

# Online Appendix - *Aftermarket Frictions and the Cost of Off-Platform Options in Centralized Assignment Mechanisms*<sup>†</sup>

## 1 Further Details on the Institutional Context

### 1.1 Higher Education Providers in Chile

This section provides additional descriptives on Chile’s higher education market. Table 1 shows the distribution of math+verbal scores of students enrolled in each G33 university in 2012. Highlighted (gray) rows indicate G8 programs. As the table shows, the distribution of G8 institutions’ selectivity overlaps with that of the G25 institutions, excluding two G25 institutions at the very top. The largest G8 institutions had relatively low mean scores, toward the low end of the G33, but these programs are not outliers.

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<sup>†</sup>Last Updated on June 24th, 2022 ([See most recent version here](#)).

**Table 1: Selectivity by Institution**

Univ.	Mean Score	Std. Dev.	P10	P25	P50	P75	P90	Total Adm.	Tier
11	691.5	46.0	630.9	658.2	688.9	723.5	751.9	5424	1
12	694.7	52.5	630.6	652.8	690.4	734.2	769.9	4754	1
16	633.9	36.64	591.8	610.7	631.75	654.9	676.6	4725	2
42	656.2	37.1	610.3	633.7	655.5	678.3	702.3	1868	2
43	657.2	53.5	586.0	613.6	657.6	694.9	723.9	1221	2
13	602.8	62.8	521.3	554.3	601.8	645.0	687.4	6377	3
15	615.2	78.9	506.9	547.7	629.2	680.5	708.2	4346	3
19	596.5	54.4	532.7	558.7	589.8	625.3	671.5	3984	3
14	610.9	43.0	557.4	581.4	608.3	637.7	667.3	3448	3
17	582.6	58.1	514.5	541.6	577.1	617.0	660.4	3076	3
44	605.9	55.0	537.8	566.2	603.0	636.1	673.9	2903	3
38	617.2	40.0	571.2	590.0	613.3	638.85	668.6	2726	3
30	592.7	56.1	525.1	549.9	586.3	627.1	669.2	2354	3
18	589.3	55.3	520.4	546.3	583.6	625.1	664.8	2322	3
34	616.1	52.1	551.4	582.9	615.0	646.9	685.9	1904	3
35	581.5	48.0	527.8	546.1	573.2	606.7	644.3	1639	3
45	595.6	38.8	547.4	567.5	593.9	619.6	646.8	1539	3
40	580.9	59.6	509.7	540.8	575.7	613.7	660.0	1145	3
20	605.7	33.6	570.0	580.7	603.3	624.95	649.0	1092	3
41	550.3	53.7	484.5	511.9	546.8	583.7	618.6	12615	4
39	571.8	56.5	501.4	529.0	568.0	604.4	645.8	4895	4
36	563.2	51.9	502.7	525.6	557.6	593.4	625.4	2451	4
29	577.5	43.7	523.4	544.5	572.4	604.35	639.7	2449	4
21	548.7	36.0	503.2	524.5	548.3	574.25	595.0	2392	4
37	543.9	43.0	492.0	509.6	540.0	571.35	598.8	2206	4
25	575.4	45.0	521.4	542.5	569.4	605.7	635.4	1791	4
26	556.0	37.4	512.0	530.0	553.0	578.55	604.0	1748	4
22	557.0	51.9	493.7	517.1	549.0	594.4	627.8	1720	4
24	566.1	65.2	487.2	515.7	558.3	606.7	662.1	1433	4
23	550.8	47.1	493.8	514.0	541.3	583.8	616.4	928	4
27	550.6	50.0	486.8	511.9	549.5	585.7	616.6	815	4
32	541.0	44.7	484.4	508.2	535.8	571.0	603.7	785	4
33	554.0	50.7	492.1	516.1	547.4	586.0	626.0	499	4

Note: This table summarizes the admission scores for each on-platform institution in 2012. Universities are grouped by “tiers”, where tier 1 is defined as universities with average scores in [660, 700), tier 2 in [620, 660), tier 3 in [580, 620), and tier 4 in [540, 680). Within each tier, universities are sorted by the number of applicants they admitted.

Table 2 shows growth in the number of students attending higher education programs in Chile around our study period. The bottom panel indicates that both private and G25 enrollments were increasing during this period. Figure 1 shows that G8 enrollment grew rapidly in the period before our study, and was growing relative to G25 enrollment during the study period.

**Table 2:** Size of the sectors of higher education

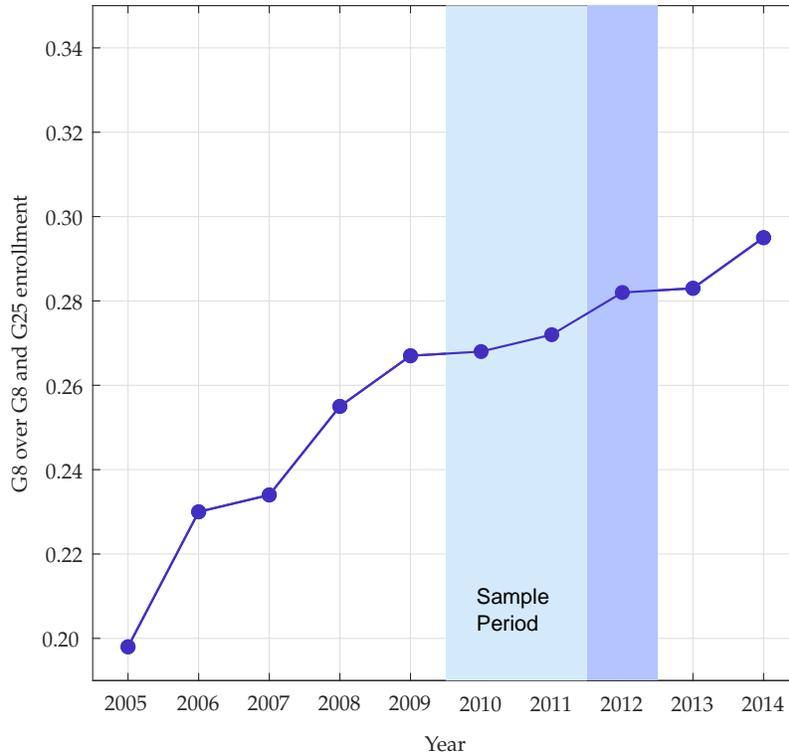
Type of Institution	2009	2010	2011	2012	2013
Universities	535,643	587,297	615,884	631,608	645,355
Professional Institutes	189,597	224,301	260,692	293,519	324,920
Technical Institutes	110,007	128,566	138,574	140,031	144,365
Total	835,247	940,164	1,015,150	1,065,158	1,114,640

Type of University	2009	2010	2011	2012	2013
G25	276,683	281,528	282,453	282,879	295,662
All Private	258,960	305,769	333,431	348,729	349,693
	535,643	587,297	615,884	631,608	645,355

Note: This table presents total matriculation over time and by type of higher education. Source is MINEDUC.

**Figure 1:** G8 enrollment as a share of G33 enrollment



Note: This figure shows the fraction of students that enroll in G8 options relative to the total enrollment in G33 options over time. The light-blue area corresponds to our pre-policy sample, whereas the dark-blue area is our post-policy sample.

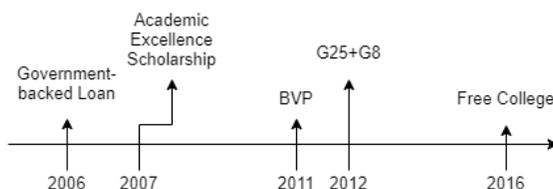
## 1.2 Financial Aid

In this subsection we discuss public and private initiatives to help finance higher education. The most important policies introduced by MINEDUC from 2006 to 2016 are included in [Figure 2](#). About 24% of students that take the college entrance exam were beneficiaries of MINEDUC loans and scholarships in 2012. Financial aid options offered by universities, in spite of being very numerous, benefit a much smaller share of students.

For the three years of our study, the complete offer of benefits offered by universities can be found in an official document put together by CRUCH and circulated in a widely-read newspaper. The aid offer expands every year, so we have summarized the most generous year of our study which is 2012. [Table 3](#) presents a list of all G25 and G8 institutions with indicators of whether they offer different types of scholarships, discounts, stipends, or supplementary help to complement MINEDUC scholarships and credits obtained by the student.

Columns 1, 5 and 7 show that most universities offer at least some form of aid. This fact can be misleading, however, because 1) there is significant variation in the generosity of their aid packages (some universities only offer modest stipends or discounts), and 2) half of them base eligibility for their most generous aid on very high PSU cutoffs, which leaves financially relevant options out of reach for most students.

**Figure 2:** Policies Expanding Access to Higher Education in Chile - Timeline



Note: This figure shows a timeline with the major policies in higher education implemented in Chile before and after the time period under study in this paper. The first one is the *Government-backed Loan (CAE)*, aid open to students applying to CRUCH or accredited non-CRUCH higher education institutions. Importantly, eligibility was not tied to participation on the centralized platform. The second one is the *Academic Excellence Scholarship*, aimed to cover part of the annual fee of students belonging to the 10% of higher achievement. They have to apply to CRUCH or accredited non-CRUCH higher education institutions, come from public or private voucher schools, belong to the 80% most vulnerable population, and enter the year right after they graduated high school. This policy is also unrelated to participation on the centralized platform. The *Teacher Scholarship (BVP)* which began in 2011 provided full scholarships for high-scoring students at eligible teacher training programs. This was unrelated to participation on the platform. Finally, the *Free College* policy established that 50% of most vulnerable students do not have to pay tuition or annual fee in CRUCH or accredited non-CRUCH higher education institutions attached to the agreement.

**Table 3: Institutional Aid Summary - 2012**

Institution	Scholarships				Discounts		Supplement
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Any	Deterministic	PSU-based	SES-based	Any	PSU-based	Any
P. Universidad Católica de Chile	Yes	Yes	Yes	No	Yes	Yes	Yes
P. Universidad Católica de Valparaiso	Yes	Yes	Yes	No	Yes	No	Yes
Universidad Adolfo Ibáñez	No	No	No	No	Yes	Yes	No
Universidad Alberto Hurtado	No	No	No	No	Yes	Yes	No
Universidad Andrés Bello	No	No	No	No	No	No	No
Universidad Austral de Chile	Yes	Yes	Yes	No	Yes	Yes	No
Universidad Católica de Temuco	No	No	No	No	No	No	No
Universidad Católica de la S. Concepción	No	No	No	No	No	No	Yes
Universidad Católica del Maule	No	No	No	No	Yes	No	Yes
Universidad Católica del Norte	No	No	No	No	No	No	Yes
Universidad Diego Portales	Yes	Yes	No	No	Yes	Yes	Yes
Universidad Finis Terrae	No	No	No	No	Yes	No	No
Universidad Mayor	No	No	No	No	Yes	Yes	No
Universidad Metrop. de Cs. de la Educación	No	No	No	No	No	No	No
Universidad Técnica Federico Santa María	Yes	Yes	Yes	Yes	Yes	Yes	No
Universidad Tecnológica Metrop.	Yes	Yes	Yes	No	Yes	Yes	No
Universidad de Antofagasta	Yes	No	Yes	No	No	No	Yes
Universidad de Arturo Prat	Yes	No	No	No	Yes	No	No
Universidad de Atacama	No	No	No	No	No	No	No
Universidad de Chile	Yes	No	Yes	Yes	No	No	No
Universidad de Concepción	Yes	No	Yes	No	Yes	Yes	No
Universidad de La Frontera	Yes	Yes	No	No	Yes	Yes	Yes
Universidad de La Serena	No	No	No	No	No	No	No
Universidad de Los Andes	No	No	No	No	No	No	No
Universidad de Los Lagos	No	No	No	No	No	No	No
Universidad de Magallanes	No	No	No	No	No	No	No
Universidad de Playa Ancha	Yes	Yes	Yes	No	Yes	No	No
Universidad de Santiago de Chile	Yes	Yes	Yes	No	No	No	Yes
Universidad de Talca	Yes	Yes	Yes	No	No	No	No
Universidad de Tarapacá	Yes	Yes	Yes	No	Yes	Yes	No
Universidad de Valparaíso	Yes	Yes	Yes	No	Yes	Yes	No
Universidad del Bío-Bío	Yes	No	Yes	No	No	No	No
Universidad del Desarrollo	No	No	No	No	Yes	No	No

Note: This table presents each institution with indicators of whether they offer different types of scholarships, discounts, stipends, or supplementary help to complement MINEDUC scholarships and credits obtained by the student. We only consider scholarships for entering (not continuing) students in this summary. Ex ante deterministic means that there is certainty about the amount of the scholarship/discount at the time of deciding where to apply. PSU-based indicates a requirement to obtain a PSU score above a high threshold, usually 1 or 2 standard deviations above the mean, depending on the institution. SES-based indicates a requirement to belong to the income quintiles 1, 2, or (sometimes) 3. PSU-based and SES-based are not mutually exclusive in most cases. Source is CRUCH's official document on university services and benefits, circulated in newspapers one month before the PSU.

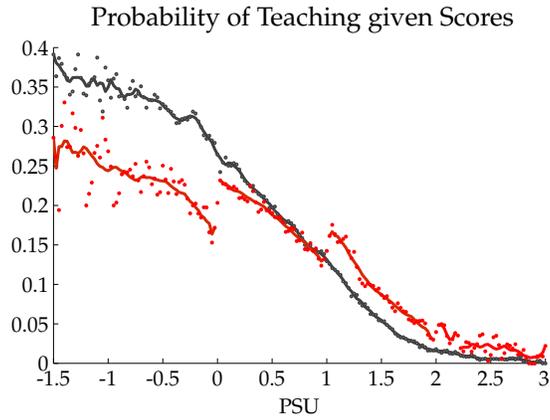
### 1.3 Further Details about *Beca Vocación Profesor*

The BVP policy was intended to induce higher-scoring students to enter the teaching profession in two ways: if a program chose to participate, then students with a simple average of at least 600 on their math and verbal scores received full scholarships. However, the policy placed a cap on the number of students with scores below 500 (the mean test score) that the program was allowed to admit. Approximately 50% of programs joined in 2011, and a handful more chose to participate in 2012. Overall almost all students who could apply to the programs on the platform saw their choice sets vary due to the policy.

Figure 3 from Gallegos, Neilson, and Calle (2019) (reproduced below) shows the probability of enrollment in teacher training programs, conditional on enrolling in some higher-education program, as a function of year and average test score. At baseline, roughly 30% of students with

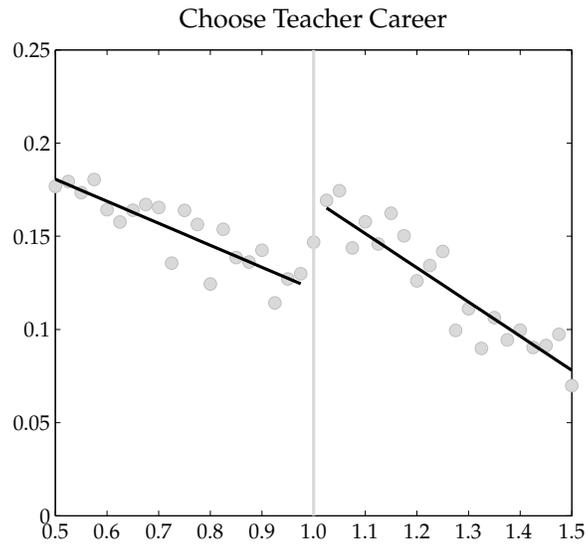
test scores near the population mean were enrolled in teaching programs. One can observe the discontinuities at 500 (the average) and 600 ( $\mu + \sigma$ ) points, as well as a level shift for high-scoring students. We use this variation later to relate the impacts of platform expansion to those of changes in prices. [Figure 4](#) shows a zoom in of the cutoff for eligibility at 600 points to show the discrete jump in choice probabilities that is induced by the policy. See more details about the policy in [Gallegos, Neilson, and Calle \(2019\)](#).

**Figure 3: Enrollment Probability and Targeted Tuition Subsidies**



Note: This figure is a reproduction from [Gallegos, Neilson, and Calle, 2019](#)). It shows the probability of enrollment in a teaching major as a function of average college entrance exam scores and time. X-axis: Standardized math+verbal PSU score, in standard-deviation units. Y-axis: probability of entering a teaching major conditional on having taken the PSU.

**Figure 4:** Enrollment Probability Around The Eligibility Cutoff

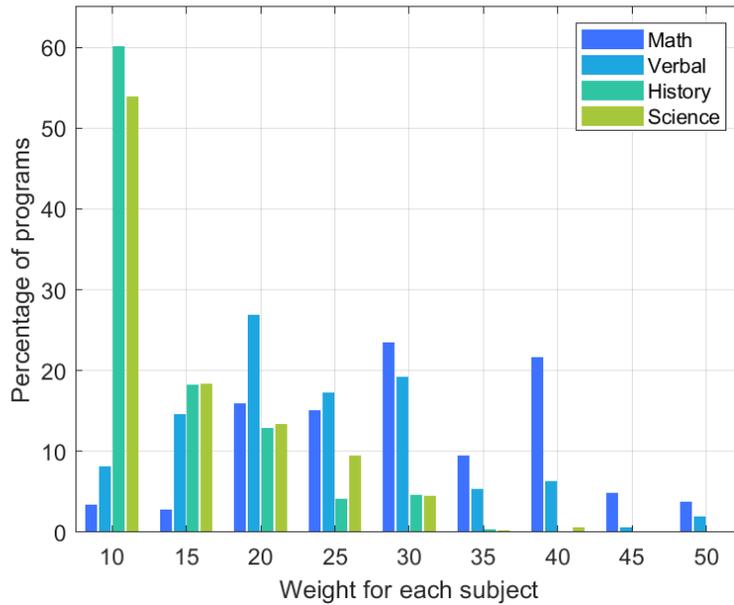


Note: This figure is a reproduction from (Gallegos, Neilson, and Calle, 2019). It shows the probability of enrollment in a teaching major as a function of average college entrance exam scores in 2011 at the cutoff.

## 1.4 Details about the Application Process

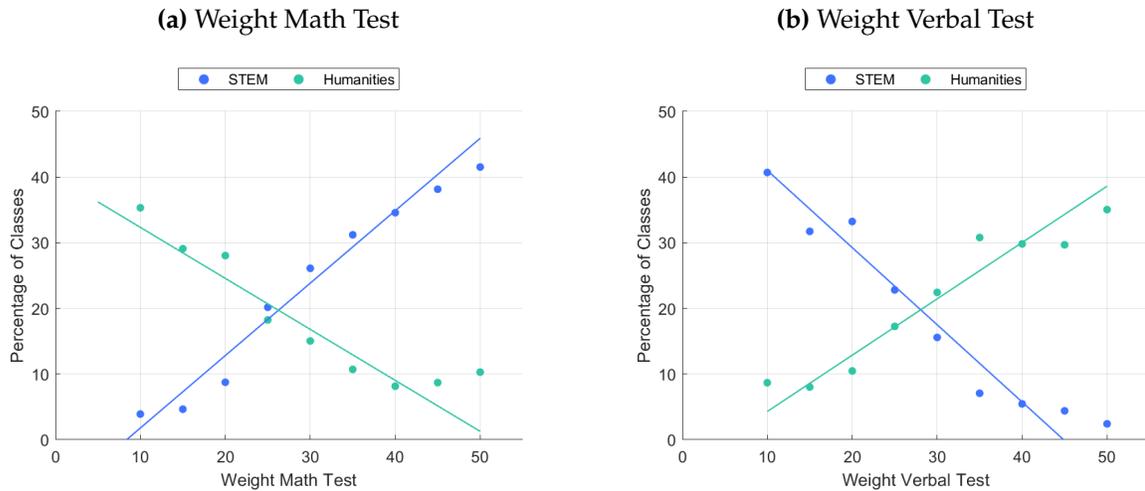
Figure 5 Shows the distribution of weights on each subject test among on-platform programs in 2011. Weights exhibit some dispersion, with programs placing 10% weight on the science exam, for instance, while others place 30%. Figure 6 shows that weights are associated with course content. The left panel shows the mean STEM and humanities course content (Y-axis) among programs with a given weight on the math PSU exam. The figure shows that as the weight on the math test increases, programs have a higher proportion of STEM content, and a lower proportion of Humanities content. For example, in the typical program with 10% weight on the math exam, less than five percent of courses have STEM content. In contrast, in programs with 50% weight on the math exam, more than 40% of courses are classified as STEM. The right panel shows that greater weight on verbal tests is associated with lower STEM course content and greater humanities course content.

