Online Appendix for:

Beware the Side Effects: Capital Controls, Trade and Misallocation

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A The Chilean "Encaje" of the 1990s

Between 1987 and 1997, Chile's economy experienced significant growth, with annual GDP growth surpassing 7.5%. This robust performance was driven by export-oriented policies and the cumulative effects of economic stabilization and liberalization measures initiated in the mid-1980s. Inflation reduction was a key achievement, supported by strict monetary and fiscal policies. The Central Bank of Chile (CBCh), which became independent in 1989, maintained tight monetary control, raising interest rates to combat inflation whenever necessary. Fiscal discipline also mattered: budget surpluses were sustained from 1987 to 1998, leading to the adoption of a structural surplus rule by 2000 (Le Fort Varela and Lehmann, 2003; Tapia and Ffrench-Davis, 2004; Cowan and Gregorio, 2005).

In this context, the early 1990s brought an unexpected surge in capital inflows, raising concerns about currency appreciation, reduced autonomy in monetary policy, and potential overheating of the economy due to the short-term nature of these inflows. The resumption of capital flows to emerging market economies after the Latin American debt crisis of the 1980s intensified these challenges. This environment made the trade-off between different macroeconomic objectives increasingly difficult and costly. In response, in 1991, the CBCh introduced the *encaje*—an unremunerated reserve requirement on selective inflows. This policy was unexpected, diverging from the broader trend of liberalization and adding an element of unpredictability to the policy framework (Tapia and Ffrench-Davis, 2004).

The *encaje* was a capital account restriction obliging private entities to hold an unremunerated fixed-term reserve deposit at the Central Bank for a fraction of the capital being brought into the country. This mechanism effectively acted as a tax per unit of time, declining with the maturity of the capital inflow (see Appendix A.1 for a detailed derivation of the equivalent tax rate).[1] The *encaje* was part of Chile's longstanding policy approach to capital controls, which dates back to the 1930s and evolved with varying levels of stringency. Following the Latin American debt crisis of

[1] The equivalent tax can also be viewed as a fee that foreign investors would need to pay the central bank upfront for borrowing from abroad, instead of placing an unremunerated reserve deposit.

the 1980s, capital controls were tightened but later relaxed as economic conditions stabilized.

The introduction of the *encaje* in 1991 occurred alongside efforts to liberalize other aspects of the capital account. For example, there were simultaneous relaxations in restrictions on outflows, such as foreign direct investment (FDI), bank lending abroad, and institutional investors' ability to invest internationally (Simone and Sorsa, 1999). Despite these broader liberalization measures, the *encaje* was a unique, market-based control that stood out as an independent policy move, aimed specifically at mitigating the adverse effects of the sudden capital influx. The total equivalent reserve deposit was economically significant, amounting to 1.9% of GDP from 1991 to 1998, and reaching 2.9% of GDP by 1997, representing 30% of that year's net capital inflows (Gallego et al., 2002).[2]

The introduction of the encaje, a relatively unexpected policy given the broader trend of liberalization and stable macroeconomic conditions, strengthens its interpretation as an exogenous shock. Although no policy is ever fully exogenous, the fact that it was not accompanied by major shifts elsewhere, and that it diverged from the prevailing direction of policy, makes it a particularly useful setting to study the effects of capital controls. This feature allows us to analyze firm- and industry-level responses in a context where identification is more credible. Appendix I further supports this interpretation by showing that our results are not driven by changes in other macroeconomic variables.

A.1 Tax-equivalent of the Chilean *encaje*

Intuitively, capital controls alter the effective interest rate faced by domestic private agents abroad, depending on whether they want to save or borrow. If they want to save, the interest rate remains equal to the risk- free interest rate r^*. But, if they want to borrow, the effective interest rate they face is higher and given by $(r^* + \tau)$, where τ is the tax-equivalent of the capital controls on funds borrowed with a g-months maturity. The methodology we describe below, based on the work of

[2]In terms of the macroeconomic effects of the *encaje*, empirical evidence suggests its most persistent impact was on the maturity structure of capital inflows, which was tilted towards longer-term investments (De Gregorio et al., 2000; Soto, 1997; Gallego and Hernández, 2003).

Table A.1: Main changes in the administration of the *Chilean encaje*

Jun-1991	20% URR introduced for all new credit Holding period (months)=min(max(credit maturity, 3),12) Holding currency=same as creditor Investors can waive the URR by paying a fix fee (Through a repo agreement at discount in favor of the central bank) Repo discount= US$ libor
Jan-1992	20% URR extended to foreign currency deposits with proportional HP
May-1992	Holding period (months)=12 URR increased to 30% for bank credit lines
Aug-1992	URR increased to 30% Repo discount= US$ libor +2.5
Oct-1992	Repo discount= US$ libor +4.0
Jan-1995	Holding currency=US$ only
Sep-1995	Period to liquidate US$ from Secondary ADR tightened
Dec-1995	Foreign borrowing to be used externally is exempt of URR
Oct-1996	FDI committee considers for approval productive projects only
Dec-1996	Foreign borrowing <US$ 200,000 (500,000 in a year) exempt of URR
Mar-1997	Foreign borrowing <US$ 100,000 (100,000 in a year) exempt of URR
Jun-1998	URR set to 10%
Sep-1998	URR set to zero

Note: URR=Unremunerated Reserve Requirement
Source: De Gregorio et al. (2000).

De Gregorio et al. (2000), constructs an estimate of τ derived from a no-arbitrage condition that factors in the requirement to make the reserve deposit at the central bank.

To compute τ, we first define r_g as the annual risk-fee return that funds borrowed for g-months invested in Chile need to yield in order for an investor to make zero profits: $r_g = r^* + \tau$. Let u be the fraction of a foreign loan that an investor has to leave as an unremunerated reserve deposit and h the period of time that this deposit must be kept at the Central Bank. Then, if the investment period is shorter than the maturity of the deposit–, i.e., $g < h$–, borrowing one dollar abroad at an annual rate of r^* to invest at an annual rate r_g in Chile for g months generates the following cash flows:

1. At $t = 0$, the entrepreneur can invest $(1 - u)$ at r_g.
2. At $t = g$, repaying the foreign loan implies the following cash flow: $-(1 + r^*)^{g/12}$.
3. At $t = h$, the reserve requirement is returned generating a cash flow u.

Because of arbitrage, it follows that r_g must be a rate such that the investor is indifferent between investing at home and abroad (computing all values as of time h, when u is returned):

$$(1-u)(1+r_g)^{g/12}(1+r^*)^{(h-g)/12}+u=(1+r^*)^{h/12}.$$

Since $r_g = r^* + \tau$, we can use this expression to solve for τ as the value that satisfies:

$$(1+r^*+\tau)^{g/12}=\frac{(1+r^*)^{g/12}-u(1+r^*)^{(g-h)/12}}{1-u}$$

If the investment horizon exceeds the term of the reserve requirement–, i.e., $h > g$–, the investor has to decide, at the end of the h-month period, whether to maintain the reserve requirement in Chile or to deposit the amount outside the country. In order to obtain closed-form solutions, we assume that the investor deposits outside the country at the risk-free interest rate. Under this assumption, the previous arbitrage condition remains the same for longer investment horizons.

Using the approximation that $(1+j)^x \approx (1+xj)$, the approximate tax-equivalent of the unremunerated reserve requirement is found by solving the following linear equation for τ:

$$1+gr^*-u(1+(g-h)r^*)=(1-u)(1+g(r^*+\tau)),$$

which yields:

$$\tau = r^*\frac{u}{1-u}\frac{h}{g}. \tag{A.1}$$

Based on the above description, computing τ requires data on the evolution of the reserve requirement (the value of u) and the length of the holding period for which the reserves had to remain at the central bank (h). These are reported in Table A.1. We also need a proxy for the risk-free interest rate at which the borrowed funds could have been invested abroad r^*, for which we used the value used in the calibration of Section 5 of the paper and a value for the targeted maturity of the funds invested in Chile g, for which we used 12 months.

B Types of Capital Controls

Capital controls on inflows can be broadly categorized into price-based (indirect controls) and quantity-based (direct controls) measures. Price-based measures include taxes on capital inflows or unremunerated reserve requirements (URR) on such flows. Quantity-based controls, on the other hand, imply authorization requirements, limits, or prohibition on certain flows. (Ostry et al., 2010). Controls may be economy-wide, sector-specific (usually the financial sector), or industry-specific (for example, strategic industries in the case of controls on FDI). Measures may apply to all flows or may differentiate by type or duration of the flow (debt, direct investment, short-term vs. medium or long-term.

Starting in the 1990s, capital controls were adopted particularly in emerging countries to strengthen the resilience of their domestic financial system. In advanced economies, these tools have been applied system-wide after the 2008 GFC (Galati and Moessner, 2018). Fernández et al. (2016) show that, on average, for 1995-2013 capital controls on inflows in middle-income countries, they are more than twice as high-income economies.

In practice, countries often introduce price and quantitative measures simultaneously. For example, Colombia (2007), Russia (2004), and Thailand (2006) adopted quantitative restrictions along with an URR (Ostry et al., 2011). Although in some cases taxes were applied, price-based capital controls were usually implemented as an URR. Besides the Chilean URR we consider in this paper, other examples of countries implementing URR are Colombia (1993, 2007), Thailand (1995, 2006), Russia (2004), and Indonesia (2010). Although an URR is conceptually like a tax as it can be computed as a tax-equivalent, the choice between the two is usually driven by administrative considerations. Typically, central banks have the authority to impose a URR but do not have the authority to levy taxes. That explains why most countries have adopted URRs. An exception is the tax on capital inflows from abroad (IOF[3]) implemented in Brazil in 2008. This tax had been implemented in the past and the Ministry of Finance had the authority to adjust its rate (Ostry

[3]Imposto Sobre Operações Financeiras

et al., 2011).

Quantity-based capital controls have also been widely implemented. The most common measures include prohibitions, or minimum maturity requirements for certain types of inflows (Brazil, 1994; Colombia, 2004), limits on banks foreign exchange positions (Mexico, 1990; Malaysia, 1994; Czech Republic, 1995; Croatia 2003; Colombia 2007; South Korea, 2009; Peru 2010,) along with administrative approvals (Czech Republic, 1995) and prohibition or limits of foreign exchange transactions (Malaysia, 1994; Iceland, 2008; Cyprus 2013; Nigeria 2015). (Magud et al., 2018)

Note that, as is often the case, capital controls are implemented through an URR or quantity restrictions, those affected by the policy typically receive no compensation. Even in Brazil, where capital controls took the form of a tax, the revenue collected was transferred to the federal treasury (Tesouro Nacional) and used to finance general public expenditures (Secretaria do Tesouro Nacional, 2018).

C Solution Method

To solve for the model's recursive stationary equilibrium, we solve for aggregate prices $\{w, p\}$, final goods output $\{y\}$, entrepreneurs' decision rules $\{c'(\nu, z), a'(\nu, z), n'(\nu, z), \tilde{m}'(\nu, z), p'_h(\nu, z), p'_f(\nu, z), y'_h(\nu, z), y'_f(\nu, z), d'(\nu, z), k'(\nu, z), e(\nu, z)\}$, lump-sum taxes $T(z)$, and value functions $v(\nu, z)$, $v^{NE}(\nu, z), v^S(\nu, z), v^E(\nu, z)$ such that equilibrium conditions (1)–(5) of Section 3.4 hold.

The productivity process $f(z)$ is discretized by means of a Gauss-Hermite quadrature algorithm. We include $n_z = 10$ nodes and use the QWLOGN algorithm from Miranda and Fackler (2004). To solve the second-stage problems of exporters, non-exporters and switchers, we use analytic solutions.[4] The first-stage problems of exporters and switchers are solved using the endogenous grids method (EGM) proposed by Carroll (2006), and for non-exporters we use the discrete-choice augmented version of EGM developed by Iskhakov et al. (2017). The algorithm exploits the fact that the entrepreneurs' problems are effectively deterministic. Two properties of

[4]Note that the FOCs yield analytic expressions in all regions for the benchmark ALBC case. For region 1 in the ELBC case, where this is not the case, we solve for the capital policy function $k'(\nu, z)$ using Newton's method.

the model yield this outcome. First, productivity is stochastic only when firms are born, and is observed before they make their first-period decisions. Second, the Blanchard-Yaari OLG structure includes an insurance environment that allows entrepreneurs to perfectly diversify this risk.

