Appendix for Natural Resources and Sovereign Risk in Emerging Economies: A Curse *and* a Blessing

Franz Hamann*
Juan Camilo Mendez-Vizcaino†
Enrique G. Mendoza‡
Paulina Restrepo-Echavarria§

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*Banco de la Republica, fhamansa@banrep.gov.co
†Banco de la Republica, jmendevi@banrep.gov.co
‡University of Pennsylvania and NBER, egme@econ.upenn.edu
§Federal Reserve Bank of St. Louis, paulinares@me.com.

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Contents

A Data

In this section we describe the data that we use and its sources. We collected data for GDP, consumption, trade balance, oil rents as a percentage of GDP, oil production (extraction), oil reserves, oil consumption, oil net exports, oil prices, total public debt, total external public debt, net foreign assets, default episodes, and country risk for the thirty largest oil-producing emerging economies in 2010. Those thirty countries are Saudi Arabia, Iran, Iraq, Kuwait, Venezuela, United Arab Emirates, Russian Federation, Libya, Nigeria, Kazakhstan, Qatar, China, Brazil, Algeria, Mexico, Angola, Azerbaijan, Ecuador, India, Oman, Sudan, Malaysia, Indonesia, Egypt, Yemen, Argentina, Syrian Arab Republic, Gabon, Colombia and Vietnam.

As an indicator of country risk we use the Institutional Investor Index (III from now on). The III country credit rating, is a measure of sovereign debt risk that is published biannually in the March and September issues of the Institutional Investor magazine. It is also commonly known as the Country Credit Survey. More specifically, the III is an indicator used to identify and measure country risk, where country risk refers to a collection of risks related to investing in a foreign country, including political risk, exchange rate risk, economic risk, sovereign risk and transfer risk. We have biannual data for the 1979-2014 period. The index is based on information provided by senior economists and sovereign-risk analysts at leading global banks and money management and securities firms. The respondents have graded each country on a scale of zero to 100, with 100 representing the least likelihood of default. Respondents responses are weighted according to their institutions' global exposure.

The data on oil reserves, oil production, oil net exports (thousands of barrels per day), and oil prices (Brent crude oil, USD per barrel) is from the US Energy Information Administration (EIA) from 1980 to 2014. For reserves, we used proved reserves. For oil prices we use the real price by deflating the Brent spot price FOB with the US CPI index for all urban consumers all items in US City average, seasonally adjusted (1982-1984=100).

Data on oil discoveries is taken from Cust et al. (2021). It refers to giant oil discoveries which are Estimated Ultimate Recovery (EUR) reserves greater or equal than 500 million barrels of oil equivalent. Data reports oil fields discoveries per country, per year between 1868 and 2018. For each country in our sample, we add all the field discoveries in each year between 1979 and 2014. The targets for the calibration are computed as follows. Frequency of oil discoveries refers to the number of years with positive discoveries relative to the total number of years in our sample for each country. Then, for a sample estimate we compute the sample weighted average of each non-defaulter country's frequency. For the average consecutive annual discoveries, we identify all the periods of positive discoveries and verify how many of those episodes have consecutive years with positive discoveries, then take an average within country. For a sample estimate we compute the sample weighted average of each non-defaulter country's consecutive years. The ratio of mean discoveries to mean extraction computes the sample average of the size of positive discoveries to the sample average of extraction.

GDP, Oil rents as a percentage of GDP, consumption, and the trade balance, are taken from the World Bank's World Development Indicators Database. Using oil rents we construct oil GDP by multiplying GDP (constant LCU) times oil rents as a percentage of GDP. Non-Oil GDP is obtained by subtracting oil GDP from total GDP. We construct gross oil output by multiplying the nominal price of oil (Brent crude oil, USD per barrel) times oil production (average number of barrels per year). When computing gross oil output as percentage of GDP, we use GDP in current USD.

Total public debt data comes from the International Monetary Fund's Historical Public Debt Database (HPDD). We have information, covering 1971-2015 period, for Gross Government Debt. Total public external debt data is taken from the World Bank Global Development Finance database (GDF), which has annual data for over 130 countries on total external debt by maturity and type of debtor (private non-guaranteed debt and publicly guaranteed debt). The data goes back as far as 1970 and is collected on the basis of public and publicly-guaranteed debt reported in the World Bank's Debtor Reporting System by each of the countries. This information is not available for Saudi Arabia, Iraq, Kuwait, United Arab Emirates, Libya, Qatar, Oman, Malaysia and Syria.

We use the updated and extended version of the "External Wealth of Nations" dataset, constructed by Lane & Milesi-Ferretti (2007) to obtain information on net foreign asset positions. It contains data for the 1970-2015 period and for 188 countries (including those in our sample), plus the euro area as a whole. Specifically, net foreign assets series are based on three alternative measures: i) the accumulated current account, adjusted to reflect the impact of capital transfers, valuation changes, capital gains and losses on equity and Foreign Direct Investment (FDI), and debt reduction and forgiveness; ii) the net external position, reported in the International Investment Positions section of the International Monetary Fund's Balance of Payments Statistics (BOPS), and net of gold holdings; iii) the sum of net equity and FDI positions (both adjusted for valuation effects), foreign exchange reserves and the difference between accumulated flows of "debt assets", and the stock of debt measured by the World Bank (or the OECD).

Default data is from Borensztein & Panizza (2009) for the 1979-2004 period. We include sovereign defaults on foreign currency bond debt and foreign currency bank debt. A sovereign default is defined as the failure to meet a principal or interest payment on the due date (or within the specified grace period) contained in the original terms of the debt issue, or an exchange offer of new debt that contains terms less favorable than the original issue. Such rescheduling agreements covering short and long term debt are considered defaults even where, for legal or regulatory reasons, creditors deem forced rollover of principal to be voluntary. We use the updated and extended version default data from Reinhart & Rogoff (2010) dataset for the 2005-2014 period. A default is defined as an external sovereign default crisis or a restructuring of external debt.

B Institutional Investor Index & Sovereign Risk Measures

In this section, we show that the Institutional Investor Index (III) is a robust measure of sovereign risk by showing that it is highly correlated with other measures of sovereign risk. We also explain how we use the III to chain the Emerging Markets Bond Index (EMBI) backwards to be able to use it to calculate the average and standard deviation of the spread used in Section 4.

B.1 Moody's and Fitch Credit Ratings

Credit ratings by agencies such as Moody's and Fitch are commonly used measures of sovereign risk. These agencies assign risk based on rating symbols. Tables B1 and B2 provide brief descriptions of what each symbol signifies about credit risk. Table B3 provides the date each agency first issued a credit risk rating to a given sovereign.

Table B1: Moody's Global Long-Term Rating Scale

Rating	Description
Aaa	Obligations rated Aaa are judged to be the highest quality, subject to the lowest level of credit risk.
Aa	Obligations rated Aa are judged to be of high quality and are subject to very low credit risk.
A	Obligations rate A are judged to be upper-medium grade and are subject to low credit risk.
Baa	Obligations rated Baa are judged to be medium-grade and subject to moderate credit and as such may possess certain speculative characteristics.
Ba	Obligations rated Ba are judged to be speculative and are subject to substantial credit risk.
B	Obligations rated B are considered speculative and are subject to high credit risk.
Caa	Obligations rated Caa are judged to be speculative of poor standing and are subject to very high credit risk.
Ca	Obligations rated Ca are very highly speculative and are likely in, or very near, default with some prospect of principal and interest.
C	Obligations rated C are the lowest rated and are typically in default, with little prospect for recovery of principal or interest.

Note: Moody's appends numerical modifiers 1, 2, and 3 to each generic rating classification from Aa through Caa. The modifier 1 indicates that the obligation ranks in the higher end of its generic rating category, the modifier 2 indicates a mid-range ranking, and the modifier 3 indicates a ranking in the lower end of that generic rating category.

Table B2: Fitch International Credit Rating Scale

Rating	Description
AAA	Highest credit quality. AAA ratings denote the lowest expectation of default risk. They are assigned only in cases of exceptionally strong capacity for payment of financial commitments. This capacity is highly unlikely to be adversely affected by foreseeable events.
AA	Very high credit quality. AA ratings denote expectations of very low default risk. They indicate very strong capacity for payment of financial commitments. This capacity is not significantly vulnerable to foreseeable events.
A	High credit quality. A ratings denote expectations of low default risk. The capacity for payment of financial commitments is considered strong. This capacity may, nevertheless, be more vulnerable to adverse business or economic conditions than is the case for higher ratings.
BBB	Good credit quality. BBB ratings indicate that expectations of default risk are currently low. The capacity for payment of financial commitments is considered adequate, but adverse business or economic conditions are more likely to impair this capacity.
BB	Speculative. BB ratings indicate an elevated vulnerability to default risk, particularly in the event of adverse changes in business or economic conditions over time; however, business or financial flexibility exists that supports the servicing of financial commitments.
B	Highly speculative. B ratings indicate that material default risk is present, but a limited margin of safety remains. Financial commitments are currently being met; however, capacity for continued payment is vulnerable to deterioration in the business and economic environment.
CCC	Substantial credit risk. Default is a real possibility.
CC	Very high levels of credit risk. Default of some kind appears probable.
C	Near default. A default or default-like process has begun, or the issuer is in standstill, or for a closed funding vehicle, payment capacity is irrevocably impaired.
RD	Restricted default.
D	D ratings indicate an issuer that in Fitch's opinion has entered into bankruptcy filings

Note: Within rating categories, Fitch may use modifiers. The modifiers "+" or "-" may be appended to a rating to denote relative status within major rating categories. Such suffixes are not added to AAA ratings and ratings below the CCC category.

Unlike the III that is updated each semester, credit rating changes can occur at any time for an individual sovereign. In order to merge credit ratings data with the III, we use the credit rating that has been assigned the longest to a sovereign during a particular semester and merge that rating with the respective semester III reading. Since the III is a continuous variable and credit rating are a discrete variable (i.e. factor variable over the ordinal ratings labels), we visualize their correlation with box plots.

Table B3: Credit Agency Rating's First Issued Date

Country	Moody's	Fitch
Argentina	11/18/1986	5/28/1997
Brazil	11/18/1986	12/1/1994
China	5/18/1988	12/11/1997
Colombia	8/4/1993	8/10/1994
Ecuador	7/24/1997	11/8/2002
Egypt	10/9/1996	8/19/1997
Gabon		10/29/2007
India	1/28/1988	3/8/2000
Iran		5/10/2002
Iraq		8/7/2015
Kazakstan	11/11/1996	11/5/1996
Kuwait	1/29/1996	12/20/1995
Malaysia	1/18/1986	8/13/1998
Mexico	12/18/1990	8/30/1995
Oman	1/29/1996	
Qatar	1/29/1996	3/6/2015
Russia	10/7/1996	10/7/1996
Saudi Arabia	1/29/1996	11/24/2004
Venezuela	12/29/1976	9/15/1997

Box plots are used to show the overall dispersion of a continuous variable over groups. In our case, the y-axis is the continuous III, and the x-axis is the agency's credit rating ranks. The credit rating ranks are ordered along the x-axis from highest to lowest credit risk (from left to right). The box plots then graphs the quartiles of III observations over each credit risk rating. The horizontal line across the middle of the box is the median. The second quartile is the region from the median line to the bottom of the box, while the third quarter is the region from the median line to the top of the box. The bottom end of the lower whisker is the smallest value excluding outliers and the top end of the upper whisker is the largest value excluding outliers. Outliers are plotted as dots above and below the whisker of the box. Outliers above the upper whisker are 1.5 times greater than the third quartile while outliers below the lower whisker are 1.5 times lower than the first quartile. Figure B1 plots the III over Moody's credit risk ratings, and figure B2 plots the III over Fitch credit risk ratings.

Figure B1: Moody's Long-Term Sovereign Credit Ratings over the III

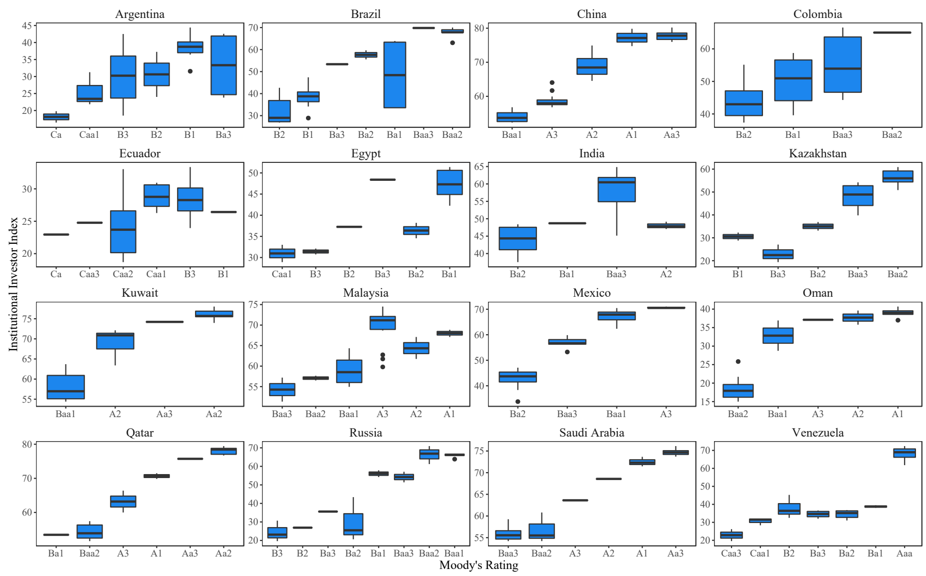
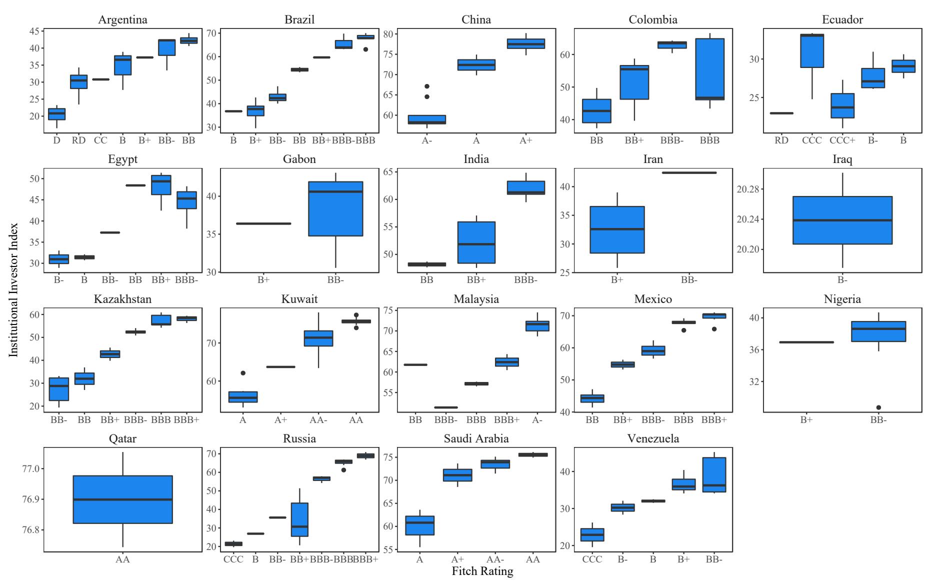


Figure B2: Fitch Long-Term Sovereign Credit Ratings over the III



We can see from the distributional characteristics of III over the Moody's and Fitch credit risk ratings that each sovereign's corresponding III measure tends to increase as its credit rating improves. This indicates that the III is correlated with credit ratings.

B.2 Emerging Markets Bond Index (EMBI)

The Emerging Market Bond Index (EMBI) is JP Morgan's index of dollar denominated bonds issued for various emerging economies. It is one of the most widely used benchmarks of emerging market sovereign debt. The index comprises of US dollar-denominated Brady bonds, loans, and Eurobonds that have a face value of $500 million dollars or more and have a maturity greater than a year. The EMBI is quoted as a spread on sovereign debt over US treasuries, and the III is a measure of sovereign risk where 0 indicates high risk of default and 100 indicates low risk of default. Thus we expect to see these two move in opposite directions if the III is a good indicator of sovereign risk. In other words we expect the EMBI to rise as sovereign risk increases. Indeed we see in table B4 that the EMBI and III are negatively correlated, moving in the same direction to indicate sovereign risk.

Table B4: Correlation Between EMBI and III

Country	Correlation
Angola	-0.570
Argentina	-0.751
Azerbaijan	0.031
Brazil	-0.789
China	0.312
Colombia	-0.740
Ecuador	-0.442
Egypt	-0.642
Gabon	-0.667
India	-0.186
Indonesia	-0.167
Iraq	-0.163
Kazakhstan	-0.293
Malaysia	-0.434
Mexico	-0.723
Nigeria	-0.666
Russian Federation	-0.686
Venezuela	-0.629
Vietnam	0.146

Since the EMBI was introduced only in 1992, we have fewer observations of the EMBI than we have for the III. Following Erb et al. (1996), we can use the fact that the EMBI and the III are correlated with each other to extend the EMBI backwards so that it starts in the same year as the III for country i.

We use the following equation to build the index for each country:

$$EMBI_t = \alpha_0 + \alpha_1 III_t + \epsilon_t \tag{B1}$$

Suppose we have observations of the $EMBI$ for country i starting at time t through T where $t < T$. We estimate (B1) using observations t through T of the $EMBI$ and III for country i. Table B5 reports the estimates for α_1 in (B1) for each country. We see that most country's estimate is negative and statistically significant. This implies that equation (B1) is an appropriate model to use to estimate values of the EMBI that are not available. We are then able to plug observation III_{t-1} into the estimated model to calculate the fitted value for $EMBI_{t-1}$. Now we re-estimate (B1) using observations $t-1$ through T of the $EMBI$ and III, and then plug observation III_{t-2} into the newly estimated model to calculate the fitted value for $EMBI_{t-2}$. We continue this back-substitution until we have exhausted all observations of the III for country i. Our final output is an index of the EMBI re-constructed to the same time as the first observation of the III for country i. Figures of our reconstructed EMBI indices are available upon request.

Table B5: α_1 Estimates on Observed Values of the EMBI

Country	Slope Coefficient	Standard Error
Angola	-61.18	35.98
Argentina	-156.10^{***}	20.91
Azerbaijan	1.44	17.76
Brazil	-21.82^{***}	2.59
China	1.75^{**}	0.81
Colombia	-14.98^{***}	2.24
Ecuador	-84.03^{***}	26.61
Egypt	-14.14^{***}	3.19
Gabon	-37.46^{***}	10.79
India	-6.00	12.96
Indonesia	-2.41	2.95
Iraq	-5.01	6.96
Kazakhstan	-18.56	15.15
Malaysia	-7.38^{***}	2.49
Mexico	-17.11^{***}	2.49
Nigeria	-42.23^{***}	7.88
Russian Federation	-41.69^{***}	7.47
Venezuela	-74.32^{***}	13.99
Vietnam	6.89	10.43

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

C Panel Estimation Approach

Before proceeding to estimate our model, we need to verify that all variables are integrated of the same order. To do so, we have used the test of the panel unit root of Im et al. (2003) IPS henceforth), which is based on averaging individual unit root test statistics for panels. Specifically, they proposed a test based on the average of augmented Dickey-Fuller statistics (ADF henceforth) computed for each group in the panel. In accordance with some survey on panel unit root tests (such as those discussed in Banerjee (1999)), this test is less restrictive and more powerful than others that do not allow for heterogeneity in the autoregressive coefficient. IPS test permit solving serial correlation problem by assuming heterogeneity between units (in this case, countries) in a dynamic panel framework, as considered here. The basic equation of IPS test is as follows:

$$\Delta y_{it} = \alpha_i + \beta_i y_{it-1} + \sum_{j=1}^{p} \phi_{ij} \Delta y_{it-j} + \epsilon_{it} \tag{C1}$$

for $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$, where N refers to the number of countries in the panel and T refers to the number of observations over time. In this case, y_i stands for each variable under consideration in our model (for example, III, oil GDP or non-oil GDP), α_i is the individual fixed effect and p is the maximum number of lags included in the test. The null hypothesis then becomes $\beta_i = 0$ for all i, against the alternative hypothesis, which is that $\beta_i < 0$ for some $i = 1, ..., N_1$ and $\beta_i = 0$ for $i = N_1 + 1, ..., N$, where N_1 denote the number of stationary panels. Therefore, IPS statistic can be written as follows:

$$\bar{t} = \frac{1}{N} \sum_{i=1}^{N} t_i^{ADF} \tag{C2}$$

where t_i^{ADF} is the ADF t-statistic for country i, taking into account the country specific ADF regression, given by (C1). The $\bar{t}$ statistic has been shown to be normally distributed under H_0. Table C1 reports the outcome for the global sample of this test.

As we can see, each variable is integrated of order one. Once the order of stationary has been defined, we estimated a country risk equation on the basis of cross-country panel data. In particular, we focus on three estimation methods which are consistent when both T and N are large. At one extreme, the usual practice is either to estimate N separate regressions

Table C1: Im et al. (2003) panel unit root test outcome: 1979-2010

	Levels		Logs	
	t-statistic	P-value	t-statistic	P-value
Inst. Inv.	0.280	0.610	0.293	0.615
Δ Inst. Inv.	-11.629	0.000	-11.645	0.000
Oil GDP	5.286	1.000	0.680	0.752
Δ Oil GDP	-11.972	0.000	-13.776	0.000
Non-oil GDP	14.801	1.000	2.247	0.988
Δ Non-oil GDP	-7.413	0.000	-10.345	0.000
Oil Reserves	4.376	1.000	2.404	0.992
Δ Oil Reserves	-13.954	0.000	-14.352	0.000
Ext. pub. debt to GDP	1.113	0.867	3.727	1.000
Δ Ext. pub. debt to GDP	-12.196	0.000	-11.045	0.000
NFA	0.117	0.546	.	.
Δ NFA	-9.364	0.000	.	.

Note: When computing NFA outcome, we excluded Iraq because of data limitations.

and compute the mean of the estimated coefficients across countries, which is called the Mean Group (MG) estimator. Pesaran & Smith (1995) show that the MG estimator will produce consistent estimates of the average of the parameters, but ignores the fact that certain parameters are the same across countries.

At the other extreme are the traditional pooled estimators (such as dynamic fixed effects estimators), where the intercepts are allow to differ across countries while all other coefficients and error variances are constrained to be the same. In this case, the model controls for all time-invariant differences between countries, so the estimated coefficient cannot be biased because of omitted time-invariant characteristics. An intermediate technique is the Pooled Mean Group (PMG) estimator, proposed by Pesaran et al. (1999), which relies on a combination of pooling and averaging of coefficients, allowing the intercepts, short-run coefficients and error variances to differ freely across countries, but the long-run coefficients are constrained to be the same.

Therefore, for the implementation of these methods we consider the following model:

$$III_{it} = \theta_{0i} + \theta_{1i}OilGDP_{it} + \theta_{2i}NonOilGDP_{it} + \theta_{3i}OilR_{it} + \theta_{4i}X_{it} + \theta_{5i}Default_{it} + \mu_i + \epsilon_{it} \quad \text{(C3)}$$

Again, each observation is subscripted for the country i and the year t. In this case, $X \in \{ExtPubD, OilDisc, NFA\}$. The variable III is the log of Institutional Investor's country credit ratings, $OilGDP$ is the log of oil GDP, $NonOilGDP$ is the log of non-oil GDP, $OilR$ is the log of oil reserves stock, $ExtPubD$ is the external public debt to GDP ratio, $OilDisc$ is the log of oil discoveries, NFA corresponds to net foreign assets to GDP ratio, and $Default$ is a dummy variable that the country is in default. Additionally, μ_i is a set of country fixed effects (such as geographical or institutional factors) and ϵ_{it} is the idiosyncratic error term.

Now, with a maximum lag of one for all variables except $Default$, we construct the autorregresive distributive lag (ARDL) (1,1,1,1,1,1,0) dynamic panel specification of (C3):

$$III_{it} = \lambda_i III_{i,t-1} + \delta_{10i}OilGDP_{it} + \delta_{11i}OilGDP_{i,t-1} + \delta_{20i}NonOilGDP_{it} + \delta_{21i}NonOilGDP_{i,t-1} + \delta_{30i}OilR_{it} + \delta_{31i}OilR_{i,t-1} + \delta_{40i}X_{it} + \delta_{41i}X_{i,t-1} + \theta_{5i}Default_{it} + \mu_i + \epsilon_{it} \quad \text{(C4)}$$

Then, the error correction equation of (C4) is:

$$\Delta III_{it} = \phi_i \Big(III_{i,t-1} - \hat{\theta_{0i}} - \hat{\theta_{1i}} OilGDP_{it} - \hat{\theta_{2i}} NonOilGDP_{it} - \hat{\theta_{3i}} OilR_{it} - \hat{\theta_{4i}} X_{it} - \hat{\theta}_{5i} Default_{it} \Big) - \delta_{11i}\Delta OilGDP_{it} - \delta_{21i}\Delta NonOilGDP_{it} - \delta_{31i}\Delta OilR_{it} - \delta_{41i}\Delta X_{it} + \epsilon_{it} \quad \text{(C5)}$$

where

$$\hat{\theta_{0i}} = \frac{\mu_i}{1-\lambda_i}; \hat{\theta_{1i}} = \frac{\delta_{10i}+\delta_{11i}}{1-\lambda_i}; \hat{\theta_{2i}} = \frac{\delta_{20i}+\delta_{21i}}{1-\lambda_i}$$
$$\hat{\theta_{3i}} = \frac{\delta_{30i}+\delta_{31i}}{1-\lambda_i}; \hat{\theta_{4i}} = \frac{\delta_{40i}+\delta_{41i}}{1-\lambda_i}; \hat{\theta_{5i}} = \frac{\theta_{5i}}{1-\lambda_i}; \phi_i = -(1-\lambda_i)$$

In this case, ϕ_i is the error correction speed of adjustment parameter, and we would expect ϕ_i to be negative if the variables exhibit a return to long-run equilibrium[1].

[1]Replacing $\hat{\theta}_i$-parameters and ϕ_i in equation (C3) we get:

$$\Delta III_{it} = -(1-\lambda_i)\Big(III_{i,t-1} - \frac{\mu_i}{1-\lambda_i} - \frac{\delta_{10i}+\delta_{11i}}{1-\lambda_i} OilGDP_{it} - \frac{\delta_{20i}+\delta_{21i}}{1-\lambda_i} NonOilGDP_{it} - \frac{\delta_{30i}+\delta_{31i}}{1-\lambda_i} OilR_{it} - \frac{\delta_{40i}+\delta_{41i}}{1-\lambda_i} X_{it} - \frac{\theta_{5i}}{1-\lambda_i} Default_{it} \Big) - \delta_{11i}\Delta OilGDP_{it} - \delta_{21i}\Delta NonOilGDP_{it} - \delta_{31i}\Delta OilR_{it} - \delta_{41i}\Delta X_{it} + \epsilon_{it}$$

Removing similar terms, the above expression is as follows:

$$\Delta III_{it} = -(1-\lambda_i) III_{i,t-1} + \mu_i + (\delta_{10i}+\delta_{11i}) OilGDP_{it} + (\delta_{20i}+\delta_{21i}) NonOilGDP_{it} + (\delta_{30i}+\delta_{31i}) OilR_{it} + (\delta_{40i}+\delta_{41i}) X_{it} + \theta_{5i} Default_{it} - \delta_{11i}\Delta OilGDP_{it} - \delta_{21i}\Delta NonOilGDP_{it} - \delta_{31i}\Delta OilR_{it} - \delta_{41i}\Delta X_{it} + \epsilon_{it}$$

Rewriting:

$$\begin{aligned} III_{it} - III_{i,t-1} = & -(1-\lambda_i) III_{i,t-1} + \mu_i + (\delta_{10i}+\delta_{11i}) OilGDP_{it} + (\delta_{20i}+\delta_{21i}) NonOilGDP_{it} + (\delta_{30i}+\delta_{31i}) OilR_{it} \\ & + (\delta_{40i}+\delta_{41i}) X_{it} - \delta_{11i}(OilGDP_{it} - OilGDP_{i,t-1}) - \delta_{21i}(NonOilGDP_{it} - NonOilGDP_{i,t-1}) \\ & - \delta_{31i}(OilR_{it} - OilR_{i,t-1}) - \delta_{41i}(X_{it} - X_{i,t-1}) + \theta_{5i} Default_{it} + \epsilon_{it} \end{aligned}$$

Again, simplifying this equality we obtain:

$$III_{it} = \lambda_i III_{i,t-1} + \delta_{10i} OilGDP_{it} + \delta_{11i} OilGDP_{i,t-1} + \delta_{20i} NonOilGDP_{it} + \delta_{21i} NonOilGDP_{i,t-1} + \delta_{30i} OilR_{it} + \delta_{31i} OilR_{i,t-1} + \delta_{40i} X_{it} + \delta_{41i} X_{i,t-1} + \theta_{5i} Default_{it} + \mu_i + \epsilon_{it}$$

Note that this expression is equivalent to (C4). For a long-run relationship to exist, we require that $\phi \neq 0$.

Table C2: Hausman test outcome: 1979-2010

	Model (1)		Model (2)		Model (3)	
	χ^2-stat	P-value	χ^2-stat	P-value	χ^2-stat	P-value
MG vs. DFE	0.02	1.000	0.01	1.000	0.06	1.000
PMG vs. DFE	0.03	1.000	0.03	1.000	0.03	1.000
MG vs. PMG	4.42	0.491	5.05	0.537	8.99	0.174

C.1 Estimation results

In this subsection we estimate the PMG, MG and DFE estimators for model (C5). In order to obtain reliable estimators and seeking to maintain a large data sample, we include information for China, India, and Brazil since these countries have large proven oil reserves, although these have not been oil net exporters in the time interval considered here. When deciding about model selection, we apply the Hausman test to see whether there are significant differences among these three estimators. The null of this test is that the difference between DFE and MG, DFE and PMG or PMG and MG is not significant. Consider, for example, the test between DFE and PMG. If the null is not rejected, the DFE estimator is recommended since it is efficient. The alternative is that there is a significant difference between PMG and DFE, and the null is rejected. Specifically, the Hausman statistic is:

$$H = \left(\beta_{DFE} - \beta_{PMG}\right)' \left[\text{var}(\beta_{DFE}) - \text{var}(\beta_{PMG})\right]^{-1} \left(\beta_{DFE} - \beta_{PMG}\right) \sim \chi^2$$

where β_j is the vector of coefficients and var(β_j) is the covariance matrix of β_j, estimated using the $j-$technique, for j =DFE, PMG. Under the null hypothesis, H has asymptotically the χ^2 distribution. Table C2 reports the results of Hausman test, in which Model (1) corresponds to equation (C5), excluding NFA from X_i, while Model (2) excludes $Default$. Model (3) includes all variables in X_i into the regressors.

Under the current specification, the hypothesis that the country risk equation (equation (C5)) is adequately modeled by a PMG or MG model is resoundingly rejected. In general, when considering Model (1) the results in table C2 suggest that it is not possible to reject the null hypothesis of the homogeneity restriction on regressors (in the short and long run), since P-values are both 1, which indicates that DFE is more efficient estimator than MG and

PMG, respectively. Notice that this conclusion holds for Model (2) and Model (3), because P-values associated to these tests are 1. Because of this, we choose to employ the DFE estimator.

The results for the unbalanced panel are found in Section 2 of the paper, and for robustness purposes Table D3 shows the results for the balanced panel.

