Internet Appendix for "Feedback and Contagion through Distressed Competition"

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Abstract

This is the supplemental material to the paper titled "Feedback and Contagion through Distressed Competition" (Chen et al., 2022*a*). It includes the rigorous mathematical proofs for all the theoretical results in Chen et al. (2022*a*), and a comprehensive summary of the direct empirical evidence on tacit collusion in the literature. It also provides many additional quantitative and empirical results.

Keywords: Competition-distress feedback loop, Distress spillover, Predatory price war, Profitability-leverage puzzle, Tacit collusion (**JEL**: C73, D43, E31, G3, L13, O33)

Contents

1 Proofs of Propositions and Corollaries in Chen et al. (2022*a*)

In the appendix section, we make the dependence of endogenous variables on ν explicit to ensure clarity. We first introduce the following technical assumptions for the simple model and the extended simple model.

Assumption 1 (Earning-based borrowing constraints)**.** *To stay running, firm $i \in \{1,2\}$ needs to make sure that the following earnings-based borrowing constraint is not violated in any period:*

$$\theta_i - b_i \geq \ell, \text{ for a constant } \ell > 0, \tag{1}$$

where θ_i and b_i are the log profitability and log coupon payment of firm i, respectively. Further, to be consistent with the fact that the covenants are typically triggered before firms choose to default, we assume that $b_i + \ell > \varphi(b_i)$ for $i \in \{1,2\}$.

The specification in equation (1) captures the prevalent earnings-based borrowing constraint, which imposes a minimum ratio of a firm's average earnings to its coupon payments, usually implemented in the form of debt covenants (e.g., Roberts and Sufi, 2009; Lian and Ma, 2020). These earnings-to-debt-ratio covenants are typically triggered before shareholders choose to default.

Assumption 2. *The log coupon level $b_i > \underline{b}$, for some $\underline{b} > \tau\nu/\bar{\varphi}$.*

This technical assumption is innocuous, because we can always set the customer base large enough to make a firm's leverage ratio close to zero for any given coupon level. We choose the lower bound $\underline{b}$ such that $\underline{b} > \tau\nu/\bar{\varphi}$ to fix the scale of the economy and ensure that increasing a firm's coupon payment has a non negligible effect on its default probability relative to the tax shield benefit.

1.1 Proof of Proposition 2.1

We first show that $P_1^N = P_2^N = 0$ is a one-shot static Nash equilibrium. No firm would like to increase the price because this deviation would lead to zero demand according to the demand system specification (equation (2) of the main text). Meanwhile, no firm would like to lower the price to some negative value because this deviation would lead to a negative profit (i.e., an operating loss). Next, we show that $P_1^N = P_2^N = 0$ is the unique one-shot static Nash equilibrium. Suppose (P_1, P_2) is a pair of prices with at least one price being positive. Without loss of generality, we assume that $P_1 \leq P_2$ and $P_2 > 0$. There are two cases. If $P_1 = P_2 > 0$, either firm can reduce its price by an infinitesimal amount to obtain a positive gain. If $P_1 < P_2$ and $P_2 > 0$, firm 1 can increase its price to a level that is lower than P_2 by an infinitesimal amount to obtain a positive gain. Therefore, $P_1^N = P_2^N = 0$ is the unique non-collusive equilibrium, in which $C_1^N = C_2^N = 1$ according to the demand system specification and $E_1^N = E_2^N = 0$.

1.2 Proof of Proposition 2.2

In the tacitly agreed scheme on product prices, two firms' prices must be the same: $P_1^C = P_2^C = P^C$. This is because no firm would ever agree to set a higher price, which leads to zero profits. As a result, both firms face unit demand per unit of customer base: $C_1^C = C_2^C = 1$. These properties, together with the assumption that $M_1 = M_2 = 1$, lead to the following equalities:

$$\lambda_i^C = \mathbb{P}\{P_i^C \widetilde{C}_i \le e^{\varphi(b_i)}\} = \mathbb{P}\{e^{\theta_i^C + z_i} \le e^{\varphi(b_i)}\}, \tag{2}$$

where $\theta_i^C \equiv \ln\left(P_i^C\right)$ for $i \in \{1,2\}$. The idiosyncratic demand shock z_i is i.i.d., following a logistic distribution, Logistic $(\mu(\nu), \nu)$. Thus, the default probability is equal to

$$\lambda_i^C = \mathbb{P}\{z_i \le \varphi(b_i) - \theta_i^C\} = \frac{1}{1 + e^{[\mu(\nu) + \theta_i^C - \varphi(b_i)]/\nu}}, \quad \text{for } i \in \{1,2\}. \tag{3}$$

1.3 Proof of Proposition 2.3

The two firms must set the same price P^C and thus set the same log profitability θ^C in the collusive equilibrium. The equity value of firm i is

$$E_i^C(b_i, b_j) = \mathbb{E}\left\{(1-\tau)\left(e^{\theta^C + z_i} - e^{b_i}\right) + E_i^C(b_i, b_j)\mathbf{1}_{\{\theta^C + z_i > \varphi(b_i)\}}\right\}, \text{ for } j \ne i.$$

where $\mathbf{1}_{\{\theta^C + z_i > \varphi(b_i)\}}$ is the indicator function for the event of $\theta^C + z_i > \varphi(b_i)$. Using the definition of the default probability $\lambda^C(\theta^C, b_i)$ in equation (4) of the main text and the distribution of z_i (i.e., $\mathbb{E}[e^{z_i}] \equiv 1$), the above equation can be rewritten as follows:

$$E_i^C(b_i, b_j) = (1-\tau)\left(e^{\theta^C} - e^{b_i}\right) + \left[1 - \lambda^C(\theta^C, b_i)\right] E_i^C(b_i, b_j), \tag{4}$$

Rearranging terms further leads to

$$E_i^C(b_i, b_j) = (1-\tau)\left(e^{\theta^C} - e^{b_i}\right)\lambda(\theta^C, b_i)^{-1}. \tag{5}$$

Plugging (3) into (5), we obtain that

$$E_i^C(b_i, b_j) = (1-\tau)\left(e^{\theta^C} - e^{b_i}\right)\left\{1 + e^{[\mu(\nu)+\theta^C-\varphi(b_i)]/\nu}\right\}. \tag{6}$$

The debt value can be characterized by the following recursive equation:

$$D_i^C(b_i, b_j) = \mathbb{E}\left\{e^{b_i} + D_i^C(b_i, b_j)\mathbf{1}_{\{\theta^C+z_i>\varphi(b_i)\}}\right\} \tag{7}$$

$$= e^{b_i} + \left[1 - \lambda^C(\theta^C, b_i)\right] D_i^C(b_i, b_j). \tag{8}$$

Rearranging terms in the equation above, we can solve the equilibrium debt value as follows:

$$D_i^C(b_i, b_j) = e^{b_i}\lambda^C(\theta^C, b_i)^{-1}. \tag{9}$$

Plugging (3) into (9), we obtain that

$$D_i^C(b_i, b_j) = e^{b_i}\left\{1 + e^{[\mu(\nu)+\theta^C-\varphi(b_i)]/\nu}\right\}. \tag{10}$$

The incentive compatibility (IC) constraint in equation (7) of the main text can be rewritten as follows:

$$\rho e^{\theta^C+\eta\theta^C} \le \left[1 - \lambda(\theta^C, b_i)\right] E_i^C(b_i, b_j) \tag{11}$$

$$= (1-\tau)\left[1 - \lambda(\theta^C, b_i)\right]\left(e^{\theta^C} - e^{b_i}\right)\left\{1 + e^{[\mu(\nu)+\theta^C-\varphi(b_i)]/\nu}\right\}. \tag{12}$$

Plugging (3) into (12), we obtain the following inequality:

$$\rho e^{\theta^C+\eta\theta^C} \le (1-\tau)\left(e^{\theta^C} - e^{b_i}\right) e^{[\mu(\nu)+\theta^C-\varphi(b_i)]/\nu}. \tag{13}$$

Taking logs on both sides, it follows that

$$\ln(\rho) + \eta\theta^C \le \ln(1-\tau) + [\mu(\nu) + \theta^C - \varphi(b_i)]/\nu + \ln\left[1 - e^{-(\theta^C - b_i)}\right]. \tag{14}$$

Therefore, the IC constraints can be written as follows:

$$\theta^C \le \left\{\ln[(1-\tau)/\rho] + [\mu(\nu) + \theta^C - \varphi(b_i)]/\nu + \ln\left[1 - e^{-(\theta^C - b_i)}\right]\right\}/\eta, \quad \text{with } i \in \{1, 2\}. \tag{15}$$

We define a function $\Psi(\theta, b)$ as follows:

$$\Psi(\theta, b) \equiv \ln[(1-\tau)/\rho] + [\mu(\nu) + \theta - \varphi(b)]/\nu + \ln\left[1 - e^{-(\theta - b)}\right]. \tag{16}$$

Thus, the two IC constraints in (15) for the pairs of log profitability (θ^C, θ^C) can be expressed in the following way:

$$\theta^C \leq \Psi(\theta^C, b_i)/\eta, \quad \text{with } i \in \{1, 2\}. \tag{17}$$

Hence, the incentive compatible region (IC region) can be characterized as follows:

$$\mathcal{C} \equiv \left\{(\theta_1, \theta_2) \ : \ \theta_i \leq \Psi(\theta_j, b_j)/\eta \ \text{ and } \ \theta_i = \theta_j, \ \text{ for } i \neq j\right\}, \tag{18}$$

for b_1 and b_2 such that $\mathcal{C}$ is not empty.

The firms try to maximize their log profitability and thus equity values. As a result, in the Pareto efficient collusive equilibrium with grim trigger (punishment) strategies, at least one of the two IC constraints in (17) must be binding, which leads to

$$\theta^C = \Psi(\theta^C, b_1 \vee b_2)/\eta, \tag{19}$$

where $b_1 \vee b_2$ denotes the maximum of b_1 and b_2.

Next, we show that the Pareto efficient collusive equilibrium exists if b_1 and b_2 satisfy certain conditions. The function $f(x) \equiv \ln\left[1 - e^{-x}\right]$ is a strictly increasing and concave function in x, it must hold that

$$\frac{1}{e^{\overline{\theta}} - 1}(\theta - b_1 \vee b_2 - \ell) \leq \ln\left[1 - e^{-(\theta - b_1 \vee b_2)}\right] - \ln\left[1 - e^{-\ell}\right] \leq \frac{1}{e^{\ell} - 1}(\theta - b_1 \vee b_2 - \ell), \tag{20}$$

for all $\theta \in \left[b_1 \vee b_2 + \ell, \overline{\theta}\right]$, where ℓ is the constant defined in Assumption 1.

We now define a function $G(\cdot, \cdot, \cdot)$ as follows:

$$G(\theta, b_1, b_2) \equiv \Psi(\theta, b_1 \vee b_2) - \eta\theta \tag{21}$$

$$= C(b_1 \vee b_2) + \theta/\nu - \eta\theta + \ln\left[1 - e^{-(\theta - b_1 \vee b_2)}\right] - \ln\left[1 - e^{-\ell}\right], \tag{22}$$

where

$$C(b) \equiv \ln[(1-\tau)/\rho] + [\mu(\nu) - \varphi(b)]/\nu + \ln\left[1 - e^{-\ell}\right]. \tag{23}$$

Based on the equilibrium characterization in (19), we know that the log profitability in the Pareto-efficient collusive equilibrium θ^C is the root of the function $G(\theta, b_1, b_2)$ if it exists. Because of the inequalities in (20), it holds that

$$\underline{G}(\theta, b_1, b_2) \leq G(\theta, b_1, b_2) \leq \overline{G}(\theta, b_1, b_2), \quad \text{for all } \theta \in \left[b_1 \vee b_2 + \ell, \overline{\theta}\right], \tag{24}$$

where

$$\underline{G}(\theta, b_1, b_2) \equiv C(b_1 \vee b_2) - (b_1 \vee b_2 + \ell)/(e^{\overline{\theta}} - 1) - \left\{\eta - \left[1/\nu + 1/(e^{\overline{\theta}} - 1)\right]\right\}\theta, \quad (25)$$

$$\overline{G}(\theta, b_1, b_2) \equiv C(b_1 \vee b_2) - (b_1 \vee b_2 + \ell)/(e^{\ell} - 1) - \left\{\eta - \left[1/\nu + 1/(e^{\ell} - 1)\right]\right\}\theta, \quad (26)$$

When η is sufficiently large and ρ is sufficiently small, both the lower-bound function $\underline{G}(\theta, b_1, b_2)$ and the upper-bound function $\overline{G}(\theta, b_1, b_2)$ have the following positive roots:

$$\underline{\theta}^*(b_1, b_2) \equiv \frac{C(b_1 \vee b_2) - (e^{\overline{\theta}} - 1)^{-1}(b_1 \vee b_2 + \ell)}{\eta - \left[\nu^{-1} + (e^{\overline{\theta}} - 1)^{-1}\right]}, \quad \text{and} \quad (27)$$

$$\overline{\theta}^*(b_1, b_2) \equiv \frac{C(b_1 \vee b_2) - (e^{\ell} - 1)^{-1}(b_1 \vee b_2 + \ell)}{\eta - \left[\nu^{-1} + (e^{\ell} - 1)^{-1}\right]}, \quad \text{respectively.} \quad (28)$$

Because of the inequalities in (24), it must hold that $\theta^C \in [\underline{\theta}^*(b_1, b_2), \overline{\theta}^*(b_1, b_2)]$.

Therefore, the equilibrium exists (i.e., θ^C exists and lies between $b_1 \vee b_2 + \ell$ and $\overline{\theta}$), as long as the parameters, including b_1 and b_2, make the following conditions hold:

$$b_1 \vee b_2 + \ell \le \underline{\theta}^*(b_1, b_2) \le \overline{\theta}^*(b_1, b_2) \le \overline{\theta}. \quad (29)$$

1.4 Proof of Proposition 2.4

We define a function $G(\cdot, \cdot, \cdot)$ as in (21). According to Proposition 2.3, the collusive equilibrium log profitability θ^C is the solution to $G(\theta, b_1, b_2) = 0$ for given b_1 and b_2. For $i \neq j \in \{1, 2\}$, if $b_i < b_j$, the function $G(\theta, b_1, b_2) \equiv \Psi(\theta, b_j) - \eta\theta$ is independent of b_i. As a result, θ^C is independent of b_i, and thus $\partial\theta^C/\partial b_i = 0$. Alternatively, if $b_i \ge b_j$, the function $G(\theta, b_1, b_2) \equiv \Psi(\theta, b_i) - \eta\theta$ depends on b_i, and thus θ^C depends on b_i. The Implicit Function Theorem implies that

$$\frac{\partial\theta^C}{\partial b_i} = -\left[\left.\frac{\partial G(\theta, b_1, b_2)}{\partial\theta}\right|_{\theta=\theta^C}\right]^{-1} \left.\frac{\partial G(\theta, b_1, b_2)}{\partial b_i}\right|_{\theta=\theta^C}, \quad (30)$$

where the derivatives, evaluated at $\theta = \theta^C$, are

$$\left.\frac{\partial G(\theta, b_1, b_2)}{\partial\theta}\right|_{\theta=\theta^C} = -\eta + (1/\nu + d_i) \quad \text{and} \quad \left.\frac{\partial G(\theta, b_1, b_2)}{\partial b_i}\right|_{\theta=\theta^C} = -\dot{\varphi}(b_i)/\nu - d_i, \quad (31)$$

with $d_i \equiv 1/[e^{\theta^C - b_i} - 1]$. Therefore,

$$\frac{\partial \theta^C}{\partial b_i} = -\frac{\dot{\varphi}(b_i)/\nu + d_i}{\eta - (1/\nu + d_i)}. \tag{32}$$

Because of Assumption 1 and the fact that $\theta^C = \ln(P^C) \leq \ln(\overline{P})$, it follows that $0 < 1/\left(\overline{P} - 1\right) < d_i \leq 1/(e^{\ell} - 1)$, which further implies that $\dot{\varphi}(b_i)/\nu + d_i > \dot{\varphi}(b_i)/\nu \geq 0$ and $\eta - (1/\nu + d_i) > \eta - \left[1/\nu + 1/(e^{\ell} - 1)\right]$, with the latter being positive if η is sufficiently large. Therefore, if η is sufficiently large, it follows from equation (32) that $\frac{\partial \theta^C}{\partial b_i} < 0$.

1.5 Proof of Corollary 2.1

Note that $\lambda_j^C = \lambda^C(\theta^C, b_j)$. It follows from the chain rule that

$$\frac{\partial \lambda^C(\theta^C, b_j)}{\partial b_i} = \left.\frac{\partial \lambda^C(\theta, b_j)}{\partial \theta}\right|_{\theta=\theta^C} \times \frac{\partial \theta^C}{\partial b_i}. \tag{33}$$

Based on equation (4) of the main text, it follows that $\partial \lambda^C(\theta, b_j)/\partial \theta < 0$. In addition, according to Proposition 2.4, it follows that $\partial \theta^C(b_1, b_2)/\partial b_i \leq 0$. Taken together, the default probability of the rival firm j is increasing in firm i's coupon level b_i, that is, $\partial \lambda^C(\theta^C, b_j)/\partial b_i \geq 0$.

1.6 Proof of Proposition 2.5

According to equation (4) of the main text, the default probability is

$$\lambda_i^C = \lambda^C(\theta^C, b_i), \;\; \text{where } \lambda^C(\theta, b) \equiv \frac{1}{1 + e^{[\mu(\nu) + \theta - \varphi(b)]/\nu}}, \tag{34}$$

where θ^C is a function of b_i and b_j in the collusive equilibrium for $i \neq j$. It holds that

$$\mathrm{d}\lambda_i^C = \left.\frac{\partial \lambda^C(\theta, b_i)}{\partial b_i}\right|_{\theta=\theta^C} \mathrm{d}b_i + \left.\frac{\partial \lambda^C(\theta, b_i)}{\partial \theta}\right|_{\theta=\theta^C} \mathrm{d}\theta^C. \tag{35}$$

The above total differential representation of $\lambda^C(\theta, b)$ is general in the sense that it does not depend on any specific economic mechanism.

According to our proposed theory, a change in b_i affects λ_i^C, which in turn affects the log profitability θ^C. Holding b_j unchanged, changes in either λ_i^C or θ^C can only be caused by variations in b_i. As a result, it follows that

$$\mathrm{d}\theta^C = \frac{\partial\theta^C(b_1,b_2)}{\partial b_i}\mathrm{d}b_i = \frac{\partial\theta^C(b_1,b_2)}{\partial b_i}\left[\frac{\partial\lambda^C(\theta^C(b_1,b_2),b_i)}{\partial b_i}\right]^{-1}\mathrm{d}\lambda_i^C = \left.\frac{\partial\theta^C}{\partial\lambda_i^C}\right|_{b_j}\mathrm{d}\lambda_i^C, \tag{36}$$

where $\partial\theta^C/\partial\lambda_i^C|_{b_j}$ is the sensitivity of log profitability θ^C to firm i's default probability λ_i^C, holding b_j unchanged, in the collusive equilibrium.

Combining (35) and (36), the feedback loop representation immediately follows:

$$\begin{bmatrix} \mathrm{d}\lambda_i^C \\ \mathrm{d}\theta^C \end{bmatrix} = \underbrace{\begin{bmatrix} \partial\lambda^C(\theta,b_i)/\partial b_i|_{\theta=\theta^C} \\ 0 \end{bmatrix}}_{\text{Initial direct effect} \geq 0}\mathrm{d}b_i + \underbrace{\begin{bmatrix} 0 & \partial\lambda^C(\theta,b_i)/\partial\theta|_{\theta=\theta^C} \\ \partial\theta^C/\partial\lambda_i^C|_{b_j} & 0 \end{bmatrix}}_{\text{Higher-order feedback effect} \leq 0}\begin{bmatrix} \mathrm{d}\lambda_i^C \\ \mathrm{d}\theta^C \end{bmatrix}. \tag{37}$$

Define $A \equiv \begin{bmatrix} 0 & \partial\lambda^C(\theta,b_i)/\partial\theta|_{\theta=\theta^C} \\ \partial\theta^C/\partial\lambda_i^C|_{b_j} & 0 \end{bmatrix}$, which captures the higher-order feedback effect of changes in b_i on λ_i^C and θ^C. The matrix A has two eigenvalues given by $\pm\sqrt{\partial\lambda^C(\theta,b_i)/\partial\theta|_{\theta=\theta^C} \times \partial\,\theta^C/\partial\lambda_i^C|_{b_j}}$. It is straightforward to show that these two eigenvalues lie on $(-1,1)$ because $\partial\lambda_i^C/\partial\theta^C|_{b_j} < \partial\lambda^C(\theta,b_i)/\partial\theta|_{\theta=\theta^C} < 0$. Intuitively, $\partial\lambda^C(\theta,b_i)/\partial\theta|_{\theta=\theta^C}$ only captures the initial direct effect of varying log profitability θ on the default probability $\lambda^C(\theta,b_i)$, holding both b_1 and b_2 unchanged. By contrast, $\partial\lambda_i^C/\partial\theta^C|_{b_j}$ captures the effect of varying b_i on λ^C through the change in the equilibrium log profitability θ^C, which reflects not only the initial direct effect of varying θ on λ^C but also the feedback effect between the financial and product markets, holding only b_j unchanged. Mathematically, according to (36), it holds that

$$\left.\frac{\partial\lambda_i^C}{\partial\theta^C}\right|_{b_j} = \left(\frac{\partial\theta^C}{\partial b_i}\right)^{-1}\frac{\partial\lambda^C(\theta^C,b_i)}{\partial b_i} \tag{38}$$

$$= \left(\frac{\partial\theta^C}{\partial b_i}\right)^{-1}\left[\left.\frac{\partial\lambda^C(\theta,b_i)}{\partial b_i}\right|_{\theta=\theta^C} + \left.\frac{\partial\lambda^C(\theta,b_i)}{\partial\theta}\right|_{\theta=\theta^C}\frac{\partial\theta^C}{\partial b_i}\right] \tag{39}$$

$$= \left(\frac{\partial\theta^C}{\partial b_i}\right)^{-1}\left.\frac{\partial\lambda^C(\theta,b_i)}{\partial b_i}\right|_{\theta=\theta^C} + \left.\frac{\partial\lambda^C(\theta,b_i)}{\partial\theta}\right|_{\theta=\theta^C}. \tag{40}$$

Because $\partial\theta^C/\partial b_i < 0$ and $\partial\lambda^C(\theta,b_i)/\partial b_i|_{\theta=\theta^C} > 0$, it follows that

$$\left.\frac{\partial\lambda_i^C}{\partial\theta^C}\right|_{b_j} - \left.\frac{\partial\lambda^C(\theta,b_i)}{\partial\theta}\right|_{\theta=\theta^C} = \left(\frac{\partial\theta^C}{\partial b_i}\right)^{-1}\left.\frac{\partial\lambda^C(\theta,b_i)}{\partial b_i}\right|_{\theta=\theta^C} < 0. \tag{41}$$

Thus, $I - A$ is invertible and the inversion is equal to

$$(I-A)^{-1} = I + A + A^2 + A^3 + \cdots . \tag{42}$$

We next show that the system is stable and invertible. According to equations (37) and (42), it follows that

$$\begin{bmatrix} \mathrm{d}\lambda_i^C \\ \mathrm{d}\theta^C \end{bmatrix} = [I + \underbrace{A + A^2 + A^3 + \cdots}_{\text{Amplification due to feedback loops}}] \underbrace{\begin{bmatrix} \partial\lambda^C(\theta, b_i)/\partial b_i\big|_{\theta=\theta^C} \\ 0 \end{bmatrix}}_{\text{Initial direct effect} \geq 0} \mathrm{d}b_i. \tag{43}$$

The powers of A, for $k = 1, 2, \cdots$, are

$$A^{2k} = \begin{bmatrix} (\partial\lambda^C(\theta, b_i)/\partial\theta\big|_{\theta=\theta^C})^k (\partial\theta^C/\partial\lambda_i^C\big|_{b_j})^k & 0 \\ 0 & (\partial\lambda^C(\theta, b_i)/\partial\theta\big|_{\theta=\theta^C})^k (\partial\theta^C/\partial\lambda_i^C\big|_{b_j})^k \end{bmatrix}, \text{ and} \tag{44}$$

$$A^{2k-1} = \begin{bmatrix} 0 & (\partial\lambda^C(\theta, b_i)/\partial\theta\big|_{\theta=\theta^C})^k (\partial\theta^C/\partial\lambda_i^C\big|_{b_j})^{k-1} \\ (\partial\lambda^C(\theta, b_i)/\partial\theta\big|_{\theta=\theta^C})^{k-1} (\partial\theta^C/\partial\lambda_i^C\big|_{b_j})^k & 0 \end{bmatrix}. \tag{45}$$

Therefore, the system is stable and invertible as shown above.

1.7 Proof of Corollary 2.2

Combining (43), (44), and (45), it follows that, for $i \in \{1, 2\}$,

$$\begin{aligned} \mathrm{d}\theta^C &= \left[\sum_{k=1}^{\infty} \left(\frac{\partial\lambda^C(\theta, b_i)}{\partial\theta}\bigg|_{\theta=\theta^C} \right)^{k-1} \left(\frac{\partial\theta^C}{\partial\lambda_i^C}\bigg|_{b_j} \right)^k \right] \frac{\partial\lambda^C(\theta, b_i)}{\partial b_i}\bigg|_{\theta=\theta^C} \mathrm{d}b_i \qquad (46) \\ &= \left[\frac{\partial\lambda_i^C}{\partial\theta^C}\bigg|_{b_j} - \frac{\partial\lambda(\theta, b_i)}{\partial\theta}\bigg|_{\theta=\theta^C} \right]^{-1} \frac{\partial\lambda^C(\theta, b_i)}{\partial b_i}\bigg|_{\theta=\theta^C} \mathrm{d}b_i. \qquad (47) \end{aligned}$$

We explicitly emphasize the dependence of θ^C on b_1 and b_2 in this proof to improve readability. Suppose that $b_1 = b_2 = b$. As both b_1 and b_2 increase simultaneously, the change in θ^C is

$$\frac{\partial \theta^C(b,b)}{\partial b} = \left.\frac{\partial \lambda(\theta,b)}{\partial b}\right|_{\theta=\theta^C} \times \sum_{i=1}^{2} \left[\left.\frac{\partial \lambda_i^C}{\partial \theta^C}\right|_{b_j=b} - \left.\frac{\partial \lambda(\theta,b_i)}{\partial \theta}\right|_{\theta=\theta^C} \right]^{-1}. \tag{48}$$

Plugging (41) into (48) leads to the following general identity:

$$\frac{\partial \theta^C(b,b)}{\partial b} = \left.\frac{\partial \theta^C(b_1,b_2)}{\partial b_1}\right|_{b_1=b_2=b} + \left.\frac{\partial \theta^C(b_1,b_2)}{\partial b_2}\right|_{b_1=b_2=b}. \tag{49}$$

As a byproduct, the derivations in (46) to (49), which eventually reach a general identity (49), verify the relations established in (43), (44), and (45). Further, plugging (32) into (49), it follows that

$$\frac{\partial \theta^C(b,b)}{\partial b} = -\frac{\bar{\varphi} b/\nu + 1/[e^{\theta^C(b,b)-b} - 1]}{\eta - \{1/\nu + 1/[e^{\theta^C(b,b)-b} - 1]\}}, \tag{50}$$

which becomes more negative when b increases.

1.8 Proof of Proposition 3.1

It suffices to show that both $e^{-\underline{z}/\nu}$ and $[\sin(\nu\pi)/(\nu\pi)]^{1/\nu}$ are decreasing in ν. Because $\underline{z} < 0$, it is obvious that $e^{-\underline{z}/\nu}$ is decreasing in ν. Let $\mathcal{H}(\nu) \equiv \ln[\sin(\nu\pi)/(\nu\pi)]^{1/\nu}$. We only need to show the following function is decreasing in ν:

$$\mathcal{H}(\nu) \equiv \frac{1}{\nu}\mu(\nu) = \frac{1}{\nu}[\ln(\sin(\nu\pi)) - \ln(\nu\pi)]. \tag{51}$$

Using the facts that $\sin x < x < \tan x$ for $x \in (0, \pi/2)$, $\lim_{x\to 0} x/\sin x = 1$, and $\lim_{x\to 0} x/\tan x = 1$, it is easy to verify that $\mathcal{H}(\nu)$ is a monotonically decreasing and strictly concave function over $(0, 1/2)$.

