Appendix for "Disaggregated Economic Accounts"

Appendix A Additional Tables and Figures

Appendix A.A Data on the Spending Matrix

Table A.I: Classification of industries

	Producer industry	Produces output	Sells directly to consumers	Pays labor compensation to consumers ("work industry")
1	Food away from home	Yes	Yes	Yes
2	Entertainment	Yes	Yes	Yes
3	Groceries	Yes	Yes	Yes
4	Personal services, pharmacies	Yes	Yes	Yes
5	Vehicles, fuel, vehicle repair, public transport	Yes	Yes	Yes
6	Hotels, rental cars	Yes	Yes	Yes
7	Airlines	Yes	Yes	Yes
8	Telecommunication, insurance	Yes	Yes	Yes
9	Online stores	Yes	Yes	Yes
10	Utilities	Yes	Yes	Yes
11	Specialized retail stores	Yes	Yes	Yes
12	Home improvement	Yes	Yes	Yes
13	Consulting, information technology, media	Yes	No	Yes
14	Business administration and janitorial services	Yes	No	Yes
15	Manufacturing	Yes	No	Yes
16	Wholesale	Yes	No	Yes
17	Finance, real estate	Yes	Yes	Yes
18	Cultural and social organizations	Yes	Yes	Yes
19	Agriculture, mining	Yes	No	Yes
20	Construction	Yes	No	Yes
21	Shipping, transport	Yes	No	Yes
22	Out of workforce and others	No	No	No
23	Retired	No	No	No
24	Health	Yes	No	Yes
25	Students	No	No	No
26	Education	Yes	No	Yes
27	Public administration	Yes	No	Yes
28	Unemployed	No	No	No
29	Private landlords	Yes	Yes	No
30	Owner-occupied housing	Yes	Yes	No
31	Government-owned housing	Yes	Yes	No

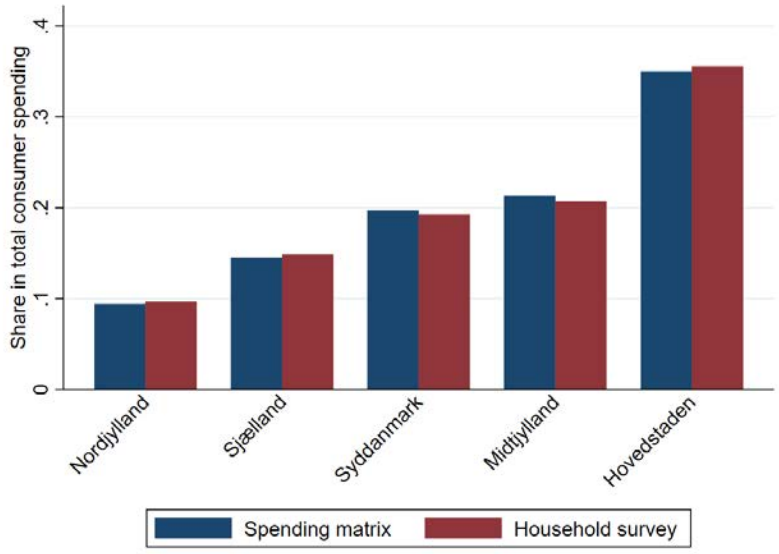
Notes: The table lists the industry classification used throughout the paper. Specialized retail stores include shops selling a specialized set of goods not listed in another industry, e.g., books, computers, shoes, clothing.

Table A.II: Summary statistics on the population and Danske Bank sample

	Full population	Danske Bank sample
Number of adults	4,367,226	858,409
Mean age	48.56	49.97
Mean income (DKK)	298,834	281,039
Age distribution		
18-39	0.35	0.34
40-59	0.35	0.32
60+	0.30	0.34
Income distribution		
Quintile 1	0.20	0.22
Quintile 2	0.20	0.23
Quintile 3	0.20	0.21
Quintile 4	0.20	0.18
Quintile 5	0.20	0.17
Ratio of liquid assets to income distribution		
Quintile 1	0.20	0.20
Quintile 2	0.20	0.21
Quintile 3	0.20	0.19
Quintile 4	0.20	0.20
Quintile 5	0.20	0.20

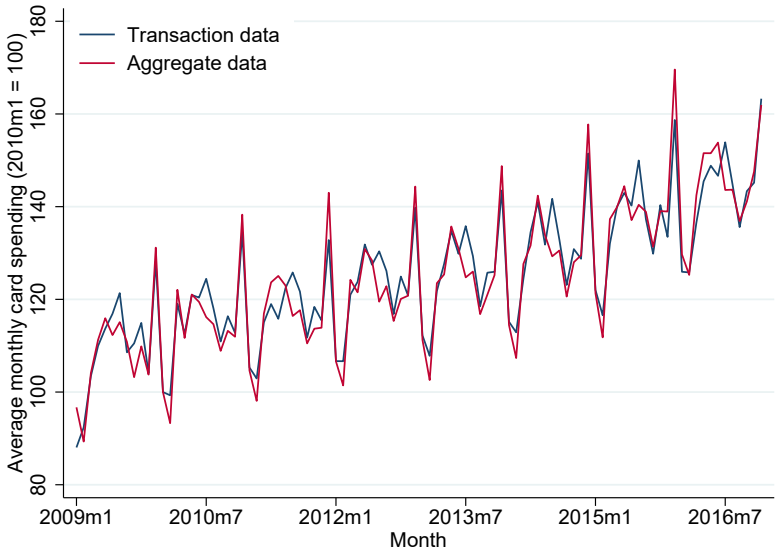
Notes: The table compares summary statistics for the Danish population from administrative registers with our sample of Danske Bank customers.

Figure A.I: Spending shares of regions in the spending matrix and household survey



Notes: The figure compares spending shares of consumer regions from the spending matrix with the Danish household budget survey accounts consumption data.

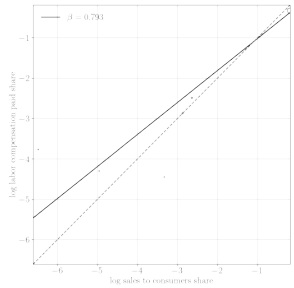
Figure A.II: Card spending over time



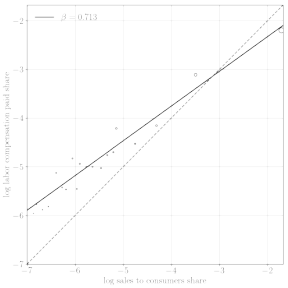
Notes: The figure compares aggregated card spending over time in data from Danske Bank and Statistics Denmark (statistikbanken.dk/MPK60).

Figure A.III: Labor income shares and spending shares by industry

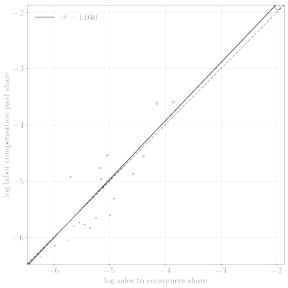
(a) Airlines



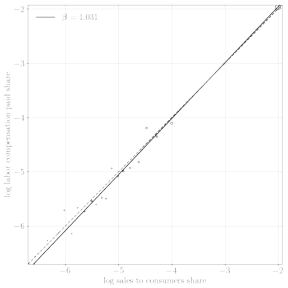
(b) Entertainment



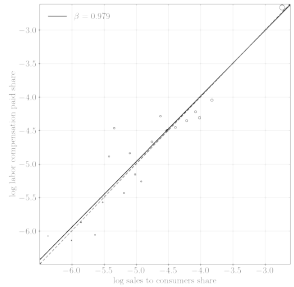
(c) Finance



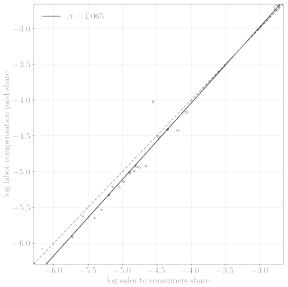
(d) Food away from home



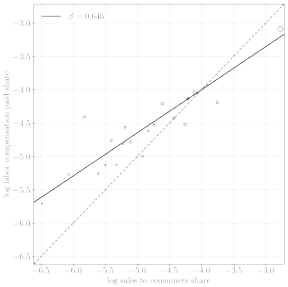
(e) Fuel, auto repair



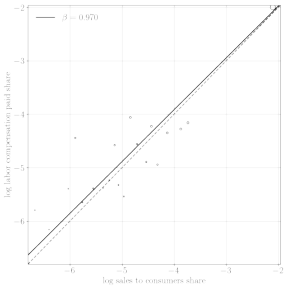
(f) Groceries



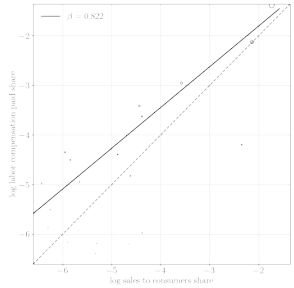
(g) Home improvement



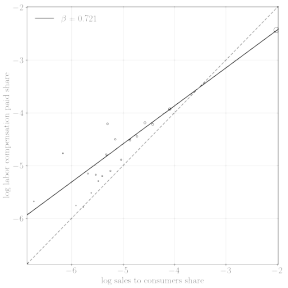
(h) Hotels, rental cars



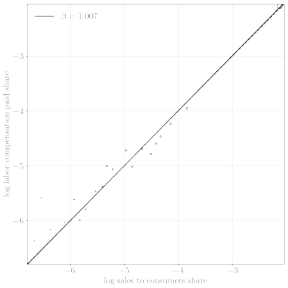
(i) Online stores



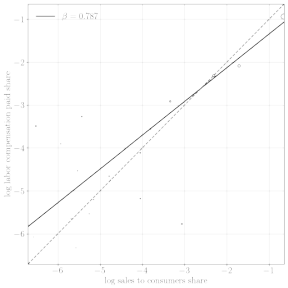
(j) Pers. services, pharmacies



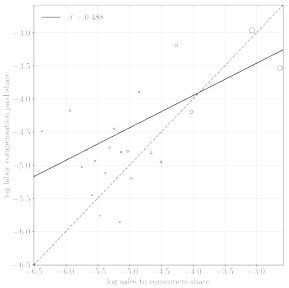
(k) Specialized retail stores



(l) Telecom., insurance



(m) Utilities

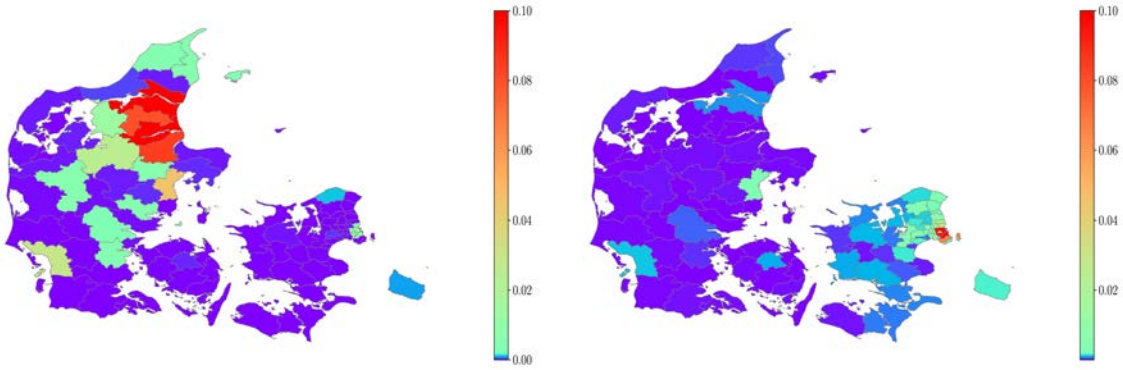


Notes: The sales-weighted binned scatters with 20 bins show the industry-specific relationships between log labor compensation paid (as share of total labor compensation in an industry) and log consumer spending received (as share of total consumer spending in an industry), across regions.

Appendix A.B Results on Regional Concentration, Sparsity, and Spending Into Cities

Figure A.IV: Labor compensation received across regions: Examples

(a) Restaurants in Mariagerfjord (b) Restaurants in Copenhagen



Note: We plot the fraction of labor compensation received by consumers in each region, from restaurants in Mariagerfjord (Panel a) and Copenhagen (Panel b). The scale is truncated at 0.10.

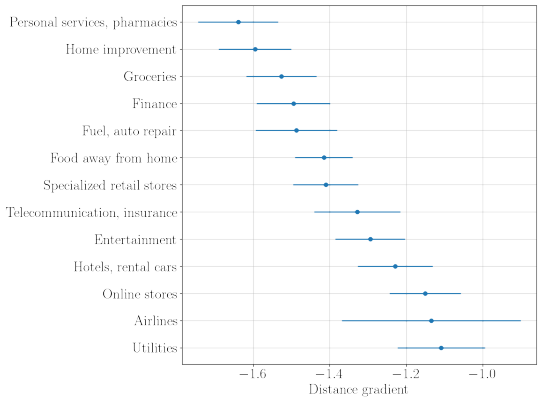
Table A.III: Gravity regressions

	log spending	log labor comp.	log intermediates	log mixed inc., surplus
	(1)	*(2)*	*(3)*	*(4)*
log distance	-1.463^{***}	-1.482^{***}	-0.557^{***}	-0.069^{***}
	(0.028)	(0.013)	(0.005)	(0.011)
Origin FE	Yes	Yes	Yes	Yes
Destination FE	Yes	Yes	Yes	Yes
Observations	2561036	1654401	5544292	5468081
R^2	0.332	0.279	0.059	0.002
F stat.	1268558.6	637191.3	348752.8	12746.1

$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

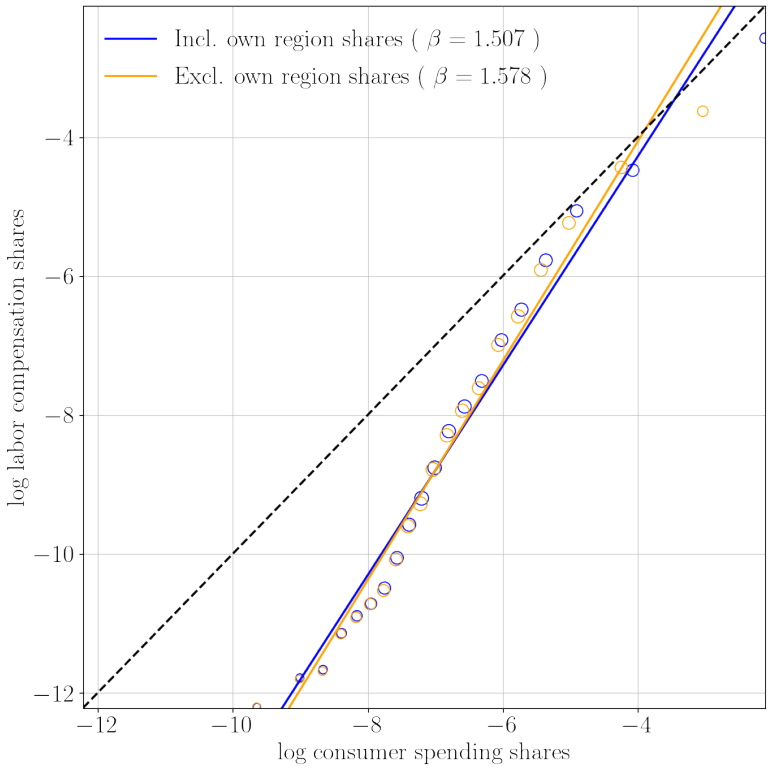
Notes: Distances are measured as driving distances between region centers.

