Appendix for Cutting Out the Middleman: The Structure of Chains of Intermediation

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A Data Appendix

A.1 Lagos Trader Survey data

This section provides a brief description of the methodology of the Lagos Trader Survey, as well as the treatment of the survey data used in the analysis. More details about the survey are provided in **?**.

A.1.1 Survey methodology

The Lagos Trader Survey (LTS) aimed to collect data from a sample of traders in Nigeria selling manufactured consumers goods. The sample frame was created from a listing of all shops located in major commercial and wholesaling areas of the city of Lagos. This listing exercise covered the markets and plazas (indoor markets in multi-story buildings) served by LAWMA, the Lagos State waste collection agency, supplemented with market areas located on Nigerian federal government land that were not included in the LAWMA list. It does not include residential or manufacturing areas, or traditional markets where vendors mainly sell food and household consumables, or several markets selling primarily used goods. Between October 2014 and April 2015, a team of research assistants counted 52,830 shops throughout these areas of Lagos, enumerating them by location (building and floor numbers) and the type of products being sold.

In the first round of the LTS (LTS1) in 2015, interviews were conducted with 1,179 respondents randomly sampled from this sample frame, reflecting a response rate of 82%. A second round of the LTS survey (LTS2) was conducted in 2016, with a 100% resample of respondents who imported in the previous round, and a 30% resample of non-importers. The response rate was 85% overall, with 75% of original respondents completing the full survey, and an additional 10% (most of whom had gone out of business or relocated) completing a shorter phone survey. A third round of the LTS was completed in 2018. The data in this paper draws mainly from LTS2; all statistics and analysis are reweighted to account for sampling probabilities.

A.1.2 Survey data

LTS2 asks retrospectively about traders' activities in the calendar year 2015. In the survey, respondents were asked questions at five different levels of observation: 1) the business, 2) source countries, 3) shipments, 4) suppliers, and 5) individual purchases. Questions about the size and tenure of the business as well as the fraction of total sales that are wholesale were collected at firm level. For imports, questions about trade costs were asked at the level of a specific shipment, from a particular country in a particular month. Questions about the cost and sale price of a particular product were asked at the transaction level, from a particular supplier within a shipment. For domestic purchases, it was not possible to recover this level of detail due to the much higher frequency of transactions. Instead, respondents are asked about a typical purchase from their two biggest suppliers, and less information is available about trade costs. For this reason, most of the analysis in Section 2 of the paper focuses on importers. Throughout the paper, analysis is done at the level of observation at which the outcome variable is observed. For instance, when the outcome is annual revenue, other variables are aggregated up to the firm level. When the outcome is the price of a specific product, other variables are distributed down to the transaction level.

Trade costs

Data on trade costs (for imports) were collected with reference to a specific shipment from a particular country. We collected information on the following categories of costs: visas, airfare, other travel costs, transportation, clearing the port, and payments to agents (e.g. to inspect the product). Importantly, respondents are asked to report these costs paid for the entire shipment, including all products purchased from all suppliers. For instance, the question about transportation asks, "How much did you pay in total for transporting/shipping ALL the products you bought on this trip?". Transaction level details (e.g. about the cost paid to a supplier) were only asked for the top two products purchased from each supplier in that shipment, and therefore may not cover the full value of items included in the shipment. Instead, we also asked about the total value of all goods purchased, e.g. "What was the total cost of ALL the goods you bought on this [date] trip to [country]? Please include both the products you just told me about and any others you purchased." Response rates on this question were much lower, and so we are only able to calculate ad valorem equivalent transport or total trade costs for a subset of shipments.

Prices and markups

Data on import purchases were collected at the level of individual transactions, for a specific product, from a specific supplier, in a specific country, on a particular date. This data includes the type of product, the quantity purchased, the cost paid to the supplier, and the average price the trader charged to customers who bought that product. In cases where a trader bought more than two products from a supplier in a particular shipment, the survey asked for details about the top product. The data may therefore not be representative of lower value purchases in cases where the trader buys many different things from a single supplier at a time.

Markups are calculated in two different ways for the purposes of the analysis in the paper. Our baseline measure of markups is simply sale price divided by purchase price, minus one. For instance, if a trader bought a pair of shoes for \$5 and sold them on average for \$7.50, we calculate the markup to be 50%. There are a limited number of cases in which purchases and sales are reported in different units (e.g. "bales" versus "sacks") and we are not able to convert between these units and so cannot calculate the markup. Our second measure of markups includes the ad valorem equivalent of transportation costs in the denominator, i.e. sale price divided by unit cost, where unit cost is purchase price plus unit transportant costs. Because we observe transportation costs at the shipment rather than the transaction level, we are only able to calculate this measure for shipments where the total cost of all products purchased in the shipment is reported. The sample for any analysis using this measure is therefore much smaller, and relies on the assumption that all transportation costs are variable costs.

There is substantial variation in the raw ratio of revenue to stock cost. In all shipment and transaction-level analysis, we trim values below the 10th and above the 90th percentile of this ratio distribution. In effect, this leaves transactions for which this ratio is over 330% or below 5% out of the analysis. Given the structure of the survey, we think that these very large and very small values are extremely likely to reflect entry errors – for instance, extra zeros in monetary values, or errors in the quantity unit recorded. However, results with trimming at the top and bottom 5% or 2% are qualitatively similar, if noisier, and are available on request.

A.2 Other data sources

The analysis in Section 5 of the paper relies on several additional data sources, beyond the Lagos Trader Survey. In this section, we describe those data sources and the way they are used.

Original survey data from Oyo state

For some of the analysis in Section 5, we use survey data that we collected from apparel traders in Ibadan, the capital of Oyo state, located approximately 130 kilometers from Lagos. This survey was conducted in 2019, and covered a representative sample of traders in used clothing markets. The survey was structured similarly to the LTS survey described above, and asked retrospectively about transactions over the preceding 6 months. We use information on the retail fraction of sales, total annual revenue, the location of suppliers, and source-specific variable trade costs. These data enable the estimation of domestic variable and fixed trade costs within Nigeria, as well as firm entry costs outside Lagos, as described in Section 5.1.1.

Living Standards Measurement Survey (LSMS) data

We use data from Wave 2 of the General Household Survey-Panel (GHS-P) covering 2012/2013. The GHS-P is a representative household panel survey covering all of Nigeria and conducted by the Nigerian National Bureau of Statistics (NBS) and the World Bank as part of the Living Standards Measurement Survey (LSMS) program. From the post-planting and post-harvest household questionnaires, we add up expenditures on apparel items to obtain total annual apparel expenditure, which we divide by household size to calculate expenditure per capita. This is the outcome variable in the regression used to estimate utility function parameters in Section 5 of the paper.

When estimate utility function parameters, we allow A_i to be a function of GDP per capita and the density of markets. We take the empirical measure of market density from the GHS-P community questionnaire. Each community in the data is asked whether they have a regular market in the community, and we average this response across communities within each state to get a measure of the fraction of localities that have a market.

We also use a measure of land cost from the GHS-P post-planting community questionnaire as an instrument for the number of sellers in the same regression. As a proxy for land costs, we use the reported price to obtain an acre of land for which the owner has full property rights (as opposed to cultivation rights or sharecropping, for instance), and take an average across all communities within each state.

CPI microdata

We are grateful to David Atkin and Dave Donaldson for sharing apparel price data from the Nigerian consumer price index data collected by the Nigerian National Bureau of Statistics. For details on data collection protocols and treatment, see **?**.

We use price data from July 2010, as the period closest to our survey data collection that is available. Prices were collected monthly at multiple outlets in the capital city and two other urban locations in each state. We use the prices of 63 reported apparel goods, and within that limit consideration to the subset of 28 products for which prices are available in all Nigerian states in order to ensure that differences are not driven by differential product availability that may be related to the intermediation chain structure we study. limit taken from CPI microdata. For the purposes of Section 5, we use the average price across these common apparel goods at the state level.

SMEDAN data

We use estimates of the number of wholesale and retail traders in each state from a report on the Small and Medium Enterprises Development Agency of Nigeria (SMEDAN) and National Bureau of Statistics (NBS) Collaborative Survey on small enterprises conducted in 2013. The report provides a count of micro and small enterprises in each state in Nigeria. It also shows the percent of businesses in each category that are engaged in wholesale and retail trade nationwide, which is 55% of microenterprises and 21% of small and medium enterprises. We assume this nationwide sector breakdown holds in each state. We take the fraction of wholesale and retail traders that sell apparel from our LTS data, and also assume that this holds in each state. These calculations allow us to estimate the number of apparel traders in each state in Nigeria.

Domestic travel times

We take average travel times by road between the capital cities of each state in Nigeria from Google Maps. Google Maps queries were made and average travel times recorded by research assistants over the course of a week in February 2019. To the extent that the sparsity or quality of the road network may covary with remoteness in Nigeria, travel time is likely to offer a better proxy for trade costs than simple distance.

A.3 Supplementary tables

Table 1: Relationship between supplier type and firm size and sourcing costs

	(1) Fixed trade cost ($US) excl. agent & "other" costs	(2) Total trade costs ($US)	(3) Total trade costs ($US)
% of purchases from wholesaler	-459.90 (314.42)	-746.86 (657.47)	-683.84 (625.48)
Total value of shipment ($US)		2.48 (1.58)	22.97 (15.63)
Square of total value of shipment ($US)			-11521.14 (7697.81)
Obs.	650	213	213
Mean of dependent variable	993.55	2970.87	2970.87
Product category FEs	x	x	x

Note: All columns have observations at the shipment level. Columns (2) and (3) have fewer observations because many shipments do not have the total value of purchases from all suppliers reported. Observations with variable profit ratios (sale price divided by purchase price) below the 10th or above the 90th percentiles are trimmed in (4) and (5). Standard errors are clustered at the firm level.

Table 2: Utility function parameter estimates

	Log apparel spending/capita ($US)		
	(1) OLS	(2) OLS	(3) IV
Log apparel price ($US)	0.35 (0.27)	0.06 (0.38)	-0.77 (0.79)
Log sellers	0.04 (0.11)	0.03 (0.11)	0.66* (0.37)
Log GDP/capita ($US)		0.11 (0.15)	0.15 (0.28)
% localities with market		0.24 (0.52)	0.62 (0.45)
Obs.	33	33	33

Note: Data on the number of sellers is not available for 3 out of 37 states (Adamawa, Borno, and Yobe), and data on average land cost is missing for an additional state (Anambra). Data is drawn from the LSMS, SMEDAN, NBS, and Google Maps.

B Material from Section 3

B.1 Restrictions on utility function

In addition to imposing that $G'(\cdot) > 0$, $G''(\cdot) < 0$, $f(0) = 0$, $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$, we assume that a second order condition on firm profits holds for firm quantity setting, and that equilibria are stable. In this subsection of the appendix, we provide conditions on $f(\cdot)$ and $G(\cdot)$ such that these additional assumptions hold. The notation is simplified by defining $Q \equiv \sum_{v'=1}^{N} f(q_{v'})$. Note that $\frac{\partial Q}{\partial q_v} = 0$ when firms are "small" with respect to the market and $\frac{\partial Q}{\partial q_v} = f'(q_v)$ otherwise.

The second order condition for firm quantity setting requires for any Q and q_v that $2\left[G''f'\frac{\partial Q}{\partial q_v} + G'f''\right] + q_v\left[G'''f'\left(\frac{\partial Q}{\partial q_v}\right)^2 + 2G''f''\frac{\partial Q}{\partial q_v} + G''f'\frac{\partial^2 Q}{\partial q_v^2} + G'f'''\right] \leq 0$. (Note that we have omitted arguments of the functions to shorten the expression).

