INTERNET APPENDIX

A Investor Heterogeneity and Thresholds in Dollar Amounts

Consider the following extension: a project is presented to a sequence of agents $i = 1, \cdots, N$ who can either support or reject it. However, each agent i is either rich ($\theta_i = H$) or poor ($\theta_i = L$), with maximum support levels $H > L > 0$ and θ drawn from an i.i.d. distribution such that $\Pr(\theta_i = H) = 1 - \Pr(\theta_i = L) = \lambda$. An H-type faces a contribution choice set given by $\{pH, pL, 0\}$; an L-type agent has a choice set $\{pL, 0\}$, choosing only between low support and rejection.[18] The pre-specified "price" $p \in (0, 1)$ is on each unit of support.

The private signal x_i is realized according to the information structure as specified in the baseline model. We abuse the notation here to use T to denote the implementation threshold in terms of total dollar amount instead of the required number of supporters. We correspondingly denote the dollar amount of support collected until agent i using $A_i = \sum_{j=1}^{i} a_j$, for $1 \le i \le N$.

If the proposal is implemented eventually, the proposer charges the pre-specified price per unit of support, and each agent receives a return V per unit of support, which is either 0 or 1. We can alternatively view pL or pH as the actual amount the entrepreneur receives, then a given threshold T maps to pT as the threshold for the actual amount collected. Our convention in labeling $\{p, T\}$ is for easy comparison with the baseline model.

The core of the issue is that agents now keep track of two statistics. One is the dollar gap between accumulated funding and the threshold, while the other one is the belief gap between current belief and the break-even belief k^*. Thus, when the dollar gap is small but the belief gap is still big, an H-type agent uses partial support to create "prolonged learning" because a partial support allows for more rounds of trials without triggering implementation or a DOWN cascade. Such episodes of prolonged learning may occur multiple times before eventually the agent returns to full support or a DOWN cascade takes place, depending on whether the break-even belief $\bar{k}(p)$ is reached. The next proposition summarizes one equilibrium featuring potential prolonged learning with partial support L, even when full support to reach the funding target sooner is feasible.

Proposition IA-1. *There exists a pair (p^*, T^*) that maximizes the proposer's expected revenue. For any given (p, T), $a_i^* = 0$ for $\theta_i \in \{H, L\}$ whenever $x_i = -1$. When $x_i = 1$, type $\theta_i = L$ supports as long as she is not the gatekeeper ($A_{i-1} \ge T - L$ & $k_{i-1} < \bar{k}(p) - 1$); type $\theta_i = H$ supports before any cascade, but can switch from full support H to partial support L if the funding gap is small relative to the belief gap, i.e., $(\bar{k}(p) - k_{i-1})L < T - A_{i-1} < H + (\bar{k}(p) - k_{i-1} - 1)L$.*

Moreover, the entrepreneur can no longer extract the full surplus even as N goes to infinity. To extract the full surplus, the proposer needs to set a price such that all agents are indifferent between supporting and rejecting. This is problematic now because if the price is high enough (i.e., $p \to V_{\bar{k}}$, a necessary condition for

[18] We use discrete choices here to reflect that in crowdfunding, investors are typically given discrete choices on the amount they can invest. This specification also allows us to best convey intuition and insight: if we allow a continuum amount would be similar but the derivation is considerably more involved.

full surplus extraction), then an agent will switch from high support to low support to prolong the campaign and information aggregation. One can show that under such a strategy, there exist some signal sequences such that an UP cascade is more likely to happen, which generates a positive payoff to the agents. Finally, note that the results hold for both endogenous and exogenous threshold and pricing. Because the minimum amount often depends on the nature of the project exogenous to the entrepreneur, it maps to an exogenous AoN target in dollar amounts in our model. In the proof part we also provide a numerical procedure to search for a model solution with endogenous (p, T).

B Contribution and Information Acquisition Costs

In practice, investing may incur an additional cost $\epsilon > 0$, which could be, for example, the opportunity cost from pre-committing the funds.[19] Notice that with the cost $\epsilon > 0$, Assumption 1 and 2 suggest that an agent supports when the expected contribution payoff equals the contribution cost ϵ.

We first show that if the cost ϵ is sufficiently small, then our informer equilibrium characterization in Proposition 1 still holds (but mapping to one with different parameters). That is, our results are qualitatively unchanged to small perturbations in the form of contributing costs.

Proposition IA-2. *Given the agent base N and threshold T, for any price $p \in [V_k, V_{k+1})$, there exists a bound $\bar{\epsilon}(p, T) > 0$ such that for $\forall \epsilon \in (0, \bar{\epsilon}(p, T))$, there exists a unique equilibrium entails the same outcomes as the one characterized in Proposition 1 under the same threshold T and a modified price $p' \equiv p + \epsilon$.*

Proof. We denote the equilibrium characterized in Proposition 1 with the same threshold T and price $p^* \equiv p + \epsilon$ as $\mathbb{G}$. Let $\chi(p, i, T, \mathcal{H}_{i-1}, x_i, a_i)$ be the expected profit of agent i with posterior $V_{k(\mathcal{H}_{i-1})+x_i}$ (conditional on her private signal as well) when the price is p in equilibrium $\mathbb{G}$. Define $\bar{\epsilon}(p, T)$ as

$$\bar{\epsilon}(p, T) = \min\{\min\{\chi(p, i, T, \mathcal{H}_{i-1}, x_i, a_i = 1))\}_{\{\chi > 0\}}, V_{k+1} - p\} > 0. \tag{IA-1}$$

Notice that by construction $\bar{\epsilon}(p, T) > 0$ because agent i always has non-negative expected profit when the price is $p + \epsilon > p$. Now we verify that $\mathbb{G}$ still holds. If there is already an UP cascade as in $\mathbb{G}$, given other agents' strategies, the project would be implement for sure if agent i supports, then by construction agent i's expected profit (conditional on her private signal) would be at least

$$V_{k+1} - p \geq \epsilon. \tag{IA-2}$$

So agent i will support regardless of her private signal.

Consider the case when there is no UP cascade yet and Agent i observes a negative signal. Then agent i's expected profit would be negative if she chooses support, and she finds it optimal to reject the project.

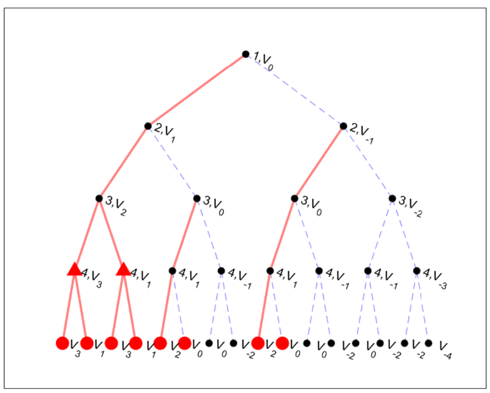
If there is no UP cascade yet and agent i's private signal is positive, then by the construction of $\bar{\epsilon}(p, T)$, agent i would choose support. Therefore, when $\epsilon < \bar{\epsilon}$, our findings are qualitatively unchanged. □

Having said that, note that the equilibrium may not be unique. In particular, Assumption 1 does not rule out the bad equilibrium in which nobody invests because everyone believes others will not contribute and does not want to incur ϵ to contribute, due to agents' strategic complementarity.

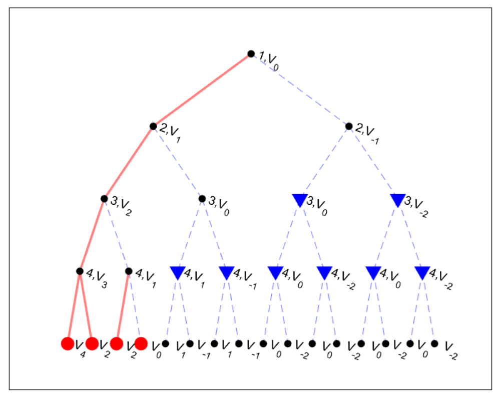
[19] We thank one anonymous referee for pointing this out.

For a larger ϵ, some agents may find the expected profit of investment lower than ϵ, which causes these agents to reject even if they observe positive private signals. These no contribution decisions hinder information aggregation and in turn reduces the number of potential contributors, discourages other agents to contribute, further lowering the possibility of project implementation. As the cost increases, the equilibrium becomes less informative and eventually reach the trivial no contribution scenario. In Internet Appendix, We provided a numerical example to show how larger contribution cost may hinder both contribution and information aggregation, and also analyze a similar case when agents incur a cost to acquire information.

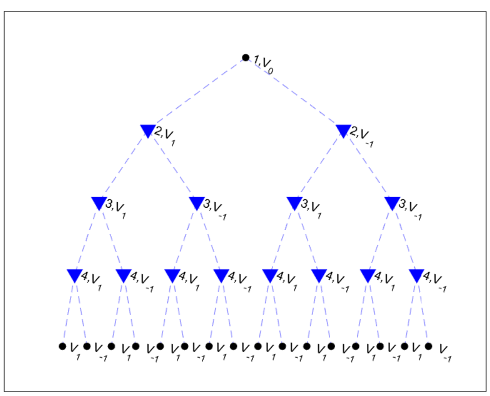
In general, a larger ϵ is associated with a lower chance of project implementation and less information aggregation. We use an example to illustrate how ϵ bounds the informativeness of the equilibria and affects information cascades. Consider the case with $N = 4, T = 2$, $\nu < p = V_1$, where the public posterior is $V_k = \frac{q^k}{q^k + (1-q)^k}$ as in Proposition 1. The equilibrium is fully characterized by Proposition 1.



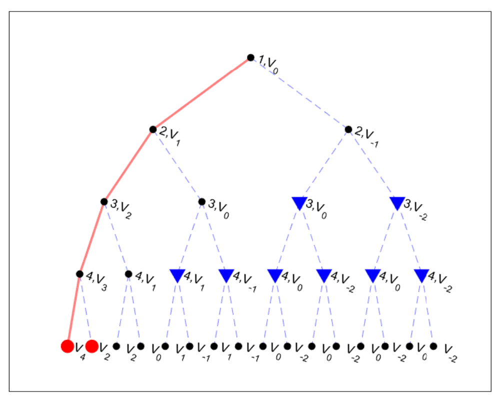
(a) The case with $\epsilon = 0$



(b) The case with $0 < \epsilon \leq V_2 - V_1$



(c) The case with $\epsilon > V_2 - V_1$



(d) How endogenous price helps when $\epsilon > V_2 - V_1$

Figure 4: Equilibrium dynamics as a function of contribution cost

Figure 4 depicts the equilibrium dynamics with binomial trees. Nodes 1-4 index the agents, and the value V_k alongside each node indicates the public prior belief of $\Pr(V = 1)$. A left branch indicates a good private signal ($x_i = 1$), while a right branch indicates a bad one ($x_i = -1$). The big red terminal nodes indicate successful implementations and the corresponding values are the final posterior beliefs of $\Pr(V = 1|\mathcal{H}_N)$ after the branch sequence of signals. A red upward triangle indicates an UP cascade at that node, while a blue downward one indicates a DOWN cascade.

Figure 4a corresponds to $\epsilon = 0$. If we take the sixth terminal node in Figure 4a, the sequence of signals is given by: $\{1, -1, 1, -1\}$. Agents 1 and 3 choose to support the proposal, while Agents 2 and 4 reject it. The final posterior is V_0, which is the belief held by Agent 4 after receiving a bad signal $x_4 = -1$. However, for Agent 3, the expected value from supporting the project is V_1 because the project would not be implemented at terminals 7 and 8 (from the left). Note that, for Agent 1, when she chooses to support the proposal (if $x_1 = 1$), her expected payoff is a linear combination of V_2 and V_1, where V_2 is the equilibrium payoff when $x_2 = 1$ and V_1 the equilibrium payoff when $x_2 = -1$. Moreover, UP cascades occur after the signal sequence terminal node 1-4. Figure 4b displays the contribution nodes when the contribution costs $\epsilon \in (0, V_2 - V_1)$. Relative to Figure 4a, the proposal is less likely to be implemented (only the first four terminal nodes). There are also more DOWN cascades.

Finally, in Figure 4c, with a sufficiently high contribution cost, the equilibrium becomes uninformative and a DOWN cascade starts at the very beginning. Figure 4d highlights the importance of endogenous pricing and threshold design in improving financing and information aggregation. Compared to Figure 4c, if the proposer increases the threshold from $T = 2$ to $T = 3$, then the proposal can still be implemented after the signal sequences $\{1, 1, 1, 1\}$ and $\{1, 1, 1, -1\}$ which previously lead to implementation failures. The intuition for our baseline model applies here as well: a higher implementation threshold ensures that the project is not implemented with a sequence of negative signals which are more likely generated by low quality projects. This protection makes financing feasible and avoids some DOWN cascades to aggregate useful information.

Information acquisition cost Besides the absence of contribution cost, every agent acquires a private signal for free in the baseline model. But in reality, acquiring private signals may cost effort and agents may forego producing any information. Let ε be the positive information production cost, and assume that an agent produces the information signal even when she is indifferent between costly learning or not (and that Assumption 1 still holds). The following proposition shows that when ε is sufficiently small, the equilibrium resembles the one characterized in Proposition 1.