Table C3: BALANCED PANEL- Dynamic Fixed Effects Regression Results for Institutional Investor Index

	Δ Inst. Investor Index		
	Model (1)	Model (2)	Model (3)
Convergence Coefficient			
Inst. Investor Index (-1)	-0.233***	-0.234***	-0.236***
	(0.032)	(0.032)	(0.032)
Short-Run Coefficients			
Δ Oil Production	0.012	0.011	0.019
	(0.032)	(0.031)	(0.031)
Δ Non-Oil GDP	0.112	0.071	0.061
	(0.075)	(0.076)	(0.075)
Δ Oil Reserves	0.028	0.027	0.025
	(0.024)	(0.023)	(0.023)
Δ Ext. pub. debt to GDP	-0.082	-0.220**	-0.238***
	(0.073)	(0.091)	(0.090)
Δ Oil Discoveries	-0.003	-0.002	-0.002
	(0.004)	(0.004)	(0.004)
Δ NFA		-0.193***	-0.187***
		(0.056)	(0.055)
Long-Run Coefficients			
Oil Production	0.217**	0.256***	0.250***
	(0.085)	(0.085)	(0.083)
Non-Oil GDP	0.529***	0.480***	0.495***
	(0.174)	(0.172)	(0.169)
Oil Reserves	-0.217***	-0.195***	-0.196***
	(0.060)	(0.061)	(0.060)
Ext. pub. debt to GDP	-1.024***	-1.436***	-1.205***
	(0.217)	(0.275)	(0.274)
Default	-0.225**		-0.213**
	(0.091)		(0.088)
Oil Discoveries	0.026	0.021	0.022
	(0.024)	(0.024)	(0.023)
NFA		-0.067	-0.082
		(0.148)	(0.146)
Constant	-2.577**	-2.219*	-2.355**
	(1.218)	(1.205)	(1.194)

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

C.2 Estimation Results with Oil Prices and no TFE

In our benchmark specification, the time fixed effects (TFE) capture country-invariant-time-varying features, such as (but not restricted to) the price of oil. We opted for this specification because we did not want to restrict ourselves to just having the oil price at the expense of missing other potentially important variables that are common to all the countries in the sample (such as the Federal Funds Rate). We understand that this approach can have two downfalls. One, that the time fixed effects can only be included in the short run because they have no economic meaning in the long run, and second, that we can't speak to the exact coefficient of the oil price. Given the pros and cons of using TFE discussed above (and for completeness), we report the results of a DFE exercise if we introduce oil prices (in both the short and the long run) instead of TFE. Table C4 shows the results.

Table C4: Regression Results for Institutional Investor Index Dynamic Fixed Effects: Time Fixed Effects vs Real Oil Price

	TFE	Real Oil Price
Long-Run Coefficients		
Oil Production	0.038	0.048
	(0.041)	(0.038)
Non-Oil GDP (LCU Deflated)	0.101	0.135**
	(0.100)	(0.057)
Oil Reserves	-0.141***	-0.142***
	(0.050)	(0.044)
Ext. pub. debt to GDP	-1.001***	-0.922***
	(0.178)	(0.164)
NFA	-0.119	-0.106
	(0.116)	(0.110)
Default	-0.379***	-0.363***
	(0.068)	(0.060)
Oil Discoveries	0.039	0.030
	(0.027)	(0.025)
Real Oil Price		-0.036
		(0.052)
Convergence Coefficient		
Inst. Investor Index (-1)	-0.183***	-0.205***
	(0.020)	(0.020)
Short-Run Coefficients		
Δ Oil Production	0.055**	0.037*
	(0.022)	(0.022)
Δ Non-Oil GDP (LCU Deflated)	0.198***	0.246***
	(0.057)	(0.058)
Δ Oil Reserves	0.010	0.017
	(0.020)	(0.020)
Δ Ext. pub. debt to GDP	-0.107**	-0.052
	(0.051)	(0.052)
Δ NFA	-0.046	-0.004
	(0.034)	(0.035)
Δ Oil Discoveries	-0.003	-0.001
	(0.003)	(0.004)
Δ Real Oil Price		0.059***
		(0.019)
Constant	0.209	0.056
	(0.559)	(0.339)
Observations		

Standard errors in parentheses
$^{*}\ p < 0.1$, $^{**}\ p < 0.05$, $^{***}\ p < 0.01$

We obtain a positive and significant coefficient of the oil price in the short run and a negative, albeit non-significant coefficient in the long-run. The coefficients of the other variables do not change drastically. Specifically, oil reserves are still negative and significant in the long run and non significant in the short run. Changes are related to the role of Non-Oil GDP in the long run, which results positive and significant, and the change of external public debt to GDP which is still negative but loses its significance. Importantly, the convergence coefficient is still negative and highly significant, and involves a faster convergence than the model with DFE and TFE.

These results highlight the importance of oil prices in the short run for country risk, but don't discredit the results obtained in the DFE with TFE specification. Additionally, the fact that the oil price is significant in the short run and not in the long run, further supports the use of the TFE (which can only be introduced in the short run).

D Oil Price Upswings and Downswings

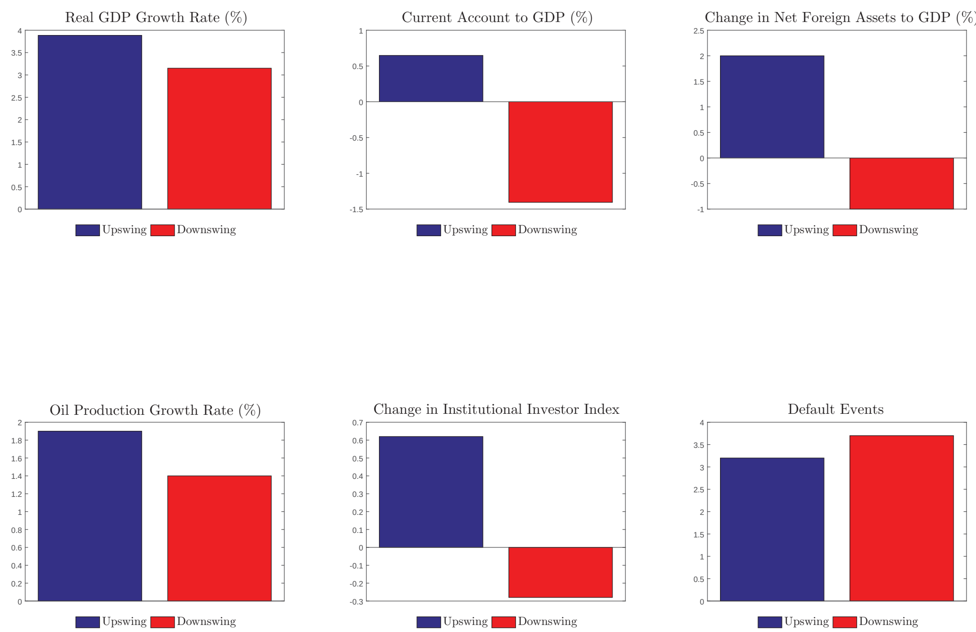
Table D1: Oil Price Upswings and Downswings

Downswings		Upswings	
Period	Number of Months	Period	Number of Months
NOV 75 - OCT 78	36	NOV 78 - JAN 81	27
FEB 81 - JUL 86	66	AUG 86 - JUL 87	12
AUG 87 - NOV 88	16	DEC 88 - OCT 90	23
NOV 90 - DEC 93	38	JAN 94 - OCT 96	34
NOV 96 - DEC 98	26	JAN 99 - SEP 00	21
OCT 00 - DEC 01	15	JAN 02 - JUL 08	79
AUG 08 - MAY 10	22		
TOTAL	219	TOTAL	196

Figure D1 provides a complementary view of the association between oil-price movements and macroeconomic fluctuations to that provided by business cyce moments. It shows the differential performance of macro variables across oil price upswings and downswings. To construct this figure we divided our panel dataset into two groups, one for all years in which oil prices rose (oil-price upswings) and one for all years in which they fell (oil-price downswings). Table E1 shows how each year in the time-series corresponds to a downswing or an upswing. We then averaged the changes in the different macroeconomic variables over the upswings and downswings and provide in Figure D1 plots of the average change in each macro variable over the upswings and downswings of oil prices.

Oil price upswings are associated with higher growth in GDP and oil production, trade balance improvement, and lower sovereign risk (higher III). Likewise, oil price downswings are associated with lower growth in oil extraction and GDP, trade balance deterioration, and higher sovereign risk.

Figure D1: Oil Price Swings and Macro Performance



E Are all Oil Exporting Countries Price Takers?

This appendix examines whether the countries in our sample are price takers in the world market of oil.[2] We examine causality between a country's extraction and oil prices using two strategies, both in a bivariate context. First, we test on the levels, using a modified version of the Granger causality test proposed by Toda & Yamamoto (1995). Second, we test causality using the Granger test on the first differences of both series.

For the causality test a modified Wald test (MWALD) is used as proposed by Toda & Yamamoto (1995) that avoids the problems associated with the ordinary Granger causality test by ignoring any possible non-stationary or cointegration between series when testing for causality. [3]The Toda & Yamamoto (1995) approach fits a standard vector autoregressive

[2]We are grateful to Norberto Rodriguez-NiÒo from the Banco de la República de Colombia for his assistance with this analysis.

[3]As quoted from Wolde-Rufael (2005) "... given that unit root and cointegration tests have low power against

model in the levels of the variables (rather than the first differences, as the case with Granger causality tests) thereby minimizing the risks associated with the possibility of wrongly identifying the order of integration of the series.

The basic idea of this approach is to artificially augment our bivariate VAR or order k, by the maximal order of integration, one in this case. Once this is done, a (k+1)-th order of VAR is estimated and the coefficients of the last one lagged vector is ignored. The application of the Toda & Yamamoto (1995) procedure ensures that the usual Wald test statistic for Granger causality has the standard asymptotic distribution hence valid inference can be done.

Lag length for VAR is chosen based on information criteria (Akaike, Schwarz and Hanna-Quinn), the Portmanteau (bivariate Ljung-Box statistic) test is used to decide. This statistics joint with its p-values are contained and third and four columns of tables E1 and E2.

E.1 Data

We used monthly data of crude oil for the 20 major exporting countries; the sample period cover from January 2002 to November 2016. The data source is Joint Oil Data Initiative (JODI) Database (available at http://www.jodidb.org/TableViewer/tableView.aspx). For Colombia, the figures have source Banco de la República and are based on DIAN-DANE. Units are thousand barrels per period. Exports the top 20 countries accounted for approximately 96% of reported crude oil exports at the JODI base in 2015.

E.2 Results

Unit root test results (not presented here but available up to request) show that all the variables are integrated of order one.

Table E1 shows the results for the TY test. It is worth to remain that the null hypothesis in this as next table is that of non-causality. Table E2 presents results for Granger causality test, for the series in differences. Results in both tables coincide signaling oil exports from United Arab Emirates, Oman, Brazil and Azerbaijan causing (in Granger sense) oil prices.

the alternative, these tests can be misplaced and can suffer from pre-testing bias (see Pesaran et al. (2001); Toda & Yamamoto (1995)). Moreover, as demonstrated by Toda & Yamamoto (1995), the conventional F-statistic used to test for Granger causality may not be valid as the test does not have a standard distribution when the time series data are integrated or cointegrated."

TY shows that exports from Canada also G-cause prices, and model in differences indicated that Kuwait G-cause oil prices.

Table E1: Taro-Yamamoto test results for series in levels

Country	Lag	Lung-Box		Jarque-Bera		Taro-Yamamoto		
		Q-Stat	P-Value	Stat	P-Value	Statistic	P-Value	Decision
Saudi Arabia	2	26.75	0.32	164.12	0.00	1.47	0.48	
Russia	2	30.14	0.18	75.23	0.00	1.90	0.39	
Iraq	2	28.48	0.24	50.22	0.00	1.28	0.53	
U. Arab Emir.	2	29.43	0.20	31.25	0.00	17.32	0.00	Cause
Canada	2	26.31	0.34	70.50	0.00	7.30	0.03	Cause
Nigeria	2	17.33	0.83	13.42	0.01	0.99	0.61	
Kuwait	2	21.64	0.60	23.36	0.00	1.20	0.55	
Angola	4	23.88	0.09	17.30	0.00	7.86	0.10	
Venezuela	2	23.61	0.48	46.25	0.00	5.17	0.08	
Iran	2	27.83	0.27	66.94	0.00	5.00	0.08	
Mexico	2	21.95	0.58	14.50	0.01	4.19	0.12	
Norway	3	18.43	0.56	6.47	0.17	3.45	0.33	
Oman	2	18.92	0.76	4320.42	0.00	9.10	0.01	Cause
Brasil	7	3.17	0.53	24.80	0.00	16.69	0.02	Cause
Azerbaijan	2	20.98	0.64	1171.78	0.00	13.11	0.00	Cause
Uni. Kingdom	2	28.88	0.22	22.93	0.00	0.10	0.95	
Algeria	2	29.15	0.21	12.62	0.01	4.89	0.09	
Qatar	2	20.04	0.69	127.50	0.00	1.84	0.40	
USA	3	21.69	0.36	332.23	0.00	1.19	0.76	
Colombia	3	25.90	0.17	13.44	0.01	0.81	0.85	

Table E2: Granger tets results for series in diferences

Country	Lag	Lung-Box		Jarque-Bera		Taro-Yamamoto		
		Q-Stat	P-Value	Stat	P-Value	Statistic	P-Value	Decision
Saudi Arabia	7	6.69	0.15	49.06	0.00	10.77	0.15	
Russia	6	14.45	0.07	132.17	0.00	7.56	0.27	
Iraq	2	34.37	0.08	6.76	0.15	3.41	0.18	
U. Arab Emir	6	9.38	0.31	71.06	0.00	18.78	0.00	Cause
Canada	6	12.82	0.12	5.86	0.21	7.58	0.27	
Nigeria	1	37.67	0.10	25.24	0.00	0.33	0.57	
Kuwait	6	6.57	0.58	14.00	0.01	13.63	0.03	Cause
Angola	6	8.01	0.43	342.62	0.00	10.84	0.09	
Venezuela	1	26.57	0.54	16.27	0.00	2.14	0.14	
Iran	2	34.39	0.08	95.29	0.00	2.96	0.23	
Mexico	2	28.31	0.25	32.99	0.00	2.65	0.27	
Norway	2	32.64	0.11	20.19	0.00	3.26	0.20	
Oman	6	10.13	0.26	13053.21	0.00	26.42	0.00	Cause
Brazil	7	8.94	0.06	265.77	0.00	18.39	0.01	Cause
Azerbaijan	2	32.15	0.12	1029.34	0.00	12.68	0.00	Cause
Uni. Kingdom	6	14.49	0.07	27.07	0.00	5.27	0.51	
Algeria	2	33.76	0.09	7.44	0.11	3.82	0.15	
Qatar	6	7.20	0.51	87.55	0.00	12.24	0.06	
USA	3	35.85	0.02	33.07	0.00	2.56	0.46	
Colombia	2	29.64	0.20	18.90	0.00	1.43	0.49	

F Model Variants under Commitment

In this Appendix we analyze three variants of the model under the assumption that there is no default in equilibrium. The first one is the case of financial autarky, the second one is the case where the planner takes the price of bonds as given, and the third one is the case where the bond pricing function is endogenous and depends on the planner's debt and reserves. The autarky case coincides with the solution of the default payoff if default triggers permanent exclusion from credit markets.

The generic planner's problem in sequential form is the following:

$$\max_{c_t, x_t, b_{t+1}, s_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \tag{F1}$$

s.t.

$$c_t + e\left(x_t, s_t\right) = y_t + p_t x_t - q\left(s_{t+1}, b_{t+1}\right) b_{t+1} + b_t \tag{F2}$$

$$s_{t+1} = s_t - x_t + \kappa \tag{F3}$$

$$x_t \geq 0 \tag{F4}$$

$$x_t \leq s_t + \kappa. \tag{F5}$$

The first constraint is the resource constraint, where $q\left(s_{t+1}, b_{t+1}\right)$ is an ad-hoc pricing function of bonds that is assumed to be the equilibrium pricing function of the model with default and satisfies the following assumptions: $q\left(\cdot\right)$ is continuously differentiable, strictly concave and increasing in b_{t+1} for $b_{t+1} \in [-\overline{b}(s_{t+1}), 0]$, where $-\overline{b}(s_{t+1})$ is the threshold debt above which default is certain for a given s_{t+1} (i.e., $D(\overline{b}(s_{t+1}), s_{t+1})$ includes all (y_{t+1}, p_{t+1}) pairs, which exists because of Proposition 1), with $q\left(\cdot\right) = q^*$ for $b_{t+1} \geq 0$ and $q\left(\cdot\right) = 0$ for $b_{t+1} \leq \overline{b}(s_{t+1})$. $q\left(\cdot\right)$ is also increasing and concave in s_{t+1} for $s_{t+1} \in [\tilde{s}(b_{t+1}), s_t + \kappa]$, where $\tilde{s}(b_{t+1}) = max[s_t + \kappa - s_t(p_t/\psi)^{(1/\gamma)}, \overline{s}(b_{t+1})]$, $s_t + \kappa - s_t(p_t/\psi)^{(1/\gamma)}$ is the minimum s_{t+1} needed for profits to be non-negative, and $\overline{s}(b_{t+1})$ is the threshold oil reserves below which default is certain for a given b_{t+1} (i.e., $D(b_{t+1}, \overline{s}(b_{t+1}))$ includes all (y_{t+1}, p_{t+1}) pairs, which exists because of Proposition 4). We also assume that $\overline{b}(s_{t+1})$ is increasing in s_{t+1} and $\overline{s}(b_{t+1})$ is increasing in b_{t+1}. In addition, we assume shocks are i.i.d so that $q(\cdot)$ is independent of p_t and y_t. The second constraint is the law of motion of reserves. The third and fourth constraints are the feasibility boundaries of oil extraction.

The first-order conditions are:

$$\lambda_t = u'(c_t) \tag{F6}$$

$$\lambda_t [p_t - e_x(x_t, s_t)] + \psi_t^l = \mu_t + \psi_t^u \tag{F7}$$

$$u'(c_t)[p_t - e_x(x_t, s_t) + q_s(s_{t+1}, b_{t+1}) b_{t+1}] + \psi_t^l - \psi_t^u = \beta E_t \left[u'(c_{t+1})(p_{t+1} - e_x(x_{t+1}, s_{t+1}) - e_s(x_{t+1}, s_{t+1})) + \psi_{t+1}^l \right] \tag{F8}$$

$$u'(c_t)[q(s_{t+1}, b_{t+1}) + q_b(s_{t+1}, b_{t+1}) b_{t+1}] = \beta E_t \left[u'(c_{t+1}) \right]. \tag{F9}$$

where λ_t is multiplier on the resource constraint, μ_t is the multiplier on the law of motion of reserves, and ψ_t^h and ψ_t^l are the multipliers on the upper and lower feasibility constraints on oil extraction.

Defining the planner's return on bonds as $R^b(s_{t+1}, b_{t+1}) \equiv \frac{1}{q(t+1)+q_b(t+1)b_{t+1}}$, which is decreasing in b_{t+1} (i.e. the planner's real interest rate increases with debt) because of the assumed properties of $q(\cdot)$, the Euler equation for bonds (eq (F9)) implies:[4]

$$u'(c_t) = R^b(s_{t+1}, b_{t+1}) \beta E_t \left[u'(c_{t+1}) \right]. \tag{F10}$$

Notice that, as implied by the definition of R^b, in evaluating the marginal benefit of borrowing in the right-hand-side of this expression, the planner internalizes that borrowing more (i.e. making b_{t+1} "more negative") increases the cost of borrowing.

The rate of return on oil extraction is defined as $R_{t+1}^O \equiv \frac{q_{t+1}^O + d_{t+1}^O}{q_t^O}$, where q_t^O is the asset price of oil defined as $q_t^O \equiv p_t - e_x(t) + \Delta\tilde{\psi}_t$ (where $\Delta\tilde{\psi}_t \equiv \tilde{\psi}_{t+1}^l - \tilde{\psi}_{t+1}^h$ and $\tilde{\psi}_t^i = \psi_t^i / u'(t)$ for $i = h, l$) and d_{t+1}^O is the dividend from oil extraction at $t+1$ defined as $d_{t+1}^O \equiv -e_s(t+1) + \tilde{\psi}_{t+1}^h$. Notice that $d_{t+1}^O > 0$ because $e_s(t+1) < 0$ and $\tilde{\psi}_{t+1}^h \geq 0$. The Euler equation for oil reserves (eq. (F8)) can then be rewritten as:

$$u'(c_t) \left[1 + \frac{q_s(s_{t+1}, b_{t+1}) b_{t+1}}{q_t^O} \right] = \beta E_t \left[u'(c_{t+1}) R_{t+1}^O \right]. \tag{F11}$$

The left-hand-side of this expression shows that in evaluating the marginal cost of accumulating additional reserves, the planner internalizes the fact that higher s_{t+1} increases the price of bonds, so that if it plans to issue debt ($b_{t+1} < 0$), the higher price at which it can

[4]The derivative of $R^b(\cdot)$ w.r.t. b_{t+1} is $R_b^b(\cdot) = \frac{-\left(2q_b(\cdot)+q_{bb}(\cdot)b_{t+1}\right)}{\left(q(\cdot)+q_b(\cdot)b_{t+1}\right)^2}$, and the properties that $q(s_{t+1}, b_{t+1}) = q^*$ for $b_{t+1} \geq 0$ and $q(s_{t+1}, b_{t+1})$ is strictly concave and increasing in b_{t+1} for $b_{t+1} \in [-\bar{b}(s_{t+1}), 0]$ imply that $-(2q_b(\cdot) + q_{bb}(\cdot)b_{t+1}) > 0$ and hence $R_b^b(\cdot) < 0$ in that same interval.

be sold reduces the marginal cost of building reserves. Hence, we can also express the Euler equation of reserves redefining the rate of return on oil to impute this extra gain:

$$u'(c_t) = \beta E_t \left[u'(c_{t+1}) \tilde{R}^O_{t+1} \right], \tag{F12}$$

where $\tilde{R}^O_{t+1} \equiv \frac{q^O_{t+1} + d^O_{t+1}}{\left[q^O_t + q_s(s_{t+1}, b_{t+1}) b_{t+1}\right]}$ is the rate of return on oil inclusive of the benefit of higher reserves increasing the price at which newly-issued debt is sold.

The above Euler equation can be used to solve forward for the asset price of oil. To this end, rewrite the equation as follows:

$$q^O_t + z_t = E_t \left[\frac{\beta u'(c_{t+1})}{u'(c_t)} \left(q^O_{t+1} + d^O_{t+1} \right) \right] \tag{F13}$$

where $z_t \equiv q_s(t)\, b_{t+1}$ and $q_s(t)$ is the derivative with respect to reserves of the price of bonds sold at date t, which is a function of (b_{t+1}, s_{t+1}). Notice $z_t \leq 0$ because $q_s(\cdot) > 0$ for $b_{t+1} < 0$ and otherwise $q_s(\cdot) = 0$. Adding and subtracting z_{t+1} to q^O_{t+1} in the right-hand-side of this equation and solving forward yields:

$$q^O_t + z_t = E_t \left[\sum_{s=t+1}^{\infty} \beta^{s-t} \frac{u'(s)}{u'(t)} [d^O_s - z_s] \right] > 0 \tag{F14}$$

The expression in the right-hand-side is positive because marginal utility is positive, $d^0_s > 0$ and $z_s \leq 0$. It follows then that $q^O_t + z_t > 0$, and since $z_s \leq 0$ we obtain $q^O_t > -z_t \geq 0$. Thus, the asset price of oil equals the expected present discounted value (discounted with the planner's stochastic discount factors) of the revenue stream composed of oil dividends plus the marginal revenue of selling bonds at a higher price when reserves increase. Or, the asset price of oil with this marginal revenue imputed, $\tilde{q}^0_t$ equals the expected present discounted value of the stream of oil dividends with the stream of these marginal revenues included $\tilde{q}^O_t = E_t \left[\sum_{s=t+1}^{\infty} \beta^{s-t} \frac{u'(s)}{u'(t)} \tilde{d}^O_s \right]$, where $\tilde{d}^O_s \equiv d^O_s - z_s$.

Combining the Euler equations for bonds and reserves yields the following expression for the excess return on oil (the oil risk premium):

$$E_t \left[R^o_{t+1} \right] - R^b_{t+1}(s_{t+1}, b_{t+1}) \left[1 + \frac{q_s(t+1) b_{t+1}}{q^O_t} \right] = - \frac{\text{cov}_t \left(u'(c_{t+1}), R^o_{t+1} \right)}{E_t \left[u'(c_{t+1}) \right]}. \tag{F15}$$

The left-hand-side is the excess return relative to the yield on bonds inclusive of the effect of higher reserves on the resources generated by borrowing. Defined in this way, the excess return takes the standard form of an equity premium determined by the covariance of the planner's marginal utility and the rate of return on oil. Defining the return on oil with the effect of higher reserves increasing bond prices imputed, the excess return is:

$$E_t\left[\tilde{R}^o_{t+1}\right] - R^b_{t+1}\left(s_{t+1}, b_{t+1}\right) = -\frac{\text{cov}_t\left(u'\left(c_{t+1}\right), \tilde{R}^o_{t+1}\right)}{E_t\left[u'\left(c_{t+1}\right)\right]}. \tag{F16}$$

We explore next three cases of this generic setup. First, a case in which the economy is in permanent financial autarky but can export oil. Second, a small-open-economy case in which the economy has access to a world credit market at a constant, exogenous price of bonds q^*, which is akin to an RBC model with oil extraction. Third, a case in which the planner faces the exogenous bond pricing function $q(b_{t+1}, s_{t+1})$. In each instance we discuss results with and without uncertainty.

F.1 Financial Autarky

Consider first the case in which the economy is in financial autarky and there is no uncertainty. The Euler equation of reserves implies:

$$\frac{q^o_{t+1} + d^o_{t+1}}{q^o_t} = \frac{u'\left(c_t\right)}{\beta u'\left(c_{t+1}\right)}. \tag{F17}$$

In turn, solving forward this condition yields a standard asset-pricing condition by which the asset price of oil equals the present discounted value of oil dividends discounted with the intertemporal discount factors:

$$q^O_t = \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{u'(s)}{u'(t)} d^O_s \tag{F18}$$

Note that since $d^0_s > 0$ and $u'(s), u'(t) > 0$, it follows that $q^O_t > 0$.

In this case, the optimal extraction and reserves plans equate R^o_t with the endogenous domestic real interest rate represented by the intertemporal marginal rate of substitution, each represented by the left- and right-hand-side of the reserves Euler equation, respectively. Oil extraction and reserves are used to smooth consumption.

The deterministic steady state is characterized by these two conditions:

$$\beta\left(q^{Oss} + d^{Oss}\right) = q^{Oss} \Rightarrow \frac{d^{Oss}}{q^{Oss}} = \rho,$$

$$x^{ss} = \kappa,$$

where ρ is the rate of time preference. Using the definitions of d^O and q^O and assuming an internal solution for extraction yields the following steady-state equilibrium condition:

$$-e_s(ss) = \rho\left[p^{ss} - e_x(ss)\right].$$

Using the functional form for extraction costs, $e = \psi\left(\frac{x_t}{s_t}\right)^\gamma x_t$, the above condition becomes:

$$\gamma\psi\left(\frac{\kappa}{s}\right)^{1+\gamma} = \rho\left[p^{ss} - (1+\gamma)\psi\left(\frac{\kappa}{s}\right)^\gamma\right]$$

which can be rewritten as:

$$\psi\left(\frac{\kappa}{s}\right)^\gamma\left[\gamma\left(\frac{\kappa}{s}\right) + \rho(1+\gamma)\right] = \rho p^{ss}. \tag{F19}$$

The steady state oil reserves s^{ss} is the value of s that solves the above equation. Since the left-hand-side is a decreasing, convex function of s, the condition determines a unique value of s^{ss} that rises as p^{ss} falls. Hence, a permanent decline in oil prices causes a permanent increase in oil reserves.

In the stochastic version of this setup, the planner uses oil reserves for self insurance, since there are no state-contingent claims to hedge oil-price shocks and no credit market of non-state-contingent international bonds. The Euler equation becomes: $u'(c_t) = \beta E_t\left[R^O_{t+1}u'(c_{t+1})\right]$. The asset price of oil is still positive and given by $q^O_t = E_t\left[\sum_{s=t+1}^{\infty}\beta^{s-t}\frac{u'(s)}{u'(t)}d^O_s\right]$. Because of self insurance, the long-run average of reserves in this economy will be larger than s^{ss} (i.e., the planner builds a buffer stock of precautionary savings in the form of oil reserves).

In Appendix H, we present the recursive formulation of this financial autarky setup and derive key properties of the associated dynamic programming problem. In particular, we show that non-negativity of oil profits and a coefficient ψ in the extraction cost function larger than the largest realization of p guarantee that the decision rule on reserves $s'(s, p, y)$ is increasing is s and that the lower bound on s_{t+1} (i.e., the upper bound on x_t) is never binding.

F.2 Exogenous q

Consider next the small-open-economy case with a constant, world-determined real interest rate such that $R^b(s_{t+1}, b_{t+1}) = R^*$. Without uncertainty, the Euler equations for bonds and reserves yield the following no-arbitrage condition for the real returns on bonds and oil:

$$R^o_{t+1} = \frac{u'(c_t)}{\beta u'(c_{t+1})} = R^*. \tag{F20}$$

Using the law of motion of reserves and the definitions of the asset price of oil and oil dividends, this no-arbitrage condition yields the following condition (assuming an internal solution for x_t for simplicity):

$$\frac{p_{t+1} - e_x\left(s_{t+1} - s_{t+2}, s_{t+1}\right) - e_s\left(s_{t+1} - s_{t+2}, s_{t+1}\right)}{p_t - e_x\left(s_t - s_{t+1}, s_t\right)} = R^*. \tag{F21}$$

This is a second-order difference equation in s that pins down the optimal decisions for $\{x_t, s_{t+1}\}_{t=0}^{\infty}$ as functions of oil prices and reserves only (and the parameter values of the extraction cost function and R^*). Hence, this setup is akin to the deterministic small-open-economy model with capital accumulation in which there is "Fisherian separation" of the investment and production decisions from the consumption and savings plans. Here, the same happens with the optimal plans for oil extraction and accumulation of oil reserves: they are determined independently of those for consumption and debt.

Assuming $\beta R^* = 1$, consumption is perfectly smooth for all t, while reserves and extraction follow the dynamics governed by the above second-order difference equation. The sovereign adjusts bond holdings as necessary so that consumption is perfectly smooth while extraction follows its transitional dynamics towards its steady state. This determines the present value of oil income net of extraction costs, and given that the perfectly smooth level of consumption is determined so as to satisfy the intertemporal resource constraint (i.e. the present value of constant consumption equals the present value of oil plus non-oil GDP plus initial bond holdings).

Since $e(\cdot)$ is increasing in x_t and decreasing in s_t, the above condition implies that, when p_{t+1} rises relative to p_t, the planner reallocates extraction from t to $t+1$ (i.e. increases the accumulation of reserves at t). This is a key incentive that is also a work in the model with default risk, but there it interacts with the planner's incentives to default and to affect the price of issuing new debt by adjusting reserves. As we demonstrate in Appendix F, default incentives strengthen when oil prices are low and the set of pairs of income and oil prices at which default is preferable shrinks as reserves grow.

This model's deterministic steady state is analogous to the one of the financial autarky case, except that the net world real interest rate $r^* = R^* - 1$ replaces the rate of time preference. Hence, the condition pinning down the deterministic steady state of reserves becomes:

$$\psi\left(\frac{\kappa}{s}\right)^{\gamma}\left[\gamma\left(\frac{\kappa}{s}\right) + r^*(1+\gamma)\right] = r^* p^{ss}.$$

As in the case of financial autarky, there is a unique deterministic steady state for s^{ss} and it increases as the steady-state price of oil falls.