Stability requires that marginal revenue for all traders is weakly falling in Q, i.e. $\frac{\partial MR}{\partial Q} \leq 0$. Formally, this requires that, for any Q and q_v, $G''f' + q_v\left(G'''f'\frac{\partial Q}{\partial q_v} + G''f''\right) \leq 0$. (Note that we have omitted arguments of the functions to shorten the expression). In the event that traders are "small" with respect to the market, this condition will hold with the assumptions made on the first two derivatives of $f(\cdot)$ and $G(\cdot)$, while if firms are large with respect to the market, this places an additional constraint on $G'''(\cdot)$.

B.2 Derivation of equation 6

To derive Equation 6, we take a second order approximation to profits around the equilibrium level of variable costs, c, and fixed costs, holding all other firms' behavior fixed

$$\pi\left(c', F'\right) \approx \pi(c, F) + \frac{\partial \pi(c, F)}{\partial c}\left(c' - c\right) + \frac{1}{2}\frac{\partial^2 \pi(c, F)}{\partial c^2}\left(c' - c\right)^2 + F' - F$$

It follows immediately from the envelope theorem that $\frac{\partial \pi}{\partial c} = -q$, and thus

$$\begin{aligned}\frac{\partial^2 \pi}{\partial c^2} &= \frac{\partial q}{\partial p}\frac{\partial p}{\partial c} \\ &= \frac{q}{c}\left(\varepsilon_p^q - 1\right)\rho\end{aligned}$$

where ρ is the equilibrium passthrough rate and ε_p^q is the price elasticity of demand. The second line follows from firms' profit-maximizing quantity choices.

Re-arranging the original second order approximation

$$\pi\left(c'\right) - \pi(c) = -q\left(c' - c\right)\left[1 + \frac{1}{2}\frac{\left(c' - c\right)}{c}\left(\varepsilon_p^q - 1\right)\rho\right] + F' - F$$

Thus, for an indirect sourcing equilibrium to hold, it must be that a change from indirect to direct sourcing yields (weakly) decreasing profits, giving the condition presented in the text.

B.3 Proof of Lemma 1

Lemma 1 A small change in the level of fixed cost around the level in the free market equilibrium yields the following comparative statics: $\frac{d}{dF}q(F_{ME}) > 0$, $\frac{d}{dF}N(F_{ME}) < 0$, and $\frac{d}{dF}\theta(F_{ME}) = 0$, where F_{ME} is the level of fixed cost chosen in the free market equilibrium.

Proof As discussed in the text, the free market equilibrium is defined by

$$\begin{aligned} \pi(q,\theta,F) &= 0 \\ \frac{\partial}{\partial q}\pi(q,\theta,F) &= 0 \\ \frac{\partial}{\partial F}\pi(q,\theta,F) &= 0 \end{aligned}$$

where in the second and third terms, the partial derivatives are holding the other two arguments of the profit function fixed. The first two conditions hold even when F is chosen by the planner, while the third condition characterizes the free market equilibrium.

Notably, when the planner chooses F, the first two equations determine q and N as a function of F. Taking the derivative of the free entry condition with respect to F, we find

$$\begin{aligned} 0 &= \frac{d}{dF}\pi(q,\theta,F) \\ &= \frac{\partial\pi}{\partial q}\frac{dq}{dF} + \frac{\partial\pi}{\partial\theta}\frac{d\theta}{dF} + \frac{\partial\pi}{\partial F} \end{aligned}$$

At the free market equilibrium, $\frac{\partial\pi}{\partial F} = 0$, and $\frac{\partial\pi}{\partial q} = 0$ via optimal choice of quantity (i.e. it is an envelope condition). And $\frac{\partial\pi}{\partial\theta} = q\frac{\partial p}{\partial\theta} < 0$. Thus, it must be that $\frac{d}{dF}\theta(F_{ME}) = 0$.

Next, we take the derivative of the condition for optimal choice of quantity with respect to F around the free market equilibrium:

$$\begin{aligned} 0 &= \frac{d}{dF}\left[\frac{\partial}{\partial q}\pi(q,\theta,F)\right] \\ &= \frac{\partial^2\pi}{\partial q^2}\frac{dq}{dF} + \frac{\partial^2\pi}{\partial q\partial\theta}\frac{d\theta}{dF} + \frac{\partial^2\pi}{\partial q\partial F} \end{aligned}$$

And $\frac{\partial^2\pi}{\partial q\partial F} = -c'(F) > 0$, $\frac{\partial^2\pi}{\partial q^2} < 0$ by the second order condition, while we just showed

$\frac{d\theta}{dF} = 0$. Consequently

$$\frac{d}{dF} q\left(F_{ME}\right) > 0$$

Finally since $\frac{d}{dF}\theta\left(F_{ME}\right) = 0$ and $\frac{d}{dF}q\left(F_{ME}\right) > 0$, it must be that $\frac{d}{dF}N\left(F_{ME}\right) < 0$.

B.4 Equation 8

Equation (8) follows immediately from the first order condition of consumer surplus with respect to fixed cost and free entry. We start from the expression for welfare

$$W = U - Npq$$

We then take the derivative of this expression with respect to F to obtain

$$\begin{aligned}\frac{dW}{dF} &= \frac{\partial U}{\partial N}\frac{dN}{dF} + \frac{\partial U}{\partial N}\frac{dq}{dF} - pq\frac{dN}{dF} - Np\frac{dq}{dF} - Nq\frac{dp}{dF} \\ &= \left(\frac{\partial U}{\partial N} - pq\right)\frac{dN}{dF} - Nq\frac{dp}{dF}\end{aligned}$$

and from the zero-profit condition

$$\begin{aligned}p &= c + \frac{F}{q} \\ \frac{dp}{dF} &= c'\left(F\right) + \frac{1}{q} - \frac{F}{q^2}\frac{dq}{dF} \\ q\frac{dp}{dF} &= \left(c'\left(F\right)q + 1\right) - \left(p - c\right)\frac{dq}{dF}\end{aligned}$$

so that

$$\frac{dW}{dF} = \left(\frac{\partial U}{\partial N} - pq\right)\frac{dN}{dF} + N\left(p - c\right)\frac{dq}{dF} - N\left(c'\left(F\right)q + 1\right)$$

B.5 Proof of Proposition 1

Proposition 1 If the assumptions on the form of demand from Section 3.1.1 hold, and there is a continuous sourcing cost frontier, the planner's preferred sourcing technology may have either a higher or lower fixed cost than the market equilibrium technology.

Proof The proof is by example. For brevity, we use Equation 8 in the text (derived earlier in this Appendix) rather than re-deriving it here.

We start with expressions for $\frac{dq}{dF}$ and $\frac{dN}{dF}$ at the market equilibrium. In the proof of

Lemma 1, we established that around the market equilibrium

$$\frac{dq}{dF} = \frac{c'}{\frac{\partial^2 \pi}{\partial q^2}}$$

so that if we define N_{-v} as the number (or measure) of competitors, depending on whether firms are large or small with respect to the market, then it follows from $\frac{d\theta}{dF} = 0$ at the market equilibrium that

$$\begin{aligned} 0 = \frac{d\theta}{dF} &= N_{-v} f' \frac{dq}{dF} + f \frac{dN}{dF} \\ \frac{\frac{dN}{dF}}{\frac{dq}{dF}} &= -N_{-v} \frac{f'}{f} \frac{dq}{dF} \end{aligned}$$

We then turn to Equation 8 in the text. Using $U = G(Nf(q))$ (and dropping the arguments of functions for brevity), we can write

$$\begin{aligned} \frac{dW}{dF} &= G' \cdot (f - qf') \frac{\partial N}{\partial F} + N (G' f' - c) \frac{\partial q}{\partial F} \\ &= \left[-\frac{N_{-v}}{N} p \left(1 - q \frac{f'}{f} \right) + (p - c) \right] N \frac{\partial q}{\partial F} \end{aligned}$$

We will focus on the sign of $-\frac{N_{-v}}{N} p \left(1 - q\frac{f'}{f}\right) + (p - c)$ as $N\frac{\partial q}{\partial F} > 0$.

First, consider a cournot setting where firms are large, $G(Q) = A\left(Q - \frac{B}{2}Q^2\right)$, and $f(q) = q$. The particular $c(F)$ is arbitrary – it does not matter as long as it satisfies the conditions laid out in the text. In this setting, $1 - q\frac{f'}{f} = 0$, while $p - c > 0$ as fixed costs are strictly positive. Consequently, $\frac{dW}{dF} > 0$.

Alternatively, consider a setting where firms are small with respect to the market, $G(Q) = \ln Q$, $f(q) = q^{\frac{1}{2}} + q$, and $c(F) = \exp(-F)$. There is no cost of entry. It can be shown numerically that at the equilibrium $q \approx 1.2039$, $N \approx 0.6424$, and with these values and setup that

$$\begin{aligned} -\frac{N_{-v}}{N} p \left(1 - q \frac{f'}{f} \right) + (p - c) &= p \cdot q \frac{f'}{f} - c \\ &\approx -0.0806 \end{aligned}$$

Consequently, $\frac{dW}{dF} < 0$.

B.6 Lemma 2

Lemma 2 A discrete change to a higher fixed cost technology starting from the market equilibrium leads to a constrained equilibrium with higher quantity per traders, fewer traders, and lower competition: $\Delta q > 0$, $\Delta N < 0$, and $\Delta\theta \leq 0$.

Proof First, we establish that $\Delta\theta \leq 0$. This follows from revealed preference. Holding θ fixed, if traders chose to source from j, then

$$\pi\left(q_j, \theta_j, j\right) \geq \pi\left(q'_{j'}, \theta_j, j'\right)$$

where, as in the text, $q'_{j'}$ denotes the quantity which maximizes profits given θ_j and the cost structure of j'. In turn, by revealed preference, since when deviating firms choose $q'_{j'}$ and not $q_{j'}$ (which is the quantity which would maximize profits given $\theta_{j'}$ and the cost structure of j')

$$\pi\left(q'_{j'}, \theta_j, j'\right) \geq \pi\left(q_{j'}, \theta_j, j'\right)$$

wConsequently,

$$\pi\left(q_j, \theta_j, j\right) \geq \pi\left(q_{j'}, \theta_j, j'\right)$$

However, due to the zero profit condition, $0 = \pi\left(q_j, \theta_j, j\right)$, and it must be that $0 = \pi\left(q_{j'}, \theta_{j'}, j'\right)$. Since profits are strictly decreasing in θ, it must be that $\theta_{j'} \leq \theta_j$, and so $\Delta\theta \leq 0$.