1.9 Proof of Proposition 3.2

We first show that any $\{P_1^N, P_2^N, M_1^N, M_2^N\}$ that satisfies $P_1^N = P_2^N = 0$ and $M_1^N + M_2^N \leq a/\varepsilon$ constitutes a one-shot static Nash equilibrium. No firm would like to increase the price in the second stage because this deviation would lead to zero demand according to the demand system specification. Meanwhile, no firm would like to decrease the price to some negative value in the second stage because this deviation would lead to a negative profit (i.e., an operating loss). Expecting the equilibrium prices to be zero in the second stage, the customer-base choices in the first stage are arbitrarily made by the firms as long as the industry-level demand curve is satisfied: $M_1^N + M_2^N \leq a/\varepsilon$. Similar to the proof of Proposition 2.1, we can show that $\{P_1, P_2, M_1, M_2\}$ cannot be a non-collusive equilibrium if $P_1 > 0$ or $P_2 > 0$.

1.10 Proof of Proposition 3.3

Given the log profitability θ_1^C and θ_2^C, the equity and debt value are derived in a way similar to the proof of Proposition 2.3. We do not repeat here. As discussed in Section 3.2 of the main text, it must hold that $P_1^C = P_2^C$ in the collusive equilibrium. Thus, we focus only on the situation in which $P_1^C = P_2^C$ and $C_1^C = C_2^C = 1$.

The IC constraint in equation (7) of the main text can be rewritten as follows:

$$\rho e^{\theta_i^C + \eta\theta_j^C} \leq \left[1 - \lambda(\theta_i^C, b_i)\right] E_i^C(b_i, b_j) \tag{52}$$

$$= (1-\tau)\left[1 - \lambda(\theta_i^C, b_i)\right]\left(e^{\theta_i^C} - e^{b_i}\right)\left\{1 + e^{[\mu(\nu)+\theta_i^C - \varphi(b_i)]/\nu}\right\}. \tag{53}$$

According to Proposition 2.2, it follows that

$$\rho e^{\theta_i^C + \eta\theta_j^C} \leq (1-\tau)\left(e^{\theta_i^C} - e^{b_i}\right) e^{[\mu(\nu)+\theta_i^C - \varphi(b_i)]/\nu}. \tag{54}$$

Taking logs on both sides, it follows that

$$\ln(\rho) + \eta\theta_j^C \leq \ln(1-\tau) + [\mu(\nu) + \theta_i^C - \varphi(b_i)]/\nu + \ln\left[1 - e^{-(\theta_i^C - b_i)}\right]. \tag{55}$$

Therefore,

$$\theta_j^C \leq \left\{\ln[(1-\tau)/\rho] + [\mu(\nu) + \theta_i^C - \varphi(b_i)]/\nu + \ln\left[1 - e^{-(\theta_i^C - b_i)}\right]\right\}/\eta. \tag{56}$$

We define a function $\Psi(\theta, b)$ as follows:

$$\Psi(\theta, b) \equiv \ln[(1-\tau)/\rho] + [\mu(\nu) + \theta - \varphi(b)]/\nu + \ln\left[1 - e^{-(\theta - b)}\right]. \tag{57}$$

Thus, the two IC constraints in (56) for the pairs of log profitability (θ_1^C, θ_2^C) can be expressed as follows:

$$\theta_j^C \leq \Psi(\theta_i^C, b_i)/\eta, \quad \text{with } i \neq j \in \{1, 2\}. \tag{58}$$

Hence, the IC region can be characterized as follows:

$$\mathcal{C} \equiv \left\{(\theta_1, \theta_2) \ : \ \theta_i \leq \Psi(\theta_j, b_j)/\eta, \ \text{ for } i \neq j\right\}, \tag{59}$$

for b_1 and b_2 such that $\mathcal{C}$ is not empty.

The firms try to maximize their log profitability and thus equity values. We first show that in the Pareto efficient collusive equilibrium with grim trigger strategies, as characterized by the equilibrium pair of log profitability (θ_1^C, θ_2^C), it must hold that both firms' IC constraints are binding:

$$\theta_1^C = \Psi(\theta_2^C, b_2)/\eta \text{ and } \theta_2^C = \Psi(\theta_1^C, b_1)/\eta, \tag{60}$$

where

$$\Psi(\theta, b) \equiv \ln[(1-\tau)/\rho] + [\mu(\nu) + \theta - \varphi(b)]/\nu + \ln\left[1 - e^{-(\theta - b)}\right]. \tag{61}$$

We prove it by contradiction. The idea is that there is a way to achieve a Pareto improvement if one firm's IC constraint is not binding. Suppose that firm 2's IC constraint satisfies $\theta_1^C < \Psi(\theta_2^C, b_2)/\eta$ and firm 1's IC constraint satisfies $\theta_2^C \leq \Psi(\theta_1^C, b_1)/\eta$. Firm 1 can increase its log profitability θ_1^C by a tiny amount $\epsilon_1 > 0$ so that the IC constraint for firm 2 still holds, i.e., $\theta_1^C + \epsilon_1 \leq \Psi(\theta_2^C, b_2)/\eta$. Because $\Psi(\theta, b)$ is increasing in θ, the IC constraint for firm 1 is also satisfied, i.e., $\theta_2^C \leq \Psi(\theta_1^C + \epsilon_1, b_1)/\eta$. Thus, (θ_1^C, θ_2^C) is not a Pareto efficient collusive equilibrium, because $(\theta_1^C + \epsilon_1, \theta_2)$ is a collusive equilibrium that Pareto-dominates (θ_1^C, θ_2^C).

Next, we show that the Pareto efficient collusive equilibrium exists if b_1 and b_2 satisfy certain conditions. According to Assumption 1, it holds that $b_i + \ell \leq \theta_i$ for $i \in \{1, 2\}$. Moreover, according to equation (19) of the main text, we know that $e^{\theta_1} + e^{\theta_2} = [a - \varepsilon(M_1 + M_2)](M_1 + M_2) \leq a^2/(4\varepsilon)$, leading to $\theta_i \leq \overline{\theta} \equiv \ln(a^2/(4\varepsilon))$. Finally, the function $\ln[1 - e^{-x}]$ is a strictly increasing and concave function in x. Taken together, for $\theta - b - \ell > 0$, it must hold that

$$\frac{1}{e^{\overline{\theta}} - 1}(\theta - b - \ell) \leq \ln\left[1 - e^{-(\theta - b)}\right] - \ln\left[1 - e^{-\ell}\right] \leq \frac{1}{e^{\ell} - 1}(\theta - b - \ell). \tag{62}$$

Therefore, $\Psi(\theta, b)/\eta$ is sandwiched by two functions that are linear in θ as follows:

$$\underline{\alpha}(b) + \underline{\beta}\theta \leq \Psi(\theta, b)/\eta \leq \overline{\alpha}(b) + \overline{\beta}\theta, \tag{63}$$

where

$$\underline{\beta} \equiv [1/\nu + 1/(e^{\overline{\theta}} - 1)]/\eta, \text{ and } \overline{\beta} \equiv [1/\nu + 1/(e^{\ell} - 1)]/\eta, \tag{64}$$

and the functions $\underline{\alpha}(b)$ and $\overline{\alpha}(b)$ are

$$\underline{\alpha}(b) \equiv \{C(b) - (b + \ell)/(e^{\overline{\theta}} - 1)\}/\eta, \tag{65}$$

$$\overline{\alpha}(b) \equiv \{C(b) - (b + \ell)/(e^{\ell} - 1)\}/\eta, \tag{66}$$

where $C(b)$ is defined in (23).

We define a function $F(\cdot,\cdot,\cdot)$ as follows:

$$F(\theta, b, b') \equiv \Psi(\Psi(\theta, b)/\eta, b')/\eta - \theta. \tag{67}$$

By definition, θ_i^C is the solution to $F(\theta, b_i, b_j) = 0$ for $i \neq j \in \{1,2\}$. Using the inequalities in (63), we can obtain the following inequalities:

$$\underline{F}(\theta, b_i, b_j) \leq F(\theta, b_i, b_j) \leq \overline{F}(\theta, b_i, b_j), \quad \text{for all } \theta \in \left[b_i + \ell, \overline{\theta}\right], \tag{68}$$

where $i \neq j \in \{1,2\}$, and the upper-bound and lower-bound functions are as follows:

$$\begin{aligned}
&\underline{F}(\theta, b_i, b_j) \equiv \underline{\alpha}(b_j) + \underline{\beta}\underline{\alpha}(b_i) - \left\{1 - \underline{\beta}^2\right\}\theta, \text{ and} \\
&\overline{F}(\theta, b_i, b_j) \equiv \overline{\alpha}(b_j) + \overline{\beta}\overline{\alpha}(b_i) - \left\{1 - \overline{\beta}^2\right\}\theta, \text{ for } i \neq j \in \{1,2\}.
\end{aligned} \tag{69}$$

When η is sufficiently large and ρ is sufficiently small, both the lower-bound function $\underline{F}(\theta, b_i, b_j)$ and the upper-bound function $\overline{F}(\theta, b_i, b_j)$, for $i \neq j \in \{1,2\}$, have the following positive roots:

$$\underline{\theta}_i^*(b_i, b_j) \equiv \frac{\underline{\alpha}(b_j) + \underline{\beta}\underline{\alpha}(b_i)}{1 - \underline{\beta}^2} \text{ and } \overline{\theta}_i^*(b_i, b_j) \equiv \frac{\overline{\alpha}(b_j) + \overline{\beta}\overline{\alpha}(b_i)}{1 - \overline{\beta}^2}, \text{ respectively.} \tag{70}$$

Therefore, the equilibrium exists as long as the parameters, including b_1 and b_2, make the following conditions hold:

$$b_i + \ell \leq \underline{\theta}_i^*(b_i, b_j) \leq \overline{\theta}_i^*(b_i, b_j) \leq \overline{\theta}. \tag{71}$$

It follows from the inequalities in (68) that the log profitability θ_i^C of the collusive equilibrium lies between $\underline{\theta}_i^*(b_i, b_j)$ and $\overline{\theta}_i^*(b_i, b_j)$ for $i \neq j \in \{1,2\}$.

1.11 Proof of Proposition 3.4

We first show that, given b_1 and b_2 fixed, the log profitability in the collusive equilibrium, denoted by $\theta_i^C(b_i, b_j)$ with $i \neq j \in \{1,2\}$, decreases with idiosyncratic left-tail risk ν. We define functions $G_i(\theta_i, \theta_j, b_i, b_j)$ with $i = 1,2$ as follows:

$$\begin{bmatrix} G_1(\theta_1, \theta_2, b_1, b_2) \\ G_2(\theta_2, \theta_1, b_2, b_1) \end{bmatrix} \equiv \begin{bmatrix} \eta\theta_1 - \Psi(\theta_2, b_2) \\ \eta\theta_2 - \Psi(\theta_1, b_1) \end{bmatrix}, \tag{72}$$

where $\Psi(\pi, b)$ is defined in equation (11) of the main text. According to Proposition 3.3, it follows that, given b_1 and b_2 fixed, the log profitability $\theta_1^C(b_1, b_2)$ and $\theta_2^C(b_2, b_1)$ satisfy

$$G_i(\theta_i^C(b_i, b_j), \theta_j^C(b_j, b_i), b_i, b_j) \equiv 0, \tag{73}$$

with $i \neq j \in \{1, 2\}$ for all b_1, b_2, and ν. The Implicit Function Theorem implies that

$$\frac{\partial(\theta_1^C, \theta_2^C)}{\partial \nu} = -\left[\frac{\partial(G_1, G_2)}{\partial(\theta_1^C, \theta_2^C)}\right]^{-1} \frac{\partial(G_1, G_2)}{\partial \nu}. \tag{74}$$

The Jacobian matrices are

$$\frac{\partial(G_1, G_2)}{\partial(\theta_1^C, \theta_2^C)} = \begin{bmatrix} \eta & -1/\nu - d_2 \\ -1/\nu - d_1 & \eta \end{bmatrix} \quad \text{and} \quad \frac{\partial(G_1, G_2)}{\partial \nu} = \begin{bmatrix} -\dot{\mathcal{H}}(\nu) + [\theta_2^C(b_2, b_1) - \varphi(b_2)]/\nu^2 \\ -\dot{\mathcal{H}}(\nu) + [\theta_1^C(b_1, b_2) - \varphi(b_1)]/\nu^2 \end{bmatrix}, \tag{75}$$

where $d_i \equiv 1/\left[e^{\theta_i^C(b_i, b_j) - b_i} - 1\right]$ for $i = 1, 2$ and $\mathcal{H}(\nu)$ is defined in (51).

The proof of Proposition 3.1 in Online Appendix 1.8 shows that $\dot{\mathcal{H}}(\nu) < 0$. In addition, the earnings-based borrowing constraint in Assumption 1 ensures that $\theta_i^C(b_i, b_j) - b_i \geq \ell$ for $i \in \{1, 2\}$. Thus, it must hold that $\partial G_i/\partial \nu > 0$ for $i \in \{1, 2\}$, that is, all elements of $\partial(G_1, G_2)/\partial \nu$ are positive. The inverse of the Jacobian matrix $\partial(G_1, G_2)/\partial(\theta_1^C, \theta_2^C)$ is

$$\left[\frac{\partial(G_1, G_2)}{\partial(\theta_1^C, \theta_2^C)}\right]^{-1} = \frac{1}{H} \begin{bmatrix} \eta & 1/\nu + d_2 \\ 1/\nu + d_1 & \eta \end{bmatrix}, \tag{76}$$

where $H \equiv \eta^2 - \prod_{i=1}^{2}(1/\nu + d_i)$. Specifically, if η is sufficiently large so that $\eta > 1/\nu + 1/(e^\ell - 1)$, all elements of $\left[\partial(G_1, G_2)/\partial(\theta_1^C, \theta_2^C)\right]^{-1}$ are positive. Taken together, all elements of $\partial(\theta_1^C, \theta_2^C)/\partial \nu$ are negative, that is, $\partial \theta_i^C(b_i, b_j)/\partial \nu < 0$ for $i \in \{1, 2\}$.

Next, we show that the best response functions $\mathbf{b_i}(b_j)$ shift up when ν increases for $i \neq j \in \{1, 2\}$. The firm value in the collusive equilibrium is $V_i^C(b_i, b_j) \equiv E_i^C(b_i, b_j) + D_i^C(b_i, b_j)$. The equity value $E_i^C(b_i, b_j)$ and debt value $D_i^C(b_i, b_j)$ are characterized in Proposition 3.3. Thus, the firm value can be expressed as follows:

$$V_i^C(b_i, b_j) \equiv \Phi(\theta_i^C(b_i, b_j), b_i), \tag{77}$$

where $\theta_i^C(b_i, b_j)$ is characterized in Proposition 3.3, and the function $\Phi(\theta, b)$ is specified as follows:

$$\Phi(\theta, b) \equiv \underbrace{(1-\tau)e^{\theta}\lambda^{C}(\theta, b)^{-1}}_{\text{Asset value}} + \underbrace{\tau e^{b}\lambda^{C}(\theta, b)^{-1}}_{\text{Value of tax shield}}, \tag{78}$$

$$= (1-\tau)e^{\theta} + (1-\tau)e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \tau e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} + \tau e^{b}. \tag{79}$$

We define a new function, denoted by $\Upsilon_i^C(b_i, b_j)$, as follows:

$$\Upsilon_i^C(b_i, b_j) \equiv \frac{\partial V_i^C(b_i, b_j)}{\partial b_i} = \Phi_\theta(\theta_i^C(b_i, b_j), b_i)\frac{\partial \theta_i^C(b_i, b_j)}{\partial b_i} + \Phi_b(\theta_i^C(b_i, b_j), b_i), \tag{80}$$

where the first-order partial derivatives of $\Phi(\theta, b)$ are:

$$\Phi_\theta(\theta, b) \equiv (1-\tau)e^{\theta} + (1-\tau)\left(1+\frac{1}{\nu}\right)e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \frac{\tau}{\nu}e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}, \tag{81}$$

$$\Phi_b(\theta, b) \equiv -(1-\tau)\frac{\dot{\varphi}(b)}{\nu}e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \tau\left[1-\frac{\dot{\varphi}(b)}{\nu}\right]e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} + \tau e^{b}. \tag{82}$$

The first-order condition and the concavity condition to characterize the best response function $\mathbf{b_i}(b_j)$ are

$$\Upsilon_i^C(\mathbf{b_i}(b_j), b_j) \equiv 0 \ \text{ and } \ \left.\frac{\partial \Upsilon_i^C(b_i, b_j)}{\partial b_i}\right|_{b_i=\mathbf{b_i}(b_j)} < 0, \ \text{ respectively.} \tag{83}$$

It follows immediately from (81) that $\Phi_\theta(\theta, b) > 0$. By the definition of $\Upsilon_i^C(b_i, b_j)$ in (80) and the first-order condition in (83), we obtain the relation below:

$$\left.\Phi_b(\theta_i^C(b_i, b_j), b_i)\right|_{b_i=\mathbf{b_i}(b_j)} = -\left.\Phi_\theta(\theta_i^C(b_i, b_j), b_i)\frac{\partial \theta_i^C(b_i, b_j)}{\partial b_i}\right|_{b_i=\mathbf{b_i}(b_j)}. \tag{84}$$

Proposition 1.13 shows that $\frac{\partial \theta_i^C(b_i, b_j)}{\partial b_i} < 0$. Thus, it must hold that

$$\left.\Phi_b(\theta_i^C(b_i, b_j), b_i)\right|_{b_i=\mathbf{b_i}(b_j)} > 0. \tag{85}$$

According to the Implicit Function Theorem, the derivative of the best response function with respect to ν can be written as

$$\frac{\partial \mathbf{b_i}(b_j)}{\partial \nu} = -\frac{\left.\frac{\partial \Upsilon_i^C(b_i, b_j)}{\partial \nu}\right|_{b_i=\mathbf{b_i}(b_j)}}{\left.\frac{\partial \Upsilon_i^C(b_i, b_j)}{\partial b_i}\right|_{b_i=\mathbf{b_i}(b_j)}}. \tag{86}$$

To show that $\frac{\partial \mathbf{b_i}(b_j)}{\partial \nu} > 0$, it suffices to show that $\left.\frac{\partial \Upsilon_i^C(b_i, b_j)}{\partial \nu}\right|_{b_i=\mathbf{b_i}(b_j)} > 0$ because the optimality condition (83) ensures that $\left.\frac{\partial \Upsilon_i^C(b_i, b_j)}{\partial b_i}\right|_{b_i=\mathbf{b_i}(b_j)} < 0$. The partial derivative $\frac{\partial \Upsilon_i^C(b_i, b_j)}{\partial \nu}$ has the following expression:

$$\begin{aligned}\frac{\partial \Upsilon_i^C(b_i, b_j)}{\partial \nu} = {} & \Phi_{\theta\theta}(\theta_i^C, b_i)\frac{\partial \theta_i^C}{\partial b_i}\frac{\partial \theta_i^C}{\partial \nu} + \Phi_{\theta\nu}(\theta_i^C, b_i)\frac{\partial \theta_i^C}{\partial b_i} + \Phi_{\theta}(\theta_i^C, b_i)\frac{\partial^2 \theta_i^C}{\partial b_i \partial \nu} \\ & + \Phi_{b\theta}(\theta_i^C, b_i)\frac{\partial \theta_i^C}{\partial \nu} + \Phi_{b\nu}(\theta_i^C, b_i),\end{aligned} \tag{87}$$

where $\theta_i^C = \theta_i^C(b_i, b_j)$ and the partial derivatives of $\Phi(\theta, b)$ are

$$\Phi_\theta(\theta, b) \equiv (1-\tau)e^\theta + (1-\tau)\left(1+\frac{1}{\nu}\right)e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \frac{\tau}{\nu}e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}, \tag{88}$$

$$\Phi_{\theta\theta}(\theta, b) \equiv \Phi_\theta(\theta, b) + \widetilde{\Phi}_{\theta\theta}(\theta, b), \quad \text{with} \tag{89}$$

$$\widetilde{\Phi}_{\theta\theta}(\theta, b) \equiv (1-\tau)\frac{1}{\nu}\left(1+\frac{1}{\nu}\right)e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \frac{\tau}{\nu}\left(\frac{1}{\nu}-1\right)e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}, \tag{90}$$

$$\begin{aligned}\Phi_{\theta\nu}(\theta, b) \equiv & -\frac{1-\tau}{\nu^2}\left\{1+\left(1+\frac{1}{\nu}\right)[\mu(\nu)+\theta-\varphi(b)-\nu\dot{\mu}(\nu)]\right\}e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu}, \\ & -\frac{\tau}{\nu^2}\left\{1+\frac{1}{\nu}[\mu(\nu)+\theta-\varphi(b)-\nu\dot{\mu}(\nu)]\right\}e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b},\end{aligned} \tag{91}$$

$$\Phi_b(\theta, b) \equiv -(1-\tau)\frac{\dot{\varphi}(b)}{\nu}e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \tau\left[1-\frac{\dot{\varphi}(b)}{\nu}\right]e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} + \tau e^b, \tag{92}$$

$$\Phi_{bb}(\theta, b) \equiv (1-\tau)\left\{\left[\frac{\dot{\varphi}(b)}{\nu}\right]^2 - \frac{\ddot{\varphi}(b)}{\nu}\right\}e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} \tag{93}$$

$$+\tau\left\{\left[\frac{\dot{\varphi}(b)}{\nu}-1\right]^2 - \frac{\ddot{\varphi}(b)}{\nu}\right\}e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} + \tau e^b, \tag{94}$$

$$\Phi_{b\theta}(\theta, b) \equiv -(1-\tau)\frac{\dot{\varphi}(b)}{\nu}\left(1+\frac{1}{\nu}\right)e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \frac{\tau}{\nu}\left[1-\frac{\dot{\varphi}(b)}{\nu}\right]e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}, \tag{95}$$

$$\Phi_{b\nu}(\theta, b) \equiv \frac{\dot{\varphi}(b)}{\nu^2}\left[(1-\tau)e^{\theta-b}+\tau\right]e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} \tag{96}$$

$$-\left[\Phi_b(\theta, b) - \tau e^b\right]\left\{[\theta-\varphi(b)]/\nu^2 - \dot{\mathcal{H}}(\nu)\right\}. \tag{97}$$

Below, we show that $\Phi_{b\theta}(\theta_i^C, b_i)\big|_{b_i=\mathbf{b_i}(b_j)} < \Phi_b(\theta_i^C, b_i)\big|_{b_i=\mathbf{b_i}(b_j)}$. Using equations (92) and (95), we can obtain the following inequality:

$$\begin{aligned}\Phi_b(\theta, b) - \Phi_{b\theta}(\theta, b) = & \frac{1}{\nu}(1-\tau)\frac{\dot{\varphi}(b)}{\nu}e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \tau e^b \\ & + \tau\left(1-\frac{1}{\nu}\right)\left[1-\frac{\dot{\varphi}(b)}{\nu}\right]e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b},\end{aligned}$$

where $\dot{\varphi}(b) = \bar{\varphi}b > \tau\nu$ according to Assumption 2. Moreover, because $\nu \in (0, 1/2)$ (which implies $1 - 1/\nu < 0$), we have

$$\begin{aligned}\Phi_b(\theta, b) - \Phi_{b\theta}(\theta, b) &> \frac{1}{\nu}(1-\tau)\tau e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} + \tau\left(1-\frac{1}{\nu}\right)(1-\tau)e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}, \\ &> \frac{1}{\nu}(1-\tau)\tau\left[e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} - e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b}\right] \\ &> 0,\end{aligned}$$

where the last inequality is due to Assumption 1 (which implies $\theta > b$).

Now, because $\Phi_{b\theta}(\theta_i^C, b_i)|_{b_i=\mathbf{b_i}(b_j)} < \Phi_b(\theta_i^C, b_i)|_{b_i=\mathbf{b_i}(b_j)}$ and $\frac{\partial\theta_i^C}{\partial\nu} < 0$, equation (87) implies that

$$\begin{aligned}\frac{\partial Y_i^C(b_i,b_j)}{\partial\nu} > \Phi_{\theta\theta}(\theta_i^C, b_i)\frac{\partial\theta_i^C}{\partial b_i}\frac{\partial\theta_i^C}{\partial\nu} + \Phi_{\theta\nu}(\theta_i^C, b_i)\frac{\partial\theta_i}{\partial b_i} + \Phi_\theta(\theta_i^C, b_i)\frac{\partial^2\theta_i^C}{\partial b_i\partial\nu} \\ + \Phi_b(\theta_i^C, b_i)\frac{\partial\theta_i^C}{\partial\nu} + \Phi_{b\nu}(\theta_i^C, b_i).\end{aligned} \tag{98}$$

According to (84), it follows that, when $b_i = \mathbf{b_i}(b_j)$, the inequality below must hold:

$$\begin{aligned}\frac{\partial Y_i^C(b_i,b_j)}{\partial\nu} &> \Phi_{\theta\theta}(\theta_i^C, b_i)\frac{\partial\theta_i^C}{\partial b_i}\frac{\partial\theta_i^C}{\partial\nu} + \Phi_{\theta\nu}(\theta_i^C, b_i)\frac{\partial\theta_i^C}{\partial b_i} + \Phi_\theta(\theta_i^C, b_i)\frac{\partial^2\theta_i^C}{\partial b_i\partial\nu} \\ &\quad - \Phi_\theta(\theta_i^C, b_i)\frac{\partial\theta_i^C}{\partial b_i}\frac{\partial\theta_i^C}{\partial\nu} + \Phi_{b\nu}(\theta_i^C, b_i) && (99) \\ &= \widetilde{\Phi}_{\theta\theta}(\theta_i^C, b_i)\frac{\partial\theta_i^C}{\partial b_i}\frac{\partial\theta_i^C}{\partial\nu} + \Phi_{\theta\nu}(\theta_i^C, b_i)\frac{\partial\theta_i^C}{\partial b_i} + \Phi_\theta(\theta_i^C, b_i)\frac{\partial^2\theta_i^C}{\partial b_i\partial\nu} + \Phi_{b\nu}(\theta_i^C, b_i). && (100)\end{aligned}$$

The term $\widetilde{\Phi}_{\theta\theta}(\theta_i^C, b_i)\frac{\partial\theta_i^C}{\partial b_i}\frac{\partial\theta_i^C}{\partial\nu}\Big|_{b_i=\mathbf{b_i}(b_j)}$ is positive, because $\widetilde{\Phi}_{\theta\theta}(\theta_i^C, b_i) > 0$, $\frac{\partial\theta_i^C}{\partial b_i} < 0$, and $\frac{\partial\theta_i^C}{\partial\nu} < 0$. The term $\Phi_{\theta\nu}(\theta_i^C, b_i)\frac{\partial\theta_i^C}{\partial b_i}\Big|_{b_i=\mathbf{b_i}(b_j)}$ is positive, because $\Phi_{\theta\nu}(\theta_i^C, b_i) < 0$ and $\frac{\partial\theta_i^C}{\partial b_i} < 0$. The term $\Phi_\theta(\theta_i^C, b_i)\frac{\partial^2\theta_i^C}{\partial b_i\partial\nu}\Big|_{b_i=\mathbf{b_i}(b_j)}$ is positive, because $\Phi_\theta(\theta_i^C, b_i) > 0$ and $\frac{\partial^2\theta_i^C}{\partial b_i\partial\nu} > 0$. The term $\Phi_{b\nu}(\theta_i^C, b_i)|_{b_i=\mathbf{b_i}(b_j)}$ is positive, because of Assumption 2. Therefore, $\frac{\partial Y_i^C(b_i,b_j)}{\partial\nu}\Big|_{b_i=\mathbf{b_i}(b_j)} > 0$, and thus, $\frac{\partial\mathbf{b_i}(b_j)}{\partial\nu} > 0$. We elaborate more on why $\Phi_{b\nu}(\theta_i^C, b_i)|_{b_i=\mathbf{b_i}(b_j)} >$

0 below:

$$\Phi_{b\nu}(\theta,b) \equiv \frac{\dot{\varphi}(b)}{\nu^2}\left[(1-\tau)e^{\theta-b}+\tau\right]e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} - \left[\Phi_b(\theta,b)-\tau e^b\right]\left\{[\theta-\varphi(b)]/\nu^2 - \dot{\mathcal{H}}(\nu)\right\}$$
$$> -\left[\Phi_b(\theta,b)-\tau e^b\right]\left\{[\theta-\varphi(b)]/\nu^2-\dot{\mathcal{H}}(\nu)\right\}. \tag{101}$$

In the above equation, we have

$$\begin{aligned}
-\left[\Phi_b(\theta,b)-\tau e^b\right] &= (1-\tau)\frac{\dot{\varphi}(b)}{\nu}e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} - \tau\left[1-\frac{\dot{\varphi}(b)}{\nu}\right]e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} \\
&\geq (1-\tau)\tau e^{\theta+[\mu(\nu)+\theta-\varphi(b)]/\nu} - \tau(1-\tau)e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} \\
&> (1-\tau)\tau e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} - \tau(1-\tau)e^{[\mu(\nu)+\theta-\varphi(b)]/\nu+b} \\
&= 0,
\end{aligned} \tag{102}$$

where the first inequality is due to Assumption 2, which implies $\dot{\varphi}(b) = \bar{\varphi}b_i > \bar{\varphi}\underline{b} \geq \tau\nu$). The second inequality is due to Assumption 1.