**Figure 5: Weights on each subject**



Note: This figure shows the distribution of weights that each program in 2011 considers for PSU's different subjects.

**Figure 6: Weights on Math and Verbal tests by type of classes**



Note: This figure shows the conditional means of course contents by test weight in 2011.

Table 4 shows the distribution of assignments by application ranking. The maximum length increased from 8 to 10 programs in 2012. However, fewer than 1% of students were assigned to programs that they ranked eighth or greater.

**Table 4:** Assignment Rates by Application Ranking

Application Rank	2010	2011	2012
1	37.4%	44.1%	48.2%
2	18.0%	18.6%	21.2%
3	10.8%	10.2%	11.7%
4	5.9%	5.1%	5.7%
5	3.5%	2.8%	3.2%
6	1.9%	1.5%	1.7%
7	1.1%	0.8%	1.0%
8	0.7%	0.5%	0.6%
9	-	-	0.3%
10	-	-	0.2%

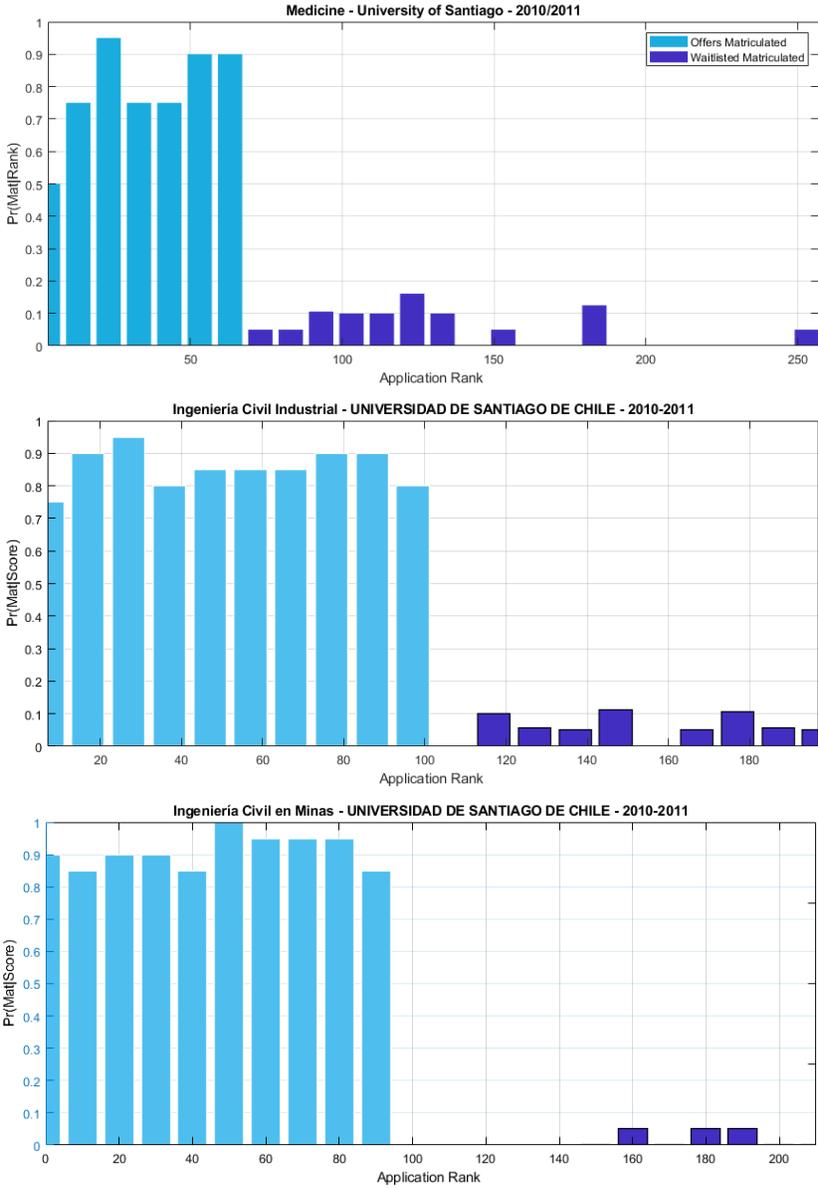
Note: This table presents the distribution of list length and rank of assignment for each year in our study. Although the maximum list length increased from 8 to 10 in the sample period, fewer than 1% of applicants were assigned to programs below their 7th ranked option.

## 1.5 Additional Case Studies of Waitlist Enrollment

Figure 7 provides additional case studies of enrollment probabilities at large majors that admitted some students off of waitlists. We rank applicants from 1 (highest-scoring student placed in the program) to N (final waitlisted student who enrolls). The pattern of long tails of students admitted off of waitlists, and discrete drops in the probability of enrollment in the target program at the initial cutoff score, is common in such programs.

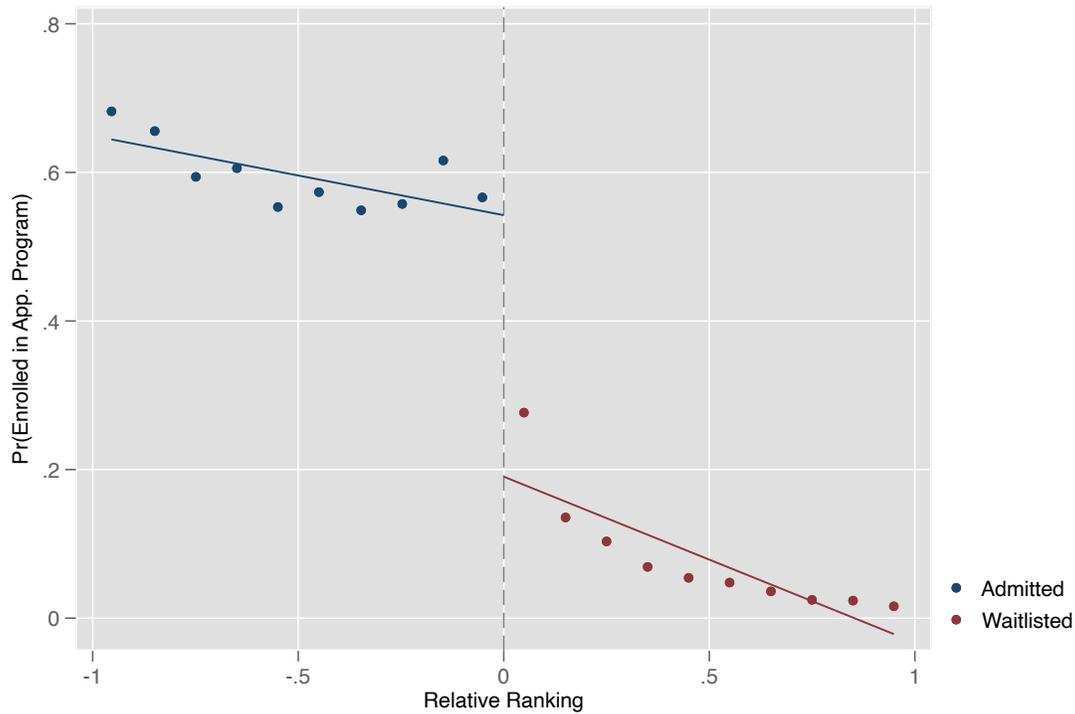
Figure 8 conducts a similar exercise, stacking all programs that enrolled some student off of a waitlist. Because programs are of different sizes, and waitlists are of different lengths, we report enrollment probabilities as a function of rank quantile within the set of admitted students (left side, [0-1]) or waitlisted students (1 denotes the final waitlisted student who ultimately enrolled). We find a similar pattern.

**Figure 7: Case Studies: Enrollment Probability at Selected Majors - 2010/2011**



Note: These figures show the probability of enrolling students who are admitted or waitlisted as a function of their rank. The x-axis shows the student rank (from 1 being the highest to the last admit). The y-axis shows the probability that students will enroll, shown in bins of 10 students.

**Figure 8:** Enrollment probability by relative ranking



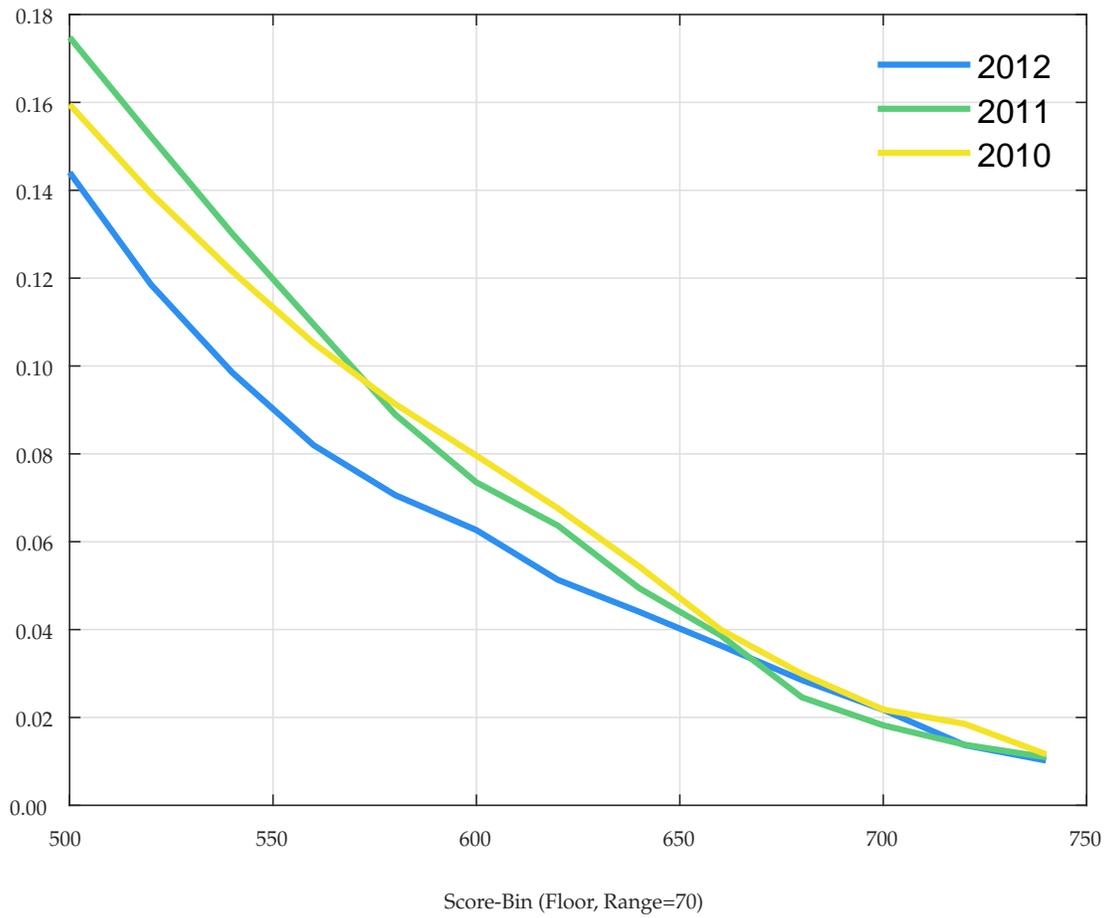
Note: This figure shows bin-scattered enrollment probabilities by *relative ranking*. For each program, admitted students were ranked in  $[-1,0)$ , where -1 represents the top-scoring admitted student, and waitlisted students were ranked in  $[0,1)$ , where 0 represents the top-scoring waitlisted student. Enrollment probabilities were pooled across programs in 2010 using equally-spaced bins. We only considered programs that enrolled over 15 students from their waitlists.

## 2 More Results on the Policy Change

### 2.1 Impacts of the Policy on First-Year Dropout

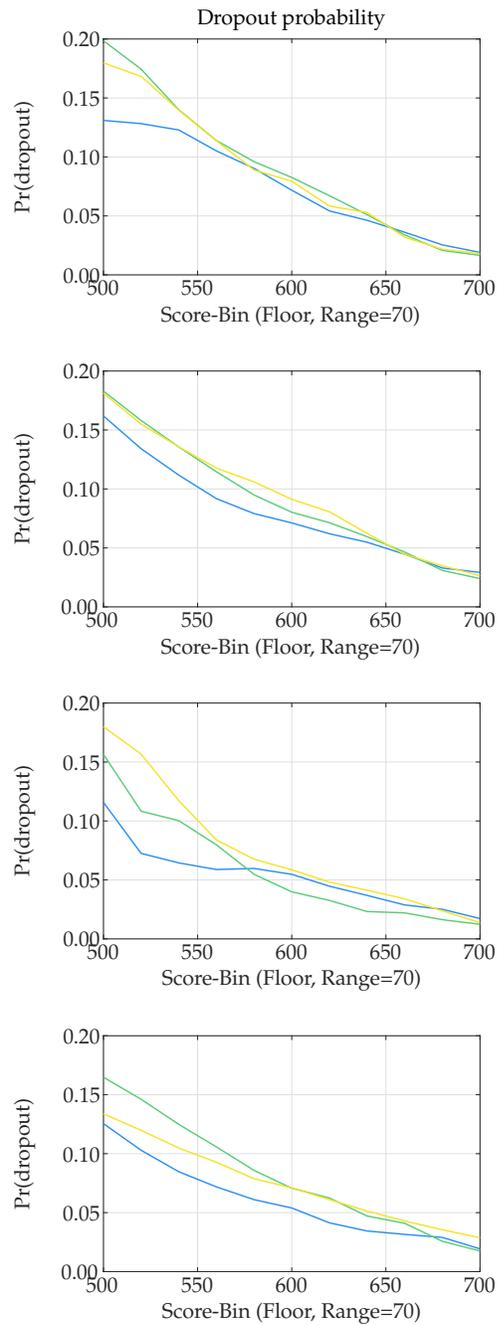
This section evaluates impacts of the policy change on first-year dropout rates among students enrolling in G25 programs. The analyses are analogous to those presented in the main body of the paper evaluating impacts on near-on-time graduation rates. We find that dropout rates fell in 2012, post-expansion, relative to 2010 and 2011. Figure 9 indicates that effects are larger for lower-scoring students. For students with scores between 500 and 550 we see a roughly 4-point reduction in dropout rates. Figure 10 indicates that this pattern is present within all observable types (In order from top to bottom: private school men; public school men; private school women; public school women).

**Figure 9:** Freshmen dropout rate, G25



Note: The figure show the probability that a student assigned to an option on the platform drops out by the end of first year. The lines show conditional means within 70 points, and the “floor” of the range is shown in the x-axis (e.g. 600 corresponds to the range [600, 670]).

**Figure 10:** Freshmen dropout rates for G25, By Gender/SES.



Note: The figures show the probability that a student enrolled in a G25 option drops out of college within one year after enrolling. The lines show conditional means within 70 points, and the “floor” of the range is shown in the x-axis (e.g. 600 corresponds to the range [600, 670]).

## 2.2 Rematching as a Descriptive Measure of Inefficiency

Applicants who renege on on-platform offers in order to enroll at off-platform options may impose externalities on other applicants. To describe how renegers affect other applicants' placements, we perform a simulation exercise. For each year, we first identify the set of matched applicants that ex-post decide to renege on their on-platform offer. Then, we (ex-ante) drop this set of ex-post-decliners and run the DA procedure with the remaining applicants to determine their new offers, in absence of renegers. Finally, we compute the fraction of the remaining applicants that improve their match each year, i.e. those that are rematched to an option ranked higher than their original placement.

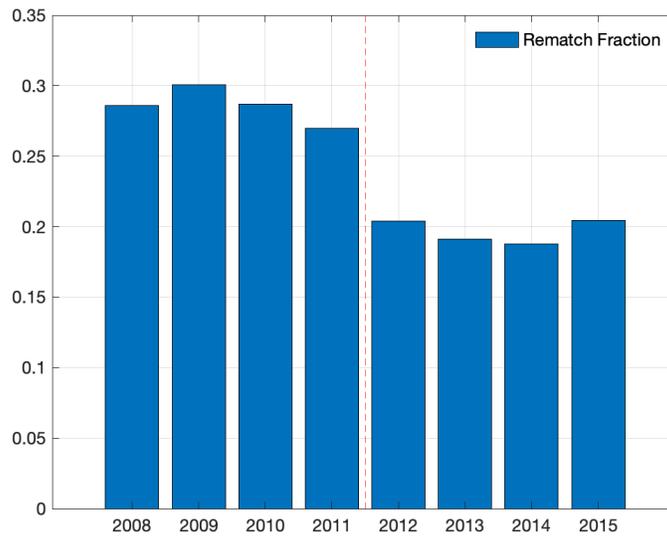
This rematch fraction, which we denote  $r$ , constitutes a measure of inefficiency in the initial match: had the renegers not participated in first place, a fraction  $r$  of the remaining applicants would have been matched to a better option. We expect the rematch fraction to increase when there are more declined offers, and when these offers are more desirable. If the declined offers are at undesired options that do not fill up, their removal should not impact other applicants.

Platform expansion reduced the set of off-platform options that potential renegers could deviate to. In the main text, we show that this led to a higher share of admitted students enrolled in their placed option. Accordingly, we should expect that fewer students would have gained from dropping all renegers post-expansion.

Figure 11 shows that the rematch fraction  $r$  sharply declines after the policy took place. Around 27% of applicants could be rematched to a better option before the policy, while post-policy levels of  $r$  fall to 20%. This 7 percentage point decline (a 26% fall) shows that the policy moved the allocation closer to an ex-post Pareto-efficient situation.

In practice, one purpose of the aftermarket is to attempt to improve students' matches by filling the chains of vacancies left by students who renege. In the paper we show that this process is subject to frictions. Moreover, we show that the marginal benefit of friction reductions becomes smaller when the G8 institutions join the platform. The reduction in the rematch fraction shows that this may be because there is less work for the aftermarket to do when more options are on-platform.

**Figure 11:** Rematch fraction when (ex-ante) dropping students who decline placements



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Note: This figure shows the fraction of match participants by year, other than those who renege on offers, whose initial assignment would change if students who decline their offers are removed from the match ex ante.

### 2.3 Additional Event Study Results

This section provides additional event study analyses. Our main specification includes linear controls for test scores. Tables 5 and 6 show estimates from type-specific event-study specifications without controls, and with flexible controls consisting of score-decile indicators, respectively. Results are similar to our main specification.