Proposition IA-3. *Given the agent base N and threshold T, for any $p \in (V_{k-1}, V_k]$, there exists a bound $\bar{\varepsilon}(p, T) > 0$ such that for $\forall \varepsilon \in (0, \bar{\varepsilon}(p, T))$, there exists a unique equilibrium is as characterized in Proposition 1 with the same threshold T and price p.*

Costly information acquisition differs from the case of contribution costs in that agents need to make the information acquisition decision before their contribution decisions, and it may be the case that one agent chooses to reject even after she pays the learning cost. If ε is large, agents may choose not to acquire any information. Similar to the contribution cost, this may lead to a lower expected contribution profit and a smaller number of contributions, which in turn may further discourage information production by other agents. As the information cost ε increases, the equilibrium becomes less informative and the project is less likely to be implemented. The key takeaway is that contribution and information acquisition costs matter, but the economic mechanisms we highlight are not driven by knife-edge cases or the omission of these costs.

C Discussion on Asymmetric Signal Structure

In this extension, we consider the case with asymmetric private signals:

$$\Pr(x_i = 1|V = 1) = q \in (1/2, 1) \quad \text{and} \quad \Pr(x_i = -1|V = 0) = \tilde{q} \in (1/2, 1). \tag{IA-3}$$

We specified $q > \tilde{q}$ here without loss of generality. If $\tilde{q} = q$, then it reduces to the benchmark case.

Unlike Proposition 1, the gap between the inferred positive signals and negative signals is no longer a sufficient statistics for the equilibrium strategy. Essentially, we need to keep track of both the number of inferred positive signals and that of inferred negative signals. As in most information cascade models, the additional dimension of the state variable often renders the problem intractable. That is the case for explicit derivations of the endogenous pricing and threshold implementation. However, we can still characterize the equilibria to a good extent and distill the intuition.

First, the basic insights and results from Proposition 1 all go through, as summarized in the next proposition. The proof is almost identical to that of Proposition 1 and is thus omitted here. Let $l_i = l(\mathcal{H}_i)$ be the inferred negative signal based on history $\mathcal{H}_i$, and $k_i = k(\mathcal{H}_i)$ the difference between the number of inferred positive signals and the number of negative signals based on history $\mathcal{H}_i$. Define $\bar{k}(p, l) = \min\left\{k : p \leq \frac{q^k}{q^k + \left(\frac{\tilde{q}(1-\tilde{q})}{q(1-q)}\right)^l (1-\tilde{q})^k}\right\}$. We have:

Proposition IA-4. *Given (p, T), there exists an essentially unique informer equilibrium with $a_i^\star(x_i, \mathcal{H}_{i-1}) \equiv a^\star(x_i, k(\mathcal{H}_{i-1}), l(\mathcal{H}_{i-1}), A(\mathcal{H}_{i-1}))$ and posteriors $P(V = 1|\mathcal{H}_i) = V_{\{k^*(\mathcal{H}_i), l^\star(\mathcal{H}_i)\}}$, where:*

$$a_i^\star(x_i, k_{i-1}, l_{i-1}, A_{i-1}) = \begin{cases} x_i & \text{if} \quad A_{i-1} < T - 1 \ \ \& \ \ k_{i-1} \leq \bar{k}(p, l_{i-1}) \\ 1 & \text{if} \quad k_{i-1} > \bar{k}(p, l_{i-1}) \\ -1 & \text{if} \quad A_{i-1} \geq T - 1 \ \ \& \ \ k_{i-1} < \bar{k}(p, l_{i-1}) - 1 \\ x_i & \text{if} \quad A_{i-1} \geq T - 1 \ \ \& \ \ k_{i-1} \in \{\bar{k}(p, l_{i-1}), \bar{k}(p, l_{i-1}) - 1\} \end{cases} \tag{IA-4}$$

$$k_i^*(\mathcal{H}_i) = \begin{cases} k_{i-1} + a_i & \text{if} \quad A_{i-1} < T - 1 \ \ \& \ \ k_{i-1} \leq \bar{k}(p, l_{i-1}) \\ k_{i-1} & \text{if} \quad k_{i-1} > \bar{k}(p, l_{i-1}) \\ k_{i-1} & \text{if} \quad A_{i-1} \geq T - 1 \ \ \& \ \ k_{i-1} < \bar{k}(p, l_{i-1}) - 1 \\ k_{i-1} + a_i & \text{if} \quad A_{i-1} \geq T - 1 \ \ \& \ \ k_{i-1} \in \{\bar{k}(p, l), \bar{k}(p, l_{i-1}) - 1\} \end{cases} \tag{IA-5}$$

$$l_i^*(\mathcal{H}_i) = \begin{cases} l_{i-1} + \mathbf{1}(a_i = -1) & \text{if} \quad A_{i-1} < T - 1 \ \ \& \ \ k_{i-1} \leq \bar{k}(p, l_{i-1}) \\ l_{i-1} & \text{if} \quad k_{i-1} > \bar{k}(p, l_{i-1}) \\ l_{i-1} & \text{if} \quad A_{i-1} \geq T - 1 \ \ \& \ \ k_{i-1} < \bar{k}(p, l_{i-1}) - 1 \\ l_{i-1} + \mathbf{1}(a_i = -1) & \text{if} \quad A_{i-1} \geq T - 1 \ \ \& \ \ k_{i-1} \in \{\bar{k}(p, l), \bar{k}(p, l_{i-1}) - 1\} \end{cases} \tag{IA-6}$$

where $k_0 = 0$, $l_0 = 0$ and $A_0 = 0$.

This means all the main results with exogenous pricing and threshold implementation still hold under this asymmetric signal structure. When pricing and AoN thresholds are endogenous, the equilibrium exists featuring efficient implementation and information aggregation. To see the existence, note that the arguments for existence in Proposition 2 goes through via a global search over a finite grid of triplets of (k, l, T), instead of pairs (k, T) as in the symmetric signal case.[20] Moreover, the law of large numbers implies that the efficient project implementation and information aggregation would ensue.

Specifically, on the one hand, under any equilibrium the proposer's profit should not exceed $\frac{1}{2}(1-\nu)*N$. This is because, the total output from the project is $N*E[V] = \frac{1}{2}N$, and the total cost is $\frac{1}{2}\nu N$. This implies that the total surplus is $\frac{1}{2}(1-\nu)*N$. However, to incentivize the investors to participate in the project, the expected payoff cannot be negative. Hence, the expected profit for the proposer cannot exceed the net total surplus, i.e.,

$$\frac{1}{N}\pi(p,T) \leq \frac{1}{2}(1-\nu).$$

On the other hand, the maximum profit $\frac{1}{2}(1-\nu)*N$ is achievable. To see it, given the true state is $V=1$, then when N is sufficiently large, the total positive signal minus the total negative signals would be close to $[q-(1-q)]*N = (2q-1)*N$.[21] This implies that if we take $T(N) = k(N) = m*N$ such that $m = \frac{1}{2}(2q-1)$, the project can always be implemented under AoN when $V = 1$.[22] In contrast, when $V = 0$, if we take N large enough, the total positive signals minus negative signals will be close to $(1-2\tilde{q})*N < 0 \ll k(N)$, which means that it is almost surely impossible to implement the project under $V = 0$ when $T(N) = k(N)$. Thus, the expected profit for the proposer satisfies

$$\lim_{N\to\infty} \frac{1}{N}\pi(p^*(N), T^*(N)) \geq \lim_{N\to\infty} \frac{1}{N}\pi(V_{k(N)}, T(N)) = \lim_{N\to\infty} \frac{1}{2}(V_{k(N)} - \nu) = \frac{1}{2}(1-\nu)$$

Hence, we have $\lim_{N\to\infty} \frac{1}{N}\pi(p^*(N), T^*(N)) = \frac{1}{2}(1-\nu)$.

Now, we can show that $\lim_{N\to\infty} \mathcal{P}_N^{II} = 0$ and $\lim_{N\to\infty} \mathcal{P}_N^{I} = 0$.

To see it, note that the investors' expected payoff is given by

$$\begin{aligned}\frac{1}{2}(1-\mathcal{P}_N^I)(1-p^*(N)) + \frac{1}{2}\mathcal{P}_N^{II}(0-p^*(N)) &= \frac{1}{2}(1-\mathcal{P}_N^I)(1-p^*(N)) - \frac{1}{2}\mathcal{P}_N^{II}p^*(N)\\ &= \frac{1}{2}(1-\mathcal{P}_N^I) - \frac{1}{2}p^*(N)(1-\mathcal{P}_N^I+\mathcal{P}_N^{II})\end{aligned}$$

and that the proposer's expected profit is given by

$$\begin{aligned}\frac{1}{N}\pi(p^*(N), T^*(N)) &= \frac{1}{2}(1-\mathcal{P}_N^I)(p^*(N)-\nu) + \frac{1}{2}\mathcal{P}_N^{II}(p^*(N)-\nu)\\ &= \frac{1}{2}(p^*(N)-\nu)(1-\mathcal{P}_N^I+\mathcal{P}_N^{II})\end{aligned}$$

First, we show that $\lim_{N\to\infty} p^*(N) = 1$. If not, suppose $\lim_{N\to\infty} p^*(N) = a < 1$, then by the fact that $\lim_{N\to\infty} \frac{1}{N}\pi(p^*(N), T^*(N)) = \frac{1}{2}(1-\nu)$, we have $\lim_{N\to\infty}(1-\mathcal{P}_N^I+\mathcal{P}_N^{II}) = \frac{1-\nu}{a-\nu}$.

We can plug this into the investor's expected payoff to get

$$\lim_{N\to\infty}(1-\mathcal{P}_N^I) = \lim_{N\to\infty} p^*(N) \lim_{N\to\infty}(1-\mathcal{P}_N^I+\mathcal{P}_N^{II}) = \frac{a(1-\nu)}{a-\nu} > 1. \tag{IA-7}$$

which is a contradiction.

Second, we show that $\mathcal{P}_N^{II} \to 0$ and $\mathcal{P}_N^I = 0$. Taking the limit as $N \to \infty$, and using the fact that

[20] Note that $k \in \{-N, \cdots, N\}$, $l \in \{0, \cdots, N\}$ and $T \in \{0, \cdots, N\}$.

[21] In fact, a "large deviation" result holds: $\frac{1}{N}\sum_{j=1}^{N} x_j = [q-(1-q)] + o(N^{-1/2}(\log N)^{1/2+\delta})$, almost surely, where δ is sufficiently small.

[22] Any $m > 0$ such that $m < (2q-1)$ works, or we can even take $T_N = \log N$, $\log\log N$, $\log\log\log N$, etc.

$\lim_{N\to\infty} p^*(N) = 1$, we know that the investor's limit expected payoff is given by

$$-\frac{1}{2}\lim_{N\to\infty}\mathcal{P}_N^{II} \geq 0$$

This is because, the participation constraint requires a non-negative payoff and thus the limit payoff is also non-negative. Thus, we have $\lim_{N\to\infty}\mathcal{P}_N^{II} = 0$.

Similarly, by the facts that $\lim_{N\to\infty} p^*(N) = 1$ and that $\lim_{N\to\infty}\mathcal{P}_N^{II} = 0$, we get

$$\lim_{N\to\infty}\frac{1}{N}\pi(p^*(N), T^*(N)) = \frac{1}{2}(1-\nu)(1-\lim_{N\to\infty}\mathcal{P}_N^{I}) = \frac{1}{2}(1-\nu)$$

which implies that $\lim_{N\to\infty}\mathcal{P}_N^{I} = 0$.

D Derivations and Proofs for the Internet Appendix

Proof of Proposition IA-1 and Discussion on Numerical Procedure

First, we take (p, T) as given, to derive the subgame-perfect equilibrium. We then prove the existence of the optimal design and provide a numerical procedure for equilibrium searching, when the design of (p, T) is endogenous.

Proof. We first define the concept of *no early under-support* and use it as an equilibrium refinement to ensure tractability.

Definition IA-1. *NEU: An equilibrium satisfies the no-early under-support (NEU) property if an agent i always choose high support (i.e., $a_i = H$) whenever $T - A_{i-1} \geq H + (\bar{k}(p) - k_{i-1} - 1)L$ & $k_{i-1} \leq \bar{k}(p) - 1$, excluding the following two cases that $k_{i-1} = \bar{k}(p) - 1$ and that $k_{i-1} = \bar{k}(p) - 2$ & $T - A_{i-1} = H + L$. Equivalently, $T - A_{i-1} > H + L$.*

Intuitively speaking, the definition says that an agent should fully support whenever it is still possible to achieve threshold implementation.[23] If Agent i chooses full support H, the belief regarding project quality goes up from k_{i-1} to $k_{i-1} + 1$. Now, there is still enough room for learning since the funding gap $T - A_{i-1} - H$ is big enough if we face a sequence of $\bar{k}(p) - k_{i-1} - 1$ good signals so as to push the belief to break even without triggering threshold implementation. Moreover, NEU can be achieved by specifications on the agents' off-equilibrium actions. If an agent does not act accordingly and switch to partial support L too early, all subsequent agents coordinate on punishment by unanimously rejecting the project. One can embed the coordinated punishment as part of the equilibrium to justify NEU.