Moreover, Assumption 1 implies $[\theta-\varphi(b)]/\nu^2 > 0$. According to the proof of Proposition 3.1 in Online Appendix 1.8, we have $\dot{\mathcal{H}}(\nu) < 0$. Thus, it follows that $\Phi_{b\nu}(\theta,b) > 0$.

Now, we show that, as ν increases, the equilibrium log coupon level b_i^C increases and the equilibrium log profitability $\theta_i^C(b_i^C, b_j^C)$ decreases for $i \in \{1,2\}$. To ensure that (b_1^C, b_2^C) is a stable fixed point of the system $(\mathbf{b_1}(b_2), \mathbf{b_2}(b_1))$, it must hold that $\left|\frac{\partial \mathbf{b_1}(b_2)}{\partial b_2}\Big|_{b_2=b_2^C}\right| < 1$ and $\left|\frac{\partial \mathbf{b_2}(b_1)}{\partial b_1}\Big|_{b_1=b_1^C}\right| < 1$. Because $b_1^C = b_2^C$, it follows that $b_i^C = \mathbf{b_i}(b_i^C)$ for $i \in \{1,2\}$. According to the chain rule, it must hold that, for $i \neq j \in \{1,2\}$,

$$\frac{\partial b_i^C}{\partial \nu} = \frac{\partial \mathbf{b_i}(b_j)}{\partial b_j}\bigg|_{b_j=b_j^C}\frac{\partial b_i^C}{\partial \nu} + \frac{\partial \mathbf{b_i}(b_j)}{\partial \nu}\bigg|_{b_j=b_j^C}. \tag{103}$$

Thus, the derivative of the equilibrium log coupon level b_i^C with respect to idiosyncratic left-tail risk ν is

$$\frac{\partial b_i^C}{\partial \nu} = \left[1-\frac{\partial \mathbf{b_i}(b_j)}{\partial b_j}\bigg|_{b_j=b_j^C}\right]^{-1}\frac{\partial \mathbf{b_i}(b_j)}{\partial \nu}\bigg|_{b_j=b_j^C} > 0. \tag{104}$$

Thus, as ν increases, both b_1^C and b_2^C increase. According to Proposition OA.1 in Online Appendix 1.13, $\frac{\partial \theta_i^C(b_i,b_j)}{\partial b_i} < 0$, $\frac{\partial \theta_i^C(b_i,b_j)}{\partial b_j} < 0$. Note that we have already shown at the beginning of the proof for Proposition 3.4 that given b_1 and b_2 fixed, $\frac{\partial \theta_i^C(b_i,b_j)}{\partial \nu} < 0$. Thus, taken together, both $\theta_1^C(b_1^C, b_2^C)$ and $\theta_2^C(b_2^C, b_1^C)$ decrease with ν.

Finally, we show that, as ν increases, firm i's market leverage lev_i^C increases for $i \in \{1,2\}$. As in equation (14) of the main text, firm i's market leverage can be rewritten as

$$lev_i^C(b_i, b_j) = \frac{1}{(1-\tau)e^{\theta_i^C - b_i} + \tau}, \quad \text{with } i \neq j. \tag{105}$$

Therefore, as ν increases, lev_i^C increases for $i \in \{1,2\}$.

1.12 Proof of Proposition 3.5

We prove this proposition by contradiction. If $b_i^C < \bar{b}_i^C$, by the definition of $\bar{b}_1^C$ and $\bar{b}_2^C$ (i.e., equation (26) of the main text), we have

$$V_1^C(b_1^C, b_2^C) + V_2^C(b_2^C, b_1^C) \leq V_1^C(\bar{b}_1^C, \bar{b}_2^C) + V_2^C(\bar{b}_2^C, \bar{b}_1^C). \tag{106}$$

Then, at least one of the following two inequalities must hold

$$V_1^C(b_1^C, b_2^C) \leq V_1^C(\bar{b}_1^C, \bar{b}_2^C) \quad \text{and} \quad V_2^C(b_2^C, b_1^C) \leq V_2^C(\bar{b}_2^C, \bar{b}_1^C). \tag{107}$$

Without loss of generality, we assume that $V_1^C(b_1^C, b_2^C) \leq V_1^C(\bar{b}_1^C, \bar{b}_2^C)$, which implies

$$\begin{aligned} V_1^C(b_1^C, b_2^C) &\leq V_1^C(\bar{b}_1^C, \bar{b}_2^C) && (108)\\ &< V_1^C(\bar{b}_1^C, b_2^C) \quad \text{(because } V_i^C(b_i, b_j) \text{ is strictly decreasing in } b_j\text{)} && (109)\\ &\leq V_1^C(b_1^C, b_2^C) \quad \text{(because } (b_1^C, b_2^C) \text{ is a Nash equilibrium)} && (110) \end{aligned}$$

Taken together, the inequalities (108) to (110) lead to a contradiction, because $V_1^C(b_1^C, b_2^C)$ cannot be strictly smaller than itself. Therefore, it must hold that $b_i^C \geq \bar{b}_i^C$ for both $i = 1, 2$.

We now elaborate how the inequalities (108) to (110) are derived. The inequalities (108) and (110) are obvious, while the inequality (109) requires more explanations. As shown in the proof of Proposition 3.4 in Online Appendix 1.11, the firm value can be characterized by

$$V_i^C(b_i, b_j) \equiv \Phi(\theta_i^C(b_i, b_j), b_i), \tag{111}$$

where $\theta_i^C(b_i, b_j)$ is characterized in Proposition 3.3, and the function $\Phi(\theta, b)$ is specified in (79). Thus, it follows that

$$\frac{\partial V_i^C(b_i, b_j)}{\partial b_j} = \Phi_\theta(\theta_i^C(b_i, b_j), b_i)\frac{\partial \theta_i^C(b_i, b_j)}{\partial b_j}. \tag{112}$$

From (88), it follows that $\Phi_\theta(\theta, b_i) > 0$. Moreover, Proposition OA.1 shows that $\frac{\partial \theta_i^C(b_i, b_j)}{\partial b_j} < 0$. Thus, $\frac{\partial V_i^C(b_i, b_j)}{\partial b_j} < 0$, i.e., $V_i^C(b_i, b_j)$ is strictly decreasing in b_j. If $b_2^C < \bar{b}_2^C$, then $V_1^C(\bar{b}_1^C, \bar{b}_2^C) < V_1^C(\bar{b}_1^C, b_2^C)$.

1.13 Extended Simple Model: Feedback and Contagion Effects

Proposition OA.1 (Effects of Financial Leverage on Product Pricing Behaviors)**.** *An increase in either b_1 or b_2 reduces both log profitability θ_1^C and θ_2^C in the collusive equilibrium. The partial derivatives of (θ_1^C, θ_2^C) with respect to (b_1, b_2) are*

$$\frac{\partial(\theta_1^C, \theta_2^C)}{\partial(b_1, b_2)} = -\frac{1}{H}\begin{bmatrix} (1/\nu + d_2)[\dot{\varphi}(b_1)/\nu + d_1] & \eta[\dot{\varphi}(b_2)/\nu + d_2] \\ \eta[\dot{\varphi}(b_1)/\nu + d_1] & (1/\nu + d_1)[\dot{\varphi}(b_2)/\nu + d_2] \end{bmatrix},$$

where $H \equiv \eta^2 - \prod_{i=1}^{2}(1/\nu + d_i)$ and $d_i \equiv 1/\left(e^{\theta_i - b_i} - 1\right)$ for $i = 1, 2$. If $\eta > 1/\nu + 1/(e^\ell - 1)$, all elements of the above Jacobian matrix are negative:

$$\frac{\partial \theta_i^C}{\partial b_i} < 0 \quad \textit{and} \quad \frac{\partial \theta_i^C}{\partial b_j} < 0, \ \textit{with} \ \ i \neq j \in \{1, 2\}. \tag{113}$$

Corollary OA.1 (Financial Distress Spillovers Through Endogenous Competition)**.** *An increase in leverage for firm i, with b_j kept unchanged, increases the default probability of rival firm j in the collusive equilibrium. That is,*

$$\frac{\partial \lambda_j^C}{\partial b_i} \geq 0, \ \textit{with} \ \ i \neq j \in \{1, 2\}. \tag{114}$$

Proposition OA.2 (Feedback Loop Between Financial and Product Markets)**.** *With firm j's coupon b_j remaining unchanged, an increase in firm i's financial leverage via an increase in b_i triggers a feedback loop between the financial and product markets, amplifying the impact of b_i on the default probability λ_i^C:*

$$\begin{bmatrix} d\lambda_i^C \\ d\theta_i^C \end{bmatrix} = \underbrace{\begin{bmatrix} \partial \lambda^C(\theta, b_i)/\partial b_i\big|_{\theta=\theta_i^C} \\ 0 \end{bmatrix}}_{\textit{Initial direct effect} \geq 0} db_i + \underbrace{\begin{bmatrix} 0 & \partial \lambda^C(\theta, b_i)/\partial \theta\big|_{\theta=\theta_i^C} \\ \partial \theta_i^C/\partial \lambda_i^C\big|_{b_j} & 0 \end{bmatrix}}_{\textit{Higher-order feedback effect} \leq 0} \begin{bmatrix} d\lambda_i^C \\ d\theta_i^C \end{bmatrix},$$

where $\partial \theta_i^C/\partial \lambda_i^C\big|_{b_j}$ is the sensitivity of log profitability θ_i^C to firm i's default probability λ_i^C, with b_j kept unchanged, in the collusive equilibrium. Importantly, the system is stable because the *eigenvalues of the matrix that captures the higher-order feedback effect lie on* $(-1,1)$.

2 Direct Evidence on Tacit Collusion

Extensive empirical evidence in many industries indicates that firms compete considerably strategically in the form of tacit collusion.

Direct Evidence and Antitrust Cases. Even in countries that impose criminal penalties on cartel conduct, such as the U.S., there is evidence for implicit or explicit collusion that triggers antitrust investigations and cartel prosecutions. These antitrust cases and cartel prosecutions provide direct evidence that collusion is widespread among market leaders. For example, early statistical studies of antitrust enforcement and price fixing cases include Posner (1970) and Hay and Kelley (1974). Levenstein and Suslow (2006) survey a wide variety of empirical studies of cartels and conclude that the average duration of cartels is about 5 years. Marshall and Marx (2012) review various actual antitrust cases to understand the organization and implementation of a cartel. Harrington (2006) offers detailed and insightful case studies of 20 cartels based on European Commission decisions over 2000-2004. Connor (2001, 2008) and Harrington and Skrzypacz (2011) provide a wealth of empirical information and discussions on recent international cartels and cartel prosecutions in various industries, such as the citric acid industry (Connor, 1998), the lysine industry (Connor, 2001; White, 2001), the vitamin industry, the carbonless paper industry, and the choline chloride industry. John Connor's Private International Cartels Dataset (Connor, 2020) shows that from 1990 to 2019, 1130 cartels and more than 49,000 companies were convicted of price fixing and a large number of cartels were suspected of it and were being investigated. The estimated cartel overcharges during the period exceeded $1.5 trillion, with the majority of the corporate cartelists coming from Europe or North America. Precisely because of the prevalence of the collusive activities, government authorities such as the Department of Justice (DOJ) and the Federal Trade Commission (FTC) spend enormous resources in antitrust enforcement.[1] Moreover, because collusion is prevalent in the economy, investors express serious concerns about the threat of price wars, which can generate a substantial drop in stock prices. Concerns about price wars are frequently discussed by news media and analyst reports (see Section 2 and Online Appendix A of Dou, Ji and Wu (2021*a*) for a collection of evidence).

[1]The Thurman Arnold Project at Yale School of Management provides a comprehensive list of scholarly publications on research related to collusion and antitrust enforcement. See https://som.yale.edu/faculty-research-centers/centers-initiatives/thurman-arnold-project-at-yale.

Moreover, there are many empirical studies investigating particular industries or cartels to understand why cartels are profitable, how they are implemented, and how they affect corporate decisions and might be detected. Porter (1983) investigates price wars in a railroad cartel in the U.S. and found that although there is collusion in most of the time, reversions to noncooperative behavior did occur in some periods, during which significant decrease in market price was observed. Genesove and Mullin (2001) provide direct evidence for tacit collusion in the sugar-refining cartel in the U.S. based on a remarkable series of notes on the weekly meetings of the Sugar Institute. They present the meeting record among the refiners, which documents the rules that eliminated the differential treatment of customers and harmonized contractual practices, even though the Sugar Institute did not fix prices or output directly. In addition to fixing prices, Levenstein and Suslow (2004, 2006) show that cartels can also influence advertising, innovation, investment, barriers to entry, and concentration. Röller and Steen (2006) study how cartels decide production and the distribution of rents in the Norwegian cement industry. In the data from the DOJ Antitrust Division Web site, there are 809 information reports and 222 indictments filed for violations of Section 1 of the Sherman Act from 1985 to 2005. Based on this dataset, Miller (2009) finds that the introduction of new leniency program in 1993 increased the detection and deterrence capabilities of the DOJ. Miller (2010) presents evidence suggesting that the Airline Tariff Publishing (ATP) antitrust intervention in 1992 has limited effects on improving long-term competition in the airline industry. Using the data on cartel prosecutions by DOJ and European Commission, Levenstein and Suslow (2011, 2016) find that the financial conditions of firms and the real interest rate play an important role in determining cartel breakup, which lends direct support to the mechanism of our model. Clark and Houde (2013) study the price-fixing cases between 2002 and 2005, involving 128 gasoline stations and 64 firms in four cities of Quebec. They found that intertemporal transfers in favor of stronger players are implemented to overcome the enforcement and agreement problems of tacit collusion. Moreover, evidence of tacit collusion has also been extensively documented in the U.S. health insurance market (e.g., Lin and McCarthy, 2018) and the U.S. hospital industry (e.g., Schmitt, 2018). By exploiting the staggered passage of leniency programs in 63 countries around the world from 1990 to 2012, Dasgupta and Žaldokas (2019) find that in places with weaker antitrust enforcement, firms reduce investment and equity issuance because of tacit collusion. González, Schmid and Yermack (2019) find that managers seem to use concealment strategies to limit detection of cartel membership. They further show that managers have the incentive to participate in cartels because price fixing allows them to gain greater job security and higher compensation. Harrington and Ye (2019, Section II) discuss several litigation cases that involve coordinated announcements and collusion in the market. Alé-Chilet and Atal (2020) find that gynecologists in a Chile city achieve collusion by forming a trade association to negotiate for better prices with insurance companies.

Evidence on Focal-Point Scheme. Although the Folk Theorem provides general conditions under which tacit collusion can be sustained as an equilibrium outcome, a practical issue is how firms can successfully achieve coordination. Obviously, making a nakedly expressed offer to collude exposes firms to the risk of prosecution under antitrust law. The theory of focal points suggest that firms can resolve the coordination problem through the use of a focal point (e.g., Schelling, 1960). Many empirical studies test if the mechanism of focal points exists and facilitates tacit collusion. For example, Knittel and Stango (2003) find that tacit collusion among credit card companies at a nonbinding state-level interest rate ceilings was prevalent in the U.S. credit card market before its national integration. Wang (2009) investigates gasoline retail markets using information from the trial records of a price-fixing case in Australia. By studying a unique law that regulates the timing and frequency of price setting, Wang (2009) provides evidence for the importance of price commitment in tacit collusion. Lewis (2015) finds that gasoline stations in the U.S. disproportionately sell at prices ending in odd digits. These odd-numbered station prices are higher and change less frequently, suggesting that odd prices are used as focal points to facilitate tacit collusion. Byrne and de Roos (2019) provide well-identified evidence on how collusive agreements are initiated. Based on a unique dataset that contains the universe of station-level prices for an urban retail-gasoline market, they find that market leaders use price experiments to test rivals' willingness to collude and signal their intentions, leading to price coordination and widened profit margins. Although it is illegal to achieve collusion through the use of private communication among firms, Bourveau, She and Žaldokas (2020) and Pawliczek, Skinner and Zechman (2022) find that firms use corporate disclosure to facilitate tacit coordination by helping firms establish focal points. More generally, Harrington (2022) collects and analyzes a wide range of episodes of collusion based on public announcements, which act as a coordinating device to achieve tacit collusion. As one example, the Department of Justice alleged that in the beer industry, Anheuser Busch pre-announced its annual list price changes and then competitors tended to follow. Miller, Sheu and Weinberg (2021) estimate that such a price-leadership strategy raised firms' profits by about 20% relative to Bertrand competition.

Evidence on the Role of Communication. Communication among oligopolists facilitates collusion (Rahman, 2014) and is a fact of their economic life (Marshall and Marx, 2012; Genesove and Mullin, 2001), although antitrust law forbids such behavior (Stigler, 1964). There is ample evidence for the role of communication and the exchange of information among oligopolists in reaching collusive agreements. For example, Borenstein (2004) provides evidence that airlines announce future intended prices as a way to facilitate collusion, leading to more frequent and successful coordination. Successful collusion that involves a large number of agents is typically achieved through a trade association, which facilitates information exchanges and sets standards (Levenstein and Suslow, 2006; Alé-Chilet and Atal, 2020). Gan and Hernandez (2013) study the lodging industry using a quarterly dataset of hotels in Texas and find that hotels located close to each other are more likely to tacit collude than isolated properties, suggesting that agglomeration facilitates tacit collusion by providing opportunities for frequent interaction and exchange of information among hotel managers. He and Huang (2017) show that institutional cross-ownership can facilitate tacit collusion and collaboration among firms in product markets. Bourveau, She and Žaldokas (2020) provide evidence for the impact of public communication through corporate disclosure on tacit collusion. Bertomeu et al. (2021) find that the Big Three U.S. automobile manufacturers use voluntary disclosures to achieve tacit collusion for the 1965-1989 period. More recently, antitrust scholars, authorities, and practitioners raised concerns about algorithmic collusion, that is, the adoption of machine learning algorithms (or, broadly, artificial intelligence) in pricing decisions promotes tacit collusion without the need of resorting to overt communications (e.g., Calvano et al., 2020; Beneke and Mackenrodt, 2021). Aryal, Ciliberto and Leyden (2022) show that legacy U.S. airlines use their quarterly earnings calls as a means of communicating and establishing collusion with each other. Goncharov and Peter (2022) find that financial reporting transparency affects industry coordination and the duration of cartels on firms indicted by the European Commission for anticompetitive behavior between 1980 and 2010. Pawliczek, Skinner and Zechman (2022) find that the U.S. firms with common ownership are more likely to disclose information for monitoring defections and sustaining collusion.

Evidence Based on Structural Estimation. Because tacit collusion is difficult to detect in the data, economists also build structural models to uncover marginal costs and provide evidence for collusive behavior. For example, Ciliberto and Williams (2014) develop a structural model in which firms operate in multiple markets that are overlapped with each other. They provide empirical evidence that multimarket contact facilitates tacit collusion among airline using the 2006-2008 data from the Airline Origin and Destination Survey database. Miller and Weinberg (2017) show that the joint venture in the beer industry may have facilitated price coordination. In the French mobile telecom industry, Bourreau, Sun and Verboven (2021) present evidence showing that the three incumbents coordinate on introducing subsidiary brands to provide low-cost services to deter entry. Through a structural model, they further show that these incumbents were able to avoid tough competition in offering product varieties until a new entrant arrived. Dou, Wang and Wang (2022) estimate that in the loan markets for distressed corporate borrowers, lenders' tacit collusion contributes to a sizable component of the risk-adjusted yield spread.

3 Supplemental Materials on the Full-Fledged Quantitative Model of Chen et al. (2022*a*)

In this appendix section, we provide supplemental information on the analysis of our quantitative model developed in Section 4 of the main text. Subsection 3.1 illustrates the model's solution by showing how firms' profit margins vary with demand. Subsection 3.2 evaluates the quantitative implications of financial contagion on profit margins and default risks. Subsection 3.3 studies the impact of the competition-distress feedback on profit margins and default risks. Subsection 3.4 illustrates firms' value and optimal leverage under different market structures. Subsection 3.5 illustrates the determination of the endogenous price-war boundary. Subsection 3.6 provides detailed discussions on the smoothing pasting conditions that characterize the endogenous default boundaries. Subsection 3.7 presents the boundary condition at $z_{i,t} = +\infty$.

3.1 Firms' Profit-Margin Decisions

Figure 1 illustrates the two firms' profit-margin decisions as a function of firm i's demand intensity $e^{z_{i,0}}$ at $t = 0$, holding $z_{j,0}$ unchanged at the initial level $e^{z_{j,0}} = 1$. Thus, as we move from the left to the right along the x-axis, the share of industry demand intensity for firm i (i.e., $e^{z_{i,0}-a_0}$) increases whereas that for firm j (i.e., $e^{z_{j,0}-a_0}$) decreases.

Both firms have higher profit margins in the collusive equilibrium (solid lines) than in the non-collusive equilibrium (dotted lines). Moreover, firm i's default boundary (represented by the vertical dotted lines) in the collusive equilibrium is lower than that in the non-collusive equilibrium (i.e., $\underline{z}_i^C(z_{j,0}) < \underline{z}_i^N(z_{j,0})$). The solid line in panel A shows that firm i reduces its profit margin when it is closer to the default boundary (i.e., the distance-to-default becomes lower as $e^{z_{i,0}}$ decreases). Moreover, its competitor, firm j, also lowers its profit margin (solid line in panel B) even though $z_{j,0}$ remains unchanged. By contrast, in the non-collusive equilibrium, firm i's profit margin increases with $e^{z_{i,0}}$ (dotted line in panel A), and firm j's profit margin decreases with $e^{z_{i,0}}$ (dotted line in panel B). In other words, both firms' non-collusive profit margins increase with their own shares of industry demand. This is a standard result in the literature: non-collusive profit margins (i.e., profit margins in the one-shot Nash equilibrium) are solely determined by the short-run price elasticity of

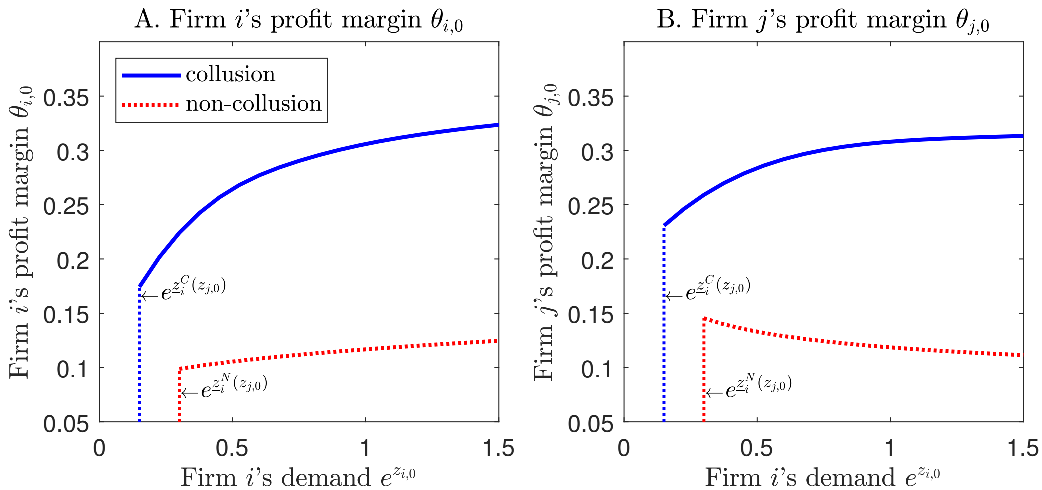

Figure 1. Firms' profit-margin decisions as a function of demand. Panels A and B plot the profit margins $(\theta_{i,0}, \theta_{j,0})$ of firms i and j, respectively, as a function of firm i's demand intensity $e^{z_{i,0}}$ at $t = 0$, holding $z_{j,0}$ unchanged at the initial level $e^{z_{j,0}} = 1$. The solid and dotted lines represent the collusive and non-collusive equilibrium. The vertical dotted lines represent the default boundaries of firm i in the collusive and (non-collusive equilibrium. The two firms' log coupon rates are set at b^C (b^N), corresponding to the optimal log coupon rate in the collusive (non-collusive) equilibrium when both firms have unit demand intensity (i.e., $e^{z_{i,0}} = e^{z_{j,0}} = 1$). Parameters are set according to Table 1 in the main text.

demand. As shown in equation (32) of the main text, firm i's short-run price elasticity of demand decreases with its share of industry demand intensity $e^{z_{i,0}-a_0}$.

3.2 Financial Contagion Through Distressed Competition

We evaluate the quantitative effect of financial contagion by studying how firm i's financial distress affects firm j's profit margin and default risk. Consider two symmetric firms with $e^{z_{i,0}} = e^{z_{j,0}} = 1$ at $t = 0$. Panel A of Figure 2 plots each firm's profit margin as a function of firm i's coupon rate e^{b_i} in the collusive equilibrium. The solid line shows that firm i reduces its profit margin $\theta_{i,0}$ as it becomes more financially distressed (indicated by a higher e^{b_i}). Due to financial contagion, firm j also reduces its profit margin $\theta_{j,0}$ when e^{b_i} increases (dashed line), even though its own coupon rate e^{b_j} does not change. The two lines intersect at the optimal equilibrium coupon rate e^{b^C} $(= e^{b_i^C} = e^{b_j^C}$ because $e^{z_{i,0}} = e^{z_{j,0}} = 1)$.

Quantitatively, when e^{b_i} increases from the optimal level $e^{b^C} = 0.016$ to 0.03 (which corresponds to an increase in firm i's leverage by about 30%), firm i's profit margin $\theta_{i,0}$ declines by about 4.8% ($\approx 26.1\% - 30.9\%$) while firm j's profit margin $\theta_{j,0}$ declines by about 4.3% ($\approx 26.6\% - 30.9\%$). Thus, the financial contagion effect of increasing e^{b_i} on $\theta_{j,0}$ is about 89.6% ($\approx 4.3/4.8$) of its direct effect on $\theta_{i,0}$. The financial contagion effect is not symmetric. When e^{b_i} decreases from the optimal level $e^{b^C} = 0.016$ to 0, both $\theta_{i,0}$ and $\theta_{j,0}$ increase slightly, indicating that financial contagion across the two firms is significant only for distressed

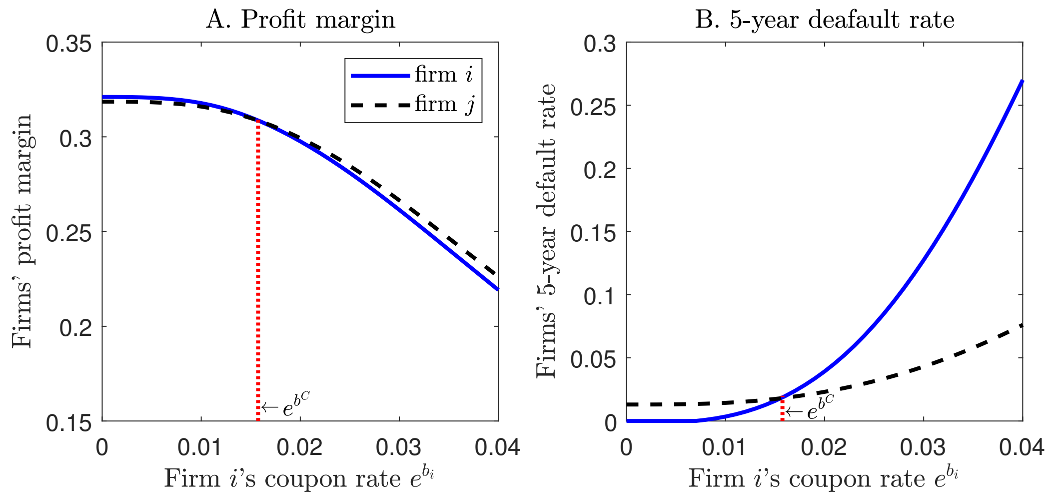

Figure 2. Financial contagion between the two firms in the same industry. Panel A plots the two firms' profit margins, $\theta_{i,0}$ and $\theta_{j,0}$, in the collusive equilibrium when firm i's coupon rate e^{b_i} varies along the x-axis, holding e^{b_j} unchanged. Panel B plots the two firms' 5-year default rates in the collusive equilibrium. The two firms' demand intensities are set at $e^{z_{i,0}} = e^{z_{j,0}} = 1$. We set $e^{b_j} = e^{b^C}$ (vertical dotted line), which corresponds to the optimal coupon rate in the collusive equilibrium when both firms have unit demand intensity (i.e., $e^{z_{i,0}} = e^{z_{j,0}} = 1$). Parameters are set according to Table 1 in the main text.

firms.

