Appendices Not for Publication

Appendix A1: Proof of Existence of the Non-myopic Equilibrium with a Minimum effort Requirement

Proof. Let e_1^* and e_2^* be the equilibrium effort level in the absence of the minimum-effort constraint. As we have shown in Lemma 4, $e_1^* > e_2^*$. If $e_1^* > e_2^* \geq \underline{e}$, this is also the equilibrium of the model when the minimum-effort requirement is imposed. Thus, it is enough to consider two remaining cases, $e_1^* > \underline{e} > e_2^*$ and $\underline{e} \geq e_1^* > e_2^*$. In the following, we suppress the dependence of $\tilde{P}_i$ on N_1 and N_2 to keep the notation concise.

Case 1: Suppose $e_1^* > \underline{e} > e_2^*$. Since $\tilde{P}_1(e_1^*, e_2^*) - c_1'(e_1^*) = 0$ and $\tilde{P}_1(e_1^*, e_2)$ is increasing in e_2 by Lemma 5, we have $\tilde{P}_1(e_1^*, \underline{e}) - c_1'(e_1^*) > 0$. That lemma also implies that $\tilde{P}_1(e_1, \underline{e}) - c_1'(e_1)$ is decreasing in e_1. Thus, if $\tilde{P}_1(e_1, \underline{e}) - c_1'(e_1) < 0$ for some e_1, continuity then implies that there must be some $\hat{e}_1 > e_1^*$ such that $\tilde{P}_1(\hat{e}_1, \underline{e}) - c_1'(\hat{e}_1) = 0$. To see the former, suppose $f_1(e_1) \to \infty$ as $e_1 \to \infty$. Then f_1' being bounded above implies

$$\tilde{P}_1(e_1, \underline{e}) = \frac{v_1 f_1'(e_1) N_2 f_2(\underline{e})}{\left(N_1 f_1(e_1) + N_2 f_2(\underline{e})\right)^2} \to 0 \text{ as } e_1 \to \infty.$$

If $f_1(e_1) \not\to \infty$, then $f_1'(e_1) \to 0$ because f_1 is increasing. Thus, $\tilde{P}_1(e_1, \underline{e}) \to 0$ either way, and $\tilde{P}_1(e_1, \underline{e}) - c_1'(e_1) < 0$ for some e_1 since c_1' is bounded below by a positive constant.

Next, since $\tilde{P}_2(e_1^*, e_2^*) - c_2'(e_2^*) = 0$ and $\tilde{P}_2(e_1^*, e_2) - c_2'(e_2)$ is decreasing in e_2 by Lemma 5, we have $\tilde{P}_2(e_1^*, \underline{e}) - c_2'(\underline{e}) < 0$. Lemma 5 also implies that $\tilde{P}_2(e_1, \underline{e})$ is decreasing in e_1, which means $\tilde{P}_2(\hat{e}_1, \underline{e}) - c_2'(\underline{e}) < 0$. This, together with $\tilde{P}_1(\hat{e}_1, \underline{e}) - c_1'(\hat{e}_1) = 0$ means that $(\hat{e}_1, \underline{e})$ is the equilibrium of the model with the minimum-effort requirement.

Case 2: Suppose $\underline{e} \geq e_1^* > e_2^*$. By the same reasoning as in case 1, there is $\hat{e}_1 > e_1^*$ such that $\tilde{P}_1(\hat{e}_1, \underline{e}) - c_1'(\hat{e}_1) = 0$ and $\tilde{P}_2(\hat{e}_1, \underline{e}) - c_2'(\underline{e}) < 0$. If $\hat{e}_1 \geq \underline{e}$, then $(\hat{e}_1, \underline{e})$ is the equilibrium, as in case 1. If $\underline{e} > \hat{e}_1$, then $\tilde{P}_1(\underline{e}, \underline{e}) - c_1'(\underline{e}) < 0$ because $\tilde{P}_1(e_1, \underline{e}) - c_1'(e_1)$ is decreasing in e_1 by Lemma 5. In addition, $P_2(\underline{e}, \underline{e}) - c_2'(\underline{e}) < 0$ since $P_2(e_1, \underline{e})$ is decreasing in e_1. Thus, $(\underline{e}, \underline{e})$ is the equilibrium of the model with the minimum-effort requirement in this case. □

Appendix A2: Proof of Comparative Statics Results with a Binding Minimum Effort Requirement for Weak Students

Proof. Let $\hat{e}_2 = \underline{e}$. If $\tilde{P}_2(\hat{e}_1, \hat{e}_2, N_1, N_2) - c_2'(\hat{e}_2) < 0$, the minimum-effort constraint on group 2 students are binding, which means $\frac{\partial \hat{e}_2}{\partial N_1} = \frac{\partial \hat{e}_2}{\partial N_2} = 0$. Suppose $\tilde{P}_2(\hat{e}_1, \hat{e}_2, N_1, N_2) - c_2'(\hat{e}_2) = 0$ instead. Then, in the absence of the minimum-effort constraint, we would have had $\frac{\partial \hat{e}_2}{\partial N_i} < 0$ for all i by Theorem . However, the minimum-effort requirement means $\hat{e}_2$ cannot decrease further since $\hat{e}_2 = \underline{e}$. Therefore, $\frac{\partial \hat{e}_2}{\partial N_i} = 0$ for all i.

Since $\hat{e}_1 > \underline{e}$, we have $\tilde{P}_1(\hat{e}_1, \hat{e_2}, N_1, N_2) - c_1'(\hat{e}_1) = 0$. Differentiating both sides of the equation with respect to N_1 yields,

$$\frac{\partial}{\partial N_1}\Big(\tilde{P}_1(\hat{e}_1, \hat{e}_2, N_1, N_2) - c_1'(\hat{e}_1)\Big) = \frac{\partial \tilde{P}_1}{\partial \hat{e}_1}\frac{\partial \hat{e}_1}{\partial N_1} + \frac{\partial \tilde{P}_1}{\partial \hat{e}_2}\frac{\partial \hat{e}_2}{\partial N_1} + \frac{\partial \tilde{P}_1}{\partial N_1} - \frac{\partial c_1'}{\partial \hat{e}_1}\frac{\partial \hat{e}_1}{\partial N_1} = 0.$$

Using $\frac{\partial \hat{e}_2}{\partial N_1} = 0$ and Lemma 5, we obtain

$$\underbrace{\left(\frac{\partial \tilde{P}_1}{\partial \hat{e}_1} - c''(\hat{e}_1)\right)}_{(-)}\frac{\partial \hat{e}_1}{\partial N_1} = \underbrace{-\frac{\partial \tilde{P}_1}{\partial N_1}}_{(-)(-)} \implies \frac{\partial \hat{e}_1}{\partial N_1} < 0.$$

Similarly, differentiating with respect to N_2 yields,

$$\frac{\partial}{\partial N_2}\Big(\tilde{P}_1(\hat{e}_1, \hat{e}_2, N_1, N_2) - c_1'(\hat{e}_1)\Big) = \frac{\partial \tilde{P}_1}{\partial \hat{e}_1}\frac{\partial \hat{e}_1}{\partial N_2} + \frac{\partial \tilde{P}_1}{\partial \hat{e}_2}\frac{\partial \hat{e}_2}{\partial N_2} + \frac{\partial \tilde{P}_1}{\partial N_2} - \frac{\partial c_1'}{\partial \hat{e}_1}\frac{\partial \hat{e}_1}{\partial N_2} = 0.$$

Using $\frac{\partial \hat{e}_2}{\partial N_2} = 0$ and Lemma 5, we obtain

$$\underbrace{\left(\frac{\partial \tilde{P}_1}{\partial \hat{e}_1} - c''(\hat{e}_1)\right)}_{(-)} \frac{\partial \hat{e}_1}{\partial N_2} = \underbrace{-\frac{\partial \tilde{P}_1}{\partial N_2}}_{(-)(+)} \implies \frac{\partial \hat{e}_1}{\partial N_2} > 0.$$

□

Appendix A3: Non-myopic Equilibrium with Swapping

We consider how the equilibrium effort in the non-myopic model changes when the members of the weaker group are swapped for those of the stronger group. That is, the stronger group's size (N_1) is increased, and the weaker group's size (N_2) is decreased while the population size is kept constant at N. To reduce confusion, we use $\hat{e}_i$ to denote the equilibrium effort when swapping is being considered: $\hat{e}_i(N_1) = e_i^*(N_1, N - N_1)$. Thus,

$$\frac{d\hat{e}_i}{dN_1} = \frac{\partial e_i^*}{\partial N_1} + \frac{\partial e_i^*}{\partial N_2}\left(\frac{dN_2}{dN_1}\right) = \frac{\partial e_i^*}{\partial N_1} - \frac{\partial e_i^*}{\partial N_2}.$$

In both the base and the peer-effect versions of the non-myopic model (Theorems and), strong students work harder when faced with an increased competition from the weak students ($\frac{\partial e_1^*}{\partial N_2} > 0$) but are discouraged by the increased competition from the fellow strong students ($\frac{\partial e_1^*}{\partial N_1} < 0$). Thus, when the swap occur, both effects work in the same direction and unambiguously reduce the strong students' effort:

$$\frac{d\hat{e}_1}{dN_1} = \underbrace{\frac{\partial e_1^*}{\partial N_1}}_{(-)} - \underbrace{\frac{\partial e_1^*}{\partial N_2}}_{(+)} < 0.$$

In contrast, weak students are discouraged by an increased competition from anyone, making the effect of a swap ambiguous:

$$\text{sign}\left(\frac{d\hat{e}_2}{dN_1}\right) = \underbrace{\frac{\partial e_2^*}{\partial N_1}}_{(-)} - \underbrace{\frac{\partial e_2^*}{\partial N_2}}_{(-)} = \text{ ambiguous.}$$

That is, an increased competition from having more strong students makes the weak students lower their effort while reduced competition from having fewer fellow weak students encourages them to raise their effort. The direction of the combined effect depends on which of the two effects dominate. The following theorem gives sufficient conditions for the effort to decline.

