denote $\widehat{g}_t = g_t - g$, and $\widehat{lp}_t = lp_t - lp$. Using the definition of stochastically detrended output, we have $\widetilde{y}_t = \ln\left(Y_t/A_t\right)$, $\Delta\ln\left(A_t\right) \equiv g_t = g + \rho_g\left(g_t - g\right) + \sigma_g \varepsilon_{g,t} \Rightarrow \widetilde{y}_t - \widetilde{y}_{t-1} = \Delta\ln\left(Y_t\right) - g_t \Rightarrow \Delta\ln\left(Y_t\right) = \widetilde{y}_t - \widetilde{y}_{t-1} + g_t = \widetilde{y}_t - \widetilde{y}_{t-1} + \widehat{g}_t + g$. Annualizing the monthly growth rates to get annualized GDP growth we have $\Delta\ln\left(GDP_t\right) \equiv 12\Delta\ln\left(Y_t\right) = 12g + 12\left(\widetilde{y}_t + \widehat{g}_t - \widetilde{y}_t\right)$. Thus the vector of observables X_t is defined as:

$$\begin{bmatrix} \Delta \ln(GDP_t) \\ Inflation \\ FFR \\ SOC(Inflation)_{12m} \\ SOC(Inflation)_{60m} \\ f_t^{(0)} \\ BC(Inflation)_{12m} \\ SPF(Inflation)_{12m} \\ Liv(Inflation)_{12m} \\ SPF(GDPDInfl)_{12m} \\ BBG(Inflation)_{12m} \\ Liv(Inflation)_{120m} \\ SPF(Inflation)_{120m} \\ BC(FFR)_{12m} \\ BC(\Delta GDP)_{12m} \\ BBG(\Delta GDP)_{12m} \\ SPF(\Delta GDP)_{12m} \\ f_t^{(n)} \\ Baa_t \\ pgdp_t \\ egdp_t \end{bmatrix} = \begin{bmatrix} 12g \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+12},\xi_t} \\ D_{\pi_{t,t+120},\xi_t} \\ D_{\pi_{t,t+120},\xi_t} \\ D_{i_{t,t+12},\xi_t^b} \\ D_{y_{t+s},\xi_t^b} \\ D_{y_{t+s},\xi_t^b} \\ D_{y_{t+s},\xi_t^b} \\ D_{i_{t+X},\xi_t} \\ C_{Baa} \\ \ln(K) + g \\ C_{egdp} \end{bmatrix} + \begin{bmatrix} 12(\widetilde{y}_t + \widehat{g}_t - \widetilde{y}_t) \\ 12\pi_t \\ 12i_t \\ \left[h + (h-1)\phi + (h-2)\phi^2 + ... + \phi^{11}\right](1-\phi)\overline{\pi}_t + \left[\phi + \phi^2 + ... + \phi^{12}\right]\pi_t \\ \left[h + (h-1)\phi + (h-2)\phi^2 + ... + \phi^{59}\right](1-\phi)\overline{\pi}_t + \left[\phi + \phi^2 + ... + \phi^{60}\right]\pi_t \\ 12i_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+12},\xi_t} S_t \\ Z_{\pi_{t,t+120},\xi_t} S_t \\ Z_{\pi_{t,t+120},\xi_t} S_t \\ Z_{i_{t,t+12},\xi_t^b} S_t \\ Z_{\xi_t^b, y_{t+s}-y_t} S_t \\ Z_{\xi_t^b, y_{t+s}-y_t} S_t \\ Z_{\xi_t^b, y_{t+s}-y_t} S_t \\ Z_{i_{t+X},\xi_t^b} S_t \\ B\widetilde{lp}_t \\ \widetilde{k}_t + pd_t + \widehat{g}_t + \widetilde{y}_t - \widetilde{y}_{t-1} \\ K\widetilde{k}_t \end{bmatrix} + U_t v_t \tag{A.13}$$

where in the last row we have used the fact that expectations for the macro agent in the model is:

$$\begin{aligned} \mathbb{E}_t^m[\pi_{t,t+h}] &= \left[h + (h-1)\phi + (h-2)\phi^2 + ... + \phi^{h-1}\right]\alpha_t^m + \left[\phi + \phi^2 + ... + \phi^h\right]\pi_t \\ &= \left[h + (h-1)\phi + (h-2)\phi^2 + ... + \phi^{h-1}\right](1-\phi)\overline{\pi}_t + \left[\phi + \phi^2 + ... + \phi^h\right]\pi_t \end{aligned}$$

The term $Inflation$ in the above stands for CPI inflation; $GDPDInfl$ refers to GDP deflator inflation. The variable $f_t^{(n)}$ refers to the time-t contracted federal funds futures market rate. Here we use $n = \{6, 10, 20, 35\}$. The variable $pgdp$ is the SP500 capitalization-to-lagged GDP ratio, i.e., P_t/GDP_{t-1}; $egdp_t$ is the SP 500 earnings-to-lagged GDP ratio, i.e., E_t/GDP_{t-1}; Baa_t is the Baa spread described above, where C_{Baa} and B and K are parameters. To allow for the fact that the true convenience yield is only a function of Baa_t, we add a constant C_{Baa} to our model-implied convenience yield lp_t and scale it by the parameter B to be estimated. Unless otherwise indicated, all survey expectations are 12 month-ahead forecasts in annualized units.