Second, we establish that $\Delta q > 0$. In both equilibria, firms choose quantities so that

$$c\left(F\right) = p\left(q, \theta\right) + q\frac{\partial p\left(q, \theta\right)}{\partial q}$$

In changes, then (as before, using j' and j subscripts to denote equilibrium levels of variables under the respective sources)

$$\begin{aligned}
\Delta c &= \Delta\left(p\left(q, \theta\right) + q\frac{\partial p\left(q, \theta\right)}{\partial q}\right) \\
&= \left[p\left(q_{j'}, \theta_{j'}\right) + q_{j'}\frac{\partial p\left(q_{j'}, \theta_{j'}\right)}{\partial q} - p\left(q_j, \theta_{j'}\right) - q_j\frac{\partial p\left(q_j, \theta_{j'}\right)}{\partial q}\right] + \ldots \\
&\ldots\left[p\left(q_j, \theta_{j'}\right) + q_j\frac{\partial p\left(q, \theta_{j'}\right)}{\partial q} - p\left(q_j, \theta_j\right) - q_j\frac{\partial p\left(q_j, \theta_j\right)}{\partial q}\right] \\
&= \int_{q_j}^{q_{j'}}\left(\frac{\partial}{\partial q}\left(p\left(q, \theta'\right) + q\frac{\partial p\left(q, \theta'\right)}{\partial q}\right)\right)dq + \int_{\theta_j}^{\theta_{j'}}\left(\frac{\partial}{\partial\theta}\left(p\left(q, \theta\right) + q\frac{\partial p\left(q, \theta\right)}{\partial q}\right)\right)d\theta
\end{aligned}$$

Since we are considering a change to a higher fixed cost source, marginal cost must fall, i.e. $\Delta c < 0$. Furthermore, our assumption that preferences are such that equilibria are stable means that for all q and θ,

$$0 \geq \frac{\partial}{\partial \theta}\left(p + q\frac{\partial p(q,\theta)}{\partial q}\right)$$

and thus as $\theta_{j'} \leq \theta_j$

$$0 \geq \int_{\theta_j}^{\theta_{j'}} \left(\frac{\partial}{\partial \theta}\left(p(q,\theta) + q\frac{\partial p(q,\theta)}{\partial q}\right)\right) d\theta$$

i.e., a (weakly) lower θ in the new equilibrium will lead to a (weakly) higher MR, quantity held equal. Consequently, it must be that

$$0 < \int_{q_j}^{q_{j'}} \left(\frac{\partial}{\partial q}\left(p\left(q,\theta'\right) + q\frac{\partial p(q,\theta')}{\partial q}\right)\right) dq$$

The second order condition for the choice of quantity implies that for any q and z,

$$0 \geq \frac{\partial}{\partial q}\left(p + q\frac{\partial p(q,\theta)}{\partial q}\right)$$

which implies $q_{j'} > q_j$.

And finally, we establish that $\Delta N < 0$. By definition

$$\theta = N_{-v} f(q)$$

so that

$$\Delta\theta = (N_{-v})_{j'} \int_{q_j}^{q_{j'}} f'(q)\, dq + f(q_j)\, \Delta N$$

And by assumption, $f'(q) \geq 0$ for all q, and we have showed that $\Delta q > 0$, so that $(N_{-v})_{j'} \int_{q_j}^{q_{j'}} f'(q)\, dq > 0$. But $\Delta\theta \leq 0$ so that it must be $f(q_j)\,\Delta N < 0$. By assumption, $f(q) \geq 0$ for all q, so that $\Delta N < 0$.

B.7 Equation 9

Equation (9) follows from expanding the difference in welfare. We start from the expression for welfare

$$W(N,q) = U(N,q) - Npq$$

We then take differences under the two levels of F (using j' and j subscripts to denote equilibrium levels of variables under the respective sources)

$$\begin{aligned}\Delta W = &\left[U\left(N_{j'},q_{j'}\right) - U\left(N_{j'},q_j\right)\right] + \left[U\left(N_{j'},q_j\right) - U\left(N_j,q_j\right)\right] - \ldots \\ &\ldots N_{j'}\Delta(pq) - (\Delta N)\,p_j q_j\end{aligned}$$

and from the zero-profit condition

$$\begin{aligned}pq &= cq + F \\ \Delta(pq) &= c_{j'}\Delta q + q_j \Delta c + \Delta F\end{aligned}$$

so that

$$\begin{aligned}\Delta W &= \left[U\left(N_{j'},q_{j'}\right) - U\left(N_{j'},q_j\right)\right] + \left[U\left(N_{j'},q_j\right) - U\left(N_j,q_j\right)\right] - \ldots \\ &\ldots N_{j'}\left(c_{j'}\Delta q + q_j\Delta c_j + \Delta F\right) - (\Delta N)\,p_j q_j \\ &= \left[U\left(N_{j'},q_j\right) - U\left(N_j,q_j\right) - (\Delta N)\,p_j q_j\right] + \ldots \\ &\ldots \left[U\left(N_{j'},q_{j'}\right) - U\left(N_{j'},q_j\right) - N_{j'}c_{j'}\Delta q\right] - N_{j'}\left(q_j\Delta c + \Delta F\right) \\ &= \int_{N_j}^{N_{j'}} \left[\frac{\partial}{\partial N}U(\nu,q_j) - p_j q_j\right] d\nu + N_{j'}\int_{q_j}^{q_{j'}} \left[p\left(N_{j'},\vartheta\right) - c_{j'}\right] d\vartheta - N_{j'}\left(q_j\Delta c + \Delta F\right)\end{aligned}$$

which is the expression provided in the text.

B.8 Proposition 2

Proposition 2 If the assumptions on the form of demand from Section 3.1.1 hold and the cost frontier is discrete, a discrete change to a higher fixed cost technology starting from the market equilibrium may increase or decrease welfare.

Proof In this setting, ambiguity can come from two sources. First, depending on the underlying functions G, f, c and the market structure, it may be bad for the planner to make an infinitesimally small increase in F as we proved in Proposition 1. If even an infinitesimally small increase in F is bad for welfare, then a discrete increase of any size will always lower welfare. But conversely, if an infinitesimally small increase in F is good for welfare, there is always a small enough discrete change which is beneficial. This is obvious, and so we omit a proof here.

However, as we discuss in the text, even when an infinitesimal increase in F would be good for welfare, a large enough change in F can be detrimental; we provide an example here. We return to the cournot setting from the proof of Proposition 1 where firms are large, $G(Q) = A\left(Q - \frac{B}{2}Q^2\right)$, and $f(q) = q$. Suppose there are two possible sourcing strategies: $c_1 = \frac{A}{5}$, $F_1 = \frac{1}{25}\frac{A}{B}$ and $c_2 = \frac{A}{10}$, $F_2 = \frac{81}{400}\frac{A}{B}$.[1]

It is well-known that the optimal cournot strategy with N firms and marginal cost c is

$$q = \frac{1}{N+1} \cdot \frac{A-c}{AB}$$

and given equilibrium prices and the free entry condition, this implies (ignoring integer constraints) that the optimal number of firms is

$$N = \frac{A-c}{\sqrt{ABF}} - 1$$

It can be shown that if firms are in an initial symmetric equilibrium from the first source, no firm will wish to deviate to sourcing from location 2.[2] Under the first source, 3 firms will operate and they will generate

$$Q = \frac{12}{20B}$$

while if the planner mandates sourcing from location 2, there will be a monopolist which produces quantity

$$q = Q = \frac{9}{20B}$$

Effectively, the fall in competition has outweighed the gains from lower marginal cost, even though welfare would be increased by a small increase in F.

[1] Note that as there are only two sources, we could easily fit both points with a cost function of the form $c(F) = a\exp(-\mu F)$ which satisfies the assumptions on $c(F)$ for the purposes of the example in Proposition 1.

[2] The easiest way to see this is that sourcing strategy two permits a monopolist to operate earning zero profits while location one permits three firms. Thus no firm will wish to deviate, as if it assumes the quantity by remaining firms is unchanged, it must earn negative profits.

C Material from Section 4

C.1 Microfoundation of consumer utility

In the main text, the utility is

$$U = q_0 + AV^{\alpha}$$
$$V = N_{ri}^{\gamma - \frac{1}{1-\sigma_r}} \left(\int_{\omega \in N_{ri}} q(\omega)^{\frac{\sigma_r - 1}{\sigma_r}} d\omega \right)^{\frac{\sigma_r}{\sigma_r - 1}}$$

In this part of the appendix, we show that this utility can be microfounded as a discrete choice problem in the style of Anderson, de Palma, and Thisse (1987).

Suppose that utility for a consumer who spends X_0 on the outside good, E_1 on the traded good, and purchases the traded good from seller ω is:

$$U = X_0 + A \left(\max_{\omega} V(\omega) \right)^{\alpha}$$
$$V(\omega) = \frac{E_1}{p(\omega)} \varepsilon(\omega)$$

where $\varepsilon(\omega)$ is an idiosyncratic iid Frechet draw of match value with seller ω, and $p(\omega)$ is the price charged by that seller. The Frechet match value has shape parameter $\frac{1}{\mu_c}$ where $\mu_c < 1$[3] and scale parameter $\frac{M_c^{\gamma - \mu_c}}{\Gamma(1-\mu_c)}$, where $\Gamma(\cdot)$ denotes the gamma function. We assume consumers make two sequential decisions: first, they observe the measure of traders and distribution of prices, and choose expenditure on the outside good to maximize expected utility, and second, they observe their idiosyncratic match values and pick the seller who maximizes $V(\omega)$ given their choice of E_1.

The payout for agent with expenditure E_1 from buying variety ω at price $p(\omega)$ with idiosyncratic match value is

$$\frac{E_1}{p(\omega)} \varepsilon(\omega)$$

where $\varepsilon(\omega)$ is distributed Frechet with shape parameter $\frac{1}{\mu_c}$ and scale parameter $\frac{N_r^{\gamma - \mu_r}}{\Gamma(1-\mu_r)}$ where N_r is the number (or measure) of retailers ω and γ is a parameter.

This gives rise to CES demands across varieties ω. Maximizing sub-utility is the same as maximizing the monotone transformation

$$\ln E_1 - \ln p(\omega) + \ln \varepsilon(\omega)$$

[3] This condition delivers consumer elasticity of demand greater than 1.

where $\ln\varepsilon(\omega)$ is distributed Gumbel with location parameter $(\gamma-\mu_r)\ln N_r-\ln\Gamma(1-\mu_r)$ and scale parameter μ_r. As shown in Anderson, de Palma, and Thisse (1987) maximizing this monotone transformation of utility gives rise to CES demands with elasticity of substitution

$$\sigma_r = 1+\frac{1}{\mu_r}$$

Next, determining the expenditure will depend on the expected value of the maximum draw. Note that because $\left(\max_\omega \frac{E_1}{p(\omega)}\varepsilon(\omega)\right)^\alpha$ is a monotone transformation of $\max_\omega \frac{E_1}{p(\omega)}\varepsilon(\omega)$, the parameter α will not affect the choice of the utility maximizing seller. And, due to the properties of the Frechet distribution, $\max_\omega \frac{E_1}{p(\omega)}\varepsilon(\omega)$ will be distributed Frechet with shape parameter $\frac{1}{\mu_r}$ and scale parameter $\left(\frac{E_1 N_r^{\gamma-\mu_r}}{\Gamma\left(1-\frac{1}{\mu_r}\right)}\right)\left(\sum_\omega p(\omega)^{-\frac{1}{\mu_r}}\right)^{\mu_r}$. Thus, $\left(\max_\omega \frac{E_1}{p(\omega)}\varepsilon(\omega)\right)^\alpha$ will be distributed Frechet with scale $\left(\frac{E_1 N_r^{\gamma-\mu_r}}{\Gamma\left(1-\frac{1}{\mu_r}\right)}\right)^\alpha\left(\sum_\omega p(\omega)^{-\frac{1}{\mu_r}}\right)^{\alpha\mu_r}$ and shape parameter $\frac{1}{\alpha\mu_r}$, so that the expected value for the maximum draw will be

$$\mathbb{E}\left[\left(\max_\omega V(\omega)\right)^\alpha\right] = \frac{E_1^\alpha}{\left(\sum_\omega\left(\frac{1}{(p(\omega))}\right)^{-\frac{1}{\mu_r}}\right)^{-\alpha\mu_r}} N_r^{\alpha(\gamma-\mu_r)}$$

If we define the price index in the usual way (and where following Anderson, de Palma, and Thisse (1987) we define $1-\sigma_r=-\frac{1}{\mu_r}$), i.e. that

$$P_r \equiv \left(\sum_\omega (p(\omega))^{1-\sigma_r}\right)^{\frac{1}{1-\sigma_r}}$$

then

$$\mathbb{E}\left[\max_\omega V(\omega)^\alpha\right] = \left(\frac{E_1}{P_r N_r^{-\frac{1}{1-\sigma_r}-\gamma}}\right)^\alpha$$

Note that this suggests that $\max_\omega V(\omega)$ is identical to standard CES utility in the event that $\gamma=\frac{1}{\sigma_r-1}$. If not, following Benassy (1996), γ is a parameter governing gains from variety.