Figure A.V: Gravity coefficients of labor compensation flows, by industry



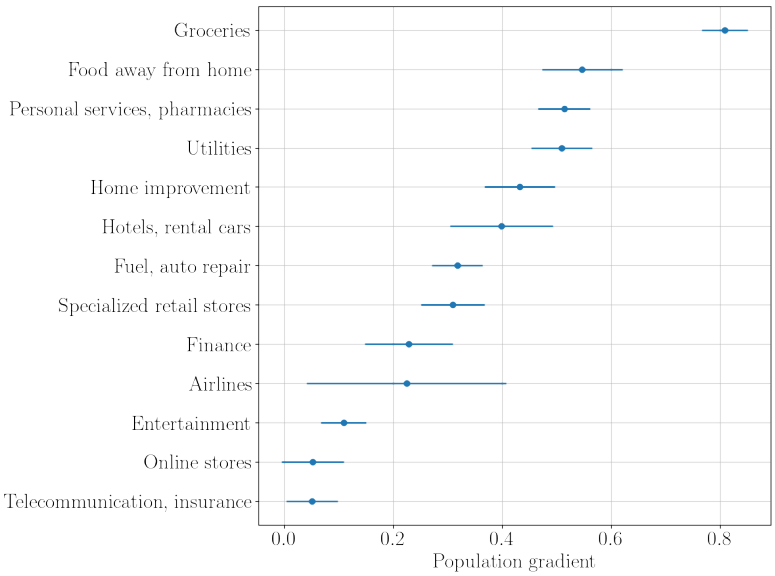
Notes: This figure shows regression coefficients for β using equation (2) for labor compensation flows by paying industry. We exclude observations with zero distance or zero flow. Distance is measured using driving distance between region centers. The solid line is the line of best fit, estimated using the cell-level data. The lines show 95% confidence intervals. All standard errors are clustered by region and industry.

Figure A.VI: Correlation between labor compensation shares and domestic consumer spending shares



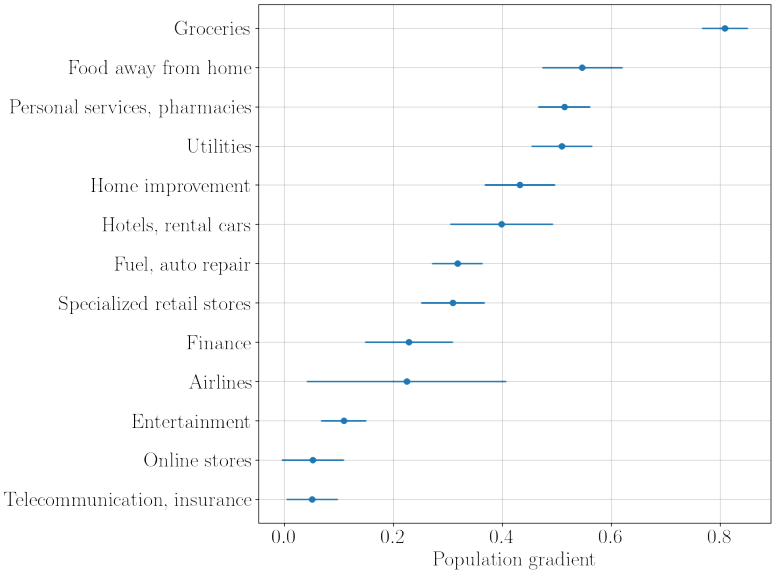
Notes: This figure shows a binned scatter plot with 20 bins between log labor compensation shares and log consumer spending shares. For each consumer cell i, we compute the distribution of labor compensation received across regions, and the distribution of consumer spending across regions. We either include or exclude own region flows. Weights are proportional to the number of bank customers in a consumer cell. The plot shows that the geographical distributions of labor compensation and consumer spending are correlated.

Figure A.VII: Population of home and receiving region: all industries



Notes: This plot shows the estimated coefficient of a regression of the average log population of regions receiving consumer spending within a specific spending category on the log population of the spending consumer cell. The error bars represent 95% robust confidence intervals.

Figure A.VIII: Population of home and receiving region: only in-person spending



Notes: This plot replicates Figure A.VII using data from a consumer spending matrix for only in-person spending.

Appendix A.C Results on Direct Foreign Spending Shares

Table A.V: Direct foreign spending shares

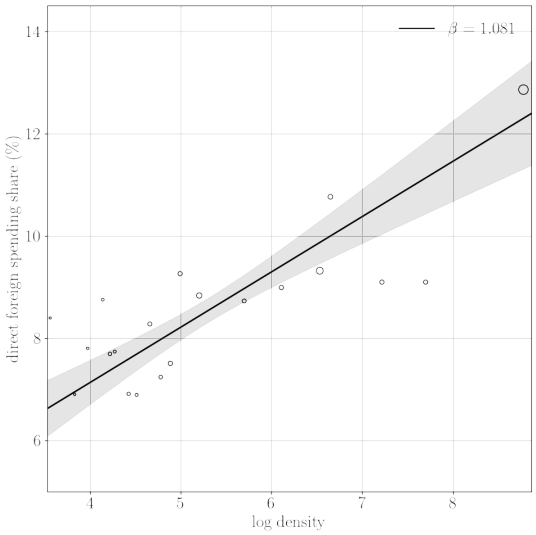
	direct foreign spending share (%)							
	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*	*(7)*	*(8)*
log population	1.622^{***}		1.766^{***}			1.664^{***}	1.616^{***}	1.648^{***}
	(0.264)		(0.244)			(0.229)	(0.225)	(0.218)
log spending per capita		3.683^{***}	4.724^{***}			3.737^{***}	3.887^{***}	5.009^{***}
		(0.595)	(0.576)			(0.641)	(0.618)	(0.960)
distance to border				-1.871^{***}		-1.329^{***}		
				(0.319)		(0.256)		
duration to border					-2.746^{***}		-1.897^{***}	-1.861^{***}
					(0.429)		(0.320)	(0.313)
log pop. $\times$ log spending p.c.								2.755^{**}
								(1.139)
const	-8.362^{***}	-34.926^{***}	-67.494^{***}	17.997^{***}	21.324^{***}	-48.573^{***}	-47.713^{***}	-61.803^{***}
	(2.820)	(7.271)	(7.333)	(1.527)	(1.931)	(7.888)	(7.731)	(11.796)
Observations	2741	2741	2741	2741	2741	2741	2741	2741
R^2	0.226	0.060	0.323	0.125	0.142	0.382	0.386	0.412
F stat.	37.7	38.3	64.0	34.3	41.0	47.1	48.5	37.3

$^{*}p < 0.1$; $^{**}p < 0.05$; $^{***}p < 0.01$

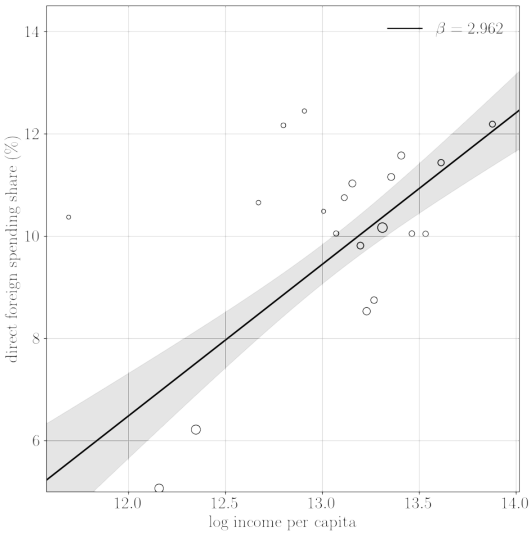
Notes: Foreign spending shares are in percent. For the log population $\times$ log income per capita interaction, the two individual variables are demeaned. We weight observations using the number of bank customers per cell. Standard errors are robust. We measure driving distance and driving duration from region centers to the nearest of eight foreign addresses (Malmø, Helsingborg, Rostock, Puttgarden, and four large border shopping centers along the Jutland-Germany border), using Google Maps API services.

Figure A.IX: Foreign spending by population density and total income

(a) By population density

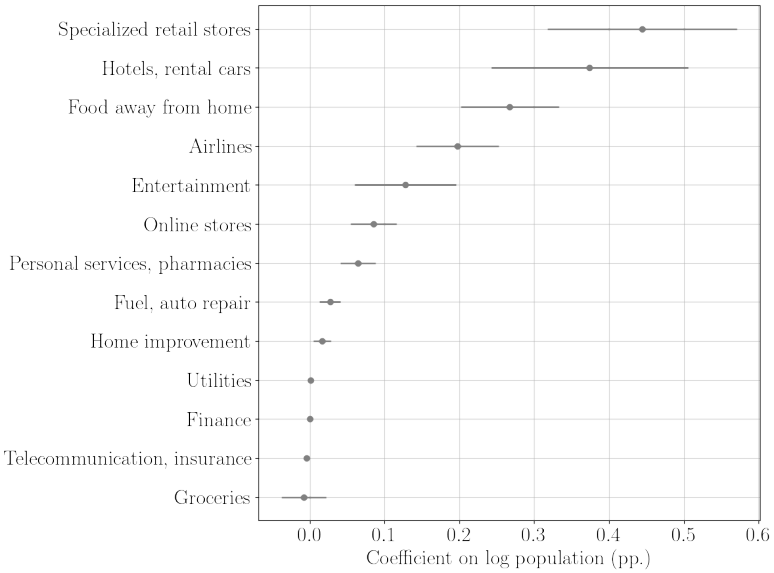


(b) By total income per capita



Notes: Panel (a) shows the direct foreign spending share of consumer cells by their home region's log population density, defined as log of population divided by area. Panel (b) shows the direct foreign spending share by log income per capita. Panel (b) excludes consumers that are "not in the workforce" as their income is likely mismeasured due to the absence of within-household transfer flows in the current iteration of the DEA. Both plots are binned scatter plots with 20 bins mirroring the number of regions in our analysis. Weights are proportional to the number of bank customers in a municipality. The shaded area are 95% robust confidence intervals.

Figure A.X: Log population slopes by foreign spending category



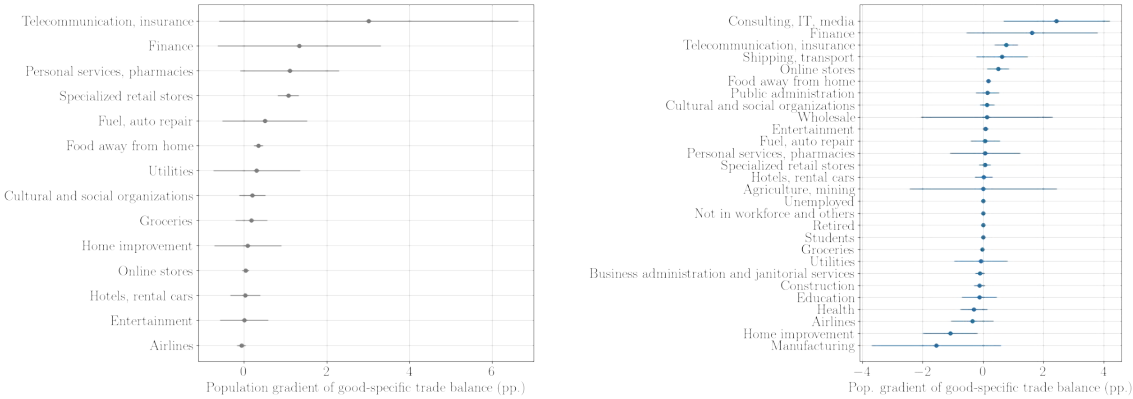
Notes: This figure shows estimated coefficients of a regression of direct foreign spending in a given category on log population. Weights are the number of bank customers in the spending cell. Coefficients are expressed in percentage points. The sum of all coefficients yields the estimated coefficient in Panel (a) of Figure VI. The error bars are 95% robust confidence intervals.

Appendix A.D Goods-Level Trade Balances

Figure A.XI: Goods-level regional trade balances and population size

(a) Consumers spending flows

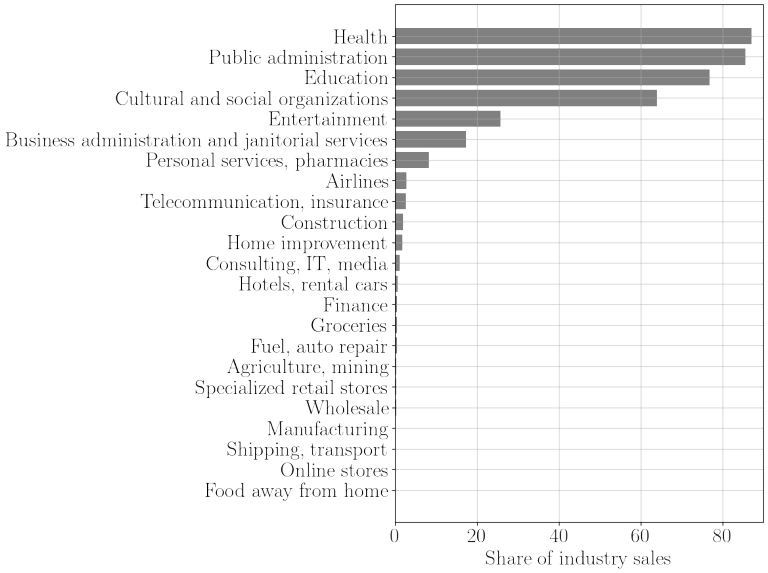
(b) Intermediate input trade flows



Notes: This plot shows estimated coefficients of a regression of goods-level trade balances (sales minus purchases by region residents elsewhere in Denmark) on log population, across regions. Weights are region GDP. Error bars are 95% robust confidence intervals.