The algorithm is as follows:

1. Initialize aggregate quantities and prices (w, p, y).

2. Given a guess for (w, p, y), solve the entrepreneur's problem for each $z \in Z$ as follows:

   (a) Initialize the entrepreneur's steady-state exporting status to $\overline{e}(z) = 1$ if $z > \hat{z}$ and $\overline{e}(z) = 0$ otherwise, where $\hat{z}$ is a guess for the highest z such that all entrepreneurs with productivity $\hat{z}$ are non-exporters for all relevant m.[5]

   (b) Given $\overline{e}(z)$, compute steady-state capital $\overline{k}(z)$, capital endowments for newborn firms $\underline{k}(z) = \kappa\overline{k}(z)$, and lump-sum taxes $T(z)$ that balance the government budget $T(z) = \rho p \underline{k}(z)$.

   (c) Define grids for future net-worth $a'_\vartheta(z)$ for each exporting state $\vartheta \in \Theta$[6,7] and compute the associated 2nd-stage grids for cash-on-hand $\widetilde{m}'_\vartheta(a', z)$; debt $d'_\vartheta(a', z)$; and collateral- and debt- constraint multipliers—$\eta'_\vartheta(a', z)$ and $\mu'_\vartheta(a', z)$—using the FOCs.

   (d) Given grids $a'_\vartheta(z)$, $\widetilde{m}'_\vartheta(a', z)$, $d'_\vartheta(a', z)$, $\eta'_\vartheta(a', z)$, and $\mu'_\vartheta(a', z)$ for $\vartheta \in \Theta$:[8]

      i. Solve for $a'_E(m, z)$ and $c_E(m, z)$ using the EGM to iterate on the 1st-stage exporter's Euler-equation. Compute the exporter's value function $v^E(m, z)$ through iteration on the converged policy functions.

[5] Note that low productivity entrepreneurs can be ruled out from exporting if they cannot afford the exporting fixed-cost in a non-exporting steady-state, that is, $\overline{e}(z) = 0$ when $\overline{m}_N(z) - (1-\rho)\overline{a}_N(z) < \frac{w}{p}F$.

[6] $\vartheta \in \Theta \equiv \{E, S, N\}$ denotes the entrepreneur's exporting-state—respectively exporters, switchers, and non-exporters—and $\mathbb{1}_S(\vartheta)$ denotes an indicator function for switching. Since exporting is irreversible and delayed by one period, the entrepreneur's exporting-state ϑ is described by: $\vartheta_t = E$ if $e_t = 1$; $\vartheta_t = S$ if $e_t = 0$ and $e_{t+1} = 1$; and $\vartheta_t = N$ otherwise.

[7] We define equally spaced grids $A'_\vartheta(z)$ over $[\kappa_0\underline{k}(z), \kappa_1\overline{a}_\vartheta(z)]$, where $\overline{a}_\vartheta(z)$ denotes the 2nd-stage policy functions' steady-state kink conditional on exporting state $\vartheta \in \Theta$, $0 < \kappa_0 < 1$ and $\kappa_1 > 1$. In most applications we use $n_{a'} = 20.000$ points and set $\kappa_0 = 0.75$ and $\kappa_1 = 1.25$.

[8] The net-worth policy functions $a'_\vartheta(m, z)$ are linearly interpolated and extrapolated, and the value functions $v^\vartheta(m, z)$ are interpolated linearly and extrapolated using cubic splines where needed.

ii. Given $a'_E(m,z)$, solve for $a'_S(m,z)$ and $c_S(m,z)$ using the EGM on the switcher's 1st-stage Euler-equation. Compute the switcher's value function $v^S(m,z)$ using $v^E(m,s)$ and these policy functions.

iii. Given $a'_S(m,z)$ and $v^S(m,z)$, solve for $a'_N(m,z)$, $c_N(m,z)$, $e'_N(m,z)$, and $v^N(m,z)$ by applying the DC-EGM to the non-exporter's 1st-stage Euler-equation iteration, and compute the cash-on-hand switching threshold $\widehat{m}(z)$ given by $v^S(\widehat{m},z) > v^N(\widehat{m},z)$.

(e) Given $k_0(z) = \underline{k}(z)$, $T(z)$, and $\widehat{m}(z)$, solve the entrepreneur's initial life-cycle states: compute the newborn firm's cash on hand $m_0(z) = \underline{m}(z)$ and determine the initial exporting state $\underline{\vartheta}(z)$ using the switching threshold $\widehat{m}(z)$.

(f) Given initial states $m_0(z) = \underline{m}(z)$ and $\vartheta_0(z) = \underline{\vartheta}(z)$; policy functions $\{a'_\vartheta(m,z), m'_\vartheta(a',z)\}_{\vartheta\in\Theta}$; and the switching threshold $\widehat{m}(z)$, map the solutions obtained for the state space (m,z,e) into (ν,z) by recursive substitution as follows: When a firm is born ($\nu = 0$), its choices are given by $a'(0,z) = a'(\underline{m}(z),z)$ and $m'(0,z) = m'(a'(0,z),z)$, respectively. Its choices at age 1 are therefore $a'(1,z) = a'(m'(0,z),z)$ and $m'(1,z) = m'(a'(1,z),z)$. Hence, for any age ν the firm's choices are $a'(\nu,z) = a'(m'(\nu-1,z),z)$ and $m(\nu,z) = m'(a'(\nu,z),z)$. For $0 \le \nu < \hat{\nu}(\hat{m}(z))$, use the non-exporter's decision rules, for $\nu = \hat{\nu}(\hat{m}(z))$, use the switcher's, and for $\nu > \hat{\nu}(\hat{m}(z))$, use the exporter's.

(g) If $e(T+1,z) = 0$, update $\bar{e}(z) = 0$ and return to step 2b.

3. Construct the following system of equations $\mathbf{h}(w,p,y) = \epsilon$ by using the market-clearing conditions of the problem:

$$\sum_\nu \sum_z n(\nu,z)\phi(\nu,z) + F\sum_z \hat{\nu}(\hat{m}(z))f(z) - 1 = \epsilon_1,$$

$$\sum_\nu \sum_z [c(\nu,z) + \rho\underline{k}(z) + x(\nu,z)]\phi(\nu,z) - y = \epsilon_2,$$

where $c(\nu,z) = m(\nu,z) - (1-\rho)a'(\nu,z) - \mathbb{1}_{\nu=\hat{\nu}(\hat{m}(z))}wF$ and $x(\nu,z) = (1-\rho)k'(\nu,z) - (1 -$

$\delta)k(\nu, z)$, and

$$\left[\sum_{\nu}[y_h(\nu,z)^{\frac{\sigma-1}{\sigma}}]\phi(\nu,z) + y_m^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - y = \epsilon_3.$$

4. Solve for $\mathbf{h}(w, p, y) \approx \mathbf{0}$ using Broyden's method, where the convergence criterion is set such that $|\epsilon| \leq 1e - 12$.

5. Check for convergence of the guess (w, p, y) used in Step 2 and the solutions from Step 4. If convergence fails, update the guess and return to Step 2.

C.1 Calibration Results

Table 1 in the main text shows the resulting parameters of the calibration. To validate these results we compare the data target moments and the same moments resulting from the model. Table C.2 shows that the model moments matches the ones in the data to the second decimal.

Table C.2: SMM Data Target Moments & Model Counterparts

Moment	Data target (1990-1991) (1)	Model solution (NCC regime) (2)
Share of exporters	0.18	0.18
Average sales (exporters/non-exporters)	8.42	8.42
Average sales (age 5 / age 1)	1.24	1.24
Aggregate exports / sales	0.21	0.21
Aggregate credit / Value added	0.33	0.33
Aggregate capital stock / wage bill	7.25	7.25
(Investment /VA)$_{exporters}$ / (Investment/VA)$_{non-exporters}$	1.87	1.87

D Marginal Revenue Products

A firm's revenue is defined by the value of its sales: $RV \equiv p_h y_h + p_f y_f$. Hence, the MRPs of labor and capital are given by $MRPN \equiv \delta RV/\delta n$ and $MRPK \equiv \delta RV/\delta k$, respectively. The results for the two MRPs used in conditions (18) and (19) are obtained as follows.

First, taking derivatives of RV with respect to n and k, we obtain:

$$MRPN = [p_h + y_h(\delta p_h/\delta y_h)](\delta y_h/\delta n) + [p_f + y_f(\delta p_f/\delta y_f)](\delta y_f/\delta n) \tag{D.1}$$

$$MRPK = [p_h + y_h(\delta p_h/\delta y_h)](\delta y_h/\delta k) + [p_f + y_f(\delta p_f/\delta y_f)](\delta y_f/\delta k) \tag{D.2}$$

Solving the demand functions faced by the entrepreneur (2)-(3) for p_h and p_f, respectively, yields $p_h = (y_h/y)^{-1/\sigma}p$ and $p_f = (y_f/y^*)^{-1/\sigma}p^*$, and from these expressions we obtain:

$$\frac{\delta p_h}{\delta y_h} = \frac{-1}{\sigma}\left(\frac{y_h}{y}\right)^{-(\frac{1}{\sigma})-1}\frac{p}{y}, \qquad \frac{\delta p_f}{\delta y_f} = \frac{-1}{\sigma}\left(\frac{y_f}{y^*}\right)^{-(\frac{1}{\sigma})-1}\frac{p^*}{y^*},$$

which multiplying by y_h and y_f, respectively, and simplifying yields:

$$\frac{\delta p_h}{\delta y_h} = \frac{-p_h}{\sigma}, \qquad \frac{\delta p_f}{\delta y_f} = \frac{-p_f}{\sigma},$$

Substituting these expressions into (D.1)-(D.2) and simplifying using the equilibrium condition $p_f = \zeta p_h$ we obtain:

$$MRPN = \frac{p_h}{\varsigma}\left(\frac{\delta y_h}{\delta n} + \zeta\frac{\delta y_f}{\delta n}\right), \qquad MRPK = \frac{p_h}{\varsigma}\left(\frac{\delta y_h}{\delta k} + \zeta\frac{\delta y_f}{\delta k}\right), \tag{D.3}$$

where, as defined in the paper, $\varsigma = \sigma/(\sigma - 1)$.

Now, differentiate the market-clearing condition $y_h + \zeta y_f = zk^\alpha n^{1-\alpha}$ with respect to n and with respect to k to obtain:

$$\frac{\delta y_h}{\delta n} + \zeta\frac{\delta y_f}{\delta n} = z(1-\alpha)\left(\frac{k}{n}\right)^\alpha, \qquad \frac{\delta y_h}{\delta k} + \zeta\frac{\delta y_f}{\delta k} = z\alpha\left(\frac{n}{k}\right)^{1-\alpha}$$

Substituting these results into those obtained in (D.3) yields the expressions used in conditions (18) and (19) of the paper:

$$MRPN = \frac{p_h}{\varsigma}z(1-\alpha)\left(\frac{k}{n}\right)^\alpha,$$
$$MRPK = \frac{p_h}{\varsigma}z\alpha\left(\frac{n}{k}\right)^{1-\alpha}.$$

E Social Planner Analysis

In this Appendix, we analyze the optimization problem of the social planner in our model and its implications. The Appendix is divided into three sections: First, we characterize the planner's problem and analyze its optimality conditions. Second, we provide the proof of Proposition 2 in the paper about the efficiency properties of the decentralized equilibrium in the absence of financial frictions. Third, we examine the implementation of the planner's optimal steady-state allocations with policy instruments, including debt taxes (i.e., capital controls). For simplicity, and since the planner removes the distortions resulting from monopolistic competition in the domestic markets of intermediate goods, we assume that the planner participates in export markets as a price-taker. In addition, since the entry cost to become an exporter is assumed to represent administrative costs, we assume that the planner incurs only the physical cost of exporting (i.e., the iceberg costs) but not the entry costs. These two assumptions are inessential for a key result of the planner's problem, namely that there is no misallocation in capital and labor across firms.