The stochastic version of the model yields a standard equity-premium expression for the excess return on oil:

$$E_t\left[R^O_{t+1}\right] - R^*_{t+1} = -\frac{\text{cov}_t\left(u'\left(c_{t+1}\right), R^O_{t+1}\right)}{E_t\left[u'\left(c_{t+1}\right)\right]},$$

This is also analogous to the expression that a standard small-open-economy RBC model would yield. Bonds are ued for self-insurance (i.e., borrowing incentives are weakened by the precautionary savings motive) and extraction and reserves play the role of investment and capital. The asset price of oil is again positive and is now given by $q^O_t = E_t\left[\sum_{s=t+1}^{\infty}(R^*)^{-(s-t)}d^O_s\right]$. Fisherian separation does not hold strictly, because the excess return on oil depends on the marginal utility of consumption, but it holds approximately because equity premia in this class of models are small (as is typical of standard consumption asset pricing models). Hence, the asset price of oil is approximately independent of consumption and savings decisions.

F.3 Endogenous q

The third case takes into account the ad-hoc bond pricing function. Without uncertainty, the Euler equations for bonds and reserves (eqs. (F10) and (F11)) imply the following no-arbitrage condition:

$$R^O_{t+1} = R^b_{t+1}\left(s_{t+1}, b_{t+1}\right)\left[1 + \frac{q_s\left(s_{t+1}, b_{t+1}\right)b_{t+1}}{q^O_t}\right]. \tag{F22}$$

Using the alternative definition of the returns on oil that imputes the effect of reserves on bond prices, and since the planner arbitrages returns on bonds and oils against the intertemporal marginal rate of substitution, we obtain that:

$$\tilde{R}^O_{t+1}\left(s_{t+1}, b_{t+1}\right) = \frac{u'\left(c_t\right)}{\beta u'\left(c_{t+1}\right)} = R^b_{t+1}\left(s_{t+1}, b_{t+1}\right). \tag{F23}$$

It follows from these conditions that this model's deterministic steady state is pinned down by a two-equation nonlinear system in (b^{ss}, s^{ss}) formed by $\tilde{R}^O\left(s^{ss}, b^{ss}\right) = 1/\beta$ and $R^b\left(s^{ss}, b^{ss}\right) = 1/\beta$. The asset price of oil is still positive in this economy, and is simply determined by the deterministic version of eq. (F14).

The conditions that characterize the equilibrium of this economy under uncertainty are the ones provided in the generic characterization of the setup. Equations (F10), (F11), (F14) and (F15) are, respectively, the Euler equations for bonds and reserves, the oil asset-pricing equation and the oil risk premium. This economy is akin to the RBC-like case where there is no default risk, except that in this case the interest rate rises as bonds and/or reserves fall, whereas in the RBC case it remains constant. It also differs in that the planner chooses bonds and reserves internalizing how those choices affect the price of bonds and thus the cost of borrowing, but all of this is done under commitment to repay. Intuitively, it is as if the government acts as a monopolist when it sells its debt.

G Theoretical Results on Debt, Reserves & Country Risk

This Section of the Appendix derives theoretical results about how country risk and default incentives are affected by the debt position, oil reserves, and the realizations of non-oil GDP and oil prices. These results show the extent to which existing results from the sovereign default literature extend to the model we proposed, and provide insights about how oil reserves and oil prices interact with country risk and default incentives. Extending the analysis of standard default models is not straightforward, because in those models the default payoff is exogenous to the sovereign's actions, whereas in our model it depends on the sovereign's optimal plans for oil reserves. As we explain below, this is particularly important for deriving results related to how default sets respond to oil reserves, what contracts are feasible under repayment when default is possible, and how shocks to y and p affect default incentives. Since some of the propositions rely on conjectures, impose parameter restrictions (i.i.d shocks, $\lambda = 0$, $\hat{p} = p$), and provide only sufficiency conditions, we evaluated numerically both the conjectures and the propositions in the calibrated model.

As reported in Table G1, all the propositions and conjectures hold in 100 percent of the possible model evaluations that apply to each, except for Conjecture 2 which holds in 98 percent of the corresponding evaluations.

Table G1: Validation of Propositions and Conjectures in the Baseline Model

Conjecture or Proposition	**Case**	**Holds in %**	**Max. Error**
Conjecture 1*	Repayment	100	
	Default	100	
Conjecture 2		98	$\tilde{c}^{nd}\left(b,s^2,p,y\right)-\tilde{c}^{nd}\left(b,s^1,p,y\right)=-0.2$
Conjecture 3		100	
Proposition 1	s	100	
	s'	100	
Proposition 2	Repayment	100	
	Default	100	
Proposition 3		100	
Proposition 4		100	
Proposition 5		100	
Proposition 6		100	

Note: *This conjecture is evaluated computing oil asset prices as the expected present value of dividends

We also evaluated the non-negativity of profits included in Conjecture 1 and the trade balance conditions that are part of Propositions 6 and 4 (see Table G2).[5] Profits are strictly positive for all optimal decision rules of s' under repayment and default. The trade balance conditions of Propositions 6 and 4 hold 97 and 100 percent of all model evaluations, respectively. Removing the trade balance conditions, the main results of those propositions, namely that default incentives strengthen at lower y (Proposition 6) or lower p (Proposition 4), both hold 100 percent of the model evaluations. Thus, in our calibrated numerical solution, lower oil prices and lower non-oil GDP *always* strengthen default incentives. Hence, the model predicts that defaults generally coincide with negative non-oil-GDP shocks but not always. Thus, although the (sufficiency) trade balance condition of Proposition 4 fails frequently, it is still the case that in the numerical results, lower oil prices *always* strengthen default incentives. This is not the case, however, for lower non-oil-GDP, since even removing the trade balance condition, default incentives strengthen for lower y only in roughly 2/3rds of the evaluations. Hence, the model predicts that defaults do not need to coincide with negative

[5]We also checked whether the boundary conditions for x (or s') bind and found that they are never binding.

non-oil-GDP shocks.

Table G2: Additional Conditions on the Validation of Propositions and Conjectures in the Baseline Model

Condition or Proposition	Case	Validation	Holds in %	Max. Error
Trade balance condition	Proposition 6	$tb\left(b^1, s^1, b\right) \geq M\left(s^1, s, p\right) - M\left(\tilde{s}^2, s, p\right)$ for $y_2 \in D\left(b, s\right)$	97	-0.05^*
	Proposition 4	$tb\left(b^1, s^1, b\right) \geq M\left(s^1, s, p_2\right) - M\left(\tilde{s}^2, s, p_2\right)$ for $p_2 \in D\left(b, s\right)$	100	
Reserves choice condition	Proposition 4	$s^1 \leq \tilde{s}^2$	100	
Proposition 6	Without trade balance condition	For all $y_1 < y_2$, and $y_2 \in D\left(b, s\right)$ then $y_1 \in D\left(b, s\right)$	100	
Proposition 4	Without trade balance condition or $s^1 \leq \tilde{s}^2$	For all $p_1 < p_2$, and $p_2 \in D\left(b, s\right)$ then $p_1 \in D\left(b, s\right)$	100	
Profits in optimal decisions	Repayment	$M^{nd}\left(s'^{nd}\left(s, p, y\right), s, p, y\right) > 0$	100	
	Default	$M^{d}\left(s'^{d}\left(s, p, y\right), s, p, y\right) > 0$	100	
$s^{nd}\left(b, s, p, y\right)$ **boundaries hit**	Lower bound	$s^{nd}\left(b, s, p, y\right) = (s+\kappa) - s(p/\psi)^{(1/\gamma)}$	0	
	Upper bound	$s^{nd}\left(b, s, p, y\right) = s + k$	0	
$s^{d}\left(s, p, y\right)$ **boundaries hit**	Lower bound	$s^{d}\left(b, s, p, y\right) = (s+\kappa) - s(p/\psi)^{(1/\gamma)}$	0	
	Upper bound	$s^{d}\left(b, s, p, y\right) = s + k$	0	

Note: *The Max. Error is computed as $tb\left(b^1, s^1, b\right) - \left[M\left(s^1, s, p\right) - M\left(\tilde{s}^2, s, p\right)\right]$

**The Max. Error is computed as $tb\left(b^1, s^1, b\right) - \left[M\left(s^1, s, p_2\right) - M\left(\tilde{s}^2, s, p_2\right)\right]$

**The Max. Error is computed as $s^1 - \tilde{s}^2$

For the analysis that follows, we define these functions:

(a) Profits from oil extraction under repayment and default (using the law of motion of reserves to express oil extraction as a function $x(s', s) = s - s' + \kappa$):

$M^{nd}(s', s, p) \equiv px(s', s) - e(x(s', s), s), \quad M^{d}(s', s, p) \equiv h(p)x(s', s) - e(x(s', s), s).$ (b) Asset prices of oil under repayment and default:[6]

$q^{Ond}(s', s, p) \equiv p - e_x(x(s', s), s), \quad q^{Od}(s', s, p) \equiv h(p) - e_x(x(s', s), s).$ (c) Trade balance under repayment:

$tb(b', s', b, y, p) \equiv q\left(b', s', y, p\right) b' - b.$

(d) Consumption under repayment and default:

$c^{nd}(b', s', b, s, y, p) \equiv y - A + M^{nd}(s', s, p) - tb(b', s', b, y, p), \quad c^{d}(s', s, y, p) \equiv y - A + M^{d}(s', s, p).$

Next, we postulate three conjectures that are used later to prove some of the propositions in this Appendix:

[6] In Appendix F, we showed that in a model without default risk $p - e_x(x(s', s), s)$ is equal to the asset price of oil (i.e., the expected present value of oil dividends discounted with the sovereign's stochastic discount factors) for internal solutions of x and it is always positive.

Conjecture 1. ***Asset prices of oil are positive under repayment and default.***

$q^{Ond}(s', s, p), q^{Od}(s', s, p) > 0$ for all p, $s \in [\underline{s}, \overline{s}] = \{s : \underline{s} \leq s \leq \overline{s}\}$, and s' in the interval $(s + \kappa) - s(p/\psi)^{(1/\gamma)} \leq s' \leq (s + \kappa)$, where $s' \geq s + \kappa - s(p/\psi)^{(1/\gamma)}$ is implied by the upper bound of x above which profits are negative and $s' \leq s + \kappa$ is the upper bound of reserves if $x = 0$.

Appendix F shows that this conjecture is an equilibrium outcome for three variants of the model under commitment. It can also be proven that $q^{Od}(\cdot) > 0$ is an equilibrium outcome in the model with default, because with permanent exclusion the planner's dynamic programming problem is the same as that with commitment to repay under financial autarky.[7]

Conjecture 2. ***If default is possible for some state $(b, \tilde{s}, y, p)$, the optimal consumption choice under repayment is nondecreasing in s in the interval $\underline{s} \leq s \leq \tilde{s} \leq \overline{s}$.***

For all $s^1, s^2 \in [\underline{s}, \tilde{s}]$ and $s^1 \leq s^2$, $\hat{c}^{nd}(b, s^2, y, p) \geq \hat{c}^{nd}(b, s^1, y, p)$, where optimal consumption under repayment is: $\hat{c}^{nd}(b, s, y, p) \equiv y - A + M^{nd}(s'(b, s, y, p), s, y, p) - tb(b'(b, s, y, p), s'(b, s, y, p), b, y, p)$, and $b'(b, s, y, p), s'(b, s, y, p)$ are the bonds and reserves decision rules under repayment, respectively.

This conjecture is also an equilibrium outcome if the sovereign is committed to repay. It is a standard result that follows from consumption being increasing in wealth but proving this property is not straightforward in the model with default, because it requires properties of decision rules under repayment that are difficult to establish since the optimization problem under repayment retains the option to default in the future and is not differentiable.

Conjecture 3. ***If default on outstanding debt is optimal at a given level of existing reserves for some realizations of income and oil prices, all the available contracts for new debt and choices of oil reserves under repayment yield a trade balance at least as large as the difference in oil profits between repayment and default.***

If for some (b, s) the default set is non-empty $D(b, s) \neq \oslash$, then for $(y, p) \in D(b, s)$ there are no contracts $\{q(b', s', y, p), b', s'\}$ available such that $tb(b', s', b, y, p) < M^{nd}(s', s, p) - M^d(s^d(s, y, p), s, p)$, where $s^d(s, y, p)$ is the optimal choice of reserves under default.

This conjecture is related to Proposition 2 in Arellano (2008). She shows that, assuming i.i.d. shocks, $\lambda = 0$, and no default income costs, if the default set is non-empty for b then

[7] We showed in Appendix F that under financial autarky and assuming an internal solution for x, $q_t^O = E_t \sum_{j=1}^{\infty} \beta^j u'(t+j)/u'(t)[-e_s(t+j)]$. This corresponds to $q^{Od}(s', s, p)$ if the probability of re-entry is zero, $\lambda = 0$.

there are no contracts $\{q(b'), b'\}$ under repayment that can yield more net resources for current consumption than the resources available under default. Under default, resources are determined by the *exogenous* realization of y, which is the same under repayment, so this result implies also that there are no contracts that can yield a trade deficit. In our model, however, the debt contracts may need to entail a trade surplus in order to match the property that they cannot generate more net resources for current consumption than what the *endogenous* choice of oil profits generates under default. This is clearer if we consider that Conjecture 3 implies: $tb(b', s', b, y, p) \geq M^{nd}(s', s, p) - M^{d}(s^{d}(s, y, p), s, p)$. If profits under repayment are larger than under default (which is the case if a lower s' is chosen under repayment, since Proposition 1 below shows that profits are decreasing in s'), all available debt contracts generate trade surpluses at least as large as the amount by which oil profits under repayment exceed those under default. A zero trade balance is not sufficient to guarantee that there are fewer net resources for consumption under repayment.[8]

Moreover, as shown in Proposition 6, this conjecture also yields the result that when $D(b, s) \neq \oslash$, the debt contracts that are available always yield lower consumption under repayment than under default (i.e. there are no contracts available to sustain a trade deficit large enough to make up for the amount by which oil profits under default exceed those under repayment).

Proposition 1. ***If asset prices of oil are positive, oil profits are increasing in s, for given s', and decreasing in s', for given s.***

Given Conjecture 1, $\underline{s} \leq s \leq \overline{s}$, then oil profits under repayment and default are increasing in $s \in [\underline{s}, \overline{s}]$, namely $M_s^{nd}(\cdot), M_s^{d}(\cdot) > 0$, and decreasing in $s' \in [s + \kappa - s(p/\psi)^{(1/\gamma)}, s + \kappa]$, namely $M_{s'}^{nd}(\cdot), M_{s'}^{d}(\cdot) < 0$.

Proof. We show first that profits are increasing in s, and then that they are decreasing in s'.

1. The derivatives of oil profits with respect to s under repayment and default are $M_s^{nd}(\cdot) = p - e_x(x(s', s), s) - e_s(x(s', s), s)$ and $M_s^{d}(\cdot) = h(p) - e_x(x(s', s), s) - e_s(x(s', s), s)$.

[8] We can show that Conjecture 3 holds as a proposition under the sufficiency condition that, if the default set is not empty for a pair (b, s), there are no available debt contracts under repayment with associated choices of oil reserves that are smaller than the reserves chosen under default (i.e., the planner cannot generate more resources by setting s' lower in repayment than in default). However, this condition fails in the majority of the state space of the numerical solution with the baseline calibration.

2. Since $q^{Ond}(s', s, p) = p - e_x(x(s', s), s)$ and $q^{Od}(s', s, p) = h(p) - e_x(x(s', s), s)$, the derivatives can be rewritten as $M_s^{nd}(\cdot) = q^{Ond}(s', s, p) - e_s(x(s', s), s)$ and $M_s^d(\cdot) = q^{Od}(s', s, p) - e_s(x(s', s), s)$ respectively.

3. Since $q^{Ond}(s', s, p), q^{Od}(s', s, p) > 0$ and $e_s(x(s', s), s) < 0$ for $s \in [\underline{s}, \overline{s}]$, it follows that $M_s^{nd}(\cdot) = q^{Ond}(s', s, p) - e_s(x(s', s), s) > 0$ and $M_s^d(\cdot) = q^{Od}(s', s, p) - e_s(x(s', s), s) > 0$.

4. The derivatives of oil profits with respect to s' under repayment and default are $M_{s'}^{nd}(\cdot) = -p + e_x(x(s', s), s))$ and $M_s^d(\cdot) = -h(p) + e_x(x(s', s), s)$.

5. Since $q^{Ond}(s', s, p) = p - e_x(x(s', s), s)$ and $q^{Od}(s', s, p) = h(p) - e_x(x(s', s), s)$, the derivatives can be rewritten as $M_s^{nd}(\cdot) = -q^{Ond}(s', s, p))$ and $M_s^d(\cdot) = -q^{Od}(s', s, p)$ respectively.

6. Since $q^{Ond}(s', s, p), q^{Od}(s', s, p) > 0$, it follows that $M_{s'}^{nd}(\cdot) = -q^{Ond}(s', s, p) < 0$ and $M_{s'}^d(\cdot) = -q^{Od}(s', s, p) < 0$.

□

Proposition 2. ***The default and repayment payoffs are non-decreasing in s.***
For all $s^1, s^2 \in [\underline{s}, \overline{s}]$ and $s^1 \leq s^2$, $v^{nd}(b, s^2, y, p) \geq v^{nd}(b, s^1, y, p)$ and $v^d(s^2, y, p) \geq v^d(s^1, y, p)$.

Proof. This proof uses the consumption functions $c^{nd}(b', s', b, s, y, p), c^d(s', s, y, p)$.

1. Since $s^1 \leq s^2$, the result that oil profits are increasing in s (Proposition 1) and the definitions of the consumption functions imply that $c^{nd}(b', s', b, s^2, y, p) \geq c^{nd}(b', s', b, s^1, y, p)$ and $c^d(s', s^2, y, p) \geq c^d(s', s^1, y, p)$ for all (b', s'). Hence, the continuation values for $s^1 \leq s^2$ satisfy:

$$\begin{aligned} v^{nd}(b, s^2, y, p) &\geq u\left(c^{nd}(b', s', b, s^2, y, p)\right) + \beta E\left[V(b', s', y', p')\right] \\ &\geq u\left(c^{nd}(b', s', b, s^1, y, p))\right) + \beta E\left[V(b', s', y', p')\right], \end{aligned}$$

for all (b', s'), which implies that $v^{nd}(b, s^2, y, p) \geq v^{nd}(b, s^1, y, p)$. Hence, $v^{nd}(b, s, y, p)$ is nondecreasing in s.

2. Similarly, the default payoffs satisfy:

$$\begin{aligned} v^d(s^2, y, p) &\geq u\left(c^d(s', s^2, y, p)\right) + \beta E\left[\lambda V(0, s', y, p) + (1-\lambda) v^d(s', y', p')\right] \\ &\geq u\left(c^d(s', s^1, y, p))\right) + \beta E\left[\lambda V(0, s', y, p) + (1-\lambda) v^d(s', y', p')\right], \end{aligned}$$

for all s', which implies that $v^d(s^2, y, p) \geq v^d(s^1, y, p)$. Hence, $v^d(s, y, p)$ is nondecreasing in s.

$\square$

Proposition 3. ***Default sets shrink as s rises (i.e. grow as reserves fall).***

Assume $\hat{p} = p$ and $\lambda = 0$ for simplicity. For all $s^1, s^2 \in [\underline{s}, \overline{s}]$ and $s^1 \leq s^2$, if default is optimal for s^2 ($d(b, s^2, y, p) = 1$) for some states (b, y, p), then default is optimal for s^1 for the same states (b, y, p) (i.e. $D(b, s^2) \subseteq D(b, s^1)$ and $d(b, s^1, y, p) = 1$).

Proof. We show first that this proposition is valid if the decision rules for oil reserves under default and repayment are such that $s^d(s^2, y, p) \leq s'(b, s^1, y, p)$, and then we show that this condition holds under Conjecture 2.[9] The proof also requires Conjectures 1 and 3.

1. Since $d(b, s^2, y, p) = 1$ implies $v^d(s^2, y, p) - v^{nd}(b, s^2, y, p) \geq 0$ and both $v^{nd}(b, s, y, p)$ and $v^d(s, y, p)$ are nondecreasing in s, in order for $v^d(s^1, y, p) - v^{nd}(b, s^1, y, p) \geq 0$ (i.e. $d(b, s^1, y, p) = 1$), we need to show that when s falls, the default payoff falls as much or less than the repayment payoff: $v^d(s^2, y, p) - v^d(s^1, y, p) \leq v^{nd}(b, s^2, y, p) - v^{nd}(b, s^1, y, p)$.

2. Using the definition of $v^d(b, s, p)$ and since $s^d(s^2, y, p)$ is the optimal reserves choice under default when $s = s^2$, it follows that the difference $v^d(s^2, y, p) - v^d(s^1, y, p)$ satisfies this condition:

$$
\begin{aligned}
&v^d(s^2, y, p) - v^d(s^1, y, p) \leq \\
&u\left(c^d(s^d(s^2, y, p), s^2, y, p)\right) + \beta E\left[\lambda V(0, s^d(s^2, y, p), y', p') + (1-\lambda) v^d(s^d(s^2, y, p), y', p')\right] \\
&- u\left(c^d(s^d(s^2, y, p), s^1, y, p)\right) + \beta E\left[\lambda V(0, s^d(s^2, y, p), y', p') - (1-\lambda) v^d(s^d(s^2, y, p), y', p')\right]
\end{aligned}
$$

which reduces to:

$$
v^d(s^2, y, p) - v^d(s^1, y, p) \leq u\left(c^d(s^d(s^2, y, p), s^2, y, p)\right) - u\left(c^d(s^d(s^2, y, p), s^1, y, p)\right)
$$

[9] Conjecture 2 could be replaced with the assumption that $s^d(s^2, y, p) \leq s'(b, s^1, y, p)$ and the last step of the proof would be unnecessary, but Conjecture 2 is more reasonable because it states a familiar property of consumption decision rules (i.e. that they are increasing in wealth) and only with respect to consumption under repayment, whereas $s^d(s^2, y, p) \leq s'(b, s^1, y, p)$ refers to decision rules for reserves under default with higher s v. repayment with lower s.

3. Using the definition of $v^{nd}(b,s,p)$ and since $b'(b,s^1,y,p)$, $s'(b,s^1,y,p)$ are the bonds and reserves decision rules under repayment when reserves are $s=s^1$, respectively, it follows that the difference $v^{nd}(b,s^2,y,p)-v^{nd}(b,s^1,y,p)$ satisfies this condition:

$$
\begin{aligned}
&v^{nd}(b,s^2,y,p)-v^{nd}(b,s^1,y,p)\geq\\
&u\left(c^{nd}(b'(b,s^1,y,p),s'(b,s^1,y,p),b,s^2,y,p)\right)+\beta E\left[V(b'(b,s^1,y,p),s'(b,s^1,y,p),y',p')\right]\\
&-u\left(c^{nd}(b'(b,s^1,y,p),s'(b,s^1,y,p),b,s^1,y,p)\right)+\beta E\left[V(b'(b,s^1,y,p),s'(b,s^1,y,p),y',p')\right]
\end{aligned}
$$

which reduces to:

$$
\begin{aligned}
&v^{nd}(b,s^2,y,p)-v^{nd}(b,s^1,y,p)\geq\\
&u\left(c^{nd}(b'(b,s^1,y,p),s'(b,s^1,y,p),b,s^2,y,p)\right)-u\left(c^{nd}(b'(b,s^1,y,p),s'(b,s^1,y,p),b,s^1,y,p)\right)
\end{aligned}
$$

4. The results in 3. and 4. imply the following sufficiency condition for $v^d(s^2,y,p)-v^d(s^1,y,p)\leq v^{nd}(b,s^2,y,p)-v^{nd}(b,s^1,y,p)$:

$$
\begin{aligned}
&u\left(c^d(s^d(s^2,y,p),s^2,y,p)\right)-u\left(c^d(s^d(s^2,y,p),s^1,y,p)\right)\leq\\
&u\left(c^{nd}(b'(b,s^1,y,p),s'(b,s^1,y,p),b,s^2,y,p)\right)-u\left(c^{nd}(b'(b,s^1,y,p),s'(b,s^1,y,p),b,s^1,y,p)\right),
\end{aligned}
$$

which, using the definitions of $c^{nd}(\cdot)$ and $c^d(\cdot)$ and noting that since $\hat{p}=p$ we can write the profit functions as $M^d(\cdot)=M^{nd}(\cdot)=M(\cdot)$, can be rearranged as follows:

$$
\begin{aligned}
&u\left(y-A+M(s^d(s^2,y,p),s^2,p)\right)\\
&\quad-u\left(y-A+M(s'(b,s^1,y,p),s^2,p)-tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p)\right)\\
&\qquad\leq u\left(y-A+M(s^d(s^2,y,p),s^1,p)\right)\\
&\quad-u\left(y-A+M(s'(b,s^1,y,p),s^1,p)-tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p))\right),
\end{aligned}
$$

and using this notation $\tilde{y}^2\equiv y-A+M(s^d(s^2,y,p),s^2,p)$, $\tilde{y}^1\equiv y-A+M(s^d(s^2,y,p),s^1,p)$, $z^2=M(s'(b,s^1,y,p),s^2,p)-tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p)-M(s^d(s^2,y,p),s^2,p)$, $z^1=M(s'(b,s^1,y,p),s^1,p)-tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p))-M(s^d(s^2,y,p),s^1,p)$ it can be re-written as:

$$
u\left(\tilde{y}^2\right)-u\left(\tilde{y}^2+z^2\right)\leq u\left(\tilde{y}^1\right)-u\left(\tilde{y}^1+z^1\right),
$$

5. The strict concavity of $u(\cdot)$ implies that the above condition holds if we can show that $\tilde{y}^2>\tilde{y}^1$ and $z^1\leq z^2\leq 0$. Since $M_s(\cdot)>0$ as shown in Proposition 1, it follows that

$\tilde{y}^2 > \tilde{y}^1$. Conjecture 3 implies that if the default set for (b, s) is not empty, then all the contracts available under repayment are such that $M(s', s, p) - tb(b', s', b, y, p) - M(s^d(s, y, p), s, p) \leq 0$, therefore $z^1, z^2 \leq 0$. Hence, $z^1 \leq z^2 \leq 0$ holds if

$$M(s^d(s^2, y, p), s^2, p) - M(s^d(s^2, y, p), s^1, p) \leq M(s'(b, s^1, y, p), s^2, p) - M(s'(b, s^1, y, p), s^1, p).$$

Since $M_{ss'}(\cdot) \geq 0$, it follows that the above condition holds if the reserves decision rules under default and repayment are such that $s^d(s^2, y, p) \leq s'(b, s^1, y, p)$.[10]

6. Finally, we show that a sufficiency condition for $s^d(s^2, y, p) \leq s'(b, s^1, y, p)$ to hold is that $\hat{c}^{nd}(b, s^2, y, p) \geq \hat{c}^{nd}(b, s^1, y, p)$, which holds because of Conjecture 2. To show this, note first that because of Conjecture 3 (if the default set for (b, s^1) is not empty) $tb(b'(b, s^1, y, p), s'(b, s^1, y, p), b, y, p)) \geq M(s'(b, s^1, y, p), s^1, p) - M(s^d(s^1, y, p), s^1, p)$, and hence $M(s^d(s^1, y, p), s^1, p) \geq M(s'(b, s^1, y, p), s^1, p) - tb(b'(b, s^1, y, p), s'(b, s^1, y, p), b, y, p)$. Moreover, in the optimization problem under full financial autarky of Appendix F (which is the same as the default problem since $\lambda = 0$) $dM(s', s, p)/ds > 0$.[11] to both sides of the above expression and simplify using the definition of $v^d(s^2, y, p)$:

$$v^d(s^2, y, p) \geq u\left(y - A + M(s'(b, s^1, y, p), s^1, p) - tb(b'(b, s^1, y, p), s'(b, s^1, y, p), b, y, p)\right) \\ + \beta E\left[\lambda V(0, s^d(s^2, y, p), y', p') + (1 - \lambda) v^d(s^d(s^2, y, p), y', p')\right]$$

Subtracting $v^{nd}(b, s^2, y, p)$ from both sides yields:

$$v^d(s^2, y, p) - v^{nd}(b, s^2, y, p) \geq \\ u\left(y - A + M(s'(b, s^1, y, p), s^1, p) - tb(b'(b, s^1, y, p), s'(b, s^1, y, p), b, y, p)\right) \\ - v^{nd}(b, s^2, y, p) + \beta E\left[\lambda V(0, s^d(s^2, p), y', p') + (1 - \lambda) v^d(s^d(s^2, y, p), y', p')\right],$$

[10] Given the functional form of $e(x, s)$, it is straightforward to show that $M^{nd}_{ss'}(\cdot) = M^{d}_{ss'}(\cdot) = e_x(\cdot)\gamma(s' - \kappa)/(xs)$. Moreover, we show in Appendix H that under financial autarky (or under default with permanent exclusion), the optimal reserves decision rule is increasing in reserves if $p^{max} < \psi$ (i.e. if the highest realization of oil prices is smaller than the coefficient ψ of the extraction costs function). Hence, $min(s' - x) = s[1 - (p/\psi)^{1/\gamma}]$ and therefore $M^{nd}_{ss'}(\cdot) = M^{d}_{ss'}(\cdot) > 0$.

[11] From the definition of $M(s', s, p)$ it follows that $dM/ds = q^{Od}(s', s, p)[1 - \partial s^d(\cdot)/\partial s] - e_s(\cdot) > 0$, because $q^O d(s', s, p) > 0$, $e_s(\cdot) < 0$ and we conjecture that $\partial s^d(\cdot)/\delta s < 1$ for local stability (Appendix H proves that $\partial s^d(\cdot)/\delta s > 0$).

which using the definitions of $v^{nd}(b, s^2, y, p)$ and $c^{nd}(b', s', b, s, y, p)$ can be written as:

$$v^d(s^2, y, p) - v^{nd}(b, s^2, y, p) \geq u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^1, y, p)\right)$$
$$-\left[u\left(c^{nd}(b'(b, s^2, y, p), s'(b, s^2, y, p), b, s^2, y, p)\right) + \beta E\left[V(b'(b, s^2, y, p), s'(b, s^2, y, p), y', p')\right]\right]$$
$$+\beta E\left[\lambda V(0, s^d(s^2, y, p), y', p') + (1-\lambda)v^d(s^d(s^2, y, p), y', p')\right],$$

and rearranging terms in the above expression yields:

$$u\left(c^{nd}(b'(b, s^2, y, p), s'(b, s^2, y, p), b, s^2, y, p)\right) - u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^1, y, p)\right)$$
$$+\beta E\left[V(b'(b, s^2, y, p), s'(b, s^2, y, p), y', p') - \lambda V(0, s^d(s^2, y, p), y', p') - (1-\lambda)v^d(s^d(s^2, y, p), y', p')\right]$$
$$\geq v^{nd}(b, s^2, y, p) - v^d(s^2, y, p).$$

Since $\lambda = 0$, and using the definition of the optimal consumption decision rule $\hat{c}^{nd}(b, s, y, p)$, this expression can be written as:

$$u\left(\hat{c}^{nd}(b, s^2, y, p)\right) - u\left(\hat{c}^{nd}(b, s^1, y, p)\right)$$
$$+\beta E\left[V(b'(b, s^2, y, p), s'(b, s^2, y, p), y', p') - v^d(s^d(s^2, y, p), y', p')\right]$$
$$\geq v^{nd}(b, s^2, y, p) - v^d(s^2, y, p).$$

This inequality holds because $d(b, s^2, y, p) = 1$ implies that the right-hand-side of this expression is non-positive $(v^{nd}(b, s^2, y, p) - v^d(s^2, y, p) \leq 0)$ while the left-hand-side is non-negative because: (a) Conjecture 2 and the fact that $u(c)$ is increasing in c imply that $u\left(\hat{c}^{nd}(b, s^2, y, p)\right) \geq u\left(\hat{c}^{nd}(b, s^1, y, p)\right)$, and (b) $E\left[V(b'(b, s^2, y, p), s'(b, s^2, y, p), y', p')\right]$ $-E\left[v^d(s^d(s^2, y, p), y', p')\right] \geq 0$ by the definition of $V(\cdot)$.