The next lemma states an incomplete subgame-perfect equilibrium characterization:

Lemma IA-1. *For any given pair of (p, T), there exists an equilibrium with strategies $a_i^*(x_i, \theta_i, \mathcal{H}_{i-1}) \equiv a^*(x_i, \theta_i, k(\mathcal{H}_{i-1}), A_{i-1}(\mathcal{H}_{i-1}))$ and posteriors $P(V = 1|\mathcal{H}_i) = V_{k^*(\mathcal{H}_i)}$, such that:*

[23] In Definition IA-1, we exclude two special cases and more discussions about this is provided after Lemma IA-1.

i) For all on equilibrium history (satisfying NEU),

$$a_i^*(x_i, H, k_{i-1}, A_{i-1}) = \begin{cases} \mathbb{1}_{\{x_i=1\}} * H & \textit{if} \quad A_{i-1} < T - H - L \\ \mathbb{1}_{\{x_i=1\}} * L & \textit{if} \quad A_{i-1} > T - H - (\bar{k}(p) - k_{i-1} - 1)L \ \& \ k_{i-1} \leq \bar{k}(p) - 2 \\ H & \textit{if} \quad k_{i-1} > \bar{k}(p) \\ 0 & \textit{if} \quad A_{i-1} \geq T - L \ \& \ k_{i-1} < \bar{k}(p) - 1 \\ \mathbb{1}_{\{x_i=1\}} * H & \textit{if} \quad A_{i-1} \geq T - L \ \& \ k_{i-1} \in \{\bar{k}(p), \bar{k}(p) - 1\} \end{cases} \tag{IA-8}$$

$$a_i^*(x_i, L, k_{i-1}, A_{i-1}) = \begin{cases} \mathbb{1}_{\{x_i=1\}} * L & \textit{if} \quad A_{i-1} < T - L \ \& \ k_{i-1} \leq \bar{k}(p) \\ L & \textit{if} \quad k_{i-1} > \bar{k}(p) \\ 0 & \textit{if} \quad A_{i-1} \geq T - L \ \& \ k_{i-1} < \bar{k}(p) - 1 \\ \mathbb{1}_{\{x_i=1\}} * L & \textit{if} \quad A_{i-1} \geq T - L \ \& \ k_{i-1} \in \{\bar{k}(p), \bar{k}(p) - 1\} \end{cases} \tag{IA-9}$$

$$k_i^*(\mathcal{H}_i) = \begin{cases} k_{i-1} + \mathbb{1}_{\{a_i \neq 0\}} - \mathbb{1}_{\{a_i = 0\}} & \textit{if} \quad A_{i-1} < T - L \ \& \ k_{i-1} \leq \bar{k}(p) - 1 \\ k_{i-1} & \textit{if} \quad k_{i-1} > \bar{k}(p) \\ k_{i-1} & \textit{if} \quad A_{i-1} \geq T - L \ \& \ k_{i-1} < \bar{k}(p) - 1 \\ k_{i-1} + \mathbb{1}_{\{a_i \neq 0\}} - \mathbb{1}_{\{a_i = 0\}} & \textit{if} \quad A_{i-1} \geq T - L \ \& \ k_{i-1} \in \{\bar{k}(p), \bar{k}(p) - 1\} \end{cases} \tag{IA-10}$$

where $k_0 = 0$ *and* $A_0 = 0$.

ii) For all off-equilibrium path histories whenever NEU is violated,
$a_j = 0$ *and* $k_j^* = k_i^*$, *for all* $j \in \{i+1, \cdots, N\}$.

Proof. The proof needs to check the incentives for two types of agents for both the on-equilibrium path histories and off-equilibrium path histories.

- *On-equilibrium path histories.*
  - Type L's incentives.

    Note that for all on equilibrium histories, the strategy and belief for type $\{\theta_i = L\}$ are essentially the same as the exogenous case in the benchmark model.
  - Type H's incentives.

    First, note that line 3-5 for the strategy specification in Eq. (IA-8) are almost identical to the exogenous case in the benchmark model and thus omitted.

    Second, in light of NEU, there are two types of on equilibrium path histories. i) *histories with big funding gap (relative to the belief gap to break even)*. Formally, this is defined as that in Definition IA-1, that is, $T - A_{i-1} \geq H + (\bar{k}(p) - k_{i-1} - 1)L$. Now we check that agent i of type $\{\theta_i = H\}$ has an incentive to choose high support H. If she rejects, the payoff is trivially zero. If she chooses low support, all subsequent agents switches to rejection and the project will never be implemented. If she chooses high support H, the payoff is strictly positive since there is enough room to push the belief to break even and the sequence of $\bar{k}(p) - k_{i-1} - 1$ successive positive signals suffices for this purpose.

ii) *histories with big belief gap (relative to funding gap to break even).* In other words, $T - A_{i-1} < H + (\bar{k}(p) - k_{i-1} - 1)L$. The payoff is zero if agent i rejects or chooses high support H. For the latter case, the belief cannot be pushed above to break even without triggering threshold implementation. This is not possible under the most favorable sequence of signals where all subsequent signals are positive. In contrast, the funding gap may be still big enough to allow for sufficient learning under low support so that more rounds of trials can be utilized.

- *Off-equilibrium path histories.* We only discuss the off-equilibrium path histories when property 1 is violated, that is, when the agents switch to low support L whereas they are supposed to use full support H. This only happens when agent i is rich and observes a good signal $x_i = 1$. The arguments here only help us pick up an equilibrium satisfying property 1, but it does not negate the existence of other informative equilibrium.

  Recall that, $A_{i-1} < T - H - L$. Now, after observing $a_i = L$ (agent i should have used H), $A_i < T - H$. Let us consider what will agent $i+1$ do in this scenario. The best belief is given by $k_i = \bar{k}(p) - 1$. Now, given all subsequent agents chooses to rejects, agent $i+1$ cannot not implement the project even if the signal $x_{i+1} = 1$, since $A_{i+1} \leq A_i + H < T$. This implies that $a_{i+1} = 0$ is incentive-compatible for agent $i+1$. The same reasoning applies to other agents.

The proof for Lemma IA-1 concludes. □

Before we proceed, we note that both the rich and the poor reject when $x_i = -1$, and that the equilibrium strategy for $\theta_i = L$ has already been specified. Hence, the following analysis only concerns the four cases above when $\theta_i = H$ and $x_i = 1$. Recall that $V_k = \frac{q^k}{q^k + (1-q)^k}$ and we use the following notation

$$f(x) = q \times x + (1-q) \times (1-x).$$

$$a \wedge b = \min\{a, b\}, \text{ and } a \vee b = \max\{a, b\}$$

For easy references, we define

$$
\begin{aligned}
p_{11}^* &= \frac{HV_{\bar{k}} - f(V_{\bar{k}})V_{\bar{k}+1}L}{H - f(V_{\bar{k}})L} \\
p_{12}^* &= \frac{HV_{\bar{k}} - f(V_{\bar{k}})V_{\bar{k}+1}L - (1-f(V_{\bar{k}}))f(V_{\bar{k}-1})V_{\bar{k}}L}{H - L[f(V_{\bar{k}}) + (1-f(V_{\bar{k}}))f(V_{\bar{k}-1})]} \\
p_{21}^* &= \frac{V_1}{V_2}, \quad p_{22}^* = \frac{V_3}{V_4}, \quad p_{23}^* = \frac{W_1}{W_2}, \quad p_{24}^* = \frac{W_3}{W_4} \quad \text{and} \quad p_{34}^* = \frac{Z_1}{Z_2} \\
V_1 &= Hf(V_{\bar{k}})V_{\bar{k}+1} + H(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})V_{\bar{k}} - Lf(V_{\bar{k}})V_{\bar{k}+1} \\
&- L(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})f(V_{\bar{k}})V_{\bar{k}+1} \\
V_2 &= Hf(V_{\bar{k}}) + H(1-f(V_{\bar{k}}))f(V_{\bar{k}-1}) - Lf(V_{\bar{k}}) - L(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})f(V_{\bar{k}}) \\
V_3 &= Hf(V_{\bar{k}})V_{\bar{k}+1} + H(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})V_{\bar{k}} - Lf(V_{\bar{k}})V_{\bar{k}+1} \\
&- L(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\Big(\lambda V_{\bar{k}} + (1-\lambda)f(V_{\bar{k}})V_{\bar{k}+1}\Big) \\
V_4 &= Hf(V_{\bar{k}}) + H(1-f(V_{\bar{k}}))f(V_{\bar{k}-1}) - Lf(V_{\bar{k}}) \\
Z_1 &= V_{\bar{k}}\Big(H - [1-f(V_{\bar{k}})]f(V_{\bar{k}-1})L - [1-f(V_{\bar{k}-1})]f(V_{\bar{k}-2})L\Big) - LV_{\bar{k}+1}f(V_{\bar{k}}) \\
Z_2 &= H - [1-f(V_{\bar{k}})]f(V_{\bar{k}-1})L - [1-f(V_{\bar{k}-1})]f(V_{\bar{k}-2})L - Lf(V_{\bar{k}})
\end{aligned}
$$

$$
\begin{aligned}
W_1 &= V_{\bar{k}+1}f(V_{\bar{k}})\Big(H - L - L(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\Big) \\
&+ V_{\bar{k}}(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\Big(H - Lf(V_{\bar{k}-1})(1-f(V_{\bar{k}})) - (1-f(V_{\bar{k}-1}))f(V_{\bar{k}-2})\Big) \\
W_2 &= f(V_{\bar{k}})\Big(H - L - L(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\Big) \\
&+ (1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\Big(H - Lf(V_{\bar{k}-1})(1-f(V_{\bar{k}})) - (1-f(V_{\bar{k}-1}))f(V_{\bar{k}-2})\Big) \\
W_3 &= V_{\bar{k}+1}f(V_{\bar{k}})\Big(H - L - L(1-f(V_{\bar{k}}))(1-\lambda)\Big) + V_{\bar{k}}(1-f(V_{\bar{k}}))f(V_{\bar{k}-1}) \\
&\times \Big(H - L\big[\lambda + (1-\lambda)(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\big] - L(1-f(V_{\bar{k}-1}))f(V_{\bar{k}-2})\Big) \\
W_4 &= f(V_{\bar{k}})\Big(H - L - L(1-f(V_{\bar{k}}))(1-\lambda)\Big) + (1-f(V_{\bar{k}}))f(V_{\bar{k}-1}) \\
&\times \Big(H - L\big[\lambda + (1-\lambda)(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\big] - L(1-f(V_{\bar{k}-1}))f(V_{\bar{k}-2})\Big)
\end{aligned}
$$

To simplify analysis and ease presentation, we further impose $H < 2L$, i.e., the budget gap is small. Note that the coordinated punishment in the proof works as long as $A_{i-1} < T-H-L$ for any arbitrary $H > L > 0$. However, when $H \geq 2L$, the incentive compatibility conditions become more complicated.

Lemma IA-1 is not complete since there exist four cases unspecified, including: 1) $k_{i-1} = \bar{k}(p) - 1$ & $A_{i-1} = T-H$; 2) $k_{i-1} = \bar{k}(p)-1, A_{i-1} \in [T-H-L, T-H)$; 3) $k_{i-1} = \bar{k}(p)-2$, $A_{i-1} = T-H-L$; and 4) $k_{i-1} = \bar{k}(p) - 1$ & $A_{i-1} > T-H$. Note that the first three cases corresponds to the two cases excluded in Definition IA-1, while the fourth case corresponds to the missing case in line 2 in (IA-8). These four cases have an important common feature, that is, we need to establish incentive compatibility when agent i is rich (i.e., $\theta_i = H$). An important ramification of this feature is that, the rich type agent is endogenously endowed with bargaining power since if the rent is too low, she can always switch to low support to accelerate learning, which can generate strictly positive payoff under a sufficiently optimistic signal sequence. On the one hand, this problem arises because coordinated punishment device is no longer applicable. On the other hand, when the belief is more optimistic and/or the funding gap is bigger, a rich agent enjoys more bargaining power and thus a higher rent generated by incentive compatibility conditions. We also note that Lemma IA-1 does not assert uniqueness of the equilibrium.

One can verify that when $x_i = -1$, we have $a_i = 0$, given the belief updating rule specified in Lemma IA-1. Specifically, under the specified equilibrium belief updating rules, all positive amounts of investments are interpreted as coming from $x_i = 1$ and all zero amounts coming from $x_i = -1$. Hence, agents with $x_i = -1$ have an incentive to choose $a_i = 0$. If $a_i = H/L$ is chosen, the following agents interpret this a_i as coming from an agent with an positive signal, which incorrectly distorts the belief upward and implies that there would be a loss when the project is implemented too early, compared to the break-even belief. Hence, when $x_i = 1$, type $\theta_i = L$ behaves according to Lemma IA-1, and type $\theta_i = H$, under the four cases mentioned above behaves as follows.