Thus, given a distribution of prices and measure of sellers which lead to the price index $\Omega_r^{-\frac{1}{1-\sigma_r}-\gamma}P_r$, following immediately from the FOC for E_1, the consumer will choose

$$E_1 = \left[\alpha A\left(N_r^{-\frac{1}{1-\sigma_r}-\gamma}P_r\right)^{-\alpha}\right]^{\frac{1}{1-\alpha}}$$

Thus consumers will have expected utility from the differentiated sector of

$$\mathbb{E}\left[\left(\max_{\omega} V\left(\omega\right)\right)^{\alpha}\right]=\left(\frac{\alpha A}{N_r^{-\frac{1}{1-\sigma_r}-\gamma}P_r}\right)^{\frac{\alpha}{1-\alpha}}$$

C.2 Generalization of results from Section 3 to encompass utility like Benassy (1996)

In this section, we consider CES utility with a divergence between the social and private value of variety as in Benassy (1996) which does not fit in the utility form described in Equation (1) and instead matches the form of Equation (11). We show that our results from Section 3 still hold in this framework.

In particular, we extend our results from Section 3 to utility of the form

$$U=\frac{1}{h\left(N\right)}G\left(\sum_{v=1}^{N}h\left(N\right)f\left(q_v\right)\right)$$

In addition to the assumptions we made before, we additionally assume $h>0$, $-\frac{h'}{h^2}\frac{G}{G'}+f+N\frac{h'}{h}f\geq 0$ (i.e. more variety never makes consumers worse off), and $N\frac{h'}{h}>-1$ (which implies that an increase in the measure or number of varieties lowers consumer prices). Note that Equation (11) fits this form: define $G\left(x\right)=AX^{\alpha\frac{\sigma}{\sigma-1}}$, $f\left(x\right)=x^{\frac{\sigma_r-1}{\sigma_r}}$, and $h\left(x\right)=x^{\gamma\left(\frac{\alpha\sigma}{(\alpha-1)\sigma+1}\right)}$ to obtain the direct utility form of Equation (11).

.

First, we adopt a slightly different definition of the level of competition; now we define (when firms are large) the level of competition faced by firm v, θ_v by

$$\theta_v\equiv h\left(N\right)\sum_{v'=1}^{N}f\left(q_{v'}\right)-f\left(q_v\right)$$

while if firms are small

$$\theta_v\equiv h\left(N\right)\int_{0}^{N}f\left(q_{v'}\right)dv'$$

We also change our notion of stability to account for the changed definition of θ so that $0\geq\frac{\partial}{\partial\theta}\left(p+q\frac{\partial p(q,\theta)}{\partial q}\right)$ as before, and place the necessary assumptions on the utility function and its components..

Next, we show that, under this definition of competition, both Lemma 1 and Lemma 2 hold. We start with Lemma 1: with this new definition of competition, we can express profits as a function of per-firm quantity, level of competition, and level of fixed cost.[4] $\frac{d\pi}{dF} = 0$ due to the free entry condition, which implies $\frac{d\theta}{dF} = 0$. This implies that $\frac{dq}{dF} > 0$ for the same reason as under the initial utility level. Furthermore, under the new definition of competition (and assumptions of the utility function and its constituent functions), it is easy to verify that $\frac{d\theta}{dq} > 0$ and $\frac{d\theta}{dN} < 0$. Thus $\frac{d\theta}{dF} < 0$. Similarly, for Lemma 2, one can make the same revealed preference argument to establish that $\Delta\theta \leq 0$. Similarly, we can establish $\Delta q > 0$ through the stability condition (the steps are unchanged). And finally, using the new definition of θ, we derive

$$\Delta\theta = \left(N_{j'} h\left(N_{j'}\right) - 1\right) \int_{q_j}^{q_{j'}} f'(q)\, dq + \int_{N_j}^{N_{j'}} \left[N_{j'} f(q_j)\, h'(N) + f(q_j)\, h(N_j)\right] dN$$

Note that by assumption, $N_{j'} h\left(N_{j'}\right) > 1$ and $f' > 0$ so the first term is positive. Since $\Delta\theta \leq 0$, the second term must be negative, and by assumption $N_{j'} f(q_j)\, h'(N) + f(q_j)\, h(N_j) > 0$. Consequently, it must be that $\Delta N < 0$.

Note that both welfare decompositions presented in the text do not depend on the functional forms of utility. Consequently, both expressions still hold, and as before there is an tradeoff between per-firm quantity gains and variety losses from movements to higher levels of fixed cost. To show that this tradeoff is ambiguous, it suffices to set $h(N) = 1$, in which case the existing examples in Propositions 1 and 2 demonstrate the ambiguity. However, it can also be shown that, for any f and G, as $-\frac{h'}{h^2}\frac{G}{G'} + f + N\frac{h'}{h} f \to 0$ (so that consumers are indifferent to additional variety), the per-firm quantity gains dominate and the planner will always choose to increase the level of fixed cost. Similarly, as $-\frac{h'}{h^2}\frac{G}{G'} + f + N\frac{h'}{h} f \to \infty$, variety gains will dominate any per-firm quantity effects and the planner will always reduce the fixed cost.

C.3 Intermediary payouts

In this section, we provide details about the distribution of intermediaries' shocks.

We assume that (for each intermediation activity u), $\xi_u(j)$ is distributed iid Gumbel

[4] The key is that under this utility and definition of competition

$$\begin{aligned} p_v &= \frac{\partial U}{\partial q_v} \\ &= G'\left(\sum_{v'=1}^{N} h(N)\, f(q_{v'})\right) f'(q_v) \\ &= G'\left(\theta_v + f(q_{v'})\right) f'(q_v) \end{aligned}$$

i.e. the price for a given firm is simply a function of own quantity and the level of competition as before.

with scale parameter s and location parameter $-s\Gamma'(1)$ where $\Gamma'(1)$ is the derivative of the gamma function evaluated at 1 (and is equal to the negative of the Euler-Mascheroni constant).

We assume that (for each intermediation activity u), $\zeta_u(z)$ is distributed iid Frechet with shape parameter $\frac{1}{\beta}$ and scale parameter $\frac{1}{\Gamma(1-\beta_u)}\left[\frac{\sum_{z\in z_{iju}}\pi_z\pi_z^{\frac{1}{\beta}}}{\sum_{z'\in z_{iju}}\pi_{z'}^{\frac{1}{\beta}}}-\left(\sum_{z\in z_{iju}}\pi_z^{\frac{1}{\beta}}\right)^{\beta}\right]$ where $\Gamma(\cdot)$ is the gamma function, π_z is the expected profits conditional on choosing chain z, and z_{iju} is the set of chains going from j to i for intermediation activity u.

Finally, we assume that $\varepsilon_z(\omega)$ is distributed iid Frechet with shape parameter $\frac{1}{\mu_t}$ and scale parameter $\frac{N_z^{\frac{1}{1-\sigma_z}}}{\Gamma(1-\mu_t)}$ where $\Gamma(\cdot)$ is the gamma function, N_z is the measure of sellers on chain z, and σ_z is the elasticity of demand at the relevant stage of chain z (note that this elasticity is solved for in the body of the paper). Note that this formulation means that the number of sellers serving a chain does not affect downstream payouts.

C.4 Consumer price index

In this section, we provide details for the derivation of the expression of the consumer price index presented in the paper, P_{ij}, and explain how it is related to the wholesale price index . This is equal to the probability a given retailer carries any particular chain times that chain's contribution to the consumer price index. Note that the probability of carrying chain z for a retailer is (taking the expression provided in Section 4.2.1 of the paper and using p_z – the retail price on chain z – in the expression for variable profits)

$$\Pr(z)=\frac{p_z^{\frac{1}{\beta}(1-\sigma_r)}}{\sum_{z'}p_{z'}^{\frac{1}{\beta}(1-\sigma_r)}}$$

where we can ignore the role of the number of sellers due to the shape parameter of seller match distribution – all that matters for trader payouts are prices. For convenience, we will define ϕ_{ij} by $\left((m_r\tau_{ij})^{\frac{1}{\beta_r}}\phi_{ij}\right)^{1-\sigma_r}=\sum_{z'}p_{z'}^{\frac{1}{\beta_r}(1-\sigma_r)}$ – i.e. it is a (transformation of the) profit index. Using this definition, we can write for the price index changed by measure 1 traders when sourcing for i from location $j\neq o$

$$
\begin{aligned}
P_{ij}^{1-\sigma_r} &= \left(M_2\tau_{ij}\tau_{jo}p_o\right)^{1-\sigma_r} \frac{\left(M_2\tau_{ij}\tau_{jo}p_o\right)^{\frac{1}{\beta}(1-\sigma_r)}}{\left(\left(m_r\tau_{ij}\right)^{\frac{1}{\beta}}\ \phi_{ij}\right)^{1-\sigma_r}} + \sum_{j'} \left(M_3\tau_{ij}\tau_{jj'}\tau_{j'o}p_o\right)^{1-\sigma_r} \frac{\left(M_3\tau_{ij}\tau_{jj'}\tau_{j'o}p_o\right)^{\frac{1}{\beta}(1-\sigma_r)}}{\left(\left(m_r\tau_{ij}\right)^{\frac{1}{\beta}}\ \phi_{ij}\right)^{1-\sigma_r}} \dots \\
&\quad + \sum_{j''}\sum_{j'} \left(M_4\tau_{ij}\tau_{jj'}\tau_{j'j''}\tau_{j''o}p_o\right)^{1-\sigma_r} \frac{\left(M_4\tau_{ij}\tau_{jj'}\tau_{j'j''}\tau_{j''o}p_o\right)^{\frac{1}{\beta}(1-\sigma_r)}}{\left(\left(m_r\tau_{ij}\right)^{\frac{1}{\beta}}\ \phi_{ij}\right)^{1-\sigma_r}} + \dots \\
&= \frac{\left(m_r\tau_{ij}\right)^{1-\sigma_r}}{\phi_{ij}^{1-\sigma_r}} \left[\left(\frac{M_2}{m_r}\tau_{jo}p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + \sum_{j'}\left(\frac{M_3}{m_r}\tau_{jj'}\tau_{j'o}p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + \sum_{j''}\sum_{j'}\left(\frac{M_4}{m_r}\tau_{jj'}\tau_{j'j''}\tau_{j''o}p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + \dots\right]
\end{aligned}
$$

and by close analogy

$$
P_{io}^{1-\sigma_r} = \frac{\left(m_c\tau_{io}\right)^{1-\sigma_r}}{\phi_{io}^{1-\sigma_r}} \left[p_o^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + \left(\frac{M_2}{m_r}\tau_{oo}p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + \sum_{j'}\left(\frac{M_3}{m_r}\tau_{oj'}\tau_{j'o}p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + \dots\right]
$$