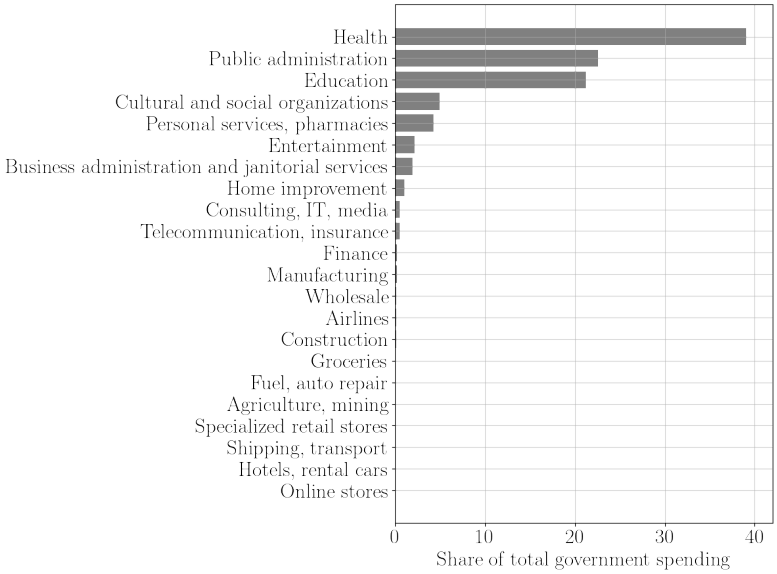
Appendix A.E Results on the Role of the Government in Denmark

Figure A.XII: Share of industry sales accounted for by government spending



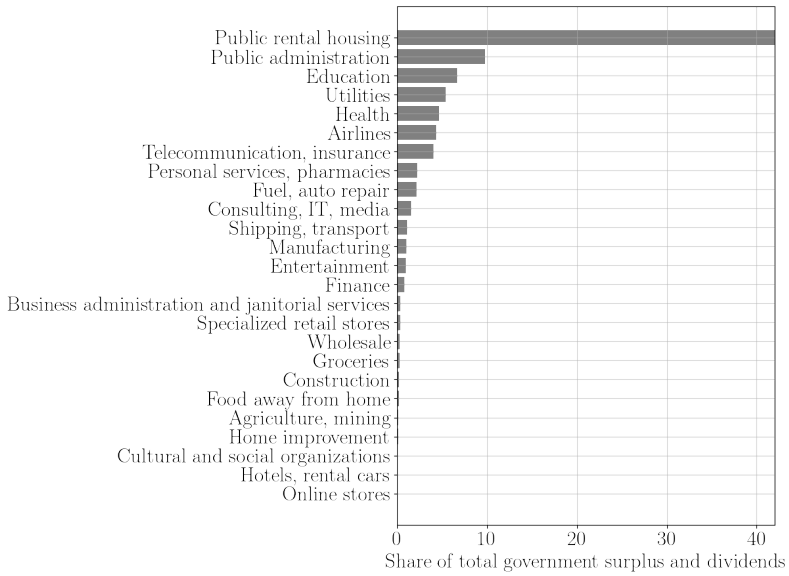
Notes: This figure shows the share of an industry's total sales accounted for by government spending.

Figure A.XIII: Distribution of government spending across industries



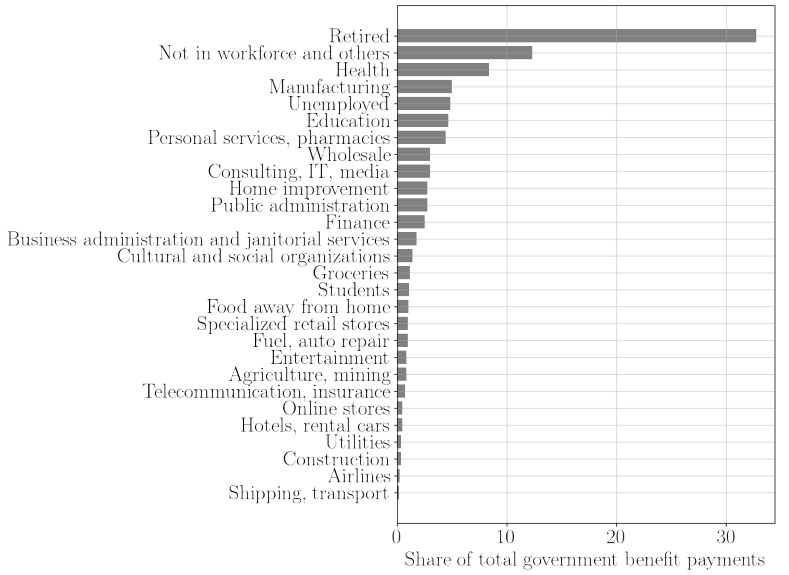
Notes: This figure shows the share of total government spending an industry accounts for.

Figure A.XIV: Shares of government surplus and dividends by industry



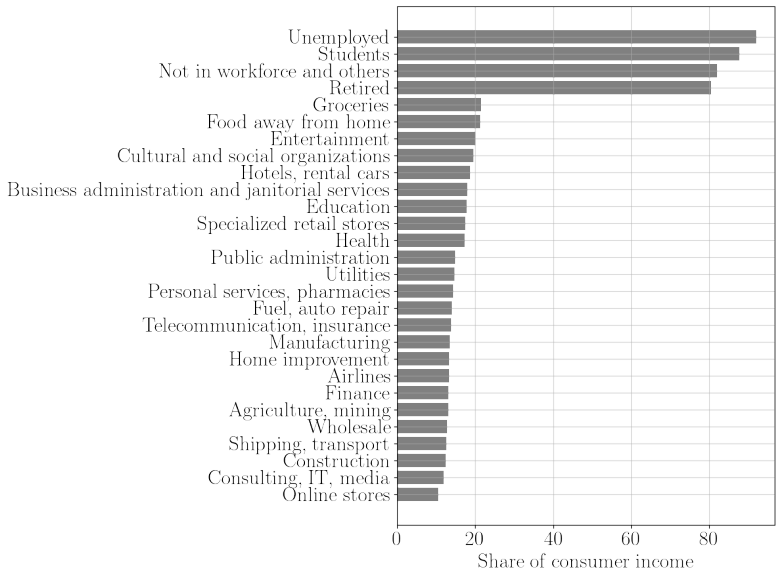
Notes: This figure shows which industries account for what share of total government surplus and dividends.

Figure A.XV: Shares of total government social benefits received by consumer industry



Notes: This figure shows which consumer industries account for what share of total government social benefits received.

Figure A.XVI: Government benefits received as shares of total consumer income



Notes: This figure shows the share of total consumer income that is accounted for by government social benefits received across consumer industries.

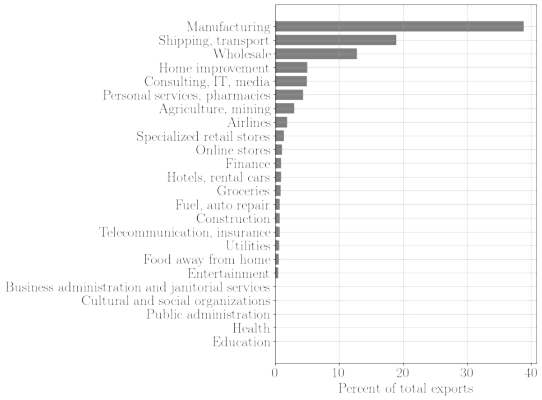
Appendix A.F Details on Calibration

Table A.VII: Calibration overview

Parameter(s)	Symbol	Calibration target	Flow numbers from Table II
Spending shares	$\alpha_{i\mathcal{R}}, \alpha_{is}, \alpha_{isj}$	Disaggregated consumer spending flows	1, 2
Labor compensation shares	λ_{ij}, γ_j^N	Disaggregated labor compensation flows	10
Profit income shares	κ_{ij}, γ_j^K	Mixed income, surplus to consumers	11, 12, 13
Income tax rate	τ_i	Consumer non-product taxes and social contributions	4, 5
Value-added tax rate	τ_i^{vat}	Consumer product taxes paid	3
Intermediate input shares, imports	$\omega_{js}, \omega_{jsj'}, \gamma^X$	Trade flows in domestic intermediates, producer imports	20, 25
Corporate tax rate	τ_j^{corp}	Dividends and surplus from gov. enterprises, taxes	21, 22, 23
Relative distribution of exports	$\tilde{x}_j/\tilde{x}_{j'}$	Producer exports	28
Gov. spending share of gov. revenue	g_j	Domestic government spending	26
Gov. import share of gov. revenue	$\tilde{g}$	Government imports	29
Gov. transfer share of gov. revenue	t_i	Consumer social benefits received, pension adjustment	14, 15
Aggregate export flows	$\tilde{x}_j$	Aggregate GDP	
Within-industry elasticity of sub. of consumers	σ	Cobb-Douglas (1) as baseline, 2 later	
Within-industry elasticity of sub. b/w labor	ζ	Cobb-Douglas (1) as baseline, 2 later	
Within-industry elasticity of sub. b/w intermediates	η	Cobb-Douglas (1) as baseline, 2 later	
Elasticity of export demand	γ	Cobb-Douglas (1) as baseline, 2 later	

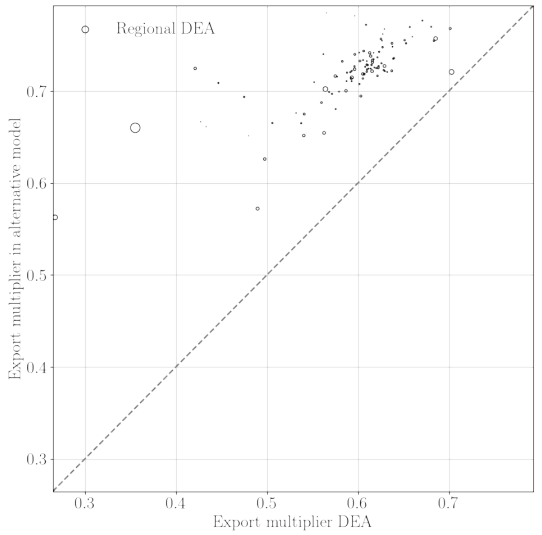
Appendix A.G Results for Application I

Figure A.XVII: Distribution of exports across industries



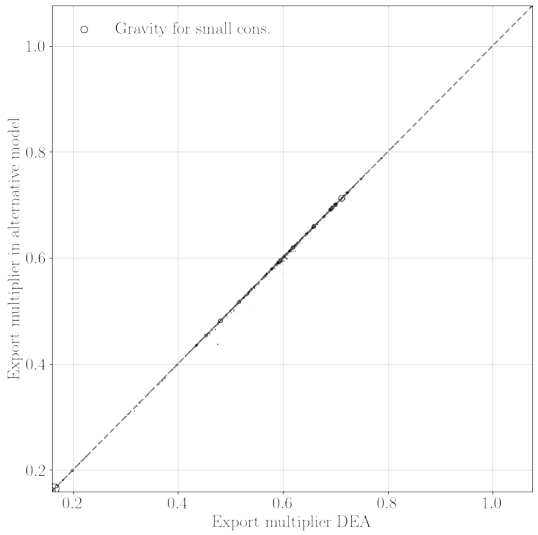
Notes: This figure shows the distribution of total exports across industries.

Figure A.XVIII: Full DEA vs regional DEA: Export multipliers across regions



Notes: This figure shows a scatter plot of export multipliers in a regional DEA model vs the full DEA model we introduced and calibrated in Section VI. The regional model is equal in all regards, and calibrated exactly as outlined in Section VI, except that it does not have an industry dimension. Node size is proportional to exports.

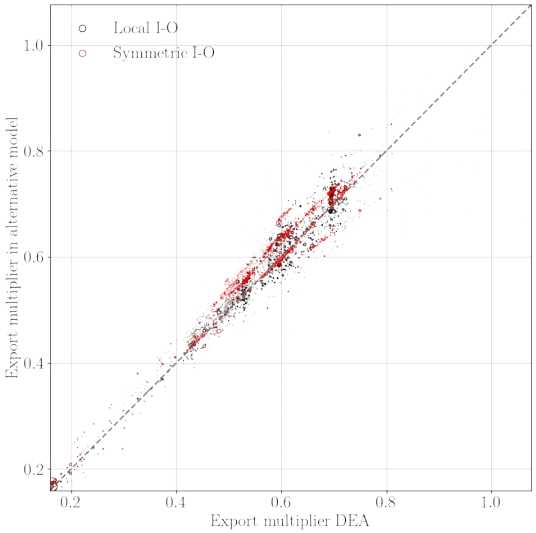
Figure A.XIX: Testing for overfitting of disaggregated consumer spending: Export multipliers



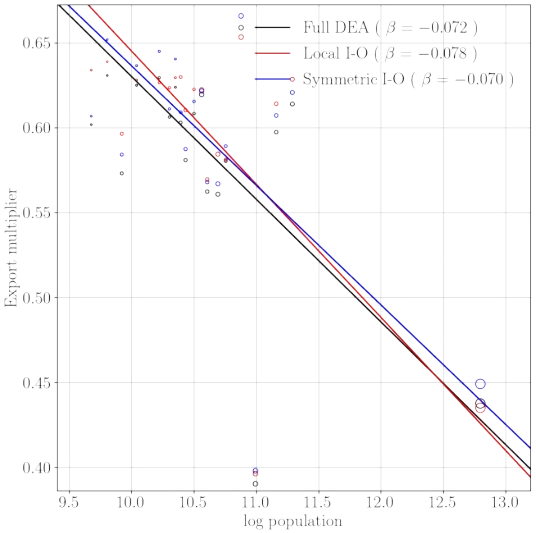
Notes: This figure shows a scatter plot of export multipliers in a modified DEA model vs the full DEA model we introduced and calibrated in Section VI. The modified model is equal in all regards, and calibrated exactly as outlined in Section VI, except that the smallest 90% of the bilateral consumer spending flows by imputed values from the standard gravity regression (2). Node size is proportional to exports.