- Case 1: $k_{i-1} = \bar{k}(p) - 1$ & $A_{i-1} = T - H$ (and $x_i = 1$ & $\theta_i = H$).

$$a_i^*(x_i, \theta_i, k_{i-1}, A_{i-1}) = \begin{cases} H, & N = i \\ H * \mathbb{1}(p \leq p_{11}^*) + L * \mathbb{1}(p > p_{11}^*), & N = i+1 \\ H * \mathbb{1}(p \leq p_{12}^*) + L * \mathbb{1}(p > p_{12}^*), & N \geq i+2 \end{cases} \tag{IA-11}$$

- Case 2: $k_{i-1} = \bar{k}(p) - 1, A_{i-1} \in [T - H - L, T - H)$ (and $x_i = 1$ & $\theta_i = H$).
  1. $k_{i-1} = \bar{k}(p) - 1, A_{i-1} \in [T - 2L, T - H)$

$$a_i^*(x_i, \theta_i, k_{i-1}, A_{i-1}) \in \{0, L, H\}$$

  2. $k_{i-1} = \bar{k}(p) - 1, A_{i-1} \in [T - H - L, T - 2L)$

$$a_i^*(x_i, \theta_i, k_{i-1}, A_{i-1}) = \begin{cases} \in \{0, L, H\}, & N = i \\ H, & N \in \{i+1, i+2\} \\ H * \mathbb{1}\big(p \leq p_{11}^* \wedge p_{22}^*\big) & +L * \mathbb{1}\big(p \in (p_{11}^* \wedge p_{22}^*, p_{11}^*]\big) \\ +H * \mathbb{1}\big(p \in (p_{11}^*, p_{11}^* \vee p_{21}^*]\big) & +L * \mathbb{1}\big(p > p_{11}^* \vee p_{21}^*\big), \\ & N = i+3 \\ H * \mathbb{1}\big(p \leq p_{12}^* \wedge p_{24}^*\big) & +L * \mathbb{1}\big(p \in (p_{12}^* \wedge p_{24}^*, p_{12}^*]\big) \\ +H * \mathbb{1}\big(p \in (p_{12}^*, p_{12}^* \vee p_{23}^*]\big) & +L * \mathbb{1}\big(p > p_{12}^* \vee p_{23}^*\big), \\ & N \geq i+4 \end{cases} \tag{IA-12}$$

- Case 3: $k_{i-1} = \bar{k}(p) - 2$, $A_{i-1} = T - H - L$ (and $x_i = 1$ & $\theta_i = H$).

$$a_i^*(x_i, \theta_i, k_{i-1}, A_{i-1}) = \begin{cases} \in \{0, L, H\}, & N = i \\ H, & N = i+1 \\ H * \mathbb{1}(p \leq p_{11}^*) + L * \mathbb{1}(p > p_{11}^*), & N = i+2 \\ H * \mathbb{1}(p \leq p_{34}^*) + L * \mathbb{1}(p > p_{34}^*), & N \geq i+3 \end{cases} \tag{IA-13}$$

- Case 4: $k_{i-1} = \bar{k}(p) - 1$ & $A_{i-1} > T - H$ (and $x_i = 1$ & $\theta_i = H$).

$$a_i^*(x_i, \theta_i, k_{i-1}, A_{i-1}) = \begin{cases} H & A_{i-1} \geq T - L \\ H & A_{i-1} \in (T-H, T-L) \text{ \& } N = i \\ H * \mathbb{1}(p \leq p_{11}^*) & +L * \mathbb{1}(p > p_{11}^*) \\ & A_{i-1} \in (T-H, T-L) \text{ \& } N = i+1 \\ H * \mathbb{1}(p \leq p_{12}^*) & +L * \mathbb{1}(p > p_{12}^*) \\ & A_{i-1} \in (T-H, T-L) \text{ \& } N \geq i+2 \end{cases} \tag{IA-14}$$

We prove it case by case.

Case 1: $k_{i-1} = \bar{k}(p) - 1$ **&** $A_{i-1} = T - H$ **(and** $x_i = 1$**).**

Case 1.1: $N = i$**.** Now, agent i is the last one, and she will choose full support H, and gets a payoff $H(V_{\bar{k}} - p)$.

Case 1.2: $N = i + 1$**.** Now, agent i is the second to last agent. By choosing high support H, she gets $H(V_{\bar{k}} - p)$. If agent i chooses low support L, with probability $f(V_{\bar{k}})$, $x_{i+1} = 1$ and it leads to UP cascade; with complementary probability $1 - f(V_{\bar{k}})$, $x_{i+1} = x_N = -1$ and the project will not be implemented since it does not break even. Thus,

$$\Big(f(V_{\bar{k}})(V_{\bar{k}+1} - p) + (1 - f(V_{\bar{k}})) \times 0\Big) \times L$$

or equivalently, agent i chooses the high support H if and only if

$$p \leq \frac{HV_{\bar{k}} - f(V_{\bar{k}})V_{\bar{k}+1}L}{H - f(V_{\bar{k}})L} \equiv p_{11}^* \tag{IA-15}$$

Case 1.3: $N \geq i + 2$**.**

- By choosing rejection, the payoff is 0.
- By choosing H, the payoff is given by $H(V_{\bar{k}} - p)$.
- By choosing L, $A_i = T - H + L$ and $k_i = \bar{k}$. The threshold implementation is not triggered, and it depends on the signals x_{i+1} and x_{i+2}.
  - $x_{i+1} = 1$ occurs with probability $f(V_{\bar{k}})$, since $V_{\bar{k}}$ is the new prior of the project being good at period $i+1$. Now, $k_{i+1} = \bar{k} + 1$, and UP cascade happens and $A_{i+1} \geq T - H + 2L \geq T$, and thus the project is implemented. Correspondingly, the payoff is $(V_{\bar{k}+1} - p)$.
  - $x_{i+1} = -1$ occurs with complementary probability $1 - f(V_{\bar{k}})$. Thus, $k_{i+1} = \bar{k} - 1$, $A_{i+1} = T - H + L$.
    * $x_{i+2} = 1$ occurs with probability $f(V_{\bar{k}-1})$. Now, independent of being rich or poor, agent $i+2$ will support, and thus $k_{i+2} = \bar{k}$ and $A_{i+2} \geq T - H + 2L \geq T$. Correspondingly, the return is $V(\bar{k}) - p$ per dollar invested.

* $x_{i+2} = -1$, which implies that $k_{i+2} = \bar{k} - 2$ and recall that $A_{i+1} = T - H + L$. These two conditions imply DOWN cascade and the return is 0.

To summarize, partial support L yields a payoff

$$\big(f(V_{\bar{k}})(V_{\bar{k}+1} - p) + (1 - f(V_{\bar{k}})f(V_{\bar{k}-1})(V_{\bar{k}} - p)\big) \times L$$

Hence, agent i of the rich type $\theta_i = H$ will switch to L whenever

$$\Big(f(V_{\bar{k}})(V_{\bar{k}+1} - p) + (1 - f(V_{\bar{k}})\{f(V_{\bar{k}-1})(V_{\bar{k}} - p) + (1 - f(V_{\bar{k}-1})) * 0\}\Big) \times L \geq H(V_{\bar{k}} - p) \quad \text{(IA-16)}$$

Equation (IA-16) highlights the endogenous bargaining power by the rich agent. It is now impossible for the proposer to subtract all surplus as that in the benchmark model since the rich type agent can switch to low support L when the coordinated punishment is not available, which may generate UP cascade and the agent can receive a positive rent. For instance, if the price $p \to V_{\bar{k}}$, then agent i will always switch to low support L, which guarantees a rent no less than $f(V_{\bar{k}})(V_{\bar{k}+1} - p) \times L$. This, in turn, implies that the full surplus extraction is not plausible.

We can simplify equation (IA-16) to find out that agent i of type $\theta_i = H$ will choose H only when

$$p \leq \frac{HV_{\bar{k}} - f(V_{\bar{k}})V_{\bar{k}+1}L - (1 - f(V_{\bar{k}}))f(V_{\bar{k}-1})V_{\bar{k}}L}{H - L[f(V_{\bar{k}}) + (1 - f(V_{\bar{k}}))f(V_{\bar{k}-1})]} \equiv p^*_{12} \quad \text{(IA-17)}$$

It is easy to check that $p^*_{12} < V_{\bar{k}}$, which negates the possibility of full surplus extraction.

Now, (IA-11) summarizes the discussion above, which concludes the analysis for case 1. ■

Case 2: $k_{i-1} = \bar{k}(p) - 1$ **and** $A_{i-1} \in [T - H - L, T - H)$ We divide it into two cases.

Case 2.1: $k_{i-1} = \bar{k}(p) - 1$**,** $T - 2L \leq A_{i-1} < T - H$ **(and** $x_i = 1$**).**

This case is very straightforward, and it is optimal for agent i of type $\theta_i = H$ to use full support H. Given that $H \in (L, 2L)$, Agent i's type is not important anymore, since either $2L$, $H + L$ or $2H$ lead to the same future threshold implementation. Hence, it is always optimal to use H as long as the return is non-negative.

Case 2.1.1: $N = i$**.** It is impossible to get enough funds to achieve threshold implementation, and agent i can choose any action from $\{0, L, H\}$.

Case 2.1.2: $N = i + 1$**.**

- By choosing rejection, the payoff is 0. Thus, $U_i(0) = 0$.
- By choosing H, then $A_i \in [T - 2L + H, T)$ and $k_i = \bar{k}$. Since the threshold is not reached, we need to check the signal x_{i+1}.

  1. x_{i+1} occurs with probability $f(V_{\bar{k}})$. Now, $k_{i+1} = \bar{k} + 1$ and $A_{i+1} = A_i + a_{i+1} \geq T$. The return per dollar invested is $V_{\bar{k}+1} - p$.
  2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}})$, which implies that $k_{i+1} = \bar{k} - 1$ and $A_{i+1} = A_i \in [T - 2L + H, T)$. The project is not implemented and the return is 0.

To summarize,

$$U_i(H) = H \times \left\{ f(V_{\bar{k}})(V_{\bar{k}+1} - p) + (1 - f(V_{\bar{k}}))f(V_{\bar{k}-1}) * 0 \right\} = Hf(V_{\bar{k}})(V_{\bar{k}+1} - p)$$

- By choosing L, then $A_i \in [T - L, T - H + L)$ and $k_i = \bar{k}$. Since the threshold is not reached, we need to check the signal x_{i+1}.

  1. x_{i+1} occurs with probability $f(V_{\bar{k}})$. Now, $k_{i+1} = \bar{k} + 1$ and $A_{i+1} = A_i + a_{i+1} \geq T$. The return per dollar invested is $V_{\bar{k}+1} - p$.
  2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}})$, which implies that $k_{i+1} = \bar{k} - 1$ and $A_{i+1} = A_i \in [T - 2L + H, T)$. The project is not implemented and the return is 0.

  To summarize,

$$U_i(H) = L \times \left\{ f(V_{\bar{k}})(V_{\bar{k}+1} - p) + (1 - f(V_{\bar{k}}))f(V_{\bar{k}-1}) * 0 \right\} = Lf(V_{\bar{k}})(V_{\bar{k}+1} - p)$$

Note that, we always have

$$U_i(H) \geq U_i(L) \geq U_i(0).$$

Henceforth, it is optimal for a rich agent i to use full support H.

Case 2.1.3: $N \geq i + 2$.

- By choosing rejection, the payoff is 0, that is, $U_i(0) = 0$.
- By choosing H, then $A_i \in [T - 2L + H, T)$ and $k_i = \bar{k}$. Since the threshold is not reached, it depends on future signals, including x_{i+1} and x_{i+2}.

  1. $x_{i+1} = 1$ occurs with probability $f(V_{\bar{k}})$, which implies that $k_{i+1} = \bar{k} + 1$ and $A_{i+1} = A_i + a_{i+1} \geq T$. Thus, the return per dollar invested is $(V_{\bar{k}+1} - p)$.
  2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}})$, which implies that $k_{i+1} = \bar{k} - 1$ and $A_{i+1} = A_i \in [T - 2L + H, T)$. For this case, project implementation depends further on x_{i+2}.
     - $x_{i+2} = 1$ occurs with probability $f(V_{\bar{k}-1})$. Now, $k_{i+2} = k_{i+1} + 1 = \bar{k}$. Agent $i + 2$ chooses to support and $A_{i+2} = A_{i+1} + a_{i+2} \geq T$. The project is implemented and the return is $(V_{\bar{k}} - p)$.
     - $x_{i+2} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. Now, $k_{i+2} = k_{i+1} - 1 = \bar{k} - 2$ and $A_{i+2} = A_{i+1} \in [T - L, T)$. Along this history, it is impossible to break even and DOWN cascade arises. Obviously, the return is 0.