Proposition 1. *In the base non-myopic model and the non-myopic model with direct peer effect, we have* $\frac{d\hat{e}_1(N_1)}{dN_1} < 0$. *Suppose in addition* $N_1 f_1 > N_2 f_2 + 2N_1 f_2$ *in the base model and* $N_1 f_1 > N_2 f_2 + 2N_1 f_2$ *or* $\frac{\partial f_2}{\partial N_1} > \frac{f_2}{N_2}$ *in the peer-effect model. Then there exists* $m > 0$ *such that* $\frac{d\hat{e}_2(N_1)}{dN_1} < 0$ *if* $c_1''(e_1^*) > m$.

The effect from having more strong students can dominate the effect from having fewer weak students if the competitive pressures from the strong students is more significant than the pressure from the weak students. The condition $N_1 f_1 > N_2 f_2 + 2N_1 f_2$, which puts a lower bound on the aggregate effort of the strong students relative to the weak students, captures this condition. Another possibility, captured by the condition $\frac{\partial f_2}{\partial N_1} > \frac{f_2}{N_2}$, is that the positive peer effect is significant enough to create an incentive to slack off. As seen in the formal proof below, either of these conditions ensures that marginal change in N_1 has a larger effect than the change in N_2 on the marginal benefits of the weak students. To understand the condition on c_1'', recall that strong students' efforts are determined by where their marginal benefit, $\widetilde{P}_1$, intersects the marginal cost, c_1'. If c_1' is very steep (c_1'' is large and positive), then the strong students reaction to changes in N_1 and N_2 will be small.[1] This in turn means that the indirect effect on the weak students that arises

[1] As an analogy, consider the demand (marginal benefit) and supply curve (marginal cost) analysis. A steeper supply curve induces a smaller change in the quantity (effort) when the demand curve shifts as a result of some parameter (group size) change.

as a reaction to the change in the strong students' effort will be small and not undo the primary effect that comes from the change in the weak students' marginal benefit.

Proof. We have shown that in both the base and the peer-effect versions of the non-myopic model, we have

$$\begin{bmatrix} \frac{\partial e_1^*}{\partial N_1} & \frac{\partial e_1^*}{\partial N_2} \\ \frac{\partial e_2^*}{\partial N_1} & \frac{\partial e_2^*}{\partial N_2} \end{bmatrix} = \frac{-1}{\text{DET}} \begin{bmatrix} \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) \\ \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) \end{bmatrix},$$

where $\text{DET} = \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right) > 0$. In the base model, we showed that

$$\frac{\partial \widetilde{P}_i}{\partial N_i} = -\frac{v_i 2 f_i' N_j f_j f_i}{\left(N_i f_i + N_j f_j\right)^3} < 0 \quad \text{and} \quad \frac{\partial \widetilde{P}_i}{\partial N_j} = \frac{v_i f_i' f_j \left(N_i f_i - N_j f_j\right)}{\left(N_i f_i + N_j f_j\right)^3} > 0 \iff N_i f_i > N_j f_j.$$

Thus,

$$\frac{\partial \widetilde{P}_2}{\partial N_1} < \frac{\partial \widetilde{P}_2}{\partial N_2} \iff \frac{v_2 f_2' f_1 \left(N_1 f_1 - N_2 f_2\right)}{\left(N_2 f_2 + N_1 f_1\right)^3} > \frac{v_2 2 f_2' N_1 f_1 f_2}{\left(N_2 f_2 + N_1 f_1\right)^3} \iff N_1 f_1 > N_2 f_2 + 2 N_1 f_2.$$

In the peer-effect model, we showed

$$\frac{\partial \widetilde{P}_2}{\partial N_1} = v_2 \left(\frac{\frac{\partial f_2'}{\partial N_1} N_1 f_1 \left(N_1 f_1 + N_2 f_2\right) - f_2' f_1 \left(N_1 f_1 - N_2 f_2\right) - 2 f_2' N_1 f_1 N_2 \frac{\partial f_2}{\partial N_1}}{\left(N_1 f_1 + N_2 f_2\right)^3} \right) < 0, \text{ and}$$

$$\frac{\partial \widetilde{P}_2}{\partial N_2} = -\frac{v_2 2 f_2' N_1 f_1 f_2}{\left(N_2 f_2 + N_1 f_1\right)^3} < 0.$$

Thus,

$$\frac{\partial \widetilde{P}_2}{\partial N_1} < \frac{\partial \widetilde{P}_2}{\partial N_2} \iff -\frac{\partial f_2'}{\partial N_1} N_1 f_1 \left(N_1 f_1 + N_2 f_2\right) + f_2' f_1 \left(N_1 f_1 - N_2 f_2\right) + 2 f_2' N_1 f_1 N_2 \frac{\partial f_2}{\partial N_1} > 2 f_2' N_1 f_1 f_2,$$

which holds if $N_1 f_1 > N_2 f_2 + 2 N_1 f_2$ or $N_2 \frac{\partial f_2}{\partial N_1} > f_2$.

In the following, we show that $\frac{\partial \widetilde{P}_2}{\partial N_1} < \frac{\partial \widetilde{P}_2}{\partial N_2}$ is consistent with $\frac{d\hat{e}_1}{dN_1} < 0$ and that $\frac{d\hat{e}_2}{dN_1} < 0$ if in addition c_1'' is positive and large. First, we have

$$\frac{\partial e_1^*}{\partial N_1} < \frac{\partial e_1^*}{\partial N_2} \iff \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) > \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right)$$

$$\iff \underbrace{\underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)}_{(-)} \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial N_1} - \frac{\partial \widetilde{P}_1}{\partial N_2}\right)}_{(-)-(+)}}_{(+)} > \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)}_{(+)} \underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial N_1} - \frac{\partial \widetilde{P}_2}{\partial N_2}\right)}_{(*)}.$$

Since $(*) < 0$ by above, the last inequality is satisfied as required. Therefore, $\frac{d\hat{e}_1}{dN_1} < 0$. Next, we have

$$\frac{\partial e_2^*}{\partial N_1} < \frac{\partial e_2^*}{\partial N_2} \iff \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) > \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right)$$

$$\iff \underbrace{\underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*}\right)}_{(-)}\underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial N_1}-\frac{\partial \widetilde{P}_2}{\partial N_2}\right)}_{(-)}}_{(+)}+c_1''\underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial N_2}-\frac{\partial \widetilde{P}_2}{\partial N_1}\right)}_{(+)} > \underbrace{\underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)}_{(-)}\underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial N_1}-\frac{\partial \widetilde{P}_1}{\partial N_2}\right)}_{(-)-(+)}}_{(+)}.$$

If c_1'' is positive and sufficiently large, the last inequality will be satisfied and $\frac{d\hat{e}_2}{dN_1} < 0$, as desired. □

Appendix A4: Nash equilibrium

We begin by showing that strong students are high achievers as expected.

Proposition 2. *At an interior equilibrium $(e_1^*, e_2^*, ..., e_N^*)$, we have*

$$f_i(e_i^*) > f_j(e_j^*) \iff \frac{v_i f_i'(e_i^*)}{c_i'(e_i^*)} > \frac{v_j f_j'(e_j^*)}{c_j'(e_j^*)}.$$

Consequently, if student i is stronger than student j, then $e_i^* > e_j^*$ and $f_i(e_i^*) > f_j(e_j^*)$. □

Proof. Rearranging the first order condition for an interior equilibrium yields,

$$\frac{v_i f_i'(e_i^*)\sum_{k\neq i} f_k(e_k^*)}{\left(\sum_k f_k(e_k^*)\right)^2} - c_i'(e_i^*) = 0 \iff \frac{v_i f_i'(e_i^*)}{c_i'(e_i^*)}\sum_{k\neq i} f_k(e_k^*) = \left(\sum_k f_k(e_k^*)\right)^2$$
$$\frac{v_j f_j'(e_j^*)\sum_{k\neq j} f_k(e_k^*)}{\left(\sum_k f_k(e_k^*)\right)^2} - c_j'(e_j^*) = 0 \iff \frac{v_j f_j'(e_j^*)}{c_j'(e_j^*)}\sum_{k\neq j} f_k(e_k^*) = \left(\sum_k f_k(e_k^*)\right)^2.$$

Thus,

$$\frac{v_i f_i'(e_i^*)}{c_i'(e_i^*)}\left(\sum_k f_k(e_k^*) - f_i(e_i^*)\right) = \frac{v_j f_j'(e_j^*)}{c_j'(e_j^*)}\left(\sum_k f_k(e_k^*) - f_j(e_j^*)\right),$$

which means

$$\frac{v_i f_i'(e_i^*)}{c_i'(e_i^*)} > \frac{v_j f_j'(e_j^*)}{c_j'(e_j^*)} \iff \sum_k f_k(e_k^*) - f_i(e_i^*) < \sum_k f_k(e_k^*) - f_j(e_i^*) \iff f_i(e_i^*) > f_j(e_j^*).$$

Therefore, if $v_i > v_j$, $f_i' \geq f_j'$, and $c_i' \leq c_j'$, then $f_i(e_i^*) > f_j(e_j^*)$. Because $f_1(0) = f_2(0)$ and $f_1' \geq f_2'$, this implies that $e_i^* > e_j^*$. □

The following lemma gives the conditions that are sufficient to guarantee that the second order condition for the individual's utility maximization problem is satisfied. Note that the condition only requires that the effective effort is weakly more concave than the cost and does not require the effective effort itself to be concave or the cost function to be convex.