We next turn to the profit indexes, and we show (for $j \neq i$, we will neglect the origin as the definition is provided in the text and by this point it is clear by analogy)

$$
\begin{aligned}
\left(\left(m_c\tau_{ij}\right)^{\frac{1}{\beta}}\ \phi_{ij}\right)^{1-\sigma_r} &= \left(M_2\tau_{ij}\tau_{jo}p_o\right)^{\frac{1}{\beta}(1-\sigma_r)} + \sum_{j'}\left(M_3\tau_{ij}\tau_{jj'}\tau_{j'o}p_o\right)^{\frac{1}{\beta}(1-\sigma_r)} \dots \\
&\quad + \sum_{j''}\sum_{j'}\left(M_4\tau_{ij}\tau_{jj'}\tau_{j'j''}\tau_{j''o}p_o\right)^{\frac{1}{\beta}(1-\sigma_r)} + \dots \\
\phi_{ij}^{1-\sigma_r} &= \left(\frac{M_2}{m_r}\tau_{jo}p_o\right)^{\frac{1}{\beta}(1-\sigma_r)} + \sum_{j'}\left(\frac{M_3}{m_r}\tau_{jj'}\tau_{j'o}p_o\right)^{\frac{1}{\beta}(1-\sigma_r)} \dots \\
&\quad + \sum_{j''}\sum_{j'}\left(\frac{M_4}{m_r}\tau_{jj'}\tau_{j'j''}\tau_{j''o}p_o\right)^{\frac{1}{\beta}(1-\sigma_r)} + \dots
\end{aligned}
$$

where since it is independent of i, we define $\phi_{ij}^{1-\sigma_r} \equiv \phi_j^{1-\sigma_r}$ shared across all i. This then lets us define the $\mathcal{P}_j$ as in the text, so that

$$
P_{ij}^{1-\sigma_r} = \left(m_r\tau_{ij}\right)^{1-\sigma_r}\mathcal{P}_j^{1-\sigma_r}
$$

yielding the expression in the text that

$$
P_{ij} = m_r\tau_{ij}\mathcal{P}_j
$$

C.5 Chain length

The expenditure-weighted length of chains going from location j to location i will reflect the share of expenditure on the route allocated to each chain times the length of that chain. Formally, for a location $j \neq o$,

$$\Lambda_{ij} = \frac{1}{\left((m_r \tau_{ij})^{\frac{1}{\beta}} \phi_{ij} P_{ij}\right)^{1-\sigma_r}} [2 \left(M_2 \tau_{ij} \tau_{jo} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + 3 \sum_{j'} \left(M_3 \tau_{ij} \tau_{jj'} \tau_{j'o} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} \dots$$
$$+ 4 \sum_{j''} \sum_{j'} \left(M_4 \tau_{ij} \tau_{jj'} \tau_{j'j''} \tau_{j''o} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + \dots$$

As with the price indeces, it is possible to factor out the i-specific components

$$\Lambda_{ij} = \frac{1}{\left(\phi_j \mathcal{P}_j\right)^{1-\sigma_c}} [2 \left(\frac{M_2}{m_r} \tau_{jo} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + 3 \sum_{j'} \left(\frac{M_3}{m_r} \tau_{jj'} \tau_{j'o} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} \dots$$
$$+ 4 \sum_{j''} \sum_{j'} \left(\frac{M_4}{m_r} \tau_{jj'} \tau_{j'j''} \tau_{j''o} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + \dots$$

so that we can define the shared part of chain length ($j \neq o$) as

$$\Lambda_j \equiv 2 \left(\frac{M_2}{m_r} \tau_{jo} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + 3 \sum_{j'} \left(\frac{M_3}{m_r} \tau_{jj'} \tau_{j'o} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} \dots$$
$$+ 4 \sum_{j''} \sum_{j'} \left(\frac{M_4}{m_r} \tau_{jj'} \tau_{j'j''} \tau_{j''o} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + \dots$$

And for chains from the origin

$$\Lambda_{io} = \frac{1}{\left((m_r \tau_{io})^{\frac{1}{\beta}} \phi_{io} P_{io}\right)^{1-\sigma_c}} [\left(M_1 \tau_{io} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + 2 \left(M_2 \tau_{oo} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} \dots$$
$$+ 3 \sum_{j'} \left(M_3 \tau_{io} \tau_{oj'} \tau_{j'o} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + \dots$$

so that we similarly simplify and define

$$\Lambda_o \equiv (p_o)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + 2 \left(\frac{M_2}{m_r} \tau_{oo} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} \dots$$
$$+ 3 \sum_{j'} \left(\frac{M_3}{m_r} \tau_{oj'} \tau_{j'o} p_o\right)^{\left(1+\frac{1}{\beta}\right)(1-\sigma_r)} + \dots$$

Using these definitions, for all ij pairs we can write

$$\Lambda_{ij} = \frac{\Lambda_j}{\left(\phi_j \mathcal{P}_j\right)^{1-\sigma_r}}$$

C.6 Proposition 3

Proposition 3 If the assumptions on the setting and consumer and trader payouts of Section 4.1 hold then the equilibrium is unique.

Proof As established in the prior subsection of this appendix, the P_{ij} are only a function of model parameters and not the equilibrium choices of intermediaries. Thus uniqueness boils down the sourcing choices of retailers and consumer price indeces, and these choices are separable across locations.

First, we show that, holding the measure of traders in a final location fixed, this implies a unique sourcing pattern. In particular, we show that when we define the function $e\left(P_{r,i}\right)$ (for notational simplicity)

$$e\left(P_{r,i}\right)=P_{r,i}^{1-\sigma_r}-N_{r,i}\frac{\sum_j\exp\left(\frac{\pi_{ij}^r}{s}\right)}{\sum_{j'}\exp\left(\frac{\pi_{ij'}^r}{s}\right)}P_{ij}^{1-\sigma_r}$$

this function is strictly decreasing in P_{ij} ; this montonicity implies a unique value of P_{ij} (which in turn implies a unique sourcing pattern). The intuition for this result is that a rise in the consumer price index will make sourcing from all locations more profitable, but disproportionately so for locations which have the lowest price indeces. Formally, we start from the expression for expenditure in terms of the price index

$$\frac{E_i}{P_{r,i}^{1-\sigma_r}}=CN_{r,i}^{\delta}P_{r,i}^{\psi}$$

where have adopted for notational convenience $C\equiv\frac{Z_i(\alpha A)^{\frac{1}{1-\alpha}}}{s\sigma_r}$,$\delta\equiv\frac{-\alpha}{1-\alpha}\left(-\frac{1}{1-\sigma_r}-\gamma\right)$ $\psi\equiv\sigma_r-1-\frac{\alpha}{1-\alpha}$. Then holding $N_{r,i}$ fixed

$$\frac{\partial}{\partial P_{r,i}}\exp\left(\frac{\pi_{ij}^r}{s}\right)=\delta CN_{r,i}^{\delta}P_{ij}^{1-\sigma_r}P_{r,i}^{\psi-1}\exp\left(\frac{\pi_{ij}^r}{s}\right)$$

Taking the derivative with respect to $P_{r,i}$ again holding $N_{r,i}$ fixed:

$$e'\left(P_{r,i}\right)=\left(1-\sigma_r\right)P_{r,i}^{-\sigma_c}-\psi N_{r,i}^{\delta}P_{r,i}^{\psi-1}N_{r,i}\cdot\sum_j P_{ij}^{1-\sigma_r}\chi_{ij}\left(P_{ij}^{1-\sigma_r}-\frac{P_{r,i}^{1-\sigma_r}}{N_{r,i}}\right)$$

where $\chi_{ij}\equiv\frac{\exp\left(\frac{\pi_{ij}^r}{s}\right)}{\sum_{j'}\exp\left(\frac{\pi_{ij'}^r}{s}\right)}$ is the sourcing share from j. Note that $N_{r,i}\sum_j\chi_{ij}P_{ij}^{1-\sigma_r}=P_{r,i}^{1-\sigma_r}$ by definition, so that

$$\sum_j P_{ij}^{1-\sigma_r}\chi_{ij}\left(P_{ij}^{1-\sigma_r}-\frac{P_{r,i}^{1-\sigma_r}}{N_{r,i}}\right)=\sum_j\chi_{ij}\left(P_{ij}^{1-\sigma_r}-\frac{P_{r,i}^{1-\sigma_r}}{N_{r,i}}\right)\left(P_{ij}^{1-\sigma_r}-\frac{P_{r,i}^{1-\sigma_r}}{N_{r,i}}\right)$$

which is simply the variance in the price index across locations and by definition weakly positive. Since restrictions on α imply $\delta > 0$ and we assumed $\sigma_r > 1$, it follows that $e'(P_{r,i}) < 0$ for all $P_{r,i}$. Thus, the solution to $e(P_{r,i}) = 0$ is unique holding the measure of firms fixed.

Second, we show that there is a unique measure of traders in equilibrium. Per-trader profits are strictly decreasing in the number of traders, such that there is only one measure of traders such that the expected profits are equal to the fixed cost of entry.

Before addressing this point directly, we develop a few expressions which will simplify future steps. We return to our expression for $\frac{E_i}{P_{r,i}^{1-\sigma_r}}$ from earlier, and now express it as

$$\frac{E_i}{P_{r,i}^{1-\sigma_r}} = CN_{r,i}^{\delta'} \left(\sum_j \chi_{ij} P_{ij}^{1-\sigma_r} \right)^{\frac{\psi}{1-\sigma_r}}$$

where $\delta' = \delta + \frac{\psi}{1-\sigma_r}$. Furthermore, it can be shown that

$$\frac{\partial \chi_{ij}}{\partial N_{r,i}} = \frac{1}{s\sigma_r} \chi_{ij} \left(P_{ij}^{1-\sigma_r} - \frac{P_{r,i}^{1-\sigma_c}}{N_{r.,i}} \right) \cdot \frac{\partial}{\partial N_{r,i}} \left(\frac{E_i}{P_{r,i}^{1-\sigma_r}} \right)$$

Using both of these expressions, we find

$$\frac{\partial}{\partial N_{r,i}} \left(\frac{E_i}{P_{r,i}^{1-\sigma_r}} \right) = \frac{\delta'}{N_{r,i}} \frac{E_i}{P_{r,i}^{1-\sigma_r}} + \frac{\frac{\psi}{1-\sigma_r} N_{r,i}}{P_{r,i}^{1-\sigma_r}} \frac{E_i}{P_{r,i}^{1-\sigma_r}} \sum_j P_{ij}^{1-\sigma_r} \frac{\partial \chi_{ij}}{\partial N_{r,i}}$$

$$\frac{\partial}{\partial N_{r,i}} \left(\frac{E_i}{P_{r,i}^{1-\sigma_r}} \right) = \frac{\frac{\delta'}{N_{r,i}} \frac{E_i}{P_{r,i}^{1-\sigma_r}}}{\left(\frac{1}{s\sigma_r} \frac{\frac{\psi}{\sigma_r - 1} N_{r,i}}{P_{r,i}^{1-\sigma_r}} \frac{E_i}{P_{r,i}^{1-\sigma_r}} V + 1 \right)}$$

where we define $V \equiv \sum_j \chi_{ij} \left(P_{ij}^{1-\sigma_r} - \frac{P_{r,i}^{1-\sigma_r}}{N_{r.,i}} \right) P_{ij}^{1-\sigma_r}$ as the variance in the P_{ij}.Note that $V > 0$. Following our assumption that $\gamma < \frac{1-\alpha}{\alpha}$, so that $\delta < -\frac{\alpha}{1-\alpha} \left(\frac{1}{\sigma_r - 1} \right) + 1$ and $\delta' < 0$. Furthermore, our assumptions on α imply $\psi > 0$. Consequently, $\frac{\partial}{\partial N_{r,i}} \left(\frac{E_i}{P_{r,i}^{1-\sigma_r}} \right) < 0$.