  To summarize, by choosing full support H, it generates a payoff

$$U_i(H) = H \times \left\{ f(V_{\bar{k}})(V_{\bar{k}+1} - p) + [1 - f(V_{\bar{k}})]\, f(V_{\bar{k}-1})(V_{\bar{k}} - p) \right\}$$

- By choosing L, then $A_i \in [T - L, T - H + L)$ and $k_i = k_{i-1} + 1 = \bar{k}$. The project is not implemented and it depends on future signals. Before we proceed, we divide the analysis into two cases.

1. x_{i+1} occurs with probability $f(V_{\bar{k}})$. Now, $k_{i+1} = k_i + 1 = \bar{k} + 1$, and there will be an UP cascade. Note that $A_{i+1} \geq A_i + L = T$, and thus the project is implemented with a return of $(V_{\bar{k}+1} - p)$.

2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}})$. Now, $k_{i+1} = \bar{k} - 1$ and $A_{i+1} = A_i \in [T - L, T - H + L)$. Hence, the threshold implementation is undetermined.

   - $x_{i+2} = 1$ occurs with probability $f(V_{\bar{k}-1})$. Now, $k_{i+2} = k_{i+1} + 1 = \bar{k}$ and $a_{i+2} \geq L$. Thus, $A_{i+2} = A_{i+1} + a_{i+2} \geq T$ and the project is implemented with a return of $(V_{\bar{k}} - p)$.
   - $x_{i+2} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. Now, $k_{i+2} = k_{i+1} - 1 = \bar{k} - 2$, and $A_{i+2} = A_i \geq T - L$. Thus, there will be DOWN cascade since the threshold implementation will be triggered with beliefs bounded by $\bar{k} - 1$, which generates a strict loss. Henceforth, the return is 0.

To summarize, by choosing low support L, the expected payoff for agent i of type $\theta_i = H$ is given by:

$$U_i(L) = L \times \left\{ f(V_{\bar{k}})(V_{\bar{k}+1} - p) + [1 - f(V_{\bar{k}})]\, f(V_{\bar{k}-1})(V_{\bar{k}} - p) \right\}$$

Note that we always have

$$U_i(H) \geq U_i(L) \geq U_i(0).$$

Hence, it is optimal to use H when agent i is rich. ■

Case 2.2: $k_{i-1} = \bar{k}(p) - 1$**,** $T - H - L \leq A_{i-1} < T - 2L$ **(and** $x_i = 1$**).**

Case 2.2.1: $N = i$**.**

It is impossible to get enough funds to achieve threshold implementation, and agent i can choose any action from $\{0, L, H\}$.

Case 2.2.2: $N = i + 1$**.**

- By choosing rejection, the payoff is 0, that is, $U_i(0) = 0$.
- By choosing H, then $A_i \in [T - L, T - 2L + H)$ and $k_i = \bar{k}$. Since the threshold is not reached, it depends on the signal in the last period, x_{i+1}.

  1. $x_{i+1} = 1$ occurs with probability $f(V_{\bar{k}})$. Now, the belief $k_{i+1} = \bar{k} + 1$, and thus $a_{i+1} \geq L$. The proposal is implemented since $A_{i+1} \geq A_i + L \geq T$. The return per dollar invested is $(V_{\bar{k}+1} - p)$.
  2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}})$. Now, the belief $k_{i+1} = \bar{k} - 1$, and thus $a_{i+1} = 0$. The proposal is abandoned and thus the return is 0.

  The expected payoff of choosing H is given by

  $$U_i(H) = H f(V_{\bar{k}})(V_{\bar{k}+1} - p)$$

- By choosing L, then $A_i \in [T - H, T - L)$ and $k_i = \bar{k}$. Since the threshold is not reached, it depends on the signal in the last period, x_{i+1}.

1. $x_{i+1} = 1$ occurs with probability $f(V_{\bar{k}})$. Now, the belief $k_{i+1} = \bar{k} + 1$. In order to reach the proposal threshold, we need $a_i > T$, which only happens when $\theta_{i+1} = H$. The return per dollar invested is $(V_{\bar{k}+1} - p)$ when $\theta_{i+1} = H$. In other words, the proposal is abandoned when $\theta_{i+1} = L$, leading to a return of 0.
2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}})$. Now, the belief $k_{i+1} = \bar{k} - 1$, and thus $a_{i+1} = 0$. The proposal is abandoned and thus the return is 0.

The expected payoff of choosing H is given by

$$U_i(L) = \lambda L f(V_{\bar{k}})(V_{\bar{k}+1} - p)$$

Henceforth, $a_i^* = H$ when $N = i + 1$.

Case 2.2.3: $N = i + 2$.

- By choosing rejection, the payoff is 0, that is, $U_i(0) = 0$.
- By choosing H, then $A_i \in [T - L, T - 2L + H)$ and $k_i = \bar{k}$. Since the threshold is not reached, it depends on future signals, including x_{i+1} and x_{i+2}.

  1. $x_{i+1} = 1$ occurs with probability $f(V_{\bar{k}})$, which implies that $k_{i+1} = \bar{k}+1$ and $A_{i+1} = A_i + a_{i+1} \geq T$. Thus, the return per dollar invested is $(V_{\bar{k}+1} - p)$.
  2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}})$, which implies that $k_{i+1} = \bar{k} - 1$ and $A_{i+1} = A_i \in [T - L, T - 2L + H)$. For this case, project implementation depends further on x_{i+2}.
     - $x_{i+2} = 1$ occurs with probability $f(V_{\bar{k}-1})$. Now, $k_{i+2} = k_{i+1} + 1 = \bar{k}$. Agent $i + 2$ chooses to support and $A_{i+2} = A_{i+1} + a_{i+2} \geq T$. The project is implemented and the return is $(V_{\bar{k}} - p)$.
     - $x_{i+2} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. Now, $k_{i+2} = k_{i+1} - 1 = \bar{k} - 2$ and $A_{i+2} = A_{i+1} \in [T - L, T)$. Along this history, it is impossible to break even and DOWN cascade arises. Obviously, the return is 0.

  To summarize, by choosing full support H, it generates a payoff

$$U_i(H) = H \times \left\{ f(V_{\bar{k}})(V_{\bar{k}+1} - p) + [1 - f(V_{\bar{k}})]\, f(V_{\bar{k}-1})(V_{\bar{k}} - p) \right\}$$

- By choosing L, then $A_i \in [T - H, T - L)$ and $k_i = k_{i-1} + 1 = \bar{k}$. The project is not implemented and it depends on future signals.

  1. $x_{i+1} = 1$ occurs with probability $f(V_{\bar{k}})$. Now, $k_{i+1} = k_i + 1 = \bar{k} + 1$, and there will be an UP cascade. If $\theta_{i+1} = H$, then the threshold implementation is triggered instantly at period $i + 1$. Otherwise, if $\theta_{i+1} = L$, then $A_{i+1} = A_i + L \in [T - H + L, T)$ and thus the project will be implemented at period $i + 2$. In either case, the return per dollar invested is $(V_{\bar{k}+1} - p)$.
  2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}})$. Now, $k_{i+1} = \bar{k} - 1$ and $A_{i+1} = A_i \in [T - H, T - L)$. Hence, the threshold implementation is undetermined.

- $x_{i+2} = 1$ occurs with probability $f(V_{\bar{k}-1})$. Now, $k_{i+2} = k_{i+1} + 1 = \bar{k}$. Since agent $i+2$ is the last one, the threshold can be reached only when $\theta_{i+2} = H$ (which happens with probability λ). The return under this case is $(V_{\bar{k}} - p)$. When $\theta_{i+2} = L$, it is impossible to reach the implementation threshold, and thus the proposal is abandoned. In this case, the return is 0.
- $x_{i+2} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. Now, $k_{i+2} = k_{i+1} - 1 = \bar{k} - 2$, and $A_{i+2} = A_i \in [T-H, T-L)$. The threshold is not reached, and thus the proposal is abandoned. The return is 0.

To summarize, the payoff of choosing L is given by

$$U_i(L) = L \times \left\{ f(V_{\bar{k}})(V_{\bar{k}+1} - p) + (1 - f(V_{\bar{k}}))f(V_{\bar{k}-1})\lambda(V_{\bar{k}} - p) \right\}$$

Note that $U_i(H) > U_i(L) > U_i(0)$. Henceforth, $a_i^* = H$ when $N = i+2$.

Case 2.2.4; $N = i + 3$.

Note that when agent i chooses rejection or high support (i.e., $a_i \in \{0, H\}$), threshold implementation will be determined within the next two periods. Hence, the analysis coincides with that in case 2.2.3.

- By choosing rejection, the payoff is 0, that is, $U_i(0) = 0$.
- By choosing H,

$$U_i(H) = H \times \left\{ f(V_{\bar{k}})(V_{\bar{k}+1} - p) + [1 - f(V_{\bar{k}})]\, f(V_{\bar{k}-1})(V_{\bar{k}} - p) \right\}$$

- By choosing L, then $A_i \in [T-H, T-L)$ and $k_i = k_{i-1} + 1 = \bar{k}$. The project is not implemented and it depends on future signals.

  1. $x_{i+1} = 1$ occurs with probability $f(V_{\bar{k}})$. Now, $k_{i+1} = k_i + 1 = \bar{k} + 1$, and there will be an UP cascade. If $\theta_{i+1} = H$, then the threshold implementation is triggered instantly at period $i+1$. Otherwise, if $\theta_{i+1} = L$, then $A_{i+1} = A_i + L \in [T - H + L, T)$ and thus the project will be implemented at period $i+2$. In either case, the return per dollar invested is $(V_{\bar{k}+1} - p)$.
  2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}})$. Now, $k_{i+1} = \bar{k} - 1$ and $A_{i+1} = A_i \in [T-H, T-L)$. Hence, the threshold implementation is undetermined.
     - $x_{i+2} = 1$ occurs with probability $f(V_{\bar{k}-1})$. Now, $k_{i+2} = k_{i+1} + 1 = \bar{k}$.
       (a) $\theta_{i+2} = H$ occurs with probability λ. Now, agent $i+2$ faces the same decision problem as agent i in case 1.2.[24] Specifically, if $p > p_{11}^*$, agent $i+2$ prefers low support L, and the return under this case is given by

$$f(V_{\bar{k}})(V_{\bar{k}+1} - p)$$

[24]In case 1.2, $k_{i-1} = \bar{k} - 1$ and $A_{i-1} = T - H$. However, the underlying decision problem coincides since H will trigger implementation, and L will not in both cases.

Otherwise, when $p \le p^*_{11}$, agent $i+2$ chooses high support H. Correspondingly, the return will be $(V_{\bar{k}} - p)$.

(b) $\theta_{i+2} = L$ occurs with probability $1-\lambda$. Agent $i+2$ will choose low support (i.e., $a_{i+2} = L$) and thus $A_{i+2} = A_{i+1} + L \in [T - H + L, T)$ and $k_{i+2} = \bar{k}$. This coincides with the case above when a rich agent $i+2$ chooses L, and thus return is given by $f(V_{\bar{k}})(V_{\bar{k}+1} - p)$.[25]

– $x_{i+2} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. Now, $k_{i+2} = k_{i+1} - 1 = \bar{k} - 2$, and $A_{i+2} = A_i \in [T - H, T - L)$. The threshold is not reached yet. However, the proposal is abandoned *de facto*, because even after a good signal $x_{i+3} = 1$, the belief cannot break even (i.e., $k_{i+3} = \bar{k} - 1$). Henceforth, the return is 0.