□

Proposition 4. ***If the trade balance is sufficiently large and reserves chosen under default at high oil prices exceed those chosen under repayment at low prices, default incentives strengthen as oil prices fall.***

Assuming i.i.d shocks, $\lambda = 0$ and $\hat{p} = p$, for all $p_1 < p_2$ and $p_2 \in D(b, s)$, if $tb(b^1, s^1, b) \geq M(s^1, s, p_2) - M(\tilde{s}^2, s, p_2)$ and $s^1 \leq \tilde{s}^2$ (where b^1, s^1 are the optimal bonds and reserves choices under repayment in state (b, s, y, p_1) and $\tilde{s}^2$ is the optimal reserves choice under default in state (s, y, p_2), then $p_1 \in D(b, s)$.

Proof. This proof requires a lower bound condition on the trade balance, but linked to the optimal decision rules of reserves under repayment with p_1 v. under default with p_2, and it also requires optimal reserves under default with p_2 to exceed those under repayment with p_1.

1. If $p_2 \in D(b,s)$ and denoting (b^2, s^2) and $\tilde{s}^2$ as the optimal choices of bonds and reserves when $p = p_2$ under repayment and default, respectively, it follows that by definition::

$$u\left(y - A + M(\tilde{s}^2, s, p_2)\right) + \beta E[v^d(\tilde{s}^2, y', p')] \geq$$
$$u\left(y - A + M(s^2, s, p_2) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')],$$

   where the profit functions under default and repayment are the same because $\hat{p} = p$.

2. To establish that $p_2 \in D(b,s) \Rightarrow p_1 \in D(b,s)$ it is sufficient to show that, denoting (b^1, s^1) and $\tilde{s}^1$ as the bonds and reserves chosen when $p = p_1$ under repayment and default, respectively, the following holds:

$$u\left(y - A + M(s^2, s, p_2) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')] -$$
$$\left[u\left(y - A + M(s^1, s, p_1) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] \geq$$
$$u\left(y - A + M(\tilde{s}^2, s, p_2)\right) + \beta E[v^d(\tilde{s}^2, y', p')] - \left[u\left(y - A + M(\tilde{s}^1, s, p_1)\right) + \beta E[v^d(\tilde{s}^1, y', p')]\right]$$

3. Given that (b^2, s^2) maximizes the repayment payoff with p_2 and $\tilde{s}^1$ maximizes the default payoff with p_1, the following two conditions hold:

$$u\left(y - A + M(s^2, s, p_2) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')]$$
$$\geq \left[u\left(y - A + M(s^1, s, p_2) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right]$$

$$u\left(y - A + M(\tilde{s}^1, s, p_1)\right) + \beta E[v^d(\tilde{s}^1, y', p')] \geq \left[u\left(y - A + M(\tilde{s}^2, s, p_1)\right) + \beta E[v^d(\tilde{s}^2, y', p')]\right]$$

4. Using the results in 3., the condition in 2. holds if:

$$\left[u\left(y - A + M(s^1, s, p_2) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] -$$
$$\left[u\left(y - A + M(s^1, s, p_1) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] \geq$$
$$u\left(y - A + M(\tilde{s}^2, s, p_2)\right) + \beta E[v^d(\tilde{s}^2, y', p')] - \left[u\left(y - A + M(\tilde{s}^2, s, p_1)\right) + \beta E[v^d(\tilde{s}^2, y', p')]\right]$$

5. The above expression simplifies to:

$$u\left(y-A+M(s^1,s,p_2)-tb(b^1,s^1,b)\right)-u\left(y-A+M(s^1,s,p_1)-tb(b^1,s^1,b)\right)$$
$$\geq u\left(y-A+M(\tilde{s}^2,s,p_2)\right)-u\left(y-A+M(\tilde{s}^2,s,p_1)\right),$$

which adding and subtracting $M(\tilde{s}^2,s,p_2)$ and $M(\tilde{s}^2,s,p_1)$ to the arguments of the repayment utility in the left- and right-hand-sides, respectively,and rearranging yields:

$$u\left(y-A+M(\tilde{s}^2,s,p_2)\right)-u\left(y-A+M(\tilde{s}^2,s,p_2)+z(p_2)\right)$$
$$\leq u\left(y-A+M(\tilde{s}^2,s,p_1)\right)-u\left(y-A+M(\tilde{s}^2,s,p_1)+z(p_1)\right)$$

where: $z(p_1)=M(s^1,s,p_1)-tb(b^1,s^1,b)-M(\tilde{s}^2,s,p_1)$ and $z(p_2)=M(s^1,s,p_2)-tb(b^1,s^1,b)-M(\tilde{s}^2,s,p_2)$. The above inequality holds because: (a) the utility function is increasing and strictly concave, (b) $M(\tilde{s}^2,s,p_2)>M(\tilde{s}^2,s,p_1)$ since profits are increasing in p, (c) $z(p_2)\leq 0$ because of the assumption that $tb(b^1,s^1,b)\geq M(s^1,s,p_2)-M(\tilde{s}^2,s,p_2)$, and (d) $z(p_1)\leq z(p_2)$ because $s^1\leq\tilde{s}^2$ (note that $z(p_1)\leq z(p_2)\leftrightarrow M(s^1,s,p_1)-M(\tilde{s}^2,s,p_1)\leq M(s^1,s,p_2)-M(\tilde{s}^2,s,p_2)$ or $M(\tilde{s}^2,s,p_2)-M(\tilde{s}^2,s,p_1)\leq M(s^1,s,p_2)-M(s^1,s,p_1)$ and using the functional form of $M(.)$ this yields $(p_2-p_1)(s-\tilde{s}^2+\kappa)\leq(s-s^1+\kappa)(p_2-p_1)$, which implies that $s^1\leq\tilde{s}^2$).

□

Proposition 5. ***The repayment payoff is non-decreasing in*** b ***and default sets shrink as*** b ***rises (i.e. grow as debt rises).***

For all $b^1\leq b^2$, $v^{nd}(b^2,s,y,p)\geq v^{nd}(b^1,s,y,p)$. *Moreover, if default is optimal for* b^2 $(d(b^2,s,y,p)=1)$ *for some states* (s,y,p) *then default is optimal for* b^1 *for the same states* (s,y,p) *(i.e.* $D(b^2,s)\subseteq D(b^1,s)$ *and* $d(b^1,s,y,p)=1$*)*

Proof. This proof follows Arellano (2008).

1. From the definition of $D(\cdot)$ and $d(b^2,s,y,p)=1$ it follows that $v^d(s,y,p)\geq v^{nd}(b^2,s,y,p)$ $\forall\{y,p\}\in D(b^2,s)$, hence:

$$v^d(s,y,p)\geq v^{nd}(b^2,s,y,p)\geq u\left(c^{nd}(b',s',b^2,s,y,p)\right)+\beta E\left[V(b',s',y',p')\right]\quad\forall(b',s')$$

2. It follows that, since $b^1 \leq b^2$ implies $c^{nd}(b', s', b^2, s, y, p) \geq c^{nd}(b', s', b^1, s, y, p)$, the continuation values for $b^1 \leq b^2$ satisfy:

$$u\left(c^{nd}(b', s', b^2, s, y, p)\right) + \beta E\left[V(b', s', y', p')\right] \geq$$
$$u\left(c^{nd}(b', s', b^1, s, y, p))\right) + \beta E\left[V(b', s', y', p')\right],$$

for all (b', s'), which implies that $v^{nd}(b, s, y, p)$ is nondecreasing in b.

3. It follows from 1. and 2. that $v^d(s, y, p) \geq v^{nd}(b^2, s, y, p) \geq v^{nd}(b^1, s, y, p)$, hence $v^d(s, y, p) \geq v^{nd}(b^1, s, y, p)$ which implies $\{y, p\} \in D(b^1, s)$ and thus $d(b^1, s, y, p) = 1$.

□

Proposition 6. ***If the trade balance is sufficiently large, default incentives strengthen as non-oil GDP falls.***

Assuming i.i.d shocks, $\lambda = 0$ and $\hat{p} = p$, for all $y_1 < y_2$, if $y_2 \in D(b, s)$ and $tb(b^1, s^1, b) \geq M(s^1, s, p) - M(\tilde{s}^2, s, p)$ (where $b^1 \equiv b'(b, s, y_1, p)$, $s^1 \equiv s'(b, s, y_1, p)$ are the optimal choices of bonds and reserves under repayment with y_1 and $\tilde{s}^2 \equiv s^d(s, y_2, p)$ is the optimal reserves choice under default with y_2), then $y_1 \in D(b, s)$.

Proof. This proof aims to extend Proposition 3 in Arellano (2008), but for this model it requires a lower bound condition on the trade balance linked to the optimal decision rules of reserves under repayment with y_1 v. under default with y_2.

1. If $y_2 \in D(b, s)$ and denoting $b^2 \equiv b'(b, s, y_2, p)$, $s^2 \equiv s'(b, s, y_2, p)$ the optimal choices of bonds and reserves when $y = y_2$ under repayment, it follows that by definition:

$$u\left(y_2 - A + M^d(\tilde{s^2}, s, p)\right) + \beta E[v^d(\tilde{s^2}, y', p')] \geq$$
$$u\left(y_2 - A + M^{nd}(s^2, s, p) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')]$$

2. To establish that $y_2 \in D(b, s) \Rightarrow y_1 \in D(b, s)$ it is sufficient to show that, denoting $\tilde{s}^1$ as reserves chosen when $y = y_1$ under default, the following holds:

$$u\left(y_2 - A + M^{nd}(s^2, s, p) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')] -$$
$$\left[u\left(y_1 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] \geq$$
$$u\left(y_2 - A + M^d(\tilde{s}^2, s, p)\right) + \beta E[v^d(\tilde{s}^2, y', p')] - \left[u\left(y_1 - A + M^d(\tilde{s}^1, s, p)\right) + \beta E[v^d(\tilde{s}^1, y', p')]\right]$$

3. Given that (b^2, s^2) maximizes the repayment payoff with y_2 and $\tilde{s}^1$ maximizes the default payoff with y_1, the following two conditions hold:

$$u\left(y_2 - A + M^{nd}(s^2, s, p) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')]$$
$$\geq \left[u\left(y_2 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right]$$

$$u\left(y_1 - A + M^d(\tilde{s}^1, s, p)\right) + \beta E[v^d(\tilde{s}^1, y', p')] \geq \left[u\left(y_1 - A + M^d(\tilde{s}^2, s, p)\right) + \beta E[v^d(\tilde{s}^2, y', p')]\right]$$

4. Using the results in 3., the condition in 2. holds if:

$$\left[u\left(y_2 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] -$$
$$\left[u\left(y_1 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] \geq$$
$$u\left(y_2 - A + M^d(\tilde{s}^2, s, p)\right) + \beta E[v^d(\tilde{s}^2, y', p')] - \left[u\left(y_1 - A + M^d(\tilde{s}^2, s, p)\right) + \beta E[v^d(\tilde{s}^2, y', p')]\right]$$

5. The above expression simplifies to:

$$u\left(y_2 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) - u\left(y_1 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right)$$
$$\geq u\left(y_2 - A + M^d(\tilde{s}^2, s, p)\right) - u\left(y_1 - A + M^d(\tilde{s}^2, s, p)\right),$$

which adding and subtracting $M^d(\tilde{s}^2, s, p)$ inside the argument of the repayment utilities and rearranging yields:

$$u\left(y_2 - A + M^d(\tilde{s}^2, s, p)\right) - u\left(y_2 - A + M^d(\tilde{s}^2, s, p) + z(y_1)\right)$$
$$\leq u\left(y_1 - A + M^d(\tilde{s}^2, s, p)\right) - u\left(y_1 - A + M^d(\tilde{s}^2, s, p) + z(y_1)\right),$$

where $z(y_1) \equiv M^{nd}(s^1, s, p) - tb(b^1, s^1, b) - M^d(\tilde{s}^2, s, p)$. The above inequality holds because: (a) the utility function is increasing and strictly concave, (b) $y_2 > y_1$ and (c) $z(y_1) < 0$ because of the assumption that $tb(b^1, s^1, b) \geq M^{nd}(s^1, s, p) - M^d(\tilde{s}^2, s, p)$.

□

H Dynamic Programming Problem under Financial Autarky

The dynamic programming problem of the planner under financial autarky, which corresponds also to the default payoff and decision rules when $\lambda = 0$, can be written as follows:

$$V^d(s,p,y) = \underset{s'\in\Gamma(s)}{max} \left\{F\left(s,s',p,y\right) + \beta E\left[V^d\left(s',p',y'\right)\right]\right\}$$
$$F\left(s,s',p,y\right) \equiv u\left(y - A + p(s - s' + \kappa) - e\left(s - s' + \kappa, s\right)\right)$$
$$\Gamma(s) \equiv \left\{s' : 0 \leq s' \leq s + \kappa\right\},$$

with first-order condition:

$$[s'] : u_c(t)\left(p - e_x(\cdot)\right) = \beta V^d_{s'}\left(s',p',y'\right)$$

or

$$-F_{s'}\left(s,s',p,y\right) = \beta V^d_{s'}\left(s',p',y'\right).$$

This Appendix shows that the period-payoff $F(s,s',p,y)$ of the above problem satisfies standard properties analogous to those of the textbook neoclassical Ramsey model, with oil reserves taking the place of the capital stock. In particular, we show that $F(s,s',p,y)$ is continuously differentiable in (s,s'), strictly increasing (decreasing) in s (s'), and strictly concave in (s,s'). We also show that the optimal decision rule $s'(s,p,y)$ is increasing in s. These properties, together with the assumptions that $F(\cdot)$ is bounded and $\Gamma(s)$ is a nonempty, compact-valued, monotone, and continuous correspondence with a convex graph, ensure that the value function $V^d(\cdot)$ that solves the above Bellman equation exists and the solution is unique, and that $V^d(\cdot)$ is strictly concave, strictly increasing and continuously differentiable.[12] The proofs of these properties are analogous to those of the textbook Ramsey model and therefore are omitted here. Existence and uniqueness follow from the contraction mapping theorem. The proof that $V^d(\cdot)$ is increasing requires $F(\cdot)$ to be increasing and $\Gamma(s)$ to be monotone, the proof that $V^d(\cdot)$ is concave requires $F(\cdot)$ to be concave, and proving the differentiability of $V^d(\cdot)$ requires $F(\cdot)$ to be continuously differentiable and concave.

[12]We also assume a standard, twice-continuously-differentiable, increasing and concave utility function. The CRRA utility function that defines $F(\cdot)$ in the numerical solution satisfies these properties but is unbounded. It can be transformed into a bounded function with a piece-wise truncation at an arbitrary small but positive consumption level (see Suen (2009). "Bounding the CRRA Utility Functions," Working Papers 200902, University of California at Riverside, Department of Economics).

1. $F(\cdot)$ **is strictly increasing in** s **(**$F_s(\cdot) > 0$**) and decreasing in** s' **(**$F_s'(\cdot) < 0$**).**

To prove these two properties, recall that $e_s(\cdot) < 0$ and that we showed in the sequential solution of the autarky model of Appendix F that the asset price of oil is positive for internal solutions of x, hence $p - e_x(\cdot) > 0$. By differentiating $F(\cdot)$ with respect to s and s' we obtain:

$$F_s(\cdot) = u_c(\cdot)(p - e_x(\cdot) - e_s(\cdot)) > 0,$$

$$F_{s'}(\cdot) = u_c(\cdot)(-p + e_x(\cdot)) = -u_c(\cdot)(p - e_x(\cdot)) < 0.$$

2. $F(\cdot)$ **is continuously differentiable.**

To prove that $F(\cdot)$ is continuously differentiable, we need to show that: (a) $F(\cdot)$ is continuous in its domain and (b) $F_s(\cdot)$ and $F_{s'}(\cdot)$ exist and are continuous in their domain. For this proof, consider the above expressions for $F_s(\cdot)$ and $F_{s'}(\cdot)$ and express the extraction cost and its derivatives as functions of s and s' using the law of motion $x = s - s' + \kappa$ as follows:

$$e(s', s) = \psi \frac{(s - s' + \kappa)^{1+\gamma}}{s^\gamma}$$

$$e_x(s', s) = (1+\gamma)\psi \left(\frac{s - s' + \kappa}{s}\right)^\gamma = (1+\gamma)\psi \left(1 - \frac{(s' - k)}{s}\right)^\gamma$$

$$e_s(s', s) = -\gamma\psi \left(\frac{s - s' + \kappa}{s}\right)^{1+\gamma} = -\gamma\psi \left(1 - \frac{(s' - k)}{s}\right)^{1+\gamma},$$

where $e_x(\cdot)$ and $e_s(\cdot)$ are continuous in the domain given by $0 \leq s' \leq s + k$ and $s > 0$ with the following upper and lower bounds:

$$e_x(0, s) = 0, \quad e_s(0, s) = 0$$

$$e_x(s + k, s) = (1+\gamma)\psi \left(\frac{s + k}{s}\right)^\gamma, \quad e_s(s + k, s) = -\gamma\psi \left(\frac{s + k}{s}\right)^{1+\gamma}$$

If in addition, oil profits are required to be non-negative, which is analogous to the non-negativity constraint on consumption (or the Inada condition in $u(c)$) in the texbook Ramsey model, the domain of the cost function and its derivatives requires $px \geq e(\cdot)$. Moreover, if oil revenue is the only income or $y - A \leq 0$, the Inada condition would imply that negative profits are never optimal and profits must always be sufficient to sustain $c > 0$. Using again the law of motion $x = s - s' + \kappa$, we obtain that with non-negative profits the lower bound of s' becomes $s' \geq \kappa + s\left[1-\left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right]$ instead of $s' > 0$. Hence, the domain of s' becomes $\kappa + s\left[1-\left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right] \leq s' \leq s+\kappa$.

The functions:

$$F(\cdot) = u\left(y - A + p(s-s'+\kappa) - e\left(s-s'+\kappa, s\right)\right)$$

$$F_s(\cdot) = u_c\left(y - A + p(s-s'+\kappa) - e\left(s-s'+\kappa, s\right)\right) \times \left[p - (1+\gamma)\,\psi\left(1-\frac{(s'-\kappa)}{s}\right)^{\gamma} + \gamma\psi\left(1-\frac{(s'-\kappa)}{s}\right)^{1+\gamma}\right],$$

$$F_{s'}(\cdot) = -u_c\left(y - A + p(s-s'+\kappa) - e\left(s-s'+\kappa, s\right)\right)\left[p - (1+\gamma)\,\psi\left(1-\frac{(s'-\kappa)}{s}\right)^{\gamma}\right],$$

are continuous and exist in the domain defined by $\kappa + s\left[1-\left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right] \leq s' \leq s+\kappa$ and $s > 0$.

3. $s'(s,p,y)$ **is increasing in** s**.**

From the first-order condition for s', this property requires that $-F_{s'}(\cdot) = u_c(\cdot)\,(p - e_x(\cdot))$ be decreasing in s, since $V^d_{s'}(\cdot)$ is independent of s. Thus, we need to show that $\frac{\partial - F_{s'}(\cdot)}{\partial s} < 0$.

$$\frac{\partial - F_{s'}(\cdot)}{\partial s} = \left[p - e_x(\cdot)\right]\left[u_{cc}(\cdot)\{p - e_x(\cdot) - e_s(\cdot)\}\right] + u_c(t)\left\{-\left[e_{xx}(\cdot) + e_{xs}(\cdot)\right]\right\}.$$

Since $e_s(\cdot) < 0$, $p - e_x(\cdot) > 0$, $u_c(\cdot) > 0$ $u_{cc}(\cdot) < 0$, the above expression is negative if $\{-\left[e_{xx}(\cdot) + e_{xs}(\cdot)\right]\} < 0$. To determine the sign of this expression, use the functional form $e(x,s) = \psi\frac{x^{1+\gamma}}{s^{\gamma}}$ to show that the derivatives $e_{xx}(\cdot)$ and $e_{xx}(\cdot)$ can be expressed as follows:

$$e_{xx}(x,s) = \gamma(1+\gamma)\,\psi\frac{x^{\gamma}}{s^{\gamma}}x^{-1} = e_x(\cdot)\,\gamma x^{-1} > 0,$$

$$e_{xs}(x,s) = -\gamma(1+\gamma)\,\psi\frac{x^{\gamma}}{s^{\gamma}}s^{-1} = -e_x(\cdot)\,\gamma s^{-1} < 0.$$

Using these expressions, we obtain:

$$\{-\left[e_{xx}(t) + e_{xs}(t)\right]\} = \left\{-\left[e_x(\cdot)\,\gamma\left(x^{-1} - s^{-1}\right)\right]\right\} < 0 \quad \text{if } x < s,$$

and using $x = s - s' + \kappa$, the condition $x < s$ implies $s - s' + \kappa < s$ which reduces to:

$$s' > \kappa.$$

Hence, $s'(s,p,y)$ is increasing in s if the choice of reserves always exceeds oil discoveries. Since the non-negativity of profits requires $s' \geq \kappa + s\left[1-\left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right]$ and existing reserves satisfy $s > 0$, the condition $s' > \kappa$ is implied by the non-negativity of profits if $p^{max} < \psi$ (i.e., ψ is larger than the largest realization of oil prices so that p/ψ is always less than 1). This result also implies that the upper bound on x never binds (since s' is always strictly positive because $s' > \kappa > 0$).

4. $F(\cdot)$ is strictly concave

To show that $F(\cdot)$ is strictly concave, let $H(\cdot)$ be the Hessian matrix of $F(\cdot)$ defined as

$$H(\cdot) = \begin{bmatrix} F_{ss}(\cdot) & F_{ss'}(\cdot) \\ F_{s's}(\cdot) & F_{s's'}(\cdot) \end{bmatrix}$$

$F(\cdot)$ is strict concave if $H(\cdot)$ is negative definite. That is

- $F_{ss}(\cdot) < 0$
- $F_{ss}(\cdot)\, F_{s's'}(\cdot) - F_{ss'}(\cdot)\, F_{s's}(\cdot) > 0$

$$F_{ss}(\cdot) = [p - e_x(\cdot) - e_s(\cdot)]\, u_{cc}(\cdot)\, [p - e_x(\cdot) - e_s(\cdot)] + u_c(\cdot)\, [-e_{xx}(\cdot) - e_{sx}(\cdot) - e_{xs}(\cdot) - e_{ss}(\cdot)]$$

Recall

$$e(x,s) = \psi \frac{x^{1+\gamma}}{s^{\gamma}} e_x(x,s) = (1+\gamma)\,\psi \left(\frac{x}{s}\right)^{\gamma} e_s(x,s) = -\gamma\psi\left(\frac{x}{s}\right)^{1+\gamma}$$

Where

1. $e_{xx}(x,s) = \gamma(1+\gamma)\,\psi \frac{x^{\gamma}}{s^{\gamma}} x^{-1} = e_x(\cdot)\,\gamma x^{-1} > 0$
2. $e_{xs}(x,s) = -\gamma(1+\gamma)\,\psi \frac{x^{\gamma}}{s^{\gamma}} s^{-1} = -e_x(\cdot)\,\gamma s^{-1} < 0$
3. $e_{sx}(x,s) = -\gamma(1+\gamma)\,\psi \left(\frac{x}{s}\right)^{1+\gamma} x^{-1} = e_s(\cdot)(1+\gamma)\, x^{-1}$
4. $e_{ss}(x,s) = \gamma(1+\gamma)\,\psi \left(\frac{x}{s}\right)^{1+\gamma} s^{-1} = -e_s(\cdot)(1+\gamma)\, s^{-1}$

Additionally, from 3. we can obtain:

$$e_{sx}(x,s) = -\gamma(1+\gamma)\,\psi\left(\frac{x}{s}\right)^{1+\gamma} x^{-1} = -e_x(\cdot)\,\gamma s^{-1}$$

Using $-e_x(\cdot)\,\gamma s^{-1} = e_s(\cdot)\,(1+\gamma)\,x^{-1}$,

$$e_x(\cdot) = -e_s(\cdot)\,\frac{(1+\gamma)}{\gamma}x^{-1}s,$$

1. $e_{xx}(x,s) = e_x(\cdot)\,\gamma x^{-1} = -e_s(\cdot)\,(1+\gamma)\,x^{-2}s$
2. $e_{xs}(x,s) = -e_x(\cdot)\,\gamma s^{-1} = e_s(\cdot)\,(1+\gamma)\,x^{-1}$
3. $e_{sx}(x,s) = e_s(\cdot)\,(1+\gamma)\,x^{-1} = e_{xs}(x,s)$
4. $e_{ss}(x,s) = -e_s(\cdot)\,(1+\gamma)\,s^{-1}$

Then

$$F_{ss}(\cdot) = [p - e_x(\cdot) - e_s(\cdot)]\,u_{cc}(\cdot)\,[p - e_x(\cdot) - e_s(\cdot)] + u_c(\cdot)\,\{-[e_{xx}(\cdot) + 2e_{xs}(\cdot) + e_{ss}(\cdot)]\}$$

$$\begin{aligned} F_{ss}(\cdot) = {} & [p - e_x(\cdot) - e_s(\cdot)]\,u_{cc}(\cdot)\,[p - e_x(\cdot) - e_s(\cdot)] \\ & + u_c(\cdot)\left\{-\left[\left\{-e_s(\cdot)\,(1+\gamma)\,x^{-2}s\right\} + 2\left\{e_s(\cdot)\,(1+\gamma)\,x^{-1}\right\} + \left\{-e_s(\cdot)\,(1+\gamma)\,s^{-1}\right\}\right]\right\} \end{aligned}$$

$$F_{ss}(\cdot) = \overset{\ominus}{u_{cc}}(\cdot)[p - e_x\overset{\oplus}{(\cdot)} - e_s(\cdot)]^2 + \overset{\oplus}{u_c}(\cdot)\left\{e_s(\cdot)\,\overset{\ominus}{(1}+\gamma)\left[x^{-2}s - 2x^{-1} + s^{-1}\right]\right\}$$

For $F_{ss}(\cdot) < 0$ to hold, $\left[x^{-2}s - 2x^{-1} + s^{-1}\right]$ must be positive

$$\begin{gathered} x^{-2}s - 2x^{-1} + s^{-1} > 0 \\ \frac{1}{x}\left(\frac{s}{x} - 2\right) + \frac{1}{s} > 0 \\ \frac{1}{x}\left(\frac{s}{x} - 2\right) > -\frac{1}{s} \\ \left(\frac{s}{x} - 2\right) > -\frac{x}{s} \\ \left(\frac{s}{x} + \frac{x}{s}\right) > 2 \\ s^2 + x^2 - 2sx > 0 \end{gathered}$$

$$(s-x)^2 > 0$$

$$\left(s'-k\right)^2 > 0$$

Which holds in domain of

$$k+s\left[1-\left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right] \leq s' \leq s+k$$

$$s > 0$$

Finally for $F_{ss}(\cdot)\, F_{s's'}(\cdot) - F_{ss'}(\cdot)\, F_{s's}(\cdot) > 0$

$$F_{s'}(\cdot) = -u_c(\cdot)\left(p - e_x(\cdot)\right)$$

$$F_{s's'}(\cdot) = -\left(-p + e_x(\cdot)\right) u_{cc}(\cdot)\left(p - e_x(\cdot)\right) + \left\{-u_c(\cdot)\left[-e_{xs'}(\cdot)\right]\right\}$$

$$F_{s's'}(\cdot) = -\left(-p + e_x(\cdot)\right) u_{cc}(\cdot)\left(p - e_x(\cdot)\right) + \left\{u_c(\cdot)\left[e_{xs'}(\cdot)\right]\right\}$$

$$e_x(x,s) = (1+\gamma)\,\psi\left(\frac{s-s'+k}{s}\right)^{\gamma}$$

$$e_{xs'}(\cdot) = -\gamma\,(1+\gamma)\,\psi\left(\frac{x}{s}\right)^{\gamma} x^{-1} = -\gamma e_x(\cdot)\, x^{-1} = -e_{xx}(\cdot) < 0$$

$$F_{s's'}(\cdot) = u_{cc}(\cdot)\left(p - e_x(\cdot)\right)^2 - \left\{u_c(\cdot)\left[e_{xx}(\cdot)\right]\right\}$$

And

$$F_s(\cdot) = u_c(\cdot)\left(p - e_x(\cdot) - e_s(\cdot)\right)$$

$$F_{ss'}(\cdot) = \left[\left(-p + e_x(\cdot)\right) u_{cc}(\cdot)\left(p - e_x(\cdot) - e_s(\cdot)\right)\right] + \left[u_c(\cdot)\left(-e_{xs'}(\cdot) - e_{ss'}(\cdot)\right)\right]$$

$$F_{ss'}(\cdot) = [(-p + e_x(\cdot))\, u_{cc}(\cdot)\, (p - e_x(\cdot) - e_s(\cdot))] - [u_c(\cdot)\, (e_{xs'}(\cdot) + e_{ss'}(\cdot))]$$

$$F_{ss'}(\cdot) = [(-p + e_x(\cdot))\, u_{cc}(\cdot)\, (p - e_x(\cdot) - e_s(\cdot))] - [u_c(\cdot)\, (-e_{xx}(\cdot) - e_{sx}(\cdot))]$$

$$F_{ss'}(\cdot) = [-(p - e_x(\cdot))\, u_{cc}(\cdot)\, (p - e_x(\cdot) - e_s(\cdot))] + [u_c(\cdot)\, (e_{xx}(\cdot) + e_{sx}(\cdot))]$$

And

$$F_{s's}(\cdot) = -(p - e_x(\cdot) - e_s(\cdot))\, u_{cc}(\cdot)\, (p - e_x(\cdot)) - u_c(\cdot)\, (-e_{xx}(\cdot) - e_{xs}(\cdot))$$

$$F_{s's}(\cdot) = -(p - e_x(\cdot) - e_s(\cdot))\, u_{cc}(\cdot)\, (p - e_x(\cdot)) + u_c(\cdot)\, (e_{xx}(\cdot) + e_{xs}(\cdot))$$

Let

$$M \equiv [p - e_x(\cdot) - e_s(\cdot)]$$

$$q^o \equiv [p - e_x(\cdot)]$$

$$A \equiv [e_{xx}(\cdot) + 2e_{xs}(\cdot) + e_{ss}(\cdot)]$$

$$B \equiv [e_{xx}(\cdot)]$$

$$C \equiv (e_{xx}(\cdot) + e_{sx}(\cdot))$$

Rewriting the system

$$F_{ss}(\cdot) = u_{cc}(\cdot)\, M^2 - u_c(\cdot)\, A$$

$$F_{s's'}(\cdot) = u_{cc}(\cdot)(q^o)^2 - u_c(\cdot)B$$

$$F_{ss'}(\cdot) = -u_{cc}(\cdot)Mq^o + u_c(\cdot)C$$

$$F_{s's}(\cdot) = -u_{cc}(\cdot)Mq^o + u_c(\cdot)C$$

Operating $F_{ss}(\cdot)F_{s's'}(\cdot)$

$$F_{ss}(\cdot)F_{s's'}(\cdot) = \left\{u_{cc}(\cdot)M^2 - u_c(\cdot)A\right\}\left\{u_{cc}(\cdot)(q^o)^2 - u_c(\cdot)B\right\}$$

$$F_{ss}(\cdot)F_{s's'}(\cdot) = [u_{cc}(\cdot)]^2[Mq^o]^2 - u_{cc}(\cdot)u_c(\cdot)BM^2 - u_{cc}(\cdot)u_c(\cdot)A[q^o]^2 + [u_c(\cdot)]^2 AB$$