Figure A.XX: Testing for measurement error in the disaggregated intermediates trade flows: Export multipliers

(a) Scatter plot of all export multipliers



(b) Binned scatter against log population

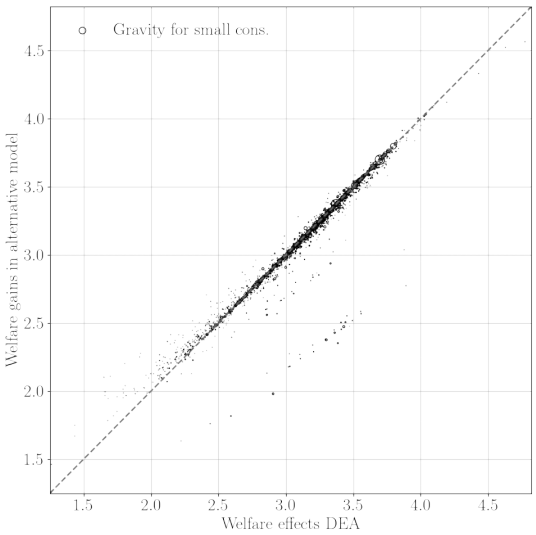


Notes: Panel (a) shows a scatter plot of export multipliers in two modified DEA models vs the full DEA model we introduced and calibrated in Section VI. Both modified models are equal in all regards, and calibrated exactly as outlined in Section VI, except that: the first modified model assumes that all intermediates trade flows are local and do not cross region borders; the second modified model assumes that all intermediates trade flows are symmetric across purchases, that is, Ω is replaced by the unique rank-1 matrix that has the same column sums and same row sums as the correct Ω matrix. Node size is proportional to exports. Panel (b) regresses the export multipliers of the three models against log population in a 20-bin scatter plot.

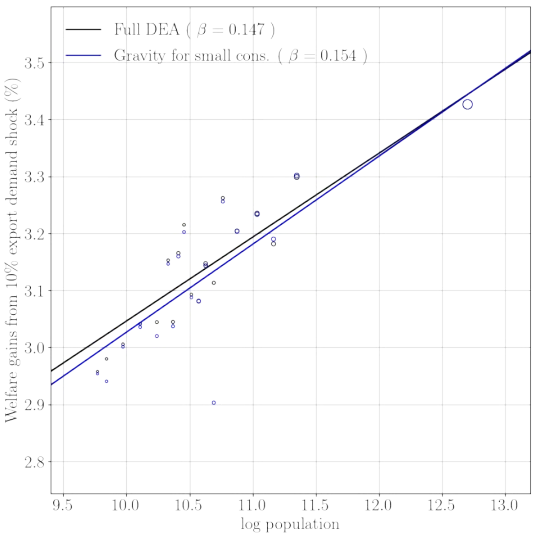
Appendix A.H Results for Application II

Figure A.XXI: Testing for overfitting of disaggregated consumer spending: Distributional gains from uniform export shock

(a) Comparing welfare gains with the DEA



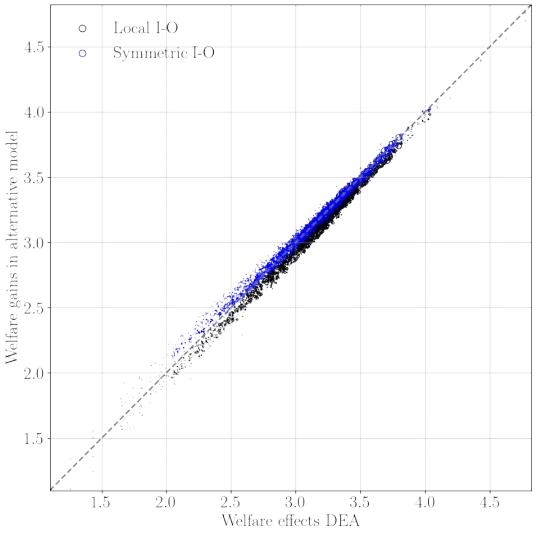
(b) Welfare gains by log population



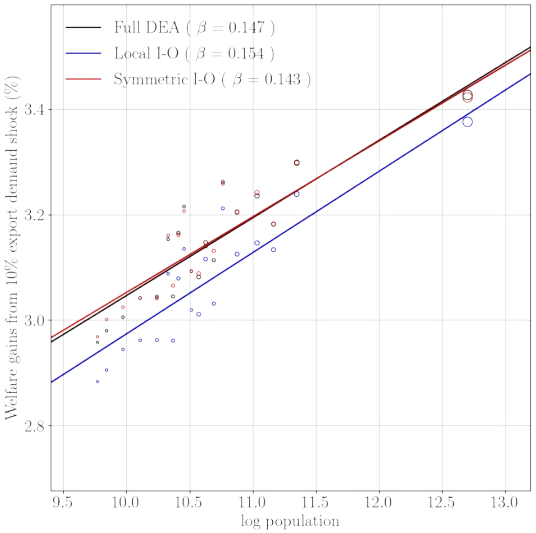
Notes: Panel (a) compares the distribution of the welfare gains from a 10% increase in overall export demand across two models: the full DEA model; and a model that is equal in all regards, and calibrated exactly as outlined in Section VI, except that the smallest 90% of the bilateral consumer spending flows by imputed values from the standard gravity regression (2). Panel (b) shows a binned scatter plot of welfare gains across log region population across the models. Scatter weights are number of bank customers by cell.

Figure A.XXII: Testing for measurement error in the disaggregated intermediates trade flows: Distributional gains from uniform export shock

(a) Comparing welfare gains with the DEA



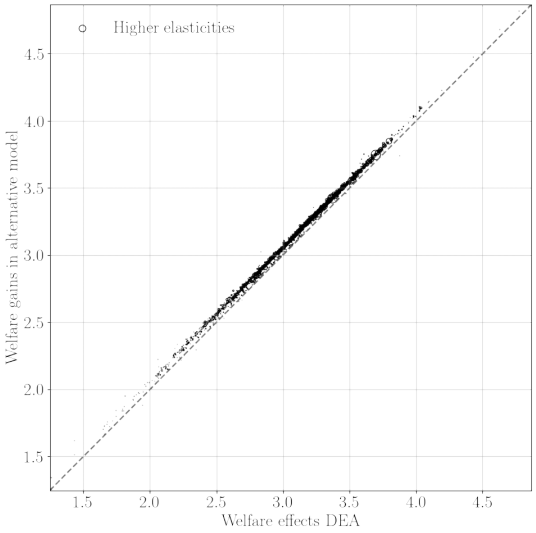
(b) Welfare gains by log population



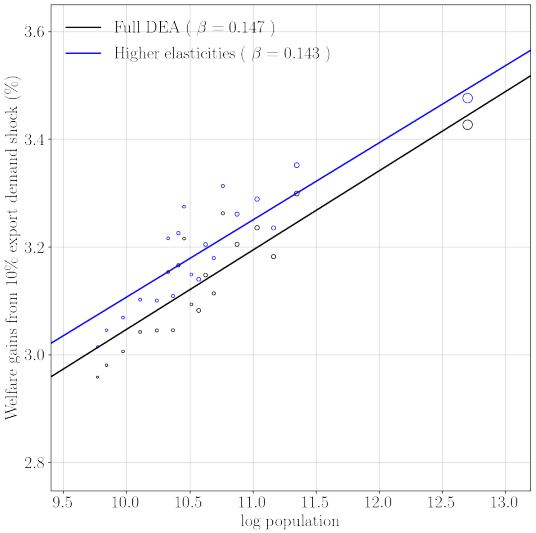
Notes: Panel (a) compares the distribution of the welfare gains from a 10% increase in overall export demand across two models: the full DEA model and two modified models. Both modified models are equal in all regards, and calibrated exactly as outlined in Section VI, except that: the first modified model assumes that all intermediates trade flows are local and do not cross region borders; the second modified model assumes that all intermediates trade flows are symmetric across purchases, that is, Ω is replaced by the unique rank-1 matrix that has the same column sums and same row sums as the correct Ω matrix. Panel (b) shows a binned scatter plot of welfare gains across log region population across the models. Scatter weights are number of bank customers by cell.

Figure A.XXIII: Comparison of distributional welfare gains under Cobb-Douglas with ones for higher elasticities

(a) Comparing welfare gains with the DEA



(b) Welfare gains by log population



Notes: Panel (a) compares the distribution of the welfare gains from a 10% increase in overall export demand across two models: the DEA model with unitary elasticity of substitution and one with higher elasticities of substitution, $\sigma = \zeta = \gamma = \eta = 2$. Panel (b) shows a binned scatter plot of welfare gains across log region population across the models. Scatter weights are number of bank customers by cell.

Appendix B Accounts Aggregated by Cell Type Based on Danish Disaggregated Economic Accounts

Table A.VIII: Aggregate consumer account

Outflows			Inflows		
Outflow	Outflow to	Value (bn DKK)	Inflow	Inflow from	Value (bn DKK)
Domestic consumer spending	Producers	771.9	Labor compensation	Producers	1141.81
Foreign consumer spending	Rest of the World	81.94	Mixed income from non-corporate producers	Producers	80.72
Consumer product taxes paid	Government	173.18	Surplus of corporate producers to consumers (dividends)	Producers	38.48
Consumer non-product taxes paid	Government	566.35	Surplus of owner-occupied housing to consumers	Producers	83.27
Consumer social contributions paid	Government	181.06	Consumer social benefits received	Government	422.2
Consumer interest paid	Capital accumulation	29.67	Consumer adjustment for pension entitlements received	Government	92.48
Consumer natural resource rents paid	Capital accumulation	3.44	Consumer interest received	Capital accumulation	5.32
Consumer other transfers paid	Capital accumulation	44.82	Consumer pension investment income	Capital accumulation	75.48
Consumer gross saving	Capital accumulation	130.05	Consumer natural resource rents received	Capital accumulation	3.44
			Consumer other transfers received	Capital accumulation	39.21
Total value outflows		1982.42	**Total value inflows**		1982.42

Table A.IX: Aggregate producer account

Outflows			**Inflows**		
Outflow	Outflow to	Value (bn DKK)	Inflow	Inflow from	Value (bn DKK)
Labor compensation	Consumers	1141.81	Domestic consumer spending	Consumers	771.9
Mixed income from non-corporate producers	Consumers	80.72	Domestic intermediates	Producers	1414.53
Surplus of corporate producers to consumers (dividends)	Consumers	38.48	Domestic government spending	Government	572.26
Surplus of owner-occupied housing to consumers	Consumers	83.27	Domestic capital accumulation spending	Capital accumulation	359.51
Domestic intermediates	Producers	1414.53	Producer exports	Rest of the World	1077.94
Dividends and surplus of government-owned/operated producers to government	Government	67.86			
Producer product taxes paid	Government	71.94			
Producer net non-product taxes paid and subsidies received	Government	82.79			
Producer net interest, transfers, and saving	Capital accumulation	422.41			
Producer imports	Rest of the World	792.33			
Total value outflows		4196.14	**Total value inflows**		4196.14

Table A.X: Aggregate government account

Outflows			Inflows		
Outflow	Outflow to	Value (bn DKK)	Inflow	Inflow from	Value (bn DKK)
Consumer social benefits received	Consumers	422.2	Consumer product taxes paid	Consumers	173.18
Consumer adjustment for pension entitlements received	Consumers	92.48	Consumer non-product taxes paid	Consumers	566.35
Domestic government spending	Producers	572.26	Consumer social contributions paid	Consumers	181.06
Government imports	Rest of the World	4.26	Dividends and surplus of government-owned/operated producers to government	Producers	67.86
Government net interest, transfers, and saving	Capital accumulation	51.99	Producer product taxes paid	Producers	71.94
			Producer net non-product taxes paid and subsidies received	Producers	82.79
Total value outflows		1143.19	**Total value inflows**		1143.19

Table A.XI: Aggregate rest of the world account

Outflows			Inflows		
Outflow	Outflow to	Value (bn DKK)	Inflow	Inflow from	Value (bn DKK)
Producer exports	Producers	1077.94	Trade surplus	Capital accumulation	100.55
			Foreign consumer spending	Consumers	81.94
			Producer imports	Producers	792.33
			Government imports	Government	4.26
			Capital accumulation imports	Capital accumulation	98.86
Total value outflows		1077.94	**Total value inflows**		1077.94

Table A.XII: Aggregate capital accumulation account

Outflows			**Inflows**		
Outflow	Outflow to	Value (bn DKK)	Inflow	Inflow from	Value (bn DKK)
Consumer interest received	Consumers	5.32	Consumer interest paid	Consumers	29.67
Consumer pension investment income	Consumers	75.48	Consumer natural resource rents paid	Consumers	3.44
Consumer natural resource rents received	Consumers	3.44	Consumer other transfers paid	Consumers	44.82
Consumer other transfers received	Consumers	39.21	Consumer gross saving	Consumers	130.05
Domestic capital accumulation spending	Producers	359.51	Producer net interest, transfers, and saving	Producers	422.41
Capital accumulation imports	Rest of the World	98.86	Government net interest, transfers, and saving	Government	51.99
Trade surplus	Rest of the World	100.55			
Total value outflows		682.38	**Total value inflows**		682.38

Appendix C Overview of Data Sources

We rely on several types of data to construct the disaggregated economic accounts. First, we use customer data from Danske Bank with information about individual customers and about their transactions such as payments and transfers. Second, we use administrative data from a range of government registers such as the population register and the income register with information about individuals and the employment register with information about employment relations. Third, we use publicly available tabulations of aggregate statistics, such as housing statistics, health statistics and securities statistics.

We use these different types of data to disaggregate the flows between consumers, producers and the government described in the national accounts to the level of region-by-industry cells. In the following appendices, we provide more details about the data sources and the various steps of the disaggregated measurement. Table A.XIII lists the disaggregated datasets, data sources, and the Appendix where laying out the details of the disaggregated measurement.

We strive to obtain consistency across all the disaggregated measurement by relying, whenever possible, on the same sample of individuals, the entire adult population in 2018, and a uniform assignment of individuals and firm establishments to cells based on government registers. The only instance where we need to deviate from these general rules is when we construct of the spending matrix. Here, we work with the sample of Danske Bank customers, roughly 20% of the national population, as we do not have data on spending for customers of other banks. Moreover, we need to assign individuals and firm establishments to cells using the bank's internal data, as confidentiality concerns prevent us from merging the bank data and government registers. Finally, we use data for both 2018 and 2019 to maximize sample size and then scale to match 2018 national accounts.