To summarize, the payoff of choosing low support L is given by

1. When $p > p^*_{11}$,

$$U_i(L|p > p^*_{11}) = L \times \Big\{ f(V_{\bar{k}})(V_{\bar{k}+1} - p) + (1 - f(V_{\bar{k}}))f(V_{\bar{k}-1})f(V_{\bar{k}})(V_{\bar{k}+1} - p) \Big\}$$

This implies that agent $i+2$ will choose H only when

$$p \le p^*_{21} := \frac{V_1}{V_2}$$

where

$$V_1 = Hf(V_{\bar{k}})V_{\bar{k}+1} + H(1 - f(V_{\bar{k}}))f(V_{\bar{k}-1})V_{\bar{k}} - Lf(V_{\bar{k}})V_{\bar{k}+1} - L(1 - f(V_{\bar{k}}))f(V_{\bar{k}-1})f(V_{\bar{k}})V_{\bar{k}+1}$$

and

$$V_2 = Hf(V_{\bar{k}}) + H(1 - f(V_{\bar{k}}))f(V_{\bar{k}-1}) - Lf(V_{\bar{k}}) - L(1 - f(V_{\bar{k}}))f(V_{\bar{k}-1})f(V_{\bar{k}})$$

2. When $p \le p^*_{11}$,

$$U_i(L|p \le p^*_{11}) = L \times \Big\{ f(V_{\bar{k}})(V_{\bar{k}+1} - p) + (1 - f(V_{\bar{k}}))f(V_{\bar{k}-1}) \times \Big(\lambda(V_{\bar{k}} - p) + (1-\lambda)f(V_{\bar{k}})(V_{\bar{k}+1} - p) \Big) \Big\}$$

This implies that agent $i+2$ will choose H only when

$$p \le p^*_{22} := \frac{V_3}{V_4}$$

[25] The threshold is not reached and thus depends on the signal x_{i+3}. Specifically, $x_{i+3} = 1$ occurs with probability $f(V_{\bar{k}})$. Now, $k_{i+3} = \bar{k} + 1$ and $a_{i+3} \ge L$. This implies that $A_{i+3} = A_{i+2} + a_{i+3} > T$. The project ends with an UP cascade and the return per dollar invested is $(V_{\bar{k}+1} - p)$. In contrast, when $x_{i+3} = -1$, then $k_{i+3} = \bar{k} - 1$. Hence, the proposal is abandoned and the return is 0.

where

$$V_3 = Hf(V_{\bar{k}})V_{\bar{k}+1} + H(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})V_{\bar{k}} - Lf(V_{\bar{k}})V_{\bar{k}+1} - L(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\Big(\lambda V_{\bar{k}} + (1-\lambda)f(V_{\bar{k}})V_{\bar{k}+1}\Big)$$

and

$$V_4 = Hf(V_{\bar{k}}) + H(1-f(V_{\bar{k}}))f(V_{\bar{k}-1}) - Lf(V_{\bar{k}}) - L(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})(\lambda + (1-\lambda)f(V_{\bar{k}}))$$

Case 2.2.5: $N \geq i+4$**.**

For reasons stated in case 2.2.4, we have

- By choosing rejection, the payoff is 0, that is, $U_i(0) = 0$.
- By choosing H,

$$U_i(H) = H \times \Big\{ f(V_{\bar{k}})(V_{\bar{k}+1} - p) + [1 - f(V_{\bar{k}})]\, f(V_{\bar{k}-1})(V_{\bar{k}} - p) \Big\}$$

- By choosing L, then $A_i \in [T-H, T-L)$ and $k_i = k_{i-1} + 1 = \bar{k}$. The project is not implemented and it depends on future signals.

  1. $x_{i+1} = 1$ occurs with probability $f(V_{\bar{k}})$. Now, $k_{i+1} = k_i + 1 = \bar{k} + 1$, and there will be an UP cascade. If $\theta_{i+1} = H$, then the threshold implementation is triggered instantly at period $i+1$. Otherwise, if $\theta_{i+1} = L$, then $A_{i+1} = A_i + L \in [T-H+L, T)$ and thus the project will be implemented at period $i+2$. In either case, the return per dollar invested is $(V_{\bar{k}+1} - p)$.
  2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}})$. Now, $k_{i+1} = \bar{k} - 1$ and $A_{i+1} = A_i \in [T-H, T-L)$. Hence, the threshold implementation is undetermined.
     - $x_{i+2} = 1$ occurs with probability $f(V_{\bar{k}-1})$. Now, $k_{i+2} = k_{i+1} + 1 = \bar{k}$.
       (a) $\theta_{i+2} = H$ occurs with probability λ. In this case, agent $i+2$ faces the same decision problem as agent i in case 1 as long as there exists two more agents (i.e., $N \geq i+4$).[26] In other words, if $p > p^*_{12}$, agent $i+2$ prefers low support L, and the return under this case is given by

       $$f(V_{\bar{k}})(V_{\bar{k}+1} - p) + (1 - f(V_{\bar{k}}))f(V_{\bar{k}-1})(V_{\bar{k}} - p)$$

       Otherwise, when $p \leq p^*_{12}$, agent $i+2$ chooses high support H. Correspondingly, the return will be $(V_{\bar{k}} - p)$.

       (b) $\theta_{i+2} = L$ occurs with probability $1 - \lambda$. Agent $i+2$ will choose low support (i.e., $a_{i+2} = L$) and thus $A_{i+2} = A_{i+1} + L \in [T-H+L, T)$ and $k_{i+2} = \bar{k}$. The threshold is not reached and thus depends on future signals.

[26] In case 1, $k_{i-1} = \bar{k} - 1$ and $A_{i-1} = T - H$. However, the underlying decision problem coincides since H will trigger implementation, and L will not in both cases.

* $x_{i+3} = 1$ occurs with probability $f(V_{\bar{k}})$. Now, $k_{i+3} = \bar{k} + 1$ and $a_{i+3} \geq L$. This implies that $A_{i+3} = A_{i+2} + a_{i+3} > T$. The project ends with an UP cascade and the return per dollar invested is $(V_{\bar{k}+1} - p)$.
* $x_{i+3} = -1$ occurs with probability $1 - f(V_{\bar{k}})$. $k_{i+3} = \bar{k} - 1$, $a_{i+3} = 0$ and $A_{i+3} \in [T - H + L, T)$.
  i. $x_{i+4} = 1$ occurs with probability $f(V_{\bar{k}-1})$. Now, $k_{i+4} = \bar{k}$, $a_{i+4} \geq L$ and $A_{i+4} > T$. The project is implemented with a return of $V_{\bar{k}} - p$.
  ii. $x_{i+4} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. Now, $k_{i+4} = \bar{k} - 2$, $a_{i+4} = 0$ and $A_{i+4} = A_{i+3} > T - L$. From now on, the DOWN cascade arises since the project can only be implemented with a belief bounded above by $\bar{k} - 1$ (with a strict loss). The return is 0.

- $x_{i+2} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. Now, $k_{i+2} = k_{i+1} - 1 = \bar{k} - 2$, and $A_{i+2} = A_i \in [T - H, T - L)$. The threshold is not reached yet and it depends on future signal x_{i+3} and x_{i+4}. Actually, only two successive good signals can implement the project. Specifically,
  (a) $x_{i+3} = 1$ occurs with probability $f(V_{\bar{k}-2})$. Now, $k_{i+3} = k_{i+2} + 1 = \bar{k} - 1$, and $a_{i+3} = L$. This is because for a rich agent $i + 3$ (i.e., $\theta_{i+3} = H$), choosing H triggers threshold implementation with a belief below the break-even point. Thus, we have $A_{i+3} \in [T - H + L, T)$.
    * $x_{i+4} = 1$ occurs with probability with $f(V_{\bar{k}-1})$. This implies that $k_{i+4} = \bar{k}$ and $a_{i+4} \geq L$. Hence, the project is implemented since $A_{i+4} = A_{i+3} + a_{i+4} \geq T - H + 2L > T$. The return is $(V_{\bar{k}} - p)$.
    * $x_{i+4} = -1$ occurs with probability with $1 - f(V_{\bar{k}-1})$. This implies that $k_{i+4} = \bar{k} - 2$ and DOWN cascade arises since the belief cannot be pushed as high as the break-even point. The return is 0.
  (b) $x_{i+3} = -1$ occurs with probability $1 - f(V_{\bar{k}-2})$. Now, $k_{i+3} = k_{i+2} - 1$, and $A_{i+3} = A_{i+2} \in [T - H, T - L)$. Note that we have a DOWN cascade along this history, since the future belief is bounded above by $\bar{k} - 1$ when the threshold T is reached. Hence, the return is 0.

To summarize, given $x_i = 1$ and $\theta_i = H$,

1. When $p > p^*_{12}$, by choosing low support L, the expected payoff is given by:

$$U_i(L|p \in (p^*_{12}, V_{\bar{k}}]) = L \times \Big\{ f(V_{\bar{k}})(V_{\bar{k}+1} - p) + [1 - f(V_{\bar{k}})] \times \Big(f(V_{\bar{k}-1}) \Big[f(V_{\bar{k}})(V_{\bar{k}+1} - p) + (1 - f(V_{\bar{k}})) f(V_{\bar{k}-1})(V_{\bar{k}} - p) \Big] + (1 - f(V_{\bar{k}-1})) f(V_{\bar{k}-2}) f(V_{\bar{k}-1})(V_{\bar{k}} - p) \Big) \Big\}$$

   Equivalently, agent i of type $\theta_i = H$ will choose H only when

$$p \leq p^*_{23} := \frac{W_1}{W_2}$$

where

$$W_1 = V_{\bar{k}+1} f(V_{\bar{k}})\Big(H - L - L(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\Big) \\ + V_{\bar{k}}(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\Big(H - Lf(V_{\bar{k}-1})(1-f(V_{\bar{k}})) - (1-f(V_{\bar{k}-1}))f(V_{\bar{k}-2})\Big)$$

and

$$W_2 = f(V_{\bar{k}})\Big(H - L - L(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\Big) \\ + (1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\Big(H - Lf(V_{\bar{k}-1})(1-f(V_{\bar{k}})) - (1-f(V_{\bar{k}-1}))f(V_{\bar{k}-2})\Big)$$

2. When $p \le p^*_{12}$, by choosing low support L, the expected payoff is given by:

$$U_i(L|p \le p^*_{12}) = L \times \Big\{ f(V_{\bar{k}})(V_{\bar{k}+1} - p) + [1-f(V_{\bar{k}})] \times \Big(f(V_{\bar{k}-1})\Big\{\lambda(V_{\bar{k}} - p) \\ (1-\lambda)\Big[f(V_{\bar{k}})(V_{\bar{k}+1}-p) + (1-f(V_{\bar{k}}))f(V_{\bar{k}-1})(V_{\bar{k}}-p)\Big]\Big\} + (1-f(V_{\bar{k}-1}))f(V_{\bar{k}-2})f(V_{\bar{k}-1})(V_{\bar{k}}-p)\Big)\Big\}$$

Equivalently, given $p \le p^*_{12}$, agent i of type $\theta_i = H$ will choose H only when

$$p \le p^*_{24} := \frac{W_3}{W_4}$$

where

$$W_3 = V_{\bar{k}+1} f(V_{\bar{k}})\Big(H - L - L(1-f(V_{\bar{k}}))(1-\lambda)\Big) + V_{\bar{k}}(1-f(V_{\bar{k}}))f(V_{\bar{k}-1}) \\ \times \Big(H - L\big[\lambda + (1-\lambda)(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\big] - L(1-f(V_{\bar{k}-1}))f(V_{\bar{k}-2})\Big)$$

and

$$W_4 = f(V_{\bar{k}})\Big(H - L - L(1-f(V_{\bar{k}}))(1-\lambda)\Big) + (1-f(V_{\bar{k}}))f(V_{\bar{k}-1}) \\ \times \Big(H - L\big[\lambda + (1-\lambda)(1-f(V_{\bar{k}}))f(V_{\bar{k}-1})\big] - L(1-f(V_{\bar{k}-1}))f(V_{\bar{k}-2})\Big)$$

Now, (IA-12) summarizes the discussion above, which concludes the analysis for case 2. ■

Case 3: $k_{i-1} = \bar{k}(p) - 2$, $A_{i-1} = T - H - L$ (and $x_i = 1$).

Case 3.1: $N = i$. It is impossible to reach AON. Agent i can choose any action from $\{0, L, H\}$.

Case 3.2: $N = i + 1$. In this case, only when $x_{i+1} = 1$ occurs, the threshold can be reached with the belief given by $k_{i+1} = 1$. Since $V_{\bar{k}} - p$ is non-negative, it is optimal to choose H and the payoff is given by

$$U_i(H) = Hf(V_{\bar{k}-1})(V_{\bar{k}} - p)$$

In contrast, if agents i chooses L, then the project will only be implemented when $x_{i+1} = 1$ and $\theta_{i+1} = H$

(which occurs with probability λ). Thus, the payoff of choosing L is given by

$$U_i(L) = L\lambda f(V_{\bar{k}-1})(V_{\bar{k}} - p)$$

Henceforth, it is always weakly dominant to choose full support (i.e., $a_i^* = H$).

Case 3.3: $N = i + 2$.

- By choosing rejection, the payoff is 0.
- By choosing H, then $k_i = \bar{k} - 1$ and $A_i = A_{i-1} + H = T - L$. Threshold implementation is not triggered and thus project implementation depends further on future signals in period $i + 1$.
  1. $x_{i+1} = 1$ occurs with probability $f(V_{\bar{k}-1})$, which implies that $k_{i+1} = \bar{k}$ and thus $A_{i+1} \geq A_i + L = T$. Under this event, the return per dollar invested is $(V_{\bar{k}} - p)$.
  2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. In this case, $k_{i+1} = \bar{k} - 2$ and thus $A_{i+1} = A_i = T - L$. Now, there is DOWN cascade since even low support L will trigger threshold implementation, but the belief is bounded above by $\bar{k} - 1$ and thus impossible to break even. In short, the return per dollar invested is 0.