And $F_{ss'}(\cdot)F_{s's}(\cdot)$

$$F_{s's}(\cdot)F_{ss'}(\cdot) = \{-u_{cc}(\cdot)Mq^o + u_c(\cdot)C\}\{-u_{cc}(\cdot)Mq^o + u_c(\cdot)C\}$$

$$F_{s's}(\cdot)F_{ss'}(\cdot) = [u_{cc}(\cdot)]^2[Mq^o]^2 - 2u_{cc}(\cdot)u_c(\cdot)CMq^o + [u_c(\cdot)]^2 C^2$$

So $F_{ss}(\cdot)F_{s's'}(\cdot) - [F_{ss'}(\cdot)]^2 > 0$

$$\begin{aligned} F_{ss}(\cdot)F_{s's'}(\cdot) - F_{ss'}(\cdot)F_{s's}(\cdot) = {} & [u_{cc}(\cdot)]^2[Mq^o]^2 - u_{cc}(\cdot)u_c(\cdot)BM^2 - u_{cc}(\cdot)u_c(\cdot)A[q^o]^2 + [u_c(\cdot)]^2 AB \\ & - [u_{cc}(\cdot)]^2[Mq^o]^2 + 2u_{cc}(\cdot)u_c(\cdot)CMq^o - [u_c(\cdot)]^2 C^2 > 0 \end{aligned}$$

$$F_{ss}(\cdot)F_{s's'}(\cdot) - F_{ss'}(\cdot)F_{s's}(\cdot) = -u_{cc}(\cdot)u_c(\cdot)\left[BM^2 - 2CMq^o + A(q^o)^2\right] + [u_c(\cdot)]^2\left[AB - C^2\right]$$

Replacing $\left[AB - C^2\right]$

$$AB = [e_{xx}(\cdot)]^2 + 2e_{xs}(\cdot)e_{xx}(\cdot) + e_{ss}(\cdot)e_{xx}(\cdot)$$

$$C^2 = [e_{xx}(\cdot)]^2 + 2e_{xx}(\cdot)\, e_{sx}(\cdot) + [e_{sx}(\cdot)]^2$$

$$\left[AB - C^2\right] = [e_{xx}(\cdot)]^2 + 2e_{xs}(\cdot)\, e_{xx}(\cdot) + e_{ss}(\cdot)\, e_{xx}(\cdot) - [e_{xx}(\cdot)]^2 - 2e_{xx}(\cdot)\, e_{sx}(\cdot) - [e_{sx}(\cdot)]^2$$

$$\left[AB - C^2\right] = e_{ss}(\cdot)\, e_{xx}(\cdot) - [e_{sx}(\cdot)]^2$$

Recall

1. $e_{xx}(x, s) = e_x(\cdot)\, \gamma x^{-1} = -e_s(\cdot)(1+\gamma)\, x^{-2} s$
2. $e_{xs}(x, s) = -e_x(\cdot)\, \gamma s^{-1} = e_s(\cdot)(1+\gamma)\, x^{-1}$
3. $e_{sx}(x, s) = e_s(\cdot)(1+\gamma)\, x^{-1} = e_{xs}(x, s)$
4. $e_{ss}(x, s) = -e_s(\cdot)(1+\gamma)\, s^{-1}$

$$\left[AB - C^2\right] = \left\{-e_s(\cdot)(1+\gamma)\, s^{-1}\right\}\left\{-e_s(\cdot)(1+\gamma)\, x^{-2} s\right\} - \left[e_s(\cdot)(1+\gamma)\, x^{-1}\right]^2$$

$$\left[AB - C^2\right] = \left\{[e_s(\cdot)]^2 (1+\gamma)^2 x^{-2}\right\} - \left[e_s(\cdot)(1+\gamma)\, x^{-1}\right]^2$$

$$\left[AB - C^2\right] = \left\{[e_s(\cdot)]^2 (1+\gamma)^2 x^{-2}\right\} - \left\{[e_s(\cdot)]^2 (1+\gamma)^2 x^{-2}\right\}$$

$$\left[AB - C^2\right] = 0$$

So the expression $F_{ss}(\cdot)\, F_{s's'}(\cdot) - F_{ss'}(\cdot)\, F_{s's}(\cdot)$ is redefined as,

$$F_{ss}(\cdot)\, F_{s's'}(\cdot) - F_{ss'}(\cdot)\, F_{s's}(\cdot) = -u_{cc}(\cdot)\, u_c(\cdot)\left[BM^2 - 2CMq^o + A\,(q^o)^2\right]$$

Then, since $-u_{cc}(\cdot)\, u_c(\cdot) > 0$, $F_{ss}(\cdot)\, F_{s's'}(\cdot) - F_{ss'}(\cdot)\, F_{s's}(\cdot) > 0$ holds if,

$$\left[BM^2 - 2CMq^o + A\left(q^o\right)^2\right] > 0$$

Let

$$Z \equiv \frac{M}{q^o}$$

$$\left[BZ^2 - 2CZ + A\right] > 0$$

Solving the inequality

$$Z > \frac{2C \pm \sqrt{4C^2 - 4AB}}{2B}$$

$$Z > \frac{C \pm 2\sqrt{C^2 - AB}}{B}$$

As we show $AB - C^2 = 0$

$$Z > \frac{C}{B}$$

Replacing $Z \equiv \frac{M}{q^o}$

$$\frac{M}{q^o} > \frac{C}{B}$$

Since $M = q^o - e_s\left(\cdot\right)$

$$\frac{q^o - e_s\left(\cdot\right)}{q^o} > \frac{C}{B}$$

$$1 - \frac{e_s\left(\cdot\right)}{q^o} > \frac{C}{B}$$

$$1 - \frac{C}{B} > \frac{e_s\left(\cdot\right)}{q^o}$$

$$\frac{B-C}{B} > \frac{e_s(\cdot)}{q^o}$$

Since $\frac{e_s(\cdot)}{q^o} < 0$ it is sufficient to show $B - C > 0$

Recall

$$B \equiv [e_{xx}(\cdot)]$$

$$C \equiv (e_{xx}(\cdot) + e_{sx}(\cdot))$$

Then

$$B - C > 0$$

$$e_{xx}(\cdot) - e_{xx}(\cdot) - e_{sx}(\cdot) > 0$$

$$-e_{sx}(\cdot) > 0$$

Recall $e_{sx}(\cdot) = e_s(\cdot)(1+\gamma)x^{-1}$

$$-e_s(\cdot)(1+\gamma)x^{-1} > 0$$

Since $e_s(\cdot) < 0$, the condition holds in domain of

$$k + s\left[1 - \left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right] \leq s' \leq s + k$$

$$s > 0$$

I Business Cycle Moments by Country

In this Appendix we present tables containing the business cycle moments by country.

	Mean	Standard Dev.	Corr(i,GDP)	Corr(i,Oil Price)	Corr(i,Reserves)	Corr(i,III)	Acorr
			Defaulters				
1. Algeria							
Oil price	0	0.18202	0.12147	1	0.60593	0.86634	0.84749
Non-oil GDP	0	0.049395	0.47551	-0.23628	-0.094443	0.11386	0.52334
GDP	0	0.024752	1	0.12147	0.15821	0.34198	0.73414
Oil production	0	0.053372	0.3074	0.18849	0.25201	0.14864	0.63562
Consumption	0	0.033709	0.50335	0.030535	0.026005	0.21056	0.56724
Gross oil output	0	0.20108	0.2301	0.42278	0.23747	0.19343	0.25491
Trade balance to GDP	0.051308	0.094454	0.096215	0.26867	0.47205	0.0025333	0.74715
Institutional Investor Index	42.7099	12.2003	0.34198	0.86634	0.38626	1	0.93769
Debt to GDP	0.32898	0.22219	-0.39214	-0.76632	-0.79507	-0.73833	0.92074
Reserves	9.9381	1.471	0.15821	0.60593	1	0.38626	0.89187
Total Debt	51.4296	28.23	-0.3002	-0.82656	-0.82417	-0.72306	0.91817
2. Angola							
Oil price	0	0.18202	0.1278	1	0.43503	0.77286	0.84749
Non-oil GDP	0	0.14135	0.5386	-0.13	-0.00055829	0.025451	0.29789
GDP	0	0.077409	1	0.1278	0.076171	0.093347	0.65645
Oil production	0	0.12037	0.63629	0.047291	0.10218	0.11615	0.6129
Consumption	0	0.029976	-0.13614	0.018858	0.057124	-0.054983	0.29984
Gross oil output	0	0.22578	0.45932	0.3572	0.075461	0.13198	0.33259
Trade balance to GDP	0.16158	0.10714	0.18158	0.21714	-0.00080032	0.030024	0.1676
Institutional Investor Index	19.1445	9.5761	0.093347	0.77286	0.80215	1	0.91153
Debt to GDP	0.73997	0.64742	-0.53112	-0.69736	-0.51013	-0.64947	0.83586
Reserves	4.6649	3.0696	0.076171	0.43503	1	0.80215	0.90737
Total Debt	72.7056	59.0513	-0.19186	-0.66452	-0.28245	-0.60741	0.70937
3. Argentina							
Oil price	0	0.18202	0.034086	1	0.31432	0.26231	0.84749
Non-oil GDP	0	0.064923	0.99358	0.0042762	-0.30157	0.56915	0.60668
GDP	0	0.061199	1	0.034086	-0.31465	0.5831	0.6004
Oil production	0	0.051743	0.30411	-0.091604	0.18964	0.50553	0.78416
Consumption	0	0.067635	0.95562	0.017355	-0.35319	0.61901	0.59212
Gross oil output	0	0.17838	0.13156	0.39824	-0.077621	0.27276	0.17326
Trade balance to GDP	0.023568	0.039506	-0.72317	0.14959	0.45455	-0.56829	0.67007
Institutional Investor Index	32.8994	11.0973	0.5831	0.26231	0.035197	1	0.71106
Debt to GDP	0.3049	0.18544	-0.64625	-0.37286	0.28406	-0.59689	0.64215
Reserves	2.4436	0.35208	-0.31465	0.31432	1	0.035197	0.80351
Total Debt	51.6696	28.6498	-0.6713	-0.15385	0.41493	-0.63168	0.67686

	Mean	Standard Dev.	Corr(i,GDP)	Corr(i,Oil Price)	Corr(i,Reserves)	Corr(i,III)	Acorr
4. Brazil							
Oil price	0	0.18202	-0.024834	1	0.49806	0.79039	0.84749
Non-oil GDP	0	0.031904	0.995	-0.049718	-0.005606	0.13673	0.53178
GDP	0	0.030964	1	-0.024834	-0.00017989	0.12743	0.51617
Oil production	0	0.098616	-0.042723	-0.049722	0.043541	-0.17762	0.74246
Consumption	0	0.023024	0.68973	-0.050018	-0.016978	0.15284	0.62325
Gross oil output	0	0.19681	-0.14609	0.36011	0.066779	0.007495	0.19901
Trade balance to GDP	0.0062973	0.02584	-0.14629	-0.072047	-0.34401	-0.46856	0.77803
Institutional Investor Index	44.1188	14.4936	0.12743	0.79039	0.7969	1	0.91109
Debt to GDP	0.17167	0.098496	-0.18291	-0.39401	-0.81942	-0.78011	0.92278
Reserves	6.4396	4.5441	-0.00017989	0.49806	1	0.7969	0.91461
Total Debt	60.4659	16.342	-0.16848	-0.19718	0.28742	-0.12239	0.61726
5. Ecuador							
Oil price	0	0.18202	0.051057	1	0.58312	0.5854	0.84749
Non-oil GDP	0	0.037235	0.54629	-0.21933	0.017453	0.14004	0.25751
GDP	0	0.020902	1	0.051057	0.056465	0.28225	0.45286
Oil production	0	0.097024	0.39881	0.033802	0.050543	0.052916	-0.046454
Consumption	0	0.028389	0.77422	0.023455	0.02264	0.22543	0.32885
Gross oil output	0	0.20848	0.24602	0.37527	0.037589	0.11712	0.34209
Trade balance to GDP	-0.013223	0.029856	-0.094194	0.21103	-0.067087	0.045609	0.24675
Institutional Investor Index	27.7376	9.1089	0.28225	0.5854	-0.022518	1	0.79698
Debt to GDP	0.42338	0.21561	-0.22603	-0.88703	-0.6235	-0.57583	0.89807
Reserves	3.0755	2.1566	0.056465	0.58312	1	-0.022518	0.8648
Total Debt	54.1452	27.8287	-0.18635	-0.80185	-0.62826	-0.58214	0.89795
6. Gabon							
Oil price	0	0.18202	-0.042988	1	0.07798	0.63564	0.84749
Non-oil GDP	0	0.12207	0.48721	-0.23237	-0.053168	0.067017	0.14097
GDP	0	0.049374	1	-0.042988	-0.0037056	-0.026527	0.42249
Oil production	0	0.10043	0.44186	0.047309	0.009268	-0.12596	0.64414
Consumption	0	0.059532	0.34465	-0.032275	-0.011857	0.1533	0.15307
Gross oil output	0	0.2277	0.17888	0.35005	-0.057871	-0.10572	0.31161
Trade balance to GDP	0.19825	0.12625	0.070059	0.47261	0.48684	-0.21946	0.65996
Institutional Investor Index	30.9989	6.8734	-0.026527	0.63564	-0.36294	1	0.87933
Debt to GDP	0.46043	0.22137	-0.019771	-0.87982	0.035243	-0.71143	0.88292
Reserves	1.4738	0.82694	-0.0037056	0.07798	1	-0.36294	0.93084
Total Debt	54.4104	25.9217	-0.055267	-0.83435	0.078494	-0.72078	0.83663

	Mean	Standard Dev.	Corr(i,GDP)	Corr(i,Oil Price)	Corr(i,Reserves)	Corr(i,III)	Acorr
7. Indonesia							
Oil price	0	0.18202	0.053508	1	-0.11502	0.45531	0.84749
Non-oil GDP	0	0.046084	0.89693	-0.034757	-0.070217	0.54932	0.57237
GDP	0	0.03981	1	0.053508	-0.083444	0.59106	0.65606
Oil production	0	0.042957	0.20162	-0.09781	-0.012229	0.081818	0.46736
Consumption	0	0.033207	0.75728	0.022585	-0.04999	0.3798	0.51527
Gross oil output	0	0.18137	0.31897	0.39463	0.064644	0.22136	0.16729
Trade balance to GDP	0.029811	0.034131	-0.59189	-0.25534	-0.1687	-0.73348	0.57345
Institutional Investor Index	46.395	10.1564	0.59106	0.45531	0.22458	1	0.89033
Debt to GDP	0.31423	0.13998	-0.45537	-0.80597	0.2782	-0.50932	0.74508
Reserves	6.2538	2.2037	-0.083444	-0.11502	1	0.22458	0.85514
Total Debt	39.2984	19.4416	-0.62056	-0.59725	-0.21956	-0.86968	0.82975
8. Iran							
Oil price	0	0.18202	0.15857	1	0.40507	0.054683	0.84749
Non-oil GDP	0	0.073453	0.60377	-0.050171	-0.10405	-0.015032	0.3807
GDP	0	0.073615	1	0.15857	0.098755	0.35133	0.28258
Oil production	0	0.12585	0.86464	0.095334	0.19691	0.40041	0.07954
Consumption	0	0.060214	0.73043	0.21046	0.030794	0.28754	0.61748
Gross oil output	0	0.24475	0.69768	0.35528	0.18975	0.30947	0.17973
Trade balance to GDP	-0.0076219	0.078019	0.29908	0.097042	0.50352	0.55082	0.5453
Institutional Investor Index	27.2039	7.7952	0.35133	0.054683	0.68527	1	0.83778
Debt to GDP	0.042387	0.04314	0.038605	-0.49961	-0.11957	0.1153	0.77461
Reserves	97.671	32.9913	0.098755	0.40507	1	0.68527	0.8819
Total Debt	25.8154	14.863	-0.38321	-0.37097	-0.52886	-0.62742	0.74972
9. Iraq							
Oil price	0	0.18202	-0.010138	1	-0.019418	0.68839	0.84749
Non-oil GDP	0	0.17174	0.26918	-0.047664	-0.097586	0.085655	0.029526
GDP	0	0.18404	1	-0.010138	-0.029922	0.080636	0.16095
Oil production	0	0.45619	0.71465	-0.061324	0.13592	0.079961	0.58805
Consumption	0	NaN	NaN	NaN	NaN	NaN	NaN
Gross oil output	0	0.45053	0.70353	0.10428	0.13296	0.11704	0.44848
Trade balance to GDP	0.045435	0.15317	0.11515	0.091827	0.50183	-0.00022145	0.46848
Institutional Investor Index	18.662	13.346	0.080636	0.68839	-0.44838	1	0.80853
Debt to GDP	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Reserves	95.4903	33.2975	-0.029922	-0.019418	1	-0.44838	0.86665
Total Debt	109.4799	98.7461	-0.020384	-0.24542	-0.10036	-0.2338	0.5729

	Mean	Standard Dev.	Corr(i,GDP)	Corr(i,Oil Price)	Corr(i,Reserves)	Corr(i,III)	Acorr
10. Mexico							
Oil price	0	0.18202	0.092198	1	-0.60096	0.72544	0.84749
Non-oil GDP	0	0.032551	0.92799	-0.011324	0.02944	0.24804	0.32856
GDP	0	0.030111	1	0.092198	0.017065	0.20485	0.34017
Oil production	0	0.077306	0.55276	0.0087796	0.033767	-0.090297	0.50157
Consumption	0	0.031051	0.93123	0.12556	-0.056781	0.27185	0.41967
Gross oil output	0	0.19909	0.45499	0.3799	-0.048464	0.10932	0.21382
Trade balance to GDP	-0.0022552	0.03185	-0.22031	0.030631	0.38487	-0.47624	0.7678
Institutional Investor Index	52.5603	14.5972	0.20485	0.72544	-0.80462	1	0.88926
Debt to GDP	0.22286	0.12516	-0.35314	-0.209	0.60275	-0.72996	0.87573
Reserves	34.5215	18.1988	0.017065	-0.60096	1	-0.80462	0.91667
Total Debt	46.7147	11.6141	-0.4022	-0.22242	0.38067	-0.61867	0.72589
11. Nigeria							
Oil price	0	0.18202	0.17039	1	0.63731	0.87623	0.84749
Non-oil GDP	0	0.069396	0.32629	-0.037719	-0.055911	0.1627	0.16864
GDP	0	0.055495	1	0.17039	0.10299	0.19702	0.70152
Oil production	0	0.089818	0.62918	0.07489	0.091378	0.0054419	0.48725
Consumption	0	0.11459	0.5226	0.088171	0.05639	0.13622	0.25979
Gross oil output	0	0.21086	0.43751	0.38738	0.18211	0.15793	0.27889
Trade balance to GDP	0.071467	0.05457	0.20338	0.096052	0.24205	-0.19605	0.14642
Institutional Investor Index	28.5439	12.419	0.19702	0.87623	0.35007	1	0.88691
Debt to GDP	0.29304	0.26822	0.070585	-0.84592	-0.64236	-0.75736	0.89848
Reserves	23.6913	8.6542	0.10299	0.63731	1	0.35007	0.92064
Total Debt	66.8785	54.7299	-0.045511	-0.78722	-0.71858	-0.65879	0.87998
12. Russia							
Oil price	0	0.18202	0.2719	1	0.55311	0.80289	0.84749
Non-oil GDP	0	0.062358	0.94979	0.23707	0.2257	0.30558	0.63282
GDP	0	0.067967	1	0.2719	0.19922	0.27066	0.61568
Oil production	0	0.062511	0.88223	0.1946	0.22979	0.26284	0.6484
Consumption	0	0.048853	0.76437	0.27278	0.23107	0.32728	0.7311
Gross oil output	0	0.21767	0.81236	0.3303	0.13583	0.22346	0.32
Trade balance to GDP	0.09276	0.044501	0.26678	-0.043356	-0.33218	-0.078509	0.64272
Institutional Investor Index	44.0281	20.5408	0.27066	0.80289	0.59573	1	0.92518
Debt to GDP	0.20467	0.13006	-0.59291	-0.66382	-0.60589	-0.76039	0.77786
Reserves	58.4132	9.512	0.19922	0.55311	1	0.59573	0.64457
Total Debt	38.8104	31.7584	-0.016431	-0.69739	-0.45332	-0.86177	0.77349

	Mean	Standard Dev.	Corr(i,GDP)	Corr(i,Oil Price)	Corr(i,Reserves)	Corr(i,III)	Acorr
13. Sudan							
Oil price	0	0.18202	0.10269	1	0.74135	0.54717	0.84749
Non-oil GDP	0	0.038045	0.52293	-0.13108	0.059501	0.23654	0.25786
GDP	0	0.051153	1	0.10269	0.14085	0.4381	0.57484
Oil production	0	0.72661	0.36305	-0.0084862	-0.082255	0.16519	0.49178
Consumption	0	0.051675	0.79625	0.07848	0.11023	0.34266	0.55617
Gross oil output	0	0.76737	0.40793	0.069806	-0.052558	0.20926	0.38924
Trade balance to GDP	-0.033098	0.036949	0.21206	0.45423	0.52063	0.41664	0.33045
Institutional Investor Index	9.435	2.4371	0.4381	0.54717	0.46449	1	0.78572
Debt to GDP	0.57049	0.30479	-0.25856	-0.68829	-0.71035	-0.82402	0.75686
Reserves	1.9854	2.2537	0.14085	0.74135	1	0.46449	0.85812
Total Debt	151.2396	97.4567	-0.13478	-0.57317	-0.58696	-0.77624	0.59737
14. Venezuela							
Oil price	0	0.18202	0.10281	1	0.43225	0.50023	0.84749
Non-oil GDP	0	0.091015	0.66055	-0.12132	-0.056011	0.15224	0.35908
GDP	0	0.058581	1	0.10281	-0.024891	0.33555	0.53322
Oil production	0	0.06455	0.59643	-0.011309	0.13291	0.28042	0.55666
Consumption	0	0.061368	0.85383	0.15415	-0.12276	0.32052	0.59518
Gross oil output	0	0.19167	0.41066	0.38726	-0.005015	0.25628	0.23017
Trade balance to GDP	0.058102	0.077381	-0.32614	-0.037448	-0.32518	-0.12908	0.36541
Institutional Investor Index	40.0426	11.0146	0.33555	0.50023	-0.30061	1	0.79414
Debt to GDP	0.30327	0.12997	-0.21128	-0.81101	-0.38869	-0.43579	0.85081
Reserves	81.6334	68.6858	-0.024891	0.43225	1	-0.30061	0.7755
Total Debt	38.5001	16.0959	-0.41222	-0.15469	0.58401	-0.69161	0.75068
15. Vietnam							
Oil price	0	0.18202	-0.063041	1	0.46626	0.73752	0.84749
Non-oil GDP	0	0.024728	0.84349	-0.18059	-0.066845	0.13148	0.66492
GDP	0	0.016654	1	-0.063041	-0.13467	0.18874	0.6636
Oil production	0	0.34284	-0.7322	0.0038434	0.0055359	-0.16185	0.35955
Consumption	0	0.023801	0.54768	0.11043	-0.19696	0.33064	0.67519
Gross oil output	0	0.41887	-0.64234	0.15838	-0.03544	-0.032436	0.34928
Trade balance to GDP	-0.068455	0.047577	-0.2561	0.30688	0.67755	0.04364	0.55626
Institutional Investor Index	35.8785	9.7368	0.18874	0.73752	0.35861	1	0.84578
Debt to GDP	0.76606	0.89076	-0.44188	-0.32025	-0.17845	-0.43097	0.73128
Reserves	1.05	1.2941	-0.13467	0.46626	1	0.35861	0.66284
Total Debt	68.1786	48.4276	-0.1675	-0.39461	-0.16143	-0.70647	0.55072

	Mean	Standard Dev.	Corr(i,GDP)	Corr(i,Oil Price)	Corr(i,Reserves)	Corr(i,III)	Acorr
16. Yemen							
Oil price	0	0.18202	-0.0444	1	-0.12154	-0.13002	0.84749
Non-oil GDP	0	0.094879	0.31162	-0.14784	-0.045711	-0.13788	0.29921
GDP	0	0.027804	1	-0.0444	-0.025765	0.46008	0.17201
Oil production	0	0.3665	0.057148	-0.0039444	0.35407	0.063962	0.33813
Consumption	0	NaN	NaN	NaN	NaN	NaN	NaN
Gross oil output	0	0.42209	0.011548	0.15062	0.33423	0.1877	0.41198
Trade balance to GDP	-0.06614	0.035184	0.23654	-0.021483	-7.29e-17	0.2341	-0.11407
Institutional Investor Index	26.062	4.6396	0.46008	-0.13002	0.11777	1	0.804
Debt to GDP	0.533	0.36362	-0.026591	-0.67409	0.25251	0.059943	0.84729
Reserves	3.3666	1.1196	-0.025765	-0.12154	1	0.11777	0.62868
Total Debt	76.747	47.0205	-0.031907	-0.57385	0.19513	-0.042079	0.6939
			Non-Defaulters				
17. Azerbaijan							
Oil price	0	0.18202	0.21181	1	0.62293	0.42364	0.84749
Non-oil GDP	0	0.14018	0.87752	0.084549	0.16139	0.16346	0.46703
GDP	0	0.14569	1	0.21181	0.28169	0.17802	0.61574
Oil production	0	0.16765	0.84413	0.13629	0.088299	0.30632	0.65709
Consumption	0	0.15356	0.86902	0.17327	0.13545	0.10888	0.46498
Gross oil output	0	0.28027	0.78874	0.29466	0.27724	0.29143	0.52646
Trade balance to GDP	0.051233	0.25159	0.66768	0.69511	0.53064	0.51948	0.75444
Institutional Investor Index	43.047	8.9829	0.17802	0.42364	0.27873	1	0.74146
Debt to GDP	0.097518	0.039908	-0.37481	-0.13065	-0.2358	-0.62979	0.71651
Reserves	5.0593	2.8241	0.28169	0.62293	1	0.27873	0.80556
Total Debt	15.5461	5.9003	-0.2411	-0.47873	-0.59967	-0.4971	0.77035
18. China							
Oil price	0	0.18202	-0.036086	1	-0.47398	0.84405	0.84749
Non-oil GDP	0	0.032768	0.96633	-0.10071	-0.28496	0.28244	0.67975
GDP	0	0.030865	1	-0.036086	-0.27416	0.3101	0.68824
Oil production	0	0.027673	0.57384	-0.060982	-0.072393	0.19003	0.57637
Consumption	0	0.017956	0.16566	0.061627	0.10645	0.047153	0.41269
Gross oil output	0	0.17455	-0.030224	0.41976	-0.12395	0.09737	0.11062
Trade balance to GDP	0.020206	0.027838	-0.096362	0.21084	-0.21181	0.19404	0.75323
Institutional Investor Index	65.2294	8.445	0.3101	0.84405	-0.55666	1	0.90512
Debt to GDP	0.066389	0.046554	0.05866	-0.78616	0.61028	-0.79031	0.9377
Reserves	21.1141	2.8723	-0.27416	-0.47398	1	-0.55666	0.80603
Total Debt	19.4289	11.9003	-0.082252	0.52356	-0.098074	0.57411	0.87207

	Mean	Standard Dev.	Corr(i,GDP)	Corr(i,Oil Price)	Corr(i,Reserves)	Corr(i,III)	Acorr
19. Colombia							
Oil price	0	0.18202	0.023415	1	-0.47787	0.77307	0.84749
Non-oil GDP	0	0.027483	0.93808	-0.048825	0.29114	0.33976	0.69423
GDP	0	0.025402	1	0.023415	0.34775	0.32465	0.72272
Oil production	0	0.12805	0.13207	-0.22519	0.38682	0.0021588	0.63081
Consumption	0	0.026964	0.87274	-0.016224	0.40004	0.40695	0.79227
Gross oil output	0	0.17366	0.2131	0.27031	0.1534	0.10656	0.08262
Trade balance to GDP	-0.017071	0.034053	-0.36393	-0.028673	-0.29959	-0.35708	0.73896
Institutional Investor Index	47.6329	9.3616	0.32465	0.77307	-0.1543	1	0.86378
Debt to GDP	0.20429	0.076394	-0.35729	-0.51183	-0.079178	-0.76895	0.88924
Reserves	1.7719	0.74994	0.34775	-0.47787	1	-0.1543	0.8471
Total Debt	33.028	8.3529	-0.42967	-0.18933	0.44906	-0.26016	0.78438
20. Egypt							
Oil price	0	0.18202	0.00098939	1	-0.18737	0.28561	0.84749
Non-oil GDP	0	0.04091	0.69954	-0.2254	-0.20686	0.14379	0.56325
GDP	0	0.021784	1	0.00098939	-0.29075	0.23407	0.65191
Oil production	0	0.042294	0.3223	-0.1803	-0.032266	-0.14919	0.42761
Consumption	0	0.014297	0.44185	-0.047312	-0.018467	-0.05899	0.33011
Gross oil output	0	0.17533	0.14789	0.38401	-0.051724	0.056272	0.030664
Trade balance to GDP	-0.076504	0.041234	-0.15671	-0.094604	0.11054	0.52514	0.75274
Institutional Investor Index	37.2455	9.2005	0.23407	0.28561	-0.43256	1	0.92446
Debt to GDP	0.4809	0.27319	0.040232	-0.30082	0.035121	-0.74259	0.9244
Reserves	3.8619	0.78127	-0.29075	-0.18737	1	-0.43256	0.60942
Total Debt	96.6575	22.9784	-0.28934	0.0050562	0.072672	-0.59136	0.85361
21. India							
Oil price	0	0.18202	0.11474	1	-0.12386	0.74406	0.84749
Non-oil GDP	0	0.014836	0.98202	0.063684	-0.013067	0.22294	0.31528
GDP	0	0.015102	1	0.11474	0.022112	0.22986	0.3558
Oil production	0	0.10639	0.2196	-0.071514	-0.038504	0.088205	0.32216
Consumption	0	0.013859	0.69138	0.021225	-0.12051	0.1975	0.50988
Gross oil output	0	0.20193	0.26979	0.33757	0.18667	0.158	0.1558
Trade balance to GDP	-0.022496	0.017243	-0.2384	-0.75674	0.029634	-0.88982	0.83753
Institutional Investor Index	50.6239	7.5071	0.22986	0.74406	0.083211	1	0.92655
Debt to GDP	0.14051	0.070829	-0.15753	-0.80208	0.3202	-0.85013	0.94031
Reserves	5.0273	1.269	0.022112	-0.12386	1	0.083211	0.81426
Total Debt	65.0445	15.2329	-0.26017	-0.078496	0.51251	0.10151	0.8038