Table A.XIII: Overview of data sources

Disaggregated dataset	Microdata from Danske Bank	Microdata from government registers	Aggregate statistics	Details
Region-by-industry cells	Customer records, transaction data on incoming transfers	Population register (BEF), labor market register (AKM), income register (IND), employment registers (BFL, IDAN)		Appendix D
Consumer spending matrix	Customer records, transaction data on card payments, cash withdrawals, bill payments and mobile applications	Income register (IND), credit register (URTE), auto register (DMR)	National Housing Statistics (BOL101)	Appendix E
Consumer taxes (product)			Input-Output table (NIO3)	Appendix F
Consumer taxes (non-product)		Income register (IND), pension contribution register (INPI), population register (BEF)		Appendix G
Consumer interest, transfers, and saving (paid)		Population register (BEF), income register (IND)		Appendix H
Labor compensation matrix		Income register (IND), pension contribution register (INPI), employment registers (BFL)		Appendix I
Consumer dividend, mixed income, and surplus matrix		Income register (IND), employment register (IDAN)	General Firm Statistics (GF5), Registered Securities Statistics (DNVPDKS)	Appendix J
Government benefits to consumers		Income register (IND), pension contribution register (INPI), population register (BEF)		Appendix K
Consumer interest, transfers, and saving (received)		Income register (IND)		Appendix L
Intermediates trade matrix			Input-Output tables (NIO1, NIO2, NIO3), Cargo transport statistics (Tables NVG23, SKIB481)	Appendix M
Government dividend and surplus income		CVR register (hand-collected)		Appendix N
Consumption of government and NPISH output	Transaction data on payments	Population register (BEF), education register (UDD), income register (IND)	Public Expenditure Statistics (SYGU1), Health Statistics (INDAMP01), Education Statistics (UDDAKT20), Child Care Statistics (BOERN4), Culture and Leisure Statistics (BIB1)	Appendix O

Appendix D Defining Region-by-Industry Cells

The unit of our measurement is region-by-industry cells, the interaction of geographical regions and economic industries. Specifically, we define 99 regions (98 Danish municipalities and one foreign region) and 31 industries (Table A.I). We choose this definition of region and industries because we can observe it consistently across all underlying datasets and because it reveals a large degree of heterogeneity in disaggregated flows.

There are 24 industries that pay labor compensation to consumers. They include retail industries selling directly to consumers (e.g., food away from home, entertainment, grocery stores, drug stores), non-retail industries transacting mostly with firms (e.g., wholesale, agriculture, manufacturing, business services), and government-operated industries (e.g., public administration, health, education). We map six-digit NACE industry codes to the 24 industries with employment (the European NACE is based on the standard United Nations ISIC). Additionally, there are 4 separate industries for the non-working parts of the population (e.g., retired, students, unemployed, out of workforce) and 3 industries providing housing without employees.

In the government registers, we assign all adults to a region based on their home address at the start of 2018 observed in the administrative population register (BEF) and to an industry based on the NACE code of the firm establishment responsible for the largest part of their 2018 labor income in the labor market register (AKM). Individuals with no labor income are assigned to an industry based on their age, observed in the administrative population register (BEF), and other income sources, observed in the income register (IND). Specifically, individuals with no labor income are assigned to the industry "retired" if they are older than 65 years; to the industry "students" if they receive a government stipend (for which higher education students are almost universally eligible); to the industry "unemployed" if they receive unemployment or cash benefits; and otherwise to the industry "out of workforce." We assign firm establishments to producer region-by-industry cells using information from the employment registers (BFL and IDAN).

In the Danske Bank data, we identify consumers' industry by extracting the name of the employer from incoming salary payments and by identifying incoming government transfers (retired, students, government stipend, unemployed). We use the banks' customer records to identify their region of residence. The address register in the bank is linked to the government's address registers and updated on a monthly basis. To ensure that moves across regions do not distort the spending patterns, we update an individual's region every month when constructing the spending matrix. However, consistent with the assignment in the government registers, we define the industry of main employment on an annual basis, as the industry paying the largest share of consumer income.

To ensure anonymity, we censor information for the (very few) cells containing fewer than 3

individuals or establishments in all datasets. In the spending matrix, we censor information for cell-to-cell flows based on fewer than 10 transactions in total (in total, less than 0.1% of national spending). We instead impute these flows, setting them equal to the average per capita spending flows of all other cells in the same region.

Appendix E Measuring the Disaggregated Consumer Spending Matrix

Appendix E.A Data and Sample

To construct the spending matrix, we rely on data from Danske Bank for 2018 and 2019. We observe transaction-level information on consumers and merchants only for this period. Our sample consists of adult customers who conducted at least one transaction per month and registered their main bank account at Danske Bank throughout 2018 and 2019.[A1] For each customer, we observe all incoming and outgoing transactions. Card payments accounted for 47% of the total value of payment transactions. The most common card was Dakotan (debit cards issued by Nets A/S), followed by MasterCard (debit and credit cards) and Visa (debit and credit cards). Cash withdrawals accounted for 7% of transaction value, while bill payments (direct debits and bank transfers to merchants) accounted for 45%. Mobile applications, such as Apple Pay or the Denmark-specific MobilePay, made up 1%.

Appendix E.B Identifying the Merchant Region

We extract the address of the merchant establishment (i.e., the store) from the string that accompanies payment transactions in the bank's internal computer system. The information for card and mobile payments differs slightly by payment type.

- Dakotan statements include a unique ID number for each merchant establishment for transactions in Denmark and the country name for transactions abroad. We match the Danish merchant IDs to the exact merchant address using a table issued by Nets A/S.
- MasterCard includes a detailed merchant address directly in the transaction string in the following format: merchant ID, shop name, street name, house number, postal code, country.
- Visa reports the merchant ID, shop name, and town for each transaction. The merchant IDs used by Visa and MasterCard generally coincide. Based on the MasterCard data, we can therefore construct a table matching merchant ID and detailed address. In very few cases, a merchant ID gets used twice for a Danish merchant and a foreign merchant. In these cases, we assume that the transaction was with the Danish merchant.

[A1] All adults register one "main" account with the Danish government, through which they conduct all financial interactions with the government.

- Some transactions in MobilePay and Apple Pay contain merchant addresses, but many do not. There is no clear pattern in which transactions contain addresses, which is likely due to the recent development of these transactions.

For all card and mobile payments, we extract the merchant address and convert it to a consistent format using an API service provided by the government agency Styrelsen for Dataforsyning og Effektivisering (dawa.aws.dk/dok/api/adgangsadresse). This conversion identifies precise geocodes for each merchant while accounting for misspelled addresses and addresses that appear twice due to minor differences in formatting or spelling. The API compares the merchant address from a transaction with its database of official addresses. It iterates in a Levenshtein manner (i.e., it calculates the number of letters/digits that must be exchanged before one string is equal to another). We force the Levenshtein process to consider only addresses with exactly identical postcodes. Municipalities are combinations of several postcodes. By restricting to the same postcode, we ensure that the Levenshtein process cannot change the municipality information, the key information that we use to construct the spending matrix.

If the API cannot match the address unambiguously (so called C-match), we remove the first line of the address, which often combines abbreviations of merchant and street name, making it difficult to automatically recognize. We also check whether the address contains the name of a shopping mall, rather than an official address. If so, we replace the name of the mall with the mall's address and rerun the API process. Finally, we manually research the official address of the 100 most common unmatched addresses.

Using this procedure, we identify the official shop address for 95% of card and mobile spending. We assume that the remaining 5% go proportionally to the same regions as other card and mobile spending. These remaining 5% also include cases where mobile applications (e.g., MobilePay) and online services (e.g., PayPal) do not directly send the purchase amount to a sales terminal, but transfer to a central account first before paying the merchant.

For bill payments, we use a slightly different approach. We directly observe the merchant's postal code for recurrent bill payments, which make up 67% of all payments. These observed postal codes allow us to infer the merchant region for the majority of remaining bill payments. Specifically, we split merchants into 48 industries. We calculate the number of transactions from each consumer region going to each of these 48 industries. To minimize noise, we keep industries where at least 50% of incoming bill payments contain postal code information and where we observe at least 200 incoming transactions from every consumer region. (Industries receiving 80% of total bill transaction value satisfy these two requirements.) For these industries, we then assume that bill payments flow to the same postal codes as bill payments with observed postal codes from the same consumer region to the same industry. For the remaining industries (covering 20% of bill transaction value), we assume that bill payments flow to the same postal codes as card

payments from the same consumer region to the same industry.

We generally do not observe how cash withdrawals are spent, but we assign them to merchants in proportion to observed card spending in Denmark if the withdrawal was in Denmark and to observed card spending abroad otherwise. In a few cases, country information is missing, so we assume that withdrawals without decimal points (e.g., 100.00 DKK) are withdrawn and spent in Denmark and that other withdrawals (e.g., 100.76 DKK) are spent abroad.

Appendix E.C Identifying the Merchant Industry

We extract the merchant category code (MCC), a classification for the type of goods sold, for each merchant in the bank transaction data. To create a disaggregated system with consistently defined producer cells, we need to map MCCs to the industry codes used in the employment and production data. However, no such mapping exists so far. We therefore create a new cross-walk between MCCs and NACE industries.

First, we observe each merchant's Danish business identification number (CVR) and MCC in the bank's system. We link the CVR to the Danish Central Business Register where we can retrieve the merchant's industry (at the level of 741 NACE codes). Second, we manually assign MCCs to our 31 industries (only 14 of which are consumer-facing and are assigned MCCs). We then identify which of our industries appear most frequently among merchants in each of the 741 NACE codes. In very few cases, two industries appear equally often and we manually research the largest firms to identify the right match.

We create an alternative mapping between a merchant classification system called PCAT and our industries. The PCAT usually appears as part of the electronic transaction information for bill payments and can easily be mapped to our industries. MCC and PCAT are missing for some bill payments, amounting to 8% of total transaction value through bills. We assume that these payments go proportionally to the same industries as other bill payments by the same consumer cell.

Appendix E.D Online Spending

We identify whether card transactions took place in a physical store or online. Dakotan transactions include a straightforward indicator for online transactions. Mastercard and Visacard transactions contain ISO 8583 information, an internationally standardized message sent by a sales terminal in a transaction. If the POS7 code (input method) equals 1, 6, K, or L or if the POS5 (cardholder) code equals 5, the transaction is online. We treat payments using mobile applications (e.g., MobilePay) and online services (e.g., PayPal) that do not report a physical merchant ID and address as online transactions. For digital payment services, such as PayPal and DoorDash, we typically see the

correct MCC of the establishment receiving the final sales.

In constructing the spending matrix, we generally treat online spending identically to spending in a physical store. That means that we identify the region-industry cell of the merchant and assign the incoming spending to this cell. However, we adjust the regional (but not the industrial) distribution of online spending for industries where we know that consumption of the final good takes place entirely in a physical location. In these cases, we assign all online spending using the regional distribution of spending on physical merchants. The online spending on these industries often goes to the central payment terminal of a parent company before being assigned to the physical merchant. For instance, online purchases of cinema tickets are often booked through a central company terminal in Copenhagen, even though consumption happens in local movie theaters. The full list of industries where we adjust the regional distribution is: food away from home; entertainment; medical and other specialized merchants; commuting; vehicle repair; hotels; rental cars; home improvement services. (These industries are generally sub-categories of our 31 final industries.)

We also adjust spending on airlines flowing into Copenhagen because we know that the airline establishments receiving the spending are actually located in the neighboring Tårnby region, which also contains the airport. Specifically, we reassign a share of each consumer cell's spending on airlines flowing into Copenhagen to airlines in Tårnby, so that Copenhagen's share in airline spending received equals its share in airline employment (see also Figure A.III).

Appendix E.E Improving the Spending Matrix with Government Data

Consumer spending on four types of goods are not captured well in bank transaction data: housing, financial services, vehicles, and water and waste services. We replace the transaction-based values for these four goods in the spending matrix with adjusted values derived from combining Danske Bank data with government registers.

Appendix E.E.1 Consumption of Housing

We use different methods to assign consumption of owner-occupied housing and rented housing. First, owner-occupied rents are notoriously difficult to measure because they involve no financial transaction. However, the administrative income register (IND) contains the imputed rental value of owner-occupied housing for every individual. We thus allocate the national consumption of owner-occupied housing to consumer cells in proportion to the imputed rental value of their owner-occupied housing. Expenditure on owner-occupied housing flows to the producer cell for owner-occupied housing (our industry number 30) located in the same region as the home owners. The operating surplus of an owner-occupied housing producer cell in a region (SNA B.2G) then

goes back to the consumer cells owning homes in that same region.

Second, we do not observe all rental payments in the bank transaction data. We instead distribute the national consumption of rented housing across consumer cells in proportion to their estimated rental costs. We observe some rental payments in the bank data, which we use to estimate:

$$rental\ payment_p = \alpha + \vartheta\, region_p + \phi\, industry_p + \psi_1\, age_p + \psi_2\, age_p^2 + \varepsilon_p, \tag{A.1}$$

where p is an individual renter. Using the estimated fixed effects ϑ and ϕ, we predict the average rental cost in each consumer cell. We observe ownership of real estate in the administrative income register (IND) and assume that all consumers who do not own real estate are renters, which allows use to calculate the number of renters by consumer cell. Combining estimated rental payments with the number of renters allows us to estimate total rental costs by consumer cell, which we use to allocate national consumption of rented housing across cells.

Finally, we use aggregate statistics to assign rental payments to producer cells. In the National Housing Statistics (Table BOL101), we observe the number of rental housing units in each region owned by different owner types: individuals, non-profit building societies, limited liability companies, housing societies, and public authorities. The surplus of individual owners accrues to the "private landlord" industry (our industry number 29) and those of corporate owners accrue to the "finance, real estate" industry (our industry number 17). As there is no information about the geographical location of the owners, we assume that the geographical distribution of individual owners of rental units in a given region follows the geographical distribution of mixed income in the region and that the geographical distribution of the individuals behind corporate owners of rental units in a given region follows the geographical distribution of dividend payments. The remaining owner types are public or non-profit organization. The surplus of these owners goes to the "government-owned housing" industry (our industry number 31) in the same region where the housing is located.

Appendix E.E.2 Consumption of Financial Services

The consumption of financial services in the national accounts is composed of the value of financial advice provided by financial firms and the interest rate spreads applied by financial firms. While we observe payments for financial advice in the bank transaction data, it is difficult to disentangle the interest rate spreads from the raw value of interest payments in transaction data. Instead, we allocate the national consumption of financial services across consumer cells in proportion to their interest expenses. The tax register (IND) contains interest expenses for every individual. We aggregate interest expenses at the level of each consumer cell.