We now turn to the main result. Following the distribution of trader shocks, retail firm profits conditional on entry are

$$\mathbb{E}[\pi] = s \ln \left(\sum_j \exp \left(\frac{\pi_{ij}^r}{s} \right) \right)$$

Thus,

$$\frac{\partial \mathbb{E}\left[\pi\right]}{\partial N_{r,i}} = CN_{r,i}^{\delta'} \sum_{j} \chi_{ij} \frac{\partial}{\partial N_{r,i}} \left(\frac{E_i}{P_{r,i}^{1-\sigma_r}} \right)$$

This will be strictly negative under the conditions provided, and there is a unique measure of traders which will enter retail.

C.7 Solving for wholesale equilibrium

This section provides explains how we solve for wholesaling decisions and calculate average chain lengths (as weighted by the number of traders on each chain.[5]. Everything in this Appendix relies on the markup approximation described in Section 5 of the text.[6]

This is organized into five subsections. The first four subsections build toward optimal entry and sourcing decisions of wholesalers, and the fifth develops expressions for average chain lengths across all traders selling in a location. First, we derive an expression for the expected payouts for a trader based in i to source from each source j (partly in terms of equilibrium outcomes we solve for in later steps). Second, we derive expressions for value of goods traded wholesale on each chain. Doing requires two components: the ultimate consumer expenditure at the end of the chain (which depends on the final price on the chain – which depends only on fundamentals of that chain – and characteristics of the final destination, including equilibrium expenditure, price index, and the sourcing decisions of its retailers), as well as the downstream markups and trade costs, which are not reflected in the value of goods upstream. Third, we find the total wholesale expenditure to a given location from each source j. Fourth, we establish that this leads to unique sourcing and entry decisions. There is a final (fifth) section which relies on the equilibrium entry and sourcing decisions of the fourth part, and develops expressions for average chain lengths across all traders selling in a location.

C.7.1 Expected payout for each source

In this section, we develop an expression for the expected payouts for a trader based in i to source from each source j (partly in terms of equilibrium outcomes we solve for in later steps).

[5]Note that this also relies on optimal retailing behavior as described in the text and elsewhere in the Appendix

[6]This assumption is not necessary – we could alternatively drop long chains (which play little role since there are sch low flows and low markups that they do not provide many wholesale profits and hence are not important for wholesaling behavior). However, the assumption simplifies the exposition substantially.

The first step is an expression for the share of traders serving an ij link on each step of each chain with an ij connection. The probability of choosing step n on chain z among the set of such objects with a link from j to i, denoted $\Pr(z,l)$, follows from the Frechet shocks across chains and is given by

$$\Pr(z,l) = \frac{\left(\pi_{z,l}^{var}\right)^{\frac{1}{\beta+1}}}{\sum_{z',l''}\left(\pi_{z',l''}^{var}\right)^{\frac{1}{\beta+1}}}$$

where $\pi_{z,l}^{var}$ is the variable profits from serving step l on chain z. in turn, the profits from serving step l on chain are given by

$$\pi_{z,l}^{var} = \frac{1}{\sigma_{l,z}}\frac{E_{z,l}}{N_{z,l}}$$

where on step l of chain z, $\sigma_{l,z}$ is the elasticity of demand across traders,[7] $E_{z,l}$ is the total expenditure, and $N_{z,l}$ is the measure of sellers. This expression follows from the standard CES profits and our assumption of symmetric sellers. And under the markups approximation,

$$\frac{1}{\sigma_{l,z}} \approx \frac{1}{\sigma_w}$$

where σ_w is shared for all wholesale steps. This, combined with the previous two expressions, gives us

$$\Pr(z,l) = \frac{E_{z,l}^{\frac{1}{\beta+1}}}{\sum_{z',l''}\left(E_{z',l''}^{\frac{1}{\beta+1}}\right)}$$

Next, we use the prior expression to solve for the profits of sourcing from each source j. We start by denoting by N_{ij}^{w} the measure of wholesalers sourcing from j for i, so that the total number of traders

$$N_{z,l} = \Pr(z,l)\,N_{ij}^{w}$$

Thus, the expected variable payout of sourcing wholesale from j to sell in i (i.e. neglecting

[7] This could be expressed in terms of step l and $L(z)$ following the formula in Section 4 of the text.

fixed costs and idiosyncratic draws), which we denote by $\pi_{ij}^{w,var}$, is given by

$$\begin{aligned}\pi_{ijw}^{var} &= \sum_{z,l} \Pr(z,l)\,\pi_{z,n} \\ &\approx \frac{1}{\sigma_w}\sum_{z,n}\frac{E_{z,n}}{N_{ij}^{w}} \\ &= \frac{1}{\sigma_w}\cdot\frac{E_{ij}^{w}}{N_{ij}^{w}}\end{aligned}$$

where E_{ijw} is the total wholesale expenditure going from j to i across all routes.

C.7.2 The value of goods on a given wholesale link

As we showed in the prior subsection, the expected variable profit for sourcing from j to sell in i as a wholesaler is a function of the total wholesale expenditure across all chains along that link. The expenditure values for all chains are simply a function of fundamentals and the choice of retail traders (which are themselves a unique function of fundamentals, as we have shown elsewhere). Thus, in this section, we develop an expression for expenditure on step l of chain z, which we denote by $E_{z,l}$ (and in the next section we will sum across all such chains). Throughout this section, we assume $l > L(z)$, i.e. this is a wholesale step on the chain.

Wholesale expenditure on step l of chain z reflects expenditure by consumers at the end of the chain, plus expenditure on markups and trade costs downstream (as these costs will not be passed up the chain). Expenditure at the end of the chain will be

$$E_{z,L(z)} = N_{r,L(z)}\chi_{rL(z),(L-1)(z)}\left(\frac{p_{z,N(z)}^{\frac{\beta+1}{\beta}}}{\phi_{L(z),(L-1)(z)}P_{r,L(z)}}\right)^{1-\sigma_r} E_{L(z)}$$

where in an abuse of notation, we use $\chi_{L(z),(L-1)(z)}^{r}$ to denote the share of retailers in the destination sourcing from the penultimate location on chain z, $P_{r,L(z)}$ for the consumer price index in the destination, and $N_{r,L(z)}$ to denote the number of retail traders in the destination.

And (in a repetition of results elsewhere in this appendix), the price at the end of the chain will be

$$p_{z,N(z)} = p_o\prod_{s=1}^{L(z)}\tau_{(s-1)(z)s(z)}m_s$$

and the downstream markups and trade costs (denoted by $C_{z,l}$) will be

$$C_{z,l} = \prod_{s=l+1}^{L(z)} \tau_{(s-1)(z)s(z)} m_s$$

where $\tau_{(l-1)(z)l(z)}$ is the trade cost incurred at step s and m_s i the markup at step s.

Taking these three expressions together, the value of expenditure flowing along chain z at step l is

$$\begin{aligned}
E_{z,L(z)} &= \frac{E_{z,L(z)}}{C_{z,l}} \\
&= \frac{N_{r,L(z)}\chi_{rL(z),(L-1)(z)}}{C_{z,l}} \left(\frac{p_{z,N(z)}^{\frac{\beta+1}{\beta}}}{\phi_{L(z),(L-1)(z)}P_{r,L(z)}} \right)^{1-\sigma_r} E_{L(z)} \\
&= p_o^{\left(1+\frac{1}{\beta_r}\right)(1-\sigma_c)} \left(\prod_{s=1}^{l} \tau_{(s-1)(z)s(z)} m_s \right)^{\frac{\beta+1}{\beta}(1-\sigma_c)} \left(\prod_{s=l+1}^{L(z)} \tau_{(s-1)(z)s(z)} m_s \right)^{\frac{1-\sigma_c(\beta_r+1)}{\beta_r}} \dots \\
&\dots \cdot \left(N_{r,L(z)}\chi_{rL(z),(L-1)(z)} E_{L(z)} \right) \left(\frac{1}{\phi_{L(z),(L-1)(z)}P_{r,L(z)}} \right)^{1-\sigma_r} E_{L(z)}
\end{aligned}$$

C.7.3 Total expenditure from each source

As the result in the first subsection shows, a critical object is the aggregate expenditure on wholesale chains passing through the j to i link, E_{ij}^{w}. We solve for that object by summing across the wholesale expenditure on each chain with an ij link following the second subsection.

We start with a definition of aggregate expenditure on wholesale chains going from j to i, which is simply the sum of expenditure on chains from j to i which feature wholesaler-to-wholesaler sales, E_{ijww}, and those which feature wholesaler-to-retailer sales, E_{ijwr}, i.e. that

$$E_{ijw} = E_{ijww} + E_{ijwr}$$

We start with wholesale-to-wholesale chains. We start with an expression for the profits from sourcing a chain from j going through i, ending up in ι from the penultimate destination φ. Furthermore, we will consider a chain z_{ww} with w_u upstream steps and w_d downstream steps until location k (so that there are $w_u + w_d + 2$ total steps: w_u steps from the origin to j, 1 step from j to i, w_d from i to φ and 1 step from φ to ι). Following the first subsection, the price of this chain will be

$$p_{z_{ww}} = \left(\tau_{o,z(2)} \prod_{n=2}^{w_u-1} \tau_{z(n)z(n+1)}\right) \tau_{ij} \left(\prod_{n=w_u+1}^{w_u+w_d} \tau_{z(n)z(n+1)}\right) \tau_{\iota\varphi} \cdot M_{w_u+w_d+2} \cdot p_o$$

and using the approximation to markups

$$p_{z_{ww}} \approx \left[(Bm_r)^{w_u} \tau_{o,z(2)} \prod_{n=2}^{w_u-1} \tau_{z(n)z(n+1)}\right] \cdot [Bm_r\tau_{ij}] \cdot \left[(Bm_r)^{w_d} \left(\prod_{n=w_u+1}^{w_u+w_d} \tau_{z(n)z(n+1)}\right) \tau_{\iota\varphi} m_r\right] p_o$$

where the term in the first set of brackets reflects trade costs and markups which occur upstream, the term in the middle brackets reflects markups and trade costs from j to i, and the term in the right brackets reflects trade costs and markups which occur downstream of the link from j to i.