E.1 Social planner's optimization problem

It is well-known from the work of Calvo and Obstfeld (1988) that a standard utilitarian social welfare function in models with overlapping generations results in time-inconsistency of optimal plans. In order to avoid it, they postulated a time-consistent social welfare function that treats symmetrically survivors and newborns by discounting the utility flows of both back to the period of their birth at the private subjective discount rate.[9] Using this time-consistent social welfare function and denoting $\hat{\beta}$ as the planner's discount factor, the planner's optimization problem is:[10]

$$\max_{\{c_{t,\iota}(z),k_{t+1,\iota}(z),y^h_{t,\iota}(z),y^f_{t,\iota}(z),n_{t,\iota}(z),y^m_t,D_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \hat{\beta}^t \sum_{\nu=0}^{\infty} \sum_{z\in Z} \left(\frac{\beta}{\hat{\beta}}\right)^{\nu} u(c_{t,t-\nu}(z))\, \phi(\nu,z), \tag{E.1}$$

[9] For survivors of the cohort born at date ι who are of age $\nu = t-\iota$ at date t and productivity z, their expected lifetime utility as of date $t=0$ is $\sum_{t=0}^{\infty}[\beta(1-\rho)]^t u(c_{t,\iota}(z))$. In the Calvo-Obstfeld social welfare function, the planner discounts this payoff by $\beta^{-\iota}$ so as to treat it symmetrically to that of newborns, which is $\sum_{t=\iota}^{\infty} \beta^{t-\iota} u(c_{t,\iota}(z))$.

[10] Alternatively, assuming commitment we could work with a simpler welfare function without the term $(\beta/\hat{\beta})^{\nu}$. Notice the two welfare functions are the same if $\beta = \hat{\beta}$.

subject to the following sequence of constraints for each $t = 0, ..., \infty$:

$$\sum_{\nu,z} \left[c_{t,t-\nu}(z) + k_{t+1,t-\nu}(z) - (1-\delta)k_{t,t-\nu}(z)\right] \phi(\nu,z) + \sum_{z} k_{t,t}(z)\, \phi(0,z) = \left[\sum_{\nu,z} y^{h}_{t,t-\nu}(z)^{\frac{\sigma-1}{\sigma}} \phi(\nu,z) + y_{m,t}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} + \rho \sum_{\nu,z} k_{t+1,t-\nu}(z)\phi(\nu,z), \quad \text{(E.2)}$$

$$\sum_{\nu,z} n_{t,t-\nu}(z)\phi(\nu,z) = 1, \quad \text{(E.3)}$$

$$\sum_{\nu,z} \hat{p}^{f}_{t,t-\nu}(z) y^{f}_{t,t-\nu}(z)\phi(\nu,z) - \hat{p}^{m}_{t} y^{m}_{t} = D_t - q^{*}_{t} D_{t+1}, \quad \text{(E.4)}$$

$$y^{h}_{t,t-\nu}(z) + \zeta y^{f}_{t,t-\nu}(z) = z k_{t,t-\nu}(z)^{\alpha} n_{t,t-\nu}(z)^{1-\alpha}, \quad \text{(E.5)}$$

$$q^{*}_{t} D_{t+1} \le \theta \sum_{\nu,z} (1-\rho) k_{t+1,t-\nu}(z)\, \phi(\nu,z), \quad \text{(E.6)}$$

where t denotes the time period, ν the age of an agent and z the agent's productivity draw at birth. D_{t+1} denotes the planner's external borrowing and $\hat{p}^{f}(z)$ and $\hat{p}^{m}$ are the world-determined relative prices in units of final goods at which the planner exports the domestic input varieties and imports foreign inputs, respectively (recall also that the debt is denominated in units of final goods).

Constraint (E.2) is the economy's resource constraint in final goods and carries the Lagrange multiplier λ^{SP}_{t}. We assume for simplicity that the planner can access at the end of period t the resources allocated to new capital of firms of cohorts that die at the start of $t+1$.[11] Since there is no world trade in final goods, all domestic production (plus the newly installed capital allocated to entrepreneurs who die at the start of $t+1$) is absorbed by domestic consumption, domestic investment and the planner's allocations of initial capital to newborn firms. Constraint (E.3) is the

[11] These resources correspond to the term $\rho \sum_{\nu,z} k_{t+1,t-\nu}(z)\phi(\nu,z)$ in the date-t resource constraint. This assumption matches the timing of the annuity payment in the decentralized equilibrium and hence makes the Euler equations of the decentralized equilibrium and the planner's problem easier to compare. This issue is absent in continuous-time models because there is no beginning- and end-of-period distinction.

aggregate labor resource constraint with multiplier ω_t^{SP}. Constraint (E.4) is the external resource constraint (with multiplier ψ_t^{SP}), which equates the trade balance (exports minus imports of intermediate goods) to the change in the external debt position net of interest. Constraint (E.5) is the technological constraint on production of each input that must hold for firms of each age and productivity, with multiplier $\varphi_t(\nu, z)$. Constraint (E.6) is the planner's borrowing constraint (with multiplier η_t^{SP}), where we assume that the planner uses the economy's *aggregate* newly-installed capital (net of the fraction allocated to dying firms which is used as income in the resource constraint) to contract aggregate debt. Note that the results would be the same if we assume instead that the planner can use all the capital as collateral. This only re-scales η_t^{SP} by the coefficient $1-\rho$.[12]

The first-order conditions of the planner's problem at each date t can be expressed as:

$$\lambda_t^{SP} = \left(\frac{\beta}{\hat{\beta}}\right)^{\nu} u'(c_{t,\iota}(z)), \tag{E.7}$$

$$\frac{\lambda_t^{SP}}{\psi_t^{SP}} = \frac{\hat{p}^f_{t,\iota}(z)}{\zeta}\left[\frac{y^h_{t,\iota}(z)}{y_t}\right]^{1/\sigma}, \tag{E.8}$$

$$\frac{\lambda_t^{SP}}{\psi_t^{SP}} = \hat{p}^m_t\left[\frac{y^m_t}{y_t}\right]^{1/\sigma}, \tag{E.9}$$

$$\psi_t^{SP}\frac{\hat{p}^f_{t,\iota}(z)}{\zeta}(1-\alpha)zk_{t,\iota}(z)^{\alpha}n_{t,\iota}(z)^{-\alpha} = \omega_t^{SP}, \tag{E.10}$$

$$\lambda_t^{SP} - \theta\eta_t^{SP} = \hat{\beta}\lambda_{t+1}^{SP}\left[(1-\delta) + \frac{\psi_{t+1}^{SP}}{\lambda_{t+1}^{SP}}\frac{\hat{p}^f_{t+1,\iota}(z)}{\zeta}\alpha z k_{t+1,\iota}(z)^{\alpha-1}n_{t+1,\iota}(z)^{1-\alpha}\right], \tag{E.11}$$

$$\psi_t^{SP} - \eta_t^{SP} = \hat{\beta}R^*\psi_{t+1}^{SP}. \tag{E.12}$$

where ι denotes the birth date of a cohort of age $\nu = t-\iota$ at date t.

[12] Quantitatively, this also makes little difference because $1-\rho$ is close to 1 and θ is small.

Conditions (E.7), (E.10) and (E.11) have three important implications. First, it is evident from (E.7) that, at any date t, the planner allocates the same consumption to all firms of the same productivity (and if $\hat{\beta} = \beta$, the same is true across firms of all ages). Second, (E.10) and (E.11) imply that at any date t there is no misallocation in labor and capital (returns in units of final goods are equalized across all firms regardless of age and productivity). To see this, use conditions (E.8), (E.10) and (E.11) to obtain these results:

$$\left[\frac{y^h_{t,\iota}(z)}{y_t}\right]^{-1/\sigma}(1-\alpha)zk_{t,\iota}(z)^{\alpha}n_{t,\iota}(z)^{-\alpha} = \frac{\omega^{SP}_t}{\lambda^{SP}_t},$$

$$\left[\frac{y^h_{t+1,\iota}(z)}{y_{t+1}}\right]^{-1/\sigma}\alpha zk_{t+1,\iota}(z)^{\alpha-1}n_{t+1,\iota}(z)^{1-\alpha} = \frac{\lambda^{SP}_t - \theta\eta^{SP}_t}{\hat{\beta}\lambda^{SP}_{t+1}} - (1-\delta).$$

Moreover, these real returns are the same as the marginal revenue products of labor and capital under perfect competition. Recall that the demand functions for each input are given by $y^h_{t,\iota}(z) = \left[\frac{p^h_{t,\iota}(z)}{p_t}\right]^{-\sigma} y_t$, hence $\left[\frac{y^h_{t,\iota}(z)}{y_t}\right]^{-1/\sigma} = \frac{p^h_{t,\iota}(z)}{p_t}$, so the above results reduce to:

$$p^h_{t,\iota}(z)(1-\alpha)zk_{t,\iota}(z)^{\alpha}n_{t,\iota}(z)^{-\alpha} = \frac{\omega^{SP}_t}{\lambda^{SP}_t}p_t,$$

$$p^h_{t+1,\iota}(z)\alpha zk_{t+1,\iota}(z)^{\alpha-1}n_{t+1,\iota}(z)^{1-\alpha} = \left[\frac{\lambda^{SP}_t - \theta\eta^{SP}_t}{\hat{\beta}\lambda^{SP}_{t+1}} - (1-\delta)\right]p_{t+1}.$$

The expressions in the left-hand-sides of these conditions match those that characterize the marginal revenue products of labor and capital, respectively, as the solution under monopolistic competition converges to perfect competition (i.e., as $\sigma \to \infty$). But these MRPs are not generally the same as the planner's because of the borrowing constraint. For instance, even if the planner's shadow value of final goods λ^{SP}_t matches p_t and the shadow value of labor ω^{SP}_t matches w_t in the decentralized equilibrium with perfect competition, the MRPKs will differ because the tightness of borrowing constraints (i.e., the multipliers of the constraint on individual firms v. the planner) differ, since the planner can pledge the aggregate capital as collateral and individual firms cannot. But in the absence of financial frictions the MRPs can be shown to be equal, as Proposition 2 demonstrates.

E.2 Proof of Proposition 4

Proposition 2 shows that, in the absence of financial frictions and assuming $\hat{\beta} = \beta$ and $\hat{\beta}R^* = 1$ (which, as shown later in Prop. EII of this Appendix, is required for the borrowing constraint not to bind at the stationary equilibrium), the planner's MRPs are constant over time and are the same as those of the competitive equilibrium without financial frictions. We start with a proposition that connects the multipliers of the domestic and external resource constraints, and two corollaries that follow from it.

Proposition EI: If foreign producers of intermediate goods specialize in inputs sold to the domestic economy, then the price faced by the social planner must satisfy $\hat{p}_t^m = (y_{m,t}/y_t)^{-1/\sigma}$ and the planner's shadow value of allocating imported inputs to domestic production of final goods equals that of enlarging the trade deficit to import them: $\lambda_t^{SP} = \psi_t^{SP}$.

Proof. Market-clearing in the market of imported inputs requires that the world-determined price is along the planner's demand function, which is the marginal product of imported inputs in the domestic production of final goods $(y_{m,t}/y_t)^{-1/\sigma}$. Hence, $\hat{p}_t^m = (y_{m,t}/y_t)^{-1/\sigma}$. Then, condition (E.9) implies $\lambda_t^{SP} = \psi_t^{SP}$. □

Corollary EI: If $\lambda_t^{SP} = \psi_t^{SP}$, the price at which the planner exports domestic intermediate goods equals the marginal product of domestic inputs in the domestic production of final goods net of the iceberg cost $\hat{p}_{t,\iota}^f(z) = \zeta \left[\frac{y_{t,\iota}^h(z)}{y_t}\right]^{-1/\sigma}$.