Panel B of Figure 2 plots the two firms' 5-year default rates in the collusive equilibrium. When e^{b_i} increases from the optimal level $e^{b^C} = 0.016$ to 0.03, firm i's 5-year default rate increases by about 10.8% ($\approx$ 12.8% $-$ 2.0%), which is due to both the decrease in its profit margin $\theta_{i,0}$ and the increase in its coupon rate e^{b_i}. Although firm j's coupon rate e^{b_j} remains unchanged, its 5-year default rate increases by about 2.4% ($\approx$ 4.4% $-$ 2.0%) when e^{b_i} increases from $e^{b^C} = 0.016$ to 0.03. Thus, the contagion effect on the competitor's default rate is about 22.2% ($\approx$ 2.4%/10.8%) of the direct effect on firm i's default rate; it is smaller than the contagion effect on the competitor's profit margin because firm j's default rate is determined both by its profit margin and its coupon rate. Hence, the increase in firm j's default rate is driven purely by the contagion effect that reduces firm j's profit margin, whereas firm j's coupon rate e^{b_j} remains unchanged.

Furthermore, we study the dynamic effects of financial contagion between the two firms within the same industry in Figure 3. Consider two firms with unit initial demand intensity $e^{z_{i,0}} = e^{z_{j,0}} = 1$ at $t = 0$, as in our calibration. Both firms' coupon rates are set at the optimal level e^{b^C} in the collusive equilibrium. Suppose that there is an unexpected financial distress shock that only hits firm i; the shock increases firm i's coupon rate from e^{b^C} to $e^{b_i^{shock}}$ at $t = 1$ while firm j's coupon rate remains unchanged at e^{b^C}. We choose $e^{b_i^{shock}}$ so that firm i's leverage ratio increases by 10% at $t = 1$ (dashed line in panel C of Figure 3). We compute the average dynamics of each firm's profit margin, 5-year default rate, and leverage ratios over 200,000 independent simulations so that the idiosyncratic shocks ($\mathrm{d}W_{i,t}$ and $\mathrm{d}W_{j,t}$)

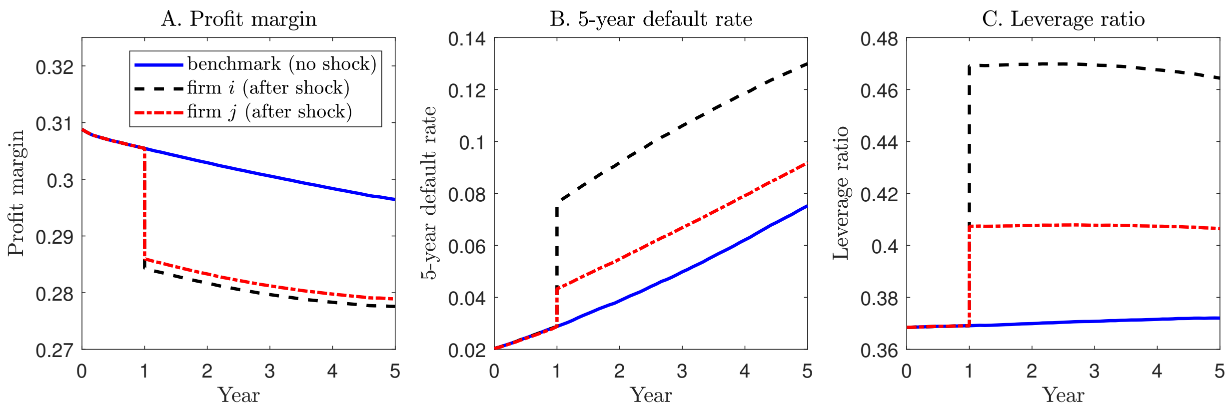

Figure 3. Dynamics of profit margins, default rates, and leverage ratios. This figure illustrates the dynamic effect of financial contagion in our full-fledged quantitative model. The solid lines in panels A, B, and C plot the average dynamics of one firm's profit margin, 5-year default rate (i.e., the expected default rate over $[t, t+5)$ conditional on not default at t), and leverage ratio, based on 200,000 independent simulations (so that idiosyncratic shocks wash out). The dashed and dash-dotted lines represent the average dynamics of firms i and j, respectively, in the scenario where firm i's log coupon rate b_i increases unexpectedly from b^C to b_i^{shock}. The value of b_i^{shock} is chosen so that firm i's leverage ratio increases by 10% at $t = 1$ (i.e., the jump size of the dashed line in panel C is 10% at $t = 1$). The initial demand intensity of the two firms is set at unit (i.e., $e^{z_{i,0}} = e^{z_{j,0}} = 1$) at $t = 0$. The two firms' coupon rates are set at e^{b^C} at $t = 0$, corresponding to the optimal coupon rate in the collusive equilibrium when both firms have unit demand intensity. Firm j's log coupon rate is fixed at b^C for $t \geq 0$ while firm i's log coupon rate is increased to b_i^{shock} at $t = 1$. Parameters are set according to Table 1 in the main text.

wash out.

The dashed and dash-dotted lines in panel A of Figure 3 plot the average dynamics of the profit margins of firms i and j, respectively. As a benchmark, the solid line represents the average dynamics of one firm's profit margin in the absence of this external financial distress shock.[2] We find that firm i's profit margin declines by about 2.1%, from 30.5% to 28.4%, in response to the increase in its coupon rate at $t = 1$. Although firm j's coupon rate remains unchanged, its profit margin also declines significantly by about 1.9%, from 30.5% to 28.6%, at $t = 1$. The contagion effect on the competitor's profit margin is about 90.5% ($\approx 1.9/2.1$) of the direct effect on firm i. The magnitude of our model-implied contagion effect is in line with the micro evidence of Dou, Johnson and Wu (2022).

Panel B of Figure 3 shows that the 5-year default of firm i increases from 2.8% to 7.6% at $t = 1$ in response to the unexpected increase in e^{b_i}. The 5-year default rate of firm j increases from 2.8% to 4.3% at $t = 1$, purely because of the significant decline in its profit margin (dash-dotted line in panel A of Figure 3). Thus, the contagion effect on the competitor's default rate is about 31.3% ($\approx (4.3\% - 2.8\%)/(7.6\% - 2.8\%)$) of the direct effect on firm i; it is smaller than the contagion effect on the competitor's profit margins for the same

[2]Because the two firms are identical at $t = 0$, their average dynamics over time are identical in the absence of the external financial distress shock. Thus, the solid line represents the average profit margin dynamics of both firm i and firm j.

reason noted for Figure 2. Panel C of Figure 3 plots the two firms' leverage ratios. Firm i's leverage ratio increases significantly by 10% at $t = 1$ through two channels: first, the increase in e^{b_i} directly increases firm i's debt value; and second, the reduction in the profit margin $\theta_{i,t}$ reduces firm i's equity value. Firm j's leverage ratio increases by 3.8% at $t = 1$ because of the decrease in its profit margin $\theta_{j,t}$ caused by intensified competition, which depresses firm j's equity value more than its debt value.

3.3 Feedback Between Competition and Financial Distress

In Subsection 3.3.1, we use the full-fledged quantitative model to quantify the competition-distress feedback effect. In Subsection 3.3.2, we show that an increase in the discount rate depresses the industry's profit margin more when firms within the industry are closer to their default boundaries, which echoes the results shown in Figure 7 in the main text.

3.3.1 Quantifying the Competition-Distress Feedback

Our model implies that increased competition leads to more financial distress, which in turn intensifies competition. In this section, we evaluate the magnitude of this competition-distress feedback at the industry level.

We focus on an industry with two identical firms (i.e., $e^{z_{i,0}} = e^{z_{j,0}} = 1$ and $b_i = b_j$). In panels A and B of Figure 4, the solid lines plot the industry's profit margin θ_0 (equal to each firm's profit margin because the firms are identical) and the firms' 5-year default rates, respectively, as a function of their coupon rates in the collusive equilibrium, holding $b_i = b_j$. It is evident that the two firms' profit margins decline and their default rates increase significantly as coupon rates increase.

To isolate the effect of financial distress on industry-level competition, the dashed line in panel A of Figure 4 plots the industry's collusive profit margin in the counterfactual scenario where firms are not allowed to default (despite negative equity values) as we vary coupon rates along the x-axis. That is, firms' default boundaries are fixed at $\underline{z}_i(z_{j,t}) = \underline{z}_j(z_{i,t}) = -\infty$, regardless of their coupon rates. In this case, firms essentially choose collusive profit margins to maximize their equity values without having the option to default and exit the industry. The dashed line is flat, implying that firms do not alter their collusive profit margins at all as coupon rates vary. This suggests that when coupon rates increase, it is the increased default/exit risk caused by financial distress, rather than the higher coupon payment (or interest expenses) per se, that causes firms to collude on lowering profit margins (i.e., the downward-sloping solid line). The gap between the solid and dashed lines quantifies the impact of financial distress on industry-level competition. When moving from $e^{b_i} = e^{b_j} = 0$ to $e^{b_i} = e^{b_j} = 0.04$, the solid lines in panels A and B jointly indicate that, on

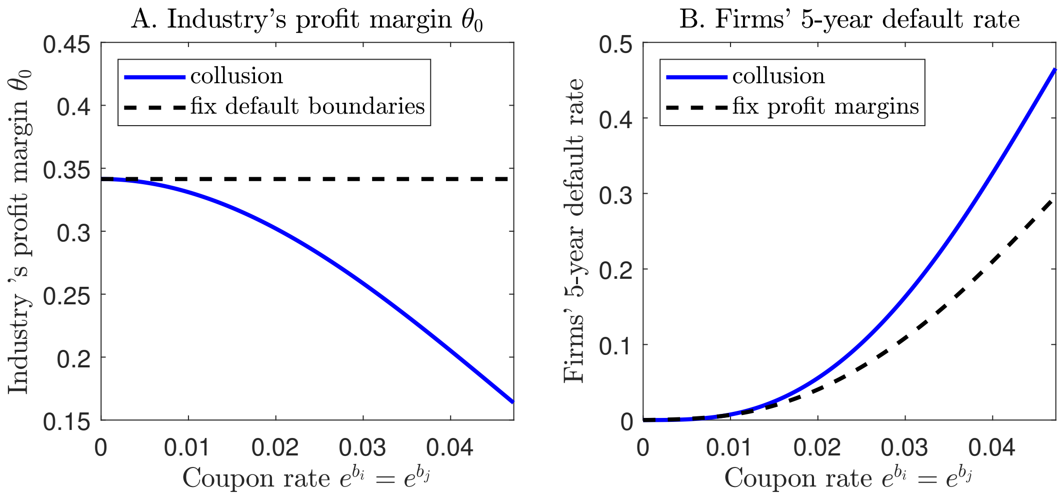

Figure 4. Positive feedback loop between competition and financial distress. We consider an industry with two identical firms (i.e., $e^{z_{i,0}} = e^{z_{j,0}} = 1$ and $b_i = b_j$). Panel A shows that financial distress can intensify competition. The solid line plots the industry's profit margin in the collusive equilibrium as a function of the two firms' coupon rates. The dashed line plots the industry's collusive profit margin in the counterfactual scenario where firms' default boundaries are fixed at $\underline{z}_i(z_{j,t}) = \underline{z}_j(z_{i,t}) = -\infty$ regardless of their coupon rates (i.e., firms are not allowed to default in this case). Panel B shows that intensified competition can exacerbate financial distress. The solid line plots the two firms' 5-year default rates in the collusive equilibrium as a function of the their coupon rates. The dashed line plots the firms' 5-year default rates in the counterfactual scenario where firms' profit margins are fixed at the levels corresponding to $e^{b_i} = e^{b_j} = 0$, regardless of their coupon rates. Parameters are set according to Table 1 in the main text.

average, firms reduce their profit margins by about 0.41% ($\approx (20.5\% - 34.1\%)/(33.0\% - 0)$) for every 1% increase in the default probability over the next 5 years.

To isolate the effect of industry-level competition on financial distress, the dashed line in panel B of Figure 4 plots the two firms' 5-year default rates in the counterfactual scenario where the level of industry-level competition does not change as we vary coupon rates along the x-axis. That is, the firms' profit margins are fixed at the levels corresponding to $e^{b_i} = e^{b_j} = 0$, regardless of their coupon rates. Therefore, the gap between the solid and dashed lines quantifies the pure impact of intensified competition (i.e., the reduction in profit margins) on firms' default risk. For example, when $e^{b_i} = e^{b_j} = 0.04$, the two firms' default rates increase by 11.9 percentage points (the gap between the two lines in panel B) because of a 13.6 percentage-point decrease in their profit margins (the gap between the two lines in panel A). Thus, on average, their 5-year default rates increase by about 0.88% ($\approx 11.9\%/13.6\%$) for every 1% decline in their profit margins.

Taking the above model-implied effects together, we can estimate the multiplier generated by the competition-distress feedback. Specifically, a 1% increase in the 5-year default rate (caused by an exogenous increase in coupon rates e^{b_i} and e^{b_j}) will increase the default rate by 0.36% ($= 0.41 \times 0.88\%$) due to the reduced profit margin, which will further reduce the profit margin and lead to a $(0.36\%)^2$ increase in the default rate, and so on. Thus,

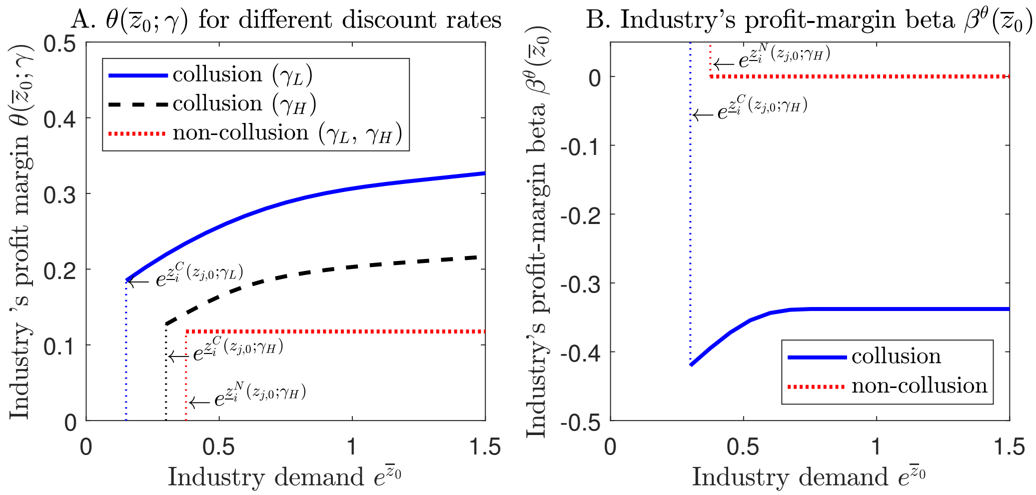

Figure 5. Sensitivity of profit margins to the discount rate. We consider an industry with two identical firms such that $z_{1,0} = z_{2,0} = \bar{z}_0$ constantly holds at $t = 0$. Panel A plots the industry's profit margin $\theta(\bar{z}_0;\gamma)$ for different discount rates γ. The solid and dashed lines represent a low γ_L and a high γ_H, respectively, in the collusive equilibrium. The dotted line represents the non-collusive equilibrium with γ_H. We set $\gamma_L = 0.4$ and $\gamma_H = 0.9$. Panel B plots the profit-margin beta to γ defined in equation (115). The solid and dotted lines represent the collusive and non-collusive equilibrium respectively. In all panels, the vertical dotted lines represent firms' default boundaries in the corresponding cases. Both firms' log coupon rates are set at b^C, corresponding to the optimal log coupon rate in the collusive equilibrium when both firms have unit demand intensity (i.e., $e^{\bar{z}_0} = 1$) and the discount rate is γ_L. Parameters are set according to Table 1 in the main text.

the multiplier of the competition-distress feedback is about 1.56 $(= 1 + 0.36/(1 - 0.36))$, implying a 56% amplification effect on firms' default risk.

3.3.2 Competition-Distress Feedback and Discount Rate

As shown by Dou, Ji and Wu (2021*a*), a rise in the discount rate intensifies industry competition, resulting in a reduction in industry-level profit margins. Thus, industry-level profit margins load negatively on the discount rate. By focusing on levered firms, our model further implies that the loadings are more negative when firms within the industry have lower distances-to-default due to a stronger competition-distress feedback effect.

We consider an industry with two symmetric firms such that $z_{1,0} = z_{2,0} = \bar{z}_0$ constantly holds. Panel A of Figure 5 plots the industry's profit margin in the collusive equilibrium of states with a low discount rate $\gamma_L = 0.4$ (solid line) and a high discount rate $\gamma_H = 0.9$ (dashed line). The industry's profit margin is lower when the discount rate is higher,[3] because a higher discount rate γ_H increases firms' impatience and leads them to focus more on short-term cash flows and less on future cooperation (Dou, Ji and Wu, 2021*a*). As

[3] Kawakami and Yoshihiro (1997) and Wiseman (2017) show that in a market with exits but no entries, firms may have less incentive to collude with each other when the discount rate is lower; instead, they may enter into a price war until only one firm remains in the industry. This is not the case in our model because we allow new firms to enter the industry.

a result, the prospect of future punishment becomes less threatening in such circumstances, and higher profit margins become more difficult to sustain. By contrast, in the non-collusive equilibrium, profit margins remain unchanged when the discount rate rises (dotted line). As discussed for Figure 1, the reason for this is that competition intensity is determined by the short-run price elasticity of demand, which remains unchanged when the firm's share of industry demand is fixed.

To illustrate how the exposure of profit margins to the discount rate varies with the distance-to-default, we calculate the industry-level profit-margin beta $\beta^{\theta}(\overline{z}_t)$ to the discount rate, defined as the percentage change in the industry's profit margin when γ increases from γ_L to γ_H:

$$\beta^{\theta}(\overline{z}_t) \equiv \theta(\overline{z}_t; \gamma_H)/\theta(\overline{z}_t; \gamma_L) - 1. \tag{115}$$

The solid line in panel B of Figure 5 shows that the profit-margin beta $\beta^{\theta}(\overline{z}_0)$ at $t = 0$ becomes increasingly negative when the industry becomes more financially distressed in the collusive equilibrium. In particular, when firms are close to the default boundary $e^{\underline{z}_i^C(z_{j,0};\gamma_H)}$, the profit-margin beta is -0.42, indicating that the industry's profit margin decreases by 8.4% ($\approx 0.42 \times 0.1/(\gamma_H - \gamma_L)$, with $\gamma_H = 0.9$ and $\gamma_L = 0.4$) or 1.8 percentage point ($\approx 8.4\% \times \theta(\overline{z}_0; \gamma_L)$, with the industry's collusive profit margin $\theta(\overline{z}_0; \gamma_L) = 22\%$) in response to a 0.1 increase in the discount rate γ. This is because, when the distance-to-default is low, the endogenously intensified competition caused by a higher discount rate is further amplified by the competition-distress feedback loop, dramatically amplifying the industry's exposure to the discount rate. By contrast, the profit-margin beta is always zero in the non-collusive equilibrium (dotted line).

3.4 Optimal Leverage Under Different Market Structures

In this subsection, we quantitatively evaluate the optimal leverage ratios of firms under different market structures

Consider two firms with unit demand intensity $e^{z_{i,0}} = e^{z_{j,0}} = 1$ at $t = 0$. As a benchmark, the dashed line in each panel of Figure 6 plots firm i's value ($V_{i,0}^N = E_{i,0}^N + D_{i,0}^N$) as a function of its leverage ratio ($D_{i,0}^N/V_{i,0}^N$) in the non-collusive equilibrium, where the variation in firm i's leverage ratio is generated by the variation in firm i's coupon rate e^{b_i}, holding firm j's coupon rate e^{b_j} fixed at e^{b^N}, which is the optimal coupon rate in the non-collusive equilibrium when both firms have unit demand intensity (i.e., $e^{z_{i,0}} = e^{z_{j,0}} = 1$).

In panel A of Figure 6, the solid line plots firm i's value ($V_{i,0}^C$) as a function of its leverage ratio ($D_{i,0}^C/V_{i,0}^C$) in the collusive equilibrium, where the variation in firm i's leverage ratio is generated by the variation in firm i's coupon rate e^{b_i}, holding firm j's coupon rate e^{b_j} fixed at e^{b^C}, which is the optimal coupon rate in the collusive equilibrium when both firms

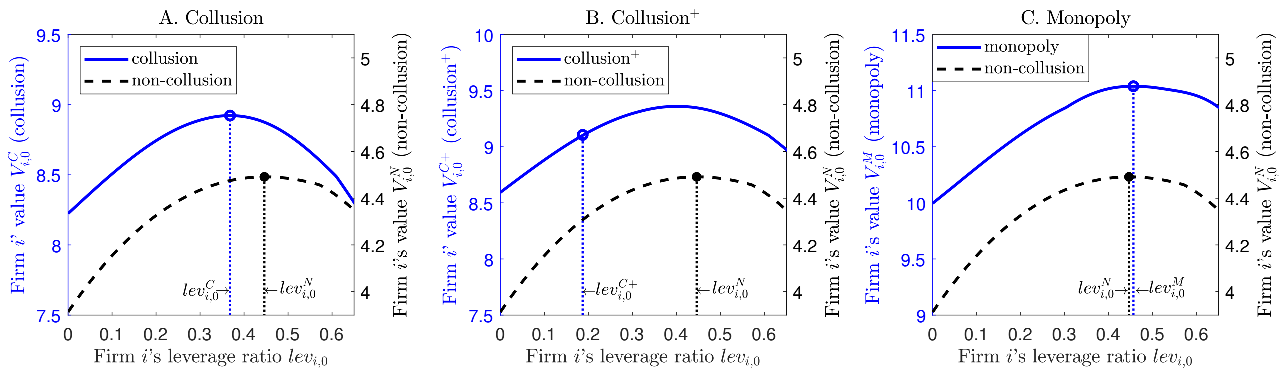

Figure 6. Implications of market structures on optimal leverage ratios. Panel A compares firm i's value as a function of its leverage ratio in the collusive and non-collusive equilibrium. The two firms' demand intensity is set at $e^{z_{i,0}} = e^{z_{j,0}} = 1$ at $t = 0$ and firm j's coupon rate e^{b_j} is fixed at the optimal value in the collusive or non-collusive equilibrium when both firms have unit demand intensity. The variation in firm i's leverage ratio along the x-axis is generated by the variation in firm i's coupon rate e^{b_i}. Panel B compares the collusive$^+$ and non-collusive equilibrium. Panel C compares the monopoly market structure and the non-collusive equilibrium of the duopoly market structure. The vertical dotted lines represent the optimal leverage ratios corresponding to each case. Parameters are set according to Table 1 in the main text.

have unit demand intensity (i.e., $e^{z_{i,0}} = e^{z_{j,0}} = 1$). The vertical dotted line indicates that the optimal leverage ratio in the non-collusive equilibrium is $lev_{i,0}^N = 44.6\%$, about 7.8% ($\approx 44.6\% - 36.8\%$) higher than that ($lev_{i,0}^C = 36.8\%$) in the collusive equilibrium (vertical dotted line) due to the absence of competition-distress feedback.

In panel B of Figure 6, the solid line focuses on the collusive$^+$ equilibrium, where we vary firm i's coupon rate e^{b_i} to generate variations in its leverage ratio along the x-axis, holding firm j's coupon rate e^{b_j} fixed at $e^{b^{C+}}$, the optimal coupon rate that maximizes the industry's total value under tacit collusion on profit margins when both firms have unit demand intensity (i.e., $e^{z_{i,0}} = e^{z_{j,0}} = 1$). The vertical dotted line represents firm i's leverage ratio, $lev_{i,0}^{C+} = 18.7\%$, when its coupon rate is $e^{b^{C+}}$, which is the value that maximizes the industry's total value. It is evident that $lev_{i,0}^{C+}$ is much lower than $lev_{i,0}^C$ in panel A of Figure 6 for the reasons discussed for panel B of Figure 11 in the main text.

In panel C of Figure 6, the solid line plots firm i's value as a function of its leverage ratio when it is the only firm in the industry, i.e., firm i is the monopoly in the industry. There is no competition-distress feedback under either the monopoly market structure (solid line) or the non-collusive equilibrium of the duopoly market structure (dashed line). Thus, the optimal leverage ratios in both cases are greater than under the collusive equilibrium of the duopoly market structure (solid line in panel A of Figure 6). The optimal leverage ratio under the monopoly market structure is $lev_{i,0}^M = 45.3\%$, which is a little higher than that ($lev_{i,0}^N = 44.6\%$) under the non-collusive equilibrium of the duopoly market structure because cash flow volatility is lower under the monopoly. The firm's profit margin is constant under the monopoly market structure, but it fluctuates in the non-collusive

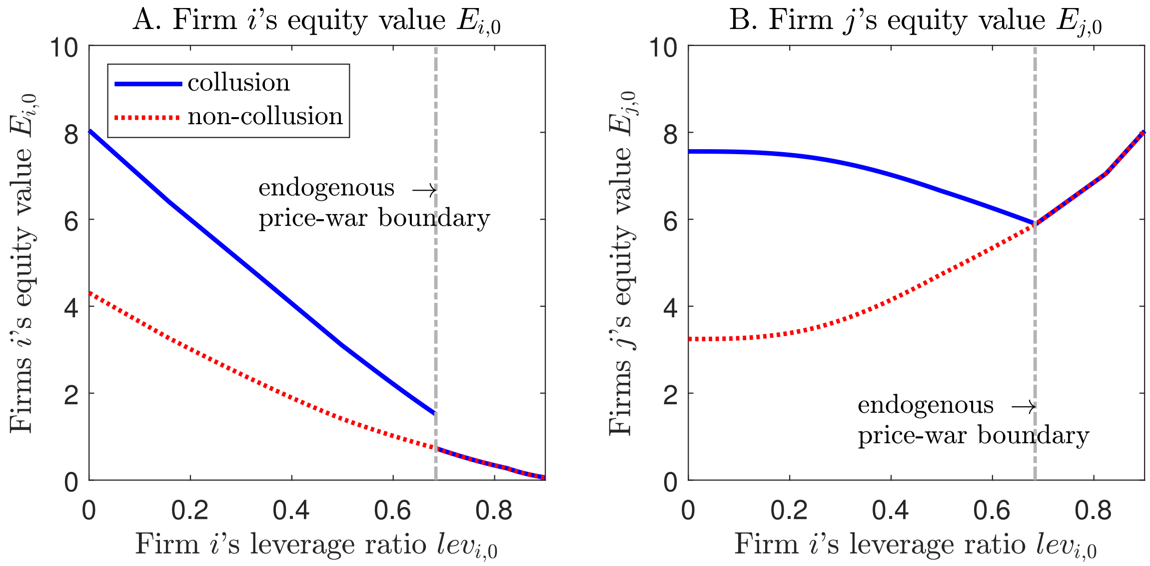

Figure 7. Illustration of the price-war boundary in the industry with $\kappa = 0$. Consider the industry with $\kappa = 0$. We consider firms with unit demand intensity (i.e., $e^{z_{i,0}} = e^{z_{j,0}} = 1$) and we set firm j's coupon rate at $e^{b_j} = e^{b^C}$, which corresponds to the optimal coupon rate in the collusive equilibrium when both firms have unit demand intensity. By varying firm i's coupon rate e^{b_i}, panel A plots firm i's equity value as a function of its own leverage ratio and panel B plots firm j's equity value as a function of firm i's leverage ratio. The solid and dotted lines represent the collusive and non-collusive equilibrium, respectively. The vertical dash-dotted line represents the boundary of the endogenous jump (i.e., the switch between collusion and non-collusion), where firm j's collusive value equals its non-collusive value (see panel B). Parameters are set according to Table 1 in the main text.

equilibrium of the duopoly market structure depending on the two firms' shares of industry demand.

3.5 Determination of the Price-War Boundary

In this section, we illustrate how the price-war boundary in Figure 8 of the main text is determined. Specifically, we consider an industry with $\kappa = 0$ and two firms with unit demand intensity (i.e., $e^{z_{i,0}} = e^{z_{j,0}} = 1$) at $t = 0$. We set firm j's coupon rate at $e^{b_j} = e^{b^C}$, which corresponds to the optimal coupon rate in the collusive equilibrium when both firms have unit demand intensity. In Figure 7, by varying firm i's coupon rate e^{b_i}, panel A plots firm i's equity value as a function of its leverage ratio and panel B plots firm j's equity value as a function of firm i's leverage ratio. The solid and dotted lines represent the collusive and non-collusive equilibrium, respectively.

In panel B of Figure 7, firm j's equity value in the collusive equilibrium (solid line) intersects with that in the non-collusive equilibrium (dotted line) when firm i's leverage ratio is about 69.1% (vertical dash-dotted line). This is the critical point when firm j's PC constraint (equation (40) in the main text) becomes binding. When firm i's coupon rate e^{b_i} is lower than the level indicated by the vertical dash-dotted line, both firms' PC constraints are always satisfied and not binding for the collusive profit-margin schemes that satisfy their IC constraints (equation (41) in the main text). Thus, both firms choose to collude with each other. However, when firm i's coupon rate e^{b_i} is above the level indicated by the vertical dash-dotted line, the PC constraint (equation (40) in the main text) becomes binding for firm j. This implies that if the two firms were to collude, the PC constraint of firm j would be violated even though the two firms' IC constraints (equation (41) in the main text) would be honored.