	Mean	Standard Dev.	Corr(i,GDP)	Corr(i,Oil Price)	Corr(i,Reserves)	Corr(i,III)	Acorr
22. Kazakhstan							
Oil price	0	0.18202	0.23532	1	0.64444	0.78508	0.84749
Non-oil GDP	0	0.072999	0.86562	0.14236	0.30521	0.13985	0.50661
GDP	0	0.07723	1	0.23532	0.23419	0.18287	0.67725
Oil production	0	0.074574	0.82865	0.13551	-0.10057	0.1705	0.5863
Consumption	0	0.094576	0.93877	0.24272	0.3022	0.1595	0.65169
Gross oil output	0	0.22091	0.69801	0.31615	0.15733	0.18155	0.29574
Trade balance to GDP	0.051226	0.085722	0.095318	0.77938	0.56057	0.88558	0.7302
Institutional Investor Index	39.8525	16.1737	0.18287	0.78508	0.56789	1	0.90127
Debt to GDP	0.080684	0.059606	-0.70165	-0.59714	-0.70868	-0.56454	0.82654
Reserves	17.139	11.913	0.23419	0.64444	1	0.56789	0.84319
Total Debt	15.7604	8.0704	-0.21697	-0.60581	-0.42935	-0.75388	0.6264
23. Kuwait							
Oil price	0	0.18202	0.095057	1	-0.15293	0.84603	0.84749
Non-oil GDP	0	0.09616	0.36697	-0.14248	0.0068189	0.012673	0.1878
GDP	0	0.074964	1	0.095057	0.04395	0.059636	0.46165
Oil production	0	0.36662	0.21689	-0.047836	0.11408	0.23503	0.20665
Consumption	0	NaN	NaN	NaN	NaN	NaN	NaN
Gross oil output	0	0.37911	0.3281	0.15145	0.093032	0.27573	0.055362
Trade balance to GDP	0.16462	0.27861	0.14304	0.60546	0.054319	0.79931	0.55377
Institutional Investor Index	64.7681	9.8694	0.059636	0.84603	-0.014489	1	0.8388
Debt to GDP	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Reserves	93.5138	11.0614	0.04395	-0.15293	1	-0.014489	0.83333
Total Debt	37.5747	43.8855	-0.012044	-0.64745	0.18429	-0.88504	0.70237
24. Libya							
Oil price	0	0.18202	0.024853	1	0.5899	0.76233	0.84749
Non-oil GDP	0	0.20823	0.76677	-0.077611	0.0064743	0.097558	-0.017665
GDP	0	0.21593	1	0.024853	0.021208	0.16785	-0.21674
Oil production	0	0.21329	0.88271	0.084693	0.064272	0.26356	-0.12773
Consumption	0	NaN	NaN	NaN	NaN	NaN	NaN
Gross oil output	0	0.29694	0.64816	0.31326	0.098807	0.32712	0.14199
Trade balance to GDP	0.13715	0.19328	0.39705	0.29135	0.18841	0.38285	0.484
Institutional Investor Index	35.8695	9.8611	0.16785	0.76233	0.44969	1	0.83257
Debt to GDP	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Reserves	30.4673	9.4812	0.021208	0.5899	1	0.44969	0.91999
Total Debt	33.4442	25.0854	-0.24876	-0.78284	-0.55695	-0.71205	0.88186

	Mean	Standard Dev.	Corr(i,GDP)	Corr(i,Oil Price)	Corr(i,Reserves)	Corr(i,III)	Acorr
25. Malaysia							
Oil price	0	0.18202	0.11833	1	-0.050013	0.72303	0.84749
Non-oil GDP	0	0.039384	0.93874	0.015836	0.43921	0.50438	0.65475
GDP	0	0.037482	1	0.11833	0.36499	0.53318	0.61325
Oil production	0	0.048213	-0.078619	-0.12784	-0.25106	-0.15053	0.34382
Consumption	0	0.051255	0.89881	0.22268	0.24793	0.60973	0.67068
Gross oil output	0	0.1813	0.44124	0.38397	-0.1751	0.28339	0.21367
Trade balance to GDP	0.094175	0.099975	-0.43144	0.069919	0.11728	-0.22257	0.8577
Institutional Investor Index	65.5677	6.4491	0.53318	0.72303	0.24617	1	0.84407
Debt to GDP	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Reserves	3.4504	0.54123	0.36499	-0.050013	1	0.24617	0.79896
Total Debt	54.9042	21.0568	-0.48645	0.0056616	-0.49443	-0.21253	0.91002
26. Oman							
Oil price	0	0.18202	-0.14766	1	-0.082668	0.55695	0.84749
Non-oil GDP	0	0.11348	0.64532	-0.24098	0.0098537	0.0078003	0.27049
GDP	0	0.045174	1	-0.14766	0.042412	0.038155	0.57016
Oil production	0	0.051852	0.49367	-0.29461	0.1083	-0.070816	0.68927
Consumption	0	0.029801	0.16066	-0.074973	-0.024208	0.009143	0.22586
Gross oil output	0	0.16562	-0.0034674	0.36529	-0.045746	0.050327	-0.0084406
Trade balance to GDP	0.15867	0.082516	-0.30959	0.65739	0.090286	0.38578	0.38047
Institutional Investor Index	56.5927	8.4563	0.038155	0.55695	0.70443	1	0.90708
Debt to GDP	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Reserves	4.6242	1.0347	0.042412	-0.082668	1	0.70443	0.91068
Total Debt	19.1806	11.3073	0.21107	-0.91822	-0.037765	-0.6253	0.8681
27. Qatar							
Oil price	0	0.18202	0.093372	1	0.61133	0.77492	0.84749
Non-oil GDP	0	0.085857	0.79249	-0.019189	-0.023466	0.023461	0.5578
GDP	0	0.07296	1	0.093372	-0.044694	0.068474	0.51897
Oil production	0	0.084106	0.21994	0.1162	0.005421	0.095879	0.26424
Consumption	0	NaN	NaN	NaN	NaN	NaN	NaN
Gross oil output	0	0.20399	0.29474	0.41535	0.03571	0.18514	0.26541
Trade balance to GDP	0.29213	0.13982	0.091686	0.52198	0.55354	0.46285	0.74384
Institutional Investor Index	61.4436	10.2191	0.068474	0.77492	0.90632	1	0.92607
Debt to GDP	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Reserves	9.6466	8.1516	-0.044694	0.61133	1	0.90632	0.89421
Total Debt	35.9414	17.3465	0.14565	-0.45207	-0.1992	-0.37198	0.77213

	Mean	Standard Dev.	Corr(i,GDP)	Corr(i,Oil Price)	Corr(i,Reserves)	Corr(i,III)	Acorr
28. Saudi Arabia							
Oil price	0	0.18202	0.093562	1	-0.076044	0.88576	0.84749
Non-oil GDP	0	0.17502	0.13306	-0.15895	-0.20685	-0.11394	0.20709
GDP	0	0.082459	1	0.093562	0.29007	-0.0019226	0.64228
Oil production	0	0.14206	0.97993	0.066585	0.31121	-0.037623	0.5881
Consumption	0	0.030485	0.050334	0.08688	0.0092598	0.14608	0.47002
Gross oil output	0	0.23984	0.69878	0.35196	0.2622	0.090359	0.48944
Trade balance to GDP	0.12552	0.13254	0.5343	0.51524	0.4465	0.223	0.79947
Institutional Investor Index	64.7	8.7672	-0.0019226	0.88576	-0.30642	1	0.84223
Debt to GDP	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Reserves	235.7527	42.8149	0.29007	-0.076044	1	-0.30642	0.89578
Total Debt	51.7547	36.5915	-0.060756	-0.80723	-0.025926	-0.70775	0.91232
29. Syria							
Oil price	0	0.18202	0.084082	1	0.18712	0.26454	0.84749
Non-oil GDP	0	0.081494	0.36677	-0.11875	0.090388	-0.002332	0.29537
GDP	0	0.10387	1	0.084082	0.094381	0.57556	0.55672
Oil production	0	0.27056	0.81357	0.085588	0.041599	0.58598	0.53976
Consumption	0	0.10915	0.86763	0.10062	0.040483	0.57075	0.4668
Gross oil output	0	0.36232	0.67032	0.27306	0.029237	0.55464	0.43158
Trade balance to GDP	-0.069884	0.10411	0.2943	-0.37683	0.30248	0.41571	0.72445
Institutional Investor Index	23.5469	5.1509	0.57556	0.26454	0.45601	1	0.70706
Debt to GDP	0.082614	0.07366	-0.76169	0.063321	6.1314e-17	-0.56729	0.61438
Reserves	2.1469	0.43027	0.094381	0.18712	1	0.45601	0.91607
Total Debt	118.7856	51.1906	-0.35132	-0.66738	-0.33057	-0.63724	0.84869
30. United Arab Emirates							
Oil price	0	0.18202	0.23249	1	-0.21656	0.64213	0.84749
Non-oil GDP	0	0.11159	0.65725	-0.017325	-0.36335	-0.023349	0.4086
GDP	0	0.063566	1	0.23249	-0.14334	0.089487	0.44501
Oil production	0	0.097505	0.10854	0.095805	0.38152	-0.016545	0.67709
Consumption	0	0.12904	0.29147	0.10815	-6.4027e-17	0.25673	0.50686
Gross oil output	0	0.22406	0.4791	0.37622	0.18714	0.14032	0.30607
Trade balance to GDP	0.17334	0.11149	-0.034899	0.35753	3.8804e-17	0.080296	0.7576
Institutional Investor Index	64.8673	7.2239	0.089487	0.64213	0.3734	1	0.89042
Debt to GDP	NaN	NaN	NaN	NaN	NaN	NaN	NaN
Reserves	82.8153	28.1244	-0.14334	-0.21656	1	0.3734	0.86403
Total Debt	8.6445	5.2709	-0.23925	0.3964	0.2324	0.51979	0.83988

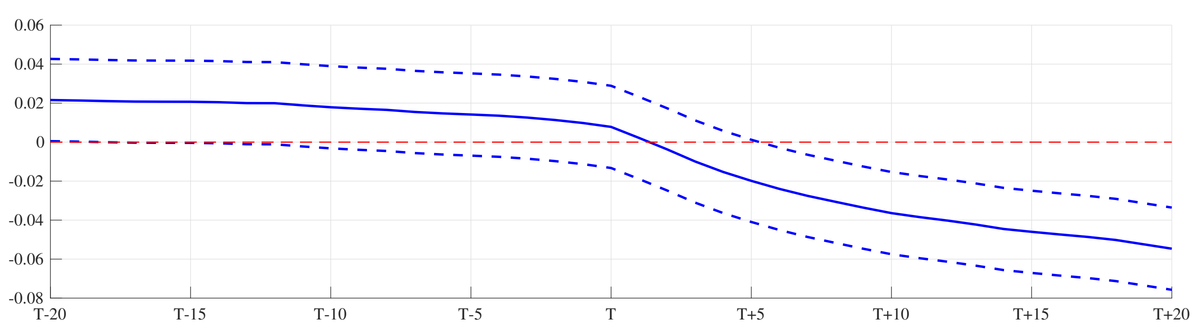
J Model Comparison with the Data

In Section 2 of the paper we produced the key empirical fact showing that, the relationship between oil reserves and sovereign risk is non-monotone over time. While in the short-run oil reserves have no significant effect on sovereign risk, in the long-run, higher oil reserves *increase* sovereign risk. In the paper we use local projections to examine the model's quantitative predictions regarding the monotonicity of the effect of oil reserves on country risk. In this Appendix we complement that analysis by first using the model-simulated data to compute the cross-correlation function of bond prices and oil reserves, and then we estimate a Vector Autoregressive (VAR) model using the model simulated data to control for debt dynamics and default.

J.1 Cross-correlation Function

In this subsection we examine the cross-correlation function of bond prices and oil reserves. Figure J1 plots the correlation between q_t and s_{t+j}, spanning the interval from $j = -20$ to $j = 20$. This plot shows that the correlation of current sovereign bond prices is more negative with future oil reserves than with current or lagged reserves. The two variables are nearly uncorrelated at the twentieth-year lag, and in fact the correlation cannot be ruled out to be zero statistically below the 10th lag. On the other hand, the correlation falls as we move into the future and converges to a statistically significant (albeit small) correlation of about -0.06 six years ahead and beyond. Thus, date-t default spreads are *positively* correlated with future oil reserves, in line with the empirical finding showing that higher future reserves increase sovereign risk in the present.

Figure J1: Cross-Correlation Function of Bond Prices and Oil Reserves in the Model



J.2 VAR

In this subsection we apply standard VAR techniques to derive the model's predictions about the dynamic relationship connecting oil reserves and country risk as the variables return to their stochastic steady states after a shock perturbs them. The goal is to complement the local projections exercise shown in Section 4.3.2 of the paper, and show that both methodological approaches render the same results.

The VAR can be viewed as an approximation to the model's dynamical system in reduced form. It has four key equations that describe the evolution of the four state variables (p_t,y_t,b_{t+1} and s_{t+1}) and a fifth equation that describes the evolution of the bond price (q_t) as a function of the state variables. The VAR is block-recursive because p_t and y_t are independent from b_{t+1} and s_{t+1}. Hence, for oil prices and non-oil GDP, we simply impose the same VAR used in the model calibration. We then use the model-simulated data to estimate a VAR for the other three equations. Using the estimated VAR, we compute the IRFs of reserves, debt, and bond prices to an oil-price shock.

The five-equation system is the following:

$$
\begin{aligned}
p_t &= a_1 p_{t-1} + a_2 y_{t-1} + c_1 + \epsilon_t^p,\\
y_t &= a_3 p_{t-1} + a_4 y_{t-1} + c_2 + \epsilon_t^y,\\
b_{t+1} &= a_5 s_t + a_6 b_t + a_7 p_t + a_8 y_t + c_3 + \Phi_1 History_t + \psi_1 Transition_t + \epsilon_t^b,\\
s_{t+1} &= a_9 s_t + a_{10} b_t + a_{11} p_t + a_{12} y_t + c_4 + \Phi_2 History_t + \psi_2 Transition_t + \epsilon_t^s,\\
q_t &= a_{13} s_t + a_{14} b_t + a_{15} p_t + a_{16} y_t + c_5 + \Phi_3 History_t + \psi_3 Transition_t + \epsilon_t^q,
\end{aligned}
$$

where b_{t+1} and s_{t+1} refer to the optimal debt and reserves decisions at time t for time $t+1$, (b_t, s_t, p_t, y_t) are the state variables at time t, and c_i are constant terms. The innovation terms, $\left(\epsilon_t^b, \epsilon_t^s, \epsilon_t^q\right)$, can be viewed as linearization errors since we are estimating a linear version of a non-linear model in which the only exogenous stochastic shocks are p_t and y_t.

We also include two dummy variables to control for the default and exclusion periods ($History_t$) and the transition towards a default ($Transition_t$). These are helpful to capture non-linearities associated with the run-ups to default events and with the defaults themselves. $History_t = 1$ when either the sovereign defaults or remains in exclusion from financial markets, and is zero otherwise. During these periods, q_t, b_t, and b_{t+1} take a value of zero in the model solution. $Transition_t$ controls for the fact that during normal times q_t fluctuates around the risk-free price but as a default approaches it falls rapidly. $Transition_t = 1$ when the bond price falls below a threshold and is typically associated with periods prior to a default event, and is zero otherwise. During these periods, q_t falls between 0.8 and 0.9 but the sovereign is still trading debt.

Table J1 shows the estimation results for the VAR and Figure J2 shows the impulse response functions of the variables to an oil-price shock. The Table shows that, contemporaneously and keeping the other state variables constant, a one-percent increase in s_t increases q_t by 0.1%. In contrast, the IRFs show that the dynamic response of both reserves and the price of bonds to an oil-price shock, taking into account all the feedback effects via the dynamics of the four state variables, results in reserves and bond prices moving together (both falling) for the first two periods, but then from $t = 3$ to $t = 20$, reserves fall as bond prices rise. After the 20th period, reserves and bond prices again move together, now both rising.

The oil-price innovation has positive persistence and takes about 50 periods to wash out. In response to the temporarily higher prices, the sovereign increases extraction and reduces reserves up to about $t = 20$, and after that reserves start to increase so as to return to their long-run average. The sovereign also responds by borrowing more on impact, because this helps finance the increase in extraction without too large a cut in consumption. After $t = 2$, however, debt starts to fall as bonds start reverting to their mean. The higher debt and lower reserves drive the initial drop in the price of bonds, but the drop is temporary as the mean-reversion of reserves and bonds then drive a recovery in bond prices.[13] Since debt starts shrinking sooner and faster than the reversal in reserves, we obtain the time interval in which reserves are still falling while bond prices are already rising (or conversely, reserves would rise while bond prices fall in the IRF to a drop in p).

All these results align exactly with the results of the local projections shown in Section 4.3.2.

[13] Note that, because of the VAR structure of the shocks, the positive oil-price shock also causes a transitory drop in non-oil GDP, which adds to the sovereign's default incentives and contributes to the drop in q.

Table J1: VAR for b_{t+1}, s_{t+1}, and q_t

	Debt (t+1)	Reserves (t+1)	Bond Price (t)
Reserves (t)	0.007***	0.983***	0.001***
	(0.000)	(0.000)	(0.0000)
Debt (t)	0.414***	-0.016***	-0.032***
	(0.004)	(0.001)	(0.0008)
Oil Price (t)	0.3799***	-0.089***	0.005***
	(0.002)	(0.001)	(0.0005)
Non-Oil GDP (t)	-0.202***	0.018***	0.024***
	(0.003)	(0.001)	(0.0008)
History	-0.119***	0.019***	-0.991***
	(0.001)	(0.001)	(0.0003)
Transition	0.027***	0.001***	-0.149***
	(0.001)	(0.000)	(0.0003)
Constant	-0.211***	0.382***	0.958***
	(0.013)	(0.005)	(0.0028)
Observations	8999	8999	8999
R-squared	0.959	0.998	0.999

Standard errors in parentheses

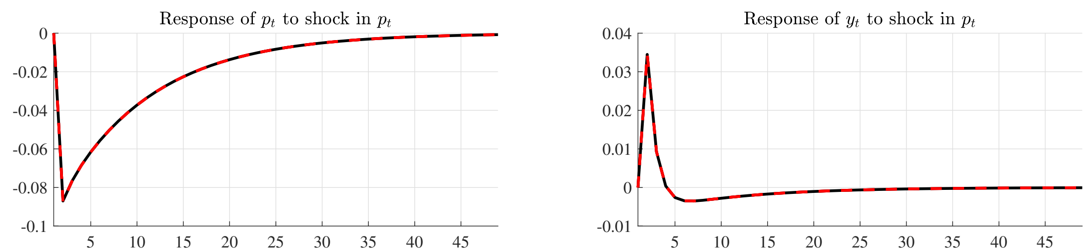
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

J.2.1 Reduced-Structural-IRFS

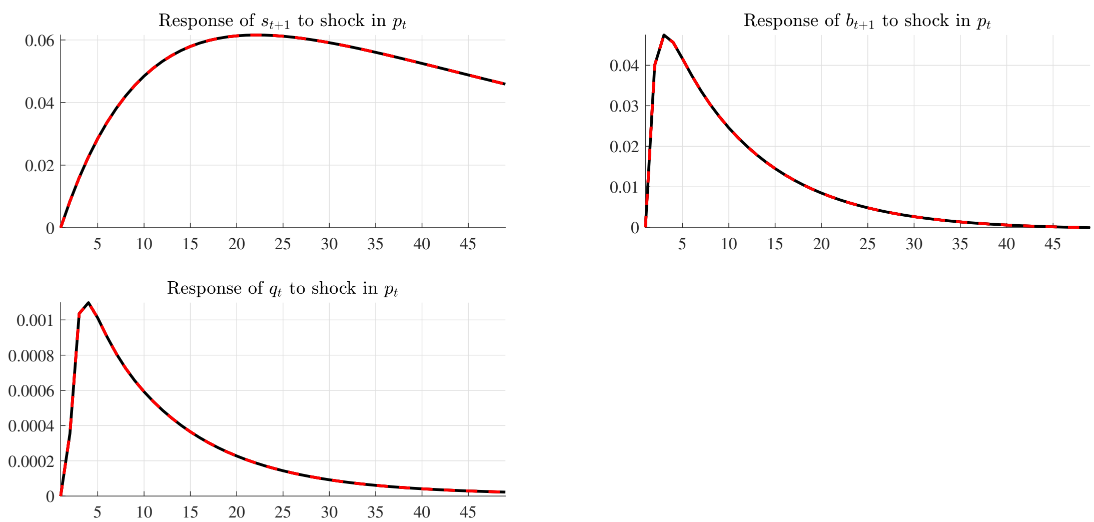
In this subsection we go deeper into the technicalities of the computation of the VAR, and we also run a specification that includes an extra dummy variable called $redemption$ using a reduced-structural VAR. As described in the previous subsection, we aim to estimate the

Figure J2: Impulse Response to an oil price innovation

a) Response of exogenous variables in the baseline model



b) Response of endogenous variables in the baseline model



structural VAR model described by,

$$
\begin{aligned}
p_t &= a_1 p_{t-1} + a_2 y_{t-1} + c_1 + \epsilon_t^p \\
y_t &= a_3 p_{t-1} + a_4 y_{t-1} + c_2 + \epsilon_t^y \\
b_{t+1} &= a_5 s_t + a_6 b_t + a_7 p_t + a_8 y_t + c_3 + \Phi_1 history_t + \psi_1 transition_t + \gamma_1 redemption_t \\
s_{t+1} &= a_9 s_t + a_{10} b_t + a_{11} p_t + a_{12} y_t + c_4 + \Phi_2 history_t + \psi_2 transition_t + \gamma_2 redemption_t,
\end{aligned}
$$

and, independently, the linearized rule for the bond's price, q_t,

$$
q_t = a_{13} s_t + a_{14} b_t + a_{15} p_t + a_{16} y_t + c_5 + \Phi_3 history_t + \psi_3 transition_t + \gamma_3 redemption_t,
$$

The treatment of the dummy variables $history_t$ and $transition_t$ is as described before. The newly introduced dummy variable called $redemption_t$ takes the value of one when the Sovereign is excluded from the financial markets but receives the signal of redemption for the next period. During these periods, the bond's price takes a positive value, but the Sovereign does not hold debt, $b_t = 0$ and starts the next period with zero debt, $b_{t+1} = 0$

$$
history_t = \begin{cases} 1 & q_t = 0, \quad b_t = 0, \quad b_{t+1} = 0 \\ 0 & otherwise, \end{cases}
$$

$$
transition_t = \begin{cases} 1 & q_t \in (0.8, 0.9), \quad b_t \leq 0, \quad b_{t+1} \leq 0 \\ 0 & otherwise, \end{cases}
$$

$$
redemption_t = \begin{cases} 1 & q_t \geq 0.9, \quad b_t = 0, \quad b_{t+1} = 0 \\ 0 & otherwise, \end{cases}
$$

As we know the VAR of the exogenous variables, oil price (p_t) and non-oil GDP (y_t) introduced in section 4.1.1 of the paper,

$$
\begin{aligned}
p_t &= a_1 p_{t-1} + a_2 y_{t-1} + c_1 + \epsilon_t^p \\
y_t &= a_3 p_{t-1} + a_4 y_{t-1} + c_2 + \epsilon_t^y,
\end{aligned}
$$

We can replace it on the structural VAR and obtain a reduced-form VAR as,

$$
\begin{aligned}
b_{t+1} &= a_5 s_t + a_6 b_t + a_7 \left(a_1 p_{t-1} + a_2 y_{t-1} + c_1 + \epsilon_t^p\right) + a_8 \left(a_3 p_{t-1} + a_4 y_{t-1} + c_2 + \epsilon_t^y\right) \\
&\quad + c_3 + \Phi_1 history_t + \psi_1 transition_t + \gamma_1 redemption_t \\
s_{t+1} &= a_9 s_t + a_{10} b_t + a_{11} \left(a_1 p_{t-1} + a_2 y_{t-1} + c_1 + \epsilon_t^p\right) + a_{12} \left(a_3 p_{t-1} + a_4 y_{t-1} + c_2 + \epsilon_t^y\right) \\
&\quad + c_4 + \Phi_2 history_t + \psi_2 transition_t + \gamma_2 redemption_t,
\end{aligned}
$$

and for the single bond's price equation,

$$
\begin{aligned}
q_t &= a_{13} s_t + a_{14} b_t + a_{15} \left(a_1 p_{t-1} + a_2 y_{t-1} + c_1 + \epsilon_t^p\right) + a_{16} \left(a_3 p_{t-1} + a_4 y_{t-1} + c_2 + \epsilon_t^y\right) \\
&\quad + c_5 + \Phi_3 history_t + \psi_3 transition_t + \gamma_3 redemption_t,
\end{aligned}
$$

grouping and simplifying similar terms, we can rewrite the reduced-form VAR as,

$$
\begin{aligned}
b_{t+1} &= a_5 s_t + a_6 b_t + A_1 p_{t-1} + B_1 y_{t-1} + C_1 + \Phi_1 history_t + \psi_1 transition_t + \gamma_1 redemption_t + \xi_t^b \\
s_{t+1} &= a_9 s_t + a_{10} b_t + A_2 p_{t-1} + B_2 y_{t-1} + C_2 + \Phi_2 history_t + \psi_2 transition_t + \gamma_2 redemption_t + \xi_t^s,
\end{aligned}
$$

and the bond's price pricing rule,

$$
q_t = a_{13} s_t + a_{14} b_t + A_3 p_{t-1} + B_3 y_{t-1} + C_3 + \Phi_3 history_t + \psi_3 transition_t + \gamma_3 redemption_t + \xi_t^q,
$$

where, $A_1 = \left(a_7 a_1 + a_8 a_3\right)$, $A_2 = (a_{11} a_1 + a_{12} a_3)$, $A_3 = (a_{15} a_1 + a_{16} a_3)$, $B_1 = \left(a_7 a_2 + a_8 a_4\right)$, $B_2 = (a_{11} a_2 + a_{12} a_4)$, $B_3 = (a_{15} a_2 + a_{16} a_4)$, $C_1 = \left(a_7 c_1 + a_8 c_2 + c_3\right)$, $C_2 = (a_{11} c_1 + a_{12} c_2 + c_4)$, $C_3 = (a_{15} c_1 + a_{16} c_2 + c_5)$, $\xi_t^b = (a_7 \epsilon_t^p + a_8 \epsilon_t^y)$, $\xi_t^s = (a_{11} \epsilon_t^p + a_{12} \epsilon_t^y)$, $\xi_t^q = (a_{15} \epsilon_t^p + a_{16} \epsilon_t^y)$.

By estimating the reduced-form VAR and the single equation for the bond's price, we obtain the results shown in Tables J2 and J3, below.

Table J2: Reduced-Form VAR for (b_{t+1}, s_{t+1})

	Debt (t+1)	Reserves (t+1)
Reserves (t)	0.007***	0.983***
	(0.000)	(0.000)
Debt (t)	0.492***	-0.014***
	(0.014)	(0.008)
Oil Price (t-1)	0.261***	-0.078***
	(0.007)	(0.005)
Non-Oil GDP (t-1)	-0.027***	0.005***
	(0.007)	(0.004)
History	-0.139***	0.025***
	(0.003)	(0.001)
Transition	-0.001***	0.008***
	(0.003)	(0.000)
Constant	-0.236***	0.378***
	(0.03)	(0.007)
Observations	8999	8999
R-squared	0.850	0.998

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table J3: Single equation for q_t

	Bond's Price (t)
Reserves (t)	0.001***
	(0.0000)
Debt (t)	-0.034***
	(0.0015)
Oil Price (t-1)	0.008***
	(0.0009)
Non-Oil GDP (t-1)	0.007***
	(0.0008)
History	-0.993***
	(0.0003)
Transition	-0.151***
	(0.0003)
Constant	0.97025***
	(0.0015)
Observations	8999
R-squared	0.999

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

To obtain the structural parameters $a_7, a_8, a_{11} a_{12}, a_{15,} a_{16}$, we use the known parameters of the exogenous variables a_1, a_2, a_3, a_4 and solve the system of equations (A_1, B_1), (A_2, B_2), (A_3, B_3).

J.3 Additional Evidence on the Elasticity of Oil Production to Oil Prices

This appendix presents additional empirical and model-based results on the elasticity of oil production with respect to oil prices.

Empirical DFE Estimates To further examine the conditional effects of oil prices on oil-related variables, we estimated an additional dynamic fixed-effects (DFE) regression, replacing the III with oil production as the dependent variable while keeping the same set of controls as in Table 2 of the paper.

It is worth noting that there is an extensive literature on the elasticity of oil production to oil prices (e.g., Bornstein et al. (2023); Caldara et al. (2019)) that finds severe identification problems resulting in biases that underestimate this elasticity. In our case, applying the same DFE approach used to identify the response of country risk proves informative for estimating the elasticity of oil production. The resulting short-run elasticity is approximately 10%, statistically significant at the 1% level (see Table 2 in the paper). This elasticity is in line with the estimate of 0.081 reported by Caldara et al. (2019) using a different cross-country panel and an identification strategy based on instrumenting oil prices with episodes of large supply disruptions. They also report that using a dynamic panel (mean-group) estimator raises the elasticity to 0.133, which is again comparable to our findings.

We explored alternative empirical strategies such as panel local projections and panel VARs, but these approaches performed poorly—consistent with what is reported in Caldara et al. (2019).

Model-Based Elasticity Estimates A natural question raised by these empirical findings is whether our model is consistent with such low elasticity estimates, given that the model generates relatively high correlations between oil prices and oil extraction. To assess this, we use model-simulated data to estimate regressions of oil extraction on oil prices analogous to those used in the empirical literature. Results are reported in Table J4.

In the baseline calibration, short-run elasticities β^S range from 0.06 to 0.09, depending on controls and lag structure. These magnitudes are very similar to the cross-country empirical estimates reported in Caldara et al. (2019), which range from 0.02 to 0.08. Long-run elasticities, defined as

$$\beta^L = \frac{\beta^S}{1-\rho^x},$$

where ρ^x is the coefficient on lagged extraction, are naturally higher and lie between 0.09 and 0.10.

Table J4: OLS estimation with model-based data: Extraction

	(I)	(II)	(III)	(IV)	(V)
Constant	0.242***	0.150***	-0.063***	-0.054***	-0.050***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
p_t	0.091***	0.061***	0.073***	0.079***	0.086***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
x_{t-1}		0.364***	0.225***	0.204***	
		(0.00)	(0.00)	(0.00)	
y_t			-0.016***	-0.018***	-0.017***
			(0.00)	(0.00)	(0.00)
b_t			-0.017***	-0.004**	-0.025***
			(0.00)	(0.00)	(0.00)
s_t			0.014***	0.014***	0.017***
			(0.00)	(0.00)	(0.00)
History				-0.016	-0.018***
				(0.00)	(0.00)
Transition				-0.002*	-0.002***
				(0.00)	(0.00)
Nobs	8999	8999	8999	8999	8999
RMSE	0.013	0.011	0.010	0.010	0.010
R-squared	0.634	0.701	0.755	0.772	0.758

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Reconciling Low Elasticities with High Correlations Why does the model produce low elasticities despite generating a high correlation between extraction and prices ($\rho_{x,p}$)? The key reason is that oil prices display substantially more variability than extraction. In a univariate regression, the relationship is:

$$\rho_{x,p} = \beta^S \frac{sd(p)}{sd(x)},$$

where $sd(p)$ and $sd(x)$ denote the standard deviations of oil prices and extraction. In the baseline model, the ratio $sd(p)/sd(x)$ is 8.75, so even a small elasticity can translate into a high correlation.

This result highlights that the baseline model's strong price–extraction correlation stems from low variability of extraction, not high elasticity. The model is calibrated to match the empirical $sd(p)$ (0.185), but it generates a much smaller $sd(x)$ than in the data, where $sd(p)/sd(x)$ is only 1.39.

This observation leads to the natural question of whether the model can jointly match the empirical correlation $\rho_{x,p}$ and the empirical variability of extraction.

Model Variants with Oil Discoveries Table J5 presents three model variants incorporating large, infrequent, and uncertain oil discoveries. These exercises were part of a broader exploration of the role of discovery shocks.