Proof. Using $\lambda_t^{SP} = \psi_t^{SP}$ in condition (E.8) yields $\hat{p}_{t,\iota}^f(z) = \zeta \left[\frac{y_{t,\iota}^h(z)}{y_t}\right]^{-1/\sigma}$. □

Corollary EII: Given the results in Prop. EI and Corollary EI, the planner's Euler equations for capital and debt ((E.11) and (E.12)) yield the following no-arbitrage condition:

$$\left[\frac{y_{t+1,\iota}^h(z)}{y_{t+1}}\right]^{-1/\sigma} \alpha z k_{t+1,\iota}(z)^{\alpha-1} n_{t+1,\iota}(z)^{1-\alpha} = r^* + \delta + (1-\theta)\frac{\eta_t^{SP}}{\hat{\beta}\lambda_{t+1}^{SP}}.$$

Proof. Applying the result that $\lambda_t^{SP} = \psi_t^{SP}$ to condition (E.11) yields:

$$\lambda_t^{SP} - \eta_t^{SP}\theta = \hat{\beta}\lambda_{t+1}^{SP}\left[(1-\delta) + \frac{\hat{p}^f_{t+1,\iota}(z)}{\zeta}\alpha z k_{t+1,\iota}(z)^{\alpha-1} n_{t+1,\iota}(z)^{1-\alpha}\right],$$

and doing the same to condition (E.12) yields:

$$\lambda_t^{SP} - \eta_t^{SP} = \hat{\beta}R^*\lambda_{t+1}^{SP}.$$

Combining the above two conditions and simplifying yields:

$$\left[\frac{\hat{p}^f_{t+1,\iota}(z)}{\zeta}\right]\alpha z k_{t+1,\iota}(z)^{\alpha-1} n_{t+1,\iota}(z)^{1-\alpha} = r^* + \delta + (1-\theta)\frac{\eta_t^{SP}}{\hat{\beta}\lambda_{t+1}^{SP}},$$

which replacing $\frac{\hat{p}^f_{t+1,\iota}(z)}{\zeta}$ with $\left[\frac{y^h_{t+1,\iota}(z)}{y_{t+1}}\right]^{-1/\sigma}$ using Corollary EI completes the proof. □

Proposition 2 *If $\hat{\beta}R^* = 1$ and $\beta = \hat{\beta}$, the marginal revenue products of capital and labor of the decentralized equilibrium without financial frictions (as $\sigma \to \infty$) match the efficient real returns on capital and labor attained by a social planner free of financial frictions. These MRPs are time-invariant, constant across firms regardless of age and productivity, and MRPK equals $p(r^* + \delta)$.*

Proof. To prove this proposition, we assume that the initial capital allocations $\underline{k}_0(z)$ are the same in the decentralized equilibrium and the planner's problem. Since $\hat{\beta}R^* = 1$ and we are assuming the planner does not face financial frictions (i.e., no borrowing constraint), condition (E.12) implies that $\psi_t^{SP} = \psi_{t+1}^{SP} = \bar{\psi}$ (the shadow value of the balance-of-payments equilibrium condition is constant across time, age and productivity). Alternatively, we can state that condition (E.12) implies that a stationary equilibrium where $\psi_t^{SP} = \psi_{t+1}^{SP} = \bar{\psi}$ requires $\hat{\beta}R^* = 1$ (see Prop. EII later in this Appendix). Because final goods are not traded internationally, however, there is no direct arbitrage of the domestic marginal rate of substitution in consumption ($\lambda_t^{SP}/\hat{\beta}\lambda_{t+1}^{SP}$) and the real interest rate R^*. Instead, we can apply the result from Prop. EI showing that $\lambda_t^{SP} = \psi_t^{SP}$, and

Corollary EII to obtain this expression for the no-arbitrage condition for returns on capital:

$$\frac{\hat{p}^f_{t+1,\iota}(z)}{\zeta}\alpha z k_{t+1,\iota}(z)^{\alpha-1}n_{t+1,\iota}(z)^{1-\alpha} = (r^* + \delta),$$

and condition (E.10) can be rewritten as:

$$\frac{\bar{\psi}^{SP}\hat{p}^f_{t,\iota}(z)}{\zeta}(1-\alpha)z k_{t,\iota}(z)^{\alpha}n_{t,\iota}(z)^{-\alpha} = \omega^{SP}_t.$$

To show that the above results match the marginal revenue product conditions of the decentralized equilibrium as it approaches the competitive equilibrium ($\sigma \to \infty$), consider that, since this decentralized equilibrium is efficient, the planner's shadow values of final goods and trade in intermediate goods must satisfy $\bar{\lambda} = \bar{\psi} = p$, the shadow value of labor must satisfy $\omega = w$, and the relative prices of the planner's problem and the decentralized equilibrium must satisfy $\hat{p}^f_{t,\iota}(z) = p^f_{t,\iota}(z)/p$, $\hat{p}^m = p^m/p$, and $p^f(z)_{t,\iota} = \zeta p^h(z)_{t,\iota}$. Under these conditions, the above optimality conditions yield these expressions:

$$p^h_{t+1,\iota}(z)\alpha z k_{t+1,\iota}(z)^{\alpha-1}n_{t+1,\iota}(z)^{1-\alpha} = p(r^* + \delta),$$

$$p^h_{t,\iota}(z)(1-\alpha)z k_{t,\iota}(z)^{\alpha}n_{t,\iota}(z)^{-\alpha} = w.$$

Hence, without financial frictions, the planner's optimality conditions and those of the decentralized equilibrium as $\sigma \to \infty$ support the same conditions equating MRPN to w and MRPK to $p(r^* + \delta)$ in all periods and across firms of different age and productivity. □

E.3 Planner's stationary equilibrium and decentralization

We characterize next the planner's stationary equilibrium, compare it vis-a-vis the decentralized equilibrium without capital controls, and study its implementation as a decentralized equilibrium with policy intervention (including capital controls). Denote the steady-state value of a given variable x as $\bar{x}$. The first result to note is that if $\hat{\beta}R^* \geq 1$, the collateral constraint does not bind at the planner's steady state. This result is established in the following proposition

Proposition EII If $\hat{\beta}R^* \geq 1$, then $\bar{\eta}^{SP} \leq 0$, namely the collateral constraint is not binding at the planner's stationary equilibrium.

Proof. Evaluating condition (E.12) at steady-state yields:

$$\bar{\psi}^{SP} - \bar{\eta}^{SP} = \hat{\beta}R^*\bar{\psi}^{SP},$$
$$\frac{\bar{\eta}^{SP}}{\bar{\psi}^{SP}} = 1 - \hat{\beta}R^*,$$

Since $\bar{\psi}^{SP} > 0$, it follows from the above result that $\hat{\beta}R^* \geq 1$ implies $\bar{\eta}^{SP} \leq 0$. □

This Proposition yields two important implications: First, $\hat{\beta}R^* < 1$ is necessary and sufficient for $\bar{\eta}^{SP} > 0$ (i.e., the constraint can only bind at the planner's steady state if the world interest rate is lower than the planner's rate of time preference). Second, if $\hat{\beta}R^* \geq 1$, the planner's stationary equilibrium is the same as that of a planner that is not subject to the collateral constraint and hence the planner's allocations are those derived from Prop. 2.

Using the results from section E.1, the social planner's steady-state allocations for entrepreneurs of different ages and productivity $\left\{c_\nu(z), n_\nu(z), y^h_\nu(z), k_{\nu+1}(z)\right\}_{\nu=0}^{\infty}$, given initial capital $\left(k_0^{SP}(z)\right)_{z\in Z}$ and steady-state aggregates and shadow prices $\left(\bar{y}, \bar{\lambda}, \bar{\omega}, \bar{\eta}\right)$, satisfy the following conditions for all $\nu \geq 0$ and $z \in Z$:

$$\text{IMRS}_\nu^{SP}(z) = R^* + \max\left\{\frac{1-\hat{\beta}R^*}{\hat{\beta}}, 0\right\}, \tag{E.13}$$

$$\text{MRPNp}_\nu^{SP}(z) = \frac{\bar{\omega}}{\bar{\lambda}}, \tag{E.14}$$

$$\text{MRPKp}_{\nu+1}^{SP}(z) = (1-\theta)\max\left\{\frac{1-\hat{\beta}R^*}{\hat{\beta}}, 0\right\} + r^* + \delta, \tag{E.15}$$

$$\left(\frac{y^h_{\nu+1}(z)}{\bar{y}}\right)^{-\frac{1}{\sigma}} = \frac{1}{z}\left(\frac{\text{MRPKp}_{\nu+1}^{SP}(z)}{\alpha}\right)^{\alpha}\left(\frac{\text{MRPNp}_{\nu+1}^{SP}(z)}{1-\alpha}\right)^{1-\alpha}, \tag{E.16}$$

$$\left(\frac{y^h_0(z)}{\bar{y}}\right)^{-\frac{1+\alpha(\sigma-1)}{\sigma}} = \frac{\bar{y}^{\alpha}}{z(k_0(z))^{\alpha}}\left(\frac{\text{MRPK}_0^{SP}(z)}{1-\alpha}\right)^{1-\alpha}, \tag{E.17}$$

where MRPKp^{SP} and MRPNp^{SP} denote marginal revenue products in units of final goods defined as $\mathrm{MRPKp}_{\nu}^{SP}(z) \equiv \alpha \frac{y_\nu(z)}{k_\nu(z)} \left(\frac{y_\nu^h(z)}{\bar{y}}\right)^{-\frac{1}{\sigma}}$ and $\mathrm{MRPNp}_{\nu}^{SP}(z) \equiv (1-\alpha) \frac{y_\nu(z)}{n_\nu(z)} \left(\frac{y_\nu^h(z)}{\bar{y}}\right)^{-\frac{1}{\sigma}}$, and $\mathrm{IMRS}_{\nu,\nu+1}^{SP}(z) \equiv \frac{u'(c_\nu(z))}{\hat{\beta} u'(c_{\nu+1}(z))}$ is the planner's intertemporal marginal rate of substitution in consumption.

Condition (E.15) characterizes the socially optimal allocation of capital to each firm and its associated MRPKp. If the collateral constraint binds for the planner, it sets a common MRPKp across all firms, but higher than $r^* + \delta$ (the optimal level in the absence of financial frictions) by the proportion $\left(\frac{1-\hat{\beta}R^*}{\hat{\beta}}\right)(1-\theta)$, because of the shadow value of the *aggregate* collateral constraint.

To compare the planner's allocations with those of the decentralized equilibrium without policy intervention, recall that conditions (19), (27) and (26) characterize MRPK and the optimal saving plans in the decentralized equilibrium. Evaluated at steady-state and for an economy without capital controls, condition (19) yields the following expression for MRPK in units of final goods in the decentralized equilibrium:[13]

$$MRPKp_{\nu}^{DE}(z) = r^* + \delta + \frac{\bar{\tilde{\eta}}_{\nu}^{DE}(z)}{\tilde{\beta} u'(\bar{c}_{\nu+1}^{DE}(z))}(1-\theta). \tag{E.18}$$

Then, the difference in MRPKp between the planner's solution and a decentralized equilibrium without capital controls is:

$$MRPKp_{\nu}^{SP}(z) - MRPKp_{\nu}^{DE}(z) = (1-\theta)\left[\max\left(\frac{1-\hat{\beta}R^*}{\hat{\beta}}, 0\right) - \frac{\bar{\tilde{\eta}}_{\nu}^{DE}(z)}{\tilde{\beta} u'(\bar{c}_{\nu+1}^{DE}(z))}\right] \tag{E.19}$$

Hence, the MRPKp differ because of differences in how the shadow values of the collateral constraint affect the social and private marginal costs of borrowing. The planner wants all firms to face the same shadow value at either $\frac{1-\hat{\beta}R^*}{\hat{\beta}}$ if the aggregate credit constraint binds or zero otherwise, while in the decentralized equilibrium the private cost varies across firms depending on the tightness of the individual collateral constraint, which varies with firm age and productivity. Hence, the misallocation of the stationary decentralized equilibrium in the absence of policy intervention is socially inefficient (i.e., it yields values of MRPKp that deviate from those that are socially opti-

[13] In this result, we use the condition $\eta/p = \tilde{\eta}/\tilde{\beta}u'(c')$. relating the multipliers from maximizing cash on hand to those maximizing lifetime utility (see Footnote 14 in the paper)

mal). If the planner is not credit-constrained, the above difference is always negative for firms that have not reached their optimal scale in the decentralized equilibrium (i.e., those that are credit constrained) and their MRPKp exceeds the efficient MRPKp, while for firms that have reached it their MRPKp matches the efficient one. If the planner is constrained, there can be firms in the decentralized equilibrium that are sufficiently close to their optimal scales so that $\frac{\bar{\tilde{\eta}}_\nu^{DE}(z)}{\tilde{\beta}u'(\bar{c}_\nu^{DE}(z))} > \frac{1-\hat{\beta}R^*}{\hat{\beta}}$, and thus have an MRPKp below the socially efficient one. This is because the planner faces the collateral constraint with the aggregate capital stock, which individual entrepreneurs do not internalize.