By contrast, panel A of Figure 7 shows that firm i's equity value in the collusive equilibrium is strictly higher than that in the non-collusive equilibrium when its leverage ratio is below the level indicated by the vertical dash-dotted line, indicating that firm i would always want to collude with firm j. Only when its leverage ratio is above the level indicated by the vertical dash-dotted line is firm i's equity value in the collusive equilibrium equal to that in the non-collusive equilibrium; in this region, as noted above firm j chooses not to collude. At the level indicated by the vertical dash-dotted line, there is an endogenous jump in firm i's equity value. Therefore, our model implies that it is the firm with a stronger financial condition (i.e., a lower leverage ratio) that wants to abandon collusion and wage a price war (which is firm j in this example).

3.6 Endogenous Default Boundary Conditions

The state space is $(z_1, z_2) \in \mathcal{Z} \subset \mathbb{R}^2$. We denote the default boundary of firm $i \in \{1, 2\}$ in the non-collusive equilibrium by $\mathcal{T}_i^N$, which is a 1-dimensional manifold in $\mathbb{R}^2$. The idea is the same for the default boundaries in the collusive equilibrium and in the characterization of the deviation value. Put simply, the default boundary $\mathcal{T}_i^N$ is a "curve" in $\mathbb{R}^2$. In general, the value matching condition for the default boundary of firm i is

$$E_i^N(z_1, z_2)\Big|_{(z_1,z_2)\in\mathcal{T}_i^N} = 0, \quad \text{for } i \in \{1, 2\}, \tag{116}$$

and the smooth pasting conditions are

$$\frac{\partial}{\partial z_i} E_i^N(z_1, z_2)\Bigg|_{(z_1,z_2)\in\mathcal{T}_i^N} = 0 \quad \text{and} \quad \frac{\partial}{\partial z_j} E_i^N(z_1, z_2)\Bigg|_{(z_1,z_2)\in\mathcal{T}_i^N} = 0, \quad \text{for } i \neq j \in \{1, 2\}. \tag{117}$$

For the default boundary $\mathcal{T}_i^N$, we can conveniently parameterize it using z_j because there is always a threshold point $\underline{z}_i^N(z_j)$ below which the shareholders of firm i would choose to default on the debt for any $z_j \in \mathbb{R}$. Thus, the default boundary $\mathcal{T}_i^N$ can naturally be represented by the function $\underline{z}_i^N(z_j)$. As a result, the value matching condition in (116) can be rewritten as

$$E_i^N(z_1,z_2)\Big|_{z_i=\underline{z}_i^N(z_j)} = 0, \ \ \forall\, z_j, \ \ \text{for } i \neq j \in \{1,2\}, \tag{118}$$

and the smooth pasting conditions in (117) can be rewritten as

$$\frac{\partial}{\partial z_i}E_i^N(z_1,z_2)\Big|_{z_i=\underline{z}_i^N(z_j)} = 0 \quad \text{and} \quad \frac{\partial}{\partial z_j}E_i^N(z_1,z_2)\Bigg|_{z_i=\underline{z}_i^N(z_j)} = 0, \ \ \forall\, z_j, \ \ \text{for } i \neq j \in \{1,2\}. \tag{119}$$

Obviously, because the specific structure of the default boundary, the second smooth pasting condition in (119) is redundant given that the first smooth pasting condition in (119) and the value matching condition in (118) hold simultaneously for arbitrary $z_j \in \mathbb{R}$. More precisely, according to the chain rule, the value matching condition in (118) implies that

$$\frac{\partial}{\partial z_i}E_i^N(z_1,z_2)\Big|_{z_i=\underline{z}_i^N(z_j)} \times \frac{\partial}{\partial z_j}\underline{z}_i^N(z_j) \ + \ \frac{\partial}{\partial z_j}E_i^N(z_1,z_2)\Bigg|_{z_i=\underline{z}_i^N(z_j)} = 0, \ \forall\, z_j, \ \ \text{for} \ \ i \neq j \in \{1,2\}, \tag{120}$$

where the first term captures the indirect effect of a small local change of z_j on $E_i^N(z_1,z_2)$ through its impact on the default boundary $\underline{z}_i^N(z_j)$, and the second term captures the direct effect of a small local change of z_j on $E_i^N(z_1,z_2)$; further, the relation established in (120) in turn leads to

$$\frac{\partial}{\partial z_j}E_i^N(z_1,z_2)\Bigg|_{z_i=\underline{z}_i^N(z_j)} = 0, \ \ \forall\, z_j, \ \ \text{for } i \neq j \in \{1,2\}, \tag{121}$$

because the first smooth pasting condition in (119) implies the following equality:

$$\frac{\partial}{\partial z_i}E_i^N(z_1,z_2)\Big|_{z_i=\underline{z}_i^N(z_j)} \times \frac{\partial}{\partial z_j}\underline{z}_i^N(z_j) = 0, \ \forall\, z_j, \ \ \text{for} \ \ i \neq j \in \{1,2\}. \tag{122}$$

Although the state space is 2-dimensional (i.e., there are two state variables z_1 and z_2), there is only one "free boundary" to be characterized, which can be parameterized by one state variable. In general, the free boundary of an optimal stopping problem with K state variables where $K \geq 2$ (i.e., a free boundary optimal control problem with K state variables), as a $(K-1)$-dimensional manifold in the K-dimensional state space, often cannot be parameterized by any $(K-1)$-subset of the state variables; in such cases, smooth pasting conditions in all different directions are needed to describe the free boundary conditions properly, similar in spirit to (116) and (117). For example, Chao and Ward (2022), Chen et al. (2022*b*), and Kakhbod et al. (2022) consider corporate liquidity models with multiple state variables in which there is an economic boundary condition for each of the state variables.

Their free boundary optimal control problems, by nature, are more complex than ours here, and as a result, they cannot simplify the smooth pasting conditions by ignoring some redundant ones, similar to what we do.

3.7 Boundary Conditions at Infinity

When $z_{i,t} = +\infty$, firm i essentially monopolizes the industry with negligible financial leverage because its competitor, firm j, has a negligible size regardless of its $z_{j,t}$. Thus, the boundary condition of firm i's equity value at $z_{i,t} = +\infty$ should satisfy

$$\lim_{z_{i,t}\to\infty} \frac{\partial}{\partial z_{i,t}} E_i^N(z_t) = \lim_{z_{i,t}\to\infty} \frac{\partial}{\partial z_{i,t}} E_i^C(z_t) = \lim_{z_{i,t}\to\infty} \frac{\partial}{\partial z_{i,t}} E_i^D(z_t) = \lim_{z_{i,t}\to\infty} \frac{\partial}{\partial z_{i,t}} U_i(z_{i,t}), \tag{123}$$

where $U_i(z_{i,t})$ is the equity value of an unlevered monopoly firm with $z_{i,t} \equiv a_t$ and price $P_{i,t} \equiv P_t$. In this industry, the demand curve facing the monopoly firm is given by equation (28) of the main text, i.e.,

$$\widetilde{C}_{i,t} \equiv \widetilde{C}_t = e^{z_{i,t}} P_{i,t}^{-\epsilon}, \tag{124}$$

and the evolution of $z_{i,t}$ is given by equation (34) of the main text,

$$e^{-z_{i,t}} \mathrm{d}e^{z_{i,t}} = (g - \varsigma\gamma)\mathrm{d}t + \varsigma \mathrm{d}W_t^{\mathbb{Q}} + \sigma \mathrm{d}W_{i,t} - \mathrm{d}J_{i,t}. \tag{125}$$

Thus, the HJB equation that determines $U(z_{i,t})$ can be written as

$$r_f U_i(z_{i,t})\mathrm{d}t = \max_{\theta_{i,t}} \quad (1-\tau)\left[\omega^{1-\epsilon}\theta_{i,t}(1-\theta_{i,t})^{\epsilon-1}e^{z_{i,t}} - e^{b_i}\right]\mathrm{d}t + \mathbb{E}_t^{\mathbb{Q}}\left[\mathrm{d}U_i(z_{i,t})\right]. \tag{126}$$

4 Full-Fledged Quantitative Model with Aggregate Shocks

In this appendix section, we extend the full-fledged quantitative model in Section 4 of the main text to incorporate time-varying discount rates to elucidate the asset pricing implications of the distressed competition mechanism. By calibrating this full-fledged quantitative model with aggregate shocks, we show that the distressed competition mechanism can help rationalize the industry-level financial distress anomaly, a puzzling empirical pattern inconsistent with the implications of the canonical frameworks of Merton (1974) and Leland (1994).

4.1 Time-Varying Discount Rates

We extend the full-fledged quantitative model in Section 4 of the main text by introducing time-varying discount rates. The countercyclical risk premium crucially allows Leland-type models to quantitatively reconcile the joint patterns of low leverage, high credit spread, and low default frequency (e.g., Chen, Collin-Dufresne and Goldstein, 2008; Chen, 2010). Motivated by the literature, we specify the stochastic discount factor (SDF) Λ_t as

$$\frac{\mathrm{d}\Lambda_t}{\Lambda_t} = -r_f \mathrm{d}t - \gamma_t \mathrm{d}W_t - \zeta \mathrm{d}Z_{\gamma,t}, \tag{127}$$

where W_t and $Z_{\gamma,t}$ are independent standard Brownian motions, r_f is the risk-free rate, and γ_t is the time-varying market price of risk evolving as follows:

$$\mathrm{d}\gamma_t = -\varphi(\gamma_t - \overline{\gamma})\mathrm{d}t - \pi \mathrm{d}Z_{\gamma,t} \quad \text{with} \quad \varphi, \overline{\gamma}, \pi > 0. \tag{128}$$

Our specification for the time-varying discount rate γ_t follows the literature on cross-sectional return predictability (e.g., Lettau and Wachter, 2007; Belo and Lin, 2012; Dou, Ji and Wu, 2021*a*). We assume that $\zeta > 0$ to capture the well-documented countercyclical market price of risk. The primitive economic mechanism driving the countercyclical market price of risk could be, e.g., time-varying risk aversion, as in Campbell and Cochrane (1999).

Valuation of Equity and Debt. In the full-fledged quantitative model with aggregate shocks, the market price of risk γ_t is also a state variable. Thus, we define $x_t \equiv \{z_{1,t}, z_{2,t}, \gamma_t\}$. The shareholder values determined by equations (38), (42), (44) of the main text can be equivalently written under the physical measure, which uses SDF Λ_t to discount future cash flows:

$$0 = \max_{\theta_{i,t}} \; (1-\tau)[\Pi_i(\theta_{i,t}, \theta_{j,t}^N)e^{z_{i,t}} - e^{b_i}]\mathrm{d}t + \Lambda_t^{-1}\mathbb{E}_t\left[\mathrm{d}(\Lambda_t E_i^N(x_t))\right], \tag{129}$$

$$0 = (1-\tau)[\Pi_i(\theta_{i,t}^C, \theta_{j,t}^C)e^{z_{i,t}} - e^{b_i}]\mathrm{d}t + \Lambda_t^{-1}\mathbb{E}_t\left[\mathrm{d}(\Lambda_t E_i^C(x_t))\right], \tag{130}$$

$$0 = \max_{\theta_{i,t}} \; (1-\tau)[\Pi_i(\theta_{i,t}, \theta_{j,t}^C)e^{z_{i,t}} - e^{b_i}]\mathrm{d}t - \xi\left[E_i^D(x_t) - E_i^N(x_t)\right]\mathrm{d}t + \Lambda_t^{-1}\mathbb{E}_t\left[\mathrm{d}(\Lambda_t E_i^D(x_t))\right]. \tag{131}$$

To generate reasonable credit spreads across industries that differ in the level of idiosyncratic left-tail risk (captured by intensity ν), we assume that when the idiosyncratic left-tail jump shock hits firm i over $[t, t+\mathrm{d}t)$ (i.e., $\mathrm{d}J_{i,t} = 1$), two outcomes are possible. With probability ϖ, the firm is wiped out with no debt recovery. With probability $1-\varpi$, the firm is restructured by the new entrant firm; old debt is retired at par value $D_i^C(x_{t_0})$, and new debt is optimally issued, where $D_i^C(x_{t_0})$ is the par value of the debt issued at the initial time t_0, when firm i enters the industry. Under this assumption, the debt value determined by equation (46) of the main text is modified to

$$\left[\varpi D_i^C(x_t) + (1-\varpi)(D_i^C(x_t) - D_i^C(x_{t_0}))\right] \nu \mathrm{d}t = e^{b_i}\mathrm{d}t + \Lambda_t^{-1}\mathbb{E}_t\left[\mathrm{d}(\Lambda_t D_i^C(x_t))\right], \tag{132}$$

The right-hand side of equation (132) is the expected gain of debtholders if the jump shock does not occur over $[t, t+\mathrm{d}t)$, and the left-hand side is the expected loss of debtholders due to the idiosyncratic left-tail jump shock. If we set $\omega = 1$, equation (132) becomes identical to equation (46) of the main text, except for characterizing $D_i^C(x_t)$ under the physical measure and expressing the intensity of idiosyncratic left-tail jump shock ν explicitly in the equation.

The boundary condition at $z_{i,t} = \infty$ (equation (47) of the main text) is modified to

$$\lim_{z_{i,t}\to+\infty} D_i^C(x_t) = \frac{e^{b_i} + (1-\varpi)\nu D_i^C(x_{t_0})}{r_f + \nu}, \tag{133}$$

which is the value of a bond with a constant coupon rate e^{b_i} until it retires at the par value with rate $(1-\varpi)\nu$ or it defaults with rate $\varpi\nu$, whichever occurs first.

4.2 Calibration and Parameter Choices

We set the within-industry elasticity of substitution at $\eta = 15$, the cross-industry price elasticity of demand at $\epsilon = 2$, the corporate tax rate at $\tau = 27\%$, the expected growth rate at $g = 1.89\%$, and the initial size of new entrants to be a fraction $\kappa = 0.3$ of the incumbent's size. The two firms' initial demand intensity is normalized to unity, i.e., $e^{z_{i,0}} = e^{z_{j,0}} = 1$. The persistence of the market price of risk is set at $\varphi = 0.13$ as in Campbell and Cochrane (1999) and the magnitude of shocks is set at $\pi = 0.12$, following Lettau and Wachter (2007).

The remaining parameters are calibrated by matching the relevant moments summarized in Table 1. We set the ex-post bond recovery rate at $\delta = 40\%$ so that the model-implied average leverage ratio is 33%. The volatility of idiosyncratic shocks is $\sigma = 25\%$ which generates a 5-year default rate of 2.5%. The marginal cost of production is set at $\omega = 2$ to match the average net profitability. We set the punishment rate at $\xi = 9\%$ so that the average gross profit margin is consistent with the data. We set $\zeta = 0.45$, $\overline{\gamma} = 0.15$, and $\varsigma = 4\%$ so that the market portfolio's equity premium is 7.27%, the Sharpe ratio is 0.42, and the credit spread is 1.63%, similar to those in the data. We calibrate the intensity of idiosyncratic left-tail jump shocks to match the difference in stock returns and credit spreads across industries sorted on the financial distress measure, $Distress_{i,t}$, constructed

Table 1: Targeted moments in the full-fledged quantitative model with aggregate shocks.

Parameter	Symbol	Value	Moments	Data	Model
Recovery ratio of assets	δ	40%	Average leverage ratio (Baa rated)	34%	33%
Volatility of idiosyncratic shocks	σ	25%	5-year default rate (Baa rated)	2.2%	2.5%
Marginal cost of production	ω	2	Average net profitability	3.9%	3.6%
Punishment rate	ξ	9%	Average gross profit margin	31.4%	26.5%
Mean market price of risk for Z_t	$\overline{\gamma}$	0.15	Market equity premium	7.36%	7.27%
Volatility of aggregate shocks	ς	4%	Market Sharpe ratio	0.40	0.42
Market price of risk for $Z_{\gamma,t}$	ζ	0.45	Credit spread (Baa-rated)	1.77%	1.63%
Idiosyncratic left-tail risk	$[\underline{\nu}, \overline{\nu}]$	$[0, 0.15]$	Diff. in excess returns (Q5−Q1)	−4.94%	−5.01%
Default rate upon jump shocks	ϖ	10%	Diff. in credit spreads (Q5−Q1)	2.35%	2.05%

in Online Appendix 6.1. In particular, we assume that the intensity of idiosyncratic left-tail jump shocks ν ranges from $\underline{\nu}$ to $\overline{\nu}$. We discretize $[\underline{\nu}, \overline{\nu}]$ into $N = 10$ grids with equal spacing so that $\nu_1 = \underline{\nu}$ and $\nu_N = \overline{\nu}$. The mass of industries associated with each value of ν is the same. We normalize $\underline{\nu} = 0$ and set $\overline{\nu} = 0.15$ to generate a stock-return difference of -5.01% across quintile portfolios of industries sorted on financial distress (Q5−Q1). We set $\varpi = 10\%$ to generate a credit spread difference of 2.05%.

4.3 Financial Distress Anomaly Across Industries

We now quantitatively examine the asset pricing implications of the endogenous distressed competition mechanism. Specifically, we show that our model can quantitatively rationalize the financial distress anomaly across industries: more financially distressed industries have lower expected equity excess returns and higher credit spreads.

In the data, we sort all SIC4 industries into quintiles based on the industry-level financial distress measure $Distress_{i,t}$ (constructed in Online Appendix 6.1). Table 2 shows that the differences in expected excess returns and credit spreads between quintile portfolios of industries sorted on financial distress (Q5−Q1) are -4.94% and 2.35%, respectively. We perform a similar portfolio-sorting analysis in the simulated data generated by our full-fledged quantitative model with aggregate shocks. The model-implied patterns are quantitatively consistent with the data. The difference in idiosyncratic left-tail risk, captured by ν, is the primary force causing the difference in financial distress across industries. Thus, sorting industries by financial distress in the model captures the cross-industry variation in idiosyncratic left-tail risk, thereby generating lower expected equity excess returns for industries that are more financially distressed. Moreover, industries with higher financial distress have higher credit spreads because they have higher idiosyncratic left-tail risk and

Table 2: Industry portfolios sorted on financial distress in the model and data.

	Data			Model		
	Q1 (low)	Q5	Q5−Q1	Q1 (low)	Q5	Q5−Q1
Equity excess return (%)	9.51	4.57	−4.94	9.88	4.87	−5.01
	[5.82, 13.16]	[−0.49, 9.64]	[−9.18, −0.69]			
Equity beta to discount rate	−5.51	4.57	10.08	−6.09	7.06	13.15
	[−9.90, −1.13]	[−1.60, 10.74]	[1.75, 18.41]			
Credit spread (%)	0.99	3.34	2.35	0.77	2.82	2.05
	[0.79, 1.20]	[2.51, 4.16]	[1.72, 2.98]			
5-year default rate (%)	0.52	5.57	5.05	0.02	7.57	7.55
	[0.25, 0.80]	[3.96, 7.19]	[3.50, 6.60]			
Leverage ratio (%)	22.59	33.56	10.97	27.96	37.87	9.91
	[20.62, 24.56]	[31.70, 35.43]	[9.20, 12.75]			

Note: The sample period of the data is from 1975 to 2021, except for the credit spread data, which is from 1988 to 2018. More information is presented in Table 8 of Online Appendix 6.4. The default event is defined in Online Appendix 6.1. The industry-level default rate is the average default rate of the top six firms in the industry. The leverage ratio is total short-term debt plus total long-term debt divided by total assets. The estimate of the equity beta to the discount rate controls for market returns. The 95% confidence intervals are reported in brackets. In the model, financial distress is measured by the 1-year default probability as in the data.

thus a higher default probability.

Table 2 also shows that, in both the data and the model, the portfolio with lower financial distress (Q1) is more negatively exposed to the discount rate than that with higher financial distress (Q5) after controlling for market portfolio returns. This explains why less financially distressed industries are associated with higher expected equity excess returns. The model implies that the 5-year default rate is about 0.02% for Q1, which is significantly lower than 7.57% for Q5. Similar patterns for default rates are observed in the data.

The model implies that more financially distressed industries (Q5) have higher leverage ratios than less financially distressed industries (Q1) because firms in the former set of industries optimally choose higher financial leverage ex ante. In our model, industries with higher financial distress are associated with higher idiosyncratic left-tail risk, resulting in a lower capacity for collusion. This lower capacity for collusion leads to a higher leverage ratio through two channels. First, as shown in Proposition 3.4 and Figure 10 in the main text, when firms have weaker collusion incentives, the competition-distress feedback becomes weaker, implying that it is less costly for firms to raise debt. Thus, firms optimally choose higher leverage ratios in equilibrium. Second, when their capacity for collusion is weaker, firms collude on lower profit margins, which are less exposed to fluctuations in the aggregate discount rate (Dou, Ji and Wu, 2021*a*,*b*). Thus, from the perspective of shareholders, the default risk caused by aggregate discount-rate shocks is lower, which motivates them to increase leverage ratios. We emphasize that the lower default risk caused by aggregate discount-rate shocks does not contradict the higher 5-year default rate in these industries. This is because a large fraction of default events is caused by idiosyncratic left-tail jump shocks due to the higher ν of these industries rather than by the volatile systematic component in cash flows. When idiosyncratic left-tail jump shocks hit, firms default with constant probability ϖ, regardless of their current financial leverage. Thus, choosing higher financial leverage ex ante does not exacerbate the default risk attributed to idiosyncratic left-tail jump shocks.

4.4 Decomposition of the Risk Premium

Because the state variable is $x_t = \{z_{1,t}, z_{2,t}, \gamma_t\}$, firm i is affected by the aggregate shocks in $z_{1,t}$, $z_{2,t}$, and γ_t. The equity return of firm i in the collusive equilibrium satisfies the following equation:

$$1 = \mathbb{E}_t \left[\frac{\Lambda_{t+\mathrm{d}t}}{\Lambda_t} (1 + \mathrm{d}R_{i,t}) \right], \tag{134}$$

where $\mathrm{d}R_{i,t}$ is firm i's equity return over $[t, t + \mathrm{d}t)$,

$$\mathrm{d}R_{i,t} = \frac{(1-\tau)[\Pi_i(\theta^C_{i,t}, \theta^C_{j,t})e^{z_{i,t}} - e^{b_i}]\mathrm{d}t + \mathrm{d}E^C_{i,t}}{E^C_{i,t}}. \tag{135}$$

Substituting equation (33) in the main text, equations (127), (128), and (135) into (134), and omitting higher-order terms, we obtain

$$\begin{aligned}
\mathbb{E}_t \left[\mathrm{d}R_{i,t} - r_f \mathrm{d}t \right] =& \mathbb{E}_t \left[(\gamma_t \mathrm{d}W_t + \zeta \mathrm{d}Z_{\gamma,t}) \mathrm{d} \ln E^C_{i,t} \right] \\
=& \mathbb{E}_t \left[(\gamma_t \mathrm{d}W_t + \zeta \mathrm{d}Z_{\gamma,t}) \left(\frac{\partial \ln E^C_{i,t}}{\partial \ln e^{z_{i,t}}} \frac{\mathrm{d}e^{z_{i,t}}}{e^{z_{i,t}}} + \frac{\partial \ln E^C_{i,t}}{\partial \ln e^{z_{j,t}}} \frac{\mathrm{d}e^{z_{j,t}}}{e^{z_{j,t}}} + \frac{\partial \ln E^C_{i,t}}{\partial \gamma_t} \mathrm{d}\gamma_t \right) \right] \\
=& \left(\frac{\partial \ln E^C_{i,t}}{\partial \ln e^{z_{i,t}}} + \frac{\partial \ln E^C_{i,t}}{\partial \ln e^{z_{j,t}}} \right) \varsigma \gamma_t \mathrm{d}t - \frac{\partial \ln E^C_{i,t}}{\partial \gamma_t} \zeta \pi \mathrm{d}t.
\end{aligned} \tag{136}$$

Equation (136) decomposes the conditional expected equity excess return of firm i into two terms. The first term captures the risk premium attributed to the exposure to the cash-flow shock W_t (which affects both $e^{z_{i,t}}$ and $e^{z_{j,t}}$), and the second-term captures the risk premium attributed to the exposure to the discount-rate shock $Z_{\gamma,t}$ (i.e., the shock in the market price of risk γ_t). Although both contribute to firm i's expected equity excess returns, we show below that it is the differential exposure to the discount-rate shock $Z_{\gamma,t}$ that generates the financial distress anomaly across industries in our full-fledged quantitative model with aggregate shocks.

Figure 8 illustrates the decomposition of a firm's conditional expected equity excess returns according to equation (136). In particular, we consider two industries with different exposures to idiosyncratic left-tail risk, as captured by the parameter ν. We set $\gamma_0 = \overline{\gamma}$ and $e^{z_{j,0}} = 1$ at $t = 0$. The two firms' coupon rates are set at the optimal level corresponding to the collusive equilibrium of each industry in which both firms have unit demand intensity (i.e., $e^{z_{i,0}} = e^{z_{j,0}} = 1$) and the discount rate is $\overline{\gamma}$. Panel A of Figure 8 plots the term $\left.\frac{\partial \ln E^C_{i,t}}{\partial \ln e^{z_{i,t}}}\right|_{t=0} \varsigma\gamma_0$ in equation (136), capturing the contribution of shocks in $e^{z_{i,t}}$ at $t = 0$, as a function of firm i's demand intensity $e^{z_{i,0}}$. The solid and dashed lines represent the industries with $\nu = 0$ and $\nu = 0.15$, respectively. In both industries, firms' conditional expected equity excess returns attributed to shocks in $e^{z_{i,t}}$ decrease with $e^{z_{i,0}}$ because a higher $e^{z_{i,0}}$ implies a lower leverage ratio. Comparing the two curves, we see that, in general, the conditional expected excess return attributed to shocks in $e^{z_{i,t}}$ in the industry with $\nu = 0.15$ (dashed line) is higher than that in the industry with $\nu = 0$. The reason is that, conditional on the same $e^{z_{i,0}}$, firms in the industry with $\nu = 0.15$ have higher leverage ratios, as explained in Online Appendix 4.3. This increases their exposure to shocks in $e^{z_{i,t}}$ in such industries. However, quantitatively, the difference becomes significant only when firms' are close to the default boundaries (i.e., low levels of $e^{z_{i,0}}$). When $e^{z_{i,0}} = 1$, which corresponds to the initial demand intensity under our calibration, the conditional expected excess return attributed to shocks in $e^{z_{i,t}}$ is about 1.05% in the industry with $\nu = 0.15$ (dashed line), which is only slightly higher than that (0.98%) in the industry with $\nu = 0$.

Panel B of Figure 8 plots the term $\left.\frac{\partial \ln E^C_{i,t}}{\partial \ln e^{z_{j,t}}}\right|_{t=0} \varsigma\gamma_0$ in equation (136), which captures the contribution of shocks in $e^{z_{j,t}}$ at $t = 0$. The two curves display similar patterns as those in panel A. Quantitatively, the difference between the two industries in terms of the conditional expected equity excess return attributed to shocks in $e^{z_{j,t}}$ is not significant when $e^{z_{i,0}} = 1$.

Panel C of Figure 8 plots the term $-\left.\frac{\partial \ln E^C_{i,t}}{\partial \gamma_t}\right|_{t=0} \zeta\pi$ in equation (136), which captures the contribution of shocks in γ_t at $t = 0$. The solid line is significantly higher than the dashed line because the competition-distress feedback effect is much stronger in the industry with $\nu = 0$, which amplifies the industry's exposure to the discount-rate shock. When $e^{z_{i,0}} = 1$, the conditional expected excess return attributed to shocks in γ_t is about 7% in the industry with $\nu = 0$, which is about 4.9% higher than that (2.1%) in the industry with $\nu = 0.15$. Moreover, the gap between the two curves increases when $e^{z_{i,0}}$ is lower because the competition-distress feedback effect becomes stronger when firms are more distressed.