The above uses multiple measures of observables on a single variable, e.g., investor expectations of inflation 12 months ahead are measured by four different surveys (BC, SPF, LIV, and BBG). In the filtering algorithm above, these provide four noisy signals on the same latent variable.

Computing the Posterior

The likelihood is computed with the Kim's approximation to the likelihood, as explained above, and then combined with a prior distribution for the parameters to obtain the posterior. A

block algorithm is used to find the posterior mode as a first step. Draws from the posterior are obtained using a standard Metropolis-Hastings algorithm initialized around the posterior mode. Here are the key steps of the Metropolis-Hastings algorithm:

- Step 1: Draw a new set of parameters from the proposal distribution: $\vartheta \sim N\left(\theta_{n-1}, c\overline{\Sigma}\right)$
- Step 2: Compute $\alpha\left(\theta^m;\vartheta\right) = \min\left\{p\left(\vartheta\right)/p\left(\theta^{m-1}\right),1\right\}$ where $p\left(\theta\right)$ is the posterior evaluated at θ.
- Step 3: Accept the new parameter and set $\theta^m = \vartheta$ if $u < \alpha\left(\theta^m;\vartheta\right)$ where $u \sim U\left([0,1]\right)$, otherwise set $\theta^m = \theta^{m-1}$
- Step 4: If $m \geq n^{sim}$, stop. Otherwise, go back to step 1

The matrix $\overline{\Sigma}$ corresponds to the inverse of the Hessian computed at the posterior mode $\overline{\theta}$. The parameter c is set to obtain an acceptance rate of around 30%. We use four chains of $540,000$ draws each (1 of every 200 draws is saved). The four chains combined are used to form an estimate of the posterior distribution from which we make draws. Convergence is checked by using the Brooks-Gelman-Rubin potential reduction scale factor using within and between variances based on the four multiple chains used in the paper.

Risk Adjustment with Lognormal Approximation

The asset pricing block of equations involves conditional subjective variance terms that are affected by Markov-switching random variables in the model. We extend the approach in Bansal and Zhou (2002) of approximating a model with Markov-switching random variables using a risk-adjustment while maintaining conditional log-normality. Consider the forward looking relation for the price-payout ratio:

$$\begin{aligned} P_t^D &= \mathbb{E}_t^b\left[M_{t+1}\left(P_{t+1}^D + D_{t+1}\right)\right] \\ \frac{P_t^D}{D_t} &= \mathbb{E}_t^b\left[M_{t+1}\frac{D_{t+1}}{D_t}\left(\frac{P_{t+1}^D}{D_{t+1}}+1\right)\right]. \end{aligned}$$

Taking logs on both sides, we get:

$$pd_t = \log\left[\mathbb{E}_t^b\left[\exp\left(m_{t+1}+\Delta d_{t+1}+\kappa_{pd,0}+\kappa_{pd,1}pd_{t+1}\right)\right]\right].$$

Applying the approximation implied by conditional log-normality we have:

$$\begin{aligned} pd_t &= \kappa_0 + \mathbb{E}_t^b\left[m_{t+1}+\Delta d_{t+1}+\kappa_{pd,1}pd_{t+1}\right] + \\ &\quad +.5\mathbb{V}_t^b\left[m_{t+1}+\Delta d_{t+1}+\kappa_{pd,1}pd_{t+1}\right]. \end{aligned}$$

To implement the solution, we follow Bansal and Zhou (2002) and approximate the conditional variance as the weighted average of the objective variance across regimes, conditional on ξ_t. Using the simpler notation of the state equation,

$$S_t = C_{\xi_t} + T_{\xi_t}S_{t-1} + R_{\xi_t}Q\varepsilon_t,$$

the approximation takes the form

$$\mathbb{V}_t^b \left[x_{t+1} \right] \approx e_x \mathbb{E}_t^b \left[R_{\xi_{t+1}} Q Q' R'_{\xi_{t+1}} \right] e_x \tag{A.14}$$

where e_x is a vector used to extract the desired linear combination of the variables in S_t. This approximation maintains conditional log-normality of the entire system. In the solution, C_{ξ_t} depends on the risk adjustment term $\mathbb{V}_t^b \left[m_{t+1} + \Delta d_{t+1} + \kappa_{pd,1} pd_{t+1} \right]$ which depends on R_{ξ_t}. To solve this fixed point problem, employ the iterative approach of Bianchi, Kung, and Tirskikh (2018). Specifically, we solve the model and get S_t for a given risk adjustment. Then, given S_t, we compute a new risk adjustment, which gives us a new $R_{\xi_{t+1}}$. If the new $R_{\xi_{t+1}}$ is the same as the $R_{\xi_{t+1}}$ that was implicitly assumed in the given risk adjustment, stop. Otherwise, use it to resolve S_t and repeat the procedure until convergence.