Using this expression, we follow the expression in the first subsection to capture the expenditure a wholesale-to-wholesale chain for the link from j to i, denoted $E_{z_{ww}}$.

$$E_{z_{ww}} = \frac{p_{z_{ww}}^{\frac{\beta+1}{\beta}(1-\sigma_r)}}{m_r\,(Bm_r)^{w_d} \prod_{n=w_u+1}^{w_u+w_d} \tau_{z(n)z(n+1)}} \cdot N_{r\iota}\chi_{r\iota\varphi} \left(\frac{1}{\phi_{\iota\varphi} P_{r,\iota}}\right)^{1-\sigma_r} E_\iota$$

Second, we consider wholesale-to-retail chains. We will think of such a chain from j going through i, ending up in l. We will denote this chain z_{wr} with w_u upstream steps and of course only 1 downstream step to l (so that there are $w_u + 2$ total steps: w_u steps from the origin to j, 1 step from j to i, 1 step from i to ι). Then, the price of this chain will be

$$p_{z_{wr}} = \left(\tau_{o,z(2)} \prod_{n=2}^{w_u-1} \tau_{z(n)z(n+1)}\right) \tau_{ij}\tau_{\iota i} \cdot M_{w_u+2} \cdot p_o$$

and using the approximation to markups

$$p_{z_{wr}} \approx \left[(Bm_r)^{w_u} \tau_{o,z(2)} \prod_{n=2}^{w_u-1} \tau_{z(n)z(n+1)}\right] \cdot [Bm_r\tau_{ij}] \cdot [\tau_{\iota i} m_r]\, p_o$$

where the term in the first set of brackets reflects trade costs and markups which occur upstream, the term in the middle brackets reflects markups and trade costs from j to i, and the term in the right brackets reflects trade costs and markups which occur downstream of the link from j to i.

Using this expression, we follow the expression in the first subsection to capture the expenditure a wholesale-to-retail chain for the link from j to i, denoted $E_{z_{wr}}$.

$$E_{z_{wr}} = \frac{p_{z_{wr}}^{\frac{\beta+1}{\beta}(1-\sigma_r)}}{m_r \tau_{\iota i}} N_{\iota i}^{r} \left(\frac{1}{\phi_{\iota i} P_{r,\iota}} \right)^{1-\sigma_r} E_{\iota}$$

With these expressions in hand, we first turn to wholesale-to-wholesale expenditure for $j \neq o$[8]

$$\begin{aligned}
E_{ijww} &= \sum_{\varphi} \sum_{\iota} \sum_{w_u=1}^{\infty} \sum_{w_d=2}^{\infty} \sum_{z_{ww} \in \{ij\varphi\iota, w_u, w_d\}} E_{z_{ww}} \\
&= (B m_r \tau_{ij} p_o)^{\frac{\beta+1}{\beta}(1-\sigma_r)} \left((B m_r \tau_{jo})^{\frac{\beta+1}{\beta}(1-\sigma_r)} + \tilde{\boldsymbol{\Upsilon}}_{j'o} \left(\mathbf{I} - \tilde{\boldsymbol{\Upsilon}} \right)^{-1} \tilde{\boldsymbol{\Upsilon}}_{jj'}^{T} \right) \dots \\
&\dots \cdot \sum_{\varphi} \sum_{\iota} \left((B m_r \tau_{\varphi i})^{\frac{\beta+1}{\beta}(1-\sigma_r)} + \tilde{\delta}_{j'i} \left(\mathbf{I} - \tilde{\delta} \right)^{-1} \tilde{\delta}_{\iota j'}^{T} \right) (m_r \tau_{\varphi \iota})^{\frac{1-\sigma_r(\beta+1)}{\beta}} N_{\iota\varphi}^{r} \left(\frac{1}{\phi_{\iota\varphi} P_{r,\iota}} \right)^{1-\sigma_r} E_{\iota}
\end{aligned}$$

where

$$\tilde{\boldsymbol{\Upsilon}}_{j'o} \equiv \begin{pmatrix} (B m_r \tau_{1o})^{\frac{\beta+1}{\beta}(1-\sigma_r)} & \dots & (B m_r \tau_{Lo})^{\frac{\beta+1}{\beta}(1-\sigma_r)} \end{pmatrix}$$

$$\tilde{\boldsymbol{\Upsilon}}_{jj'}^{T} \equiv \begin{pmatrix} (B m_r \tau_{j1})^{\frac{\beta+1}{\beta}(1-\sigma_r)} \\ \vdots \\ (B m_r \tau_{jL})^{\frac{\beta+1}{\beta}(1-\sigma_r)} \end{pmatrix}$$

$$\tilde{\boldsymbol{\Upsilon}} \equiv \begin{bmatrix} (B m_r \tau_{11})^{\frac{\beta+1}{\beta}(1-\sigma_r)} & \dots & (B m_r \tau_{L1})^{\frac{\beta+1}{\beta}(1-\sigma_r)} \\ \vdots & \ddots & \vdots \\ (B m_r \tau_{1L})^{\frac{\beta+1}{\beta}(1-\sigma_r)} & \dots & (B m_r \tau_{LL})^{\frac{\beta+1}{\beta}(1-\sigma_r)} \end{bmatrix}$$

$$\tilde{\delta}_{j'i} \equiv \begin{pmatrix} (B m_r \tau_{1i})^{\frac{1-\sigma_r(\beta+1)}{\beta}} & \dots & (B m_r \tau_{Li})^{\frac{1-\sigma_r(\beta+1)}{\beta}} \end{pmatrix}$$

$$\tilde{\delta}_{\iota j'}^{T} \equiv \begin{pmatrix} (B m_r \tau_{\iota 1})^{\frac{1-\sigma_r(\beta+1)}{\beta}} \\ \vdots \\ (B m_r \tau_{\iota L})^{\frac{1-\sigma_r(\beta+1)}{\beta}} \end{pmatrix}$$

$$\tilde{\delta} \equiv \begin{bmatrix} (B m_r \tau_{11})^{\frac{1-\sigma_r(\beta+1)}{\beta}} & \dots & (B m_r \tau_{L1})^{\frac{1-\sigma_r(\beta+1)}{\beta}} \\ \vdots & \ddots & \vdots \\ (B m_r \tau_{1L})^{\frac{1-\sigma_r(\beta+1)}{\beta}} & \dots & (B m_r \tau_{LL})^{\frac{1-\sigma_r(\beta+1)}{\beta}} \end{bmatrix}$$

[8] If $j = o$, it is necessary to add a 1 to the upstream expression for buying from the manufacturer, i.e. the first parenthetical would instead be $1 + (B m_r \tau_{jo})^{\frac{\beta+1}{\beta}(1-\sigma_r)} + \tilde{\boldsymbol{\Upsilon}}_{j'o} \left(\mathbf{I} - \tilde{\boldsymbol{\Upsilon}} \right)^{-1} \tilde{\boldsymbol{\Upsilon}}_{jj'}^{T}$; we have omitted this expression for brevity and because it is analogous to our expressions for P_{ij} in Section 5.

Similarly, for sales to retailers for $j \neq o$[9]

$$\begin{aligned} E_{ijwr} &= \sum_{\iota} \sum_{w_u=1}^{\infty} \sum_{z_{ww} \in \{ij\varphi\iota, w_u, w_d\}} E_{z_{wr}} \\ &= (Bm_r\tau_{ij}p_o)^{\frac{\beta+1}{\beta}(1-\sigma_r)} \left((Bm_r\tau_{jo})^{\frac{\beta+1}{\beta}(1-\sigma_r)} + \tilde{\boldsymbol{\Upsilon}}_{j'o} \left(\mathbf{I} - \tilde{\Upsilon} \right)^{-1} \tilde{\boldsymbol{\Upsilon}}^{T}_{jj'} \right) \dots \\ &\dots \cdot \sum_{\iota} (m_r\tau_{i\iota})^{\frac{1-\sigma_r(\beta+1)}{\beta}} \left(\frac{N_{rij}}{m_r} \left(\frac{1}{\phi_{\iota i} P_{r,\iota}} \right)^{1-\sigma_r} \right) E_{\iota} \end{aligned}$$

where $\tilde{\Upsilon}$,$\tilde{\boldsymbol{\Upsilon}}_{j'o}$, and $\tilde{\boldsymbol{\Upsilon}}^{T}_{jj'}$ are as defined above.

Thus, using the prior expressions, we can solve for the share of traders on any given route, and hence expected profits from sourcing from j for wholesalers in i as a function of fundamentals and the equilibrium number of wholesalers serving the j to i route.

C.7.4 Equilibrium sourcing

We now return to find expected sourcing shares and entry. In this subsection, we show that both of these objects are unique. Thus we can in practice construct the necessary matrices following subsection 3 to find wholesale expenditure flows between every location pair, and then solve numerically to find the unique measure of wholesalers and the sourcing pattern. Throughout this subsection, we will refer to E^{w}_{ij} without using the complicated expressions derived in the prior subsection. Note that the aggregate wholesale expenditure on routes from j to i is exogenous with respect to wholesale sourcing locations and entry – it depends only on parameters and retail sourcing and entry decisions.

The expected wholesale profits (inclusive of fixed costs) of sourcing from j for i, denoted π_{ijw}, are given by

$$\pi_{ijw} = \pi^{var}_{ijw} - F_{ij}$$

(where we have an expression for π^{var}_{ijw} from the first subsection) and given the Gumbel distribution of fixed costs and the assumptions on the location parameters, expected profits (pre-entry) will be

$$\pi^{w}_{i} = s \ln \left(\sum_{j} \exp \left(\frac{\pi_{ijw}}{s} \right) \right) - f_{e,i}$$

[9] If $j = o$, it is necessary to add a 1 to the upstream expression for buying from the manufacturer as in the wholesale-wholesale expression, i.e. the first parenthetical would instead be $1 + (Bm_r\tau_{jo})^{\frac{\beta+1}{\beta}(1-\sigma_r)} + \tilde{\boldsymbol{\Upsilon}}_{j'o} \left(\mathbf{I} - \tilde{\boldsymbol{\Upsilon}} \right)^{-1} \tilde{\boldsymbol{\Upsilon}}^{T}_{jj'}$.

By combining this expression with results from prior subsections, we can use the expression for variable profits to write

$$\chi_{wij} = \exp\left(\frac{\frac{1}{\sigma_w} \cdot \frac{E_{ijw}}{\chi_{wij} N_{wi}} - F_{ij} - f_{i,e}}{s}\right)$$

and

$$\pi_{iw} = s \ln\left(\sum_j \exp\left(\frac{\frac{1}{\sigma_w} \cdot \frac{E_{ijw}}{\chi_{wij} N_{wi}} - F_{ij}}{s}\right)\right) - f_{i,e}$$

where N_{wi} is the total measure of wholesalers in i.

Proposition A1: The measure of wholesalers and wholesale sourcing shares (for each location) are unique functions of fundamentals.

Proof: First, we show that, conditional on N_{wi}, the share of wholesalers which source from every location j is unique. And second, we show that this implies a unique measure of wholesalers which enter in location i.

We start with the expression for wholesale sourcing shares above

$$\chi_{wij} = \exp\left(\frac{\frac{1}{\sigma_w} \cdot \frac{E_{ijw}}{\chi_{wij} N_{wi}} - F_{ij} - f_{i,e}}{s}\right)$$

and we show that $\frac{\partial \chi_{wij}}{\partial N_{wi}} < 0$ for every value of N_{wi}. This implies that χ_{wij} is a unique function of N_{wi}:

$$\begin{aligned}
\frac{\partial \chi_{wij}}{\partial N_{wi}} &= \chi_{wij}\left(\frac{-\frac{1}{\sigma_w} \cdot \frac{E_{ijw}}{\chi_{wij} N_{wi}}}{s}\right)\left(\frac{1}{\chi_{ijw}}\frac{\partial \chi_{wij}}{\partial N_{wi}} + \frac{1}{N_{wi}}\right) \\
&= \frac{-\frac{E_{wij}}{N_{wi}}}{\sigma_w s N_{wi} + \frac{E_{wij}}{\chi_{wij}}} \\
&< 0
\end{aligned}$$

And second,

$$\frac{\partial \pi_{wi}}{\partial N_{wi}} = -s\frac{E_{wij}}{N_{wi}}\sum_{j}\left(\frac{\chi_{wij}}{\sigma_w sN_{wi}\chi_{wij}^2 + E_{ijw}\chi_{wij}}\right)$$
$$< 0$$

This implies that there is only one value of N_{wi} under which the free entry condition holds. Thus equilibrium is unique.