Compare next the social and private Euler equations. In the stationary decentralized equilibrium without capital controls, conditions (27) and (26) yield:

$$IMRS_\nu^{DE}(z) \equiv \frac{u'(\bar{c}_\nu^{DE}(z))}{\beta u'(\bar{c}_{\nu+1}^{DE}(z))} = R^* + \frac{\bar{\tilde{\eta}}_\nu^{DE}(z)}{\tilde{\beta}u'(\bar{c}_{\nu+1}^{DE}(z))}, \tag{E.20}$$

while for the social planner condition (E.13) yields:

$$IMRS_\nu^{SP}(z) \equiv \frac{u'(c_\nu(z))}{\hat{\beta}u'(c_{\nu+1}(z))} = R^* + \max\left\{\frac{1-\hat{\beta}R^*}{\hat{\beta}}, 0\right\}. \tag{E.21}$$

Hence, the difference between the IMRS of the social and decentralized equilibria is given by:

$$IMRS_\nu^{SP}(z) - IMRS_\nu^{DE}(z) = \max\left(\frac{1-\hat{\beta}R^*}{\hat{\beta}}, 0\right) - \frac{\bar{\tilde{\eta}}_\nu^{DE}(z)}{\tilde{\beta}u'(\bar{c}_{\nu+1}^{DE}(z))}. \tag{E.22}$$

Thus, saving decisions are also socially inefficient in the decentralized equilibrium. The planner assigns the same IMRS to all entrepreneurs, while those in the decentralized equilibrium differ according to the tightness of the collateral constraint for each entrepreneur, in a similar way as the differences in MRPKp. The wedges in MRPKp and IMRS, relative to their efficient counterparts, are proportional to each other, the former being a fraction $1-\theta$ of the latter.

We discuss next the decentralization of the allocation of the planner's stationary equilibrium with policy intervention. As policy instruments, we allow at the outset for taxes/subsidies on firm sales, labor costs, profits, and debt (i.e., capital controls) and for lump-sum taxes/transfers, all of which can vary across firms of different age and productivity.[14] Sales and labor taxes turned out to be redundant, since we can show that the former match the firm's markup coefficient ς and the latter can be set to zero because MRPNp is always common across firms (assuming newborn-firm capital allocations in the decentralized equilibrium are the same as those allocated by the planner). Hence, we can focus without loss of generality on profit taxes, debt taxes and lump-sum transfers/taxes. For simplicity, we focus on the case in which this equilibrium can be implemented without individual entrepreneurs being credit-constrained in the decentralized equilibrium with policy intervention, and consider scenarios in which the aggregate credit constraint binds at the stationary equilibrium of the social planner ($\hat{\beta}R^* < 1$).[15] Under these assumptions, optimal debt taxes are both positive and invariant across firms of different age and productivity, in line with the formulation of capital controls assumed in the model and calibrated to the Chilean data.

If the collateral constraint is not binding for any entrepreneur, the debt and profit taxes needed to support the planner's steady-state allocations are:

$$\tau_\nu(z) = \frac{1-\hat{\beta}R^*}{\hat{\beta}} > 0 \quad \text{and} \quad \tau_\nu^k(z) = -\frac{\theta\tau_\nu(z)}{(1-\theta)\,\frac{1-\hat{\beta}R^*}{\hat{\beta}} + r^* + \delta} < 0. \tag{E.23}$$

Notice these results follow readily from conditions (E.19) and (E.22), since we are assuming $\hat{\beta}R^* < 1$ and $\bar{\tilde{\eta}}_\nu^{DE}(z) = 0$ for all ν and z. This policy arrangement calls for a debt tax and a subsidy on profits, both invariant across age and productivity. The debt tax sets capital controls so as to support the IMRS of the planner in the decentralized equilibrium, and given that debt tax, the subsidy on profits supports the planner's MRPKp (common across firms).

The above tax and subsidy rates are necessary conditions to decentralize the planner's steady-state allocations, but they are not sufficient. In particular, we also need a set of lump-sum transfers/taxes that support the planner's consumption allocations across entrepreneurs and guarantee that the government budget constraint holds. In Andreasen et al. (2025), we characterize

[14] A full analysis of the decentralized equilibrium with these policy instruments in place and their design when used to implement the allocations of the planner's stationary equilibrium is provided in Andreasen et al. (2025).

[15] In the quantitative results provided in the paper using this formulation, we verified that indeed the collateral constraint does not bind for all entrepreneurs in the decentralized equilibrium with policy intervention.

the optimal schedule of transfers and show that the debt tax and profit subsidies have a combined negative effect on the fiscal balance (namely, the cost of the subsidies is larger than the revenue of the taxes).

F Model with Domestic Credit Market

This Section of the Appendix examines a simplified version of the model that introduces a domestic credit market in which entrepreneurs can buy or sell bonds, so that they can optimally choose whether they prefer to invest in their own capital or effectively lend to other firms. The model is simplified by assuming that there are no exporters, no imported inputs, and no labor market. Firms use a fixed amount of labor $\bar{n}$ so that effective productivity becomes $\tilde{z} = z\bar{n}^{1-\alpha}$, or alternatively we can think of $\bar{n}$ as non-marketable land.

Individual holdings of domestic bonds are denoted b. The price of these bonds is q^b (with return $R^b \equiv 1/q^b$). The foreign debt market is the same as in the model of the paper. The borrowing constraint is now formulated in terms of the net bond position including foreign and domestic bonds:

$$qd' - q^b b' \leq \theta k'.$$

Net worth is now defined as:

$$a' = k' - qd' + q^b b'.$$

Hence, the borrowing constraint in terms of net worth remains as before:

$$k' \leq \frac{a'}{1-\theta}.$$

In principle, there would seem to be a portfolio choice involving d' and b', but in fact, with one exception, the portfolio is always at the corners because of the following arguments:[16]

[16]The exception is when there is excess demand for credit in the domestic market at $R^b = R^* + \tau$. In this case, the gap is covered by external borrowing at the aggregate level but the portfolio structure of domestic and foreign bonds of individual borrowers is undetermined.

1. If $R^b > R^* + \tau$, all firms that borrow always borrow from abroad, and therefore, there is no supply of domestic bonds. Hence, all the debt is in d' and $b' = 0$ for all firms. Here, firms that have repaid their debt (i.e., attained $a' = \bar{k}^{cc}(\tilde{z})$) move into region 3 and accumulate net worth along the ray $k' = a'$ as assumed in the paper, because (a) there is no domestic debt market and (b) since the marginal return on saving exceeds R^* firms want to grow their net worth but can only allocate it to capital.

2. If $R^* < R^b < R^* + \tau$, all firms that borrow always borrow in the domestic market and therefore there are no capital inflows. Hence, all the debt is in b' and $d' = 0$ for all firms. At equilibrium, some firms will borrow and have $b' < 0$ and others will lend (save into bonds) and have $b' > 0$ and this bond market must clear internally at the rate R^b. Thus, capital controls move the economy to financial autarky.

3. If $R^b = R^* + \tau$, the two bonds are perfect substitutes for borrowers, they are indifferent which one they use to borrow. The portfolio composition depends on whether at $R^* + \tau$ there is excess demand or supply of credit. If there is excess supply, since lenders cannot get $R^* + \tau$ by investing abroad, the domestic interest rate falls and thus $R^b = R^* + \tau$ cannot be an equilibrium. If there is excess demand, all domestic savers buy the domestic bonds they desire at $q(\tau)$ and the excess over those that borrowers still want to sell are sold abroad paying the capital controls tax, so that the price is still $q(\tau)$. We can assume that the domestic market opens first (since after all capital controls are in place). Borrowers step in to borrow (sell bonds) and when the domestic bond demand is covered, the rest of borrowers can borrow from abroad. In this case, however, the portfolio structure of individual borrowers is undetermined. Their net position $q(\tau)[d' - b']$ is well-defined, but the breakdown between b' and d' is not. Finally, if at $R^* + \tau$ the aggregate supply and demand of bonds are equal (recalling that lenders would always lend domestically since saving into international bonds pays R^* not $R^* + \tau$), there would be nobody left to borrow from abroad after the domestic market meets and thus $d' = 0$.

4. If $R^b < R^*$, firms that save would never want to save into domestic bonds, since the return is higher abroad, and therefore no firm would be able to borrow domestically at R^b. Moreover, since $\beta R^* = 1$, it must be that $\beta R^b < 1$, and thus dynamic effects will induce firms to always want to reduce their net worth. All firms would want to borrow inducing an excess demand for credit that would cause R^b to rise. Hence, $R^b < R^*$ cannot be an equilibrium.

F.1 Firms that save prefer buying domestic bonds than investing

We start the analysis by presenting a proposition that establishes that, for any $R^* < R^b \leq R^* + \tau$, an entrepreneur with enough net worth to self-finance the pseudo-steady state of capital supported by capital controls will prefer to save its additional net worth into domestic bonds (i.e., lend it to other firms) rather than accumulate more capital.

Proof. This proof shows that the entrepreneur's increase in cash-on-hand in response to an increase in a' is larger by investing the marginal net worth into bonds than into capital, because the marginal return of the former exceeds that of the latter.

Start with the case $R^b = R^* + \tau$ (domestic bonds yield the same as the interest rate with capital controls). Since $a' \geq \bar{k}^{cc}(\tilde{z})$, the firm is not borrowing, and since $R^b > R^*$, the firm sets $d = 0$ (saving abroad by setting $d' < 0$ yields a smaller return than domestic bonds). The firm will then choose from one of two strategies: (i) $b = 0$ if it sets $k' = a'$ (this is the assumption in Region 3 of the analysis in the paper), or (ii) $b = (R^* + \tau)[a' - \bar{k}^{cc}(\tilde{z})]$ if it keeps its capital constant by setting $k' = \bar{k}^{cc}(\tilde{z})$.