Lemma 1. *The first order condition for the interior solution ($e_i > 0$) to the individual's utility maximization problem is equivalent to*

$$\frac{f_i'(e_i)}{c_i'(e_i)} = \frac{\left(\sum_k f_k(e_k)\right)^2}{v_i \sum_{k\neq i} f_k(e_k)}.$$

Suppose $\frac{f_i''(e_i)}{f_i'(e_i)} \leq \frac{c_i''(e_i)}{c_i'(e_i)}$. Then the second order condition for the utility maximization problem is satisfied at the solution to the interior first order condition. Thus, any $e_i > 0$ satisfying the interior first order condition is the optimal effort level. □

Proof. Suppose $e_i > 0$ satisfies the first order condition for utility maximization. Then, we have:

$$v_i \frac{\partial P_i(e_1, ..., e_N)}{\partial e_i} - c_i'(e_i) = 0 \iff v_i \left(\frac{f_i'(e_i) \sum_{k \neq i} f_k(e_k)}{\left(\sum_k f_k(e_k)\right)^2} \right) - c_i'(e_i) = 0 \iff \frac{f_i'(e_i)}{c_i'(e_i)} = \frac{\left(\sum_k f_k(e_k)\right)^2}{v_i \sum_{k \neq i} f_k(e_k)}.$$

We also have:

$$\begin{aligned}
v_i \frac{\partial^2 P_i}{\partial e_i^2} - c_i''(e_i) < 0 &\iff v_i \left(\frac{f_i''(e_i) \left(\sum_{k \neq i} f_k(e_k)\right)}{\left(\sum_k f_k(e_k)\right)^2} - \frac{2 f_i'(e_i) f_i'(e_i) \left(\sum_{k \neq i} f_k(e_k)\right)}{\left(\sum_k f_k(e_k)\right)^3} \right) - c_i''(e_i) < 0 \\
&\iff \frac{f_i''(e_i) \left(\sum_{k \neq i} f_k(e_k)\right)}{\left(\sum_k f_k(e_k)\right)^2} < \frac{c_i''(e_i)}{v_i} + \frac{2 f_i'(e_i) f_i'(e_i) \left(\sum_{k \neq i} f_k(e_k)\right)}{\left(\sum_k f_k(e_k)\right)^3} \\
&\iff f_i''(e_i) < c_i''(e_i) \left(\frac{\left(\sum_k f_k(e_k)\right)^2}{v_i \sum_{k \neq i} f_k(e_k)} \right) + \frac{2 f_i'(e_i) f_i'(e_i)}{\sum_k f_k(e_k)} \\
&\iff f_i''(e_i) < c_i''(e_i) \left(\frac{f_i'(e_i)}{c_i'(e_i)} \right) + \frac{2 f_i'(e_i) f_i'(e_i)}{\sum_k f_k(e_k)} \iff \frac{f_i''(e_i)}{f_i'(e_i)} < \frac{c_i''(e_i)}{c_i'(e_i)} + \underbrace{\frac{2 f_i'(e_i)}{\sum_k f_k(e_k)}}_{(+)}.
\end{aligned}$$

Thus, the second order condition is satisfied if $\frac{f_i''(e_i)}{f_i'(e_i)} \leq \frac{c_i''(e_i)}{c_i'(e_i)}$. □

Since we are interested in a symmetric equilibrium, in which individuals in the same group exert the same effort, it is enough to investigate the incentives of the representative members of the groups. Therefore, let i index the group in the following, and let e_i^* be the equilibrium effort level of the representative individual in group i. Let

$$\widetilde{P}_i(e_1^*, e_2^*, N_1, N_2) = v_i \frac{\partial P_i}{\partial e_i}\Big|_{(e_1^*, e_2^*)} = \frac{v_i f_i'(e_i^*) \left((N_i - 1) f_i(e_i^*) + N_j f_j(e_j^*) \right)}{\left(N_i f_i(e_i^*) + N_j f_j(e_j^*) \right)^2},$$

where $j \neq i$, denote the marginal benefit of effort at the equilibrium. The following result gives the sign of the partial derivatives of $\widetilde{P}_i$.

Lemma 2. *Suppose $N_i \geq 2$ for all i. Then for all i and $j \neq i$, we have:*

$$\frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial e_i^*} - c_i''(e_i^*) < 0 \quad \textit{and} \quad \frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial e_j^*} < 0.$$

$$\frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial N_i} < 0 \quad \textit{and} \quad \frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial N_j} < 0.$$

□

In the proof below and elsewhere we will drop the arguments in the function notation to conserve space when it does not interfere with clarity.

Proof. First, $N_i \geq 2$ means we have

$$\frac{(N_i - 1) f_i + N_j f_j}{N_i f_i + N_j f_j} > \frac{1}{2}. \tag{1}$$

To see this, note that:

$$\frac{(N_i - 1) f_i + N_j f_j}{N_i f_i + N_j f_j} > \frac{1}{2} \iff 2N_i f_i - 2f_i + 2N_j f_j > N_i f_i + N_j f_j \iff N_i f_i + N_j f_j > 2f_i,$$

which is always true. Inequality (1) further implies the following.

$$\frac{N_i-1}{N_i}<1<\left(\frac{2(N_i-1)f_i+N_jf_j}{N_if_i+N_jf_j}\right)\implies\frac{N_i-1}{(N_i-1)f_i+N_jf_j}<\frac{2N_i}{N_if_i+N_jf_j}. \tag{2}$$

Next, we have

$$\begin{aligned}\frac{\partial\widetilde{P}_i}{\partial e_i^*}&=v_i\left(\frac{\left(f_i''\left((N_i-1)f_i+N_jf_j\right)+f_i'(N_i-1)f_i'\right)\left(N_if_i+N_jf_j\right)^2-f_i'\left((N_i-1)f_i+N_jf_j\right)2\left(N_if_i+N_jf_j\right)N_if_i'}{\left(N_if_i+N_jf_j\right)^4}\right)\\&=v_i\left(\frac{f_i''\left((N_i-1)f_i+N_jf_j\right)}{\left(N_if_i+N_jf_j\right)^2}+\frac{(N_i-1)f_i'f_i'}{\left(N_if_i+N_jf_j\right)^2}-\frac{2N_if_i'f_i'\left((N_i-1)f_i+N_jf_j\right)}{\left(N_if_i+N_jf_j\right)^3}\right).\end{aligned}$$

Thus,

$$\begin{aligned}\frac{\partial\widetilde{P}_i}{\partial e_i^*}-c_i''<0&\iff v_i\left(\frac{f_i''\left((N_i-1)f_i+N_jf_j\right)}{\left(N_if_i+N_jf_j\right)^2}+\frac{(N_i-1)f_i'f_i'}{\left(N_if_i+N_jf_j\right)^2}-\frac{2N_if_i'f_i'\left((N_i-1)f_i+N_jf_j\right)}{\left(N_if_i+N_jf_j\right)^3}\right)-c_i''<0\\&\iff f_i''+\frac{(N_i-1)f_i'f_i'}{(N_i-1)f_i+N_jf_j}-\frac{2N_if_i'f_i'}{(N_if_i+N_jf_j)}<\frac{c_i''}{v_i}\left(\frac{\left(N_if_i+N_jf_j\right)^2}{(N_i-1)f_i+N_jf_j}\right)\\&\iff f_i''+\frac{(N_i-1)f_i'f_i'}{(N_i-1)f_i+N_jf_j}-\frac{2N_if_i'f_i'}{(N_if_i+N_jf_j)}<c_i''\left(\frac{f_i'}{c_i'}\right)\text{ by Lemma 1}\\&\iff\underbrace{\frac{(N_i-1)f_i'}{(N_i-1)f_i+N_jf_j}-\frac{2N_if_i'}{(N_if_i+N_jf_j)}}_{(-)\text{ by inequality (2)}}<\underbrace{\frac{c_i''}{c_i'}-\frac{f_i''}{f_i'}}_{(\geq 0)}.\end{aligned}$$

Since $N_i\geq 2$, the remaining derivatives are also negative:

$$\begin{aligned}\frac{\partial\widetilde{P}_i}{\partial e_j^*}&=v_i\left(\frac{f_i'N_jf_j'\left(N_if_i+N_jf_j\right)^2-f_i'\left((N_i-1)f_i+N_jf_j\right)2\left(N_if_i+N_jf_j\right)N_jf_j'}{\left(N_if_i+N_jf_j\right)^4}\right)\\&=v_i\left(\frac{N_jf_i'f_j'}{\left(N_if_i+N_jf_j\right)^2}-\frac{2N_jf_i'f_j'\left((N_i-1)f_i+N_jf_j\right)}{\left(N_if_i+N_jf_j\right)^2\left(N_if_i+N_jf_j\right)}\right)<0\quad\text{by inequality (1).}\\\frac{\partial\widetilde{P}_i}{\partial N_i}&=v_i\left(\frac{f_i'f_i\left(N_if_i+N_jf_j\right)^2-f_i'\left((N_i-1)f_i+N_jf_j\right)2\left(N_if_i+N_jf_j\right)f_i}{\left(N_if_i+N_jf_j\right)^4}\right)\\&=v_i\left(\frac{f_i'f_i}{\left(N_if_i+N_jf_j\right)^2}-\frac{2f_i'f_i\left((N_i-1)f_i+N_jf_j\right)}{\left(N_if_i+N_jf_j\right)^2\left(N_if_i+N_jf_j\right)}\right)<0\quad\text{by inequality (1).}\\\frac{\partial\widetilde{P}_i}{\partial N_j}&=v_i\left(\frac{f_i'f_j\left(N_if_i+N_jf_j\right)^2-f_i'\left((N_i-1)f_i+N_jf_j\right)2\left(N_if_i+N_jf_j\right)f_j}{\left(N_if_i+N_jf_j\right)^4}\right)\\&=v_i\left(\frac{f_i'f_j}{\left(N_if_i+N_jf_j\right)^2}-\frac{2f_i'f_j\left((N_i-1)f_i+N_jf_j\right)}{\left(N_if_i+N_jf_j\right)^2\left(N_if_i+N_jf_j\right)}\right)<0\quad\text{by inequality (1).}\end{aligned}$$