**Table 5:** Event study: Admission, Enrollment, Dropout, Graduation (without student covariates)

	Admission	Enrollment	Dropout	Graduation
Year 2010×Male×Private	-0.002 (0.006)	0.006 (0.008)	-0.003 (0.005)	0.006 (0.010)
Year 2012×Male×Private	0.095*** (0.005)	0.107*** (0.007)	-0.005 (0.005)	0.030*** (0.010)
Year 2013×Male×Private	0.104*** (0.005)	0.127*** (0.007)	0.002 (0.005)	
Year 2014×Male×Private	0.114*** (0.005)	0.125*** (0.007)	-0.003 (0.005)	
Year 2015×Male×Private	0.099*** (0.005)	0.134*** (0.007)	0.003 (0.005)	
Year 2010×Male×Public	-0.045*** (0.003)	0.014*** (0.004)	0.001 (0.003)	-0.002 (0.005)
Year 2012×Male×Public	0.082*** (0.003)	0.051*** (0.003)	-0.011*** (0.003)	0.009* (0.005)
Year 2013×Male×Public	0.084*** (0.003)	0.069*** (0.003)	-0.001 (0.003)	
Year 2014×Male×Public	0.089*** (0.002)	0.081*** (0.003)	-0.004 (0.003)	
Year 2015×Male×Public	0.058*** (0.003)	0.084*** (0.003)	-0.002 (0.003)	
Year 2010×Female×Private	-0.018** (0.007)	0.004 (0.009)	0.008 (0.005)	-0.024** (0.010)
Year 2012×Female×Private	0.090*** (0.006)	0.139*** (0.008)	0.000 (0.004)	0.022** (0.010)
Year 2013×Female×Private	0.107*** (0.006)	0.156*** (0.008)	0.006 (0.004)	
Year 2014×Female×Private	0.114*** (0.006)	0.166*** (0.008)	0.002 (0.004)	
Year 2015×Female×Private	0.096*** (0.006)	0.155*** (0.008)	0.013*** (0.005)	
Year 2010×Female×Public	-0.045*** (0.003)	0.020*** (0.004)	-0.008** (0.003)	-0.001 (0.005)
Year 2012×Female×Public	0.060*** (0.003)	0.057*** (0.004)	-0.023*** (0.003)	0.016*** (0.005)
Year 2013×Female×Public	0.072*** (0.003)	0.093*** (0.004)	-0.013*** (0.003)	
Year 2014×Female×Public	0.084*** (0.003)	0.110*** (0.004)	-0.017*** (0.003)	
Year 2015×Female×Public	0.051*** (0.003)	0.112*** (0.004)	-0.013*** (0.003)	
Constant	0.778*** (0.002)	0.691*** (0.003)	0.107*** (0.002)	0.569*** (0.003)
Observations	606280	393193	318809	163531

Note: This table shows estimates of the average difference in each outcome, for each type of student, and for each year after 2009. The base year is 2011 and the base type is Female-Public. Admission refers to the probability of being assigned a seat in the platform; Enrollment refers to the probability of enrolling in a platform program conditional on being admitted in a G25 option; Dropout refers to the probability of not being enrolled in any option the year after enrolling in a G25 program; and Graduation refers to the probability of graduating within 7 years of enrolling in a G25 program. The estimating equation does not include student covariates, except for student-type fixed effects. These estimated coefficients are not reported in the table. The results on graduation rates are constrained to years before 2013 because we do not have data after 2019. Robust standard errors in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

**Table 6:** Event study: Admission, Enrollment, Dropout, Graduation (score-decile covariates)

	Admission	Enrollment	Dropout	Graduation
Year 2010×Male×Private	-0.002 (0.006)	0.004 (0.008)	-0.003 (0.005)	0.006 (0.010)
Year 2012×Male×Private	0.117*** (0.005)	0.109*** (0.007)	-0.008* (0.005)	0.035*** (0.010)
Year 2013×Male×Private	0.125*** (0.005)	0.127*** (0.007)	-0.001 (0.005)	
Year 2014×Male×Private	0.132*** (0.005)	0.128*** (0.007)	-0.006 (0.005)	
Year 2015×Male×Private	0.119*** (0.005)	0.135*** (0.006)	0.002 (0.005)	
Year 2010×Male×Public	-0.042*** (0.003)	0.013*** (0.003)	0.001 (0.003)	-0.002 (0.004)
Year 2012×Male×Public	0.091*** (0.002)	0.058*** (0.003)	-0.017*** (0.003)	0.015*** (0.004)
Year 2013×Male×Public	0.093*** (0.002)	0.076*** (0.003)	-0.008*** (0.003)	
Year 2014×Male×Public	0.093*** (0.002)	0.087*** (0.003)	-0.010*** (0.003)	
Year 2015×Male×Public	0.062*** (0.002)	0.086*** (0.003)	-0.005* (0.003)	
Year 2010×Female×Private	-0.018*** (0.007)	0.003 (0.009)	0.009* (0.005)	-0.022** (0.010)
Year 2012×Female×Private	0.115*** (0.006)	0.141*** (0.008)	-0.001 (0.004)	0.026*** (0.010)
Year 2013×Female×Private	0.135*** (0.005)	0.163*** (0.008)	0.002 (0.004)	
Year 2014×Female×Private	0.137*** (0.005)	0.171*** (0.008)	-0.001 (0.004)	
Year 2015×Female×Private	0.126*** (0.006)	0.164*** (0.008)	0.007 (0.005)	
Year 2010×Female×Public	-0.042*** (0.003)	0.022*** (0.004)	-0.009*** (0.003)	-0.002 (0.005)
Year 2012×Female×Public	0.074*** (0.003)	0.064*** (0.004)	-0.026*** (0.003)	0.024*** (0.005)
Year 2013×Female×Public	0.087*** (0.003)	0.102*** (0.004)	-0.019*** (0.003)	
Year 2014×Female×Public	0.097*** (0.003)	0.120*** (0.004)	-0.025*** (0.003)	
Year 2015×Female×Public	0.062*** (0.003)	0.118*** (0.004)	-0.018*** (0.003)	
Constant	0.557*** (0.013)	0.464*** (0.005)	0.260*** (0.004)	0.321*** (0.007)
Observations	606280	393193	318809	163531

Note: This table shows estimates of the average difference in each outcome, for each type of student, and for each year after 2009. The base year is 2011 and the base type is Female-Public. Admission refers to the probability of being assigned a seat in the platform; Enrollment refers to the probability of enrolling in a platform program conditional on being admitted in a G25 option; Dropout refers to the probability of not being enrolled in any option the year after enrolling in a G25 program; and Graduation refers to the probability of graduating within 7 years of enrolling in a G25 program. The estimating equation includes student covariates (GPA and test score deciles) and student-type fixed effects. These estimated coefficients are not reported in the table. The results on graduation rates are constrained to years before 2013 because we do not have data after 2019. Robust standard errors in parentheses. \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

## 2.4 Additional Post-Policy Descriptive Statistics

This section provides additional descriptive statistics on the policy change. Table 7 shows that there is a high degree of persistence in programs' cutoffs over time. The year 2013 saw a change in the admissions formula to include 10% weight on a new class rank measure. Table 8 shows summary statistics for post-policy-change years. G25 and G8 enrollment probabilities, admission to the top choice, and the probability that the top choice is a G25 program are stable over time. Scores of admitted students are stable.

Finally, we ask how the characteristics of applicants that predict their propensity to enroll, drop out after one year, and graduate within seven years changed in 2012 when the platform expanded. One may ask whether increases in enrollment and graduation rates in 2012 were driven by a change in the average quality of applicants, rather than by improvements in student-program match quality or in the number of seats that get filled. The goal of this exercise is to address this question.

To do so, we first estimate applicants' propensities to enroll, to drop out, and to graduate, using pre-reform data from 2010. In particular, for each outcome  $y$ , and each conditioned sample, we estimate a Probit model of the form:

$$y_i = \beta_0 + \beta_1 \text{Male}_i + \beta_2 \text{PrivateHS}_i + \beta_3 \text{HSGPA}_i + \beta_4 \text{LangPSU}_i + \beta_5 \text{MathPSU}_i \\ + \beta_6 \text{JustGraduated}_i + \sum_{k \in \text{Region}} \beta_{7k} \text{Region}_{ik} + \varepsilon_i, \quad \varepsilon_i \sim^{iid} \mathcal{N}(0, 1).$$

We then obtain predicted values,  $\hat{y}_i$ , for all applicants in 2011 and 2012, using the estimated "2010" coefficients. Finally, we regress the predicted values  $\hat{y}$  of all 2011 and 2012 applicants on a constant and a 2012 indicator. Table 9 shows the results.

The first column considers all platform applicants. We find that the 2012 applicant pool is somewhat more positively selected on propensity to enroll in their placed options and on propensity to graduate, and has a lower propensity to drop out, although magnitudes are small. In other words, the set of students who submit any on-platform application looks slightly stronger in 2012 than in 2011, if one uses the weights that predicted success in 2010.

However, once one conditions on the event that students were admitted to some on-platform program, the opposite pattern emerges. 2012 admits are 0.7 percentage points less likely to accept offers, one percentage point more likely to drop out, and about equally likely to graduate (the point estimate is negative but small). Thus, if anything, post-reform admitted students are negatively selected on characteristics that predicted success in the pre-reform period. The same pattern emerges for dropout and graduation propensities among students who enroll.

In the paper we found that observed enrollment and graduation rates increased post-reform,

and that dropout fell. The results here suggest that these findings may be due to improved match quality rather than changes in the average observable characteristics of students who receive an admissions offer.

**Table 7:** Year-to-year regression estimates: program cutoffs

	<u>2013 Cutoff</u>	<u>2014 Cutoff</u>	<u>2015 Cutoff</u>	<u>2016 Cutoff</u>	<u>2017 Cutoff</u>
Previous Cutoff (t-1)	0.938*** (0.0109)	0.990*** (0.00687)	0.952*** (0.00651)	0.942*** (0.00684)	0.962*** (0.00641)
Constant	3448.8*** (591.8)	843.9* (394.5)	3379.4*** (380.5)	3455.6*** (395.5)	2324.8*** (377.6)
Observations	1293	1351	1331	1397	1422

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Note: This table shows the estimated  $\hat{\beta}_t$  coefficients and robust standard errors (in parentheses) obtained from regressing  $cutoff_{jt} = \alpha + \beta_t cutoff_{jt-1} + \varepsilon_j, \forall t \in \{2013, \dots, 2017\}$ .

**Table 8: Additional Descriptive Statistics on Post-Policy Periods**

	<u>Year 2012</u>		<u>Year 2013</u>		<u>Year 2014</u>		<u>Year 2015</u>	
	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.	Mean	Std.Dev.
<b>All Applicants</b>								
Top Choice is G25	0.76	0.43	0.77	0.42	0.75	0.43	0.77	0.42
Admit in Top Choice	0.48	0.50	0.50	0.50	0.53	0.50	0.49	0.50
G25 Adm. Top Choice	0.37	0.48	0.37	0.48	0.37	0.48	0.36	0.48
Enrolled in G25	0.51	0.50	0.52	0.50	0.52	0.50	0.52	0.50
Enrolled in G8	0.18	0.38	0.20	0.40	0.21	0.41	0.19	0.40
Observations	99855		100809		100602		105861	
<b>G8 Enrollees</b>								
Male	0.47	0.50	0.47	0.50	0.48	0.50	0.48	0.50
GPA	578.99	87.67	570.06	87.00	569.92	84.84	569.93	84.78
Verbal Score	596.36	72.06	588.18	77.92	585.31	80.77	584.56	77.34
Math Score	598.50	68.67	591.48	76.36	589.79	78.28	590.67	76.72
Private HS	0.43	0.50	0.39	0.49	0.38	0.49	0.39	0.49
Metro Area	0.80	0.40	0.82	0.39	0.82	0.38	0.83	0.38
Weighted Score	597.45	60.27	593.15	60.35	594.93	60.59	595.78	61.18
Observations	17934		19752		21196		20560	
<b>G25 Enrollees</b>								
Male	0.53	0.50	0.53	0.50	0.52	0.50	0.52	0.50
GPA	615.44	92.76	615.78	92.77	619.52	91.32	621.56	90.21
Verbal Score	600.51	79.56	597.80	85.57	594.73	87.19	597.19	84.63
Math Score	607.98	79.76	602.59	84.96	600.75	86.12	602.59	85.82
Private HS	0.17	0.38	0.17	0.38	0.17	0.38	0.17	0.37
Metro Area	0.58	0.49	0.57	0.50	0.57	0.50	0.56	0.50
Weighted Score	613.51	68.92	614.93	69.84	621.36	72.15	625.20	71.70
Observations	50494		52276		52358		55087	

Notes: This table shows additional post-policy descriptive statistics on the population of applicants and platform enrollees.

**Table 9: Assessing Changes In Enrollment and Outcome Propensities**

	Applicants	Admits	Enrollees
<i>Predicted Enrollment Rates</i>			
Change in 2012	0.0139*** (0.000971)	-0.00687*** (0.000644)	
2011 Baseline	0.591*** (0.000662)	0.741*** (0.000443)	
<i>Predicted Dropout Rates</i>			
Change in 2012	-0.00275** (0.000986)	0.0121*** (0.000770)	0.00478*** (0.000384)
2011 Baseline	0.286*** (0.000663)	0.208*** (0.000522)	0.0951*** (0.000267)
<i>Predicted Graduation Rates</i>			
Change in 2012	0.00436*** (0.000562)	-0.000661 (0.000607)	-0.000265 (0.000690)
2011 Baseline	0.368*** (0.000379)	0.374*** (0.000423)	0.375*** (0.000490)
Observations	163345	138802	107723

Note: This table shows the estimated  $\hat{\beta}$  coefficients and robust standard errors (in parentheses) obtained from regressing  $\hat{y}_i = \alpha + \beta 1[\text{cohort}_i = 2012] + \varepsilon_i$ , where  $\hat{y}_i$  is the predicted enrollment, dropout or graduation rate for individual  $i$ , conditional on applying through the platform, on being admitted, and on having enrolled into a program in the platform, according to coefficients obtained from pre-reform (2010) probit regressions.

### 3 Estimation Appendix

#### 3.1 Gibbs Sampler Details

##### Preliminaries

For computational convenience in estimation, we reparametrize our model as follows. We emphasize that the likelihood is unchanged under this reparameterization, and hence the reparametrized model is equivalent to our original specification.

First, without loss, we modify the location normalization by subtracting  $z_i \lambda_g^z$  from all “inside” utilities  $u_{ij}$  and all “outside” utilities  $u_{i0}^k$  for  $k = 0, 1$ . We do so because this transformation allows us to reduce the size of the matrices required to store all covariates that enter the “inside” goods’ utilities.<sup>1</sup>

<sup>1</sup>For each person  $i$  we store a  $J \times K$  matrix of match-level covariates, where  $K$  is the number of such variables. This transformation lets us avoid repeating the student-level variables  $J$  times.

For each group  $g$ , define  $\gamma_g^0 = -\lambda_g^z$ , and  $\gamma_g^1 = \gamma_g - \lambda_g^z$ . We have

$$\begin{aligned} u_{ij} &= \delta_{j,g(i)} + w_{ij}\lambda_{g(i)}^w + x_j\eta_i^x + p_{ij}\lambda_{g(i)}^p + \epsilon_{ij} \\ u_{i0} &\sim N(z_i\gamma_{g(i)}^0, \sigma_{0,0,g(i)}^2) \\ u_{i1} &\sim N(z_i\gamma_{g(i)}^1, \sigma_{0,0,g(i)}^2). \end{aligned}$$

Second, again without loss, we substitute  $u_{ij}$  into the human-capital index  $h_{ij}$  as follows: Define  $\tilde{\beta}_g = \bar{\beta}_g - \rho_g\delta_g$  and  $\tilde{\beta}_g = \beta_g - \rho_g\lambda_g$ . Whenever  $i$  enrolls in  $j$ , let  $\tilde{v}_{ij} = v_{ij} - \rho_{g(i)}(x_{j,t(i)}\eta_i^x + \epsilon_{ij})$ . We have

$$h_i = \tilde{\beta}_{g(i)} + z_i\beta_{g(i)}^z + w_{ij}\tilde{\beta}_{g(i)}^w + p_{ij}\tilde{\beta}_{g(i)}^p + u_{ij}\rho_{g(i)} + \tilde{v}_{ij},$$

where  $\tilde{v}_{ij} \sim N(0, 1)$ .

As all shocks are normal, this transformed model is equivalent to including the error term  $x_j\eta_i^x + \epsilon_{ij}$  in the human capital index, or to allowing correlation between the human-capital shock  $v_{ij}$  and the utility shock, in the sense that the likelihood is invariant to this transformation.

If person  $i$  does not enroll in any inside option, we adopt the convention  $h_{i,0} = 0$ .

## Constructing the relevant set and bound indices

Before estimation, we first restrict each person  $i$ 's rank-order list to programs in  $i$ 's relevant set,  $J_i^{\text{relevant}}$ , dropping programs outside this set. To do so, we iterate through  $i$ 's list, dropping programs that are ex-ante clearly infeasible. In the event that we reach a program  $k$  which is ex-ante clearly feasible, we keep program  $k$  but drop the remainder of the list. Let  $\ell_i$  denote the resulting restricted list.

Next, we find the indices of the programs that provide the least upper bound and greatest lower bound for each option in application data. Let  $\bar{J}_t$  be a  $(J_t \times N_t)$  matrix denoting upper bound indices. We set:

- $\bar{J}_t[j, i] = -1$ , denoting no upper bound on  $u_{ij}$ , if  $j$  was  $i$ 's first choice,  $j$  was ex-ante clearly infeasible for  $i$ , or  $j$  was off-platform in market  $t$ .
- $\bar{J}_t[j, i] = 0$ , denoting that the least upper bound on  $u_{ij}$  is given by  $u_{i0}^0$ , if  $j$  was on-platform, not ex-ante clearly infeasible, not ranked by  $i$ , and  $J_i^{\text{relevant}}$  does not contain a program that is clearly feasible for  $i$ .
- $\bar{J}_t[j, i] = k > 0$ , denoting that the least upper bound on  $u_{ij}$  is given by  $u_{ik}$ , if  $j$  is ranked in  $r$ th place for  $r \geq 2$  and  $k$  is ranked in  $r - 1$ th place within  $J_i^{\text{relevant}}$ , or if  $j$  was on-platform, not ex-ante clearly infeasible, not ranked by  $i$ , and  $J_i^{\text{relevant}}$  contains an ex-ante clearly-feasible program  $k$ .

Analogously, we construct a matrix of lower-bound indices,  $\underline{J}_t$ , as follows. We set:

- $\underline{J}_t[j, i] = -2$ , denoting that the greatest lower bound on  $u_{ij}$  is given by

$$\max\{u_{i0}^0, \max\{u_{ik} : k \text{ unranked, on-platform, not ex-ante clearly infeasible}\}\},$$

in the event  $j$  is the final program in  $J_i^{\text{relevant}}$ , and either the list is of full length or  $j$  is ex-ante clearly feasible.<sup>2</sup>

- $\underline{J}_t[j, i] = -1$ , denoting no lower bound (i.e. the greatest lower bound on  $u_{ij}$  is  $-\infty$ ), if  $j \notin J_i^{\text{relevant}}$ . This case occurs when  $j$  is available but not listed, as well as when  $j$  is off-platform or ex-ante clearly infeasible.
- $\underline{J}_t[j, i] = 0$ , denoting the greatest lower bound is  $u_{ij} < u_{i0}^0$ , when  $j$  is the final element of  $J_i^{\text{relevant}}$ , the list is not full, and  $j$  is not ex-ante clearly feasible.
- $\underline{J}_t[j, i] = k > 0$ , denoting the greatest lower bound on  $u_{ij}$  is given by  $u_{ik}$ , if  $j$  is ranked in  $r$ th place and  $k$  is ranked in  $r + 1$ th within  $J_i^{\text{relevant}}$ .