Cash-on-hand is:

$$pm' = p'^h \tilde{z} k'^\alpha + p(1-\delta)k' + p\hat{R}[a' - k']$$

Recall from the demand-determined output under monopolistic competition that $p^h/p = [\tilde{z}k'^\alpha/y]^{-1/\sigma}$, hence cash on hand simplifies to:

$$
\begin{aligned}
m' &= [\tilde{z}k'^{\alpha}/y]^{-1/\sigma}\tilde{z}k'^{\alpha} + (1-\delta)k' + (R^* + \tau)[a' - k'] \\
&= y^{1/\sigma}[\tilde{z}k'^{\alpha}]^{\frac{\sigma-1}{\sigma}} + (1-\delta)k' + (R^* + \tau)[a' - k']
\end{aligned}
$$

The additional unit of a' is invested where it yields the larger increase in cash-on-hand, which can be determined by evaluating the total derivative of m' with respect to a' under each strategy. The total derivative of cash-on-hand is:

$$
\frac{dm'}{da'} = y^{1/\sigma}\frac{\sigma-1}{\sigma}[\tilde{z}k'^{\alpha}]^{\frac{-1}{\sigma}}\alpha\tilde{z}k'^{\alpha-1}\frac{\partial k'}{\partial a'} + (1-\delta)\frac{\partial k'}{\partial a'} + (R^* + \tau)\left(1 - \frac{\partial k'}{\partial a'}\right)
$$

which using again $p^h/p = [\tilde{z}k'^{\alpha}/y]^{-1/\sigma}$ reduces to:

$$
\frac{dm'}{da'} = \frac{p'^h}{p}\frac{\sigma-1}{\sigma}\alpha\tilde{z}k'^{\alpha-1}\frac{\partial k'}{\partial a'} + (1-\delta)\frac{\partial k'}{\partial a'} + (R^* + \tau)\left(1 - \frac{\partial k'}{\partial a'}\right)
$$

Since the marginal revenue product for a firm with productivity $\tilde{z}$ and capital k' is $MRPK(k', \tilde{z}) \equiv p'^h\frac{\sigma-1}{\sigma}\alpha\tilde{z}k'^{\alpha-1}$, we obtain that the total derivative is:

$$
\frac{dm'}{da'} = \frac{MRPK(k', \tilde{z})}{p}\frac{\partial k'}{\partial a'} + (1-\delta)\frac{\partial k'}{\partial a'} + (R^* + \tau)\left(1 - \frac{\partial k'}{\partial a'}\right)
$$

The additional m' earned by investing the extra unit of a' following strategy (i) that sets $k' = a'$ is:

$$
\frac{\partial m'}{\partial a'} = \frac{MRPK(a', \tilde{z})}{p} + (1-\delta)
$$

and under strategy (ii) that sets $k' = \bar{k}^{cc}(\tilde{z})$ is:

$$
\frac{\partial m'}{\partial a'} = R^* + \tau = \frac{MRPK(\bar{k}^{cc}(\tilde{z}), \tilde{z})}{p} + (1-\delta),
$$

where the last equality follows from the optimality condition that defines the pseudo-steady state of capital $\bar{k}^{cc}(\tilde{z})$ with capital controls. Since $a' \geq \bar{k}^{cc}(\tilde{z})$ and the MRPK is decreasing in k it follows that:

$$\left.\frac{\partial m'}{\partial a'}\right|_{k'=a'} \leq \left.\frac{\partial m'}{\partial a'}\right|_{k'=\bar{k}^{cc}(\tilde{z})},$$

which holds with equality only if $a' = \bar{k}^{cc}(\tilde{z})$. Hence, the firm that has attained $a' = \bar{k}^{cc}(\tilde{z})$ still desires to increase a' because $R^* + \tau > R^*$ (so that $\beta(R^* + \tau) > 1$) but it will always prefer to keep capital constant and save at R^b than to invest in capital.

If $R^b < R^* + \tau$, there is no borrowing from abroad and hence the economy is in financial autarky. There is something akin to region 2 but defined not by $a' = \bar{k}^{cc}(\tilde{z})$ but by $a' = \bar{k}^{R^b}(\tilde{z})$, where $\bar{k}^{R^b}(\tilde{z})$ is the pseudo steady-state of capital such that $R^b = \frac{MRPK(\bar{k}^{R^b}(\tilde{z}),\tilde{z})}{p} + (1-\delta)$. Then the same argument of the case with $R^b = R^* + \tau$ applies. Any firm with $a' = \bar{k}^{R^b}(\tilde{z})$ will always prefer to save into domestic bonds at R^b keeping capital constant than investing into capital at $k' = a'$ because the marginal return of the former strategy dominates that of the latter. □

The above result shows that Region 3 as presented in the paper can only exist if either (a) the domestic credit market under financial autarky is too small, in the sense that it yields an interest rate such that $R^b > R^* + \tau$; or (b) we assume restrictions that prevent firms from saving into the domestic bond market (i.e. domestic lending) at a rate higher than R^*. For instance, the government could tax domestic bond purchases so that savers can only earn R^*. This is reasonable under the interpretation that the capital controls represent a form of financial repression, because by definition financial repression means that there are wedges that make interest rates on borrowing and saving different. Even relaxing this assumption so that the domestic bond market may exist, however, it does not follow that the static effects of capital controls on misallocation are necessarily weaker than in the paper. The outcome depends on what interest rate is generated by the financial autarky equilibrium. This point is explained in detail in the next Section but for now consider the following intuition for two extreme cases.

On one hand, if R^b is negligibly higher than R^* it is clear that Region 3 disappears (because of what Proposition 1 proved). Firms never leave Region 2 after reaching it and Region 2 converges to Region 4, so the NCC and CC regimes would have nearly identical capital decision rules and therefore capital controls would be nearly neutral. On the other hand, if R^b is negligibly lower than $R^* + \tau$, the static effects would be stronger than in the paper because Region 2 is wider and there are no regions 3 and 4. Firms would never attain their efficient optimal scale. Instead, firms that are sufficiently old or have enough net worth converge to $\bar{k}^{R^b}(\tilde{z})$ and have permanently higher MRPK than the efficient one. Hence, understanding the financial autarky equilibrium is critical for determining whether the domestic credit market would strengthen or weaken the results produced by the model presented in the paper.

F.2 Credit market equilibrium in financial autarky

We study next the general equilibrium of the model with domestic credit market and what it implies for misallocation relative to the results obtained with the benchmark model in the paper. To start the analysis, note that the arguments about portfolio choice of foreign and domestic bonds presented earlier imply that domestic borrowing emerges when capital controls are introduced only if $R^b \leq R^* + \tau$. Moreover, they also imply that when this happens all the borrowing is domestic and the economy moves to financial autarky. As we explain below, the case $R^b = R^* + \tau$ emerges only if by chance the autarky equilibrium yields a domestic interest rate equal to $R^* + \tau$, and in this case we assume the domestic bond market opens first to support the equilibrium. Hence, the main case of interest for studying domestic debt is when $R^b < R^* + \tau$. Before examining the implications of this case, we characterize the general equilibrium of the model.

The model is the same as in Section 3 of the paper, except for the following modifications. First, for simplicity, we assume that there are no exporters, no imported inputs, and no labor market. Firms use a fixed amount of labor $\bar{n}$ so that effective productivity becomes $\tilde{z} = z\bar{n}^{1-\alpha}$, or alternatively we can think of this fixed labor as non-marketable land. The entrepreneurs' production technology hence becomes $y_h = \tilde{z}k_h^\alpha$. Since there are no imported inputs, the optimization problem of final goods producers becomes:

$$\max_{y_{h,t}(i), y_{m,t}} p_t y_t - \int_0^1 p_{h,t}(i) y_{h,t}(i) di,$$

$$\text{s.t.} \qquad y_t = \left[\int_0^1 y_{h,t}(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}},$$

where $p = [\int_0^1 p_{h,t}(i)^{1-\sigma} di]^{1/(1-\sigma)}$. This problem yields the same demand functions for domestic inputs as in Section 3, $y_{h,t}(i) = \left(\frac{p_{h,t}(i)}{p_t}\right)^{-\sigma} y_t$.

Second, since we now allow for the possibility of domestic borrowing, the borrowing constraint becomes:

$$qd_{t+1} - q^b b_{t+1} \leq \theta k_{t+1},$$

Keep in mind, however, that as implied by the results from earlier in this Section, when the domestic credit market operates the economy moves to financial autarky, so the relevant case for this analysis is when $d_{t+1} = 0$.

The value of an individual firm (assuming financial autarky) is:

$$v(m, \tilde{z}) = \max_{a'} \left[u\left(m - (1-\rho)a'\right) + \tilde{\beta} v\left(\tilde{m}'(a', \tilde{z}), \tilde{z}\right) \right]$$

$$\tilde{m}'(a', \tilde{z}) = \max_{k', b', p'_h} \left[\frac{\frac{{p'}_h^{1-\sigma}}{p^{-\sigma}} y' + p(1-\delta)k' - pd' - T(\tilde{z})}{p} \right] \tag{F.1}$$

$$\text{s.t.} \left(\frac{p'_h}{p}\right)^{-\sigma} y' = \tilde{z}{k'}^{\alpha} \qquad a' = k' + q^b b' \qquad k' \leq a'/(1-\theta) \tag{F.2}$$

F.2.1 Static Effects in the Second-Stage Solution

The static effects of the borrowing constraint are determined by the first-order conditions of the second-stage problem, which determines $\tilde{m}'(a', \tilde{z})$. These conditions simplify to:

$$MRPK \equiv \frac{p'_h}{\varsigma} \alpha \tilde{z} (k')^{\alpha-1} = \left[p(r^b + \delta) + \eta(1-\theta) \right]$$

$$\left(\frac{p'_h}{p}\right)^{-\sigma} y = \tilde{z}{k'}^{\alpha} \qquad b' = R^b[a' - k']$$

where $\varsigma = \sigma/(\sigma - 1)$ is the markup of price over marginal cost and η is the multiplier on the borrowing constraint. When $\eta > 0$, the firm is borrowing and the capital and bond decision rules are:

$$k'(a') = [a'/(1-\theta)], \quad b' = -R^b \frac{\theta}{1-\theta} a',$$

When $\eta = 0$, the decision rules are:

$$k' = \bar{k}^{R^b}(\tilde{z}), \quad b' = R^b[a' - \bar{k}^{R^b}(\tilde{z})],$$

where $\bar{k}^{R^b}(\tilde{z})$ is the capital stock at which $MRPK(\tilde{z}, k') = p(r^b + \delta)$. The firm may still be borrowing, in which case $a' < \bar{k}^{R^b}(\tilde{z})$ and $b' < 0$, otherwise the firm is saving and $b' > 0$.

The value of $\bar{k}^{R^b}(\tilde{z})$ is given by:

$$\bar{k}^{R^b}(\tilde{z}) = \left[\frac{\alpha y^{1/\sigma} \tilde{z}^{1/\varsigma}}{\varsigma(r^b + \delta)} \right]^{\frac{\varsigma}{\varsigma - \alpha}} \tag{F.3}$$

which, using the Cobb-Douglas production function, implies that $\bar{y}_h^{R^b}(\tilde{z}) = \left[\frac{\alpha y^{1/\sigma}}{\varsigma(r^b+\delta)}\right]^{\frac{\varsigma}{\varsigma-\alpha}} \tilde{z}^{\varsigma/(\varsigma-\alpha)}$. Since $p^h/p = [y_h/y]^{-1/\sigma}$, at this steady state the more productive firms have higher capital, higher output and lower prices.