□

We are now ready to proceed to the main result. Expanding the interior equilibrium condition yields,

$$F(e_1^*,e_2^*,N_1,N_2)=\begin{bmatrix}\widetilde{P}_1(e_1^*,e_2^*,N_1,N_2)-c_1'(e_1^*)\\\widetilde{P}_2(e_1^*,e_2^*,N_1,N_2)-c_2'(e_2^*)\end{bmatrix}=\begin{bmatrix}\frac{v_1f_1'(e_1^*)((N_1-1)f_1(e_1^*)+N_2f_2(e_2^*))}{\left(N_1f_1(e_1^*)+N_2f_2(e_2^*)\right)^2}-c_1'(e_1^*)\\\frac{v_2f_2'(e_2^*)((N_2-1)f_2(e_2^*)+N_1f_1(e_2^*))}{\left(N_1f_1(e_1^*)+N_2f_2(e_2^*)\right)^2}-c_2'(e_2^*)\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}.$$

Thus, letting $\boldsymbol{e}^* = (e_1, e_2)$ and $\boldsymbol{N} = (N_1, N_2)$, we have

$$D_{\boldsymbol{N}}\boldsymbol{e}^* = \begin{bmatrix} \frac{\partial e_1^*}{\partial N_1} & \frac{\partial e_1^*}{\partial N_2} \\ \frac{\partial e_2^*}{\partial N_1} & \frac{\partial e_2^*}{\partial N_2} \end{bmatrix} = -\left[D_{\boldsymbol{e}^*}F(e_1^*, e_2^*, N_1, N_2)\right]^{-1}\left[D_{\boldsymbol{N}}F(e_1^*, e_2^*, N_1, N_2)\right]$$

by the implicit differentiation theorem. Then,

$$\begin{aligned} D_{\boldsymbol{N}}\boldsymbol{e}^* &= -\begin{bmatrix} \frac{\partial}{\partial e_1^*}\left(\widetilde{P}_1 - c_1'\right) & \frac{\partial}{\partial e_2^*}\left(\widetilde{P}_1 - c_1'\right) \\ \frac{\partial}{\partial e_1^*}\left(\widetilde{P}_2 - c_2'\right) & \frac{\partial}{\partial e_2^*}\left(\widetilde{P}_2 - c_2'\right) \end{bmatrix}^{-1}\begin{bmatrix} \frac{\partial}{\partial N_1}\left(\widetilde{P}_1 - c_1'\right) & \frac{\partial}{\partial N_2}\left(\widetilde{P}_1 - c_1'\right) \\ \frac{\partial}{\partial N_1}\left(\widetilde{P}_2 - c_2'\right) & \frac{\partial}{\partial N_2}\left(\widetilde{P}_2 - c_2'\right) \end{bmatrix} \\ &= -\begin{bmatrix} \frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1'' & \frac{\partial \widetilde{P}_1}{\partial e_2^*} \\ \frac{\partial \widetilde{P}_2}{\partial e_1^*} & \frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2'' \end{bmatrix}^{-1}\begin{bmatrix} \frac{\partial \widetilde{P}_1}{\partial N_1} & \frac{\partial \widetilde{P}_1}{\partial N_2} \\ \frac{\partial \widetilde{P}_2}{\partial N_1} & \frac{\partial \widetilde{P}_2}{\partial N_2} \end{bmatrix}. \end{aligned}$$

Letting $\mathrm{DET} = \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)$, we have

$$\begin{aligned} D_{\boldsymbol{N}}\boldsymbol{e}^* &= \frac{-1}{\mathrm{DET}}\begin{bmatrix} \frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2'' & -\frac{\partial \widetilde{P}_1}{\partial e_2^*} \\ -\frac{\partial \widetilde{P}_2}{\partial e_1^*} & \frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1'' \end{bmatrix}\begin{bmatrix} \frac{\partial \widetilde{P}_1}{\partial N_1} & \frac{\partial \widetilde{P}_1}{\partial N_2} \\ \frac{\partial \widetilde{P}_2}{\partial N_1} & \frac{\partial \widetilde{P}_2}{\partial N_2} \end{bmatrix} \\ &= \frac{-1}{\mathrm{DET}}\begin{bmatrix} \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) \\ \left(-\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) + \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) & \left(-\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) + \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) \end{bmatrix} \\ &= \frac{-1}{\mathrm{DET}}\begin{bmatrix} \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) \\ \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) \end{bmatrix}. \end{aligned}$$

Taking the required derivatives and determining the signs yield the following theorem.

Theorem (Theorem 2 in the main text)**.** *Suppose $N_i \geq 2$ for all i. Then we have*

$$\frac{\partial e_i^*}{\partial N_j^*} < 0 \quad \textit{for all } i = 1, 2 \textit{ and } j = 1, 2.$$

□

Proof. Let $g_i = \frac{v_i f_i''((N_i - 1)f_i + N_j f_j)}{(N_i f_i + N_j f_j)^2}$. Appealing to Lemma 1 yields,

$$g_i - c_i'' = \frac{v_i f_i''\left((N_i - 1)f_i + N_j f_j\right)}{\left(N_i f_i + N_j f_j\right)^2} - c_i'' = \frac{f_i'' c_i'}{f_i'} - c_i'' \leq 0 \quad \text{since } \tfrac{f_i''}{f_i'} \leq \tfrac{c_i''}{c_i'} \text{ by assumption.}$$

In addition, using the derivative obtained in the proof of Lemma 2, we obtain

$$\begin{aligned} \frac{\partial \widetilde{P}_i}{\partial e_i^*} - g_i < 0 &\iff \frac{v_i(N_i - 1)f_i' f_i'}{\left(N_i f_i + N_j f_j\right)^2} - \frac{v_i 2N_i f_i' f_i'\left((N_i - 1)f_i + N_j f_j\right)}{\left(N_i f_i + N_j f_j\right)^3} < 0 \\ &\iff \frac{N_i - 1}{(N_i - 1)f_i + N_j f_j} < \frac{2N_i}{N_i f_i + N_j f_j}, \quad \text{which holds by inequality (2).} \end{aligned}$$

Next, we show that $\left(\frac{\partial \widetilde{P}_i}{\partial e_i^*} - g_i\right)\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right) > \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial e_i^*}\right)$. This follows because, using the derivatives obtained in the proof of Lemma 2, we have

$$\left(\frac{\partial \widetilde{P}_i}{\partial e_i^*} - g_i\right)\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right) > \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial e_i^*}\right) \iff \frac{1}{v_i v_j}\left(\frac{\partial \widetilde{P}_i}{\partial e_i^*} - g_i\right)\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right) > \frac{1}{v_i v_j}\left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial e_i^*}\right)$$

$$\iff \left(\frac{(N_i-1)f_i'f_i'}{(N_if_i+N_jf_j)^2} - \frac{2N_if_i'f_i'\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)^3}\right)\left(\frac{(N_j-1)f_j'f_j'}{(N_if_i+N_jf_j)^2} - \frac{2N_jf_j'f_j'\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)^3}\right)$$

$$> \left(\frac{N_jf_i'f_j'}{(N_if_i+N_jf_j)^2} - \frac{2N_jf_i'f_j'\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)^3}\right)\left(\frac{N_if_j'f_i'}{(N_if_i+N_jf_j)^2} - \frac{2N_if_j'f_i'\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)^3}\right)$$

$$\iff \left((N_i-1) - \frac{2N_i\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\right)\left((N_j-1) - \frac{2N_j\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}\right)$$

$$> \left(N_j - \frac{2N_j\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\right)\left(N_i - \frac{2N_i\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}\right)$$

$$\iff (N_i-1)(N_j-1) - (N_j-1)\frac{2N_i\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)} - (N_i-1)\frac{2N_j\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}$$

$$+ \left(\frac{2N_i\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\right)\left(\frac{2N_j\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}\right)$$

$$> N_jN_i - N_i\frac{2N_j\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)} - N_j\frac{2N_i\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}$$

$$+ \left(\frac{2N_j\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\right)\left(\frac{2N_i\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}\right)$$

$$\iff (N_iN_j - N_i - N_j + 1) - (N_j-1)\frac{2N_i\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)} - (N_i-1)\frac{2N_j\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}$$

$$> N_jN_i - N_i\frac{2N_j\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)} - N_j\frac{2N_i\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}$$

$$\iff 1 + \frac{2N_i\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)} - N_i + \frac{2N_j\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)} - N_j > 0,$$

and the last inequality holds by (1). This implies that

$$\begin{aligned}
\text{DET} &= \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right) \\
&= \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1 + g_1 - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - g_2 + g_2 - c_2''\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right) \\
&= \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - g_2\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)}_{(+)} \\
&\quad + \underbrace{\left(g_1 - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - g_2\right)}_{(\geq 0)} + \underbrace{\left(g_2 - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1\right)}_{(\geq 0)} + \underbrace{\left(g_1 - c_1''\right)\left(g_2 - c_2''\right)}_{(\geq)} > 0.
\end{aligned}$$