Taken together, Figure 8 shows that firms in the industries with a lower ν are more exposed to the discount-rate shock $Z_{\gamma,t}$ (panel C) but less exposed to the cash-flow shock W_t (panels A and B). In our full-fledged quantitative model with aggregate shocks, the

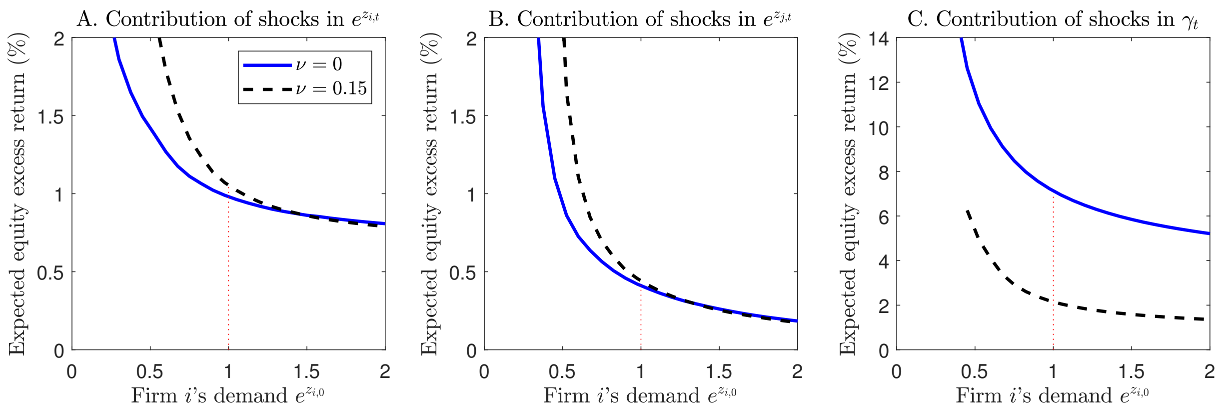

Figure 8. Decomposition of the conditional expected excess returns. This figure visualizes the components of the conditional expected equity excess returns in equation (136). Panel A plots the term $\left.\frac{\partial \ln E^C_{i,t}}{\partial \ln e^{z_{i,t}}}\right|_{t=0} \varsigma\gamma_0$, capturing the contribution of shocks in $e^{z_{i,t}}$ at $t = 0$, as a function of $e^{z_{i,0}}$. The solid and dashed lines represent the industries with $\nu = 0$ and $\nu = 0.15$, respectively. Panel B plots the term $\left.\frac{\partial \ln E^C_{i,t}}{\partial \ln e^{z_{j,t}}}\right|_{t=0} \varsigma\gamma_0$, which captures the contribution of shocks in $e^{z_{j,t}}$ at $t = 0$. Panel C plots the term $-\left.\frac{\partial \ln E^C_{i,t}}{\partial \gamma_t}\right|_{t=0} \zeta\pi$, which captures the contribution of shocks in γ_t at $t = 0$. In all three panels, we hold $\gamma_0 = \overline{\gamma}$ and $e^{z_{j,0}} = 1$ unchanged. Both firms' coupon rates are set at the optimal level corresponding to the initial demand intensity $e^{z_{i,0}} = e^{z_{j,0}} = 1$. Parameters are set according to our calibration in Online Appendix 4.2.

former channel significantly dominates the latter and, hence, our model implies that it is the differential exposure to the discount-rate shock $Z_{\gamma,t}$, not the cash-flow shock W_t, across industries that explains the industry-level financial distress anomaly. In Online Appendix 6.5, we provide empirical evidence to support this model implication. Theoretically, we obtain this result because firms in industries with lower ν endogenously collude on higher profit margins, which decline more significantly when the aggregate discount rate γ_t increases. In other words, the cash flows of these firms are more exposed to the discount-rate shock $Z_{\gamma,t}$.

In Table 3, we provide further analyses of the model mechanism. Specifically, if we assume that the discount-rate shock is not priced (i.e., by setting $\zeta = 0$, in column (3) of Table 3) or that all industries have the same level of idiosyncratic left-tail risk (i.e., by setting the same $\overline{\nu}$ across industries, in column (4) of Table 3), the model could not rationalize the financial distress anomaly. In fact, in this case, the model would predict that more distressed industries are associated with higher expected equity excess returns due to their higher financial leverage, as in the canonical frameworks of Merton (1974) and Leland (1994). Further, we emphasize that introducing idiosyncratic left-tail risk alone to the canonical framework does not help explain the industry-level financial distress anomaly. This is simply because idiosyncratic left-tail risk is not priced, so it merely increases the default rate without affecting the risk premium in the absence of the endogenous distressed

Table 3: The model's key ingredients that explain the industry-level distress anomaly.

	(1) Data	(2) Model	(3) $\zeta = 0$	(4) $\nu \equiv \overline{\nu}$
Leverage ratio	0.34	0.33	0.41	0.33
Expected equity excess return (%)	7.36	7.27	1.69	7.41
Difference in excess returns sorted on $Distress_{i,t}$ (%)	−4.94	−5.01	0.28	1.41

competition mechanism. This highlights the importance of the endogenous distressed competition mechanism of our model. Idiosyncratic left-tail risk has cross-sectional asset pricing implications precisely because it affects the strength of the endogenous distressed competition mechanism, which determines the strength of the competition-distress feedback effect.

4.5 Robustness of the Quantitative Implications

In this subsection, we discuss the robustness of the quantitative results presented in Table 2.

4.5.1 Time-Varying Jump Intensity

In our full-fledged quantitative model with aggregate shocks, the jump intensity ν of idiosyncratic left-tail risk is constant for a given industry. However, in the data, the estimated idiosyncratic left-tail risk is time varying. We conduct a robustness check to show that the model's quantitative implications in Table 2 remain robust if the jump intensity ν is time varying.

Specifically, we assume that the intensity of the Poisson process $J_{i,t}$ in equation (33) of the main text is an industry-specific variable ν_t, which evolves idiosyncratically according to a Markov chain on $\{\nu_1, \nu_2, ..., \nu_N\}$, where $\nu_N > ... > \nu_1 > 0$. The value of ν_t remains the same unless the industry is hit by a Poisson shock with rate $\chi > 0$. Conditional on receiving the Poisson shock over $[t, t + \mathrm{d}t)$, the industry randomly draws a new ν_t from the set $\{\nu_1, \nu_2, ..., \nu_N\}$, each with equal probability. The Poisson shocks to industry characteristic ν_t are idiosyncratic and independent across industries.

Following the calibration in Online Appendix 4.2, we choose $\nu_1 = \underline{\nu}$, $\nu_N = \overline{\nu}$, and the set $\{\nu_1, \nu_2, ..., \nu_N\}$ has equal space between adjacent values. The parameter χ governs the persistence of industry characteristic ν_t.[4] Thus, we set $\chi = 0.27$ so that the model-implied yearly autocorrelation in ν_t is about 0.7, consistent with the yearly autocorrelation of our

[4]If $\chi = 0$, then the intensity of idiosyncratic left-tail jump shocks is constant as in the full-fledged quantitative model with aggregate shocks.

Table 4: Robustness check: Time-varying intensity of idiosyncratic left-tail risk.

	Model			Time-varying ν_t		
	Q1 (low)	Q5	Q5−Q1	Q1 (low)	Q5	Q5−Q1
Equity excess return (%)	9.88	4.87	−5.01	9.21	5.84	−3.37
Equity beta to discount rate	−6.09	7.06	13.15	−5.17	2.95	8.12
Credit spread (%)	0.77	2.82	2.05	0.99	2.51	1.52
5-year default rate (%)	0.02	7.57	7.55	0.51	6.59	6.08
Leverage ratio (%)	27.96	37.87	9.91	28.72	36.13	7.41

Note: The three columns below "Model" present the results of the full-fledged quantitative model with aggregate shocks when ν is constant, as in Table 2. The three columns below "Time-varying ν_t" present the results of the full-fledged quantitative model with aggregate shocks when ν_t is time varying.

empirical measure for idiosyncratic left-tail risk constructed in Section 6.1 of the main text. All other parameters are calibrated at the same values as in Online Appendix 4.2.

Table 4 shows that the spread (Q5−Q1) in expected equity excess returns implied by the model with time-varying ν_t is about −3.37%, which is about 1.64% higher than that (−5.01%) of the model with a constant ν. The reason is that assuming that industries can randomly redraw a new characteristic ν_t at a certain rate χ in the future essentially makes firms' current conditions more aligned across industries with different values of ν_t. This reduces the cross-industry difference in the strength of the distressed competition mechanism, which is the key to rationalizing the financial distress anomaly in our model. The spreads in other variables, such as the equity beta to the discount rate, the credit spread, the 5-year default rate, and the leverage ratio, shrink for the same reason. Although the model with time-varying ν_t generates a less negative spread in expected equity excess returns across industries, the magnitude of this spread (−3.37%) is reasonably close to the spread (−4.94%) in the data, for which the 95% confidence interval is [−9.18%, −0.69%] (see Table 2).[5]

4.5.2 Duopolies vs. Oligopolies

For tractability and transparency, both our full-fledged quantitative model in Section 4 of the main text and the full-fledged quantitative model with aggregate shocks in Online Appendix 4.1 focus on the duopoly market structure to emphasize the endogenous strategic competition between two market leaders in an industry. The computational complexity increases exponentially with the number of firms considered because each firm's decisions

[5]When conducting this robustness check with time-varying ν_t, we hold the parameters $\underline{\nu}$ and $\overline{\nu}$ unchanged at the calibrated value in Table 1 of Online Appendix 4.2. Alternatively, we can recalibrate a larger $\overline{\nu}$ to increase the model-implied spread in expected equity excess returns so that it becomes more consistent with the spread in the data.

Table 5: Robustness check: Industries with two firms vs. industries with three firms.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Model			Three firms			Three firms (recalibrated)		
	Q1	Q5	Q5−Q1	Q1	Q5	Q5−Q1	Q1	Q5	Q5−Q1
Profit margins (%)	37.4	12.7	−24.7	27.5	12.4	−15.1	37.6	12.8	−24.8
Equity excess return (%)	9.88	4.87	−5.01	8.84	4.75	−4.09	9.79	4.87	−4.92
Equity beta to discount rate	−6.09	7.06	13.15	−5.15	7.13	12.28	−6.03	7.04	13.07
Credit spread (%)	0.77	2.82	2.05	0.79	2.83	2.04	0.78	2.82	2.04
5-year default rate (%)	0.02	7.57	7.55	0.03	7.57	7.54	0.02	7.57	7.55
Leverage ratio (%)	27.96	37.87	9.91	29.55	37.94	8.39	28.03	37.88	9.85

Note: Columns (1) to (3) present the results of the full-fledged quantitative model with aggregate shocks in which each industry has two firms, as in Table 2. Columns (4) to (6) present the results of the full-fledged quantitative model with aggregate shocks in which each industry has three firms, using the same parameter values as columns (1) to (3) (see Online Appendix 4.2). Columns (7) to (9) present the results of the full-fledged quantitative model with aggregate shocks in which each industry has three firms; the parameter ξ is recalibrated at 12% so that the implied average profit margin across all industries is 26.5%, the same as our calibration for columns (1) to (3) (see Table 1). The other parameters are set according to our calibration in Online Appendix 4.2.

are solved based on every other firm's profit margin, share of industry demand, default status, and collusion decisions. Thus, solving a generic n-firm model is NP hard. In reality, an industry could have more than two market leaders. To illustrate the impact of modeling more firms in an industry, we now evaluate how the quantitative implications would change if we consider industries with three firms, as an example.

Columns (4) to (6) of Table 5 present the results based on the three-firm model (i.e., each industry has three firms), where parameters are set at the same values as our duopoly model (see Online Appendix 4.2). Compared with the duopoly model (columns (1) to (3) of Table 5), the profit margin of portfolio Q1 declines significantly by about 9.9%, from 37.4% to 27.5% while the profit margin of portfolio Q5 declines slightly by about 0.3%, from 12.7% to 12.4%. This is because maintaining a collusive equilibrium becomes more difficult when there are more firms within an industry and thus only lower profit margins can be sustained in the equilibrium that satisfies IC constraints. Quantitatively, the profit margins of industries in portfolio Q5 change little when the number of firms in an industry increases from two to three because firms in these industries do not collude much anyway even when there are two firms. However, for industries in portfolio Q1, firms collude on high profit margins in the duopoly industry and the capacity for collusion significantly declines when the industry has three firms. The significant decrease in collusive profit margins naturally dampens the response of firms' cash flows to discount-rate shocks, resulting in lower expected equity returns. As a result, the model with three firms implies that the spread of expected equity excess returns between portfolios Q1 and Q5 is −4.09%, which is 0.92% higher than that (−5.01%) of the duopoly model. The differences in the credit spread and 5-year default rate between portfolios Q1 and Q5 do not alter greatly compared with the duopoly model because they are determined mainly by the difference in the exogenous intensity of idiosyncratic left-tail jump shocks, rather than the endogenous distressed competition mechanism. The leverage ratio of portfolio Q1 in the three-firm model is 29.55%, about 1.59% higher than that (27.96%) in the duopoly model. This is because when the industry has three firms, the distressed competition mechanism is weakened, which motivates firms to optimally choose higher leverage ratios.

The main reason that the three-firm model generates a less significant financial distress anomaly than the duopoly model is that we do not recalibrate the three-firm model to match the average profit margin in the data when conducting the quantitative exercise for columns (4) to (6) of Table 5. Because we are using the same parameters as the duopoly model, the endogenous distressed competition mechanism in the three-firm model naturally becomes weaker than the duopoly model due to the increase in competition intensity within each industry. This can be directly seen by comparing the average profit margins between the duopoly model and the three-firm model. The average profit margin across all industries is 26.5% in the duopoly model but only 19.4% in the three-firm model. By comparing columns (1) and (4) of Table 5, we can see that the lower average profit margin in the three-firm model is mainly driven by the decline in the profit margin of portfolio Q1, which consists of industries with the lowest exposure to idiosyncratic left-tail risk. In the duopoly model, the endogenous distressed competition mechanism in these industries is much stronger than that in the three-firm model because firms collude on much higher profit margins. Because profit margins are much higher, they respond more significantly to discount-rate shocks (i.e., profit margins decline more dramatically when the discount rate rises), generating more pronounced comovement between firms' cash flows and the discount rate. This is why the duopoly model can generate a more significant financial distress anomaly than the three-firm model.

In columns (7) to (9), we recalibrate the parameter ξ for the three-firm model to 12% (compared with $\xi = 9\%$ in the calibration of the duopoly model, shown in Table 1), so that the average profit margin in the three-firm model is the same as that in the duopoly model, holding all other parameter values unchanged. Intuitively, a higher ξ increases the rate at which deviant behavior can be punished, which increases firms' capacity for collusion. By comparing the results of the recalibrated three-firm model (columns (7) to (9)) with the results of the duopoly model (columns (1) and (3)), it is evident that the spreads in expected equity excess returns (Q5−Q1) have a comparable magnitude.

Due to computational constraints, we cannot extend the model to four, five, or more firms to evaluate the quantitative implications. The comparison between the duopoly model and the three-firm model in Table 5 suggests that increasing the number of firms in an industry does indeed reduce the strength of the endogenous distressed competition mechanism, resulting in a less significant financial distress anomaly implied by the model. The main reason is that, all else equal, when the industry has a greater number of firms, collusion is more difficult to sustain, and competition becomes more intensive. However, once we recalibrate the parameter ξ, which governs firms' capacity for collusion in our model, to match the average gross profit margin observed in the data, the strength of the endogenous distressed competition mechanism remains largely unchanged compared with the duopoly model despite modeling three firms in an industry. In fact, in our model, the profit margin fully reflects the competition intensity within an industry, which consequently determines the strength of the endogenous distressed competition mechanism and the significance of the financial distress anomaly. Thus, to maintain the competition intensity (to match the average profit margin in the data), the increased competition, due to a greater number of firms, needs to be offset by a higher value of ξ compared with the duopoly model, which leads to comparatively more collusion and less competition.

5 Numerical Algorithm

In this appendix section, we detail the numerical algorithm that solves the quantitative model in Section 4 of the main text. The full-fledged quantitative model with aggregate shocks in Online Appendix 4 can be solved similarly. To give an overview, our algorithm proceeds in the following steps:

(1). We solve the non-collusive equilibrium. This requires us to solve the subgame perfect equilibrium of the dynamic game played by two firms. The simultaneous-move dynamic game requires us to solve the intersection of the two firms' best response functions (i.e., optimal decisions on profit margins and default), which themselves are optimal solutions to coupled partial differential equations (PDEs).

(2). We solve the collusive equilibrium using the value functions in the non-collusive equilibrium as punishment values. Because we are interested in the highest collusive profit margins with binding IC constraints, this requires us to solve a high-dimensional fixed-points problem. Thus, we use an iteration method inspired by Abreu, Pearce and Stacchetti (1986, 1990), Ericson and Pakes (1995), and Fershtman and Pakes (2000) to solve the problem.

Note that standard methods for solving PDEs with free boundaries (e.g. finite difference or finite element) can easily lead to non-convergence of value functions. To mitigate such problems and obtain accurate solutions, we solve the continuous-time game using a discrete-time dynamic programming method, as in Dou, Ji and Wu (2021*a*). In Subsection 5.1, we present the discretized recursive formulation for the model, including firms' problems in non-collusive equilibrium, collusive equilibrium, and deviation. In Subsection 5.2, we discuss how we discretize the stochastic processes, time grids, and state variables in the model. Finally, in Subsection 5.3, we discuss the implementation details of our numerical algorithm, including searching for the equilibrium profit margins in the non-collusive equilibrium and solving the optimal collusive profit margins and default boundaries.

5.1 Discretized Dynamic Programming Problem

Because firm 1 and firm 2 are symmetric, one firm's equity value and policy functions are obtained directly given the other firm's equity value and policy functions. We first illustrate the non-collusive equilibrium and then the collusive equilibrium.

5.1.1 Non-Collusive Equilibrium

We first present the recursive formulation for the firm's equity value in the non-collusive equilibrium. Next, we present the conditions that determine the non-collusive equilibrium.

Recursive Formulation for Equity Value in Non-collusive Equilibrium. Firm i's state is characterized by two state variables, including firm i's demand intensity $z_{i,t}$ and firm j's demand intensity $z_{j,t}$. Denote by $E_i^N(z_{i,t}, z_{j,t}; b_i, b_j)$ the equity value function in the non-collusive equilibrium for $i = 1, 2$, where b_i and b_j are the two firms' log coupon rates, which will be optimally determined in the end.

To characterize the equilibrium equity value functions, it is more convenient to introduce two off-equilibrium equity value functions. Let $\widehat{E}_i^N(z_{i,t}, z_{j,t}; \theta_{j,t}, d_{j,t}; b_i, b_j)$ be firm $i(= 1, 2)$'s equity value when its competitor j's profit margin is any (off-equilibrium) value $\theta_{j,t}$ and default status is any (off-equilibrium) value $d_{j,t} = 0, 1$.

Firm $i = 1, 2$ solves the following problem:

$$\widehat{E}_i^N(z_{i,t}, z_{j,t}; \theta_{j,t}, d_{j,t}; b_i, b_j) = \max_{\theta_{i,t}, d_{i,t}} (1 - d_{i,t}) \left\{ (1-\tau) \left[\omega^{1-\epsilon} \theta_{i,t} (1-\theta_{i,t})^{\eta-1} (1-\theta_t)^{\epsilon-\eta} e^{z_{i,t}} - e^{b_i} \right] \Delta t \right.$$
$$\left. + e^{-(r_f+\nu)\Delta t} \mathbb{E}_t \left[(1-d_{j,t}) E_i^N(z_{i,t+\Delta t}, z_{j,t+\Delta t}; b_i, b_j) + d_{j,t} E_i^N(z_{i,t+\Delta t}, z_{new}; b_i, b_{new}) \right] \right\}, \qquad (137)$$

subject to the following constraints. (1) The industry's profit margin is given by

$$1-\theta_t = \left[\sum_{j=1}^{2} e^{z_{i,t}-a_t}\left(1-\theta_{i,t}\right)^{\eta-1}\right]^{\frac{1}{\eta-1}} \quad \text{with} \;\; e^{a_t} = e^{z_{i,t}} + e^{z_{j,t}}. \tag{138}$$

(2) Firms' demand evolves according to

$$e^{z_{i,t+\Delta t}} = e^{z_{i,t}} + (g-\varsigma\gamma)e^{z_{i,t}}\Delta t + \varsigma e^{z_{i,t}}\Delta W_t^{\mathbb{Q}} + \sigma e^{z_{i,t}}\Delta W_{i,t}, \tag{139}$$

$$e^{z_{j,t+\Delta t}} = e^{z_{j,t}} + (g-\varsigma\gamma)e^{z_{j,t}}\Delta t + \varsigma e^{z_{j,t}}\Delta W_t^{\mathbb{Q}} + \sigma e^{z_{j,t}}\Delta W_{j,t}. \tag{140}$$

Non-collusive Equilibrium. Denote by $\theta_i^N(z_{i,t}, z_{j,t}; b_i, b_j)$ and $d_i^N(z_{i,t}, z_{j,t}; b_i, b_j)$ the equilibrium profit margin and default functions. Denote by $\widehat{\theta}_i^N(z_{i,t}, z_{j,t}; \theta_{j,t}, d_{j,t}; b_i, b_j)$ and $\widehat{d}_i^N(z_{i,t}, z_{j,t}; \theta_{j,t}, d_{j,t}; b_i, b_j)$ the off-equilibrium profit margin and default functions.

Given firm j's profit margin $\theta_{j,t}$ and default decision $d_{j,t}$, firm i optimally sets the profit margin $\theta_{i,t}$ and makes default decision $d_{i,t}$. The non-collusive equilibrium is derived from the fixed point—each firm's profit margin and default are optimal given the other firm's optimal profit margin and default:

$$\theta_i^N(z_{i,t}, z_{j,t}; b_i, b_j) = \widehat{\theta}_i^N(z_{i,t}, z_{j,t}; \theta_j^N(z_{j,t}, z_{i,t}; b_j, b_i), d_j^N(z_{j,t}, z_{i,t}; b_j, b_i); b_i, b_j), \tag{141}$$

$$d_i^N(z_{i,t}, z_{j,t}; b_i, b_j) = \widehat{d}_i^N(z_{i,t}, z_{j,t}; \theta_j^N(z_{j,t}, z_{i,t}; b_j, b_i), d_j^N(z_{j,t}, z_{i,t}; b_j, b_i); b_i, b_j). \tag{142}$$

The equilibrium equity value functions are given by

$$E_i^N(z_{i,t}, z_{j,t}; b_i, b_j) = \widehat{E}_i^N(z_{i,t}, z_{j,t}; \theta_j^N(z_{j,t}, z_{i,t}; b_j, b_i), d_j^N(z_{j,t}, z_{i,t}; b_j, b_i); b_i, b_j). \tag{143}$$

5.1.2 Collusive Equilibrium

We first present the recursive formulation for the firm's equity value in the collusive equilibrium. Next, we present the recursive formulation for the firm's equity value when it deviates from the collusive equilibrium. Finally, we present the IC constraints to determine the equilibrium collusive profit margins. After finding the equilibrium collusive profit margin scheme, we check whether the PC constraints are satisfied. There are two cases, if the PC constraints are satisfied, the two firms will collude on the equilibrium profit margin scheme. If the PC constraints are not satisfied, the two firms will set profit margins according to their non-collusive ones.

Recursive Formulation for Equity Value in The Collusive Equilibrium. Denote by $\overline{E}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \overline{\Theta}^C(\cdot))$ firm i's equity value in the collusive equilibrium with collusive profit margin scheme $\overline{\Theta}^C(\cdot)$, for $i = 1,2$. Denote by $\widehat{E}_i^C(z_{i,t}, z_{j,t}; d_{j,t}; b_i, b_j; \overline{\Theta}^C(\cdot))$ firm i's equity value in the collusive equilibrium with collusive profit margin scheme $\overline{\Theta}^C(\cdot)$ when its competitor j's default status is any (off-equilibrium) value $d_{j,t} = 0, 1$.

Firm i solves the following problem:

$$\widehat{E}_i^C(z_{i,t}, z_{j,t}; d_{j,t}; b_i, b_j; \overline{\Theta}^C(\cdot)) = \max_{d_{i,t}}(1 - d_{i,t}) \left\{ (1-\tau) \left[\omega^{1-\epsilon} \overline{\theta}_{i,t}^C (1 - \overline{\theta}_{i,t}^C)^{\eta-1} (1 - \overline{\theta}_t^C)^{\epsilon-\eta} e^{z_{i,t}} - e^{b_i} \right] \Delta t \right.$$
$$\left. + e^{-(r_f+\nu)\Delta t} \mathbb{E}_t \left[(1 - d_{j,t}) \overline{E}_i^C(z_{i,t+\Delta t}, z_{j,t+\Delta t}; b_i, b_j; \overline{\Theta}^C(\cdot)) + d_{j,t} \overline{E}_i^C(z_{i,t+\Delta t}, z_{new}; b_i, b_{new}; \overline{\Theta}^C(\cdot)) \right] \right\}, \tag{144}$$

subject to the following constraints. (1) The industry's profit margin is given by

$$1 - \overline{\theta}_t^C = \left[\sum_{j=1}^{2} e^{z_{i,t} - a_t} \left(1 - \overline{\theta}_{i,t}^C \right)^{\eta - 1} \right]^{\frac{1}{\eta-1}} \quad \text{with} \quad e^{a_t} = e^{z_{i,t}} + e^{z_{j,t}}. \tag{145}$$

(2) Firms' demand evolves according to

$$e^{z_{i,t+\Delta t}} = e^{z_{i,t}} + (g - \varsigma\gamma) e^{z_{i,t}} \Delta t + \varsigma e^{z_{i,t}} \Delta W_t^{\mathbb{Q}} + \sigma e^{z_{i,t}} \Delta W_{i,t}, \tag{146}$$
$$e^{z_{j,t+\Delta t}} = e^{z_{j,t}} + (g - \varsigma\gamma) e^{z_{j,t}} \Delta t + \varsigma e^{z_{j,t}} \Delta W_t^{\mathbb{Q}} + \sigma e^{z_{j,t}} \Delta W_{j,t}. \tag{147}$$

Denote by $\overline{d}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \overline{\Theta}^C(\cdot))$ the equilibrium default function. Denote by $\widehat{d}_i^C(z_{i,t}, z_{j,t}; d_{j,t}; b_i, b_j; \overline{\Theta}^C(\cdot))$ the off-equilibrium default function. The default decisions are determined in Nash equilibrium. In particular, given firm j's default decision $d_{j,t}$, firm i optimally makes default decision $d_{i,t}$. The Nash equilibrium is derived from the fixed point—each firm's default is optimal given the other firm's optimal default:

$$\overline{d}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \overline{\Theta}^C(\cdot)) = \widehat{d}_i^C(z_{i,t}, z_{j,t}; \overline{d}_j^C(z_{j,t}, z_{i,t}; b_j, b_i; \overline{\Theta}^C(\cdot)); b_i, b_j; \overline{\Theta}^C(\cdot)). \tag{148}$$

The equilibrium equity value functions are given by

$$\overline{E}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \overline{\Theta}^C(\cdot)) = \widehat{E}_i^C(z_{i,t}, z_{j,t}; \overline{d}_j^C(z_{j,t}, z_{i,t}; b_j, b_i; \overline{\Theta}^C(\cdot)); b_i, b_j; \overline{\Theta}^C(\cdot)). \tag{149}$$

Recursive Formulation for Equity Value upon Deviation. The deviation equity value is obtained by assuming that firm i optimally sets its profit margin conditional on firm j setting the profit margin according to the collusive profit margin scheme, i.e., $\overline{\theta}_j^C(z_{j,t}, z_{i,t}; b_j, b_i; \overline{\Theta}^C(\cdot))$ and default decision $\overline{d}_j^C(z_{j,t}, z_{i,t}; b_j, b_i; \overline{\Theta}^C(\cdot))$.