Similarly, we construct vectors  $\overline{O}$  and  $\underline{O}$ , of length  $N_t$ , giving the indices of the programs providing bounds on  $u_{i0}^0$ . If  $i$ 's rank-order list is empty we have  $\overline{O}[i] = -1$ , denoting no upper bound, otherwise  $\overline{O}[i] = k$  where  $k$  is the final element of  $i$ 's list. If  $i$ 's list is not of full length and does not contain an ex-ante clearly feasible program, we have  $\underline{O}[i] = -2$ , denoting that the greatest lower bound on  $u_{i0}^0$  is given by  $\max\{u_{ik} : k \text{ unranked, on-platform, not ex-ante clearly infeasible}\}$ , otherwise we have  $\underline{O}[i] = -1$ , indicating that there is no lower bound.

Next, we construct upper and lower bounds,  $\overline{A}$  and  $\underline{A}$  respectively, on aftermarket availability. If  $i$  is assigned to  $j$  in the match, or enrolls in  $j$ , we have  $\overline{A}[j, i] = \underline{A}[j, i] = 1$ , indicating that the program is always available. If  $i$  enrolls in on-platform program  $k$ , and ranked  $j$  ahead of  $k$  on her rank-order list, then  $j$  must not have offered  $i$  a position, and hence we have  $\overline{A}[j, i] = \underline{A}[j, i] = 0$ . In the remaining cases, we have  $\overline{A}[j, i] = 1$  and  $\underline{A}[j, i] = 0$ .

Finally, let

$$J_i^{\text{uncertain}} = \{j \in J_{t(i)} : j \text{ ex-post aftermarket-feasible for } i\} \setminus (\{0\} \cup \{\text{placement}_i\})$$

denote the set of programs which are available to  $i$  if and only if there is a positive realization of  $a_{ij}^*$ . We use this set to determine which  $(i, j)$  cells are relevant for the steps in which we draw  $a_{ij}$  and  $\alpha$ .

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<sup>2</sup>We know that the final listed program is preferred to the outside option and to everything that was not listed. The subtlety is that in this case we do not know how the outside option  $u_{i0}^0$  compares to the "inside" options that could have been listed but were not.

## Starting values

We find feasible starting values for utility and availability as follows. If  $j$  is on-platform and is ranked in the  $n_j$ th position on  $i$ 's relevant rank-order list,  $\ell_i$ , then  $u_{ij} = 1/n_j$ . If  $j$  is on-platform but not ranked, then  $u_{ij} = -1$ . If  $j$  is off-platform and  $i$  does not enroll in it, then  $u_{ij} \sim \text{Uniform}[-1e^{-4}, 1e^{-4}]$ . If  $j$  is off-platform and  $i$  enrolls in it, then  $u_{ij} = 2$ .

At all programs for which  $a_{ij} = 1$  is possible given these utilities, we set  $a_{ij}^* = 1.0$ . Otherwise, we set  $a_{ij}^* = -1.0$ . We set  $u_{i0}^0 = -0.001$  unless there is a clearly-feasible school on  $i$ 's rank-order list, in which case we have  $u_{i0}^0 = .001$ . We set  $u_{i0}^1 = 0.002$  for all students.

These starting values ensure that  $i$  enrolls in his most-preferred program among those that he could have attended, and that the rank-order list is truthful after restricting to the relevant set.

We start with  $h_{i, \text{enroll}_i} = 1$  if  $i$  graduated, and  $h_{i, \text{enroll}_i} = -.5$  if  $i$  did not. We draw initial random coefficients  $\eta_i^x \sim N(0, I)$ , i.i.d. across people.

## Estimation procedure

Our procedure iterates the following draws from conditional posterior distributions.

1. For each market  $t$ , for each type  $g \in G$ , for each  $i \in \{1, \dots, N_t\}$  of type  $g$ :
  - (a) Draw  $u_{i0}^0 | u_i, u_{i0}^1, \ell_i, \text{enroll}_i, \gamma_{g(i)}^0, \sigma_{0,0,g(i)}^2$ .
  - (b) Draw  $u_{i0}^1 | u_i, u_{i0}^0, \ell_i, \text{enroll}_i, \gamma_{g(i)}^1, \sigma_{0,1,g(i)}^2$ .
  - (c) for each  $j \in \{1, \dots, J_t\}$ 
    - If  $j \neq \text{enroll}_i$ :
      - i. Draw  $u_{ij} | a_i, u_{i0}^0, u_{i0}^1, u_{i,-j}, \ell_i, \text{enroll}_i, \eta_i^x, \delta_{j,g(i)}, \lambda_{g(i)}$ .
      - ii. Draw  $a_{ij}^* | u_i, u_{i0}^0, u_{i0}^1, \ell_i, \text{enroll}_i, \alpha_{g(i)}$ .
    - Else:
      - i. Draw  $u_{ij} | a_i, u_{i0}^0, u_{i0}^1, u_{i,-j}, \ell_i, \text{enroll}_i, \eta_i^x, \delta_{j,g(i)}, \lambda_{g(i)}, h_{ij}, \tilde{\beta}_{g(i)}, \tilde{\beta}_{g(i)}^{w,p}, \beta_{g(i)}^z$ .
      - ii. Draw  $h_{ij} | \text{graduate}_{ij}, u_{ij}, \tilde{\beta}_{g(i)}, \tilde{\beta}_{g(i)}^{w,p}, \beta_{g(i)}^z$ .
      - iii. Draw  $a_{ij}^* | u_i, u_{i0}^0, u_{i0}^1, \ell_i, \text{enroll}_i, \alpha$ .
  - (d) Draw  $\eta_i^x | u_i, \Sigma_{g(i)}^{rc}$
2. for each type  $g \in G$ :
  - (a) Draw  $\sigma_{0,0,g}^2 | \{u_{i0}^0 : g(i) = g\}, \gamma_g^0$ .
  - (b) Draw  $\sigma_{0,1,g}^2 | \{u_{i0}^1 : g(i) = g\}, \gamma_g^1$ .
  - (c) Draw  $\gamma_g^0 | \{u_{i0}^0 : g(i) = g\}, \sigma_{0,0,g}^2$ .

- (d) Draw  $\gamma_g^1 | \{u_{i0}^1 : g(i) = g\}, \sigma_{0,1,g}^2$ .
- (e) Draw  $(\delta_g, \lambda_g) | \{(u_i, \eta_i^x) : g(i) = g\}$ .
- (f) Draw  $\Sigma^{rc} | \{\eta_i^x : g(i) = g\}$
- (g) Draw  $\alpha | \{a_{ij}^* | g(i) = g\}$ .
- (h) Draw  $(\tilde{\beta}_g, \tilde{\beta}_g^{w,p}, \beta_g^z, \rho_g) | \{h_{i,\text{enroll}_i}, u_i : g(i) = g\}$ .

## Priors

We use standard conjugate priors. We choose the prior parameters to be uninformative. Program effects and other linear-index parameters  $(\delta, \lambda, \eta, \gamma^1, \gamma^0, \tilde{\beta}, \tilde{\beta}^{w,p}, \beta^z, \alpha)$  have independent  $Normal(0, 10 * I)$  priors, where  $I$  is the identity matrix. Scalar variances have  $InverseGamma(10, 10)$  priors. Variance-covariance matrices of size  $(k, k)$  have  $InverseWishart(k + 1, 10 * I)$  priors.

## Counterfactuals, Model Fit, and Convergence

We conduct 7500 iterations. We throw out the first 2500 as burn-in. After dropping the initial burn-in draws, we simulate all counterfactuals and model-fit exercises at every 200th iteration. To simulate the counterfactuals discussed in Section 5.3 of the paper, we use the latent utility values  $(u, u_0)$  associated with the relevant MCMC iteration, but draw aftermarket frictions  $a^*$  and human-capital shocks  $h$  freshly from their distribution conditional on  $\alpha$  and human-capital parameters, respectively.

For the model fit exercises in Section 5.2 of the paper, in contrast, we do not condition on observed choices or on estimated latent availability, human capital, or utility draws consistent with those choices. Rather, we draw utilities, human-capital shocks, and availability shocks from their distributions conditional on parameters.<sup>3</sup> We conduct two draws of latent variables at every 200th iteration along the Markov chain and compute model fit under each draw. We then report averages over parameters and latent utility/availability/human-capital draws.

To obtain starting values for parameters for our main chain of 7500 iterations, we first conduct 20000 iterations on a random 15% subsample. Taking the final parameter draw from this procedure, we construct starting values of latent variable for the full sample, then iterate the Gibbs steps that update the latent variables 2500 times, holding the parameters fixed. Once we've obtained starting values in this way, we begin the 7500 iterations described above.

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<sup>3</sup>Re-simulating all latent variables is important for assessing fit. If we were to use the utility values that were drawn in the Markov chain, which are by construction consistent with optimality of the applicant's observed behavior, we would exactly replicate the initial match, and the characteristics of students matched to each program in simulations would be identical to those in the data.

To check convergence, see [subsection 4.4](#) where we provide trace plots for different parameters and specifications.

## Details on updating latent variables

Conditional on other variables and parameters, the terms  $a_{ij}^*$ ,  $u_{i0}$ ,  $u_{ij}$ , and  $h_{i,\text{enroll}_i}$  are drawn from truncated normal distributions. Truncation bounds on  $u_{ij}$  and  $u_{i0}$  come from two sources: optimality of the submitted application and optimality of the enrollment decision. Bounds on  $h$  come from the observed graduation outcome. Constraints on  $a$  come from  $i$ 's choices and utilities, as we describe below. Our approach here builds on [McCulloch and Rossi \(1994\)](#) to allow for partial rank-order data as well as the constraints implied by the enrollment decision when there is a latent “availability set”.

When updating  $u_{i0}^0$ , the construction of bounds is as follows. The first outside option  $u_{i0}^0$  is bounded below by the maximum over the utilities of all programs that are not clearly infeasible and are not listed, and by  $\max\{u_{ik} : a_{ik} = 1\}$  whenever  $\text{enroll}_i = 0$  and  $u_{i0}^1 < \max\{u_{ik} : a_{ik} = 1\}$ .  $u_{i0}^0$  is bounded above by the utility of the final listed program, if any, and by  $u_{i,\text{enroll}_i}$  if  $\text{enroll}_i \neq 0$ . We draw  $u_{i0}^0$  from a  $\text{Normal}(z_i \gamma_{g(i)}^0, \sigma_{0,0,g(i)}^2)$  distribution truncated at the min of the upper bounds and the max of the lower bounds.

The second outside option,  $u_{i0}^1$ , is bounded below by  $\max\{u_{ik} : a_{ik} = 1\}$  whenever we have  $\text{enroll}_i = 0$  and  $u_{i0}^0 < \max\{u_{ik} : a_{ik} = 1\}$ , and is bounded above by  $u_{i,\text{enroll}_i}$  if  $\text{enroll}_i \neq 0$ . We draw  $u_{i0}^1$  from a  $\text{Normal}(z_i \gamma_{g(i)}^1, \sigma_{0,1,g(i)}^2)$  distribution truncated at these bounds.

Applications are optimal if and only if the following constraints on  $u_{ij}$  are satisfied: If  $j \in J_i^{\text{relevant}}$  was ranked  $m$ th on  $\ell_i$ , then  $u_{ij}$  is bounded above by the utility of the  $m - 1$ th program whenever  $m > 1$ , and bounded below by the utility of the  $m + 1$ th option if one exists. If  $j$  is the final option in  $\ell_i$ , then  $u_{ij}$  is bounded below by  $u_{i0}$  if  $\ell_i$  is not of full length and  $j$  is ex-ante marginal, and otherwise by the max of  $u_{i0}^0$  and highest-utility program in  $J_i^{\text{relevant}}$  that was not listed. If  $j$  was on-platform, not ex-ante clearly infeasible, and not listed, its utility is bounded above by  $u_{i0}^0$  if  $\ell_i$  does not contain an ex ante strictly feasible program, and by the final listed program otherwise.

These constraints are captured by the matrices  $\bar{J}$  and  $\underline{J}$ .

The decision to enroll in  $j$  implies that it is preferred to all other programs  $k$  for which  $a_{ik} = 1$ , providing additional constraints. If  $j = \text{enroll}_i$  then

$$u_{ij} > \max\{u_{ik} : k \geq 0, k \neq j, k \text{ ex-post feasible for } i, a_{ik} = 1\}.$$

If  $j \neq \text{enroll}_i$  then whenever  $a_{ij} = 1$  we must have  $u_{ij} < u_{i,\text{enroll}_i}$ . We adopt the convention  $a_{i0} = 1$ .

The lower bound on  $u_{ij}$  is the maximum of the lower bound from the enrollment decision (if

any) and the lower bound from applications (if any). The upper bound is analogous.

let

$$\mu_{ij}^u \equiv \delta_{j,g(i)} + w_{ij}\lambda_{g(i)}^w + x_{j,t(i)}\eta_i^x + p_{ij}\lambda_{g(i)}^p.$$

If  $j > 0$  is not the program in which  $i$  enrolls, then  $u_{ij}$  is drawn from a truncated Normal( $\mu_{ij}^u, 1$ ) distribution with truncation bounds given above.

To update  $u_{i,\text{enroll}_i}$ , we must condition on  $h_{i,\text{enroll}_i}$ , which affects the mean and variance of the truncated normal as follows. Fixing  $j = \text{enroll}_i$ , let

$$\mu_{ij}^h \equiv h_{ij} - u_{ij}\rho_{g(i)} = \tilde{\beta}_{j,g(i)} + z_i\beta_{g(i)}^z + w_{ij}\tilde{\beta}_{g(i)}^w + p_{ij}\tilde{\beta}_{g(i)}^p + \tilde{v}_{ij}$$

denote the portion of  $h_{ij}$  that is independent of utility shocks, conditional on observables. The likelihood of  $u_{ij}|h_{ij}$  is proportional to  $\phi(u_{ij} - \mu_{ij}^u)\phi(h_{ij} - \mu_{ij}^h)$ . With some algebra one can show:

$$u_{ij}|h_{ij}, \eta_i^x, \delta, \lambda \sim N\left((\mu_{ij}^u + \rho_{g(i)}(h_{ij} - \mu_{ij}^h))\tilde{\sigma}^2, \tilde{\sigma}^2\right),$$

where  $\tilde{\sigma}^2 = \frac{1}{\rho_{g(i)} + 1}$ .

Conditional on enrolling in  $j > 0$  and on  $u_{ij}$ , human capital  $h_{ij}$  is distributed according to a Normal( $\mu_{ij}^h + u_{ij}\rho_{g(i)}, 1$ ) distribution truncated from above at zero if  $i$  does not graduate and from below at zero if  $i$  graduates. An advantage of our “tilde” reparameterization is that  $u_{ij}$  may be treated as an ordinary covariate at this step.

Some elements of  $a_{ij}$  are observed. If  $i$  enrolls in program  $j$  or was placed in  $j$  then  $a_{ij}^* > 0$ . If  $i$  enrolls in his original match or in a waitlist offer, then  $a_{ik} = 0$  for all waitlisted programs  $k$  that  $i$  ranks above where he enrolls. When  $a_{ij}$  is not observed, and is potentially relevant because  $j$  is off-platform or  $i$  is waitlisted at  $j$ , we have  $a_{ij} = 0$  whenever  $u_{ij} > u_{i,\text{enroll}_i}$ , and  $a_{ij} \sim \text{Bernoulli}(1 - \alpha)$  otherwise.

We update  $a_{ij}$  only for  $(i, j)$  cells for which  $j \in J_i^{\text{uncertain}}$ , and use only these values of  $(i, j)$  when updating the availability parameters  $\alpha$ .

## “Special” Admissions

There are a handful of cases in the data in which people enroll in on-platform options that were infeasible, that they did not apply to via the platform, or that they ranked below their placed option. These cases receive special treatment as follows.

Let program  $j$  denote the program in which person  $i$  enrolls, and suppose that  $j$  is on platform but is not part of the relevant set (e.g. because  $i$  did not apply to it, or because it was ex-ante clearly infeasible), or is relevant but ranked below  $i$ 's placed program. We assume that the constraints

implied by optimal enrollment decisions continue to apply, and  $j$  is assumed to be available (i.e. to have  $a_{ij} = 1$ ) and maximize utility among available options. However, the application decision does not impose any constraints involving  $j$ . In the event that  $j$  was part of the relevant set, it is removed from this set. In addition, because selection into enrollment may differ for these programs, the student’s outcome at this program is not used when updating graduation/human capital parameters, nor do we use this person-program pair to update friction parameters.

A leading example is the case in which person  $i$  enrolls in some program  $j$  that was ex-ante clearly infeasible. In most cases, this event occurs because the person applied to the program, and was admitted off of the waitlist, but the program’s cutoff was outside of the person’s bandwidth. In a handful of additional cases, people match to programs via alternative channels such as the BEA program, which reserves some seats for students with high class rank.<sup>4</sup> We emphasize that our assumptions here are weaker than if we had treated every ex-post waitlist-feasible program as ex-ante marginal, in the sense that we impose fewer constraints on utilities.<sup>5</sup>

### Details on updating other parameters

The remaining steps are standard. We provide details here for completeness. Similar derivations are available for linear parameters and covariance matrices in the online appendix to [Agarwal and Somaini \(2018\)](#).

**Posterior distribution of regression coefficients:** We first state a standard result which we use below. Suppose we have a multilinear regression model of the form

$$v_i = X_i b + e_i,$$

where  $v \in \mathbb{R}^k$  for  $i = 1, \dots, N$  is a vector of “outcomes”,  $x_i$  is a matrix of “regressors” of dimension  $(J, K)$ , and the error term is distributed as  $e_i \sim \text{MvNormal}(0, \Sigma)$  for some positive-definite matrix  $\Sigma$ , iid across  $i$ . Let us place a  $\text{MvNormal}(\bar{b}, A^{-1})$  prior on  $b$ , for some positive-definite  $k$ -by- $k$  matrix  $A$  and  $k$ -vector  $\bar{b}$ . Then the posterior distribution of “regression coefficients”  $b$  is given by:<sup>6</sup>

$$(b | \{v_i, X_i\}_{i=1, \dots, N}) \sim \text{MvNormal}(\tilde{b}, V),$$

---

<sup>4</sup>In a very small number of cases, people enroll in programs that were part of the relevant set but that they dispreferred to their placed program. It is impossible to enroll via the standard process in an option that was ranked below the placed program, but it may be possible for people to do so if they have received a BEA offer, or other offer outside of the match at the dispreferred program, in addition to their “standard” on-platform offer.

<sup>5</sup>We have estimated a version of the model in which all ex-post waitlist-feasible programs that would have been ex-ante clearly infeasible are coded as ex-ante marginal for the relevant applicants. This version makes use of more waitlist data because “long-shot” admissions offers off of waitlists are not treated as “special admissions”. Results are quantitatively very similar.