F.2.2 General equilibrium

The definition of this model's equilibrium is analogous to that of the model in the paper, except that we need to add the market-clearing condition of the domestic bond market. Aggregating over net worth and $\tilde{z}$ using the stationary distribution $\phi(a', \tilde{z})$, the market-clearing condition is :

$$\sum_{a'} \sum_{\tilde{z}} \phi(a', \tilde{z}) b'(a', \tilde{z}) = 0.$$

Since $R^b > R^*$ and $\beta R^* = 1$, the dynamic effect drives all firms to grow their net worth. At the threshold net worth $\tilde{a'}(\tilde{z}) = \bar{k}^{R^b}(\tilde{z})$, firms attain zero debt and become lenders/savers. All firms with $a' < \tilde{a'}(\tilde{z})$ are borrowers and can be divided into two groups. First, in the interval

$0 \leq a' \leq (1-\theta)\bar{k}^{R^b}(\tilde{z})$, firms borrow $b' = -R^b\theta a'/(1-\theta)$. Second, in the interval $(1-\theta)\bar{k}^{R^b}(\tilde{z}) < a' < \bar{k}^{R^b}(\tilde{z})$, firms borrow $b' = R^b[a' - \bar{k}^{R^b}(\tilde{z})] < 0$. All firms with $a' > \tilde{a}'(\tilde{z})$ are savers with $b' = R^b[a' - \bar{k}^{R^b}(\tilde{z})] > 0$. Hence, we can rewrite the market-clearing condition as expressing that the aggregate supply of bonds (aggregate debt) must equal the aggregate demand for bonds (aggregate credit). Thus, the negative of the sum of all negative bond positions must equal the sum of all positive bond positions:

$$\sum_{a'=0}^{(1-\theta)\bar{k}^{R^b}(\tilde{z})} \sum_{\tilde{z}} \phi(a',\tilde{z})\theta a'/(1-\theta) + \sum_{a'=(1-\theta)\bar{k}^{R^b}(\tilde{z})}^{\bar{k}^{R^b}(\tilde{z})} \sum_{\tilde{z}} \phi(a',\tilde{z})[a' - \bar{k}^{R^b}(\tilde{z})] = \sum_{a'>\bar{k}^{R^b}(\tilde{z})} \sum_{\tilde{z}} \phi(a',\tilde{z})[a' - \bar{k}^{R^b}(\tilde{z})]. \quad \text{(F.4)}$$

The above condition can be rewritten in terms of the distribution of age and productivity: $\phi(\nu,\tilde{z}) = \rho(1-\rho)^{\nu} f(\tilde{z})$, where ρ is the probability of death and $f(.)$ is the pdf of firm productivity drawn at birth. Define $\nu_1(\tilde{z})$ as the firm age threshold at which a firm of productivity $\tilde{z}$ builds enough net worth to reach $(1-\theta)\bar{k}^{R^b}(\tilde{z})$ (this is analogous to the vertex connecting regions 1 and 2 in the original model), and $\nu_2(\tilde{z})$ as a similar age threshold at which net worth reaches $\bar{k}^{R^b}(\tilde{z})$ (this is analogous to the vertex connecting regions 2 and 3 in the original model). The market-clearing condition can then be rewritten as:

$$\sum_{\nu=0}^{\nu_1(\tilde{z})} \sum_{\tilde{z}} (1-\rho)^{\nu} f(\tilde{z})\theta a'(\nu,\tilde{z};R^b)/(1-\theta) + \sum_{\nu=\nu_1(\tilde{z})+1}^{\nu_2(\tilde{z})} \sum_{\tilde{z}} (1-\rho)^{\nu} f(\tilde{z})[a'(\nu,\tilde{z};R^b) - \bar{k}^{R^b}(\tilde{z})] = \sum_{\nu=\nu_2(\tilde{z})+1}^{\infty} \sum_{\tilde{z}} (1-\rho)^{\nu} f(\tilde{z})[a'(\nu,\tilde{z};R^b) - \bar{k}^{R^b}(\tilde{z})]. \quad \text{(F.5)}$$

In this expression, $a'(\nu,\tilde{z};R^b)$ denotes that a' changes with age and productivity and depends on the interest rate on bonds that firms took as given in solving their optimization problems. At the equilibrium interest rate, R^b needs to be such that this market clearing condition holds.[17] Note that all firms aged $\nu > \nu_2(\tilde{z})$ continue to grow their net worth indefinitely, but as long as their net worth

[17] The aggregate variables p, y are also determinants of $a'(\cdot)$ but are omitted for simplicity, and the market clearing conditions of the markets for intermediate goods and final goods are also part of the general equilibrium solution.

grows at a rate less than the exponential decay of $(1-\rho)^{\nu}$, the sum converges and aggregate demand for domestic bonds is well-defined even if very old firms have infinitely large bond positions.

The graph used to describe the static effects of capital controls can be modified to draw a diagram that illustrates the equilibrium of the domestic bond market. The diagram shown in Figure F.1 assumes for simplicity that there is no borrowing constraint and no differences in productivity. The shaded area in black represents the aggregate demand for domestic credit and the one in red the aggregate supply. To be an equilibrium interest rate, R^b must be such that the two are equal.[18] The threshold $\hat{a}'$ is the value of a' at which the infinite (but converging) sum in the right-hand-side of the above market-clearing condition converges. This bounds how far to the right in the horizontal axis we need to go to pin down the supply of credit. If the area in black were bigger (smaller) than the area in red, there would be excess demand (supply) of credit and R^b would rise (fall). The equilibrium interest rate R^b is the financial autarky interest rate, because it is determined entirely within the domestic economy and all credit is financed internally.

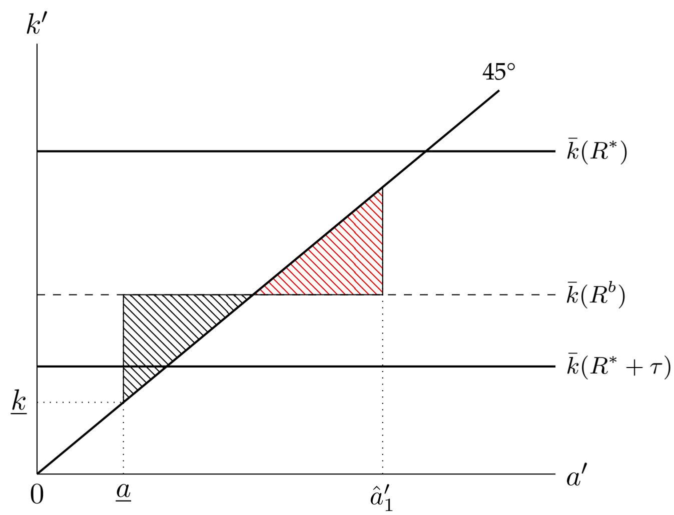

Figure F.1: Equilibrium in Domestic Bond Market

[18] Mathematically, the aggregate demand and supply of credit do not correspond to the entire shaded areas but to the part of them determined by the sums of the discrete elements formed by the optimal choices of net worth determined by the decision rule $a'(\nu; R^b)$.

F.2.3 Effects of capital controls on misallocation

Because of Proposition 1, if $R^* < R^b < R^* + \tau$, the capital controls cause the domestic credit market to emerge and the economy to move to financial autarky. Without capital controls, the fact that $R^* < R^b$ rules out borrowing at the autarky rate and all firms that borrow do so from abroad, and since $\beta R^* = 1$, firms stop growing net worth when they reach $a' = \bar{k}^{R^*}(\tilde{z})$, which is their optimal scale consistent with the world interest rate (or the rate of time preference since they are the same). Hence, without capital controls all firms are borrowers that carry non-negative debt positions and they optimally choose to keep net worth, debt and capital constant when they reach their optimal scale. In contrast, with capital controls, $R^{b*} + \tau$ rules out any borrowing from abroad and therefore the economy moves to the financial autarky equilibrium. Moreover, as noted in the previous subsection, firms that reach $a' = \bar{k}^{R^b}(\tilde{z})$ still want to grow their net worth, because $\beta R^b > 1$.

How do effects of capital controls on misallocation vary because of the domestic credit market? As Figure F.1 shows, one important result is that in this environment (with $R^{b*} + \tau$) capital controls cause permanent effects on misallocation for firms of all ages. In the model of the paper, when capital controls are present, firms that build sufficient net worth reach the efficient capital stock $\bar{k}^{R^*}(\tilde{z})$ and do not have misallocation, nor do they carry any debt or savings. But if the domestic credit market exists, capital controls move the economy to financial autarky, firms stay with the lower capital stock given by $\bar{k}^{R^b}(\tilde{z})$ permanently, and some firms are creditors and others debtors.

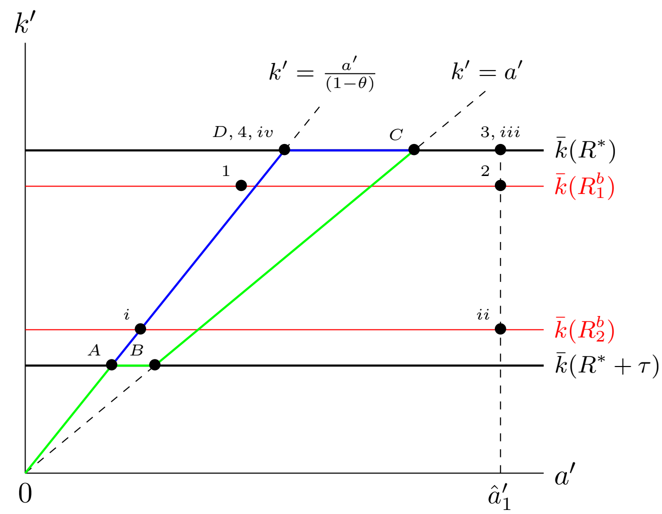

Figure F.2: Effects of Capital Controls with Domestic Bond Market

Whether capital controls cause more or less misallocation in this setup with domestic credit market than in the model of the paper hinges on the value of R^b. Capital controls move the economy to financial autarky and because of this Region 3 disappears but Region 2 widens. Two possible (extreme) outcomes are illustrated in Figure F.2, which is again a variant of Figure 1 in the paper. The piece-wise linear function in blue is the capital decision rule without capital controls and the one in green is the one with capital controls and no domestic market, as in the paper. The magnitude of the effect of capital controls on misallocation is reflected in the size of the overall decline in capital induced by the capital controls, which is measured by the trapezoid formed by the vertexes A-B-C-D.

Consider now the case in which the domestic credit market exists and yields an interest rate R_1^b just above R^*. As Figure F.2 shows, the effect of capital controls on misallocation is now reflected by the loss of capital measured by the trapezoid formed by the vertexes 1-2-3-4.[19] We have more firms in Region 2 than in the model of the paper but in this region the fall in capital and rise in misallocation are small (since R^b and R^* are close). Firms that were in the original regions 2 and most of 3 have much less misallocation, and firms close to region 4 and in region 4

[19]The vertexes 3 and 4 are determined by the upper bound of net worth $\hat{a}'$ at which the sum that defines the aggregate supply of bonds converges.

will have slightly more misallocation. Hence, the overall misallocation is likely to be smaller than in the model of the paper, as comparing the size of the trapezoids A-B-C-D and 1-2-3-4 suggests.

Now consider the case in which the domestic credit market yields an interest rate R_2^b just below $R^* + \tau$. The effect of capital controls on misallocation is now reflected by the loss of capital measured by the trapezoid formed by the vertexes i-ii-iii-iv. Again there are more firms in Region 2 but in this region misallocation is still large (just slightly smaller than in the model of the paper). Firms that were in the original region 2 and in region 3 close to 2 have slightly less misallocation but all the rest of firms in the original regions 3 and 4 have much higher misallocation. Hence, the overall misallocation is likely to be larger than in the model of the paper, as comparing the size of the trapezoids A-B-C-D and i-ii-iii-iv suggests. Thus, capital controls may induce even stronger misallocation effects with than without a domestic credit market if the latter is relatively small (i.e., if it clears at an interest rate sufficiently close to $R^* + \tau$).

The severity of the capital controls also matters. Given the financial autarky equilibrium, stricter capital controls will yield larger misallocation effects in the model of the paper than in the model with domestic debt market.

G Summary Statistics of Firm-level Panel and Macro Data

Table G.3: Summary statistics of firm-level panel

VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
Payroll	90,051	0.388	1.438	0	80.36
Fixed Capital	90,051	2.245	30.72	0	5,707
Exporters	90,051	0.202	0.401	0	1
Misallocation	90,051	1.958	1.601	0	13.00
OSG	90,051	0.664	0.364	0	1.000
TFP	90,051	2.098	0.155	0.0727	2.908

Note: Payroll and fixed capital are reported in millions of Chilean Pesos. The export status takes the value of cero when the firm does not export in the current period and 1 if it does export. TFP is calculated following the methodology of Wooldridge (2009).OSG is the percentage gap between the fixed capital of the firm and the year-industry average of fixed capital for firms that are older than 10 years old.

Table G.4: Summary Statistics: Macroeconomic Indicators 1990-2007

VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
Capital Controls$^{(*)}$	8	1.981	0.703	0.688	2.649
Libor 12m	18	5.149	2.082	1.357	9.305
Inflation	18	8.269	7.479	1.070	27.33
Growth	18	5.890	2.766	-0.412	11.17
RER	18	94.29	10.02	78.01	112.7
Private Credit/GDP	18	0.613	0.107	0.442	0.743
World Growth	18	3.024	0.967	1.369	4.476

Note: Variables are annual averages unless otherwise indicated. (*) Capital Controls are measured following the methodology of De Gregorio et al. (2000). The reported statistics for this variable correspond to the 1991–1998 period, during which capital controls were active. Inflation, RER, Growth and World Growth are from the Central Bank of Chile. RER_dev is calculated as the yearly variation of the real exchange rate, which is defined as the inverse of the nominal exchange rate multiplied by an international price index relevant for Chile and deflated by the Chilean price index. The Private Credit to GDP ratio is from the Financial Structure Database (see Beck et al. (2000)). The 12-month Libor interest rate is obtained from the FRED Economic Data.