Table J5: Alternative Calibrations and Oil Price- Extraction Correlations

	Baseline (I)	(II)	(III)	(IV)
γ	3.10	4.00	3.10	4.00
ψ	32.76	36.68	32.76	36.68
κ	Deterministic	Deterministic	Stochastic	Stochastic
β	0.831	0.71	0.831	0.71
$\hat{p}$	0.645	0.604	0.645	0.604
$\rho_{p,x}$	0.79	0.67	0.22	0.35
ρ_x	0.74	0.70	0.30	0.32

We calibrated discoveries to match the frequency and magnitude of discoveries in the data, which then required re-calibrating the full model. However, if we only add the discoveries without re-calibrating the model (Col. (III)), we obtain an example in which the value of $\rho_{x,p}$ is only 0.22 (v. 0.79 in the baseline case). The autocorrelation of x drops sharply to 0.30 (v. 0.74 in baseline) and the relative variability of x rises to 3.28. In a second example, we re-calibrate the full model with discoveries (Col. (IV)). Relative to the baseline parameters, the curvature parameter of extraction costs rises from 3.1 to 4, the discount factor drops to 0.71 from 0.83 and the default penalty rises as the penalty price $\hat{p}$ drops to 0.6 from 0.65. The results yield again a low $\rho_{x,p}$ (0.35), the autocorrelation of x is again low (0.32), and the relative variability of x is high again (3.28). In addition, price elasticities of x are still low, albeit higher than without discoveries. Hence, these two examples show that the large, infrequent discoveries bring the model closer to the data by reducing $\rho_{x,p}$ and increasing $sd(x)$ (regardless of whether the model's parameters are recalibrated). In contrast, our third example shows that using all the parameters of the fully recalibrated model with discoveries but returning to constant discoveries (Col. (II)) still maintains a high correlation of x and p. For a detailed analysis of these results, see Appendix K.

K Variations on Baseline Model: Stochastic Discoveries and Recalibration

In this appendix we present the comparative results between the baseline model, and intermediate steps towards the recalibrated model with stochastic discoveries presented in Section 4.3.2. Relative to the baseline model, the recalibrated model with stochastic discoveries differs in two ways: i) It includes a stochastic process for the discoveries, and ii) following the same two-step calibration strategy as in the baseline model, it recalibrates oil technology parameters, discount factor, and oil penalty. For understanding the mechanisms of each of the two, we opt to present intermediate steps as variations of the baseline model (from now on in this appendix, Model I). In the first variation, we keep deterministic discoveries and change the parameterization (from now on in this appendix Model II). In this variation we are interested in exploring how changing the cost structure, the time preference, and default penalty affect the decisions of baseline model sovereign.

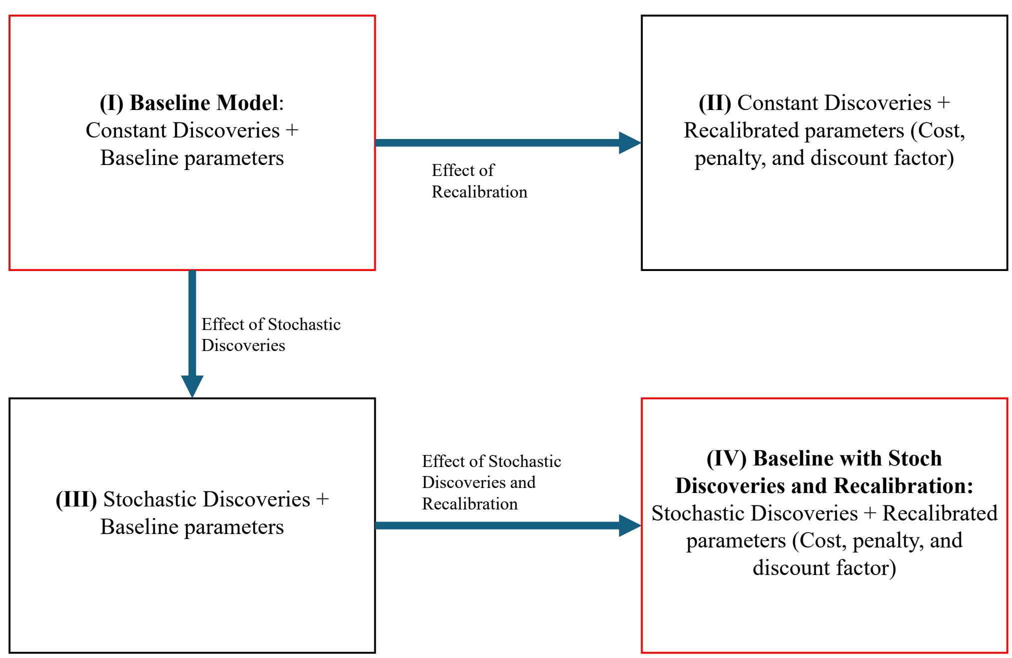
In the second variation we include stochastic discoveries but keep the baseline model parameterization (from now on in this appendix Model III). In this variation, we are interested in exploring stochastic discoveries as a counterfactual of the baseline model. Particularly, how do results change when including stochastic discoveries, without altering the cost structure or the penalty and impatience of a sovereign that may default. As we explored theoretically (See Appendix L), when analyzing an Small Open Economy with Foreign investors (which is the economy in our first step in the calibration), including stochastic discoveries does not play a significant role in the extraction decisions. When included in the baseline model, with debt and default incentives, it has a large effect on the variances of the model, but the results are qualitatively the same as in the baseline model. Finally, the recalibrated model with stochastic discoveries (from now on, Model IV), combines the novel features of Models II and III.

We use discoveries from the data on giant oil discoveries constructed by Cust et al. (2021). Giant discoveries are defined as those with Estimated Ultimate Recoverable (EUR) reserves of at least 500 million barrels, where EUR is the sum of proven reserves at a specific date plus cumulative extraction up to that date. Cust et al. (2021) report data for specific oil fields per country, per year for the 1868-2018 period. For each country in our sample, we added the discoveries of each year over the 1979-2014 sample period. See Table K6 for the summary of the parameterization changes and nature of discoveries, and see Figure K3 for a graphical illustration of the variations.

Table K6: Calibration and Simulated Moments: Variations on Baseline model

		Baseline (I)	(II)	(III)	(IV)
	γ	3.10	4.00	3.10	4.00
	$\hat{\psi} = \psi^{\frac{1}{\gamma}}$	32.76	36.68	32.76	36.68
Discoveries	κ	Deterministic	Deterministic	Stochastic	Stochastic
Discount Factor	β	0.831	0.71	0.831	0.71
Penalty	$\hat{p}$	0.645	0.604	0.645	0.604
Default Rate		1.18%	0.79%	0.79%	1.14%
Debt to GDP ratio	$-b/y$	22.4%	30.9%	15.9%	22.3%
Extraction CV relative to oil price CV.	σ_x/σ_p	0.35	0.25	3.28	3.28
Gross Oil Output CV relative to oil price CV.	σ_x/σ_p	1.36	1.27	3.75	3.97
GDP CV relative to oil price CV.	σ_x/σ_p	0.55	0.54	0.63	0.63
Corr. Price-Extraction	$\rho_{p,x}$	0.79	0.67	0.22	0.35
Autocorr. Extraction	ρ_x	0.74	0.70	0.30	0.32
Corr. Trade Balance - Disp. Income	$\rho_{tb,di}$	0.03	-0.06	0.22	0.03

Figure K3: Graphical illustration of Appendix



The results we present in this appendix encompass the main battery of results of the paper. Namely, long-run moments of the economy, default sets, Impulse-Response Functions from Local Projection models, default costs disentanglement, and event-windows analysis.

In Table K6, we present the main effects of the variations on the simulated long-run moments considered in this appendix.[14] With respect to the long-run moments, relative to baseline model, Model II major changes are such that extraction variability relative to oil price decreases from 0.35 to 0.25, as the new parameterization imply higher per unit cost of extraction ($\gamma = 4, \hat{\psi} = 36.68$), which in turns decreases Gross Oil Output volatility (1.36 to 1.27) and GDP volatility (0.55 to 0.54) relative to oil price. In addition, it weakens the correlation between extraction and oil price (0.79 to 0.67) and with respect to disposable income (0.57 to 0.50). This change generates a slightly countercyclical trade balance (-0.06). Referring to default rates, changing the parameterization results in a lower default rate (1.18% to 0.79%). The mechanism behind this result involves three forces with opposing effects. On the one side increasing the per-unit cost of extraction holding constant discoveries, and keeping discount factor and default penalty as in baseline (not reported), decreases default rate from 1.18% to 0.99%. This effect is due to an endogenous lower volatility in extraction, and ultimately in disposable income, which reduces the incentives of the sovereign for defaulting through a lower uncertainty in the fluctuations of her income. The remaining drop in the default rate (0.99% to 0.79%) has two opposing forces. On the one hand, a lower discount factor implies a more impatient sovereign which, as it is well studied in sovereign default literature, is consistent with a sovereign that can hold less debt and faces higher default rates. On the other hand, the penalty for defaulting is higher (lower price cap $\hat{p}$), which reduces the incentives of the sovereign to default as it becomes more costly in all states of the economy. Together, they account for the default rate of 0.79%, and allow the government to hold more debt (from 22.5% to 30.9%).

On the second dimension of variations, relative to baseline model, Model III includes stochastic reserves keeping the parameterization of the baseline model unchanged. Stochastic discoveries are calibrated following the procedure described in Section 4.1 of the paper. As we mentioned before, the major changes are evidenced in the volatility of variables. Specifically, extraction volatility relative to oil price increases by a factor of 10, from (0.35 to 3.28). Which implies that even if the parameters are calibrated to an SOE with Foreign Investors Economy with constant discoveries, the interaction with debt, and default incen-

[14] In Table K8 we present a more detailed summary of the results.

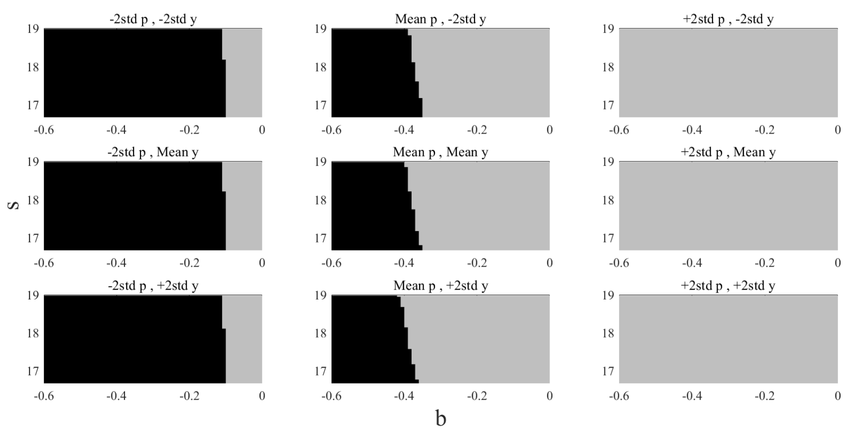
tives increases extraction volatility considerably. It is worth noting, however, that Gross Oil Output coefficient of variation increases by 2.75 (from 1.36 to 3.75), and GDP by 1.14 (from 0.55 to 0.63, relative to oil price). This higher volatility considerably weakens the positive contemporaneous correlation between oil price and and extraction (from 0.79 in baseline, to 0.22 in Model III), and the autocorrelation of extraction (from 0.74 to 0.30). Finally, this variation makes trade balance to GDP more procyclical (0.028 to 0.22) and decreases the autocorrelation of reserves (0.998 to 0.937). This variation also decreases default rate, from (1.18% to 0.98%), although for different reasons than in Model II. Particularly, when including stochastic reserves, a new source of exogenous variation is included in the model, and increasing volatility might increase the likelihood of the sovereign to default if bad states are prevalent. However, on the other side, this higher variance pushes for a precautionary asset holding motive, making the sovereign want less debt and, therefore, reducing the default frequency and holding less debt (22.5% to 15.9%).

Finally, when combining the two dimensions, recalibration and stochastic discoveries, we can analyze Model IV. As described in the paper, model IV follows the same calibration as in the baseline model. That is, a two-step calibration with stochastic discoveries in which the first step matches the extraction volatility, years of reserves, and share of Gross Oil Output in total GDP of the sample of non-defaulters. Then, using the technology of the extraction as given by this first step, we calibrate the discount factor and penalty cost to match the debt-to-GDP ratio, and default rate. By construction of this exercise, relative to baseline, Model IV also matches default rate and debt to GDP. However, extraction volatility is considerably higher (3.28 vs. 0.35 in baseline). This is particularly interesting, because in the first step of calibration, both models match extraction volatility, but when stochastic discoveries interact with debt/saving and default decisions, extraction volatility is considerably higher. This, however increases GDP volatility to 0.63 relative to oil price (vs. 0.55 in the baseline model). Reserve years also become considerably higher (from 53 in baseline to 83 in Model IV, consistent with the precautionary motive we argued in Model II). As in Model III, the two variations together lower correlation of extraction and oil price (0.79 to 0.35), and makes trade balance less procyclical (0.03). Each of the two variations separately accounted for a lower default rate, and for different directions of the debt-to-GDP ratio. However, when interacting together, default rate and debt are the same as in baseline model. This effect is mostly explained by a higher impatience (lower discount factor, β) together with a higher volatility of income due the stochastic discoveries, which offset the effects of a higher penalty costs (lower penalty, $\hat{p}$) and the precautionary savings induced by the higher volatility included by the stochastic discoveries.

The main effects on the default sets are summarized in Figure K4. Relative to baseline model, Model II keeps the same properties of the default sets, shrinking in oil price, assets and reserves. As explained before, a lower default rate and higher debt-to-GDP holdings, are consistent with a shrinkage in the default set relative to baseline. Models with stochastic discoveries (Models III and IV) do not hold all the properties as in baseline. Particularly, default sets shrink in oil prices and debt, but there are non-monotonicities in reserves and non-oil GDP. Additionally, default sets shrink in the state of positive extra discoveries.

Figure K4: Default Sets

a) Baseline Default Sets



b) Model II Default Sets

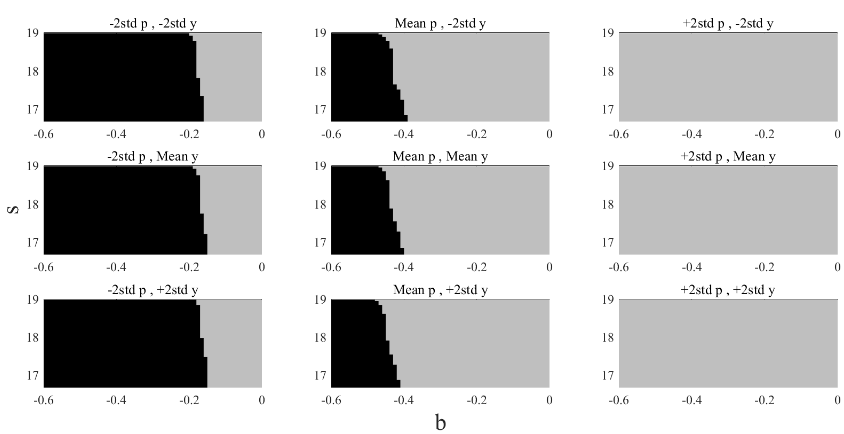
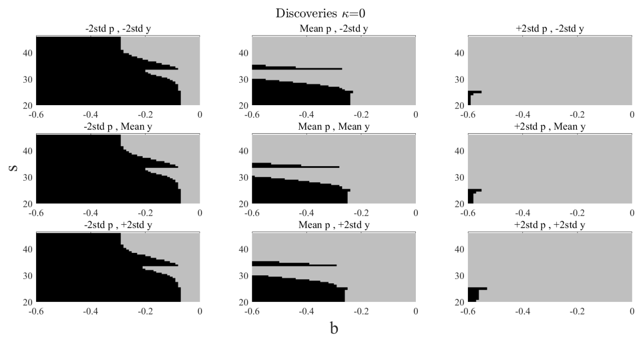
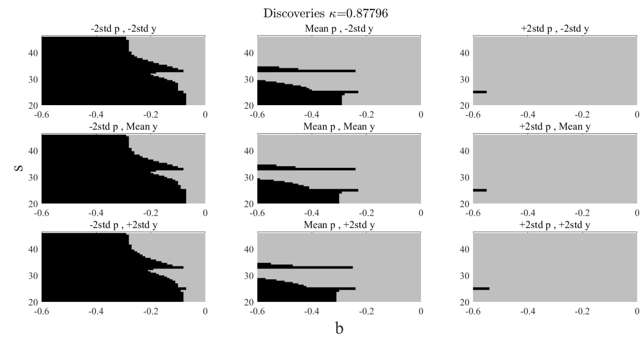


Figure L6: Default Sets (cont'd)

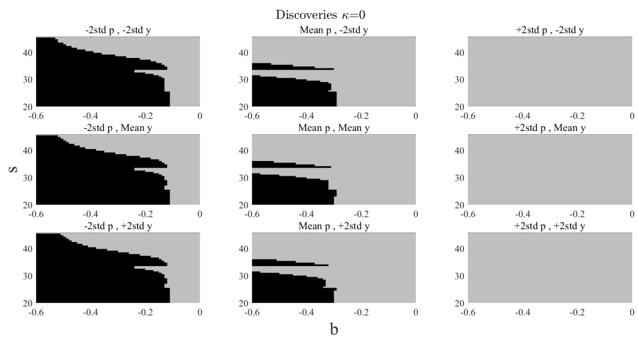
c) Model III Default Sets: Low Discoveries



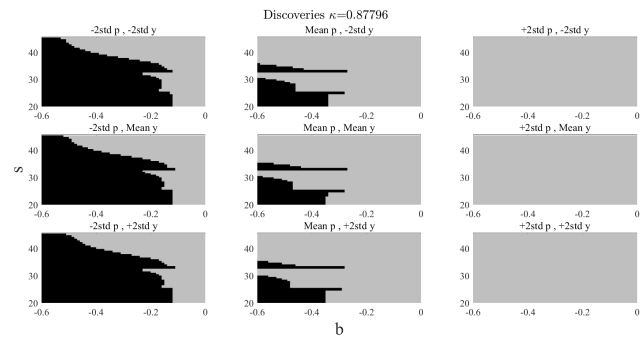
d) Model III Default Sets: High Discoveries



e) Model IV Default Sets: Low Discoveries



f) Model IV Default Sets: High Discoveries

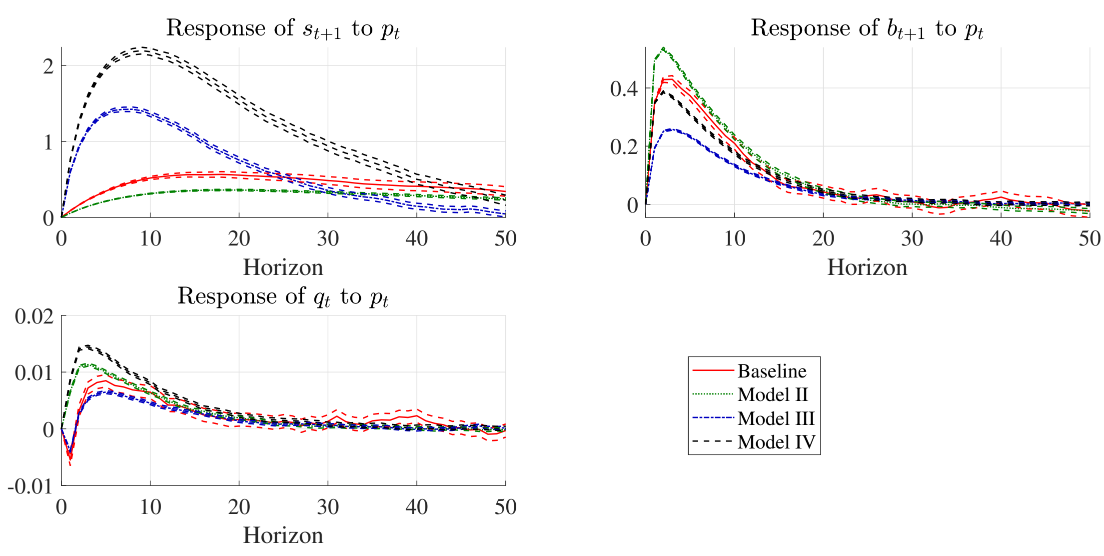


As part of the main results presented in the paper, we document the results on IRFs based on local projections on the variations over the baseline model (See Figure K5). As in the paper, we focus on IRFs on oil price. The local projections specifications are as in baseline, for the IRF of oil prices, we estimate,

$$x_{t+h} = \beta_0 + \beta_1 s_t + \beta_2 b_t + \beta_3^h p_t + \beta_4 y_t + \beta_5 history_{t+h} + \beta_6 transition_{t+h} + \epsilon_{t+h},$$

for $x_t = \{b_{t+1}, s_{t+1}, q_t\}$, where the IRF is built fromβ_3^h in $h = 0, ..., T$, with $T = 50$. With respect to assets and reserves (b_{t+1}, and s_{t+1}), the results are qualitatively the same between the four models. Particularly, after a negative fluctuation in oil prices at period t, both assets and reserves increase above the pre-shock scenario. Quantitatively, the greatest effects on reserves occur in models with stochastic discoveries, Models III and IV (1.41, and 2.22 at the highest point), while the highest positive effect occurs in Model II (0.39 at lowest). In terms of timing, models with stochastic discoveries reach the highest response in around 6-8 periods (consistent with the dynamic structure observed in unconditional cross-correlations in data for extraction), after 8 periods, it starts converging to the pre-shock scenario, which is reached in about 40-50 periods. Models with deterministic discoveries take longer to reach the highest point of the IRFs (about 20-25 periods), and their effect takes longer to dissipate. With respect to debt, the response is qualitatively the same in terms of direction and timing among the four models, all imply a lower debt (higher assets) that reach a maximum after 2-3 periods and then converge after 30 periods to pre-shock scenario. The greatest effect here occurs in Model II, which suggests that this sovereign adjusts via less debt and more reserves. Finally, with respect to bond prices, the effects are different between calibrations. In models with baseline calibration (Baseline and Model III), the effect of oil prices is having a higher country risk (lower bond prices) in impact, while for models with the recalibrated parameters (Models II and IV), the effect is positive in impact, and stronger with stochastic discoveries. These results suggest that having higher costs, penalty and impatience, result in an enhancing of the country risk in impact. It is worth noting, however, that after the different dynamics in impact, all four models converge from above. These dynamics are consistent with our predictions of the baseline model, in which country risk starts recovering, while reserves are being depleted (periods 5-23 in baseline, periods 3-8 in Model IV).

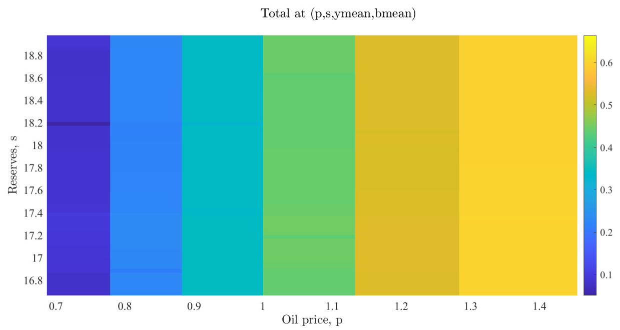
Figure K5: IRFs based on Local Projections: Oil Prices



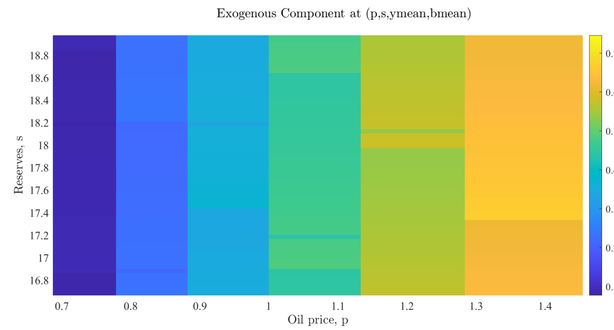
The next set of results refer to the default costs. As in Section 4.2.2 in the paper, we present the total default costs and disentangle their mechanisms through its endogenous and exogenous mechanisms. As opposed to the results presented in the paper, in this appendix we illustrate the total costs and its components by analyzing both dimensions oil price and oil reserves. Although in models with deterministic discoveries the results are akin to those of the paper, models with stochastic discoveries will have interesting features depending on the oil discovery regime of the economy. Starting with baseline model, the results are straightforward as in the paper. Increasing oil prices results in an increasing total cost of default. The cost ranges between 0.08 and 0.66. The effect when varying the the reserves is small. At the mean oil prices the difference between the total cost at the highest and the lowest level of reserves is 0.005. The results are mainly guided by the exogenous component, which ranges between 0.08 and 0.71. One of the most relevant results in the paper are the endogenous gains of default captured by the endogenous component. As we showed in the paper, increasing oil prices increases default gains through endogenous channels. Ranging from 0 to -0.04 when evaluated at mean reserves. Interestingly, the gains are larger at higher levels of reserves when oil prices are at or below the mean, while are larger at lower levels of reserves at prices above the mean.

Figure K6: Default Costs in Baseline Model

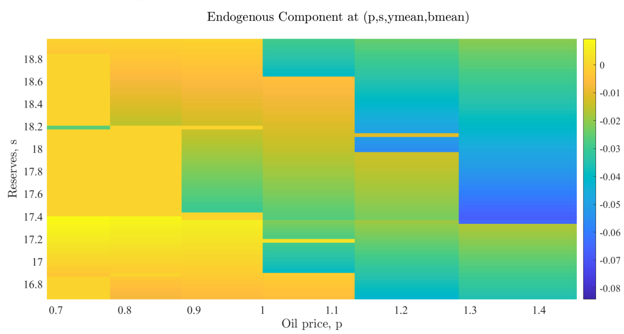
a) Total



b) Exogenous Component

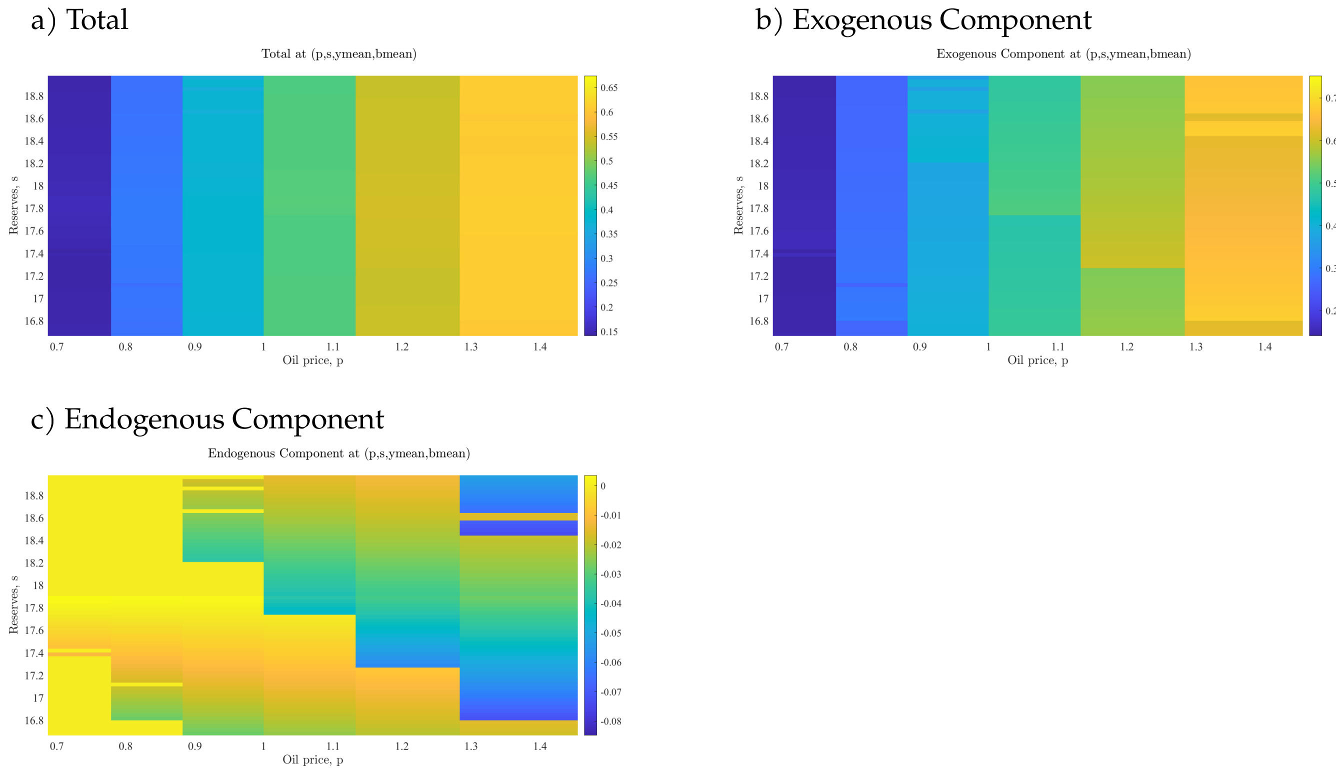


c) Endogenous Component



Results are qualitatively the same for Model II but change quantitatively. That is, increasing total costs due the exogenous component, ranging from 0.14 to 0.66, and endogenous default gains component that reaches its maximum at 0.06 for low reserves and high prices.

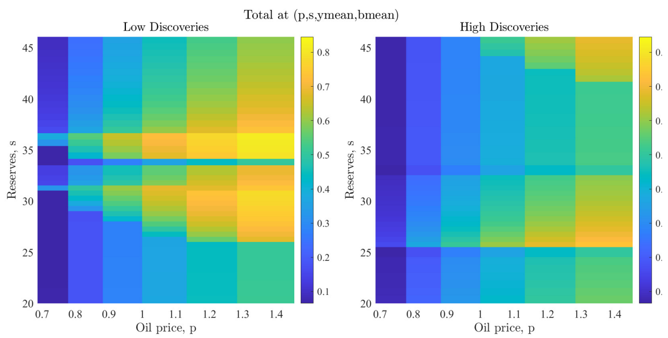
Figure K7: Default Costs in Model II



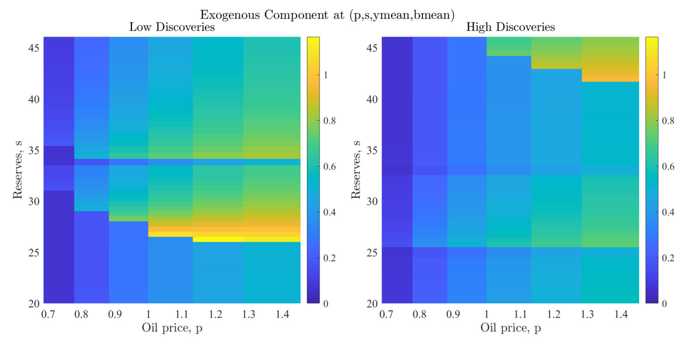
Models with stochastic discoveries feature a different default cost structure on both the oil price and oil reserves space, and the discoveries regime. In Model III total costs range between 0.1 and 0.83, and are increasing in oil prices. However, the results are non-monotone in the levels of reserves, depending on the discoveries regime. For low discoveries, at mean oil prices, lowest reserves ($s = 20$) feature a lower total cost (0.37) than highest reserves ($s = 46$) of 0.44, however, medium reserves ($s = 34$) feature a higher total cost of 0.67. The maximum cost (0.83) is at medium reserves $s = 35$, and highest oil prices. For high discoveries the total costs are still increasing in oil prices, but the non-monotonicity of reserves is less markedly as in the low discoveries. That is, for low oil prices, costs do not vary much between low and high reserves, but at high oil prices, highest reserves, and medium low reserves ($s = 20$)feature the highest default costs. This non-monotonicity is mainly guided by the exogenous component, however costs now range between 0.2 and 1.09. The endogenous component, also displays an overall default gain as in baseline. However, at low oil prices, low discoveries, and medium reserves, there are positive default costs (negative gains) due the endogenous component. At higher prices and lower reserves, those gains become evident. The high discoveries regime features zero default costs or gains in almost the whole (p, s) space, but in high reserves and high prices, which reaches default gains of 0.33. As shown in Appendix L for a Small Open Economy with Foreign Investors, having high discoveries switches the dynamics of reserves decisions to higher realizations of the state.