<sup>6</sup>This result is standard. e.g. see [Gelman et al. \(2013\)](#).

where

$$\begin{aligned}
V &= (X^* X^{*'} + A)^{-1} \\
\tilde{b} &= V(X^* v^* + A\bar{b}) \\
X^* &= \begin{bmatrix} X_1^* \\ \vdots \\ X_N^* \end{bmatrix} \\
X_i^* &= CX_i \\
v_i^* &= Cv_i,
\end{aligned}$$

and  $C$  is the upper-triangular Cholesky factor from the Cholesky decomposition of  $\Sigma$ .

We now give details about specific steps:

- For each  $i$ , we draw  $\eta_i^x$  using the “regression coefficients” formula above in which the coefficients are  $b = \eta_i^x$ , the “outcomes” are  $v_i = u_i - (\delta_{j,g(i)} + z_i \lambda_{g(i)}^z + w_{ij} \lambda_{g(i)}^w + p_{ij} \lambda_{g(i)}^p)$ , the regressors are  $X_{j,t(i)}$ , the covariance of utility shocks is the identity matrix and hence  $C' = I$ , and the prior is  $\eta_i^x \sim N(0, \Sigma^{g(i)})$
- For each type  $g \in G$ :
  - Draw  $\sigma_{0,0,g}^2 | \{u_{i0}^0 : g(i) = g\}$ : When the prior is  $\sigma_{0,0,g}^2 \sim \text{InverseGamma}(\alpha, \beta)$ , the posterior is distributed as  $\text{InverseGamma}(\alpha + \sum_t N_{gt}, \beta + \sum_t (u_{i0} - z_i \gamma^0)^2)$  where  $N_{gt}$  denotes the number of people in cohort  $t$  who are of type  $g$ .
  - Draw  $\sigma_{0,1,g}^2 | \{u_{i0}^0 : g(i) = g\}$ : When the prior is  $\sigma_{0,1,g}^2 \sim \text{InverseGamma}(\alpha, \beta)$ , the posterior is distributed as  $\text{InverseGamma}(\alpha + \sum_t N_{gt}, \beta + \sum_t (u_{i0} - z_i \gamma^1)^2)$ .
  - Draw  $\gamma_g^0 | \{u_{i0}^0 : g(i) = g\}, \sigma_{0,0,g}^2$ : use the “regression coefficients” formula above, with “outcomes”  $u_{i0}^0$ , “regressors”  $z_i$ , and error terms distributed according to a  $N(0, \sigma_{0,0,g_i}^2)$ .
  - Draw  $\gamma_g^1 | \{u_{i0}^1 : g(i) = g\}, \sigma_{0,1,g}^2$ : use the “regression coefficients” formula above, with “outcomes”  $u_{i0}^1$ , “regressors”  $z_i$ , and error terms distributed according to a  $N(0, \sigma_{0,1,g_i}^2)$ .
  - Draw  $(\delta_g, \lambda_g) | \{(u_i, \eta_i^x) : g(i) = g\}$ : use the “regression coefficients” formula above, with outcomes  $(u_i - x_{j,t(i)} \eta_i^x)$ , regressors  $(I, z_i, w_{ij}, p_{ij})$ , and  $\Sigma = I$ .
  - Draw  $\Sigma^g | \{\eta_i^x : g(i) = g\}$ : when the prior distribution is  $\Sigma^g \sim IW(v_0, S)$ , the posterior is
$$\Sigma^g | \{\eta_i^x : g(i) = g\} \sim IW \left( v_0 + \sum_t N_{gt}, S + \sum_t \sum_{i=1}^{N_{gt}} \eta_i^x \eta_i^{x'} \right).$$
  - Draw  $\alpha_g | \{a_{ij}^* | g(i) = g\}$ : Restrict attention to  $(i, j)$  cells for which  $j \in J_i^{\text{uncertain}}$ . Use the “regression coefficients” formula above, with iid standard normal error terms.

- Draw  $(\tilde{\beta}_g, \tilde{\beta}_g, \rho_g) | \{h_{i,\text{enroll}_i}, u_i, \text{graduate}_i : g(i) = g\}$ : use the “regression coefficients” formula above, restricting to people who enrolled in some  $j > 0$ , with outcomes  $h_{i,\text{enroll}_i}$ , regressors  $(I, z_i, w_{i,\text{enroll}_i}, p_{i,\text{enroll}_i}, u_{i,\text{enroll}_i})$ , and  $\Sigma = I$ .

### 3.2 Notation table

Notation Table - Model

Indices	
$i$	Student
$j$	Program
$t$	Application cohort
$g$	Observable student group; interaction of student gender and type of high school attended
$N_t$	Set of students in year $t$
G8, G25	Denotes whether program belongs to G8 or G25 university
$J_t^{\text{on}}, J_t^{\text{off}}$	On-platform options in year $t$ , Off-platform options in year $t$
$J_t = J_t^{\text{on}} \cap J_t^{\text{off}}$	Set of all inside options in year $t$
Observables	
$w_{i,j}$	Observed student-program match-level observables
$x_{j,t}$	Program-year characteristics
$z_i$	Vector of student-level observables
$p_{i,j}$	Price that student $i$ would pay to attend program $j$ after government and institutional aid
$v_{i,j}$	Observed student-level and student-program match-level variables that affect aftermarket availability
$\text{placement}_i$	Program $j$ that student $i$ was assigned in the match, 0 if none
$\text{enroll}_i$	Program $j$ in which student $i$ enrolled, 0 if student $i$ did not enroll
$\text{graduate}_i$	graduation outcome of student $i$
$\text{index}_{i,j}$	Application score of eligible student $i$ at program $j$ ; weighted sum of exam scores; 0 if ineligible
$\pi_{j,t}$	Minimum value of $\text{index}_{i,j}$ among students placed in program $j$ during the initial match
$\underline{\pi}_{j,t}, \bar{\pi}_{j,t}$	Cutoff values that determine whether program $j$ is ex-ante infeasible, marginal, or clearly feasible
$J_i^{\text{relevant}}$	On-platform programs that are relevant for $i$ : not ex-ante infeasible or dispreferred to the best clearly-feasible option
$\ell_i$	(Restricted) submitted rank-order list; consists of all elements of $J_i^{\text{relevant}}$ ranked according to preferences
$J_i^{\text{uncertain}}$	Programs that are ex-post aftermarket-feasible for $i$ but not ex-post match-feasible; admit $i$ if and only if $a_{ij}^* > 0$
Shocks and Latent Variables <sup>7</sup>	
$\epsilon_{i,j}, \aleph_{i,j}, \nu_{i,j}$	iid match-level shocks to preferences, aftermarket availability, human capital

<sup>7</sup>Bolded variables are stored, updated, and tracked during estimation. Non-bolded shocks and indicators are constructed as needed from the values of the parameters and tracked latent variables.

$\eta_i^x$	Unobserved random tastes for program-year characteristics
$u_{ij}$	Utility that student $i$ derives from attending program $j$
$u_{i0}$	Utility that student $i$ derives from their outside option. Max of $u_{i0}^0$ and $u_{i0}^1$
$u_{i0}^0$	Value of student $i$ 's first outside option; known to $i$ before applications are due
$u_{i0}^1$	Value of student $i$ 's second outside option; learned during the aftermarket
$a_{ij}^*$	Latent variable determining whether $j$ is able to successfully contact $i$ in the aftermarket
$a_{ij}$	Indicator for whether program $j$ is available to student $i$
$h_{ij}$	Human capital index determining whether student $i$ graduates from program $j$

---

**Parameters**

$\alpha_g$	Aftermarket friction parameters
$\bar{\beta}_{j,g}, \beta_g^z, \beta_g^w, \beta_g^p$	Coefficients of human capital function
$\delta_{j,g}$	Mean utility of program $j$ for students in group $g$
$\lambda_g^w$	Group specific coefficients for match-level term $w_{ij}$
$\lambda_g^z$	Group specific coefficients on student-level terms $z_i$ , which shift the value of all inside options relative to the value of attending a non-G33 program
$\lambda_g^p$	Group specific coefficient for price $p_{i,j}$
$\gamma_g^z$	Group specific coefficients on student-level terms $z_i$ that enter the second outside option component.
$\rho_g$	Group specific correlation between utility shock and human-capital shock
$\Sigma_g$	Group specific covariance matrix of random coefficients

---

### 3.3 Choosing a Bandwidth

This section describes the tradeoffs involved in our choice of bandwidth in constructing the set of *relevant* applications for each student. In practice we choose a bandwidth

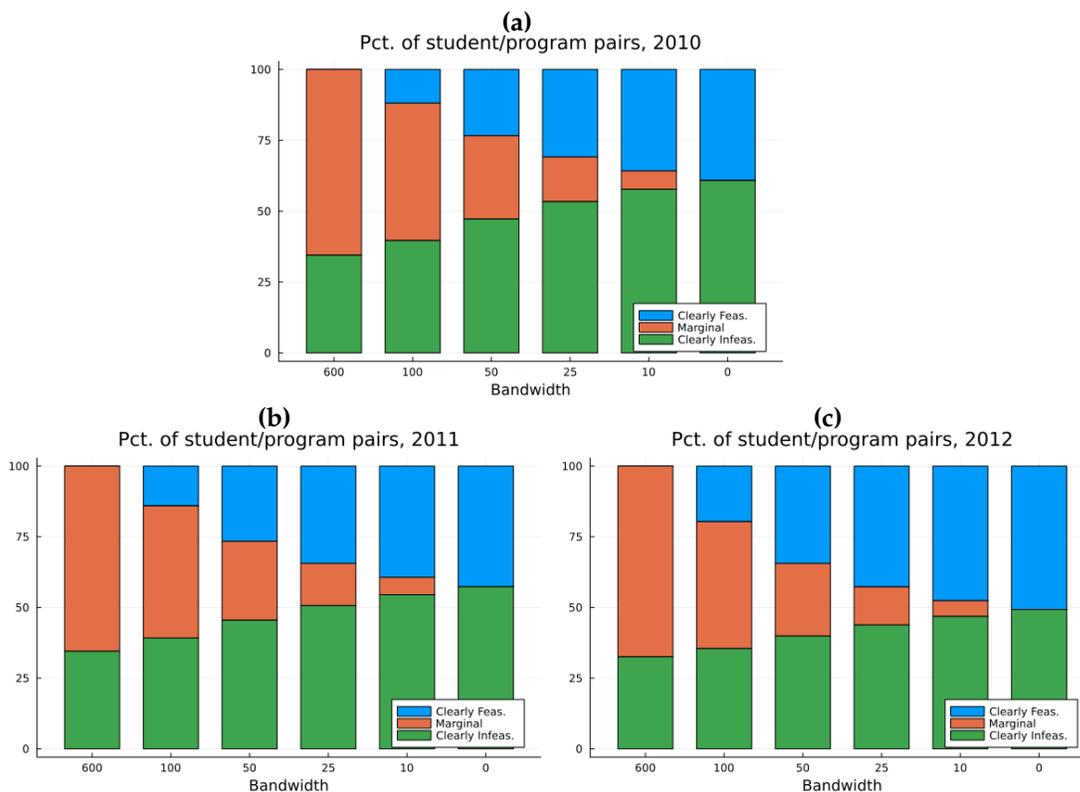
$$\bar{\pi}_{jt} - \pi_{jt} = \pi_{jt} - \underline{\pi}_{jt} = 25$$

for all  $j, t$  (recall that the standard deviation of student test scores is 110).

In this section, we consider bandwidth specifications of 600, 100, 50, 25, 10, and 0 points. At a bandwidth of 600 points, Every program at which  $i$  is eligible to apply is ex-ante marginal, and hence only programs in which the student violates an eligibility rule are clearly infeasible. As bandwidths shrink, we approach the case in which the relevant set contains at most one program: namely, the most-preferred ex-post feasible program, if there is one that is preferred to the outside option.

We first show how the share of ex-ante strictly-feasible, strictly infeasible, and marginal programs varies with bandwidth. Figure 12 shows that as the bandwidth shrinks, fewer program-student pairs are classified as ex-ante marginal. Roughly 40% of program-student pairs are impossible because the students do not meet program-specific eligibility rules; these pairs are always ex-ante strictly infeasible, even in the case of bandwidth=600. As the bandwidth approaches zero, every program becomes either strictly infeasible or strictly feasible.

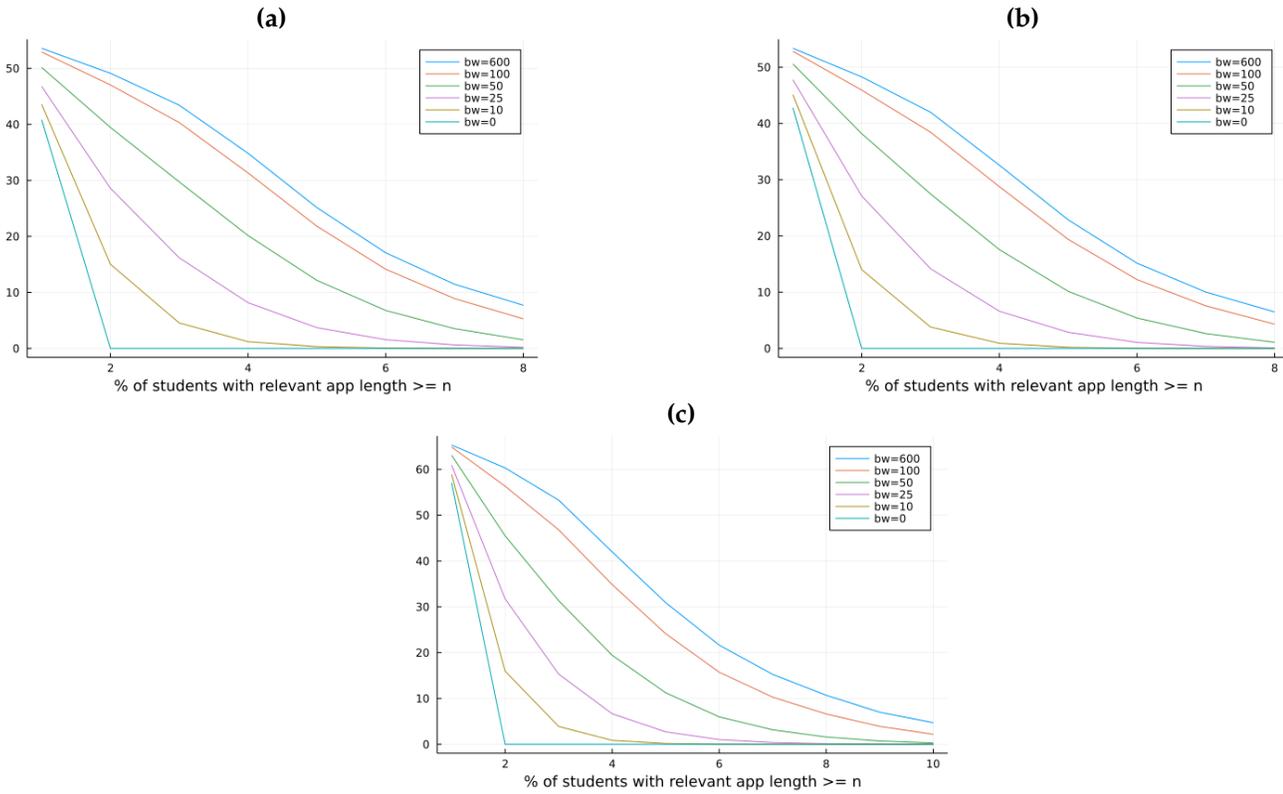
**Figure 12:** Ex-ante classification of (program,student) pairs



Note: the bars show the share of student-program pairs which are ex-ante clearly feasible, ex-ante marginal, and ex-ante clearly infeasible as a function of year and bandwidth choice.

Next, consistent with the shrinking share of programs that are ex-ante marginal, we show that the length of relevant applications declines as the bandwidth shrinks. Figure 13 shows inverse-CDFs of relevant application length by year and bandwidth. An important observation is that, for bandwidths less than 50, the list length constraint very rarely binds.

**Figure 13: Length of relevant application lists**



Note: lines show the probability that the application contains at least  $n$  programs, by year and bandwidth.

Finally, table 11 describes the share of students for whom list length constraints might possibly bind after dropping irrelevant applications. There are two relevant measures here. We say that “list length may bind” for estimation if the restricted application is of the maximum length and does not contain an ex-ante clearly feasible program. In this case, it is possible that a student who reports elements of his relevant set in order of true preferences could run out of space to rank all options. Second, we say an application is “possibly unstable” if the restricted application is of maximum length and does not contain an ex-post match-feasible program. Because every ex-ante clearly feasible program is ex-post match-feasible, this set is a subset of the set of applications for which list length may bind. A person whose application is not “possibly unstable” is necessarily matched to his most-preferred match-feasible program, if any, and hence does not have justified envy. Therefore, the share of “possibly unstable” applications places an upper bound on the amount of justified envy. We find that this share is negligible under a 25-point bandwidth specification. The table shows that there is at most one person, who applied in 2010, who could have justified envy.

**Table 11: When do Constraints on List Length Bind**

Year	Bandwidth	% List Length Binds	% Possibly Unstable	N Possibly Unstable
2010	600	7.7	0.85	1387
2010	100	5.03	0.33	543
2010	50	1.28	0.03	45
2010	25	0.13	0.0	1
2010	10	0.0	0.0	0
2010	0	0.0	0.0	0
2011	600	6.48	0.51	804
2011	100	4.11	0.19	296
2011	50	0.85	0.01	16
2011	25	0.05	0.0	0
2011	10	0.0	0.0	0
2011	0	0.0	0.0	0
2012	600	4.71	0.23	366
2012	100	1.99	0.06	103
2012	50	0.21	0.0	1
2012	25	0.01	0.0	0
2012	10	0.0	0.0	0
2012	0	0.0	0.0	0

This table shows the share of students for whom list length constraints may bind after restricting to relevant applications  $J_i^{\text{relevant}}$ . Bandwidth: symmetric bandwidth around  $\pi_{jt}$ ;  $j$  is ex-ante marginal for  $i$  if  $i$  is eligible to apply to  $j$  and  $|\text{index}_{ij} - \pi_{j,t(i)}| < \text{Bandwidth}$ . “% List Length Binds”: percentage of all students with  $\ell_i$  of full length and containing no ex-ante strictly-feasible program. “% Possibly Unstable”, “N Possibly Unstable”: Percentage and count of students with  $\ell_i$  of full length and containing no ex-post match-feasible program.

## 4 Additional Model Estimates and Results

### 4.1 Additional Estimation Results

This section reports parameters from the estimated model. We divide the results into three tables, displaying inside-option preference and friction parameters, outside-option parameters, and parameters of the graduation production function, respectively. Posterior means and standard deviations are reported.