  To summarize, the expected payoff from choosing H is given by

$$H\left\{f(V_{\bar{k}-1})(V_{\bar{k}} - p) + (1 - f(V_{\bar{k}-1})) * 0\right\} = f(V_{\bar{k}-1})(V_{\bar{k}} - p)H$$

- By choosing L, then $k_i = \bar{k} - 1$ and $A_i = A_{i-1} + L = T - H$.
  1. $x_{i+1} = 1$ occurs with probability $f(V_{\bar{k}-1})$. Now, $k_{i+1} = \bar{k}$. Note the independence between realization of signals and the type of the agent.
     - $\theta_{i+1} = H$ occurs with probability λ. If agent $i + 1$ chooses high support H, then $k_{i+1} = \bar{k}$ and $A_{i+1} = A_i + H = T$. The return per dollar invested is $(V_{\bar{k}} - p)$. If agent $i + 1$ chooses low support L, then $k_{i+1} = \bar{k}$ and $A_{i+1} = A_i + L = T - H + L$. In this case, we need to consider the signal x_{i+2}.
       (a) $x_{i+2} = 1$ occurs with probability $f(V_{\bar{k}})$. Now, the posterior $k_{i+2} = k_{i+1} + 1 = \bar{k} + 1$, which triggers UP cascade. Hence, the return per dollar invested is $(V_{\bar{k}+1} - p)$.
       (b) $x_{i+2} = -1$ occurs with probability $1 - f(V_{\bar{k}})$. Now, the posterior $k_{i+2} = k_{i+1} - 1 = \bar{k} - 1$, and $A_{i+2} = A_{i+1} = T - H + L$. The proposal is abandoned and thus the return is 0.
     - $\theta_{i+1} = L$ occurs with probability $1 - \lambda$. Now, $k_{i+1} = \bar{k}$ and $A_{i+1} = A_i + L = T - H + L$.
       (a) $x_{i+2} = 1$ occurs with probability $f(V_{\bar{k}})$. Then, $k_{i+2} = \bar{k} + 1$, which implies UP cascade. The return per dollar invested is $V_{\bar{k}+1} - p$.
       (b) $x_{i+2} = -1$ occurs with probability $1 - f(V_{\bar{k}})$. Then, $k_{i+2} = \bar{k} - 1$ and $A_{i+2} = A_{i+1} = T - H + L$.
  2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. Now, $k_{i+1} = \bar{k} - 2$ and $A_{i+1} = A_i = T - H$. Since it is impossible to break even and the belief is bounded above by $\bar{k} - 1$ even if $x_{i+2} = 1$. Hence, the project is abandoned and the return is 0 under this history.

To summarize, when $p > p_{11}^*$ (i.e., agent $i+1$ of type $\theta_{i+1} = H$ chooses L after $x_{i+1} = 1$), then the payoff of choosing L for agent i is given by

$$U_i(L|p > p_{11}^*) = Lf(V_{\bar{k}-1})f(V_{\bar{k}})(V_{\bar{k}+1} - p)$$

This implies that agent i of type $\theta_i = H$ after signal $x_i = 1$ will choose H when

$$p \leq \frac{HV_{\bar{k}} - Lf(V_{\bar{k}})V_{\bar{k}+1}}{H - Lf(V_{\bar{k}})} := p_{31}^* = p_{11}^*$$

Otherwise, when $p \leq p_{11}^*$ (i.e., agent $i+1$ of type $\theta_{i+1} = H$ chooses H after $x_{i+1} = 1$), then the payoff of choosing L is given by

$$U_i(L|p \leq p_{11}^*) = Lf(V_{\bar{k}-1})\Big[\lambda(V_{\bar{k}} - p) + (1-\lambda)f(V_{\bar{k}})(V_{\bar{k}+1} - p)\Big]$$

This implies that agent i of type $\theta_i = H$ after signal $x_i = 1$ will choose H if

$$p \leq \frac{HV_{\bar{k}} - \lambda LV_{\bar{k}} - (1-\lambda)Lf(V_{\bar{k}})V_{\bar{k}+1}}{H - \lambda L - (1-\lambda)Lf(V_{\bar{k}})} := p_{32}^*$$

Actually, one can verify that $p_{32}^* > p_{11}^*$. Hence, $a_i^* = H$ for any $p \leq p_{11}^*$.

Case 3.4: $N \geq i+3$.

- By choosing rejection, the payoff is 0.
- By choosing H, then $k_i = \bar{k} - 1$ and $A_i = A_{i-1} + H = T - L$. Threshold implementation is not triggered and thus project implementation depends further on future signals in period $i+1$.
    1. $x_{i+1} = 1$ occurs with probability $f(V_{\bar{k}-1})$, which implies that $k_{i+1} = \bar{k}$ and thus $A_{i+1} \geq A_i + L = T$. Under this event, the return per dollar invested is $(V_{\bar{k}} - p)$.
    2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. In this case, $k_{i+1} = \bar{k} - 2$ and thus $A_{i+1} = A_i = T - L$. Now, there is DOWN cascade since even low support L will trigger threshold implementation, but the belief is bounded above by $\bar{k} - 1$ and thus impossible to break even. In short, the return per dollar invested is 0.

  To summarize, the expected payoff from choosing H is given by

$$H\Big\{f(V_{\bar{k}-1})(V_{\bar{k}} - p) + (1 - f(V_{\bar{k}-1})) * 0\Big\} = f(V_{\bar{k}-1})(V_{\bar{k}} - p)H$$

- By choosing L, then $k_i = \bar{k} - 1$ and $A_i = A_{i-1} + L = T - H$. From the perspective of agent $i+1$, it reduces to Case 1 discussed above (depending on how many agents left). By the same token, project implementation depends on future signals, including those from period $i+1$ to $i+3$.
    1. $x_{i+1} = 1$ occurs with probability $f(V_{\bar{k}-1})$. Now, $k_{i+1} = \bar{k}$. Note the independence between realization of signals and the type of the agent.

- $\theta_{i+1} = H$ occurs with probability λ. If agent $i+1$ chooses high support H, then $k_{i+1} = \bar{k}$ and $A_{i+1} = A_i + H = T$. The return per dollar invested is $(V_{\bar{k}} - p)$. If agent $i+1$ chooses low support L, then $k_{i+1} = \bar{k}$ and $A_{i+1} = A_i + L = T - H + L$. In this case, we need to consider two more rounds as below.
  - (a) $x_{i+2} = 1$ occurs with probability $f(V_{\bar{k}})$. Now, the posterior $k_{i+2} = k_{i+1} + 1 = \bar{k} + 1$, which triggers UP cascade. Hence, the return per dollar invested is $(V_{\bar{k}+1} - p)$.
  - (b) $x_{i+2} = -1$ occurs with probability $1 - f(V_{\bar{k}})$. Now, the posterior $k_{i+2} = k_{i+1} - 1 = \bar{k} - 1$, and $A_{i+2} = A_{i+1} = T - H + L$.
    - $x_{i+3} = 1$ occurs with probability $f(V_{\bar{k}-1})$. Then, $k_{i+3} = \bar{k}$ and $A_{i+3} \geq A_{i+2} + L \geq T$. The return per dollar invested is $(V_{\bar{k}} - p)$.
    - $x_{i+3} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. Then, $k_{i+3} = \bar{k} - 2$ and $A_{i+3} = A_{i+2} = T - H + L$. We have DOWN cascade under this history since the belief is bounded above by $\bar{k} - 1$ when the threshold implementation is triggered. Hence, the return per dollar is 0.
- $\theta_{i+1} = L$ occurs with probability $1 - \lambda$. Now, $k_{i+1} = \bar{k}$ and $A_{i+1} = A_i + L = T - H + L$.
  - (a) $x_{i+2} = 1$ occurs with probability $f(V_{\bar{k}})$. Then, $k_{i+2} = \bar{k}+1$, which implies UP cascade. The return per dollar invested is $V_{\bar{k}+1} - p$.
  - (b) $x_{i+2} = -1$ occurs with probability $1 - f(V_{\bar{k}})$. Then, $k_{i+2} = \bar{k} - 1$ and $A_{i+2} = A_{i+1} = T - H + L$. By the same token, we consider one more round.
    - $x_{i+3} = 1$ occurs with probability $f(V_{\bar{k}-1})$. Then, $k_{i+3} = \bar{k}$ and $A_{i+3} \geq A_{i+2} + L \geq T$. The return per dollar invested is $(V_{\bar{k}} - p)$.
    - $x_{i+3} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. Then, $k_{i+3} = \bar{k} - 2$ and $A_{i+3} = A_{i+2} = T - H + L$. We have DOWN cascade under this history since the belief is bounded above by $\bar{k} - 1$ when the threshold implementation is triggered. Hence, the return per dollar is 0.

2. $x_{i+1} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. Now, $k_{i+1} = \bar{k} - 2$ and $A_{i+1} = A_i = T - H$.
   - $x_{i+2} = -1$ occurs with probability $1 - f(V_{\bar{k}-2})$. Now, $k_{i+2} = \bar{k} - 3$. Under this history, DOWN cascade happens eventually since there is no enough room to push the belief to break even without triggering project implementation. Hence, the return under this case is 0.
   - $x_{i+2} = 1$ occurs with probability $f(V_{\bar{k}-2})$. Now, $k_{i+2} = \bar{k} - 1$. Given the independence of agent type and signal realization,
     - $\theta_{i+2} = H$ occurs with probability λ. First, note that it is suboptimal to choose H. Otherwise, we have $k_{i+2} = \bar{k} - 1$ and $A_{i+2} = A_{i+1} + H = T$. The project is implemented with a strict loss. Now, we know for sure that $a_{i+2} = L$, then $k_{i+2} = \bar{k}-1$ and $A_{i+2} = T - H + L$. Since project implementation is undetermined, we need to consider one more round.
       - (a) $x_{i+3} = 1$ occurs with probability $f(V_{\bar{k}-1})$. In this case, $k_{i+3} = \bar{k}$ and $A_{i+3} \geq A_{i+2} + L = T - H + 2L > T$. The return per dollar invested is $(V_{\bar{k}} - p)$.

(b) $x_{i+3} = -1$ occurs with probability $1 - f(V_{\bar{k}-1})$. Now, $k_{i+3} = \bar{k} - 2$ and $A_{i+3} = A_{i+2} = T - H + L$, which triggers DOWN cascade. The return per dollar is 0.

* $\theta_{i+2} = L$ occurs with probability $(1 - \lambda)$. The analysis is almost identical with the discussion when $\theta_{i+2} = H$ because the rich type will not choose H.

Hence, given that $x_{i+2} = 1$, the realization of type θ_{i+2} is irrelevant. Thus, with probability $f(V_{\bar{k}-1})$, $x_{i+3} = 1$, the return per dollar invested is $(V_{\bar{k}} - p)$; With complementary probability $1 - f(V_{\bar{k}-1})$, $x_{i+3} = -1$, the return is 0 due to the DOWN cascade.

To summarize, when $p > p^*_{12}$ (i.e., agent $i+1$ of type $\theta_{i+1} = H$ chooses L after $x_{i+1} = 1$), then the payoff of choosing L for agent i is given by

$$U_i(L|p > p^*_{12}) = L \times \Big\{ f(V_{\bar{k}-1}) \Big[f(V_{\bar{k}})(V_{\bar{k}+1} - p) + (1 - f(V_{\bar{k}}))f(V_{\bar{k}-1})(V_{\bar{k}} - p) \Big] + (1 - f(V_{\bar{k}-1}))f(V_{\bar{k}-2})f(V_{\bar{k}-1})(V_{\bar{k}} - p) \Big\}$$

Otherwise, when $p \leq p^*_{12}$ (i.e., agent $i+1$ of type $\theta_{i+1} = H$ chooses H after $x_{i+1} = 1$), then the payoff of choosing L is given by

$$U_i(L|p \leq p^*_{12}) = L \times \Big\{ f(V_{\bar{k}-1}) \Big[f(V_{\bar{k}})(V_{\bar{k}+1} - p) + (1 - f(V_{\bar{k}}))f(V_{\bar{k}-1})(V_{\bar{k}} - p) \Big] + (1 - f(V_{\bar{k}-1}))f(V_{\bar{k}-2})f(V_{\bar{k}-1})(V_{\bar{k}} - p) \Big\}$$

Henceforth, given that $p \in (p^*_{12}, V_{\bar{k}}]$, agent i of type $\theta_i = H$ will choose full support H only when

$$U_i(H) \geq U_i(L|p > p^*_{12})$$

or equivalently,

$$p \leq \frac{V_{\bar{k}}\Big(H - Lf(V_{\bar{k}-1})[1 - f(V_{\bar{k}})] - L[1 - f(V_{\bar{k}-1})]f(V_{\bar{k}-2})\Big) - Lf(V_{\bar{k}})V_{\bar{k}+1}}{H - Lf(V_{\bar{k}}) - L[1 - f(V_{\bar{k}})]f(V_{\bar{k}-1}) - L[1 - f(V_{\bar{k}-1})]f(V_{\bar{k}-2})} \equiv p^*_{33} \tag{IA-18}$$

Actually, we can verify that $p^*_{33} < p^*_{12}$.[27] Hence, for all $p \in (p^*_{12}, V_{\bar{k}}]$, agent i of type $\theta_i = H$ will choose L.