Next, we show that $\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_i}\right) > \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_i}\right)$. This follows because, using the derivatives obtained in the proof of Lemma 2, we have

$$\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_i}\right) > \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_i}\right) \iff \frac{1}{v_j v_i}\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_i}\right) > \frac{1}{v_i v_j}\left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_i}\right)$$

$$
\begin{aligned}
&\iff \left(\frac{(N_j-1)f_j'f_j'}{(N_if_i+N_jf_j)^2}-\frac{2N_jf_j'f_j'\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)^3}\right)\left(\frac{f_i'f_i}{(N_if_i+N_jf_j)^2}-\frac{2f_i'f_i\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)^3}\right)\\
&\quad>\left(\frac{N_jf_i'f_j'}{(N_if_i+N_jf_j)^2}-\frac{2N_jf_i'f_j'\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)^3}\right)\left(\frac{f_j'f_i}{(N_if_i+N_jf_j)^2}-\frac{2f_j'f_i\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)^3}\right)\\
&\iff \left((N_j-1)-\frac{2N_j\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}\right)\left(1-\frac{2\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\right)\\
&\quad>\left(N_j-\frac{2N_j\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\right)\left(1-\frac{2\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}\right)\\
&\iff (N_j-1)-\frac{2N_j\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}-N_j\frac{2\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}+\frac{2\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\\
&\qquad\qquad+\left(\frac{2N_j\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}\right)\left(\frac{2\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\right)\\
&\quad>N_j-\frac{2N_j\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}-N_j\frac{2\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}\\
&\qquad\qquad+\left(\frac{2N_j\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\right)\left(\frac{2\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}\right)\\
&\iff -1+\frac{2\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}>0,
\end{aligned}
$$

and the last inequality holds by (1). This implies that

$$
\begin{aligned}
\frac{\partial e_i^*}{N_i}&=\frac{-1}{\mathrm{DET}}\left(\left(\frac{\partial\widetilde{P}_j}{\partial e_j^*}-c_j''\right)\left(\frac{\partial\widetilde{P}_i}{\partial N_i}\right)-\left(\frac{\partial\widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial\widetilde{P}_j}{\partial N_i}\right)\right)\\
&=\frac{-1}{\mathrm{DET}}\left(\left(\frac{\partial\widetilde{P}_j}{\partial e_j^*}-g_j+g_j-c_j''\right)\left(\frac{\partial\widetilde{P}_i}{\partial N_i}\right)-\left(\frac{\partial\widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial\widetilde{P}_j}{\partial N_i}\right)\right)\\
&=\underbrace{\frac{-1}{\mathrm{DET}}}_{(-)}\left(\underbrace{\left(\frac{\partial\widetilde{P}_j}{\partial e_j^*}-g_j\right)\left(\frac{\partial\widetilde{P}_i}{\partial N_i}\right)-\left(\frac{\partial\widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial\widetilde{P}_j}{\partial N_i}\right)}_{(+)}+\underbrace{\left(g_j-c_j''\right)\left(\frac{\partial\widetilde{P}_i}{\partial N_i}\right)}_{(\geq 0)}\right)<0.
\end{aligned}
$$

Next, we show that $\left(\frac{\partial\widetilde{P}_j}{\partial e_j^*}-g_j\right)\left(\frac{\partial\widetilde{P}_i}{\partial N_j}\right)>\left(\frac{\partial\widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial\widetilde{P}_j}{\partial N_j}\right)$. This follows because, using the derivatives obtained in the proof of Lemma 1, we have

$$
\begin{aligned}
&\left(\frac{\partial\widetilde{P}_j}{\partial e_j^*}-g_j\right)\left(\frac{\partial\widetilde{P}_i}{\partial N_j}\right)>\left(\frac{\partial\widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial\widetilde{P}_j}{\partial N_j}\right)\\
&\iff \left(\frac{(N_j-1)f_j'f_j'}{(N_if_i+N_jf_j)^2}-\frac{2N_jf_j'f_j'\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)^3}\right)\left(\frac{f_i'f_j}{(N_if_i+N_jf_j)^2}-\frac{2f_i'f_j\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)^3}\right)\\
&\quad>\left(\frac{N_jf_i'f_j'}{(N_if_i+N_jf_j)^2}-\frac{2N_jf_i'f_j'\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)^3}\right)\left(\frac{f_j'f_j}{(N_if_i+N_jf_j)^2}-\frac{2f_j'f_j\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)^3}\right)\\
&\iff \left((N_j-1)-\frac{2N_j\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}\right)\left(1-\frac{2\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\right)\\
&\quad>\left(N_j-\frac{2N_j\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\right)\left(1-\frac{2\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}\right)\\
&\iff (N_j-1)-\frac{2N_j\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}-N_j\frac{2\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}+\frac{2\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}
\end{aligned}
$$

$$
\begin{aligned}
&\quad + \left(\frac{2N_j\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}\right)\left(\frac{2\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\right)\\
>&N_j - N_j\frac{2\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)} - \frac{2N_j\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\\
&\quad + \left(\frac{2N_j\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)}\right)\left(\frac{2\left((N_j-1)f_j+N_if_i\right)}{(N_if_i+N_jf_j)}\right)\\
\iff& -1 + \frac{2\left((N_i-1)f_i+N_jf_j\right)}{(N_if_i+N_jf_j)} > 0,
\end{aligned}
$$

and the last inequality holds by (1). This implies that

$$
\begin{aligned}
\frac{\partial e_i^*}{N_j} &= \frac{-1}{\text{DET}}\left(\left(\frac{\partial\widetilde{P}_j}{\partial e_j^*}-c_j''\right)\left(\frac{\partial\widetilde{P}_i}{\partial N_j}\right)-\left(\frac{\partial\widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial\widetilde{P}_j}{\partial N_j}\right)\right)\\
&= \frac{-1}{\text{DET}}\left(\left(\frac{\partial\widetilde{P}_j}{\partial e_j^*}-g_j+g_j-c_j''\right)\left(\frac{\partial\widetilde{P}_i}{\partial N_j}\right)-\left(\frac{\partial\widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial\widetilde{P}_j}{\partial N_j}\right)\right)\\
&= \underbrace{\frac{-1}{\text{DET}}}_{(-)}\left(\underbrace{\left(\frac{\partial\widetilde{P}_j}{\partial e_j^*}-g_j\right)\left(\frac{\partial\widetilde{P}_i}{\partial N_j}\right)-\left(\frac{\partial\widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial\widetilde{P}_j}{\partial N_j}\right)}_{(+)}+\underbrace{\left(g_j-c_j''\right)\left(\frac{\partial\widetilde{P}_i}{\partial N_j}\right)}_{(\geq 0)}\right)<0.
\end{aligned}
$$

□

Appendix A5: Non-myopic equilibrium

As in the Nash equilibrium model, we let i and j, $i \neq j$, index the two groups and analyze the incentives of the representative members. We start by showing that the second order condition for the optimal effort problem is satisfied in the non-myopic equilibrium model when $\frac{f_i''(e_i)}{f_i'(e_i)} \leq \frac{c_i''(e_i)}{c_i'(e_i)}$, which we assume.

Lemma 3. *The first order condition for the interior solution ($e_i > 0$) to the individual's utility maximization problem is equivalent to*

$$\frac{f_i'(e_i)}{c_i'(e_i)} = \frac{\left(N_if_i(e_i)+N_jf_j(e_j)\right)^2}{v_iN_jf_j(e_j)}.$$

Suppose $\frac{f_i''(e_i)}{f_i'(e_i)} \leq \frac{c_i''(e_i)}{c_i'(e_i)}$. Then the second order condition for the utility maximization problem is satisfied at the solution to the interior first order condition. Thus, any $e_i > 0$ satisfying the interior first order condition is the optimal effort level. □

Proof. Suppose $e_i > 0$ satisfies the first order condition for utility maximization. Then, we have:

$$v_i\frac{\partial P_i(e_1,...,e_N)}{\partial e_i} - c_i'(e_i) = 0 \iff v_i\frac{f_i'(e_i)N_jf_j(e_j)}{\left(N_if_i(e_i)+N_jf_j(e_j)\right)^2} - c_i'(e_i) = 0 \iff \frac{f_i'(e_i)}{c_i'(e_i)} = \frac{\left(N_if_i(e_i)+N_jf_j(e_j)\right)^2}{v_iN_jf_j(e_j)}.$$

We also have:

$$
\begin{aligned}
v_i\frac{\partial^2 P_i}{\partial e_i^2} - c_i''(e_i) < 0 &\iff v_i\left(\frac{f_i''(e_i)N_jf_j(e_j)}{\left(N_if_i(e_i)+N_jf_j(e_j)\right)^2} - \frac{f_i'(e_i)N_jf_j(e_j)2N_if_i'(e_i)}{\left(N_if_i(e_i)+N_jf_j(e_j)\right)^3}\right) - c_i''(e_i) < 0\\
&\iff \frac{f_i''(e_i)N_jf_j(e_j)}{\left(N_if_i(e_i)+N_jf_j(e_j)\right)^2} < \frac{c_i''(e_i)}{v_i} + \frac{f_i'(e_i)N_jf_j(e_j)2N_if_i'(e_i)}{\left(N_if_i(e_i)+N_jf_j(e_j)\right)^3}\\
&\iff f_i''(e_i) < c_i''(e_i)\left(\frac{\left(N_if_i(e_i)+N_jf_j(e_j)\right)^2}{v_iN_jf_j(e_j)}\right) + \frac{f_i'(e_i)2N_if_i'(e_i)}{\left(N_if_i(e_i)+N_jf_j(e_j)\right)}
\end{aligned}
$$

$$\iff f_i''(e_i) < c_i''(e_i)\left(\frac{f_i'(e_i)}{c_i'(e_i)}\right) + \frac{f_i'(e_i)2N_i f_i'(e_i)}{(N_i f_i(e_i) + N_j f_j(e_j))}$$
$$\iff \frac{f_i''(e_i)}{f_i'(e_i)} < \frac{c_i''(e_i)}{c_i'(e_i)} + \underbrace{\frac{2N_i f_i'(e_i)}{(N_i f_i(e_i) + N_j f_j(e_j))}}_{(+)}.$$

Thus, the second order condition is satisfied if $\frac{f_i''(e_i)}{f_i'(e_i)} \leq \frac{c_i''(e_i)}{c_i'(e_i)}$. □

The next lemma shows that strong students have a higher achievement than weak students both individually and in the aggregate and enjoy a higher (expected) utility in equilibrium.