**Table 12:** Preference estimates: inside-option parameters

Parameters	Male Private	Male Public	Female Private	Female Public
Preferences ( $\psi^o$ )				
Price	-0.17 (0.012)	-0.146 (0.004)	-0.109 (0.008)	-0.142 (0.003)
Same City	1.296 (0.012)	1.388 (0.002)	1.289 (0.012)	1.418 (0.003)
STEM x Math	0.331 (0.012)	0.277 (0.004)	0.254 (0.008)	0.336 (0.002)
Humanities x Math	-0.067 (0.004)	-0.105 (0.002)	-0.062 (0.008)	-0.034 (0.003)
STEM x Verbal	-0.071 (0.005)	-0.065 (0.002)	-0.035 (0.008)	-0.075 (0.003)
Humanities x Verbal	0.179 (0.007)	0.182 (0.002)	0.145 (0.011)	0.133 (0.003)
Score*Teaching	-0.258 (0.02)	-0.113 (0.007)	-0.15 (0.02)	-0.17 (0.005)
Score2*Teaching	-0.086 (0.019)	-0.145 (0.005)	-0.045 (0.022)	-0.222 (0.007)
Score3*Teaching	0.072 (0.008)	0.077 (0.003)	0.04 (0.007)	0.121 (0.003)
Aftermarket frictions ( $\alpha$ )				
G25	-1.551 (0.03)	-0.88 (0.014)	-1.349 (0.035)	-0.84 (0.014)
G8 On	-0.827 (0.042)	-0.616 (0.037)	-0.739 (0.045)	-0.595 (0.032)
G8 Off	0.255 (0.024)	-0.435 (0.016)	0.183 (0.021)	-0.739 (0.01)
Local	0.232 (0.027)	-0.046 (0.014)	0.187 (0.029)	0.041 (0.01)
SD of program FE				
$\sigma_{FE}$	1.2 (0.041)	0.803 (0.005)	1.429 (0.054)	0.853 (0.01)
RC covariance matrix ( $\psi^u$ )				
STEM	0.318 (0.019)	0.248 (0.003)	0.483 (0.011)	0.403 (0.004)
Humanities	0.337 (0.022)	0.226 (0.002)	0.39 (0.017)	0.264 (0.004)
Humanities vs STEM ( $\rho$ )	0.166 (0.013)	0.09 (0.001)	0.28 (0.01)	0.178 (0.003)

Note: Preference parameters were estimated via Gibbs sampling and include program fixed effects. The number of observations used for the estimation are 484549 and the number of options are 1334 over three years.

**Table 13:** Preference estimates: outside-option and individual-level parameters

Parameters	Male Private	Male Public	Female Private	Female Public
First Outside Option ( $\gamma^0$ )				
Constant	2.602 (0.08)	2.405 (0.01)	3.153 (0.062)	2.553 (0.01)
Math	0.099 (0.018)	-0.087 (0.006)	-0.012 (0.017)	-0.136 (0.006)
Verbal	-0.008 (0.015)	-0.041 (0.006)	0.028 (0.015)	0.032 (0.005)
Big City	0.891 (0.033)	0.688 (0.008)	0.886 (0.028)	0.679 (0.008)
Current Cohort	-0.117 (0.027)	0.127 (0.008)	-0.107 (0.024)	0.113 (0.008)
1(2011)	0.063 (0.031)	0.078 (0.009)	0.046 (0.024)	0.082 (0.008)
1(2012)	-0.095 (0.032)	0.14 (0.009)	-0.074 (0.026)	0.154 (0.009)
Scholarship Amount	-0.29 (0.118)	0.032 (0.014)	-0.185 (0.104)	0.011 (0.013)
Score2	0.303 (0.027)	0.371 (0.008)	0.216 (0.031)	0.354 (0.007)
Score3	-0.078 (0.009)	-0.086 (0.004)	-0.046 (0.012)	-0.095 (0.005)
$\sigma_{0,0}$	0.569 (0.012)	0.584 (0.003)	0.546 (0.009)	0.578 (0.003)
Second Outside Option ( $\gamma^1$ )				
Constant	1.845 (0.089)	2.135 (0.027)	2.378 (0.075)	2.339 (0.019)
Math	-0.33 (0.039)	-0.41 (0.017)	-0.402 (0.033)	-0.386 (0.012)
Verbal	-0.211 (0.03)	-0.223 (0.01)	-0.092 (0.02)	-0.138 (0.009)
Big City	0.807 (0.036)	0.884 (0.014)	0.697 (0.035)	0.671 (0.009)
Current Cohort	-0.607 (0.044)	-0.001 (0.01)	-0.392 (0.032)	0.036 (0.008)
1(2011)	-0.064 (0.043)	0.063 (0.012)	0.027 (0.038)	0.096 (0.013)
1(2012)	0.608 (0.041)	0.4 (0.012)	0.572 (0.038)	0.525 (0.01)
Scholarship Amount	0.259 (0.109)	0.344 (0.017)	-0.103 (0.1)	0.183 (0.014)
Score2	0.233 (0.048)	0.225 (0.01)	0.219 (0.038)	0.175 (0.012)
Score3	-0.018 (0.015)	0.007 (0.005)	-0.018 (0.014)	0.008 (0.007)
$\sigma_{0,1}$	1.815 (0.093)	38 1.277 (0.036)	1.54 (0.055)	1.205 (0.024)

Note: Preference parameters were estimated via Gibbs sampling and include program fixed effects. The number of observations

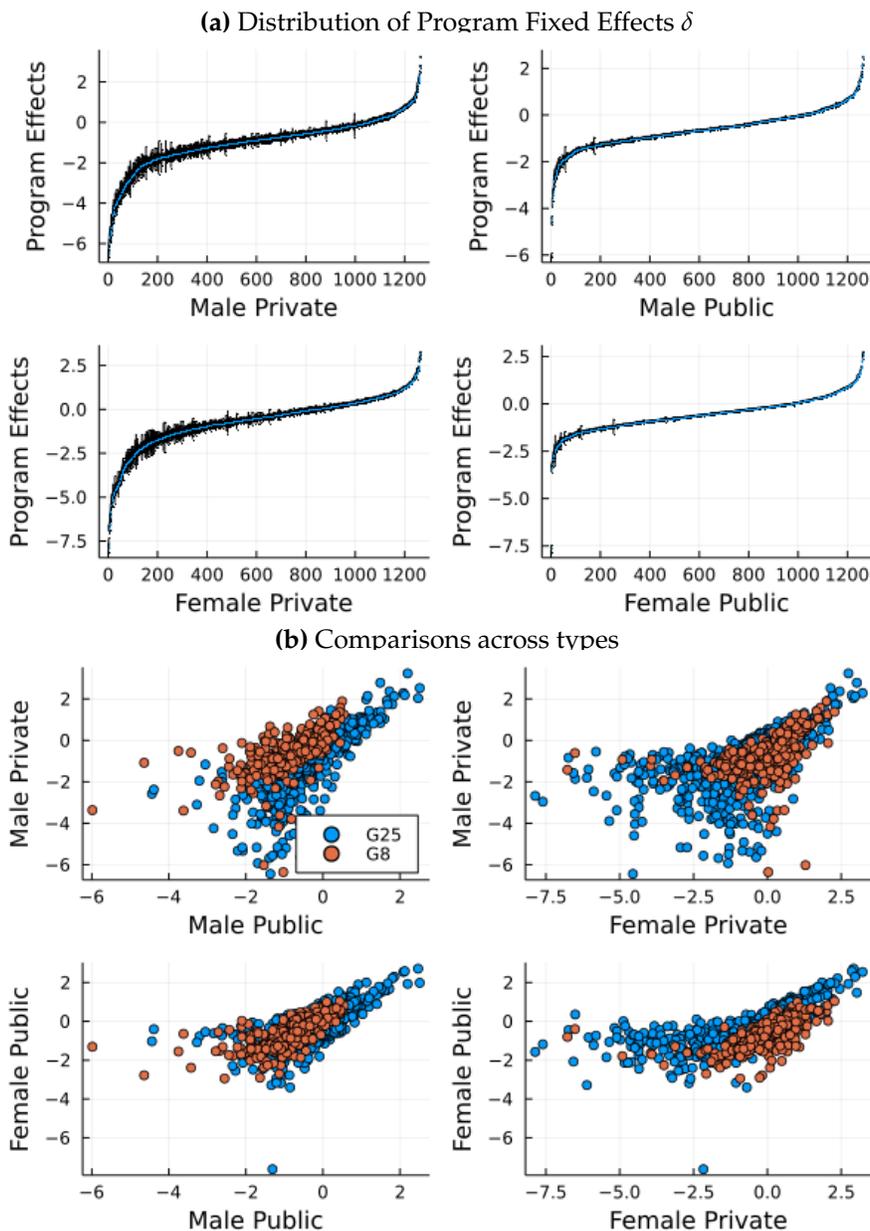
**Table 14:** Parameter estimates: outcomes

Parameters	Male Private	Male Public	Female Private	Female Public
Production Function				
utility shock	0.571 (0.372)	0.402 (0.026)	1.15 (1.117)	0.321 (0.031)
Constant	-2.608 (1.299)	-1.851 (0.118)	-4.57 (4.191)	-1.319 (0.129)
Math	0.279 (0.049)	0.298 (0.012)	0.327 (0.043)	0.361 (0.012)
Verbal	0.095 (0.03)	0.031 (0.01)	0.117 (0.053)	0.043 (0.01)
Big City	-0.196 (0.217)	-0.183 (0.02)	-0.753 (0.711)	-0.203 (0.025)
Current Cohort	0.208 (0.036)	0.144 (0.011)	0.288 (0.114)	0.124 (0.01)
1(2011)	-0.0 (0.025)	-0.012 (0.012)	0.055 (0.03)	0.022 (0.013)
1(2012)	0.036 (0.041)	0.022 (0.014)	0.147 (0.065)	0.059 (0.014)
Scholarship Amount	0.073 (0.107)	0.138 (0.019)	0.111 (0.125)	0.121 (0.015)
Score2	-0.161 (0.064)	-0.133 (0.018)	-0.313 (0.204)	-0.102 (0.019)
Score3	0.019 (0.017)	0.014 (0.006)	0.044 (0.048)	-0.005 (0.008)
Price	-0.117 (0.08)	-0.065 (0.025)	-0.066 (0.09)	-0.015 (0.019)
Same City	0.614 (0.366)	0.492 (0.032)	1.268 (1.136)	0.424 (0.038)
STEM x Math	0.076 (0.093)	0.059 (0.015)	0.239 (0.214)	0.085 (0.019)
Humanities x Math	-0.07 (0.03)	-0.093 (0.015)	-0.035 (0.049)	-0.068 (0.017)
STEM x Verbal	0.024 (0.028)	0.035 (0.014)	0.006 (0.037)	0.005 (0.015)
Humanities x Verbal	0.102 (0.056)	0.081 (0.014)	0.124 (0.137)	0.056 (0.013)
Score*Teaching	-0.149 (0.164)	-0.107 (0.049)	-0.101 (0.264)	-0.15 (0.043)
Score2*Teaching	-0.04 (0.189)	-0.004 (0.059)	0.057 (0.15)	0.008 (0.043)
Score3*Teaching	0.019 (0.079)	0.012 (0.028)	-0.024 (0.062)	0.015 (0.03)

Note: Preference parameters were estimated via Gibbs sampling and include program fixed effects. The number of observations used for the estimation are 484549 and the number of options are 1334 over three years.

Figure 14 shows the distribution of program mean utility terms  $\delta$ , which are estimated separately by type. These vary from roughly -3 to 2, relative to the idiosyncratic utility shock which is normalized to have variance 1. Panel (b) of this figure indicates that the types disagree about the relative ranking of G8 vs G25 programs, with students from private schools (a proxy for SES) systematically exhibiting stronger preferences for G8 programs, relative to G25 programs, than students who attended public schools. In addition, while students of all types tend to rank top programs similarly, the scatter plots indicate disagreement about middle- and lower-ranked programs.

**Figure 14:** Distribution of Program Fixed Effects ( $\delta$ )



Note: Figures display estimates of program fixed effects  $\delta$ . Parameters are estimated separately by type. Left panel: sorted within each type, black lines represent 95% posterior probability intervals. Right panel: scatter plots comparing means of each program across types. Blue indicates G25, red indicates G8.

## 4.2 Model Fit

This section provides additional model fit figures. Figure 15 shows scatter plots of average math + verbal test scores among students placed in G25 programs in the years 2010-2012. These figures

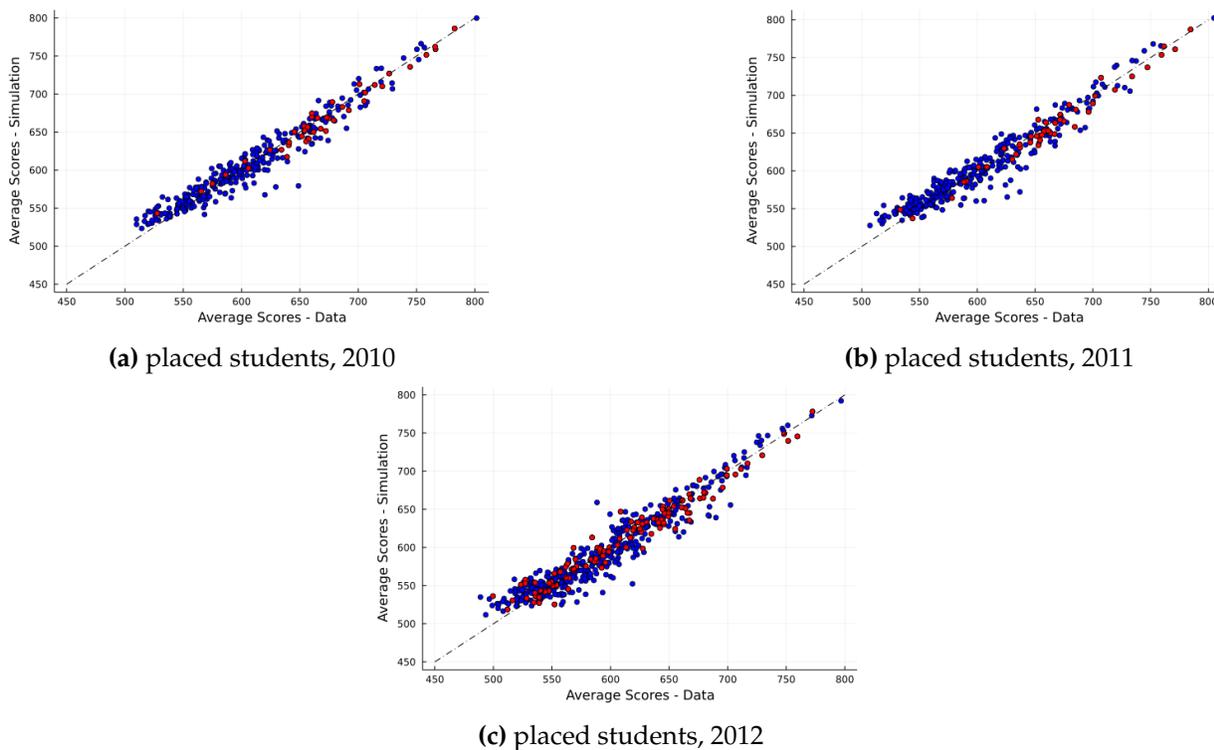
complement the analysis of characteristics of enrolled students that we present in the main text. Blue circles denote programs with 50 to 100 seats. Red circles are large programs with at least 100 seats. In general, the model fits well, especially for elite programs but throughout the distribution as well.

Figure 16 decomposes the differences in the probability of enrolling in teaching programs, presented in the main body of the paper, into 2010 and 2011 probabilities. We match not only the differences but also the levels pre- and post-BVP for students with scores above 500.

Figure 17 examines fit along students' propensity to enroll in, and graduate from, G25 institutions as a function of their own mean math + verbal test scores and their cohort. The model matches the general increase in enrollment and in graduation (not conditional on enrollment) as a function of scores.

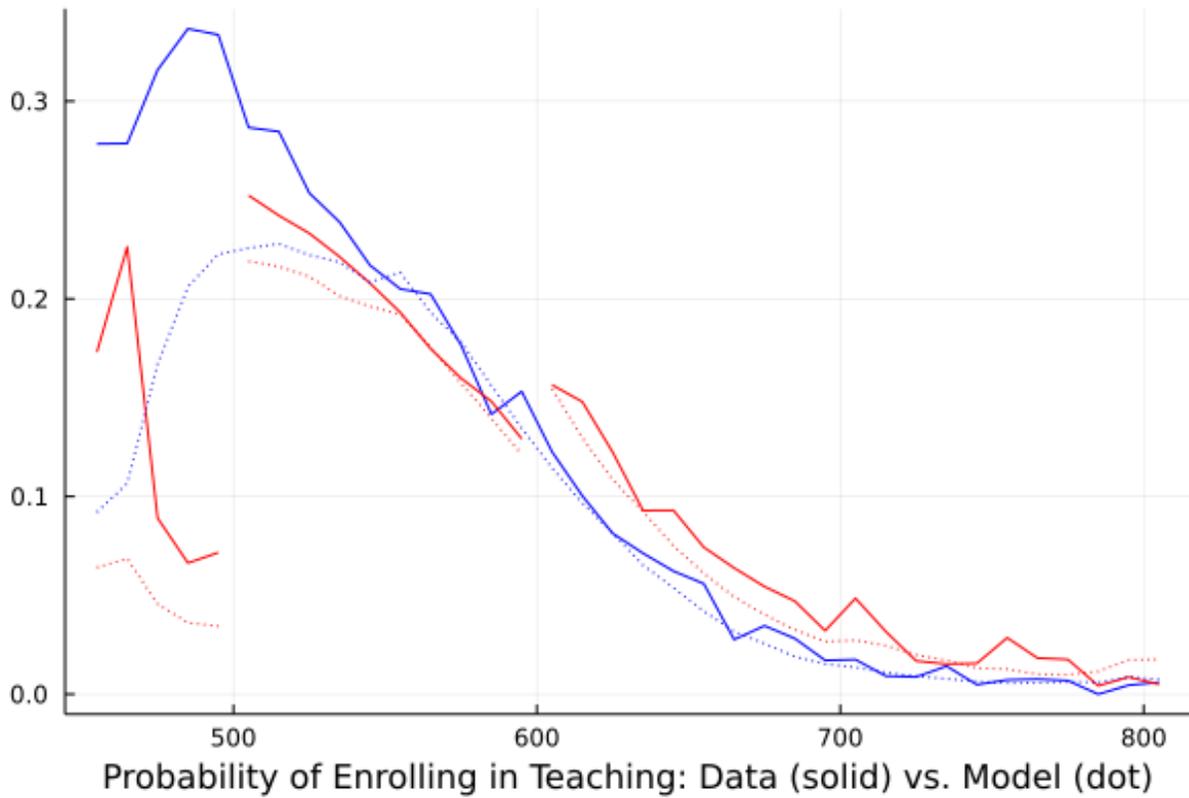
Finally, Table 15 compares type-specific event studies of G33 enrollment and graduation probabilities to event studies conducted on simulated data, complementing the non-type-specific event study presented in the main text. We find that the model fits well within each type. Table 16 shows the same set of outcomes within G25 institutions.

**Figure 15: Model Fit - Selectivity**



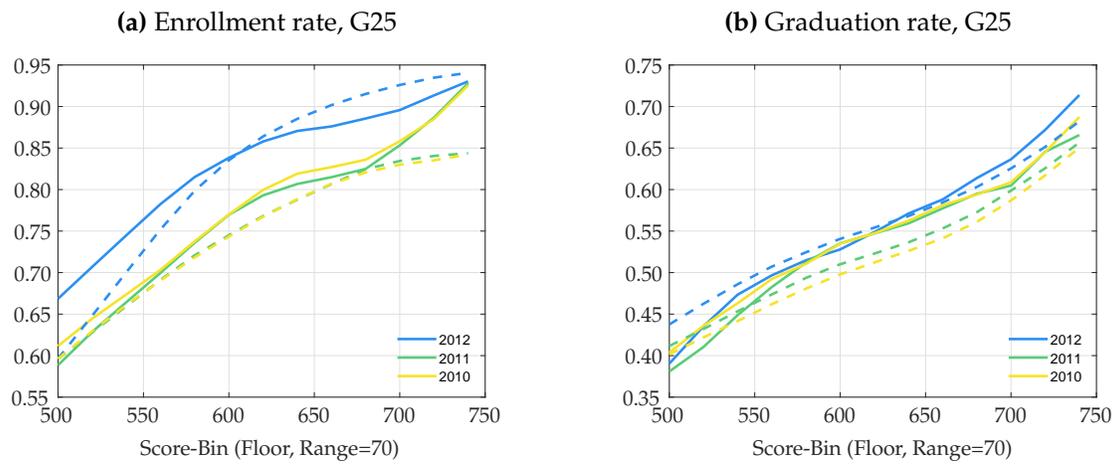
Note: each panel shows, for each on-platform program, the mean math and verbal scores of students **placed** in the program via the initial match in the data (X-axis) and in simulations (Y-axis).