C.7.5 Wholesaling chain length

The last part of this subsection is to describe wholesaling chain lengths. Once we describe the expenditure flowing on chains of a particular length, then the remaining steps are trivial.

The result will have the form of a 4-touple: wholesale flows through j to i with ν upstream steps and υ downstream steps, denoted $E_{ij\nu\upsilon w}$. For the purposes of calculating this expression, we develop two scalars: $M_{uj\nu}$ which captures upstream costs, and $M_{di\varphi\upsilon}$.

If $\nu = 1$, then $M_{uo1} = 1$ (note that the only j with 1 upstream step is the origin, as manufacturer sourcing is not possible from other places).

If $\nu = 2$, then $M_{uj2} = (Bm_r\tau_{jo})^{\frac{\beta+1}{\beta}(1-\sigma_r)}$.

If $\nu \geq 3$, then $M_{uj\nu} = \tilde{\mathbf{\Upsilon}}_{j'o}\left(\tilde{\Upsilon}\right)^{\nu-3}\tilde{\mathbf{\Upsilon}}_{jj'}^{T}$ (where $\tilde{\mathbf{\Upsilon}}_{j'o}$, $\tilde{\Upsilon}$, and $\tilde{\mathbf{\Upsilon}}_{jj'}^{T}$ are defined as in the prior subsection).

If $\upsilon = 2$, then $M_{di\varphi 2} = (Bm_r\tau_{\varphi i})^{\frac{1-\sigma_r(\beta+1)}{\beta}}$ (since this is a wholesale transaction, it is impossible for $\upsilon = 1$).

If $\upsilon \geq 3$, then $M_{di\varphi 3} = \tilde{\delta}_{j'i}\tilde{\delta}^{\nu-3}\tilde{\delta}_{\varphi j'}^{T}$. (again where the vectors and matrices are defined as in the prior subsection).

With these definitions in hand, we can then describe

$$E_{ij\nu\upsilon w} = (Bm_r\tau_{ij}p_o)^{\frac{\beta+1}{\beta}(1-\sigma_r)} M_{uj\nu}\sum_{\varphi}\sum_{\iota} M_{di\varphi\upsilon}\left(m_r\tau_{\varphi\iota}\right)^{\frac{1-\sigma_r(\beta+1)}{\beta}} N_{\iota\varphi}^{r}\left(\frac{1}{\phi_{\iota\varphi}P_{r,\iota}}\right)^{1-\sigma_r} E_{\iota}$$

D Material from Section 5

D.1 Markups approximation

We start with the expression for the markup at step l and find

$$\frac{\sigma_l}{\sigma_l - 1} = m_r\left[1 - \frac{1}{\sigma_r}\left(1 - \left(\frac{\mu_w + 1}{\mu_w}\right)^{1-n}\right)\right]$$

and then substitute into the expression for the log of the aggregate markup

$$\ln M_L = L \ln m_r + \sum_{l=1}^{L} \ln \left(1 - \frac{1}{\sigma_r}\left(1 - \left(\frac{\mu_w + 1}{\mu_w}\right)^{1-l}\right)\right)$$

We next take a second order approximation to $\ln M_L$ around $L = 2$ and $\frac{\mu_w+1}{\mu_w} = 1$ (i.e. as μ_w gets very large) to obtain

$$\ln M_L \approx L \ln m_r - \frac{1}{\sigma_r}\left(\frac{1}{\mu_w}\right)(L-1) + \frac{1}{2}\left(\frac{1}{\mu_w}\right)^2 \left[\frac{m_r + 1}{\sigma_r m_r}\right]$$

By exponentiating both sides we obtain

$$M_L = \exp\left(\frac{1}{2}\left(\frac{1}{\mu_w}\right)^2 \left[\frac{m_r + 1}{\sigma_r m_r}\right]\right) m_r \left(m_r e^{-\frac{1}{\mu_w \sigma_r}}\right)^{L-1}$$

which implies a wholesale markup between any two steps L and $L-1$ (for $L > 0$)

$$m_w = \frac{M_L}{M_{L-1}} = m_r e^{-\frac{1}{\mu_w \sigma_r}}$$

In general in the text we will treat $M_L \approx m_r \left(m_r e^{-\frac{1}{\mu_w \sigma_r}}\right)^{L-1}$. We do this for two reasons. First, it avoids needlessly distorting the retail markup (which would otherwise take place as the approximation implies a markup at $L = 1$ of $\exp\left(\frac{1}{2}\left(\frac{1}{\mu_w}\right)^2 \left[\frac{m_r+1}{\sigma_r m_r}\right]\right) m_r$). And second, in practice $\exp\left(\frac{1}{2}\left(\frac{1}{\mu_w}\right)^2 \left[\frac{m_r+1}{\sigma_r m_r}\right]\right)$ is quite close to 1. Note that omitting this factor does not affect our estimation of the wholesale markup, and under our parameter values,

$$\exp\left(\frac{1}{2}\left(\frac{1}{\mu_w}\right)^2 \left[\frac{m_r + 1}{\sigma_r m_r}\right]\right) \approx 1.037$$

so omitting this factor is of little practical importance.

D.2 Price index approximation

Using our markup approximation (and defining $B \equiv m_r e^{-\frac{1}{\mu_w \sigma_r}}$), we are able to obtain closed-form solutions our expressions for the $\mathcal{P}_j$. Starting from the expression for P_j in the text for a non-origin source

$$\mathcal{P}_j^{1-\sigma_r} \approx \frac{1}{\phi_j^{1-\sigma_r}}[(Bm_r\tau_{jo}p_o)^{\left(1+\frac{1}{\beta_r}\right)(1-\sigma_r)} + \sum_{j'}\left((Bm_r)^2\,\tau_{j'o}\tau_{jj'}p_o\right)^{\left(1+\frac{1}{\beta_r}\right)(1-\sigma_r)}\dots$$
$$+\sum_{j''}\sum_{j'}\left((Bm_r)^3\,\tau_{j''o}\tau_{j'j''}\tau_{jj'}p_o\right)^{\left(1+\frac{1}{\beta_r}\right)(1-\sigma_r)} + \dots]$$

Then,

$$\begin{aligned}\mathcal{P}_{j\neq o}^{1-\sigma_c} &\approx \frac{p_o^{\left(1+\frac{1}{\beta_r}\right)(1-\sigma_r)}}{\phi_j^{1-\sigma_r}}\left[(Bm_r\tau_{jo})^{\frac{1}{\beta_r}(1-\sigma_r)} + \mathbf{T}_{jo}\left(\sum_{n=0}^{\infty}\mathbf{T}^n\right)\mathbf{T}_{jj'}^T\right] \\ &= \frac{p_o^{\left(1+\frac{1}{\beta_r}\right)(1-\sigma_r)}}{\phi_o^{1-\sigma_r}}\left[(Bm_c\tau_{jo})^{\frac{1}{\beta_r}(1-\sigma_r)} + \mathbf{T}_{jo}\left(\mathbf{I}-\mathbf{T}\right)\mathbf{T}_{jj'}^T\right]\end{aligned}$$

where we define

$$\mathbf{T} \equiv \begin{pmatrix} (Bm_r\tau_{11})^{\left(1+\frac{1}{\beta_r}\right)(1-\sigma_r)} & \cdots & (Bm_r\tau_{L1})^{\left(1+\frac{1}{\beta_r}\right)(1-\sigma_r)} \\ \vdots & \ddots & \vdots \\ (Bm_r\tau_{1L})^{\left(1+\frac{1}{\beta_r}\right)(1-\sigma_r)} & \cdots & (Bm_r\tau_{LL})^{\left(1+\frac{1}{\beta_r}\right)(1-\sigma_r)} \end{pmatrix}$$

and we use $\mathbf{T}_{jo}$ to denote the first row of $\mathbf{T}$ and $\mathbf{T}_{jj'}^T$ to denote the transpose of the jth row of $\mathbf{T}$.

Essentially identical steps can be taken to obtain

$$\begin{aligned}\mathcal{P}_o &= \frac{p_o^{\left(1+\frac{1}{\beta_r}\right)(1-\sigma_r)}}{\phi_o^{1-\sigma_r}}\left[1 + (Bm_c\tau_{oo})^{\frac{1}{\beta_r}(1-\sigma_r)} + \mathbf{T}_{jo}\left(\mathbf{I}-\mathbf{T}\right)^{-1}\mathbf{T}_{oj'}^T\right] \\ (j\neq o)\ \phi_j^{1-\sigma_r} &= p_o^{\frac{1}{\beta_r}(1-\sigma_r)}\left[(Bm_r\tau_{jo})^{\frac{1}{\beta_r}(1-\sigma_r)} + \hat{\mathbf{T}}_{j'o}\left(\mathbf{I}-\hat{\mathbf{T}}\right)^{-1}\hat{\mathbf{T}}_{jj'}^T\right] \\ \phi_o^{1-\sigma_r} &= p_o^{\frac{1}{\beta_r}(1-\sigma_r)}\left[1 + (Bm_r\tau_{oo})^{\frac{1}{\beta_r}(1-\sigma_r)} + \hat{\mathbf{T}}_{j'o}\left(\mathbf{I}-\hat{\mathbf{T}}\right)^{-1}\hat{\mathbf{T}}_{oj'}^T\right]\end{aligned}$$

where we define

$$\hat{\mathbf{T}} \equiv \begin{pmatrix} (Bm_r\tau_{11})^{\frac{1}{\beta_r}(1-\sigma_r)} & \cdots & (Bm_r\tau_{L1})^{\frac{1}{\beta_r}(1-\sigma_r)} \\ \vdots & \ddots & \vdots \\ (Bm_r\tau_{1L})^{\frac{1}{\beta_r}(1-\sigma_r)} & \cdots & (Bm_r\tau_{LL})^{\frac{1}{\beta_r}(1-\sigma_r)} \end{pmatrix}$$

and we use $\hat{\mathbf{T}}_{jo}$ to denote the first row of $\hat{\mathbf{T}}$ and $\hat{\mathbf{T}}_{jj'}^T$ to denote the transpose of the jth row of $\mathbf{T}$.

Similarly, the expressions for expenditure-weighted chain length in Section 4.3.4 can be approximated as

$$(j \neq o)\ \Lambda_j = \frac{p_o^{\frac{1}{\beta_r}(1-\sigma_r)}}{\left(\tilde{P}_j \phi_j\right)^{1-\sigma_r}} \left[2 \left(B m_r \tau_{jo}\right)^{\left(1+\frac{1}{\beta_r}\right)(1-\sigma_r)} + \mathbf{T}_{jo} \left(\sum_{n=0}^{\infty} (n+3)\, \mathbf{T}^n \right) \mathbf{T}_{jj'}^T \right]$$

$$\Lambda_o = \frac{p_o^{\frac{1}{\beta_r}(1-\sigma_r)}}{\left(\tilde{P}_o \phi_o\right)^{1-\sigma_r}} \left[1 + \mathbf{T}_{jo} \left(\sum_{n=0}^{\infty} (n+2)\, \mathbf{T}^n \right) \mathbf{T}_{oj'}^T \right]$$

where $\mathbf{T}$, $\mathbf{T}_{jo}$, $\mathbf{T}_{jj'}^T$, $\mathcal{P}_j$ and ϕ_j are as previously defined.