Lemma 4. *Suppose group 1 is stronger than group 2 in the sense given in Definition 3, and (e_1^*, e_2^*) is the corresponding interior equilibrium. Then $N_1 f_1(e_1^*) > N_2 f_2(e_2^*)$, $f_1(e_1^*) > f_2(e_2^*)$, and $u_1(e_1^*, e_2^*, N_1, N_2) > u_2(e_1^*, e_2^*, N_1, N_2)$.* □

Proof. Rearranging the first order condition for an interior equilibrium yields,

$$\frac{v_1 f_1'(e_1^*) N_2 f_2(e_2^*)}{\left(N_1 f_1(e_1^*) + N_2 f_2(e_2^*)\right)^2} - c_1'(e_1^*) = 0 \iff N_2 f_2(e_2^*) = \left(\frac{c_1'(e_1^*)}{v_1 f_1'(e_1^*)}\right)\left(N_1 f_1(e_1^*) + N_2 f_2(e_2^*)\right)^2$$
$$\frac{v_2 f_2'(e_2^*) N_1 f_1(e_1^*)}{\left(N_1 f_1(e_1^*) + N_2 f_2(e_2^*)\right)^2} - c_2'(e_2^*) = 0 \iff N_1 f_1(e_1^*) = \left(\frac{c_2'(e_2^*)}{v_2 f_2'(e_2^*)}\right)\left(N_1 f_1(e_1^*) + N_2 f_2(e_2^*)\right)^2.$$

Thus,

$$\frac{N_1 f_1(e_1^*)}{N_2 f_2(e_2^*)} = \frac{\frac{v_1 f_1'(e_1^*)}{c_1'(e_1^*)}}{\frac{v_2 f_2'(e_2^*)}{c_2'(e_2^*)}} \iff \frac{f_1(e_1^*)}{f_2(e_2^*)} = \frac{\frac{v_1 f_1'(e_1^*)}{N_1 c_1'(e_1^*)}}{\frac{v_2 f_2'(e_2^*)}{N_2 c_2'(e_2^*)}}.$$

Therefore, if $v_1 > v_2$, $\frac{v_1}{N_1} > \frac{v_2}{N_2}$, $f_1' \geq f_2'$, and $c_1' \leq c_2'$, then $N_1 f_1(e_1^*) > N_2 f_2(e_2^*)$ and $f_1(e_1^*) > f_2(e_2^*)$.

We now show that $u_1(e_1^*, e_2^*, N_1, N_2) > u_2(e_1^*, e_2^*, N_1, N_2)$. Suppose $e_1^* \leq e_2^*$. Then $c_1(e_1^*) \leq c_1(e_2^*) \leq c_2(e_2^*)$. Therefore,

$$u_1(e_1^*, e_2^*, N_1, N_2) = \frac{v_1 f_1(e_1^*)}{N_1 f_1(e_1^*) + N_2 f_2(e_2^*)} - c_1(e_1^*) > \frac{v_2 f_2(e_2^*)}{N_1 f_1(e_1^*) + N_2 f_2(e_2^*)} - c_2(e_2^*) = u_2(e_1^*, e_2^*, N_1, N_2)$$

since $v_1 \geq v_2$ and by above $f_1(e_1^*) > f_2(e_2^*)$. Next, suppose $e_1^* > e_2^*$. Then $f_1(e_1^*) > f_1(e_2^*)$ since f_1 is increasing in effort. Mover, because e_1^* is the best response of group 1 individuals to group 2 individuals choosing e_2^*, group 1 individuals will receive a (weakly) lower utility if they switch their effort to e_2^*. That is, $u_1(e_1^*, e_2^*, N_1, N_2) \geq u_1(e_2^*, e_2^*, N_1, N_2)$. Thus,

$$\begin{aligned} u_1(e_1^*, e_2^*, N_1, N_2) \geq u_1(e_2^*, e_2^*, N_1, N_2) &= \frac{v_1 f_1(e_2^*)}{N_1 f_1(e_2^*) + N_2 f_2(e_2^*)} - c_1(e_2^*) \\ &> \frac{v_2 f_2(e_2^*)}{N_1 f_1(e_1^*) + N_2 f_2(e_2^*)} - c_2(e_2^*) = u_2(e_1^*, e_2^*, N_1, N_2). \end{aligned}$$

The last inequality follows because group 1 being stronger than group 2 means $v_1 \geq v_2$, $f_1(e_2^*) \geq f_2(e_2^*)$, and $c_1(e_2^*) \leq c_2(e_2^*)$ by definition, and $f_1(e_1^*) > f_1(e_2^*)$ by above. □

Let

$$\widetilde{P}_i(e_1^*, e_2^*, N_1, N_2) = v_i \frac{\partial P_i}{\partial e_i}\Big|_{(e_1^*, e_2^*)} = \frac{v_i f_i'(e_i^*) N_j f_j(e_j^*)}{\left(N_i f_i(e_i^*) + N_j f_j(e_j^*)\right)^2},$$

where $j \neq i$, denote the marginal benefit of effort at the equilibrium. The following result gives the signs of the partial derivatives of $\widetilde{P}_i$.

Lemma 5. *Letting $i \neq j$, we have the following:*

$$\frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial e_i^*} - c_i''(e_i^*) < 0, \quad \frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial N_i} < 0,$$

$$\textit{and} \quad \frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial e_j^*} > 0 \iff N_i f_i(e_i^*) > N_j f_j(e_j^*) \iff \frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial N_j} > 0.$$

□

Proof. We have

$$\begin{aligned}
\frac{\partial \widetilde{P}_i}{\partial e_i^*} - c_i'' &= v_i \left(\frac{f_i'' N_j f_j \left(N_i f_i + N_j f_j\right)^2 - 2 f_i' N_j f_j \left(N_i f_i + N_j f_j\right) N_i f_i'}{\left(N_i f_i + N_j f_j\right)^4} \right) - c_i'' \\
&< 0 \iff \frac{f_i'' N_j f_j}{\left(N_i f_i + N_j f_j\right)^2} - \frac{2 f_i' N_j f_j N_i f_i'}{\left(N_i f_i + N_j f_j\right)^3} < \frac{c_i''}{v_i} \\
&\iff f_i'' < \frac{c_i''}{v_i} \left(\frac{\left(N_i f_i + N_j f_j\right)^2}{N_j f_j} \right) + \frac{2 f_i' N_i f_i'}{\left(N_i f_i + N_j f_j\right)} \\
&\iff f_i'' < \frac{c_i'' f'}{c'} + \frac{2 f_i' N_i f_i'}{\left(N_i f_i + N_j f_j\right)} \text{ by Lemma 3} \\
&\iff \frac{f_i''}{f_i'} < \frac{c_i''}{c'} + \frac{2 N_i f_i'}{\left(N_i f_i + N_j f_j\right)}, \text{ which holds since } \frac{f_i''}{f_i'} \leq \frac{c_i''}{c'}.
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \widetilde{P}_i}{\partial e_j^*} &= v_i \left(\frac{f_i' N_j f_j' \left(N_i f_i + N_j f_j\right)^2 - 2 f_i' N_j f_j \left(N_i f_i + N_j f_j\right) N_j f_j'}{\left(N_i f_i + N_j f_j\right)^4} \right) \\
&= v_i \left(\frac{f_i' N_j f_j' (N_i f_i + N_j f_j) - 2 f_i' N_j f_j N_j f_j'}{\left(N_i f_i + N_j f_j\right)^3} \right) = \frac{v_i \left(N_i f_i - N_j f_j\right) f_i' N_j f_j'}{\left(N_i f_i + N_j f_j\right)^3} > 0 \iff N_i f_i > N_j f_j.
\end{aligned}$$

$$\frac{\partial \widetilde{P}_i}{\partial N_i} = v_i \left(\frac{0 - 2 f_i' N_j f_j \left(N_i f_i + N_j f_j\right) f_i}{\left(N_i f_i + N_j f_j\right)^4} \right) = - \frac{v_i 2 f_i' N_j f_j f_i}{\left(N_i f_i + N_j f_j\right)^3} < 0.$$

$$\begin{aligned}
\frac{\partial \widetilde{P}_i}{\partial N_j} &= v_i \left(\frac{f_i' f_j \left(N_i f_i + N_j f_j\right)^2 - 2 f_i' N_j f_j \left(N_i f_i + N_j f_j\right) f_j}{\left(N_i f_i + N_j f_j\right)^4} \right) = v_i \left(\frac{f_i' f_j \left(N_i f_i + N_j f_j\right) - 2 f_i' N_j f_j f_j}{\left(N_i f_i + N_j f_j\right)^3} \right) \\
&= \frac{v_i f_i' f_j \left(N_i f_i - N_j f_j\right)}{\left(N_i f_i + N_j f_j\right)^3} > 0 \iff N_i f_i > N_j f_j.
\end{aligned}$$

□

Given that the second order condition is satisfied, the interior equilibrium condition is given by the first order condition as in the Nash equilibrium setting:

$$F(e_1^*, e_2^*, N_1, N_2) \equiv \begin{bmatrix} \widetilde{P}_1(e_1^*, e_2^*, N_1, N_2) - c_1'(e_1^*) \\ \\ \widetilde{P}_2(e_1^*, e_2^*, N_1, N_2) - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} \frac{v_1 f_1'(e_1^*) N_2 f_2(e_2^*)}{\left(N_1 f_1(e_1^*) + N_2 f_2(e_2^*)\right)^2} - c_1'(e_1^*) \\ \\ \frac{v_2 f_2'(e_2^*) N_1 f_1(e_1^*)}{\left(N_1 f_1(e_1^*) + N_2 f_2(e_2^*)\right)^2} - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} 0 \\ \\ 0 \end{bmatrix}.$$

Let $\boldsymbol{e^*} = (e_1^*, e_2^*)$ and $\boldsymbol{N} = (N_1, N_2)$. Appealing to the implicit differentiation theorem yields,

$$D_{\boldsymbol{N}} \boldsymbol{e^*} = \begin{bmatrix} \frac{\partial e_1^*}{\partial N_1} & \frac{\partial e_1^*}{\partial N_2} \\ \\ \frac{\partial e_2^*}{\partial N_1} & \frac{\partial e_2^*}{\partial N_2} \end{bmatrix} = - \left[D_{\boldsymbol{e^*}} F(e_1^*, e_2^*, N_1, N_2) \right]^{-1} \left[D_{\boldsymbol{N}} F(e_1^*, e_2^*, N_1, N_2) \right]$$

$$= \frac{-1}{\text{DET}} \begin{bmatrix} \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) \\ \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right) & \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) \end{bmatrix},$$

where $\text{DET} = \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)$.