Producer cells in our "finance, real estate" industry (number 17) receive consumers' expenditures on financial services. To identify which regional producer cells receive expenditures from which consumer cells, we use loan-level data from the administrative credit register (URTE). Specifically, for each bank loan and each interest payment, the credit register contains an identifier for the bank branch that has recorded the loan and the interest payment. There are around 3,000 bank branches in Denmark. We do not observe the location of branches directly, so we define the region of each branch as the most common region of consumers holding loans recorded at the branch. We then compute how the interest payments of each consumer cell are distributed across bank branches in different regions. Finally, we assume that a consumer cells' expenditure of financial services is distributed across producer cells in proportion to its distribution of interest payments.

Appendix E.E.3 Vehicle Purchases

We do not observe all vehicle purchases in the bank transaction data because many purchases do not flow directly to the vehicle producer but through financial firms. We therefore use a top-down approach to assign national vehicle purchases to consumer cells in proportion to each cell's share of total spending on new cars. We estimate each consumer cell's total spending on new cars by combining bank transaction data on annual spending at vehicle dealers with information on vehicle registrations from the administrative auto register (DMR). We use data over the period 2014-2016, as this is the most recent period where we can combine transaction data from Danske Bank and administrative data from the auto register. We therefore assume that relative levels of vehicle spending are unchanged between 2014-2016 and 2018.

We first estimate the average vehicle price in each consumer cell. We use a sample of individuals where we observe just one new car registration in a given year and spending at vehicle dealers of at least 50,000 DKK in the bank transaction data in the same year. We then regress individual-level spending at vehicle merchants on industry-by-year and region-by-year fixed effects:

$$spending_{p,y}^{vehicle\_merchants} = \theta_{i(p),y} + \eta_{r(p),y} + \epsilon_p.$$

where p is an individual in industry $i(p)$ and region $r(p)$ and year is y. We predict the average price of new cars in each consumer cell using the estimated fixed effects. We can directly calculate the number of newly registered vehicles in the government vehicle registers. Combining the estimated price with the number gives an estimate of total spending on new cars by consumer cell for each year in 2014-2016. We compute each cell's share of total spending on new cars in each year and then average across years. We use these shares to allocate national vehicle purchases in 2018 across consumer cells.

Producer cells in our "cars, fuel, car repair, public transport" industry (number 5) receive consumers' expenditures on vehicles. To identify which regional producer cells receive spending from which consumer cells, we use Danske Bank data on card transactions. Since vehicle purchases are relatively infrequent, we collapse to the regional level and assume that all consumers within the same region distribute their vehicle spending across regions in the same proportion.[A2]

Appendix E.E.4 Water and Waste Services

Rental payments often include consumption of water and waste services in Denmark, which implies that we cannot separately identify spending on water and waste in the transaction data. We therefore allocate the national consumption of water and waste services to consumer cells in proportion to their spending on other utilities. We assume that water and waste is produced locally, setting the region of the producer cell receiving the payments equal to region of the consumer cell.

Appendix F Measuring Consumer and Producer Product Taxes Paid

Product taxes are paid by buyers upon the purchase of a good. The most important product taxes are import taxes, product-specific taxes (e.g., on fuel, energy, cigarette, and alcohol), and valued added taxes (VAT).

We first focus on measuring product taxes paid by consumers. We observe import and product-specific taxes paid by consumers in the Danish national accounts table NIO3. It reports the total amount of import and product-specific taxes paid by consumers on purchases of goods produced by 117 distinct industries. These 117 industries do not map directly into our 31 industries. We therefore break down the 117 industries into the most granular grouping in the Danish national accounts (741 industries), assume that taxes paid are proportional to employment shares, and aggregate to our 31 industries. We then calculate the implied tax rate by combining information on total consumer spending on each industry from the consumer spending matrix with the tax data. The implied import and product-specific tax rates range from 0 percent (e.g., for "personal services, pharmacies" and "cultural and social organizations") to 50 percent in utilities (due to Denmark's very high energy tax rates).

The standard VAT rate in Denmark is 25 percent. We set this tax rate for consumer spending on all industries except a few industries whose products are VAT-exempt: airlines, finance, health, education, public administration, and all housing. In addition, spending on two of our industries

[A2] If there are less than 50 transactions for a region, we group that region with a neighboring region. This leads us to group Læsø with Frederikshavn; Langeland and Ærø with Svendborg; Fanø with Esbjerg; Ringkøbing-Skjern with Herning; Lemvig with Holstebro; Morsø with Thisted; and Samsø with Odder.

is partially exempt: insurance (part of our industry 8, "telecommunication and insurance") and culture (part of our industry 18 "cultural and social organizations"). We use data from NIO3 on total VAT paid by consumers by industry and calculate that the average VAT rate is 7 percent (for industry 8) and 5 percent (for industry 18).

We observe product taxes paid by Danish producers, the government and capital accumulation cells, and foreign countries in the Danish Input-Output table, described in detail in Appendix M. Notable facts are that VAT does not get paid on exports and that firms get reimbursed for any VAT they paid on intermediates.

Appendix G Measuring Disaggregated Consumer Non-Product Taxes

Consumer taxes comprise national accounting flows from consumer cells to the government. First, current taxes on income, wealth, etc., payable (SNA D.5) includes income taxes paid directly by consumers as well as a tax on pension wealth returns paid by pension funds on their behalf. We disaggregate each part separately. In the income register (IND), we observe total annual income tax liabilities, excluding pension returns taxes, for each individual in the population. We use this as the disaggregated measure of directly paid income taxes, scaling it by a factor **1.09** to match the national accounts value when aggregating across consumer cells. For the pension returns taxes paid by pension funds, we have no direct micro level measure. We therefore apply a top-down approach assuming that DKK pension wealth returns, and hence also returns taxes, are proportional to total accumulated pension contributions since 1995, which is the first year for which we have microdata on pension contributions in the pension contribution register (INPI).

Second, we use a bottom-up approach to disaggregate social contributions, payable (SNA D.61). In the income register (IND) and the pension contributions register (INPI), we extract information on total annual pension contributions, including contributions to a mandatory retirement savings program (ATP), as well as on membership fee payments to unemployment insurance funds. We aggregate these variables to the cell level and scale the cell totals by a factor **1.2** to make the national total match the value from national accounts.

Appendix H Measuring Disaggregated Consumer Interest and Transfers Paid

Consumer financial and rental expenditures include consumers' interest payments (SNA D.41) and payments related to renting of land and subsoil resources (SNA D.45). We disaggregate the former using a bottom-up approach. From the administrative income register (IND), we have information on each individual's interest payments on all financial liabilities. The sum of these interest payments exceeds the value of position D.41 in the national accounts because the micro-level measure includes the full nominal amounts paid by consumers to lenders, whereas the

national accounts value is net of Financial Services Indirectly Measured (FISIM). We handle this by **scaling** the micro-level variable so that it matches the national accounts value in the aggregate, the implicit assumption being that the ratio between FISIM and total nominal interest payment is the same across consumer cells. Since we have no microdata information about payments related to renting of land and subsoil resources, we use a top-down approach to disaggregate this national accounts flow. Each consumer cell is assigned a share of the aggregate value corresponding to its population share derived from the administrative population register (BEF). Other current transfers, payable (SNA D.7) includes non-life insurance premium payments and miscellaneous current transfers. We disaggregate it top-down, assigning to each cell a share of the national total equal to its population share derived from the administrative population register (BEF).

Appendix I Measuring the Disaggregated Labor Compensation Matrix

The labor compensation matrix records flows of labor income from producer cells to consumer cells. We use the income register (IND) and the pension contributions register (INPI) to measure total annual labor compensation, including employer-paid contributions to pension schemes, received by each individual. We define the producer cells based on the industry of the individual's main employer observed in the labor market register (AKM) and the region of the firm establishment where most of the payments come from observed in the employment register (BFL). We then aggregate these payments to a matrix at the level of consumer and producer cells. We scale all values by a factor of **1.02** to match the national accounts flow Compensation of employees, receivable (SNA D.1).

Appendix J Measuring the Disaggregated Dividend, Mixed Income, and Surplus Matrix

The matrix reveals dividends, mixed income, and surplus flowing from each producer cell to each consumer cell. We first calculate how each producer cell is split into a "corporate" and a "non-corporate" part and then, for each part separately, estimate the distribution of dividends, mixed income, and surplus across consumer cells.

In the first step, we use aggregate statistics to split firms in each industry into a corporate and a non-corporate part (Table GF5 in "General Firm Statistics"). We categorize sole proprietorships as "non-corporate," corporations with limited liability and partnerships as "corporate," and associations, funds and other rare legal forms as "other." For each industry, we are thus able to compute the share of total sales attributable to "non-corporate" and "corporate" firms, respectively. The sole exception is the financial industry where we compute the split based on employment, as

sales are not reported. We convert the industries reported in Table GF5 to our industries using employment shares as weights.

In the next step, we estimate how corporate ownership is distributed across consumer cells. We measure foreign ownership using aggregate statistics (Table "DNVPDKS" in the "Registered Securities Statistics"). The share of Danish stocks held by foreigners was 50% in 2018. We assume that this fraction is constant across all producer cells. We then distribute the ownership of the remaining 50% across domestic consumer cells in proportion to their stock market wealth. To implement this distribution, we aggregate the market value of stocks from the administrative income register (IND) for each consumer cell. We thereby assume that all consumer cells hold a fully diversified portfolio of Danish corporations.

Next, we determine how non-corporate ownership is distributed across consumer cells. Here, we construct a self-employment matrix similar to the labor supply matrix discussed above. To construct the matrix, we link information about establishments operated by sole proprietorships from the employment register (IDAN), including the municipality where the establishments are located, to the business income of the self-employed individual operating the establishment.

We distribute the income of stock corporations and privately-owned firms using distinct methods. For stock corporations, we rely on data from the income register (IND) about individual stock dividend income, which we use to disaggregate distributed income of corporations, receivable (SNA D.42). We **scale** so that the total across consumer cells matches the national aggregate.

Finally, the national accounting flow operating surplus, gross (SNA B.2G) corresponds to operating surplus from owner-occupied housing. We aggregate individual-level imputed rental values of owner-occupied housing as reported in the income register (IND), which produces the industry's total output. We scale this output by a **factor 0.67** to match the national value for B.2G. The implicit assumption is that the ratio between gross operating surplus and output in the owner-occupied dwellings industry is constant across consumer cells.

Appendix K Measuring Disaggregated Government Benefits to Consumers

National accounts describe three types of transfers to consumers. First, we aggregate all government income transfers and private pension savings payouts from the income register (IND) to calculate the cell-level measure of social benefits other than social transfers in kind, receivable (SNA D.62). We scale by a factor **1.03** to match the national accounts value. Second, other current transfers, receivable (SNA D.7) consist of miscellaneous current transfers, for example disaster and accident relief. We disaggregate this position top-down using cell population shares obtained from the administrative population register (BEF). Third, adjustment for the change in pension entitlements, receivable (SNA D.8) represents an accounting adjustment to make net changes

in pension entitlements on which consumers have a definite claim appear in the calculation of consumer saving. We disaggregate it by combining data from the income register and the pension contributions register (INPI) to construct a micro-level measure of pension contributions net of payouts. We then scale this measure to match the national accounts aggregate value.

Appendix L Measuring Disaggregated Consumer Interest and Transfers Received

First, we disaggregate interest, receivable (SNA D.41) bottom-up by using individual-level information on interest income from the income register (IND) and scaling so that the total across consumer cells matches the national accounts aggregate. Second, other investment income, receivable (SNA D.44) includes investment income from insurance policies and pension entitlements. We disaggregate this using a top-down approach where each consumer cell is assigned a share of the national accounts value proportional to its pension contributions accumulated since 1995. Third, rent, receivable (SNA D.45) consists of income from renting land and subsoil resources. We disaggregate it top-down using population shares.

Appendix M Measuring the Disaggregated Intermediates Trade Matrix

The disaggregated intermediates matrix describes how producers in one cell are connected to producers in other region-industry cells through their demand for production inputs. The Disaggregated Government Consumption Production Vector describes how producer cells contribute to the production of goods and services used for government consumption. To construct them, we start from a standard input-output table for the Danish economy at the most disaggregated level with 117 industries (Tables NIO1, NIO2, and NIO3 at statbank.dk). The input-output table illustrates how the output produced in one industry is used as intermediate inputs in other industries or in final use categories such as private consumption, government consumption, capital formation, or exports. It also shows how output from a given industry is produced from intermediate inputs from other domestic industries and from abroad (net of product taxes and import duties, which are added as additional primary inputs), and gross value added.

To convert the standard input-output table to a format suitable for our purposes, we need to address four challenges. First, the 117 industries do not map directly into our industry classification. Second, the input-output table has no geographical dimension: It describes inputs from firms in industry s to firms in industry t at the national level, but not from firms in industry s and region i to firms in region j and industry t. Third, the national accounting convention of defining production in the retail industry as the net trade margins makes the table unsuitable for combining with the disaggregated consumer spending matrix (which reports actual money flows) in order to follow the flow of money from consumers to retailers and further on to upstream producers. Fourth,

the input-output table does not distinguish between residents' and non-residents' spending in the moneys flows from private consumers to domestic producers, bringing it at odds with the disaggregated consumer spending matrix. The following subsections describe how we handle each of these challenges.

Appendix M.A Adapting the IO table to DEA Industry Classification

To address the first challenge, we first disaggregate the input-output table based on the national accounts (NA) classification to a more granular subindustry grouping. We later re-aggregate to the industry classification used in our system of disaggregated economic accounts (DEA). For the initial disaggregation, we use that the industry codes in the microdata (which have 744 categories including the three zero-employment housing services industries) maps directly into both classifications.

Formally, let lower-case letters $\{a, b, c...\}$denote the NA industries in the standard input-output table and let upper-case letters $\{A, B, C...\}$denote the 27 output-producing DEA industries shown in Table A.I. Let j denote a granular industry at the level used in the microdata. Consider a particular NA industry $x \in \{a, b, c...\}$ and a particular DEA industry $Y \in \{A, B, C...\}$: We define subindustry x_Y as the set of granular-level industries that are subindustries of both x and Y, $x_Y = \{j | j \subset x, j \subset Y\}$. This produces 173 non-empty subindustries. These represent the highest level of industry aggregation compatible with both the NA classification and the DEA classification.