Denote by $\overline{E}_i^D(z_{i,t}, z_{j,t}; b_i, b_j; \overline{\Theta}^C(\cdot))$ firm i's deviation equity value. Firm i solves the following problem:

$$\overline{E}_i^D(z_{i,t}, z_{j,t}; b_i, b_j; \overline{\Theta}^C(\cdot)) = \max_{\theta_{i,t}, d_{i,t}} (1 - d_{i,t}) \left\{ (1-\tau) \left[\omega^{1-\epsilon} \theta_{i,t} (1-\theta_{i,t})^{\eta-1} (1 - \overline{\theta}_t^D)^{\epsilon-\eta} e^{z_{i,t}} - e^{b_i} \right] \Delta t \right.$$
$$+ e^{-(r_f+\nu)\Delta t} \mathbb{E}_t \left[d_{j,t} \left((1-\xi\Delta t) \overline{E}_i^D(z_{i,t+\Delta t}, z_{new}; b_i, b_{new}; \overline{\Theta}^C(\cdot)) \right. \right.$$
$$\left. + \xi \Delta t E_i^N(z_{i,t+\Delta t}, z_{new}; b_i, b_{new}) \right) + (1 - d_{j,t}) \left(\xi \Delta t E_i^N(z_{i,t+\Delta t}, z_{j,t+\Delta t}; b_i, b_j) \right.$$
$$\left. \left. \left. + (1-\xi\Delta t) \overline{E}_i^D(z_{i,t+\Delta t}, z_{j,t+\Delta t}; b_i, b_j; \overline{\Theta}^C(\cdot)) \right) \right] \right\}, \tag{150}$$

subject to the following constraints. (1) The industry's profit margin is given by

$$1 - \overline{\theta}_t^D = \left[e^{z_{i,t} - a_t} (1 - \theta_{i,t})^{\eta-1} + e^{z_{j,t} - a_t} \left(1 - \overline{\theta}_{j,t}^C \right)^{\eta-1} \right]^{\frac{1}{\eta-1}} \quad \text{with} \quad e^{a_t} = e^{z_{i,t}} + e^{z_{j,t}}. \tag{151}$$

(2) Firms' demand evolves according to

$$e^{z_{i,t+\Delta t}} = e^{z_{i,t}} + (g - \varsigma\gamma) e^{z_{i,t}} \Delta t + \varsigma e^{z_{i,t}} \Delta W_t^{\mathbb{Q}} + \sigma e^{z_{i,t}} \Delta W_{i,t}, \tag{152}$$
$$e^{z_{j,t+\Delta t}} = e^{z_{j,t}} + (g - \varsigma\gamma) e^{z_{j,t}} \Delta t + \varsigma e^{z_{j,t}} \Delta W_t^{\mathbb{Q}} + \sigma e^{z_{j,t}} \Delta W_{j,t}. \tag{153}$$

Solving for Equilibrium Profit Margins. The collusive equilibrium is a subgame perfect Nash equilibrium if and only if the collusive profit margin scheme $\overline{\Theta}^C(\cdot)$ satisfies the following PC and IC constraints:

$$\overline{E}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \overline{\Theta}^C(\cdot)) \geq E_i^N(z_{i,t}, z_{j,t}; b_i, b_j), \tag{154}$$
$$\overline{E}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \overline{\Theta}^C(\cdot)) \geq \overline{E}_i^D(z_{i,t}, z_{j,t}; b_i, b_j; \overline{\Theta}^C(\cdot)), \tag{155}$$

for all $z_{i,t}$ and $i = 1, 2$.

There exist infinitely many subgame perfect Nash equilibria. We focus on the collusive equilibrium with the collusive profit margins lying on the "Pareto efficient frontier" (denoted by $\Theta^C(\cdot)$), which are obtained when all IC constraints are binding, i.e.,

$$\overline{E}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \Theta^C(\cdot)) = \overline{E}_i^D(z_{i,t}, z_{j,t}; b_i, b_j; \Theta^C(\cdot)), \tag{156}$$

for all $z_{i,t}$ and $i = 1, 2$. The collusive equilibrium is solved by finding the collusive profit margin scheme $\Theta^C(\cdot)$ such that the PC constraint (154) and the IC constraint (156) are satisfied simultaneously.

We denote $E_i^C(z_{i,t}, z_{j,t}; b_i, b_j)$ as firm i's value in the collusive equilibrium with the collusive profit margin scheme $\Theta^C(\cdot)$. In solving the equilibrium, we first ignore the PC constraint (154) and solve for $\Theta^C(\cdot)$ that satisfies the IC constraint (156). Then given $\Theta^C(\cdot)$, for each value of $z_{i,t}$ and $z_{j,t}$, we check whether the PC constraint (154) is satisfied for both i and j. If it is satisfied, the equity value and profit margin in the collusive equilibrium are determined according to the collusive value

$$E_i^C(z_{i,t}, z_{j,t}; b_i, b_j) = \overline{E}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \Theta^C(\cdot)), \tag{157}$$

$$\theta_i^C(z_{i,t}, z_{j,t}; b_i, b_j) = \overline{\theta}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \Theta^C(\cdot)), \tag{158}$$

$$E_j^C(z_{j,t}, z_{i,t}; b_j, b_i) = \overline{E}_j^C(z_{j,t}, z_{i,t}; b_j, b_i; \Theta^C(\cdot)), \tag{159}$$

$$\theta_j^C(z_{j,t}, z_{i,t}; b_j, b_i) = \overline{\theta}_j^C(z_{j,t}, z_{i,t}; b_j, b_i; \Theta^C(\cdot)). \tag{160}$$

If it is not satisfied, there are two cases. First, if firm i's PC constraint is not satisfied, then we search for the endogenous collusion boundary $\lambda_i(z_{j,t}; b_i, b_j)$ at which firm i's PC constraint just becomes binding. This is done through guess-and-verify iterations. Then, given $z_{j,t}$, for all $z_{i,t} \leq \lambda_i(z_{j,t}; b_i, b_j)$, the profit margins in the collusive equilibrium are determined according to the non-collusive value

$$\theta_i^C(z_{i,t}, z_{j,t}; b_i, b_j) = \theta_i^N(z_{i,t}, z_{j,t}; b_i, b_j), \tag{161}$$

$$\theta_j^C(z_{j,t}, z_{i,t}; b_j, b_i) = \theta_j^N(z_{j,t}, z_{i,t}; b_j, b_i). \tag{162}$$

For all $z_{i,t} > \lambda_i(z_{j,t}; b_i, b_j)$, the profit margins in the collusive equilibrium are determined according to the collusive value

$$\theta_i^C(z_{i,t}, z_{j,t}; b_i, b_j) = \overline{\theta}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \Theta^C(\cdot)), \tag{163}$$

$$\theta_j^C(z_{j,t}, z_{i,t}; b_j, b_i) = \overline{\theta}_j^C(z_{j,t}, z_{i,t}; b_j, b_i; \Theta^C(\cdot)). \tag{164}$$

Based on these profit margins, we compute the collusive equity value $\overline{E}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \Theta^C(\cdot))$ again and set

$$E_i^C(z_{i,t}, z_{j,t}; b_i, b_j) = \overline{E}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \Theta^C(\cdot)), \tag{165}$$

$$E_j^C(z_{j,t}, z_{i,t}; b_j, b_i) = \overline{E}_j^C(z_{j,t}, z_{i,t}; b_j, b_i; \Theta^C(\cdot)). \tag{166}$$

Second, if firm j's PC constraint is not satisfied, then we search for the endogenous collusion boundary $\lambda_j(z_{i,t}; b_j, b_i)$ at which firm j's PC constraint just becomes binding. This is done through guess-and-verify iterations. Then, given $z_{i,t}$, for all $z_{j,t} \leq \lambda_j(z_{i,t}; b_j, b_i)$, the profit margins in the collusive equilibrium are determined according to the non-collusive value

$$\theta_i^C(z_{i,t}, z_{j,t}; b_i, b_j) = \theta_i^N(z_{i,t}, z_{j,t}; b_i, b_j), \tag{167}$$
$$\theta_j^C(z_{j,t}, z_{i,t}; b_j, b_i) = \theta_j^N(z_{j,t}, z_{i,t}; b_j, b_i). \tag{168}$$

For all $z_{j,t} > \lambda_j(z_{i,t}; b_j, b_i)$, the profit margins in the collusive equilibrium are determined according to the collusive value

$$\theta_i^C(z_{i,t}, z_{j,t}; b_i, b_j) = \overline{\theta}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \Theta^C(\cdot)), \tag{169}$$
$$\theta_j^C(z_{j,t}, z_{i,t}; b_j, b_i) = \overline{\theta}_j^C(z_{j,t}, z_{i,t}; b_j, b_i; \Theta^C(\cdot)). \tag{170}$$

Based on these profit margins, we compute the collusive value $\overline{E}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \Theta^C(\cdot))$ again and set

$$E_i^C(z_{i,t}, z_{j,t}; b_i, b_j) = \overline{E}_i^C(z_{i,t}, z_{j,t}; b_i, b_j; \Theta^C(\cdot)), \tag{171}$$
$$E_j^C(z_{j,t}, z_{i,t}; b_j, b_i) = \overline{E}_j^C(z_{j,t}, z_{i,t}; b_j, b_i; \Theta^C(\cdot)). \tag{172}$$

Value of Debt. Firm i's debt value in the collusive equilibrium is given by

$$\begin{aligned} D_i^C(z_{i,t}, z_{j,t}; b_i, b_j) =& (1 - d_i^C(z_{i,t}, z_{j,t}; b_i, b_j)) \left\{ e^{b_i}\Delta t + e^{-(r_f+\nu)\Delta t}\mathbb{E}_t \left[D_i^C(z_{i,t+\Delta t}, z_{j,t+\Delta t}; b_i, b_j) \right] \right\} \\ &+ d_i^C(z_{i,t}, z_{j,t}; b_i, b_j)\nu A_i^C(z_{i,t}, z_{j,t}; b_j), \end{aligned} \tag{173}$$

where $d_i^C(z_{i,t}, z_{j,t}; b_i, b_j)$ and $A_i^C(z_{i,t}, z_{j,t}; b_j)$ are the optimal default decision and the unlevered asset value in the collusive equilibrium under the collusive profit margin scheme $\Theta^C(\cdot)$. Firm i's debt value in the non-collusive equilibrium is determined similarly using the optimal default decision and the unlevered asset value in the non-collusive equilibrium.

Optimal Choice of Debt. When firm i enters the market at time t_0, it optimally chooses b_i to maximize equity value (which is equal to the firm value after debt issuance) by solving the following problem:

$$\max_{b_i} = E_i^C(z_{i,t_0}, z_{j,t_0}; b_i, b_j) + D_i^C(z_{i,t_0}, z_{j,t_0}; b_i, b_j), \tag{174}$$

where the competitor's log coupon rate b_j is given because firms can choose coupon rates only at the beginning, when they enter the market.

At $t = 0$, we need to solve a fixed point problem in terms of b_i and b_j because the two firms choose coupon rates simultaneously. Moreover, because the two firms are symmetric at the very beginning ($z_{i,0} = z_{j,0}$), the initial optimal debt choice is also the same, i.e., $b_i = b_j$.

5.2 Discretization

The time line is discretized into intervals with length Δt. We use collocation methods to solve each firm's problem. Let $S_z \times S_z \times S_\gamma \times S_b \times S_b$ be the grid of collocation nodes for a firm's equilibrium equity value, $S_z \times S_z \times S_\theta \times S_d \times S_b \times S_b$ be the grid of collocation nodes for a firm's off-equilibrium equity value in the non-collusive equilibrium, and $S_z \times S_z \times S_d \times S_b \times S_b$ be the grid of collocation nodes for a firm's off-equilibrium equity value in the collusive equilibrium. We have $S_z = \{e^{z_1}, e^{z_2}, ..., e^{z_{n_z}}\}$, $S_\theta = \{\theta_1, \theta_2, ..., \theta_{n_\theta}\}$, $S_d = \{0, 1\}$, and $S_b = \{e^{b_1}, e^{b_2}, ..., e^{b_{n_b}}\}$.

We approximate the firm's value function $E_i(\cdot)$ and $D_i(\cdot)$ on the grid of collocation nodes using a linear spline with coefficients corresponding to each grid point. We first form a guess for the spline's coefficients, and then we iterate to obtain a vector that solves the system of Bellman equations.

5.3 Implementation

The numerical algorithms are implemented using C++. The program is run on the server of the Economics Department at MIT, supply.mit.edu and demand.mit.edu, which are built on Dell PowerEdge R910 (64 cores, Intel(R) Xeon(R) CPU E7-4870, 2.4GHz) and Dell PowerEdge R920 (48 cores, Intel(R) 4 Xeon E7-8857 v2 CPUs). We use OpenMP for parallelization when iterating value functions and simulating the model.

Selection of Grids. We set $n_z = 101$, $n_\theta = 11$, $n_b = 21$, $\Delta t = 1/24$. The grid of consumer demand S_z is discretized into 101 nodes from $e^{z_1} = 0$ to $e^{z_{n_z}} = 3$, with equal space in terms of e^z. The upper bound $e^{z_{n_z}} = 3$ is sufficiently large to ensure that the first derivative of $E_{i,t}$ with respect to $e^{z_{i,t}}$ is a constant. This guarantees that the boundary condition at $z_{i,t} = +\infty$ is accurately solved and satisfied. The time interval Δt is set to be 1/24 (i.e., half a month). A higher Δt implies faster convergence for the same number of iterations but lower accuracy. We check that the solution is accurate enough for $\Delta t = 1/24$, further reducing Δt brings little improvement for the accuracy of the solution. With $\Delta = 1/24$, 5000 iterations allow us to achieve convergence in value functions. The profit margin grid is discretized into 11 nodes from $1/\eta$ to $1/\epsilon$ with equal spaces. The lowest coupon rate e^{b_1} is set to zero to represent unlevered firms. The highest coupon rate is set to be $e^{b_{n_b}} = 0.05$.

Solving the Non-Collusive Equilibrium. Given the value functions from the previous iteration, we use the golden section search method to find the optimal profit margins. The computational complexity of this algorithm is at the order of $\log(n)$, much faster and more accurate than a simple grid search. The optimal default decisions can be trivially solved by checking two cases with $d^N_{i,t} = 0$ and $d^N_{i,t} = 1$.

Searching for the equilibrium profit margin is challenging because we have to solve a fixed-point problem that involves both firms' simultaneous profit margin decisions. Our solution technique is to iteratively solve the following three steps. First, given $E^N_i(z_{i,t}, z_{j,t}; b_i, b_j)$, we solve the off-equilibrium equity value $\widehat{E}^N_i(z_{i,t}, z_{j,t}; \theta_{j,t}, d_{j,t}; b_i, b_j)$ and policy functions $\widehat{\theta}^N_i(z_{i,t}, z_{j,t}; \theta_{j,t}, d_{j,t}; b_i, b_j)$ and $\widehat{d}^N_i(z_{i,t}, z_{j,t}; \theta_{j,t}, d_{j,t}; b_i, b_j)$. Second, for each $(z_{i,t}, z_{j,t}; b_i, b_j) \in S_z \times S_z \times S_b \times S_b$, we solve equations (141) and (142) and obtain the equilibrium decisions $\theta^N_i(z_{i,t}, z_{j,t}; b_i, b_j)$ and $d^N_i(z_{i,t}, z_{j,t}; b_i, b_j)$. Third, we solve equation (143) and obtain equilibrium equity value functions $E^N_i(z_{i,t}, z_{j,t}; b_i, b_j)$.

Solving the Collusive Equilibrium. To solve the collusive equilibrium, we have to simultaneously solve the endogenous default boundaries and the endogenous collusive profit margins within the default boundaries (as well as the endogenous collusion boundary in extreme industries, e.g., $\kappa = 0$). We implement a nested iteration method. First, we guess the default boundaries. Second, we solve for the highest collusive profit margins above the default boundaries using the iteration algorithm below. The profit margins associated with the states below the default boundaries are indeterminate because firms are in default. For these states, we set firms' profit margins at the non-collusive profit margins $\theta^N_i(z_{i,t}, z_{j,t}; b_i, b_j)$. Third, we check whether the implied default boundaries are consistent with our guessed boundaries. If not, we update our guess and resolve the highest collusive profit margins.

We modify the golden section search method to find the highest collusive profit margins $\theta^C_i(z_{i,t}, z_{j,t}; b_i, b_j)$ above the default boundaries (i.e., when first do not default) by iterations. For each iteration, we guess collusive profit margins $\overline{\theta}^C_i(z_{i,t}, z_{j,t}; b_i, b_j)$, and given the guessed profit margins, we solve firms' collusive equity value and deviation equity value using standard recursive methods. We update the guessed collusive profit margins until the IC constraints (156) are binding for all states.

There is a key difference between our method and a standard golden section search method. To increase efficiency, we guess and update the collusive profit margin scheme $\overline{\theta}^C_i(z_{i,t}, z_{j,t}; b_i, b_j)$ simultaneously for all $(z_{i,t}, z_{j,t}) \in S_z \times S_z$, instead of doing it one by one for each state. A natural problem introduced by the simultaneous updating is that there might be overshooting. For example, if for some particular state $(z^*_{i,t}, z^*_{j,t})$, we updated a collusive profit margin $\overline{\theta}_i^C(z_{i,t}^*, z_{j,t}^*; b_i, b_j)$ too high in the previous iteration, the collusive profit margin for some other states $(z_{i,t}, z_{j,t}) \neq (z_{i,t}^*, z_{j,t}^*)$ might be affected in this iteration and might never achieve a binding IC constraint. Eventually, this may lead to non-convergence.

We solve this problem by gradually updating the collusive profit margin scheme. In particular, in each round of iteration, we first compute the updated collusive profit margin scheme $\overline{\theta}_i^{C'}(z_{i,t}, z_{j,t}; b_i, b_j)$ implied by the golden section search method. Then, instead of changing the upper search bound or lower search bound to $\overline{\theta}_i^{C'}(z_{i,t}, z_{j,t}; b_i, b_j)$ directly, we change it to $(1 - adj) \times \overline{\theta}_i^C(z_{i,t}, z_{j,t}; b_i, b_j) + adj \times \overline{\theta}_i^{C'}(z_{i,t}, z_{j,t}; b_i, b_j)$, i.e., a weighted average of the current iteration's collusive profit margin $\overline{\theta}_i^C(z_{i,t}, z_{j,t}; b_i, b_j)$ and the updated one $\overline{\theta}_i^{C'}(z_{i,t}, z_{j,t}; b_i, b_j)$. If $adj = 0.5$, our algorithm is essentially the same as bisection search algorithm. A lower adj is more suitable to solve the problem in which different states have a higher degree of interdependence. We set a relatively low $adj = 0.05$ to ensure convergence.

6 Supplemental Materials on Empirical Analyses

In this appendix section, we provide supplemental information on the empirical analyses. Subsection 6.1 constructs additional empirical measures for the empirical and quantitative analyses in Online Appendix 6. Subsection 6.2 presents the estimated regression coefficients for constructing the measure of idiosyncratic left-tail risk. Subsection 6.3 presents the contagion effect on credit spreads. Subsection 6.4 tests the cross-industry asset pricing implications of the full-fledged quantitative model with aggregate shocks developed in Online Appendix 4. Subsection 6.5 presents evidence to support the model's prediction that the industry-level distress spread is explained by the differential loadings on discount-rate shocks across industries. Subsection 6.6 presents the financial distress spread across firms. Finally, Subsection 6.7 shows that the financial distress spread across industries cannot be explained by the financial distress spread across firms, indicating that these two spreads are likely to be explained by different theoretical mechanisms.

6.1 Additional Empirical Measures

In addition to the empirical measures constructed in Section 6.1 of the main text, we construct the following empirical measures for the supplementary empirical analyses in Online Appendix 6.

Asset Price Data. We obtain firm-level stock returns from the Center for Research in Security Practices (CRSP). Industry-level stock returns are the average firm-level stock returns weighted by market capitalization. As in Chen et al. (2018), our credit spread data combine the Mergent Fixed Income Securities Database (FISD) from 1973 to 2004 and the Trade Reporting and Compliance Engine (TRACE) database from 2005 to 2018. We clean the Mergent FISD and TRACE data following Collin-Dufresne, Goldstein and Martin (2001) and Dick-Nielsen (2009). Following Blanco, Brennan and Marsh (2005), we construct credit spreads as the difference between the corporate bond yield and the rate of interest rate swaps, which is regarded as the best parsimonious proxy for the risk-free rate (Feldhütter and Lando, 2008). The interest rate swap data are from Bloomberg, and are available from 1988. The credit spread data span the period from 1988 to 2018 and cover a cross section of 400-750 firms. We obtain the spreads of credit default swaps (CDS) from Markit for the period from 2001 to 2018. Industry-level credit spreads and CDS spreads are the average firm-level credit spreads and CDS spreads weighted by the par value of bonds, respectively.

For predictive analyses that involve both market and annual accounting data, we follow the practice of Fama and French (1993) and assume that accounting information becomes available at the end of June each year. For quarterly accounting data, we follow the practice of Campbell, Hilscher and Szilagyi (2008) and implement a 2-month lag (e.g., quarterly accounting data with the period end date of March 31 are matched with market data at the end of May).

Measure of Default Event. We retrieve and merge the information on Chapter 7 and Chapter 11 bankruptcies filed by large, public, nonfinancial U.S. firms over the period from 1981 to 2014 using the databases from New Generation Research's Bankruptcydata.com, the UCLA LoPucki Bankruptcy Research Database, Public Access to Court Electronic Records (PACER), National Archives at various locations, and U.S. Bankruptcy Courts for various districts, following Ma, Tong and Wang (2020) and Dou et al. (2021). Similar to Campbell, Hilscher and Szilagyi (2008), we define a default event as the first of the following events: Chapter 7 or Chapter 11 bankruptcy filing, delisting due to insolvency (delisting code 572), and a default or selective default rating by a rating agency. This expanded measure of failure (relative to measuring only bankruptcy filings) allows us to capture some instances in which firms fail but reach an agreement with creditors before an actual bankruptcy filing, such as pre-court liquidation or pre-court reorganization (e.g., Gilson, John and Lang, 1990; Gilson, 1997; Dou et al., 2021).

Measure of Financial Distress. The firm-level financial distress measure is constructed as the 12-month failure probability, following Campbell, Hilscher and Szilagyi (2008). The industry-level financial distress measure for industry i and period t, denoted by $Distress_{i,t}$, is the average firm-level financial distress measure weighted by firms' sales.

Table 6: Summary statistics and estimated coefficients for constructing $\nu_{i,t}$.

Panel A: Summary statistics of the dependent variables

	$NIMTA$	$EXRET$	$SIGMA$	$RSIZE$	$CASHMTA$	MB	$PRICE$	Observations
Mean	0.006	−0.006	0.404	−8.472	0.075	1.960	2.491	6,166
Stdev	0.008	0.021	0.161	1.258	0.051	0.967	0.277	6,166
Min	−0.052	−0.125	0.162	−13.325	0.005	0.415	0.365	6,166
Max	0.026	0.097	1.273	−6.265	0.344	6.064	2.708	6,166

Panel B: Regression coefficients in the estimation of idiosyncratic left-tail risk

	$NIMTA$	$EXRET$	$SIGMA$	$RSIZE$	$CASHMTA$	MB	$PRICE$	$constant$
Coefficients	-2.776^{***}	-0.368^{**}	0.090^{**}	0.001	-0.145^{***}	-0.017^{***}	-0.081^{***}	0.373^{***}
	[−4.76]	[−2.21]	[2.25]	[0.17]	[−2.77]	[−4.33]	[−4.16]	[5.48]

Note: Panel A presents the summary statistics for the industry-level variables included in the vector $X_{i,t}$ of specification (50) in Section 6.1 of the main text. These are the sales-weighted average of the following firm-level variables: weighted average of net income over market value of total assets ($NIMTA$), weighted average of log of gross excess return over value-weighted S&P 500 return ($EXRET$), square root of the sum of squared firm stock returns over a 3-month period ($SIGMA$) annualized, log of firm's market equity over the total valuation of S&P 500 ($RSIZE$), stock of cash and short-term investments over the market value of total assets ($CASHMTA$), market-to-book ratio of the firm (MB), and log of price per share winsorized above \$15 ($PRICE$). Panel B presents the estimated coefficients for specification (50) in Section 6.1 of the main text. The sample spans the period from 1971 to 2021. t-statistics robust to heteroskedasticity and autocorrelation are reported in brackets. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively.

Measure of Idiosyncratic Shocks. As noted in Section 6.1 of the main text, we construct firm-level idiosyncratic shocks using two methods, $M1$ and $M2$. We provide more details for $M1$ here. First, we compute the annual sales growth of each firm, censoring the rare instances in which the sales of the firm in the last year are negative. Second, we compute average sales growth based on a panel consisting of the top 100 firms (ranked by sales) in each year. The firm-year observations are winsorized at 30% above and below the average sales growth. Third, we compute aggregate sales growth as the average winsorized sales growth of the top 100 firms in each year. The firm-level idiosyncratic shocks are their winsorized sales growth minus aggregate sales growth.

6.2 Supplemental Information for Constructing $\nu_{i,t}$

Panel A of Table 6 presents the summary statistics for the industry-level variables included in the vector $X_{i,t}$ of regression specification (50) in Section 6.1 of the main text. Panel B presents the estimated coefficients for this regression specification.

Table 7: Financial contagion effect on credit spreads and CDS spreads within an industry.

$Idio\_Shock_{i,t}^{(H)}$	(1)	(2)	(3)	(4)
	Credit spread		CDS spread	
	$Credit\_Spread_{i,t}^{(L)}$	$Credit\_Spread_{i,t}^{(H)}$	$CDS\_Spread_{i,t}^{(L)}$	$CDS\_Spread_{i,t}^{(H)}$
$M1$	-1.004^{**}	-2.245^{***}	-0.232	-7.492^{***}
	[−2.19]	[−2.76]	[−0.55]	[−5.78]
$M2$	-1.132^{*}	-1.818^{**}	-0.957^{*}	-7.765^{***}
	[−1.96]	[−2.10]	[−1.99]	[−3.83]

Note: This table studies the financial contagion effect on credit spreads within an industry. Column (1) reports the coefficient β_H in specification (175), which captures the contagion effect on credit spreads, namely, the effect of idiosyncratic shocks to firms in group H (financially distressed firms) on the credit spread of firms in group L (financially healthy firms). Column (2) reports the coefficient β_H in specification (176), which captures the effect of idiosyncratic shocks to firms in group H on these firms' own credit spreads. $M1$ and $M2$ represent the two methods for constructing firms' idiosyncratic shocks (see Section 6.1 of the main text). Columns (3) and (4) report the estimated effects on CDS spreads based on specifications similar to columns (1) and (2), respectively. The sample is yearly for all columns. In columns (1) and (2), the sample spans the period from 1988 to 2018 and the number of observations is 1,079. In columns (3) and (4), the sample spans the period from 2001 to 2018 and the number of observations is 143. t-statistics robust to heteroskedasticity and autocorrelation are reported in brackets. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively.

6.3 Contagion Effects on Credit Spreads

Our model implies that the financial contagion effect among market leaders within the same industry is reflected in firms' credit spreads. To test this prediction, we run regressions according to specification (53) of the main text, except for using the group-level credit spreads on both sides:

$$Credit\_Spread_{i,t}^{(L)} = \sum_{k\in\{H,L\}} \beta_k Idio\_Shock_{i,t}^{(k)} + \sum_{s=1}^{5} \gamma_s Credit\_Spread_{i,t-s}^{(L)} + \delta_t + \ell_i + \epsilon_{i,t}. \quad (175)$$

The coefficient β_H captures the effect of idiosyncratic shocks to firms in group H (i.e., firms that are more financially distressed) on the credit spreads of firms in group L (i.e., firms that are financially healthier), reflecting the contagion effect on credit spreads. Column (1) of Table 7 shows that the coefficient β_H is negative and statistically significant for the idiosyncratic shocks constructed using method $M1$, indicating that positive idiosyncratic shocks to group H reduce the credit spread of group L. Consistent with the small estimate for the financial contagion effect on profit margins (see Table 3 of the main text), our estimate for the financial contagion effect on credit spreads is also small. According to the estimates based on the method $M1$, in response to a one percentage-point positive idiosyncratic shock in the sales growth of group H, the credit spread of group L decreases by 1.004 basis points. For large idiosyncratic shocks, our estimate indicates that a two standard-deviation increase in the sales growth of group H would reduce the credit spread of group L by 22 basis points, which is sizable given that the standard deviation of group H's sales growth is about 11 percentage points.

As an alternative way to interpret the economic magnitude of the financial contagion effect, we compare the impacts of idiosyncratic shocks to firms in group H on the credit spreads of group L with the impacts of idiosyncratic shocks to firms in group H on the credit spreads of group H itself. The former captures the spillover effects, which are estimated in specification (175); and the latter captures the direct effects on group H itself, which are estimated using the following regression:

$$Credit\_Spread_{i,t}^{(H)} = \sum_{k \in \{H,L\}} \beta_k Idio\_Shock_{i,t}^{(k)} + \sum_{s=1}^{5} \gamma_s Credit\_Spread_{i,t-s}^{(H)} + \delta_t + \ell_i + \epsilon_{i,t}, \quad (176)$$

which differs from specification (175) in that $Credit\_Spread_{i,t}^{(L)}$ is replaced with $Credit\_Spread_{i,t}^{(H)}$ on both sides of the regression. Column (2) of Table 7 shows that for group H, a one percentage-point positive idiosyncratic shock in its sales growth is associated with a 2.245 basis points decrease in its credit spread, according to the estimates based on the method $M1$. Taking the estimates in columns (1) and (2) together, we find that the contagion effect on credit spreads has a high pass-through rate of about 44.7% (= 1.004/2.245).

In columns (3) and (4), we report the estimated effects on CDS spreads based on specifications (175) and (176), respectively. In column (3), the estimated contagion effect on CDS spreads is insignificant based on the method $M1$ for constructing idiosyncratic shocks; and the estimate is significant and has comparable magnitude as the estimated contagion effect on credit spreads based on the method $M2$. The estimates for financial contagion effects using CDS spreads are not as robust as those using credit spreads mainly due to the small sample size. In the cross section, the information of CDS spreads is not available for many firms whose credit spreads are available. Moreover, the CDS spread data are available after 2001 whereas credit spreads data are available since 1988. As a result, the number of observations used for columns (3) and (4) is 143, as compared with 1,079 for columns (1) and (2).