**Figure 16: Model Fit - Selectivity**



Note: This figure shows the share of test-takers enrolling in G-33 teaching majors in 2010 and 2011 (blue: 2010; red: 2011) in the data (solid lines) and in model fit simulations (dotted lines) as a function of mean math+verbal test scores. Lines are means within 10-point bins. Students with scores above 600 in 2011 were eligible for full scholarships at teaching majors. Students with scores below 500 in 2011 were restricted from entering most G33 teaching programs.

**Figure 17:** Enrollment and Graduation probability with model-simulated outcomes, G25



Note: The figures show the probability that a student assigned to an option on the platform, accepts and enrolls in that option (left panel) and graduates no more than 6 years after first year (right panel). The lines show conditional means within 70 points, and the “floor” of the range is shown in the x-axis (e.g. 600 corresponds to the range [600, 670]). The dashed lines show the model-simulated outcome in each case.

**Table 15: Event study outcomes by type: G33 Enrollment and Graduation**

	Data Enroll G33	Model Enroll G33	Data Graduate G33	Model Graduate G33	Data Graduate   Enroll G33	Model Graduate   Enroll G33
All						
Constant	0.186 (0.0012)	0.162 (0.0011)	0.067 (0.0011)	0.05 (0.0011)	0.354 (0.0026)	0.359 (0.0035)
Math	0.215 (0.0011)	0.201 (0.0013)	0.116 (0.001)	0.123 (0.0009)	0.015 (0.0019)	0.055 (0.0021)
Language	0.149 (0.0011)	0.132 (0.0017)	0.078 (0.001)	0.077 (0.0012)	0.024 (0.0019)	0.036 (0.0023)
GPA	0.055 (0.0008)	0.075 (0.0008)	0.072 (0.0007)	0.056 (0.0009)	0.108 (0.0015)	0.05 (0.0021)
2010	-0.003 (0.0015)	-0.003 (0.0013)	-0.007 (0.0013)	-0.004 (0.0012)	-0.013 (0.0027)	-0.01 (0.0028)
2012	0.011 (0.0015)	0.022 (0.002)	0.024 (0.0013)	0.025 (0.0019)	0.026 (0.0027)	0.034 (0.0034)
Obs.	484549		484549		203596	
Male Private						
Constant	0.345 (0.0055)	0.315 (0.0105)	0.129 (0.006)	0.074 (0.0072)	0.358 (0.009)	0.321 (0.0106)
Math	0.172 (0.0037)	0.122 (0.0065)	0.11 (0.0041)	0.126 (0.0049)	0.041 (0.0054)	0.1 (0.0063)
Language	0.12 (0.0039)	0.078 (0.0069)	0.044 (0.0043)	0.041 (0.0053)	-0.003 (0.0055)	0.023 (0.0071)
GPA	-0.012 (0.0034)	0.08 (0.0039)	0.064 (0.0037)	0.049 (0.0058)	0.099 (0.0049)	0.013 (0.0088)
2010	-0.005 (0.0054)	-0.008 (0.0096)	-0.009 (0.0059)	-0.007 (0.0066)	-0.009 (0.0075)	-0.01 (0.0075)
2012	-0.004 (0.0053)	0.036 (0.0089)	0.012 (0.0058)	0.03 (0.0078)	0.018 (0.0074)	0.021 (0.0106)
Obs.	37042		37042		25609	
Male Public						
Constant	0.172 (0.0018)	0.16 (0.0019)	0.055 (0.0015)	0.04 (0.0018)	0.272 (0.004)	0.283 (0.0072)
Math	0.225 (0.0017)	0.216 (0.0019)	0.102 (0.0014)	0.118 (0.0012)	0.03 (0.0031)	0.08 (0.0036)
Language	0.141 (0.0016)	0.131 (0.0026)	0.054 (0.0014)	0.055 (0.0014)	0.018 (0.003)	0.018 (0.003)
GPA	0.071 (0.0012)	0.08 (0.0011)	0.066 (0.001)	0.041 (0.0008)	0.092 (0.0023)	0.025 (0.002)
2010	-0.004 (0.0022)	-0.006 (0.0029)	-0.005 (0.0019)	-0.003 (0.0019)	-0.009 (0.0042)	-0.005 (0.0043)
2012	0.02 (0.0023)	0.024 (0.0034)	0.023 (0.0019)	0.021 (0.0027)	0.022 (0.0042)	0.03 (0.0058)
Obs.	199203		199203		77729	
Female Private						
Constant	0.368 (0.0055)	0.302 (0.0086)	0.205 (0.0062)	0.14 (0.0072)	0.534 (0.0085)	0.523 (0.0139)
Math	0.178 (0.0045)	0.118 (0.0076)	0.148 (0.0051)	0.125 (0.0088)	0.047 (0.0062)	0.078 (0.0127)
Language	0.123 (0.0043)	0.086 (0.0074)	0.057 (0.0048)	0.054 (0.0061)	-0.019 (0.0058)	0.01 (0.0078)
GPA	-0.01 (0.0038)	0.083 (0.0039)	0.038 (0.0043)	0.062 (0.0096)	0.068 (0.0053)	0.015 (0.0174)
2010	0.004 (0.0056)	-0.004 (0.0096)	-0.024 (0.0063)	-0.015 (0.0084)	-0.026 (0.0075)	-0.022 (0.0091)
2012	-0.009 (0.0055)	0.034 (0.0093)	0.02 (0.0062)	0.049 (0.0082)	0.042 (0.0075)	0.043 (0.0092)
Obs.	34081		34081		23519	
Female Public						
Constant	0.162 (0.0018)	0.136 (0.0018)	0.069 (0.0016)	0.055 (0.0014)	0.421 (0.0042)	0.428 (0.005)
Math	0.221 (0.0018)	0.212 (0.0018)	0.141 (0.0016)	0.15 (0.0019)	0.048 (0.0037)	0.101 (0.0042)
Language	0.166 (0.0017)	0.152 (0.002)	0.087 (0.0015)	0.089 (0.0016)	-0.003 (0.0032)	0.018 (0.0032)
GPA	0.068 (0.0012)	0.082 (0.0012)	0.07 (0.0011)	0.055 (0.0009)	0.092 (0.0025)	0.031 (0.002)
2010	-0.002 (0.0022)	0 (0.0023)	-0.006 (0.002)	-0.003 (0.0016)	-0.013 (0.0043)	-0.013 (0.0048)
2012	0.01 (0.0022)	0.016 (0.0023)	0.024 (0.002)	0.021 (0.0019)	0.027 (0.0043)	0.035 (0.0045)
Obs.	214223		214223		76739	

Note: this table shows estimates of each outcome, for each type of student, for the years 2010-2012. The base year is 2011. We consider the events that a student enrolls in, and graduates within seven years from enrolling in some G33 program. Regressions are pooled (top) and by observable type. "Model" columns: we use the results of 52 simulation draws in which we draw utilities, availability indicators, human-capital indices, and parameters from their estimated posterior joint distribution. In each draw, we simulate the market, then estimate the relevant linear models. We report means and standard deviations of parameter estimates from the relevant models over these draws.

**Table 16: Event study outcomes by type: G25 Enrollment and Graduation**

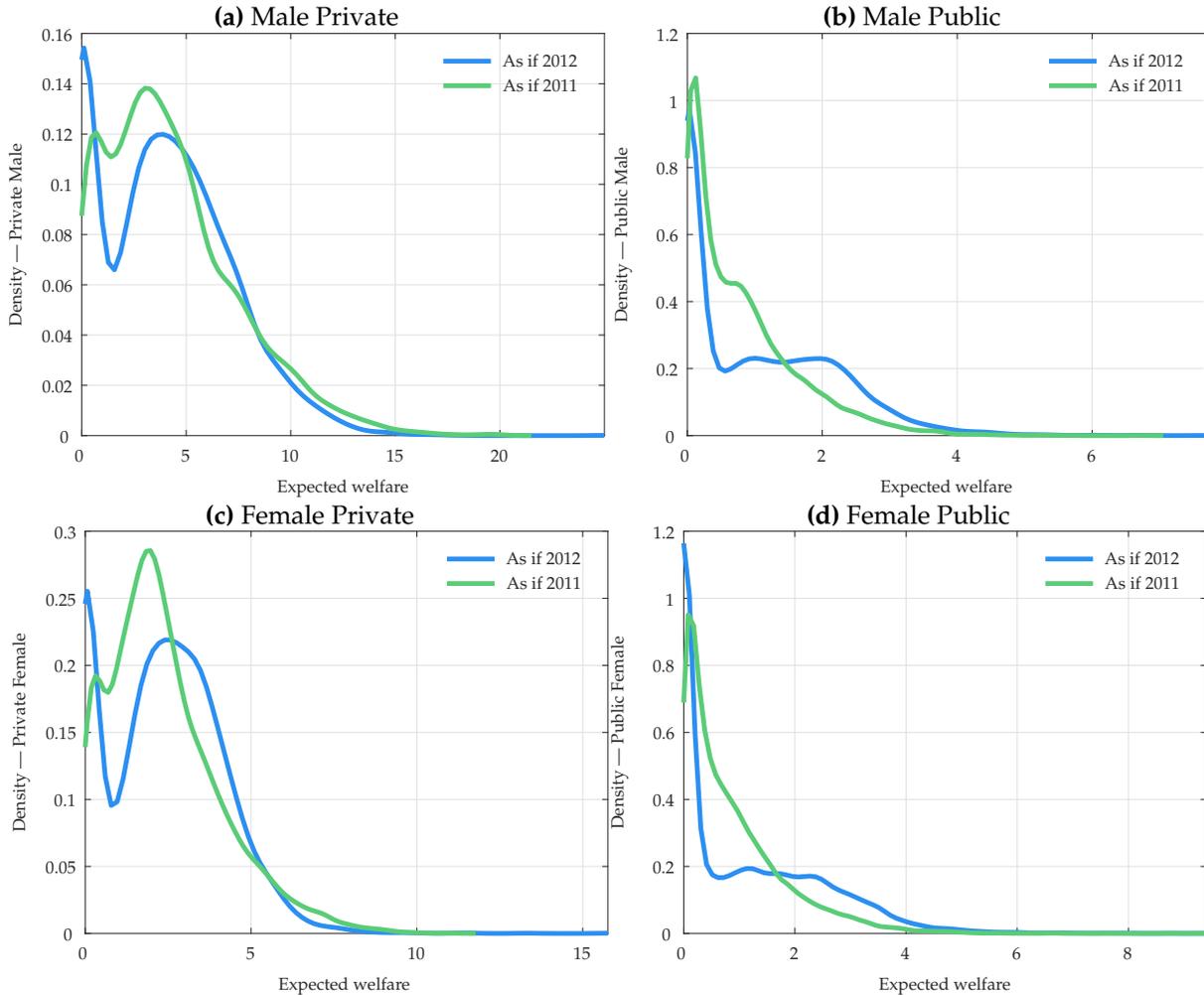
	Data Enroll G25	Model Enroll G25	Data Graduate G25	Model Graduate G25	Data Graduate   Enroll G25	Model Graduate   Enroll G25
All						
Constant	0.116 (0.0012)	0.105 (0.0009)	0.029 (0.0009)	0.028 (0.001)	0.317 (0.003)	0.336 (0.0048)
Math	0.156 (0.001)	0.154 (0.001)	0.072 (0.0008)	0.088 (0.0008)	-0.007 (0.0021)	0.043 (0.0023)
Language	0.095 (0.0011)	0.086 (0.0013)	0.053 (0.0009)	0.049 (0.0008)	0.026 (0.0022)	0.031 (0.0025)
GPA	0.098 (0.0008)	0.096 (0.0008)	0.084 (0.0006)	0.063 (0.0006)	0.13 (0.0017)	0.065 (0.0018)
2010	0.001 (0.0014)	-0.003 (0.0009)	-0.001 (0.0012)	-0.003 (0.0009)	-0.01 (0.0031)	-0.01 (0.0029)
2012	-0.004 (0.0014)	-0.014 (0.0015)	0.006 (0.0012)	0.002 (0.0012)	0.025 (0.0031)	0.029 (0.0031)
Obs.	484549		484549		150728	
Male Private						
Constant	0.013 (0.0056)	0.003 (0.0053)	-0.048 (0.0047)	-0.054 (0.0037)	0.188 (0.0132)	0.186 (0.0143)
Math	0.151 (0.0038)	0.138 (0.0045)	0.087 (0.0032)	0.108 (0.0032)	0.044 (0.0067)	0.118 (0.0059)
Language	0.078 (0.0039)	0.049 (0.0052)	0.038 (0.0033)	0.022 (0.0039)	0.024 (0.0072)	0.028 (0.0094)
GPA	0.101 (0.0034)	0.121 (0.0031)	0.092 (0.0029)	0.067 (0.003)	0.135 (0.0066)	0.039 (0.0089)
2010	0.012 (0.0054)	0.003 (0.0055)	0.007 (0.0046)	-0.0 (0.004)	0.0 (0.01)	-0.009 (0.0079)
2012	-0.008 (0.0053)	0.023 (0.0062)	0.009 (0.0045)	0.016 (0.0041)	0.039 (0.0099)	0.027 (0.009)
Obs.	37042		37042		14048	
Male Public						
Constant	0.126 (0.0017)	0.119 (0.0017)	0.029 (0.0013)	0.027 (0.0016)	0.253 (0.0045)	0.269 (0.0084)
Math	0.2 (0.0017)	0.196 (0.0018)	0.083 (0.0013)	0.104 (0.0012)	0.022 (0.0033)	0.077 (0.0034)
Language	0.104 (0.0016)	0.103 (0.0021)	0.044 (0.0012)	0.043 (0.0013)	0.019 (0.0032)	0.017 (0.0035)
GPA	0.1 (0.0012)	0.098 (0.001)	0.071 (0.0009)	0.048 (0.0007)	0.104 (0.0025)	0.033 (0.0025)
2010	0.001 (0.0022)	-0.006 (0.0025)	-0.002 (0.0017)	-0.003 (0.0017)	-0.01 (0.0046)	-0.006 (0.0045)
2012	0.008 (0.0022)	-0.005 (0.0028)	0.009 (0.0017)	0.007 (0.002)	0.018 (0.0046)	0.027 (0.0053)
Obs.	199203		199203		64721	
Female Private						
Constant	-0.018 (0.0055)	-0.025 (0.0048)	-0.043 (0.0049)	-0.042 (0.0044)	0.414 (0.0146)	0.431 (0.0217)
Math	0.107 (0.0045)	0.088 (0.0055)	0.076 (0.004)	0.079 (0.0045)	0.021 (0.0083)	0.075 (0.0109)
Language	0.123 (0.0042)	0.083 (0.0045)	0.073 (0.0038)	0.048 (0.0039)	0.007 (0.0083)	0.007 (0.0113)
GPA	0.099 (0.0038)	0.117 (0.0034)	0.087 (0.0034)	0.079 (0.0043)	0.11 (0.0083)	0.042 (0.0165)
2010	-0.003 (0.0056)	-0.002 (0.0049)	-0.01 (0.005)	-0.007 (0.0043)	-0.028 (0.0109)	-0.022 (0.0105)
2012	-0.019 (0.0055)	0.008 (0.0061)	-0.003 (0.0049)	0.017 (0.0051)	0.029 (0.0108)	0.048 (0.0105)
Obs.	34081		34081		11421	
Female Public						
Constant	0.103 (0.0017)	0.092 (0.0014)	0.035 (0.0015)	0.035 (0.0011)	0.409 (0.0049)	0.42 (0.005)
Math	0.17 (0.0017)	0.175 (0.0017)	0.102 (0.0015)	0.121 (0.0015)	0.027 (0.0041)	0.093 (0.0039)
Language	0.13 (0.0016)	0.124 (0.0015)	0.07 (0.0014)	0.073 (0.001)	-0.002 (0.0036)	0.017 (0.0031)
GPA	0.097 (0.0012)	0.093 (0.0011)	0.079 (0.001)	0.06 (0.0008)	0.102 (0.0028)	0.038 (0.0021)
2010	0.001 (0.0021)	-0 (0.002)	-0.001 (0.0018)	-0.003 (0.0014)	-0.008 (0.0048)	-0.014 (0.0048)
2012	-0.004 (0.0021)	-0.025 (0.0019)	0.006 (0.0018)	-0.005 (0.0015)	0.028 (0.0049)	0.034 (0.0043)
Obs.	214223		214223		60538	

Note: this table shows estimates of each outcome, for each type of student, for the years 2010-2012. The base year is 2011. We consider the events that a student enrolls in, and graduates within seven years from enrolling in some G25 program. Regressions are pooled (top) and by observable type. "Model" columns: we use the results of 52 simulation draws in which we draw utilities, availability indicators, human-capital indices, and parameters from their estimated posterior joint distribution. In each draw, we simulate the market, then estimate the relevant linear models. We report means and standard deviations of parameter estimates from the relevant models over these draws.

### 4.3 Additional Counterfactuals

We now show additional results from counterfactual simulations. We first display the impacts of the policy change on the distribution of student welfare.

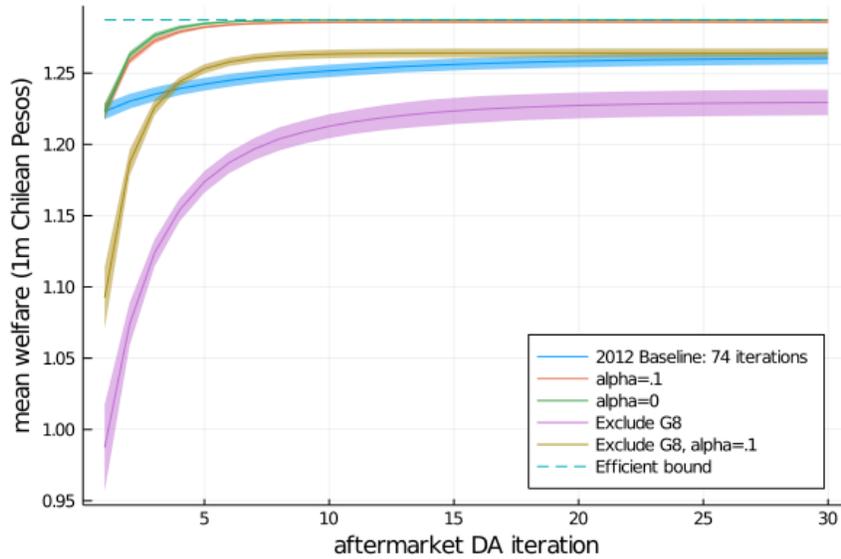
**Figure 18:** Distributional change in welfare after policy change in 2012



Note: The densities plotted in the figure are the estimated enrollment utilities for students in years 2011 and 2012. For each plot, the x-axis corresponds to the average utility levels across draws for each individual and the y-axis indicates the conditional kernel density estimates.