Moreover, given that $p \leq p^*_{12}$, agent i of type $\theta_i = H$ will choose full support H if $U_i(H) \geq U_i(L|p \leq p^*_{12})$, or equivalently,

$$p \leq \frac{V_{\bar{k}}\Big(H - [1 - f(V_{\bar{k}})]f(V_{\bar{k}-1})L - [1 - f(V_{\bar{k}-1})]f(V_{\bar{k}-2})L\Big) - LV_{\bar{k}+1}f(V_{\bar{k}})}{H - [1 - f(V_{\bar{k}})]f(V_{\bar{k}-1})L - [1 - f(V_{\bar{k}-1})]f(V_{\bar{k}-2})L - Lf(V_{\bar{k}})} \equiv p^*_{34}. \tag{IA-19}$$

We can verify that $p^*_{34} < p^*_{12}$. Hence, for any $p \leq p^*_{12}$, agent i of type $\theta_i = H$ will choose H when $p \leq p^*_{34}$ and choose L when $p \in (p^*_{34}, p^*_{12}]$.

Now, (IA-13) summarizes the discussion above, which concludes the analysis for case 3. ■

Case 4: $k_{i-1} = \bar{k}(p) - 1$, $A_{i-1} > T - H$ (given that $x_i = 1$ and $\theta_i = H$). We divide this case into two

[27] The calculation for this claim is available upon request.

sub-cases.

Case 4.1: $A_{i-1} \geq T - L$**.** This case is simple. Even choosing low support $a_i = L$ will trigger threshold implementation, thus agent i of type $\theta_i = H$ has an incentive to choose H as long as the return is non-negative.

Case 4.2: $A_{i-1} \in (T - H, T - L)$**.** Basically, it coincides with Case 1 under the assumption that $H \in (L, 2L)$, since choosing H will instantly implement the project, while choosing L allows for one more period of learning. We omit the analysis here and just characterize the optimal decision rule.

Now, (IA-14) summarizes the discussion above, which concludes the analysis for case 4. ■

We then prove the existence of the optimal AoN design and pricing when they are endogenous. Note that Proposition IA-1 features a rent linked to the incentive compatibility conditions. This is very intuitive. To get full rent extraction, the proposer needs to set a price such that all agents are indifferent between support and rejection. Now, this is problematic since if the price is so high (i.e., $p \to V_{\bar{k}}$, which is a necessary condition for full surplus extraction), then the agent will switch from high support to low support so as to accommodate more periods of trials. Under such a strategy, there exist some signal sequences such that an UP cascade is more likely to happen, which generates a positive payoff.[28]

Finally, the next proposition discusses the existence of an optimal pair (p^*, T^*) that maximizes the proposer's expected revenue and provides a numerical procedure for finding them.

Proposition IA-5. *Given agents' equilibrium strategies specified in Proposition IA-1, there exists a pair* (p^*, T^*) *that maximizes the proposer's expected revenue. Specifically, there exists* $k^* \in \{-1, 0, \cdots, N\}$ *such that* $p^* \in \{p^*_{11}(k^*), p^*_{12}(k^*), p^*_{21}(k^*), p^*_{22}(k^*), p^*_{23}(k^*), p^*_{24}(k^*), p^*_{34}(k^*), V_{k^*}\}$ *and* $T = m^*L + n^*H$ *for some* $0 \leq m^*, n^* \leq N$ *and* $m^* + n^* \leq N$.

Proof. The proof consists of two steps.

i) we show that $T^* \in \mathbb{T} := \{mH + nL : m \in \mathbb{Z}_+, m, n \leq N\}$, where $\mathbb{Z}_+$ is the set of all non-negative integers. Rearranging the set $\mathbb{T}$ in an increasing order and keep only one copy if there exists duplicates. For instance, if $m_1 * H + n_1 * L = m_2 * H + n_2 * L = T_0$, then we just need to keep T_0. Recall that the proposer's expected utility is

$$E\left[(p - \nu) A_N \mathbb{1}_{A_N \geq T} \middle| \{a_i^*\}_{i=1,\cdots,N}\right]$$

Fix any two successive T's, say, $T_i, T_{i+1} \in \mathbb{T}$. Consider any $T \in (T_i, T_{i+1}]$. It induces the same equilibrium outcome at that under $T^* = T_{i+1}$. Since $p > \nu$, any threshold $T \in (T_i, T_{i+1})$ is strictly dominated by T_{i+1}. This implies the optimal threshold $T^* \in \mathbb{T}$. Note that the cardinality $|\mathbb{T}| \leq (N + 1)^2$.

[28]Note that the no DOWN cascade before approaching the threshold is a by-product of Assumption 1 in the baseline model. The same situation arises here. Consider the "prolonged learning" episode where $A_{i-1} > T - H - (\bar{k}(p) - k_{i-1} - 1)L$. When $T - A_{i-1} < (\bar{k}(p) - k_{i-1})L$ is satisfied (which is consistent with the case here), it is impossible to achieve the break-even belief without triggering threshold implementation, so every agent should abstain from supporting the project. However, the agents still choose to (partially) support, because the decision is delegated to the agent with $A_{i-1} \geq T - L$. This prevents the DOWN cascade before approaching the threshold. One caveat is such that this cannot be supported by the equilibrium refinement by lowering the price p by a tiny amount, unless the break-even belief is changed.

ii) We show that $p^* \in \mathbb{W}$, where

$$\mathbb{W} := \bigcup_{k=-1}^{N} \{p_{11}^*(k), p_{12}^*(k), p_{11}^*(k) \wedge p_{22}^*(k), p_{11}^*(k) \vee p_{21}^*(k), p_{12}^*(k) \wedge p_{24}^*(k), p_{12}^*(k) \wedge p_{23}^*(k), p_{34}^*(k), V_k\}$$

$$\subseteq \bigcup_{k=-1}^{N} \{p_{11}^*(k), p_{12}^*(k), p_{21}^*(k), p_{22}^*(k), p_{23}^*(k), p_{24}^*(k), p_{34}^*(k), V_k\}$$

First, note that $p < V_{-1}$ is suboptimal since it induces UP cascade from the very beginning. Meanwhile, $p > V_N$ is also suboptimal, because it is impossible to get enough support after a sequence of all positive signals. Hence, $p \in [V_{-1}, V_k]$.

Second, consider any $p \in (V_{k-1}, V_k]$, $\forall k \in \{0, \cdots, N\}$. Note that, there are only eight discrete points defined as in the set $\mathbb{W}$. Note that the decision rules only depend on $\bar{k}(p)$. Since any $p \in (V_{k-1}, V_k)$ induces the same $\bar{k}(p)$ and is dominated by $p = V_k$. This implies the optimum must be achieved at $p = V_k$ when we only consider $p \in (V_{k-1}, V_k]$.

Similarly, we can consider the decision rules define in equations (55), (56), (57), and (58). For instance, in equation (55), when $N = i + 1$, H is optimal when $p \leq p_{11}^*$, and L is optimal when $p > p_{11}^*$ (and $p \leq V_k$). In the case, $p^* \in \{p_{11}^*, V_k\}$. To see it, if H generates a higher payoff for the proposer, then we should choose $p^* = p_{11}^*$, because any other $p \in (V_{k-1}, p_{11}^*)$ induces the same outcome as that under $p = p_{11}^*$. In contrast, if L generates a higher payoff for the proposer, then $p^* = V_k$ is optimal, since any other $p \in (p_{11}^*, V_k)$ induces the same outcome as that under $p = V_k$. Thus, in light of the decision rule for case 1 in equation (55), we have three discrete points $\{p_{11}^*, p_{12}^*, V_k\}$, which we need to check for optimality. By the same token, for case 2, 3, and 4, we need to check the cutoff points in the respective decision rule. In total, we need to check $8 * (N + 1) + 1$ discrete points in equations (56), (56), (57), and (58), because we only need to check V_{-1} when $k = -1$.

To summarize, we need to check $|\mathbb{T}| \times |\mathbb{W}| \leq (N + 1)^2 * (8N + 9)$ discrete points, which ensures the existence of the optimal solution (p^*, T^*). The proof concludes. □

Finally, we describe the numerical procedure for searching for the optimal design, i.e.,the global optimum for (p^*, T^*):

1. Set the values for all parameters, including N, λ, ν, and q.
2. Generate the set $\mathbb{T} = \{mH + nL : m \in \mathbb{Z}_+, m, n \leq N\}$, where $\mathbb{Z}_+$ are all non-negative integers. Rearrange $\mathbb{T}$ in an increasing order, that is, $\mathbb{T} = \{T_l | T_l < T_{l+1}, \forall l\}$.
3. Generate the set $\mathbb{W}$, where

$$\mathbb{W} = \bigcup_{k=-1}^{N} \{p_{11}^*(k), p_{12}^*(k), p_{11}^*(k) \wedge p_{22}^*(k), p_{11}^*(k) \vee p_{21}^*(k), p_{12}^*(k) \wedge p_{24}^*(k), p_{12}^*(k) \wedge p_{23}^*(k), p_{34}^*(k), V_k\}$$

For each k, delete any points $p_j^*(k) \notin (V_{k-1}, V_k]$, for $j \in \{11, 12, 21, 22, 23, 24, 34\}$.

Rearranging $\mathbb{W}$ in an increasing order such that $\mathbb{W} = \{p_j | p_j < p_{j+1}\}$.

4. Generate all the possible sequence of signals x_i and types θ_i, for $i \in \{1, ..., N\}$.

5. Fix a pair $(T_l, p_j) \in \mathbb{T} \times \mathbb{W} := \{(T_l, p_j) : 1 \leq l \leq |\mathbb{T}|, 1 \leq j \leq |\mathbb{W}|\}$.

   - For each sequence, find out the decision s by each $i \in \{1, ..., N\}$ and then calculate A_N and check whether $A_N \geq T_l$. Return $A_N(p - \nu)\mathbb{1}(A_N \geq T_l)$.
   - Aggregate $A_N(p - \nu)\mathbb{1}(A_N \geq T_l)$ to form the expected payoff $EU_{T_l,p_j} = E[A_N(p - \nu)\mathbb{1}(A_N \geq T_l)|\{T_l, p_j\}]$.

6. Return $(T_l^*, p_j^*) \in \underset{(T_l,p_j)\in\mathbb{T}\times\mathbb{W}}{\arg\max} EU_{T_l,p_j}$, and return $EU_{T_l^*,p_j^*}$.

□

Proof of Proposition IA-3

Proof. We denote the equilibrium characterized in Proposition 1 with agent base N, threshold T, and price p as $\mathbb{G}_1$. Define the expected investment profit of agent i with posterior $V_{k(\mathcal{H}_{i-1})}$ in equilibrium G conditional on a positive and negative signal as $\chi_1(p, i, T, \mathcal{H}_{i-1})$ and $\chi_0(p, i, T, \mathcal{H}_{i-1})$, respectively. Let $\mathbb{K}$ be the set of agent i and associated history $\mathcal{H}_{i-1}$ such that:

$$\mathbb{K} \equiv \{i, \mathcal{H}_{i-1} | \chi_1(p, i, T, \mathcal{H}_{i-1}) > 0, \chi_0(p, i, T, \mathcal{H}_{i-1}) < 0\}. \tag{IA-20}$$

Define $\overline{\varepsilon}(p, T)$ as

$$\overline{\varepsilon}(p, T) = \min \left\{V_{k(\mathcal{H}_{i-1})}\chi_1(p, i, T, \mathcal{H}_{i-1}), -(1 - V_{k(\mathcal{H}_{i-1})})\chi_0(p, i, T, \mathcal{H}_{i-1})\right\}_{\mathbb{K}}. \tag{IA-21}$$

Now we verify that when $p \in (V_{k-1}, V_k)$, then $\mathbb{G}$ still holds.

1. If there is an UP cascade as in $\mathbb{G}$, then agent i finds it optimal to contribute regardless of her potential private signal. Agent i chooses not to produce information and contribute.

2. If there is no UP cascade yet, and in the equilibrium $\mathbb{G}$ agent i contribute if she observes a good signal, then $\chi_1(p, i, T, \mathcal{H}_{i-1}) > 0$. If agent i chooses not to acquire the private signal and contribute anyway, then her expected profit is

$$V_{k(\mathcal{H}_{i-1})}\chi_1 - \varepsilon > V_{k(\mathcal{H}_{i-1})}\chi_1(p, i, T, \mathcal{H}_{i-1}) + (1 - V_{k(\mathcal{H}_{i-1})})\chi_0(p, i, T, \mathcal{H}_{i-1}). \tag{IA-22}$$

   Because by construction $V_{k(\mathcal{H}_{i-1})}\chi_1 - \varepsilon > 0$, agent i finds it optimal to acquire the information and contribute if and only if she observes a positive signal.

3. If there is no UP cascade yet, and in the equilibrium G agent i finds it impossible to reach the threshold and chooses not to contribute, then $\chi_1(p, i, T, \mathcal{H}_{i-1}) = 0$. Agent chooses not to acquire information and not to contribute.

Finally we verify that case for $p = V_k$. The only difference is that if there is no UP cascade yet, and it is impossible to reach an UP cascade in the future, then $\chi_1(p, i, T, \mathcal{H}_{i-1}) = 0$, and agent chooses not to acquire information and not to contribute. □