In the microdata, we aggregate salary payouts within each non-empty subindustryx_Y and compute the subindustry's salary share within NA industry x. We then disaggregate the flows reported in the original input-output table using these shares. For example, the original input-output table reports the value of flows from NA industry a to NA industry b. We assume that the flow stemming from subindustry a_Y is proportional to its share of total salary payouts to workers in NA industry a. Similarly, we assume that the flow to subindustry b_Z is proportional to its share of total salary payouts to workers in NA industry b. Concretely, we compute the flow from a_Y to b_Z as the total flow from a to b multiplied by the product of the salary shares of a_Y and b_Z within their respective NA industries.[A3]

The input-output table also reports money flows from final use categories (private consumption, government consumption exports, capital accumulation, etc) to domestic producers, and from domestic producers to tax collectors and foreign producers. In both cases, we disaggregate to the subindustry level on the domestic producer side. As a general rule, we again use salary shares for the disaggregation procedure, but with one important exception: The shipping industry (NA

[A3] We make an exception to this general rule for the three zero-employment housing services industries where we use production shares rather than salary shares for the disaggregation.

industry 500000) is large in Denmark, accounting for 16% of both imports and exports in 2018. It has two subindustries, one related to transport of passengers and one related to transport of goods, and the latter pays about twice as much in total salary payments as the former. Detailed foreign trade statistics show that trade values are about 100 times larger for goods than for passengers. On this background, we assign all imports and exports to/from the shipping industry to the goods transport subindustry.

It should be noted that the disaggregation procedure may introduce considerable measurement error if flows are not proportional to salary shares. But to the extent that granular industries that belong to the same industry in the NA classification also belong to the same industry in our DEA classification, which is very often the case, such errors will vanish when we later re-aggregate to the DEA industry level.

Appendix M.B Adding Geography

The next challenge is to disaggregate the national flows between subindustries and final use categories by pairs of regions. To do this, we use aggregate cargo transport statistics describing general trade patterns across localities, as well as micro-level data on the economic size of each region within a subindustry. Specifically, we assume that regions send and receive flows in proportion to their shares of total salary payouts in each subindustry, except in the three zero-employment housing services industries where we use regional spending shares from the disaggregated consumer spending matrix (see Appendix E). Further, we take the regions' geographical locations into account by imposing a gravity structure that assigns larger flows to pairs of regions that are close to each other than to those that are far apart.

The first step in this procedure is to obtain an estimate of the elasticity of intra-country trade with respect to distance. For that purpose, we use data from publicly available cargo transport statistics (Tables NVG23 and SKIB481) to estimate the following gravity model:

$$log(Goods_{ij}) = \alpha + \gamma log(Empl)_i + \eta log(Empl)_j + \beta log(Distance_{ij}) + \epsilon_{it} \qquad \text{(A.2)}$$

where $Goods_{ij}$ denotes the weight of goods transported between provinces i and j by either road or sea; $Empl_i$ and $Empl_j$denote employment in provinces i and j respectively; and $Distance_{ij}$denotes the average driving distance between regions within province i and regions within province j (where all intra-province distances are set to 1km). We conduct the analysis at the province-level as the cargo data is not available at the more disaggregated region-level. To accommodate zeroes, we estimate the equation on its multiplicative form using a Poisson pseudo-maximum-likelihood estimator (Silva and Tenreyro, 2006). The model yields an estimated elasticity of cargo transport with respect to distance of -0.61 (s.e. 0.03).

Equipped with this elasticity estimate, we disaggregate the national flows between subindustries to the regional level in the origin dimension as well as the destination dimension. Formally, let a_Y and b_Z denote subindustries and let i and j denote regions. We then assume that the flow from firms in subindustry a_Y, region i to firms in subindustry b_Z, region j is

$$Flow_{a_Y,b_Z,i,j} = Flow_{a_Y,b_Z} * \theta_{a_Y,b_Z,i} * \eta_{a_Y,b_Z,j} * Distance_{ij}^{-0.6} \tag{A.3}$$

where $Flow_{a_Y,b_Z}$is the flow from subindustry a_Y to subindustry b_Z at the national level and $Distance_{ij}$ is the distance between region i and region j.[A4] The parameters $\theta_{a_Y,b_Z,i}$ and $\eta_{a_Y,b_Z,j}$ are origin- and destination region fixed effects within the a_Y-b_Z subindustry pair. These are set so that *i*) region i's total share of national a_Y-b_Z flows (i.e.$\sum_j Flow_{a_Y,b_Z,i,j}/Flow_{a_Y,b_Z}$) matches its share of total salary payouts in subindustry a_Z, and *ii*) region j's total share of industry a_Y-b_Z flows (i.e. $\sum_i Flow_{a_Y,b_Z,i,j}/Flow_{a_Y,b_Z}$) matches its share of total salary payouts in subindustry b_Z. In practice, we accomplish this through an iterative numerical procedure: Starting from initial guesses for $\theta_{a_Y,b_Z,i}$ and $\eta_{a_Y,b_Z,j}$, we compute the implied value of each $Flow_{a_Y,b_Z,i,j}$. We then adjust the guesses by a multiplicative constant that ensures that these implied values add up to their national level counterpart, $Flow_{a_Y,b_Z}$. Next, we update the guesses by multiplying them with the ratios of the regions' salary shares to the implied shares of national a_Y-b_Z flows. We repeat this procedure iteratively until the implied share of national a_Y-b_Z flows converges toward the relevant salary share for each origin and destination region.

For money flows from final use categories (private consumption, non-profits, government consumption, capital formation, exports) to domestic subindustries, we add a geographic dimension on the destination side only. Here, we simply assign each region a share of the total national flow equal to its share of total subindustry salary payouts. Conversely, for flows going from domestic subindustries to tax collectors and foreign producers, we add a geographic dimension on the origin side only: Once again, each region is assigned a share of the national flow equal to its salary share within the subindustry.

After disaggregating to the subindustry-region level, we aggregate to the industry-region cell level described in Appendix D by summing over subindustries belonging to the same DEA industry within each region.

Appendix M.C Redirecting Flows From Consumers Through Retailers

National accounting conventions imply that production in retail industries is given by trade margins, i.e. sales net of acquisition costs. Thus, if a retailer buys a product from a non-retail

[A4]Distances are measured as the distance between the two regions' centroids. We use Google Maps API service to calculate actual travel distances. We define a region's distance to itself as 1.

producer at price p_1 and sells it to a consumer at price p_2, the input-output table will display two money flows: 1) a flow of $p_2 - p_1$from consumers to retail, and 2) a flow of p_1 from consumers to the non-retail producer's industry. This makes the standard input-output table unsuitable for tracing the circular flow of spending starting from the Disaggregated Consumer Spending Matrix. The reason is that the spending matrix shows actual cash-flows from consumers to producers. Thus, in the example above, the spending matrix would show a flow of p_2 from the consumer to the retailer, but no flow from the consumer to the non-retail producer. Tracing the circular flow of spending from consumers and through the production network therefore requires an adjustment to make the two matrices compatible.

We adjust the input-output table by re-coding all flows from consumers to non-consumer facing industries (defined as those not selling directly to consumers) and foreign producers (except flows related to Danish residents' tourist spending abroad, see subsection Appendix M.D). We replace each of these flows with two equal-sized sets of flows: One from a retail industry to the non-retail industry in question (or foreign producers in the case of imports), and one from private consumers to the same retail industry. Each set splits the value of the original flow across retailers' regions, using the geographic distribution of existing retail-to-non-retail flows of the same type. The choice of retail industry depends on the type of consumption good traded. Exploiting that the input-output table breaks flows from private consumers down on 72 consumption categories, we manually assign each category to the retail industry most likely to sell products within that category.[A5]

Note that this adjustment increases the total sum of flows in the table; for retail industries the total sum of inflows thus no longer corresponds to the total value of production as defined in national accounts. One way to think of this is that we apply a different definition of production for retailer's sales to consumers: The output value of such sales is now defined as the full amount of the sale (net of taxes such as VAT), while the retailer's acquisition cost is treated as an intermediate input from the non-retail producer of the traded product.

Appendix M.D Foreign Residents' Spending in Denmark

In the original input-output matrix, consumers' cross-border spending is captured by two separate flows from consumers to foreign producers: a positive representing residents' spending abroad, and a negative representing non-residents' spending in Denmark. The latter adjusts for the fact that the flows from consumers to domestic producers include spending from foreign residents and

[A5]For example, if we observe a flow of 1000 from private consumers to manufacturers in region X as payment for cheese,we replace it with 1) flows from grocery retailers in each of the 98 regions to manufacturers in region X, where the size of each flow is 1000 times the origin region's share of total existing flows from grocery retailers to manufacturers in region X; 2) flows from private consumers to grocery retailers in each of the 98 regions, where the size of each flow matches the corresponding retailer-to-manufacturer flow.

thus ensures that the total sum of flows from private consumers matches the national accounts value of resident consumers' private consumption expenditure (SNA P.31). To ensure consistency with national accounts aggregates for imports (SNA P.7) and exports (SNA P.6), the matrix also contains a positive, same-sized flow from foreign consumers (i.e. exports) to foreign producers (i.e. imports).

For our purposes, it is important that the recorded flows from domestic consumers to domestic producers represent the spending of resident consumers only. If not, consistency with the disaggregated consumer spending matrix is compromised. Sales to foreign tourists, on the other hand, should be recorded as exports for the producer cells making the sales. To achieve this, we adjust the flows from private consumers to domestic producers downwards so that the total adjustment across producer cells matches the aforementioned negative flow from consumers to foreign producers. We then adjust the total flow from consumers to foreign producers upwards by the same amount. To distribute the total adjustment across producer cells, we draw on two data sources: First, we use the industry distribution of Danish residents' spending abroad from the disaggregated consumer spending matrix to compute a proxy for each industry's share of foreign tourist spending, thus assuming that Danish tourists' spending behavior in other countries is indicative of foreign tourists' spending in Denmark. Second, for the distribution across regions, we use regional data from visitdenmark.dk on foreigners' overnight stays at hotels and hostel to compute a proxy for each region's share of foreign tourist spending. In an initial round of adjustments, we then allocate the total amount to be adjusted on cells in proportion to the product of these two shares, but subject to a constraint that the adjusted flow must be non-negative. The non-negativity constraint implies that less than the required amount is allocated in the first round, so we distribute the residual amount on the cells with positive tourist spending shares in proportion to their remaining private consumption flows. For each producer cell, we also adjust the flow from foreign consumers (i.e. exports) upwards by the exact same amount so that total flows from consumers (domestic and foreign) to the cell are unchanged. To complete the adjustment process, we adjust the flow from foreign consumers to foreign producers downwards by the same total amount.

Combined, these steps ensure that total consumer spending and total exports are both unaffected, but the split between flows to domestic producers and flows to foreign producers has changed for both. Similarly, total imports are unchanged, and so is total output for each producer cell, but the split between flows from domestic consumers and flows from foreign consumers is different in both cases.

Appendix N Measuring Disaggregated Government Dividend and Surplus Income

The matrix describes the income that the government receives from each producer cell. We start with a list of firms that sell to consumers and producers at market prices are (full or partly) owned by the government.[A6] We manually collect annual turnover for every firm from annual reports. We also collect information on establishment-level employment from the Danish business register (CVR). We combine these two datasets and split annual turnover regionally using each firm's distribution of employment across regions. We finally aggregate across industries (using the industry-code of the parent firm) and regions to get to our level of producer cells. We assume that the share of surplus received by the government is equal to the share of turnover by government-owned establishments in each producer cell.

Appendix O Measuring Disaggregated Consumption of Government and NPISH Output

We measure which consumer cells use different types of government services. The government purchases these services from producer cells (specifically, government-operated establishments in each producer cell) and provides them to consumers free of charge. We assume a uniform per capita consumption for collective public goods (including police, national defense, and public administration). We use individual-level data on actual uses of public services to allocate individual public consumption (including education, healthcare, and social protection), as detailed below.

Appendix O.A Education

We assign the aggregate consumption of education services observed in the national accounts to consumer cells according to the number of students in primary, secondary, and tertiary education in a cell. The education register ("UDD") contains information about the education program in which each individual is currently enrolled (if any) as well as each individual's highest completed education. We categorize individuals as primary school students if they are currently enrolled in a program and have no completed education; as secondary school students if they are currently enrolled in a program and their highest completed education is primary school (10 years); as tertiary education students if they are currently enrolled in a program and their highest completed education is secondary school (13 years); and as non-students if they are currently not enrolled in a program.

[A6] See fm.dk/arbejdsomraader/statens-selskaber/organisering.

We aggregate the number of students in primary, secondary and tertiary education to the level of consumer cells. As the cells only include the adult population, we assign the education consumption of minors to adults in the same household before aggregating, drawing on the intra-household links in the population register (BEF). For instance, two parents with three children, two of which are in primary school and one is in secondary school, each consume the equivalent of one year of primary education and half a year of secondary education.

Finally, we allocate the aggregate government consumption of education to cells in proportion to their share of students in primary, secondary and tertiary education and total government expenditure on education at each of these levels. Specifically, the estimated consumption of education services in region-by-industry cell i is given by

$$C_i^{edu} = \sum_{q=p,s,t} \frac{\#students_{i,q}}{\#students_q} \times expenditure_q$$

where q is the level of education (with p, s and t indicating primary, secondary, and tertiary, respectively) and expenditure is government consumption of education of level q.

Appendix O.B Healthcare

Government consumption of healthcare falls into six categories: outpatient services; hospital services; medical products, appliances and equipment; public health services; research and development; and other. We allocate government healthcare consumption summed over all six categories, as reported in national accounts, to consumer cells using publicly available statistics for the first two categories, which make up around 85% of the total.

"Outpatient services" capture government consumption flowing to primary healthcare providers, like general practitioners, specialist doctors, psychiatrists, and dentists. We obtain average primary healthcare expenditures by age, gender, and municipality (Table SYGU1 in the Public Expenditure Statistics). Based on a regression of average primary healthcare expenditure on a set of indicators for age, gender and municipality, we predict primary healthcare expenses for each individual in the population. We then aggregate the predicted expenditures to the level of consumer cells. Since children account for a non-negligible part of healthcare spending, we include the full population by assigning minors to the same consumer cells as the adults cohabiting with the child. If parents live together but work in different industries, we split the child's predicted healthcare expenditure equally between the two cells.

"Hospital services" capture government expenditure related to hospital treatments, including emergency room and outpatient hospital treatments. Again, we obtain information on the average number of days spent in hospital by age, gender, and municipality (Table INDAMP01 in the Health

Statistics). Regressing average hospital days on a set of indicators for age, gender, and municipality, we predict the number of hospital days for each individual in the population. We then aggregate the predicted hospital days to the consumer cell level, again allocating the hospital days of minors to their parents' cells.