6.4 Idiosyncratic Left-Tail Risk and Financial Distress Anomaly

We now test the cross-industry asset pricing implications of the full-fledged quantitative model with aggregate shocks developed in Online Appendix 4. In Subsection 6.4.1, we show that industries with higher financial distress or higher idiosyncratic left-tail risk are associated with lower expected equity excess returns and lower CAPM alphas, but higher credit spreads and higher CDS sprads. In Subsection 6.4.2, we show that industries with higher financial distress or idiosyncratic left-tail risk are less negatively exposed to discount-rate shocks. Finally, in Subsection 6.4.3, we show that the industry-level distress spread becomes statistically insignificant after controlling for the industry-level measure of idiosyncratic left-tail risk.

6.4.1 Equity Returns and Credit Spreads in the Cross Section

We sort all SIC4 industries into quintiles based on the industry-level financial distress measure $Distress_{i,t}$. Panel A of Table 8 shows that the expected equity excess returns and CAPM alphas of industries with high $Distress_{i,t}$ (i.e., industries in quintile group 5 (Q5 in the table)) are significantly lower than those with low $Distress_{i,t}$ (i.e., industries in quintile group 1 (Q1 in the table)). The difference in annualized expected equity excess returns is -4.935% (Q5$-$Q1) and is significant both statistically and economically. The empirical results on equity returns remain robust after controlling for the market factor. In particular, we find that more financially distressed industries have lower CAPM alphas. These industry-level patterns are consistent with the financial distress anomaly documented at the firm level (e.g., Campbell, Hilscher and Szilagyi, 2008). In terms of bond returns, panel A of Table 8 shows that industries with higher $Distress_{i,t}$ are associated with higher credit spreads and CDS spreads, which is in sharp contrast to the lower expected equity excess returns and CAPM alphas associated with these industries.

Panel A of Table 9 shows that similar results are obtained if we sort industries on the empirical measure of idiosyncratic left-tail risk ($\nu_{i,t}$). In particular, industries with higher $\nu_{i,t}$ have significantly lower expected equity excess returns and CAPM alphas but significantly higher credit spreads and CDS spreads than industries with lower $\nu_{i,t}$.

6.4.2 Equity Betas to the Discount Rate

Our full-fledged quantitative model with aggregate shocks in Online Appendix 4 suggests that industries with higher financial distress or idiosyncratic left-tail risk have lower expected equity excess returns because they are less negatively exposed to aggregate discount-rate shocks. We test this prediction by estimating the equity beta to the discount rate. Specifically, in each quintile group k of industries sorted on $Distress_{i,t}$, we run the following time-series regression:

$$R_{k,t} - R_{f,t} = \alpha_k + \beta_k \Delta Discount\_rate_t + \gamma_k Mkt_t + \epsilon_{k,t}, \tag{177}$$

Table 8: Industry portfolios sorted on financial distress.

$Distress_{i,t}$	(1) Q1 (low)	(2) Q2	(3) Q3	(4) Q4	(5) Q5 (high)	(6) Q5−Q1
	Panel A: Equity returns, credit spreads, and CDS spreads					
Excess return	9.508***	11.022***	10.869***	9.650***	4.573	−4.935**
	[5.11]	[5.53]	[5.79]	[4.78]	[1.77]	[−2.28]
CAPM alpha	0.921	1.945	1.146	−0.884	−6.680***	−7.601***
	[0.83]	[1.63]	[0.78]	[−0.49]	[−3.17]	[−3.44]
Credit spread	0.992***	1.253***	1.430***	1.814***	3.338***	2.346***
	[9.40]	[7.08]	[7.47]	[7.72]	[7.95]	[7.33]
CDS spread	0.298***	0.409***	0.530***	0.812***	2.046***	1.748***
	[6.14]	[5.07]	[5.14]	[4.11]	[5.86]	[5.51]
	Panel B: Equity betas to the discount rate					
$\Delta Discount\_rate_t$	−5.510**	−1.587	−0.544	1.094	4.567	10.077**
	[−2.48]	[−0.62]	[−0.19]	[0.33]	[1.46]	[2.39]
Mkt_t	0.909***	1.104***	1.189***	1.269***	1.463***	0.554***
	[12.41]	[13.83]	[28.05]	[18.15]	[18.83]	[4.61]

Note: Panel A reports the expected equity excess returns, CAPM alphas, credit spreads, and CDS spreads of industry portfolios sorted on financial distress ($Distress_{i,t}$). Panel B reports the equity beta to the discount rate, estimated using specification (177). All numbers are in annualized percentage units. For panel A, the sample is monthly and spans the period from 1975 to 2021, except for the credit spread and CDS spread rows, which spans the period from 1988 to 2018 and from 2001 to 2018, respectively. The number of observations is 42,594 except for the credit spread and CDS spread rows, where the number of observations is 12,963 and 2,499, respectively. For panel B, the sample is quarterly and spans the period from 1975 to 2021. The number of observations is 14,020. t-statistics robust to heteroskedasticity and autocorrelation are reported in brackets. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively.

where $R_{k,t}$ is the equity return of quintile group k, $R_{f,t}$ is the risk-free rate, $\Delta Discount\_rate_t$ is the discount-rate shock, and Mkt_t is the market's excess return over the risk-free rate.

Panel B of Table 8 shows that the expected equity excess returns of industry portfolios of low financial distress (Q1) are significantly more negatively exposed to the discount rate compared to those of high financial distress (Q5). Panel B of Table 9 shows that the results are similar if we sort industries based on the measure of idiosyncratic left-tail risk ($\nu_{i,t}$).

6.4.3 Financial Distress Anomaly After Controlling For $\nu_{i,t}$

In our full-fledged quantitative model with aggregate shocks in Online Appendix 4, the financial distress spread across industries can be explained by industries' heterogeneous exposure to idiosyncratic left-tail risk. We now empirically examine whether the financial distress anomaly becomes significantly less pronounced after controlling for idiosyncratic

Table 9: Industry portfolios sorted on idiosyncratic left-tail risk.

$\nu_{i,t}$	(1) Q1 (low)	(2) Q2	(3) Q3	(4) Q4	(5) Q5 (high)	(6) Q5−Q1
	Panel A: Equity returns, credit spreads, and CDS spreads					
Excess return	9.947***	10.284***	10.059***	9.777***	3.890	−6.057***
	[5.24]	[5.78]	[4.72]	[4.68]	[1.54]	[−2.59]
CAPM alpha	1.552*	1.465	0.296	−0.553	−7.184***	−8.736***
	[1.68]	[1.39]	[0.22]	[−0.35]	[−2.86]	[−3.61]
Credit spread	1.191***	1.349***	1.504***	1.709***	3.101***	1.910***
	[9.48]	[8.32]	[7.87]	[7.61]	[7.07]	[5.41]
CDS spread	0.389***	0.511***	0.555***	0.825***	1.866***	1.478***
	[8.96]	[5.99]	[4.86]	[4.61]	[5.40]	[4.55]
	Panel B: Equity beta to the discount rate					
$\Delta Discount\_rate_t$	−4.364**	−3.322	−0.967	2.534	5.369	9.733**
	[−1.99]	[−1.55]	[−0.37]	[0.81]	[1.28]	[2.04]
Mkt_t	0.956***	1.042***	1.192***	1.321***	1.460***	0.504***
	[14.60]	[25.10]	[21.03]	[20.71]	[13.75]	[4.08]

Note: This table performs the same analysis as that in Table 8, except for sorting industries on the measure of idiosyncratic left-tail risk ($\nu_{i,t}$). All numbers are in annualized percentage units. For panel A, the sample is monthly and spans the period from 1976 to 2021, except for the credit spread and CDS spread rows, which spans the period from 1988 to 2018 and from 2001 to 2018, respectively. The number of observations is 42,385 except for the credit spread and CDS spread rows, where the number of observations is 12,963 and 2,499, respectively. For panel B, the sample is quarterly and spans the period from 1976 to 2021. The number of observations is 13,943. t-statistics robust to heteroskedasticity and autocorrelation are reported in brackets. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively.

left-tail risk. Specifically, we perform a double-sort analysis, where we first sort industries into quintiles based on the measure of idiosyncratic left-tail risk ($\nu_{i,t}$). Then, for industries within each quintile portfolio, we further sort them into quintiles based on the financial distress measure ($Distress_{i,t}$). Panel A of Table 10 shows that the differences in expected equity excess returns and CAPM alphas between high distress (Q5) and low distress (Q1) industry portfolios become statistically insignificant. The magnitude of the differences (Q5−Q1) in expected equity excess returns and CAPM alphas changes from −4.935% and −7.601%, respectively, in panel A of Table 8 to −1.778% and −2.563%, respectively, in panel A of Table 10.

Panel B of Table 10 performs an additional test by sorting industries on an adjusted financial distress measure ($Distress\_adjusted_{i,t}$) which controls for idiosyncratic left-tail risk $\nu_{i,t}$. Specifically, $Distress\_adjusted_{i,t}$ is the residuals of regressing the financial distress measure ($Distress_{i,t}$) on the measure of idiosyncratic left-tail risk ($\nu_{i,t}$) and a constant term

Table 10: Financial distress anomaly after controlling for idiosyncratic left-tail risk.

	(1)	(2)	(3)	(4)	(5)	(6)
		Panel A: Double sort on $Distress_{i,t}$ and $\nu_{i,t}$				
$Distress_{i,t}$	Q1 (low)	Q2	Q3	Q4	Q5 (high)	Q5−Q1
Excess return	9.354***	9.578***	7.463***	9.875***	7.576***	−1.778
	[5.28]	[4.57]	[3.68]	[4.71]	[3.79]	[−1.23]
CAPM alpha	0.164	−0.052	−2.239**	−0.010	−2.398*	−2.563
	[0.10]	[−0.03]	[−2.07]	[−0.01]	[−1.71]	[−1.64]
		Panel B: Industry portfolios sorted on $Distress\_adjusted_{i,t}$				
$Distress\_adjusted_{i,t}$	Q1 (low)	Q2	Q3	Q4	Q5 (high)	Q5−Q1
Excess return	8.526***	7.740***	9.743***	9.961***	7.791***	−0.735
	[4.04]	[3.39]	[5.54]	[6.00]	[3.40]	[−0.46]
CAPM alpha	−1.747	−2.221	0.335	1.014	−2.087*	−0.339
	[−0.85]	[−1.62]	[0.25]	[0.85]	[−1.68]	[−0.22]

Note: This table studies the financial distress anomaly after controlling for idiosyncratic left-tail risk. In panel A, we perform a double-sort analysis. We first sort industries into quintiles based on $\nu_{i,t}$. Then, we further sort industries within each quintile portfolio into quintiles based on $Distress_{i,t}$. Panel B performs a single-sort analysis based on the adjusted financial distress measure ($Distress\_adjusted_{i,t}$), which is computed as the residuals of regressing $Distress_{i,t}$ on $\nu_{i,t}$ and a constant term in the cross section of industries in each month t. All numbers are in annualized percentage units. The sample is monthly and spans the period from 1976 to 2021. The number of observations is 42,385. t-statistics robust to heteroskedasticity and autocorrelation are reported in brackets. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively.

in the cross section of industries in each month t. It is evident that both of the differences (Q5−Q1) in expected equity excess returns and CAPM alphas become statistically insignificant once we sort industries on $Distress\_adjusted_{i,t}$.

6.5 Economic Force Captured by the Industry-Level Distress Spread

In Online Appendix 4.4, our full-fledged quantitative model with aggregate shocks suggests that the economic force that generates the industry-level distress spread, i.e., the returns of the long-short portfolio of industries sorted on $Distress_{i,t}$, is the industries' differential exposure to discount-rate shocks ($Z_{\gamma,t}$), rather than their differential exposure to cash-flow shocks (W_t). Ideally, we would empirically test this prediction by investigating whether the industry-level distress spread is explained by its loading on discount-rate shocks or its loading on cash-flow shocks. However, perfect empirical measures for discount-rate shocks

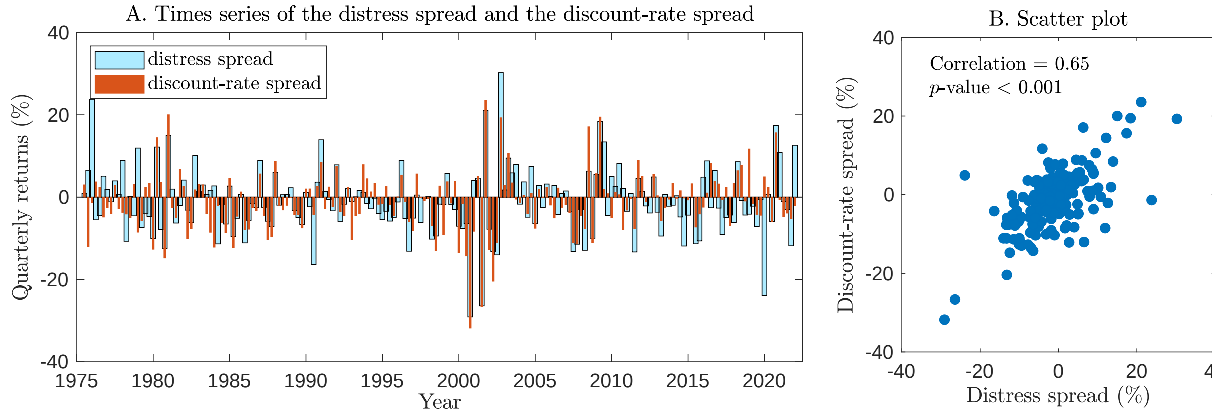

Figure 9. Correlation between the industry-level distress spread and the industry-level discount-rate spread.

or cash-flow shocks do not exist, making it difficult to implement this ideal test.

In this appendix section, we provide a complementary empirical test for this prediction. Specifically, we estimate the expected loadings of industry returns on discount-rate shocks. We show that the returns of the long-short portfolio of industries sorted on their expected loadings on discount-rate shocks comove with the industry-level distress spread. The two time series have a high correlation coefficient of 0.65. Furthermore, the industry-level distress spread becomes statistically insignificant after controlling for the returns of this long-short portfolio.

We first estimate the expected loadings of industry returns on discount-rate shocks. This is achieved in three steps. First, we estimate the realized loading of industry returns on discount-rate shocks using a moving window regression. For each industry i in quarter t, we estimate the following regression over a four-quarter horizon from $\tau = t - 3$ to $\tau = t$, consistent with the horizon used by Campbell, Hilscher and Szilagyi (2008) for the estimation of failure probability:

$$R^e_{i,\tau} = a_t + b^1_{i,t} \Delta Discount\_rate_\tau + b^2_{i,t} Mkt_\tau + \varepsilon_{i,\tau}, \tag{178}$$

where $R^e_{i,\tau} = R_{i,\tau} - R_{f,\tau}$ is the excess return of industry i over the risk-free rate in quarter τ, and $\Delta Discount\_rate_\tau$ and Mkt_τ are the discount-rate shock and the market's excess return over the risk-free rate in quarter τ, respectively. The estimated coefficient $\widehat{b}^1_{i,t}$ is the realized loading of industry i in quarter t (i.e., over the four-quarter horizon from $t - 3$ to t).

Second, we relate the realized loading of industry returns on discount-rate shocks in quarter $t + 4$ to industry characteristics in quarter t by estimating the following panel regression using industry-quarter observations between 1975 and 2021:

$$\widehat{b}^1_{i,t+4} = \alpha + \beta X_{i,t} + \epsilon_{i,t+4}, \tag{179}$$

Table 11: The industry-level distress spread can be explained by the industry-level discount-rate spread.

	(1)	(2)	(3)	(4)
Intercept	-4.935^{**}	-1.467	-7.601^{***}	-2.965
	[-2.28]	[-0.67]	[-3.44]	[-1.37]
Industry-level discount-rate spread		0.645^{***}		0.596^{***}
		[9.59]		[9.36]
Mkt			0.316^{***}	0.146^{***}
			[3.64]	[3.09]

Note: This table shows that the industry-level discount-rate spread can explain the industry-level distress spread. Column (1) reports the expected return of the industry-level distress spread. Column (2) reports the coefficients of regressing the industry-level distress spread on the industry-level discount-rate spread and a constant. In columns (3) and (4), we perform regressions similar to those in columns (1) and (2), except for additionally controlling for the market excess return over the risk-free rate. The sample is monthly and spans the period from 1975 to 2021. The number of observations for constructing the industry-level distress spread (discount-rate spread) is $42,594$ $(45,032)$. All numbers are in annualized percentage units. t-statistics robust to heteroskedasticity and autocorrelation are reported in brackets. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively.

where $X_{i,t}$ is a vector of industry-level variables, constructed as the sales-weighted average of firm-level variables used by Fama and French (2015) in constructing the Fama-French five-factor model and those used by Campbell, Hilscher and Szilagyi (2008) in constructing firms' failure probability. Specifically, these firm-level variables include firm's net income over market value of total assets ($NIMTA$), total liabilities over market value of total assets ($TLMTA$), log of firm's gross excess return over value-weighted S&P 500 return ($EXRET$), square root of the sum of squared firm's stock returns over a 3-month period ($SIGMA$) annualized, log of firm's market equity over the total valuation of S&P 500 ($RSIZE$), firm's stock of cash and short-term investments over the market value of total assets ($CASHMTA$), market-to-book ratio of the firm (MB), log of firm's price per share winsorized above \$15 ($PRICE$), revenue minus cost of goods sold, minus selling, general, and administrative expenses, minus interest expense divided by book equity (OP), and the change in total assets from the last year to the current year divided by total assets (INV).

Third, we construct the measure of the expected loading of industry returns on discount-rate shocks for industry i in month t as

$$Discount\_loading_{i,t} = \widehat{\alpha} + \widehat{\beta} X_{i,t}. \tag{180}$$

Next, we sort industries into quintiles based on $Discount\_loading_{i,t}$.[6] We construct the industry-level discount-rate spread as the returns of the long-short portfolio of industries sorted on $Discount\_loading_{i,t}$, i.e., the returns of quintile group 5 (industries with a high $Discount\_loading_{i,t}$) minus the returns of quintile group 1 (industries with a low $Discount\_loading_{i,t}$). Then, we relate this industry-level discount-rate spread to the industry-level distress spread, which is constructed as the returns of the long-short portfolio of industries sorted on $Distress_{i,t}$, i.e., the returns of quintile group 5 (industries with high $Distress_{i,t}$) minus the returns of quintile group 1 (industries with low $Distress_{i,t}$). Figure 9 shows that the industry-level distress spread is strongly correlated with the industry-level discount-rate spread, with a correlation coefficient equal to 0.65. This supports our model's prediction that more distressed industries are associated with higher loadings on discount-rate shocks (i.e., less negatively exposed to discount-rate shocks) and lower expected equity returns.

In Table 11, we further examine whether the industry-level discount-rate spread can explain the industry-level distress spread. Columns (1) and (3) of Table 11 show that the expected return and CAPM alpha of the industry-level distress spread are negative and statistically significant, similar to the results shown in Table 8. Columns (2) and (4) of Table 11 show that the magnitude of the intercepts drops significantly and becomes statistically insignificant after controlling for the industry-level discount-rate spread. Moreover, the industry-level distress spread has a positive and significant loading on the industry-level discount-rate spread in both columns (2) and (4). These results lend further support to our model prediction that the financial distress spread across industries is attributed to the difference in industries' loadings on discount-rate shocks.

6.6 Financial Distress Anomaly Across Firms

Our full-fledged quantitative model with aggregate shocks in Online Appendix 4 and empirical analyses in Online Appendix 6.4 focus on the financial distress anomaly across industries. In this subsection, we present the empirical results at the firm level. In panel A of Table 12, we sort all firms into quintiles based on the firm-level financial distress measure and examine the returns of each portfolio. It is clear that the financial distress anomaly is significant at the firm level. The difference in annualized expected equity excess

[6] Note that it is reasonable to perform the portfolio sorting based on the predicted loading (i.e., $Discount\_loading_{i,t}$ estimated in equation (180)) rather than the realized loading (i.e., $\widehat{b}^{1}_{i,t}$ estimated in equation (178)) of industry returns on discount-rate shocks. The realized loading could be noisy as we use a 4-quarter moving window when estimating equation (180), following Campbell, Hilscher and Szilagyi (2008). We do not use a longer moving window in our estimation because in this case, the estimated loadings cannot very well capture the time-varying component in industries' exposure to discount-rate shock.

returns between quintile groups 5 and 1 (i.e., Q5−Q1) is −10.239% and is significant both statistically and economically. Compared with panel A of Table 8, the financial distress spread across firms is about twice that across industries.

Table 12: Excess returns in the cross section of firms.

	(1)	(2)	(3)	(4)	(5)	(6)
	Panel A: Portfolios sorted on financial distress across firms					
	Q1 (low)	Q2	Q3	Q4	Q5 (high)	Q5−Q1
Excess return	9.721***	9.149***	7.009**	3.253	−0.518	−10.239**
	[4.37]	[4.82]	[2.49]	[0.84]	[−0.11]	[−2.26]
CAPM alpha	2.012**	0.105	−4.147**	−9.702***	−15.444***	−17.456***
	[2.50]	[0.11]	[−2.22]	[−3.11]	[−3.58]	[−3.79]
	Panel B: Portfolios sorted on financial distress across firms within each industry					
	Q1 (low)	Q2	Q3	Q4	Q5 (high)	Q5−Q1
Excess return	10.174***	7.462***	7.630**	7.019**	1.803	−8.371**
	[4.43]	[3.86]	[2.77]	[2.02]	[0.40]	[−1.97]
CAPM alpha	2.225**	−1.203	−3.061**	−5.616**	−12.756***	−14.981***
	[2.32]	[−1.49]	[−1.98]	[−1.99]	[−3.29]	[−3.67]
	Panel C: Portfolios sorted on industry-adjusted financial distress across firms					
	Q1 (low)	Q2	Q3	Q4	Q5 (high)	Q5−Q1
Excess return	9.279***	9.601***	8.467***	6.453*	1.589	−7.690*
	[3.70]	[4.06]	[3.93]	[1.84]	[0.34]	[−1.90]
CAPM alpha	−0.640	1.403	−0.102	−5.881**	−13.146***	−12.506***
	[−0.46]	[1.52]	[−0.13]	[−2.33]	[−3.20]	[−3.39]

Note: This table reports expected equity excess returns and CAPM alphas of portfolios sorted on financial distress across firms. In panel A, we sort all firms based on the firm-level financial distress measure. In panel B, we sort firms within each industry based on the firm-level financial distress measure. In panel C, we construct an industry-adjusted financial distress measure for each firm, which is firm-level distress in excess of the average distress of firms within the same industry over the same time period. We then sort all firms based on the industry-adjusted financial distress measure. All numbers are in annualized percentage units. The sample is monthly and spans the period from 1975 to 2021. The number of observations is 1,129,379 in each panel. t-statistics robust to heteroskedasticity and autocorrelation are reported in brackets. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively.

In panel B of Table 12, we present the within-industry financial distress spread, constructed in spirit similar to the "industry-neutral" portfolios of Moskowitz and Grinblatt (1999). In particular, we sort firms within each industry into quintile portfolios based on the firm-level financial distress measure. Then, we pool all firms within the same quintile group of each industry together and compute the value-weighted expected equity excess returns and CAPM alphas. The difference in annualized expected equity excess returns between quintile groups 5 and 1 (i.e., Q5−Q1) captures the within-industry financial distress spread, which has a magnitude of −8.832% and is significant both statistically and economically. This magnitude is larger than the financial distress spread across industries reported in panel A of Table 8.

In panel C of Table 12, we present the within-industry financial distress spread using an alternative sorting method. Inspired by the "excess-industry" portfolios constructed by Moskowitz and Grinblatt (1999), we construct an industry-adjusted financial distress measure for each firm, which is firm-level distress in excess of the average distress of firms within the same industry over the same period. Intuitively, our industry-adjusted financial distress measure controls for the cross-industry variation in the degree of financial distress, and it captures each firm's degree of financial distress relative to its peers in the same industry. We sort all firms into quintile portfolios based on the industry-adjusted financial distress measure. Panel C shows that the difference in annualized expected equity excess returns between quintile groups 5 and 1 (i.e., Q5−Q1) is −7.690%, which is comparable with the one estimated in panel B of Table 12.

Overall, our results in Table 12 and panel A of Table 8 jointly indicate that the financial distress anomaly is significant across both firms and industries.

6.7 Potential Mechanisms to Explain the Distress Anomaly Across Firms

In this subsection, we present empirical evidence to show that the firm-level and industry-level financial distress spreads are likely to be explained by different theoretical mechanisms. Our full-fledged quantitative model with aggregate shocks in Online Appendix 4 provides an explanation for the financial distress spread across industries, but not for the one across firms.

Specifically, we test whether the financial distress spread across industries can be explained by the financial distress spread across firms in a time-series regression. As noted in Online Appendix 6.5, the industry-level distress spread is the returns of the long-short portfolio of industries sorted on $Distress_{i,t}$. We denote it by $industry\_distress\_spread_t$.

We construct the firm-level financial distress spread that excludes industry components following the idea of Moskowitz and Grinblatt (1999). Specifically, we construct an industry-adjusted financial distress measure for each firm, which is firm-level distress in excess of the average distress of firms within the same industry over the same period. This industry-adjusted financial distress measure captures firms' financial distress relative to

Table 13: The distress anomaly across industries is not explained by that across firms.

	(1)	(2)	(3)
	$industry\_distress\_spread_t$	$industry\_distress\_spread_t$	$industry\_distress\_spread_t$
Intercept	-4.146^{**}	-6.023^{***}	-3.920^{***}
	[−2.44]	[−4.34]	[−3.02]
$firm\_distress\_spread_t$	0.276^{***}	0.228^{***}	0.094^{**}
	[11.49]	[3.94]	[2.38]
Mkt_t	0.158^{***}	0.186^{***}	0.157^{***}
	[2.94]	[3.69]	[3.72]
HML_t		0.270^{**}	0.135
		[2.16]	[1.18]
SMB_t		0.226	0.419^{***}
		[1.56]	[4.29]
MOM_t			-0.415^{***}
			[−7.87]
Observations	558	558	558
R-squared	0.323	0.368	0.498

Note: This table tests whether the financial distress anomaly across industries can be explained by that across firms. We regress the time-series of $industry\_distress\_spread_t$ on $firm\_distress\_spread_t$, controlling for standard risk factors. The variable $industry\_distress\_spread_t$ is the returns of the long-short portfolio of industries sorted on $Distress_{i,t}$. To construct $firm\_distress\_spread_t$, we sort all firms into quintiles based on an industry-adjusted financial distress measure, which is firm-level distress in excess of the average distress of firms within the same industry over the same period. The variable $firm\_distress\_spread_t$ is the returns of the long-short portfolio of firms sorted on the industry-adjusted financial distress measure. All numbers are in annualized percentage units. The sample is monthly and spans the period from 1975 to 2021. The number of observations for constructing the industry-level (firm-level) distress spread is 42,594 (1,129,379). All numbers are in annualized percentage units. t-statistics robust to heteroskedasticity and autocorrelation are reported in brackets. *, **, and *** indicate statistical significance at 10%, 5%, and 1%, respectively.

the industry benchmark, and is similar in spirit to the method used by Moskowitz and Grinblatt (1999) in constructing "excess-industry" portfolios. We then sort all firms into quintiles based on the industry-adjusted financial distress measure.[7] The firm-level distress spread, denoted by $firm\_distress\_spread_t$, is the returns of the long-short portfolio of firms sorted on the industry-adjusted financial distress measure.

We then regress the time-series of $industry\_distress\_spread_t$ on $firm\_distress\_spread_t$, controlling for standard risk factors. In columns (1) to (3) of Table 13, we gradually

[7] We do not sort firms based on firm-level financial distress directly, because firm characteristics also reflect industry effects (Moskowitz and Grinblatt, 1999).

add market excess returns over the risk-free rate, the value, size, and momentum factors as controls. All three columns show that $industry\_distress\_spread_t$ loads positively on $firm\_distress\_spread_t$. However, column (3) shows that the loading of $industry\_distress\_spread_t$ on $firm\_distress\_spread_t$ is not strong and the magnitude is much smaller than its loadings on other risk factors. The estimated intercept in all three regression specifications is statistically significant, indicating that $industry\_distress\_spread_t$ is not fully explained by $firm\_distress\_spread_t$ or standard risk factors in our sample from 1975 to 2021.

This empirical finding suggests that the industry- and firm-level distress anomalies are due to different economic mechanisms. Our full-fledged quantitative model with aggregate shocks in Online Appendix 4 provides a potential mechanism to explain the financial distress spread across industries, but not the one across firms. In the literature, several papers have proposed mechanisms that focus on explaining the firm-level distress anomaly rather than the industry-level distress anomaly (e.g., Garlappi and Yan, 2011; Boualam, Gomes and Ward, 2020; Chen, Hackbarth and Strebulaev, 2022).

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