H PE and GE effects of counterfactual exercises

Table H.5: Effects of Capital Controls on Real Profits (%)

	Model		Counterfactuals			
	Baseline		LTV Reg.		Higher τ	
	PE	GE	PE	GE	PE	GE
	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)
All firms	-1.46%	-0.70%	-0.65%	-0.06%	-3.55%	-0.38%
Exp. status in CC regime						
Exporters	-1.92%	-0.79%	-1.67%	-0.97%	-8.24%	-4.89%
Non-exporters	-1.38%	-0.68%	-0.43%	0.14%	-2.69%	0.53%
OSG						
Large	-1.52%	-0.74%	-0.68%	-0.09%	-3.71%	-0.53%
Small	-0.33%	0.05%	-0.06%	0.50%	-0.31%	2.76%
Productivity						
High ($z \geq 7$)	-1.08%	-0.02%	-1.32%	-0.72%	-7.27%	-4.10%
Low ($z \leq 4$)	-0.71%	-0.40%	-0.22%	0.29%	-1.28%	1.46%
1	-0.14%	0.24%	-0.02%	0.55%	-0.14%	2.97%
2	-0.28%	0.09%	-0.06%	0.49%	-0.38%	2.63%
3	-0.51%	-0.17%	-0.13%	0.40%	-0.78%	2.09%
4	-0.75%	-0.44%	-0.23%	0.27%	-1.36%	1.36%
5	-0.81%	-0.51%	-0.34%	0.15%	-1.98%	0.67%
6	-2.62%	-1.33%	-0.85%	-0.13%	-4.45%	-0.58%
7	-1.10%	-0.03%	-1.32%	-0.72%	-7.23%	-4.05%
8	-0.98%	0.06%	-1.33%	-0.75%	-7.57%	-4.42%
9	-0.94%	0.09%	-1.34%	-0.76%	-7.61%	-4.47%
10	-0.93%	0.10%	-1.34%	-0.76%	-7.62%	-4.49%

Table H.6: Effects of Capital Controls on Misallocation

	Model		Counterfactuals			
	Baseline		LTV Reg.		Higher τ	
	PE	GE	PE	GE	PE	GE
	(1a)	(1b)	(2a)	(2b)	(3a)	(3b)
All firms	$0.47pp$	$0.53pp$	$0.31pp$	$0.37pp$	$1.79pp$	$2.15pp$
Exp. status						
Exporters	$1.33pp$	$1.30pp$	$1.16pp$	$1.07pp$	$5.71pp$	$5.55pp$
Non-exporters	$0.31pp$	$0.38pp$	$0.14pp$	$0.22pp$	$1.07pp$	$1.47pp$
OSG						
Large	$0.49pp$	$0.55pp$	$0.33pp$	$0.39pp$	$1.87pp$	$2.24pp$
Small	$0.23pp$	$0.24pp$	$0.04pp$	$0.05pp$	$0.22pp$	$0.28pp$
Productivity						
High	$0.75pp$	$0.72pp$	$0.91pp$	$0.90pp$	$5.04pp$	$5.01pp$
Low	$0.49pp$	$0.55pp$	$0.15pp$	$0.20pp$	$0.89pp$	$1.19pp$
1	$0.10pp$	$0.11pp$	$0.02pp$	$0.02pp$	$0.10pp$	$0.14pp$
2	$0.19pp$	$0.20pp$	$0.04pp$	$0.06pp$	$0.26pp$	$0.37pp$
3	$0.35pp$	$0.39pp$	$0.09pp$	$0.12pp$	$0.54pp$	$0.75pp$
4	$0.52pp$	$0.57pp$	$0.16pp$	$0.21pp$	$0.94pp$	$1.25pp$
5	$0.56pp$	$0.62pp$	$0.23pp$	$0.29pp$	$1.37pp$	$1.73pp$
6	$0.25pp$	$0.35pp$	$0.20pp$	$0.29pp$	$1.14pp$	$1.72pp$
7	$0.76pp$	$0.73pp$	$0.91pp$	$0.90pp$	$5.01pp$	$4.97pp$
8	$0.68pp$	$0.67pp$	$0.92pp$	$0.92pp$	$5.25pp$	$5.23pp$
9	$0.65pp$	$0.65pp$	$0.93pp$	$0.93pp$	$5.27pp$	$5.27pp$
10	$0.65pp$	$0.64pp$	$0.93pp$	$0.93pp$	$5.28pp$	$5.28pp$

I Robustness of empirical results

In this section, we conduct a set of tests that document the robustness of our empirical findings. In particular, we show that our results are robust to: (i) using total sales instead of value added to measure misallocation; (ii) winsorizing the top and bottom 1% observations of our database with respect to alternative dimensions–i.e., dependent variable, controls, and sectors' productivity; (iii) excluding firms born around the Russian crisis; (iv) introducing alternative classifications of exporters, i.e., backward- and forward-looking; (v) considering a binary measure of capital controls; and (vi) introducing the interaction of alternative macroeconomic controls with our firms' characteristics.

Using Total Sales vs. Value Added to Measure Misallocation: Column 1 of Table (I.7) shows that the regression results are consistent whether misallocation is measured by value added or total sales. Fewer observations are available when using total sales, as over 3,000 firm-year entries report value added but no total sales. Notably, results remain robust when these observations are excluded from the value-added regressions as well.

Winsorization Analysis: To ensure that potential outliers do not skew our results, columns (2)–(4) of Table I.7 show robustness checks based on winsorizing the top and bottom 5% of observations across different dimensions. Column (2) reports the baseline regression results with winsorization applied to the dependent variable; column (3) applies winsorization to the control variables; and column (4) restricts winsorization to firms within sectors whose average productivity lies at the distribution's extremes. These robustness checks confirm that our results are not influenced by outliers related to the dependent variable, control variables, or sector-level productivity.

Robustness to Exclusion of Firms Born Around the Russian Crisis: Column (5) of Table I.7 demonstrates that the results remain robust when the sample is restricted to firms established outside the period surrounding the 1998 Russian crisis, during which Chile experienced a Sudden Stop. The three interaction coefficient estimates are consistent, showing similar values and t-statistics. This is especially relevant given findings by Ates and Saffie (2021), which show that firms born during this crisis period exhibit notable differences from average firms, being approximately 30% smaller in size and 64% more productive, underscoring the distinct characteristics of firms established during economic turmoil.

Alternative Exporter Definitions: To ensure our exporter definition does not bias results, columns (6) and (7) of Table I.7 replicate the baseline regression using two alternative classifications: backward- and forward-looking definitions. In the backward-looking approach (column 6), exporters are defined as firms reporting exports at least once in the prior two years, aiming to capture the capital-intensive nature and productivity of exporters in steady-state conditions. The forward-looking approach (column 7) defines exporters as firms reporting exports within the following two years, aiming to capture investment requirements for future export plans, which could increase exposure to the debt tax. Our results remain robust across these alternative classifications.

Binary Definition of Capital Controls: Column (8) of Table I.7 examines the robustness of the results using a binary indicator for capital controls, set to 1 when capital controls are in place and 0 otherwise. The findings remain consistent with this simplified binary measure, which excludes the time-varying intensity of capital controls.

Interaction with macroeconomic controls: A potential concern is that the estimated interaction effects with capital controls may be confounded by interactions between TFP_{ijt}, OSG_{ijt}, and Exp_{ijt} and other macroeconomic variables. To address this, Table I.8 reports results from a series of regressions that augment the baseline specification by adding interactions between each firm-level variable and a set of macroeconomic controls (one macro variable at a time). The macro variables considered are: the LIBOR rate, domestic inflation, domestic growth, the real exchange rate, the ratio of private credit to GDP, and world growth. All macro variables are lagged by one period. Summary statistics are provided in Table G.4.

The results show that the coefficients on the capital control interactions remain similar in magnitude, sign, and statistical significance across all specifications, with the sole exception of the interaction with $High\_OSG$ in the regression that includes interactions with domestic growth.

Table I.7: Heterogeneous Effects of the Chilean *Encaje*:
Robustness

VARIABLES	(1) Total sales	(2) Winsorize MRPK	(3) Winsorize Controls	(4) Winsorize Sectors	(5) Excluding crisis cohort	(6) Backward Exp. Status	(7) Forward Exp. Status	(8) Binary CC
CC*High TFP	0.108***	0.169***	0.147***	0.143***	0.145***	0.144***	0.145***	0.370***
	(0.014)	(0.011)	(0.013)	(0.013)	(0.013)	(0.013)	(0.013)	(0.026)
CC*Exp	0.178***	0.111***	0.152***	0.146***	0.145***			0.378***
	(0.018)	(0.014)	(0.017)	(0.017)	(0.018)			(0.035)
CC*High OSG	0.048***	0.029**	0.026*	0.040**	0.042***	0.042***	0.030*	0.106***
	(0.017)	(0.013)	(0.016)	(0.016)	(0.016)	(0.016)	(0.016)	(0.033)
CC*B_Exp						0.159***		
						(0.017)		
CC*F_Exp							0.117***	
							(0.017)	
Observations	86,367	90,055	90,055	90,055	89,064	90,055	90,055	90,055
R-squared	0.595	0.611	0.602	0.598	0.599	0.599	0.598	0.600
Firm FE	YES	YES	YES	YES	YES	YES	YES	YES
Time FE	YES	YES	YES	YES	YES	YES	YES	YES
Controls	YES	YES	YES	YES	YES	YES	YES	YES

Note: This table tests the robustness of the interactions between capital controls and firm characteristics—TFP, OSG, and Exp—on misallocation. Column (1) presents results when using total sales versus value added to calculate misallocation. Columns (2)–(4) conduct winsorization tests, trimming the top and bottom 1% of (2) the dependent variable, (3) control variables, and (4) firms in sectors with extreme productivity. Column (5) excludes firms born around the 1998 Russian crisis. Columns (6) and (7) use backward- and forward-looking exporter definitions, and column (8) employs a binary indicator for capital controls (1 when they are in place). All regressions include a constant, firm and time fixed effects, with errors clustered at the firm level (in parentheses). ***, **, and * denote significance at the 1%, 5%, and 10% levels.

This sensitivity likely reflects the correlation between firm size and growth over the business cycle, which may partially overlap with the optimal-scale-gap channel.

Table I.8: Interaction with macroeconomic controls

VARIABLES	(1) Libor	(2) Inflation	(3) Growth	(4) RER	(5) PrivCreditGDP	(6) WorldGrowth
CC*High TFP	0.137***	0.148***	0.131***	0.186***	0.109***	0.137***
	(0.012)	(0.013)	(0.014)	(0.014)	(0.013)	(0.013)
CC*High OSG	0.040***	0.044***	0.023	0.070***	0.032**	0.039**
	(0.015)	(0.016)	(0.018)	(0.017)	(0.016)	(0.016)
CC*Exp	0.132***	0.154***	0.122***	0.241***	0.080***	0.138***
	(0.017)	(0.017)	(0.020)	(0.017)	(0.018)	(0.018)
Observations	90,055	90,055	90,055	90,055	90,055	90,055
R-squared	0.600	0.602	0.599	0.600	0.602	0.599
Firm FE	YES	YES	YES	YES	YES	YES
Time FE	YES	YES	YES	YES	YES	YES
Controls	YES	YES	YES	YES	YES	YES

Note: This table examines the robustness of the interaction between capital controls and variables of interest—TFP_{ijt}, OSG_{ijt}, and Exp_{ijt}—on misallocation, with stepwise inclusion of their interactions with lagged macroeconomic variables. The macroeconomic variables included are the Libor rate, inflation, growth, real exchange rate (RER), private credit to GDP, and global growth. The interactions of these macroeconomic variables with the firm level variables are included within the control vector. All regressions contain a constant term, firm and time fixed effects, and errors clustered at the firm level (in parentheses). ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

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