Figure K8: Default Costs in Model III

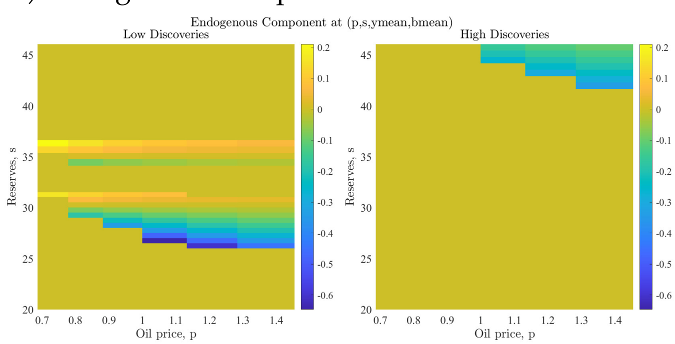
a) Total



b) Exogenous Component



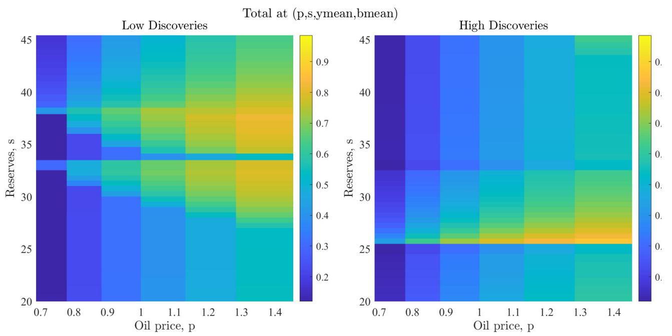
c) Endogenous Component



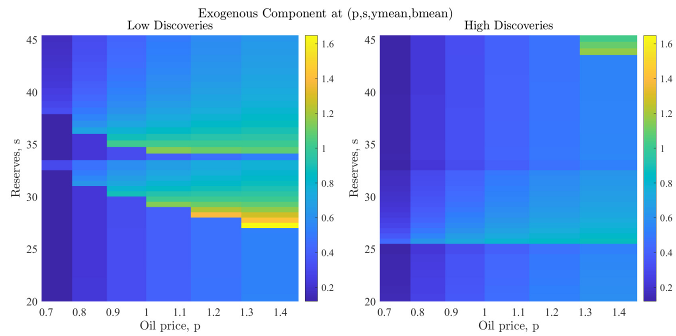
The results of Model IV are qualitatively the same as in Model III, but quantitatively differ a little. Particularly, the total costs range between 0.12 and 0.9, while exogenous and endogenous range between 0.2 and 1.45, and -0.75 and 0.09, respectively.

Figure K9: Default Costs in Model IV

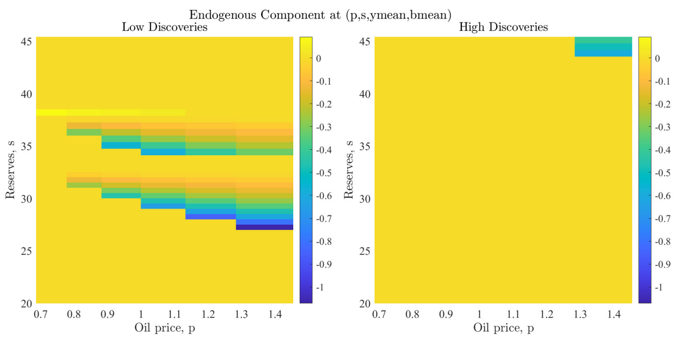
a) Total



b) Exogenous Component



c) Endogenous Component



Lastly, we present the results related to the dynamics of the variables on event windows around the default episodes. In the paper, we underline the differences of an economy with default risk and the possibility to manage reserves, with either an economy without default risk, or an economy with constant extraction. In this section of the appendix we present the levels of the dynamics for each of the four models. Naturally, since the realizations of the exogenous processes (oil prices, non-oil GDP, and oil discoveries) will differ across models. It is interesting to note that previous the event date of a default, all models show a lower than the mean Non-Oil GDP and a oil price that falls to the penalty price. At the moment in which default is decided, non-oil GDP is relatively larger to the pre-default period episode. The first difference between models with stochastic (Models III, and IV) and deterministic discoveries (Models I, and II), is evident in the extraction dynamics. While in all models sovereign adjust extraction to a lower level facing the drop in oil prices, the adjustment is considerably higher (-35%) than in the models with deterministic reserves (-10%). It is worth noting that default episodes in stochastic discoveries model occur when there has been a long-spell of low discoveries. The differences between the models remain in the gross oil output dynamics, however, with respect to Oil GDP, the adjustment of extraction, with a convex cost function implies less lower differences between the four models. The adjustment of GDP and consumption then follow closely the dynamics of the components, falling by 15% and 20% relative to the mean in an episode of default. In terms of debt to GDP it is worth noting that both baseline model and Model IV display a quantitatively very similar evolution, while, as introduced before, Model II holds more debt, but Model III holds less debt. The effects are thus evident on the interest rate, as baseline economy displays the highest interest rate at the moment previous to the default (11.4%), followed by Model IV (9.7%).

Figure K10: Event Windows around a default episode

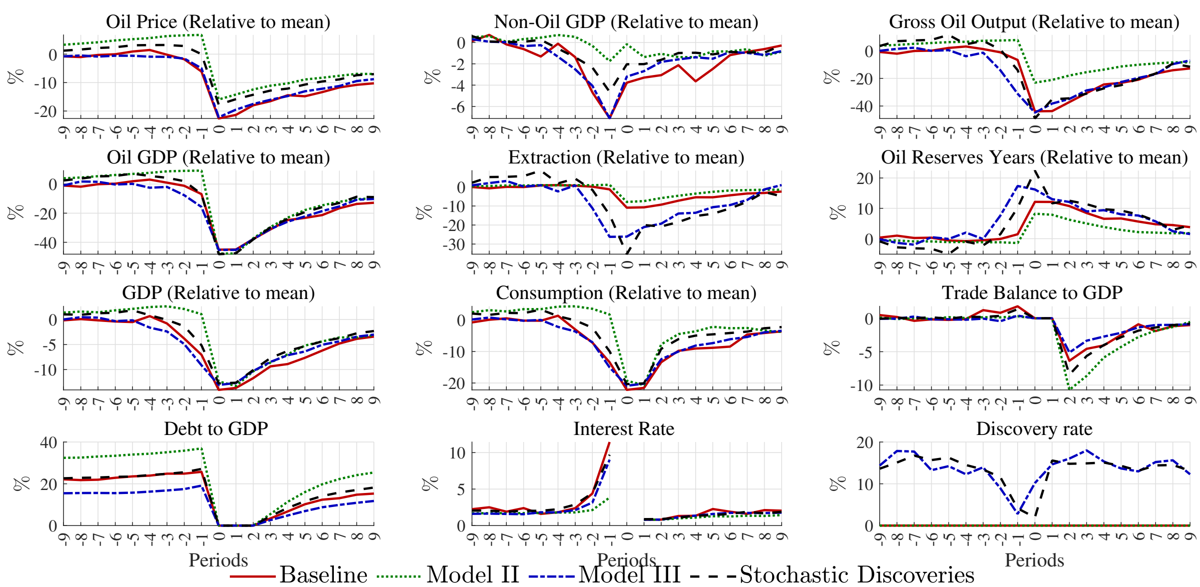


Table K7: Data and Model Moments

Description	Data	Model			
		Baseline BSL	Model II	Model III	Model IV SD
Average External Debt to GDP	0.225	0.225	0.309	0.159	0.223
Default Rate	1.14%	1.19%	0.79%	0.79%	1.13%
Variability of Gross Oil Output1	23.99%	24.73%	23.18%	68.8%	72.8%
Average Reserves (in years)2	62	53	55	78	83

The variability of gross oil output is the standard deviation of the cyclical component of px in the data and the corresponding coefficient of variation in the model.

Oil reserves data are proven reserves from the US Energy Information Administration.

Table K8: Business Cycle moments - Data v. Model Variations

	Variability relative to Oil Price					Correlation with DI					Correlation with Oil Price					Autocorrelation				
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)
	Data	BSL	(II)	(III)	(IV)	Data	BSL	(II)	(III)	(IV)	Data	BSL	(II)	(III)	(IV)	Data	BSL	(II)	(III)	(IV)
Gross Oil Output	1.32	1.36	1.27	3.75	3.97	0.51	0.72	0.71	0.63	0.61	0.34	0.96	0.95	0.45	0.57	0.29	0.86	0.86	0.41	0.45
Total GDP	0.38	0.55	0.54	0.63	0.63	0.62	1.00	1.00	1.00	1.00	0.11	0.71	0.70	0.59	0.58	0.52	0.66	0.65	0.54	0.50
Disposable Income (DI)	0.56	0.87	0.85	1.02	1.01	1.00	1.00	1.00	1.00	1.00	0.12	0.72	0.70	0.59	0.58	0.42	0.66	0.65	0.54	0.50
Extraction	0.67	0.35	0.25	3.28	3.28	0.52	0.58	0.50	0.50	0.52	0.04	0.80	0.67	0.22	0.35	0.50	0.74	0.70	0.30	0.32
Consumption	0.27	0.95	1.01	0.98	1.07	0.34	0.91	0.87	0.93	0.92	0.12	0.74	0.67	0.66	0.60	0.52	0.60	0.45	0.62	0.49
Trade Balance/GDP	0.49	0.25	0.31	0.22	0.26	0.39	0.03	−0.06	0.22	0.03	0.19	−0.18	−0.15	−0.15	−0.15	0.63	0.11	0.05	−0.00	−0.00
Reserves	1.78	0.12	0.09	0.16	0.22	0.04	−0.20	−0.16	−0.15	−0.17	0.13	−0.31	−0.28	−0.48	−0.49	0.83	0.99	0.99	0.93	0.97
Spread	3.53	15.21	10.29	16.15	13.67	−0.09	−0.29	−0.00	−0.28	−0.20	−0.46	−0.15	0.15	−0.14	−0.04	0.66	0.29	0.22	0.27	0.33

* Actual data are for the 1979-2014 period, logged and HP-detrended, except for the TB/GDP, Debt/GDP ratios and the EMBI, which is in levels (basis points). BSL- Baseline, Model (II)- , Model (III), Model (IV). Data for the four models are not detrended because the models are stationary by construction. Variability ratios for the three models are ratios of coefficients of variation divided by the standard deviation of oil prices.

L Small Open Economy with Foreign Investors with and without Stochastic Discoveries

In Section 4.4.2 of the paper we argue that introducing rare, large discoveries in the SOE-FI model makes little difference. In this appendix we derive analytic solutions that show that this feature is caused by the homogeneity of the unitary extraction costs, which makes the marginal cost of extraction with respect to oil production and reserves and the Euler equation depend on x_t/s_t but not on κ_t.

Our oil extraction cost function is given by,

$$e\left(x_t, s_t\right) = \phi\left(\frac{x_t}{s_t}\right)^{\gamma} x_t,$$

implying that marginal cost functions are given by

$$e_x\left(x_t, s_t\right) = \phi\left(1+\gamma\right)\left(\frac{x_t}{s_t}\right)^{\gamma},$$

and

$$e_s\left(x_t, s_t\right) = -\phi\left(\gamma\right)\left(\frac{x_t}{s_t}\right)^{1+\gamma}.$$

If we assume an internal solution for extraction (x_t), then the system of equations consists of the Euler equation,

$$\frac{p_{t+1} - e_x\left(x_{t+1}, s_{t+1}\right) - e_s\left(x_{t+1}, s_{t+1}\right)}{p_t - e_x\left(x_t, s_t\right)} = \frac{1}{\beta},$$

and the law of motion for reserves,

$$s_{t+1} = s_t + k_t - x_t,$$

where $p_t, s_t, k_t, p_{t+1}, p_t, \beta, \phi, and \gamma$ are known.

Lets define $\theta_t = \frac{x_t}{s_t}$, such that we can re-write the marginal cost functions as

$$e_x\left(\theta_t\right) = \phi\left(1+\gamma\right)\left(\theta_t\right)^{\gamma}$$

and

$$e_s\left(\theta_t\right) = -\phi\left(\gamma\right)\left(\theta_t\right)^{1+\gamma},$$

and the Euler equation as

$$\frac{p_{t+1} - e_x(\theta_{t+1}) - e_s(\theta_{t+1})}{p_t - e_x(\theta_t)} = R,$$

where $R = \frac{1}{\beta}$. Note that written in this form, the Euler equation is is a differential equation of θ_t as a function of θ_{t+1}:

$$p_{t+1} - e_x(\theta_{t+1}) - e_s(\theta_{t+1}) = R(p_t - e_x(\theta_t)).$$

Furthermore we can substitute in functional forms,

$$p_{t+1} - \phi(1+\gamma)(\theta_{t+1})^{\gamma} + \phi(\gamma)(\theta_{t+1})^{1+\gamma} = R(p_t - \phi(1+\gamma)(\theta_t)^{\gamma}),$$

and bring back uncertainty (up to now we had assumed perfect foresight), such that

$$(\theta_t)^{\gamma} = \frac{1}{R}\left\{AB_t + E\left[(\theta_{t+1})^{\gamma}\right] - CE\left[(\theta_{t+1})^{1+\gamma}\right]\right\},$$

where $A = \frac{1}{\phi(1+\gamma)}$, $B_t = Rp_t - E[p_{t+1}]$, and $C = \frac{\gamma}{(1+\gamma)}$.

This equation can be iterated forward to obtain an expression of the form

$$(\theta_t)^{\gamma} = f\left(\{B_{t+s}\}_{s=0}^{\infty}, E[\theta_{\infty}]\right),$$

where $\{B_{t+s}\}$ is a sequence of future prices, $E[\theta_{\infty}] = n$, and n is the inverse of the reserve years in steady state, which depends only on exogenous realizations of p_t.

Substituting back $\theta = \frac{x}{s}$ in the equation above, we can obtain the policy for extraction as a share of reserves

$$x_t = \left(f\left(\{B_{t+s}\}_{s=0}^{\infty}, n\right)\right)^{\frac{1}{\gamma}} s_t,$$

and then we can substitute this last equation in the reserves accumulation constraint to obtain an expression for reserves in $t+1$

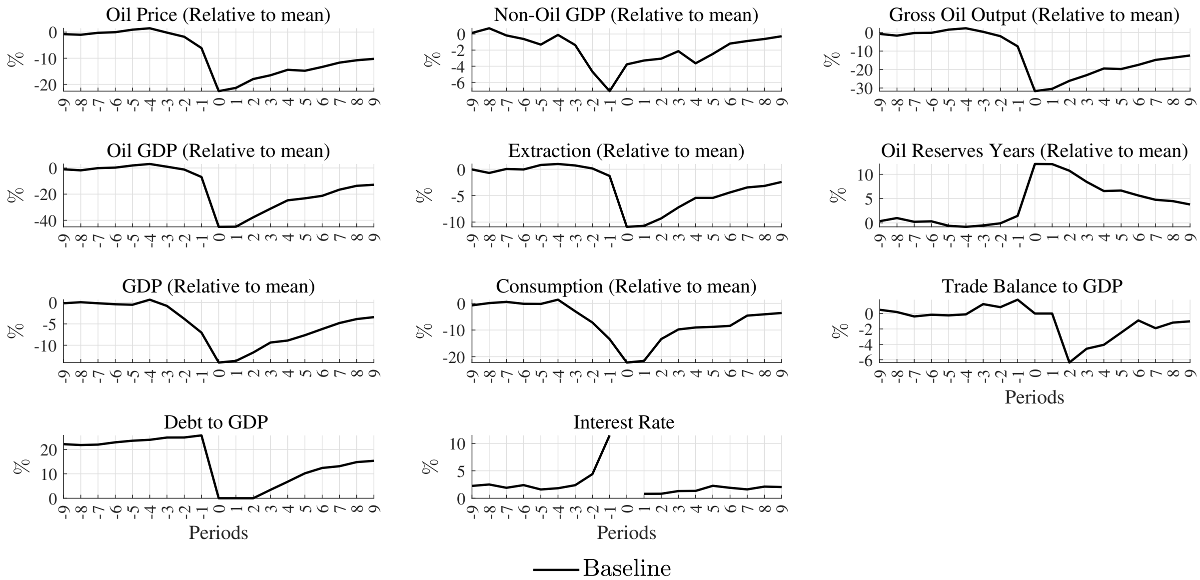
$$s_{t+1} = \left(1 - \left(f\left(\{B_{t+s}\}_{s=0}^{\infty}, n\right)\right)^{\frac{1}{\gamma}}\right) s_t + k_t.$$

Note that the policy function for extraction x_t doesn't depend on discoveries at all. This implies that extraction will be the same in the SOE-FI model with and without stochastic discoveries, and the expression for reserves tomorrow, does depend on discoveries but it does so in a linear way. They move one to one, meaning that discoveries only have a level effect on reserves tomorrow, maintaining all the intuition behind the mechanisms in the SOE-FI model without stochastic discoveries.

M Event Analysis: Levels

In Section 4.4 of the paper, we underline the differences of an economy with default risk and the possibility to manage reserves, with either an economy without default risk, or an economy with constant extraction. In this appendix we present the levels of the endogenous variables around an default episode. As shown in the paper, one period before a default episode, oil prices are below the mean by 4.27 p.p., while Non-Oil GDP is on average 7.2 pp below the level of 9 years before default. While both have a negative trend, the one in Non-Oil GDP is more markedly such that all periods before a default, it is below the mean. At the moment in which sovereign chooses default optimally, oil price is capped by the penalty $(\hat{p})$, and remains excluded on average 3 periods. At the moment of default, Non-Oil GDP increases, but it is still below the 9th period before default (-3.89%) and continues growing until reaching the same level in about 10 periods after the default. With these dynamics on the exogenous processes, the optimal decision of the sovereign is to reduce its extraction by 10pp relative to the long-run mean $(E\left[x\right] = 0.335)$, this drop in extraction together with the drop in oil prices, result in a straightforward contraction of Gross-Oil Output that reaches a minimum at the default date of 43pp below the long-run mean. As a result, Oil GDP drops between four periods before the default and the default date by 48pp (from 3pp above the mean to 45pp below the mean) and recovers as extraction increases, and after 9 periods of the default episode it is still 13pp below the long-run mean. The relatively low Non-Oil GDP level together with the drop in Oil GDP result in a lower GDP level of about 14pp below the mean. In this baseline model, the sovereign defaults on its debt when its level before the default is above the long-run mean (25.73%), but the interest rate reaches levels of around 11.45%. As a response, reducing extraction, and not being able to issue debt, the sovereign increases its years of reserves in about 6 years (from 54 years before default to 60 during default) and remains above 54 years in the next 9 periods after the default. With the default consumption cannot adjust smoothly and it displays a drop relative to the long-run mean which reaches its minimum at the default period of 22pp below the mean. Lastly, trade-balance to GDP jumps from a surplus of 1.8% to a deficit of -6.4% in 3 years, then it slowly recovers, still being negative after 9 periods post-default.

Figure M11: Event Windows around a default episode: Baseline Model



N Refining the Grids

In this Appendix we show the robustness of the results to a refinement in the grids. Particularly, we are interested in analyzing the impact of considering a finer grid on oil prices, or a finer grid on reserves. For the first we consider an increase in the number of nodes in the Markov discretization of oil price from $n^p = 7$ in the baseline model, to a $n^p = 11$ nodes. For the second exercise, due computational reasons related to the curse of dimensionality, we cannot increase the number of nodes in reserves considerably without reducing the number of nodes in other state variables. To illustrate the effects we thus proceed in two steps: 1) We set non-oil GDP to its long-run value, which implies that the only source of exogenous fluctuations are the oil prices, which we assume as coarse as in baseline, that is, $n^p = 7, n^y = 1$. 2) After understanding the implications of shutting down the fluctuations in non-oil GDP, we increase the number of nodes in reserves from $n^s = 70$ to $n^s = 200$. In this section we present results on the long-run moments, default sets, and default costs.

Table N9: Calibration and Simulated Moments: Variations on grid Refinements

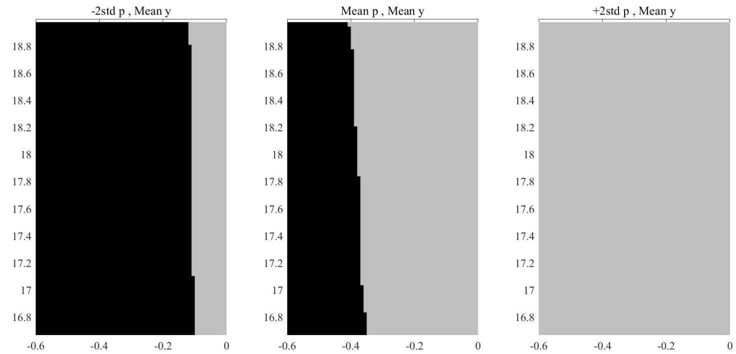
		Baseline (I)	Fine P	No Y	No Y, Fine S
Nodes for oil price	n^p	7	11	7	7
Nodes for non-oil GDP	n^y	5	5	1	1
Nodes for reserves	n^s	70	70	70	200
Default Rate		1.18%	1.47%	0.78%	1.01%
Debt to GDP ratio	$-b/y$	22.4%	21.7%	23.7%	23.8%
Extraction CV relative to oil price std.	σ_x/σ_p	0.35	0.36	0.33	0.31
Gross Oil Output CV relative to oil price std.	σ_x/σ_p	1.36	1.40	1.33	1.34
GDP CV relative to oil price std.	σ_x/σ_p	0.55	0.57	0.34	0.34
Corr. Price-Extraction	$\rho_{p,x}$	0.79	0.77	0.78	0.87
Autocorr. Extraction	ρ_x	0.74	0.72	0.75	0.86
Corr. Trade Balance - Disp. Income	$\rho_{tb,di}$	0.03	0.06	-0.20	-0.20

In Table N9, we present the main effects on the simulated long-run moments considered in this section. The effects of considering a finer grid on oil prices are small. The main mechanisms of the model remain unchanged, and quantitatively the results are not that different from those in the baseline model. With respect to the long-run moments, relative to baseline model, using a finer grid on oil prices has little impact on the long-run moments of this economy related to volatility and cross correlation. Particularly, relative to oil price, extraction, Gross Oil Output, and GDP volatility and correlation with respect to oil price change little, while trade balance becomes more procyclical (0.06). The most notable change, however occurs in the targeted moments, as this finer oil price grid results in a slightly higher default rate (1.4%) in an economy in which the sovereign can hold less debt (21.7%) relative to the baseline economy. When mechanically shutting down the fluctuations in non-oil GDP, the major changes are that GDP has a lower volatility relative to oil price (from 0.52 in baseline to 0.32 in the model without fluctuations in y), which lowers default incentives to sovereign resulting in a drop in default rates (from 1.18% to 0.78%), and allowing the sovereign to hold more debt than in baseline (23.7%). Finally, considering an economy with no fluctuations due non-oil GDP but finer grid for reserves, results in a similar debt to GDP ratio as in the one with coarser grid, with a higher default rate (1.01%) but still lower than in baseline economy.

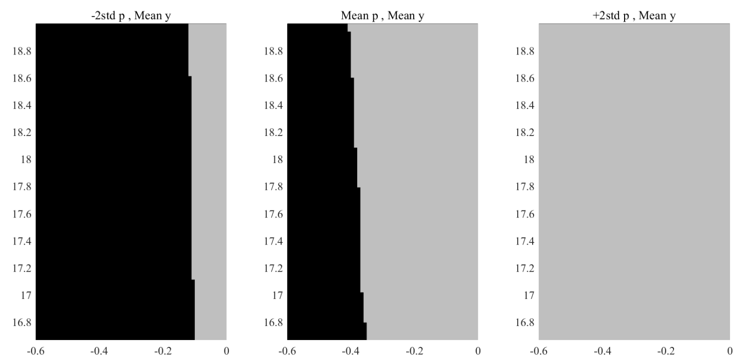
Default sets and event windows remain broadly unchanged. Relative to baseline model, the default sets of the variations in this section have the same properties of the default sets, shrinking in oil price, assets and reserves, while the event windows feature the same qualitative and broadly unchanged quantitativeproperties.

Figure N12: Default sets: Grids Refinement

a) No Y



b) No Y, Fine S



c) Fine P

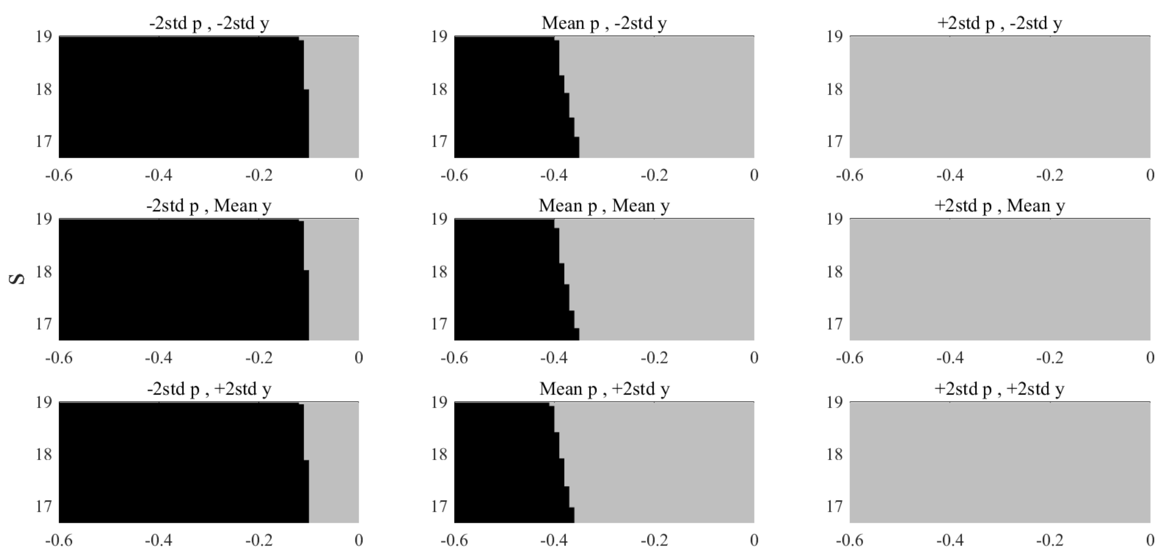
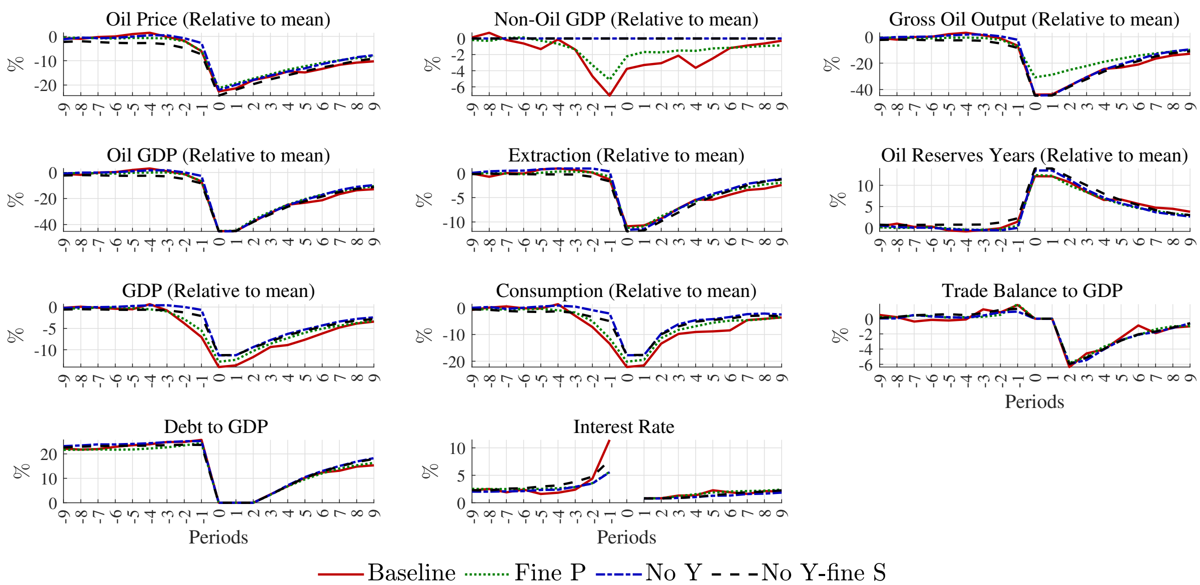
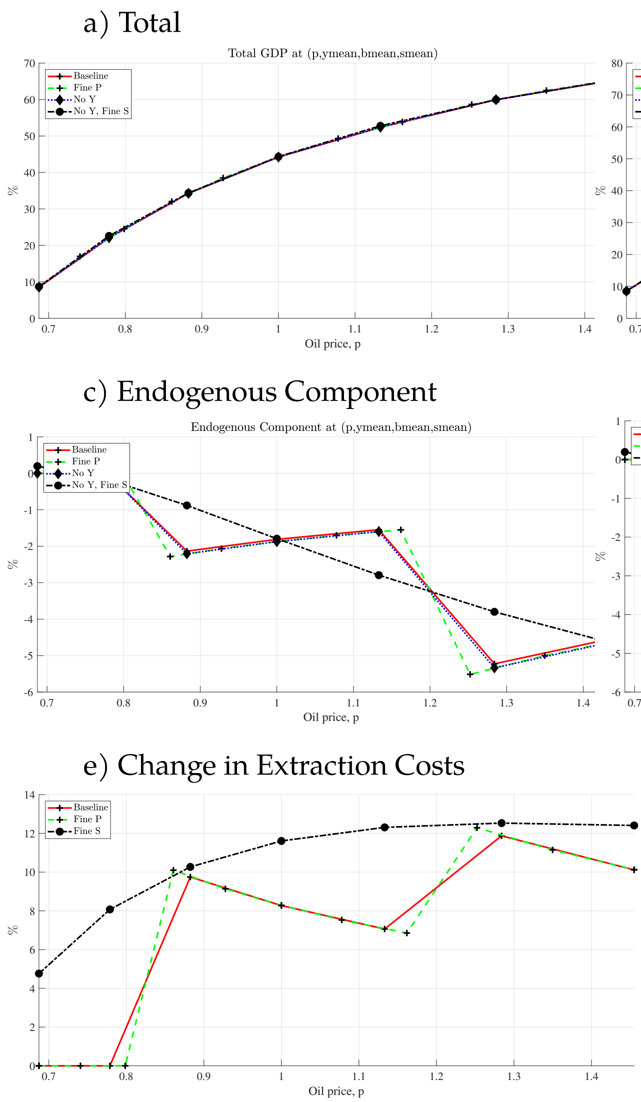


Figure N13: Event Windows around a default episode: Grids Refinement



With respect to default costs, results suggest minor differences between baseline model and model with finer grid on oil prices, and without fluctuations in non-oil GDP. The biggest difference arises from the model with a finer grid on reserves. As suggested in the paper, default costs can be distinguished between an exogenous and an endogenous component. The latter considers changes in extraction costs and losses from oil revenues. In the baseline model, there is a negative trend with some non-monotone regions around the long-run price. This non-monotonicity does not arise from the coarseness of oil prices grid but from the coarseness of reserves. Particularly, from considering a grid of oil reserves with steps of the size of 10% of average extraction ($s_i - s_{i-1} = 0.1E\,[x]$) in baseline model with 70 nodes, to a grid with step size of 5% of average extraction ($s_i - s_{i-1} = 0.05E\,[x]$) in model with no fluctuations in non-oil GDP and finer grid for reserves, total costs and the exogenous component are broadly unchanged between the two models, but the endogenous component features a smooth gain of default which keeps the trend in baseline, however the differences are not quantitatively large, but the finer grid appears as an interpolated version of the coarser solution. The difference between the two models is greatly explained by the Discrete Space Value Function Iteration solution method, which implies that the policy function must take values in the grid of reserves. Those differences are studied in Mendoza & Villalvazo (2020), and is one of the reasons our calibration strategy involves using Fix Point Iteration (FiPIt) method in the first stage of the calibration in the model of Small Open Economy with Foreign investors.

Figure N14: Default Costs: Grids Refinement



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