Next, figure 19 illustrates the length of the aftermarket process. In our counterfactuals, we simulate the DA process until convergence, which occurs after roughly 70 rounds when all programs are on platform, longer when some programs are excluded. This process occurs within a few weeks after the match is announced before classes begin. Figure 19 illustrates that, if this process were to stop early, welfare losses relative to a frictionless benchmark would be larger, especially when frictions are large and some programs do not join the platform.

**Figure 19: Welfare by Round of Aftermarket DA Process**



Note: Mean welfare at baseline and under counterfactuals if aftermarket DA process were to terminate at round  $t$ .

Figure 20 examines heterogeneous impacts of our main counterfactuals by student mean math and verbal exam score. Effects of platform expansion on welfare, enrollment, and graduation are small for students with very high test scores, and larger for students with scores below 650. One might worry that the positive enrollment impacts of platform expansion are concentrated at programs with low selectivity, as proxied by baseline average test scores of enrollees. However, panel (D) shows that effects on the average selectivity of program attended, conditional on enrolling in some G33 program, are small.

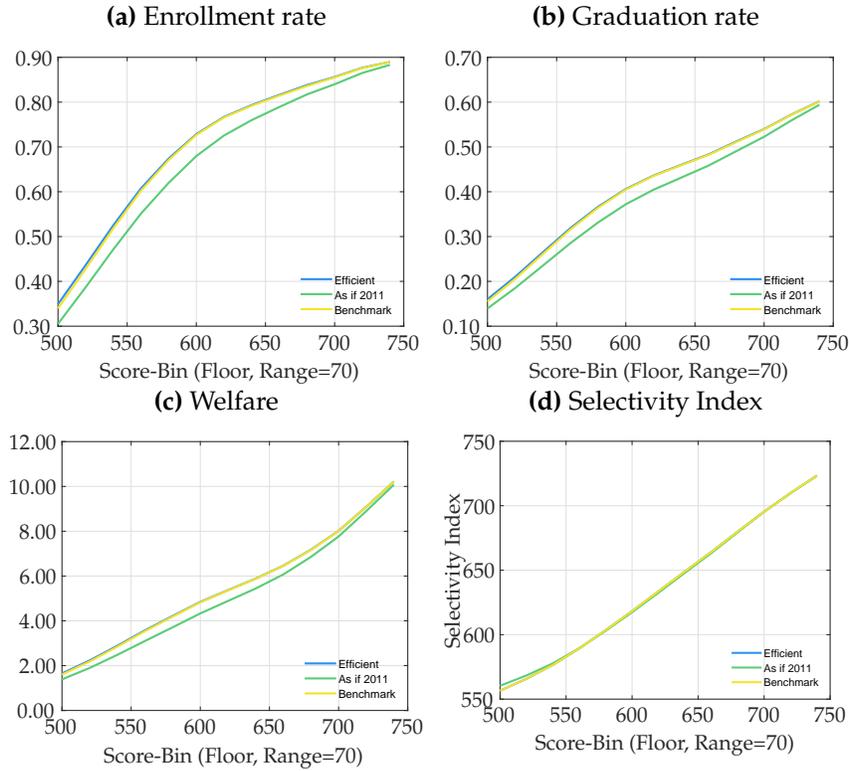
Finally, we compare the impacts of our main counterfactual—what if the G8 had not joined the platform in 2012—to that of increasing the number of seats. We do this by multiplying the total (true + extra seat) capacity of each on-platform program by  $(1 + x)$ , for values  $x \in \{0.05, 0.10, \dots, 0.30\}$  within a counterfactual in which the G8 institutions are off-platform, then rounding each program’s capacity to the nearest integer. The additional seats are treated “extra” seats rather than “true” seats, in the sense we describe in section 2 of the paper. The idea is that programs may be able to compensate for a greater share of declined offers by overenrolling more in the initial match.

Figure 21 shows the results. The orange dashed line shows welfare under the baseline scenario; the blue dashed line shows welfare under the scenario in which the G8 is absent and no extra seats are added. We find that platform expansion has large impacts on welfare. To obtain similar welfare impacts to those of platform expansion, the figure shows that one would have to increase the total number of seats available in the match by roughly 30%, which we interpret as a large number.

In this exercise we consider a fairly crude way to increase capacity. Targeted expansions in highly desirable programs could have larger impacts with fewer additional seats. Nonetheless, the

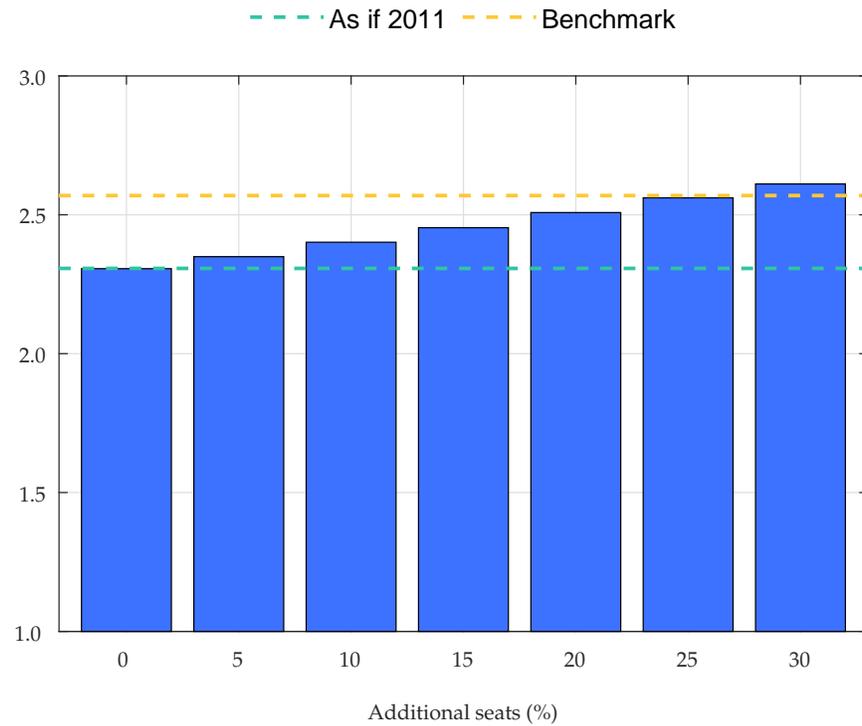
results indicate that small tweaks to capacities cannot compensate for the absence of G8 programs from the platform.

**Figure 20:** Enrollment, Graduation probability, and Welfare - Counterfactuals



Note: These figures show (a) the probability that a student assigned to an option on the platform accepts and enrolls in that option; (b) the probability that a student assigned to an option graduates no more than 6 years after first year; (c) mean welfare among PSU-takers; and (d) a selectivity index of the program enrolled among students who enroll in any G33 option. The selectivity of program  $j$  is defined to be the mean math + verbal exam score of students enrolling in program  $j$  in the 2012 data. The lines show conditional means within 70 points, and the “floor” of the range is shown in the x-axis (e.g. 600 corresponds to the range [600, 670]).

**Figure 21:** Counterfactuals with additional seats - Welfare.

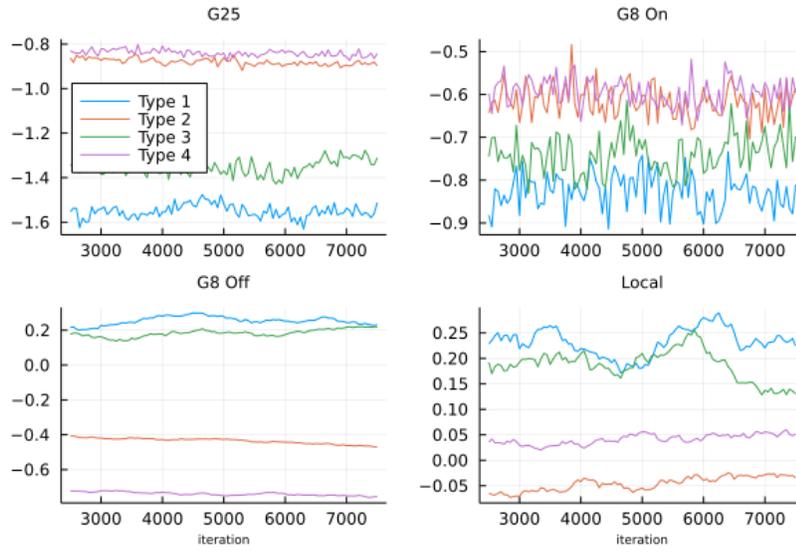


Note: The figure shows mean welfare, in units of 1 million CLP per capita, under counterfactual simulations in which total on-platform seats are increased 0 to 30%, uniformly across on-platform G25 programs, with the G8 institutions off platform. Orange dashed line: mean welfare under the 2012 benchmark. Blue dashed line: welfare under the counterfactual in which the G8 is absent from the platform, and no seats are added.

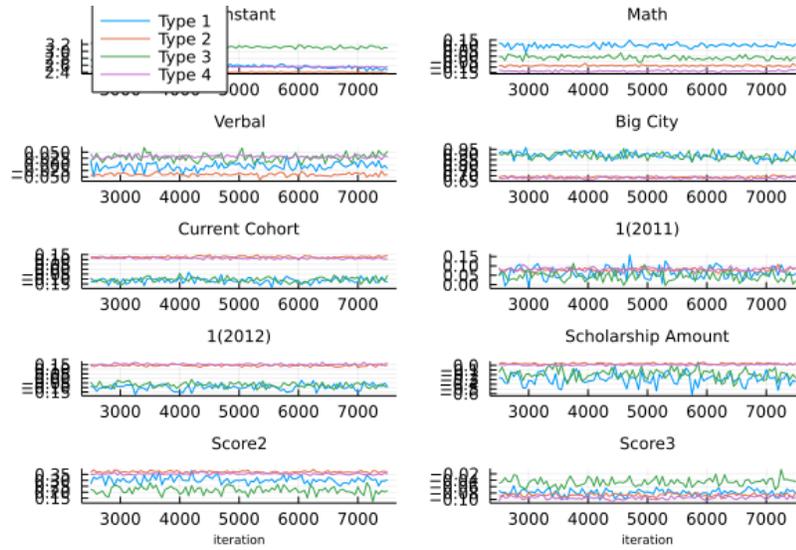
#### 4.4 Convergence

This section shows trace plots for estimated parameters.

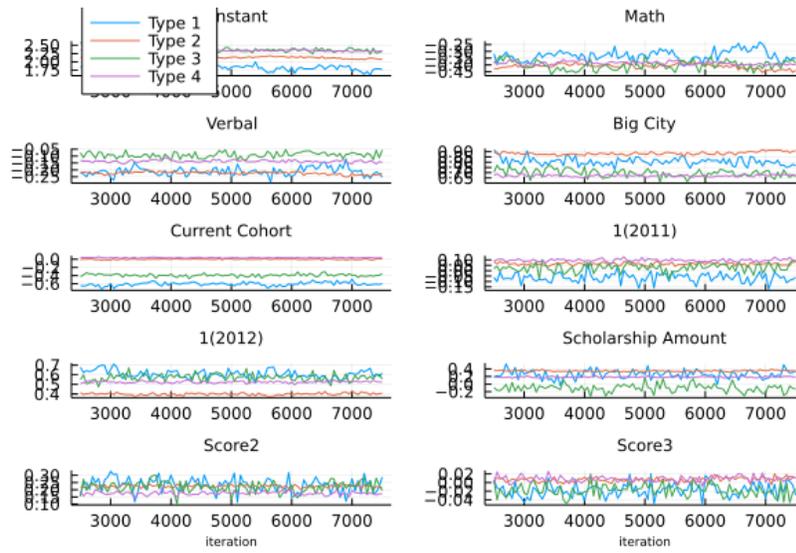
**Figure 22: Friction Parameters**



**Figure 23: Student-Level Parameters / First Outside Option Mean**



**Figure 24: Second Outside Option Mean**



**Figure 25: Match-level Utility Terms**

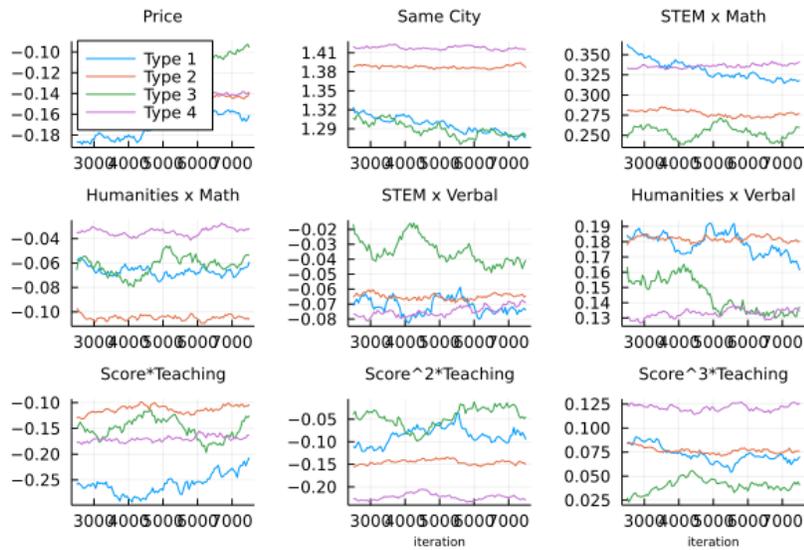


Figure 26: Outside Option Variance

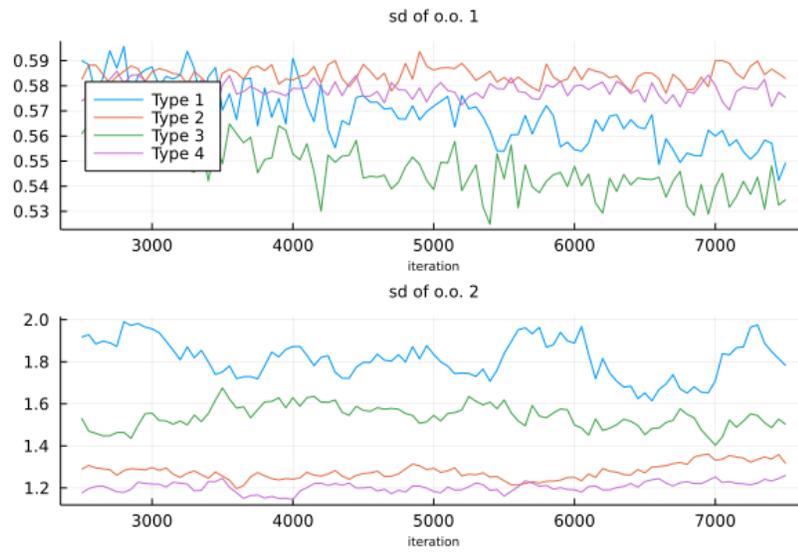
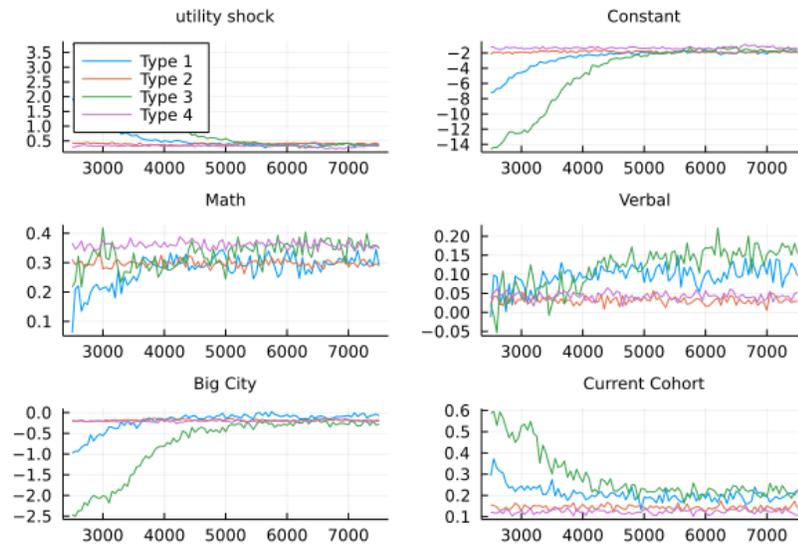
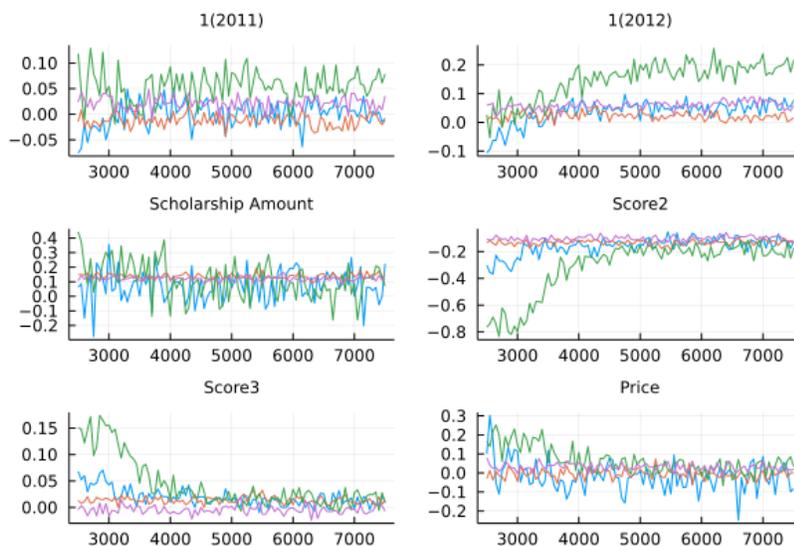


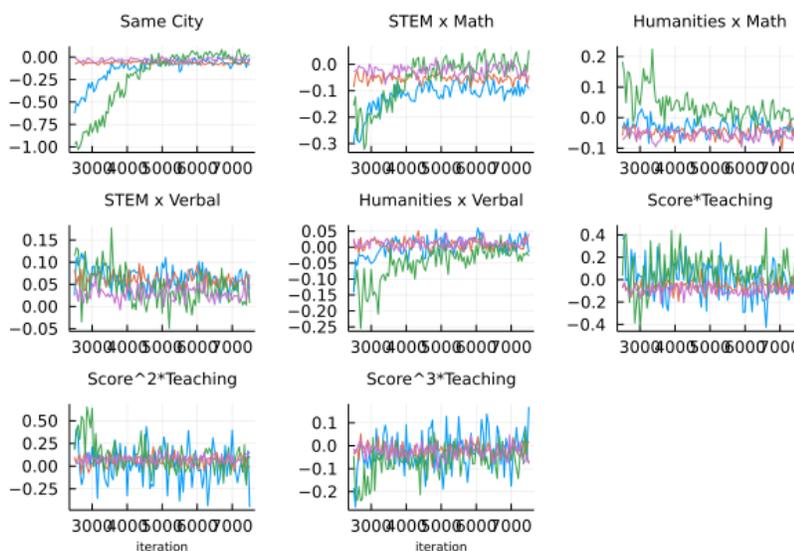
Figure 27: Human-Capital Terms 1



**Figure 28: Human Capital Terms 2**



**Figure 29: Human Capital Terms 3**



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