As in the Nash equilibrium, taking the required derivatives and determining the signs yield the comparative statics result.

Lemma 6. *We have, $\frac{\partial e_i^*}{\partial N_i} < 0$ for all i. Let $i \neq j$. If $\frac{f_i''(e_i)}{f_i'(e_i)} < \frac{c_i''(e_i)}{c_i'(e_i)}$, then $\frac{\partial e_i^*}{\partial N_j} > 0 \iff N_i f_i > N_j f_j$. If instead $\frac{f_i''(e_i)}{f_i'(e_i)} = \frac{c_i''(e_i)}{c_i'(e_i)}$, then $\frac{\partial e_i^*}{\partial N_j} = 0$.* □

Proof. By Lemma 5, we have

$$\text{DET} = \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)}_{(-)}\underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)}_{(-)} - \underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)}_{(\leq 0)} > 0$$

$$\text{and} \quad \frac{\partial e_i^*}{\partial N_i} = \underbrace{\frac{-1}{\text{DET}}}_{(-)}\left(\underbrace{\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - c_j''\right)}_{(-)}\underbrace{\left(\frac{\partial \widetilde{P}_i}{\partial N_i}\right)}_{(-)} - \underbrace{\left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_i}\right)}_{(\leq 0)}\right) < 0.$$

Next, let $g_j = \frac{v_j f_j'' N_i f_i}{(N_i f_i + N_j f_j)^2}$. Appealing to Lemma 3 yields,

$$g_j - c_j'' = \frac{v_j f_j'' N_i f_i}{(N_i f_i + N_j f_j)^2} - c_j'' = \frac{f_j'' c_j'}{f_j'} - c_j'' \leq 0 \quad \text{since } \tfrac{f''}{f'} \leq \tfrac{c''}{c'} \text{ by assumption.}$$

As a preliminary to determining the sign of $\frac{\partial e_i^*}{\partial N_j}$, we first show that $\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right) = \left(\frac{\partial \widetilde{P}_j}{\partial N_j}\right)\left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)$. This follows because, using the derivatives found in the proof of Lemma 5, we have

$$\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right) = \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_j}\right)$$

$$\iff v_j\left(-\frac{2f_j' N_i f_i N_j f_j'}{(N_i f_i + N_j f_j)^3}\right) v_i\left(\frac{f_i' f_j \left(N_i f_i - N_j f_j\right)}{(N_i f_i + N_j f_j)^3}\right) = v_i\left(\frac{\left(N_i f_i - N_j f_j\right) f_i' N_j f_j'}{(N_i f_i + N_j f_j)^3}\right) v_j\left(-\frac{2f_j' N_i f_i f_j}{(N_i f_i + N_j f_j)^3}\right)$$

$$\iff \left(-2f_j' N_i f_i N_j f_j'\right)\left(f_i' f_j \left(N_i f_i - N_j f_j\right)\right) = \left(\left(N_i f_i - N_j f_j\right) f_i' N_j f_j'\right)\left(-2f_j' N_i f_i f_j\right),$$

the last line of which is clearly true. Thus,

$$\begin{aligned} \frac{\partial e_i^*}{\partial N_j} &= \frac{-1}{\text{DET}}\left(\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j + g_j - c_j''\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right) - \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_j}\right)\right) \\ &= \frac{-1}{\text{DET}}\left(\left(\frac{\partial \widetilde{P}_j}{\partial e_j^*} - g_j\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right) - \left(\frac{\partial \widetilde{P}_i}{\partial e_j^*}\right)\left(\frac{\partial \widetilde{P}_j}{\partial N_j}\right) + \left(g_j - c_j''\right)\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right)\right) \\ &= \underbrace{\frac{-1}{\text{DET}}}_{(-)}\underbrace{\left(g_j - c_j''\right)}_{(\leq 0)}\underbrace{\left(\frac{\partial \widetilde{P}_i}{\partial N_j}\right)}_{(+) \iff N_i f_i > N_j f_j} \quad \text{is} \begin{cases} (+) \iff N_i f_i > N_j f_j & \text{if } \frac{f_i''}{f_i'} < \frac{c_i''}{c_i'} \\ 0 & \text{if } \frac{f_i''}{f_i'} = \frac{c_i''}{c_i'}. \end{cases} \end{aligned}$$

□

As seen in Lemma 6, the effect of the change in the group size on the equilibrium effort level depends on the relative magnitude of the aggregate achievements. Combining this result with Lemma 4 yields our main result:

Theorem (Theorem 4 in the main text)**.** *Suppose* $\frac{f_i''(e_i)}{f_i'(e_i)} < \frac{c_i''(e_i)}{c_i'(e_i)}$ *for all* i*. Suppose in addition that group 1 is stronger than group 2. Then*

$$\frac{\partial e_1^*}{\partial N_1} < 0, \quad \frac{\partial e_2^*}{\partial N_1} < 0, \quad \frac{\partial e_1^*}{\partial N_2} > 0, , \quad \textit{and} \quad \frac{\partial e_2^*}{\partial N_2} < 0.$$

□

Proof. Immediate. □

Appendix A6: Non-myopic equilibrium with a direct peer effect

The sole difference between the non-myopic model with peer effect and the base non-myopic model is that the achievement functions are now $f_1(e_1, N_1) = a_1(N_1)b_1(e_1)$ and $f_2(e_2, N_1) = a_2(N_1)b_2(e_2)$, where $a_i(e_i) > 0$ and $a_i'(N_i) > 0$. This means that expressions involving differentiation with respect to e_1, e_2, or N_2 do not change from the base model. In particular, the first and the second order conditions for the individual's optimal effort problem and the equilibrium condition remain the same, aside from the achievement functions having additional dependence on N_1. Thus, the marginal benefits of effort at the equilibrium are:

$$\widetilde{P}_1(e_1^*, e_2^*, N_1, N_2) = \frac{v_1 f_1'(e_1^*, N_1) N_2 f_2(e_2^*, N_1)}{\left(N_1 f_1(e_1^*, N_1) + N_2 f_2(e_2^*, N_1)\right)^2} \text{ and } \widetilde{P}_2(e_1^*, e_2^*, N_1, N_2) = \frac{v_2 f_2'(e_2^*, N_1) N_1 f_1(e_1^*, N_1)}{\left(N_1 f_1(e_1^*, N_1) + N_2 f_2(e_2^*, N_1)\right)^2},$$

and the equilibrium condition is:

$$F(e_1^*, e_2^*, N_1, N_2) = \begin{bmatrix} \widetilde{P}_1(e_1^*, e_2^*, N_1, N_2) - c_1'(e_1^*) \\ \\ \widetilde{P}_2(e_1^*, e_2^*, N_1, N_2) - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} \frac{v_1 f_1'(e_1^*, N_1) N_2 f_2(e_2^*, N_1)}{\left(N_1 f_1(e_1^*, N_1) + N_2 f_2(e_2^*, N_1)\right)^2} - c_1'(e_1^*) \\ \\ \frac{v_2 f_2'(e_2^*, N_1) N_1 f_1(e_1^*, N_1)}{\left(N_1 f_1(e_1^*, N_1) + N_2 f_2(e_2^*, N_1)\right)^2} - c_2'(e_2^*) \end{bmatrix} = \begin{bmatrix} 0 \\ \\ 0 \end{bmatrix}.$$

In addition, Lemma 3 and Lemma 4 from the base model also hold for the peer-effect model because neither involves differentiation with respect to N_1. The partial derivatives of the marginal benefits are given below.