Finally, we combine the two indicators of healthcare usage to disaggregate total consumption of healthcare services of consumer cell i as:

$$C_i^{health} = \sum_{q=o,h} \left(\frac{usage_{i,q}}{\sum_i usage_{i,q}} \times \frac{exp_q}{exp_o + exp_h} \right) \times exp, \tag{A.4}$$

where q indexes the type of healthcare (with o and h indicating outpatient and hospital services, respectively), $usage_{i,q}$ denotes cell *i's* usage of type q (expenditure on primary care and the number of hospital days), exp_q is national government consumption of healthcare of type q, and exp is national government consumption of healthcare summed over all six categories.

Appendix O.C Social Protection

Government consumption of social protection falls into five categories: sickness and disability; old age; family and children; unemployment; and other. We allocate social protection services to consumer cells based on government microdata. Specifically, we allocate the category "sickness and disability" to cells in proportion to the number of individuals on sick leave or disability pension as observed in the income register (IND); the category "old age" in proportion to the number of individuals aged 80 or older as observed in the population register (BEF); the category "family and children" in proportion to the number of pre-school children as observed in the population register (BEF); the category "unemployment" in proportion to the number of unemployed; and the category "other" by population shares. The estimated consumption of social protection services in consumer cell i is thus:

$$C_i^{social} = \sum_{q=s,o,d,u,z} \frac{usage_{i,q}}{\sum_i usage_{i,q}} \times exp_q, \tag{A.5}$$

where q indexes the type of social protection (with s, o, d, u and z denoting sickness/disability, old age, family/children, unemployment and other, respectively), $usage_{i,q}$ denotes the relevant indicator for cell i's usage of type q (see above), and exp_q is national government consumption of social protection services of type q.

Appendix O.D Measuring Disaggregated Consumption of NPISH Output

We measure the consumption of output provided by non-profit organizations (NPISH) for different consumer cells. NPISH output falls into five categories: education; social work activities; libraries; museums and other cultural activities; sports activities (non-market); and activities of membership organizations.

We first disaggregate usage of the first four categories by consumer region. For education, we use regional data on the share of children attending private schools relatively to public schools (Table "UDDAKT20" in the Education Statistics). For social work, we use regional data on the share of privately-owned (as opposed to government-operated) daycare institutions (Table "BOERN4" in the Child Care Statistics). For libraries, we use regional data on library usage per capita (Table "BIB1" in the Culture and Leisure Statistics). For museums and other cultural activities, we use regional data on members of sports associations per capita.

To infer usage by industry of work, we rely on the Danske Bank data. For education, we proxy use of NPISH education with payments to private schools. For social work, we calculate payments to private child-care institutions. For libraries, we use payments to libraries. For sports activities, we use membership payments to sports associations. For all categories, we count the number of transactions relative to the number of bank customers in each industry. We thereby estimate how likely consumers in each industry are to consume a given type of NPISH output.

We combine the information on consumption shares of NPISH by consumer region and industry to calculate consumption of NPISH output by consumer cell:

$$npish^{q}_{r,i} = \sum_{q} \frac{npish_{q,r} \times npish_{q,i} \times pop_{r,i}}{\sum_{r,s} npish_{q,r} \times npish_{q,i} \times pop_{r,i}} \times expenditure_{q},$$

where q is the NPISH category, r is region, i is industry, $pop_{r,i}$ is the cell's population, and expenditure is national NPISH output of type q.

For the final type of NPISH consumption, activities of membership organizations, we rely on Danske Bank data. This category consists mostly of trade unions and a small component of political or religious organizations. We disaggregate national consumption using as weights the share of individuals making payments to trade unions in each cell multiplied by the cell's population.

Appendix P Details on Model Derivations

Appendix P.A Derivation of the Circular Flow Matrix $\mathbb{M}$

To derive (13), we proceed column by column.

The first $\mathcal{J}$ columns of $\mathbf{M}$ describe producer cells' cost shares:

- Producer cell j has an intermediate input share of γ_j^X, of which share $\Omega_{j'j}$ goes to producer cell j'. In matrix notation, this gives $\Omega \cdot \mathbf{D}(\gamma^X)$.
- Producer cell j has a capital share of γ_j^K. In matrix notation, this means the flows from all producer cells to all $\mathcal{J}$ capital stocks is given by $\mathbf{D}(\gamma^K)$.
- Producer cell j has a labor share of γ_j^N. Of that, a share λ_{ij} is going to consumer cell i's labor. Together, this gives $\Lambda \cdot \mathbf{D}(\gamma^N)$ in matrix notation.

The second $\mathcal{J}$ columns of $\mathbf{M}$ describe the beneficiaries of capital income accruing to the $\mathcal{J}$ types of capital.

- A fraction τ_j^{corp} is paid to the government.
- A fraction $\kappa_{ij} \cdot \left(1 - \tau_j^{corp}\right)$ is paid to consumer cell i.

The third $\mathcal{I}$ columns capture the beneficiaries of labor income accruing to the $\mathcal{I}$ types of labor in the economy.

- The government receives a share τ_i of that income.
- Everything else goes to consumer cell i.

The fourth $\mathcal{I}$ columns of $\mathbf{M}$ captures the spending behavior of consumer cells.

- Consumer cell i allocates a fraction $\mathbf{A}_{ji}$ of spending to producer cell j. Of that $\frac{1}{1+\tau_i^{vat}}$ actually reaches producer cell j.
- The remainder, $\sum_{j \in \mathcal{J}} \mathbf{A}_{ji} \cdot \frac{\tau_i^{vat}}{1+\tau_i^{vat}}$ goes to the government in terms of product tax revenue. In matrix notation, this is given by the row vector $\mathbf{1}'\mathbf{A}\mathbf{D}\left(\frac{\tau^{vat}}{1+\tau^{vat}}\right)$.

The fifth column of $\mathbf{M}$ captures the allocation of spending of the government.

- A fraction g_j is spent on producer cell j's goods.
- A fraction t_i is transferred to consumer cell i.

This concludes our derivation and description of $\mathbf{M}$.

Appendix P.B Formula for change in GNE (14)

With welfare weights equal to inverse marginal utilities, the first order change in welfare is given by

$$\begin{aligned}
\sum_{i\in\mathcal{I}} P_i C_i \cdot dU_i &= \sum_{i\in\mathcal{I}} P_i dC_i + \sum_{i\in\mathcal{I}} \frac{P_i C_i}{G_i}\chi_i dG_i \\
&= \sum_{i\in\mathcal{I}} P_i dC_i + \sum_{i\in\mathcal{I}} \frac{\nu_i \cdot \sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j G_j}{G_i} dG_i \\
&= \sum_{i\in\mathcal{I}} P_i dC_i + \frac{\sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j G_j}{v\left(\{G_j\}\right)} \sum_{i\in\mathcal{I}} dG_i \\
&= \sum_{i\in\mathcal{I}} P_i dC_i + \sum_{i\in\mathcal{I}} \mathcal{P}(\{P_j\}) dG_i \\
&= \sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j dc_{ij} + \sum_{j\in\mathcal{J}\cup\{\mathcal{R}\}} P_j dG_j \\
&= dGNE.
\end{aligned}$$

Appendix P.C Details on the Vertical Economy in Section VII.A

The vertical economy has a set $\mathcal{I} = \{1, \ldots, N\}$ of consumers and a set $\mathcal{J} = \{1, \ldots, N\}$ of producers. The circular flow matrix for this economy is given by

$$\mathbf{M} = \begin{pmatrix} 0 & 0 & 0 & \mathbf{A} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \mathbf{I} & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathbf{I} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \tag{A.6}$$

where

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

(A.6) has this specific form because there is no government; no capital; all production in producer cell j is done using labor of consumer cell $i = j$; and because consumer cell i spends on producer cell $j = i - 1$, unless $i = 1$ in which case consumer cell i spends abroad.

Observe that the exposure of factor i to a shock to producer cell j is given by

$$(\mathbf{I}-\mathbf{M})^{-1}_{N+N+i,j} = 1_{\{i\leq j\}},$$

where $1_{\{...\}}$ is an indicator function. This implies that Proposition 1 is a special case of Proposition 2.

Appendix P.D Proof of Proposition 2

We consider the case of a general export demand shock $d\tilde{\mathbf{x}}$ hitting the economy. Since the world economy in our model is frictionless, the change in domestic welfare must be coming from an increase in the price of exports. That is,

$$dGNE = \mathbf{x}' d\log \mathbf{P}$$

where $\mathbf{x} = (x_j)_{j\in\mathcal{J}}$, $d\log\mathbf{P} = (d\log P_j)_{j\in\mathcal{J}}$. How do prices change? Linearizing the unit cost function associated with (7), we find that

$$d\log P_j = \gamma_j^X \sum_{j'} \Omega_{j'j} d\log P_{j'} + \gamma_j^N \sum_i \lambda_{ij} d\log W_i + \gamma_j^K d\log\left(P_j Q_j\right),$$

where $d\log(P_jQ_j)$ appears as the return to the fixed factor. In matrix and vector notation, this becomes

$$d\log\mathbf{P} = \mathbf{D}(\gamma^X)\Omega' d\log\mathbf{P} + \mathbf{D}(\gamma^N)\Lambda' d\log\mathbf{W} + \mathbf{D}(\gamma^K) d\log\mathbf{PQ}, \tag{A.7}$$

where we use the notation $d\log\mathbf{PQ} \equiv (d\log(P_jQ_j))$ for the vector of changes in nominal output across producer cells; similarly, $d\log\mathbf{W} = (d\log W_i)$. Solving (A.7) for $d\log\mathbf{P}$, we find

$$d\log\mathbf{P} = \left(\mathbf{I} - \mathbf{D}(\gamma^X)\Omega'\right)^{-1}\left(\mathbf{D}(\gamma^N)\Lambda' d\log\mathbf{W} + \mathbf{D}(\gamma^K) d\log\mathbf{PQ}\right). \tag{A.8}$$

Given the definitions for $\mathbf{x}^K$ and $\mathbf{x}^N$ in (18) and (19), we can then write

$$dGNE = \mathbf{x}' d\log\mathbf{P} = \left(\mathbf{x}^N\right)' \mathbf{D}\left(NW\right) d\log\mathbf{W} + \left(\mathbf{x}^K\right)' \mathbf{D}\left(\Pi^{pre}\right) d\log\mathbf{PQ}.$$

Given that $\Pi_j^{pre} = \gamma_j^K P_j Q_j$, this simplifies to

$$dGNE = \mathbf{x}' d\log\mathbf{P} = \left(\mathbf{x}^N\right)' d\mathbf{NW} + \left(\mathbf{x}^K\right)' d\mathbf{\Pi}^{pre}, \tag{A.9}$$

where $d\mathbf{NW} = d\left(N_iW_i\right)$ is the change in labor income paid to consumer cell i in response to the shock; and $d\mathbf{\Pi}^{pre}$ is the change in pre-tax profits in response to the shock. Observe that both can

be computed directly via the circular flow matrix:

$$\begin{pmatrix} * \\ d\mathbf{\Pi}^{pre} \\ d\mathbf{NW} \\ * \\ * \end{pmatrix} = (\mathbf{I} - \mathbf{M})^{-1} \begin{pmatrix} d\tilde{\mathbf{x}} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \tag{A.10}$$

where $*$ indicates some other entries which we are not further characterizing. Putting (A.10) together with (A.9), we find

$$dGNE = \begin{pmatrix} 0 & \mathbf{x}^K & \mathbf{x}^N & 0 & 0 \end{pmatrix} (\mathbf{I} - \mathbf{M})^{-1} \begin{pmatrix} d\tilde{\mathbf{x}} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Since $d\tilde{\mathbf{x}}$ was arbitrary, this establishes (20) and proves Proposition 2.

Appendix P.E Proof of Proposition 3

The proof of this proposition is straightforward. A uniform export demand shock, increasing all exports by $d\log\tilde{x}$, scales up all nominal flows by $d\log\tilde{x}$. This implies that nominal consumption of consumer cell i is scaled up by $d\log\tilde{x}$,

$$\frac{d\log\left(\sum_j P_j c_{ij}\right)}{d\log\tilde{x}} = 1.$$

Real consumption therefore moves according to

$$\frac{d\log C_i}{d\log\tilde{x}} = 1 - \sum_j \mathbf{A}_{ji} \frac{d\log P_j}{d\log\tilde{x}}. \tag{A.11}$$

Moreover, nominal labor income and nominal output scale with the shock, $d\log W_i = d\log(P_j Q_j) = d\log\tilde{x}$. Applying (A.8) to the uniform shock, we find price responses

$$d\log\mathbf{P} = \left(\mathbf{I} - \mathbf{D}(\gamma^X)\Omega'\right)^{-1} \left(\mathbf{D}(\gamma^N)\Lambda'\mathbf{1} + \mathbf{D}(\gamma^K)\mathbf{1}\right).$$

Since Λ is column stochastic, $\sum_{i\in\mathcal{I}} \lambda_{ij} = 1$, this can be simplified further,

$$\begin{aligned} d\log \mathbf{P} &= \left(\mathbf{I} - \mathbf{D}(\gamma^X)\Omega'\right)^{-1}\left(\mathbf{1} - \mathbf{D}(\gamma^X)\mathbf{1}\right) \\ &= \left(\mathbf{I} - \mathbf{D}(\gamma^X)\Omega'\right)^{-1}\left(\mathbf{1} - \mathbf{D}(\gamma^X)\Omega'\mathbf{1} - \mathbf{D}(\gamma^X)\left(\mathbf{1} - \Omega'\mathbf{1}\right)\right) \\ &= \mathbf{1} - \left(\mathbf{I} - \mathbf{D}(\gamma^X)\Omega'\right)^{-1}\mathbf{D}(\gamma^X)\left(\mathbf{1} - \Omega'\mathbf{1}\right). \end{aligned} \tag{A.12}$$

Substituting (A.12) into (A.11), we find

$$\begin{aligned} \frac{d\log \mathbf{C}}{d\log \tilde{x}} &= \mathbf{1} - \mathbf{A}' d\log \mathbf{P} \\ &= \mathbf{1} - \mathbf{A}'\mathbf{1} + \mathbf{A}'\left(\mathbf{I} - \mathbf{D}(\gamma^X)\Omega'\right)^{-1}\mathbf{D}(\gamma^X)\left(\mathbf{1} - \Omega'\mathbf{1}\right), \end{aligned}$$

which is precisely (22).

Appendix References

Silva, J. S., and S. Tenreyro (2006): "The Log of Gravity," *Review of Economics and Statistics*, 88, 641–658.