Lemma 7. *Suppose* $\frac{a_1'(N_1)}{a_1(N_1)} > \frac{a_2'(N_1)}{a_2(N_1)}$*. Then, we have the following:*

$$\begin{aligned}
&\frac{\partial \widetilde{P}_i(e_1^*, e_2^*, N_1, N_2)}{\partial e_i^*} - c_i'' < 0 \textit{ for all } i, \\
&\frac{\partial \widetilde{P}_1(e_1^*, e_2^*, N_1, N_2)}{\partial e_2^*} > 0 \iff N_1 f_1(e_1^*, N_1) > N_2 f_2(e_2^*, N_1) \iff \frac{\partial \widetilde{P}_2(e_1^*, e_2^*, N_1, N_2)}{\partial e_1^*} < 0, \\
&\frac{\partial \widetilde{P}_1(e_1^*, e_2^*, N_1, N_2)}{\partial N_1} < 0 \textit{ if } N_1 f_1(e_1^*, N_1) > N_2 f_2(e_2^*, N_1), \\
&\frac{\partial \widetilde{P}_1(e_1^*, e_2^*, N_1, N_2)}{\partial N_2} > 0 \iff N_1 f_1(e_1^*, N_1) > N_2 f_2(e_2^*, N_1), \\
&\frac{\partial \widetilde{P}_2(e_1^*, e_2^*, N_1, N_2)}{\partial N_1} < 0 \textit{ if } N_1 f_1(e_1^*, N_1) > N_2 f_2(e_2^*, N_1), \\
\textit{and} \quad &\frac{\partial \widetilde{P}_2(e_1^*, e_2^*, N_1, N_2)}{\partial N_2} < 0.
\end{aligned}$$

□

Proof. To clarify, in this proof and elsewhere a_i' denotes differentiation with respect to N_1 and b_i' denote differentiation with respect to e_i. The partial derivatives of $\widetilde{P}_i$ with respect to e_1^*, e_2^*, and N_2 are as in the proof of Lemma 5. That is, we have

$$\frac{\partial \widetilde{P}_i}{\partial e_i^*} - c_i'' = v_i \left(\frac{f_i'' N_j f_j \left(N_1 f_1 + N_2 f_2\right) - 2 f_i' N_j f_j N_i f_i'}{\left(N_1 f_1 + N_2 f_2\right)^3} \right) - c_i'' < 0$$

$$\iff \frac{f_i''}{f_i'} < \frac{c_i''}{c'} + \frac{2 N_i f_i'}{\left(N_1 f_1 + N_2 f_2\right)}, \text{ which holds since } \frac{f_i''}{f_i'} \leq \frac{c_i''}{c'}.$$

$$\frac{\partial \widetilde{P}_i}{\partial e_j^*} = \frac{v_i \left(N_i f_i - N_j f_j\right) f_i' N_j f_j'}{\left(N_1 f_1 + N_2 f_2\right)^3} > 0 \iff N_i f_i > N_j f_j.$$

$$\frac{\partial \widetilde{P}_1}{\partial N_2} = \frac{v_1 f_1' f_2 \left(N_1 f_1 - N_2 f_2\right)}{\left(N_1 f_1 + N_2 f_2\right)^3} > 0 \iff N_1 f_1 > N_2 f_2.$$

$$\frac{\partial \widetilde{P}_2}{\partial N_2} = -\frac{v_2 2 f_2' N_1 f_1 f_2}{\left(N_1 f_1 + N_2 f_2\right)^3} < 0.$$

The partial derivatives with respect to N_1 are derived below.

$$\frac{\partial \widetilde{P}_1}{\partial N_1} = v_1 \left(\frac{\left(\frac{\partial f_1'}{\partial N_1} N_2 f_2 + f_1' N_2 \frac{\partial f_2}{\partial N_1} \right) \left(N_1 f_1 + N_2 f_2\right)^2 - 2 f_1' N_2 f_2 \left(N_1 f_1 + N_2 f_2\right) \left(f_1 + N_1 \frac{\partial f_1}{\partial N_1} + N_2 \frac{\partial f_2}{\partial N_1} \right)}{\left(N_1 f_1 + N_2 f_2\right)^4} \right)$$

$$= v_1 N_2 \left(\frac{\left(\frac{\partial f_1'}{\partial N_1} f_2 + f_1' \frac{\partial f_2}{\partial N_1} \right) \left(N_1 f_1 + N_2 f_2\right) - 2 f_1' f_2 \left(f_1 + N_1 \frac{\partial f_1}{\partial N_1} + N_2 \frac{\partial f_2}{\partial N_1} \right)}{\left(N_1 f_1 + N_2 f_2\right)^3} \right)$$

$$= v_1 N_2 \left(\frac{\left(a_1' b_1' a_2 b_2 + a_1 b_1' a_2' b_2\right) \left(N_1 f_1 + N_2 f_2\right) - 2 a_1 b_1' a_2 b_2 \left(N_1 a_1' b_1 + N_2 a_2' b_2\right) - 2 f_1' f_2 f_1}{\left(N_1 f_1 + N_2 f_2\right)^3} \right)$$

$$= v_1 N_2 \left(\frac{b_1' b_2 \Big(\left(a_1' a_2 + a_1 a_2'\right) \left(N_1 f_1 + N_2 f_2\right) - 2 \left(a_1 a_2 N_1 a_1' b_1 + a_1 a_2 N_2 a_2' b_2\right) \Big) - 2 f_1' f_2 f_1}{\left(N_1 f_1 + N_2 f_2\right)^3} \right)$$

$$= v_1 N_2 \left(\frac{b_1' b_2 \Big(\left(a_1' a_2 + a_1 a_2'\right) \left(N_1 f_1 + N_2 f_2\right) - 2 \left(a_1' a_2 N_1 f_1 + a_1 a_2' N_2 f_2\right) \Big) - 2 f_1' f_2 f_1}{\left(N_1 f_1 + N_2 f_2\right)^3} \right)$$

$$= v_1 N_2 \left(\frac{b_1' b_2 \Big(a_1' a_2 \left(N_2 f_2 - N_1 f_1\right) + a_1 a_2' \left(N_1 f_1 - N_2 f_2\right) \Big) - 2 f_1' f_2 f_1}{\left(N_1 f_1 + N_2 f_2\right)^3} \right)$$

$$= v_1 N_2 \left(\frac{b_1' b_2 \overbrace{\left(a_1' a_2 - a_1 a_2'\right)}^{(+)} \overbrace{\left(N_2 f_2 - N_1 f_1\right)}^{(-)} - \overbrace{2 f_1' f_2 f_1}^{(+)}}{\left(N_1 f_1 + N_2 f_2\right)^3} \right) < 0 \quad \text{if } \frac{a_1'}{a_1} > \frac{a_2'}{a_2} \text{ and } N_1 f_1 > N_2 f_2.$$

Next, we have,

$$\frac{\partial \widetilde{P}_2}{\partial N_1} = v_2 \left(\frac{\left(\frac{\partial f_2'}{\partial N_1} N_1 f_1 + f_2' f_1 + f_2' N_1 \frac{\partial f_1}{\partial N_1} \right) \left(N_1 f_1 + N_2 f_2\right)^2 - 2 f_2' N_1 f_1 \left(N_1 f_1 + N_2 f_2\right) \left(f_1 + N_1 \frac{\partial f_1}{\partial N_1} + N_2 \frac{\partial f_2}{\partial N_1} \right)}{\left(N_1 f_1 + N_2 f_2\right)^4} \right)$$

$$= v_2 \left(\frac{\left(\frac{\partial f_2'}{\partial N_1} N_1 f_1 + f_2' f_1 + f_2' N_1 \frac{\partial f_1}{\partial N_1} \right) \left(N_1 f_1 + N_2 f_2\right) - 2 f_2' N_1 f_1 \left(f_1 + N_1 \frac{\partial f_1}{\partial N_1} + N_2 \frac{\partial f_2}{\partial N_1} \right)}{\left(N_1 f_1 + N_2 f_2\right)^3} \right)$$

$$= v_2 \left(\frac{\left(\frac{\partial f_2'}{\partial N_1} N_1 f_1 + f_2' N_1 \frac{\partial f_1}{\partial N_1} \right) \left(N_1 f_1 + N_2 f_2\right) - 2 f_2' N_1 f_1 \left(N_1 \frac{\partial f_1}{\partial N_1} + N_2 \frac{\partial f_2}{\partial N_1} \right) + f_2' f_1 \left(N_2 f_2 - N_1 f_1\right)}{\left(N_1 f_1 + N_2 f_2\right)^3} \right)$$

$$
\begin{aligned}
&= v_2\left(\frac{\left(a_2'b_2'N_1a_1b_1+a_2b_2'N_1a_1'b_1\right)\left(N_1f_1+N_2f_2\right)-2a_2b_2'N_1a_1b_1\left(N_1a_1'b_1+N_2a_2'b_2\right)+f_2'f_1\left(N_2f_2-N_1f_1\right)}{\left(N_1f_1+N_2f_2\right)^3}\right)\\
&= v_2\left(\frac{b_2'N_1b_1\Big(\left(a_2'a_1+a_2a_1'\right)\left(N_1f_1+N_2f_2\right)-2\left(a_2a_1N_1a_1'b_1+a_2a_1N_2a_2'b_2\right)\Big)+f_2'f_1\left(N_2f_2-N_1f_1\right)}{\left(N_1f_1+N_2f_2\right)^3}\right)\\
&= v_2\left(\frac{b_2'N_1b_1\Big(\left(a_1a_2'+a_1'a_2\right)\left(N_1f_1+N_2f_2\right)-2\left(a_1'a_2N_1f_1+a_1a_2'N_2f_2\right)\Big)+f_2'f_1\left(N_2f_2-N_1f_1\right)}{\left(N_1f_1+N_2f_2\right)^3}\right)\\
&= v_2\left(\frac{b_2'N_1b_1\Big(a_1a_2'\left(N_1f_1-N_2f_2\right)+a_1'a_2\left(N_2f_2-N_1f_1\right)\Big)+f_2'f_1\left(N_2f_2-N_1f_1\right)}{\left(N_1f_1+N_2f_2\right)^3}\right)\\
&= v_2\left(\frac{b_2'N_1b_1\overbrace{\left(a_1a_2'-a_1'a_2\right)}^{(-)}\overbrace{\left(N_1f_1-N_2f_2\right)}^{(+)}+\overbrace{f_2'f_1\left(N_2f_2-N_1f_1\right)}^{(-)}}{\left(N_1f_1+N_2f_2\right)^3}\right)<0 \quad \text{if } \frac{a_1'}{a_1}>\frac{a_2'}{a_2} \text{ and } N_1f_1>N_2f_2.
\end{aligned}
$$

□

The main comparative statics results for the non-myopic model with peer effect is given below.

Theorem (Theorem 7 in the main text)**.** *Suppose* $\frac{f_i''(e_i,N_1)}{f_i'(e_i,N_1)}<\frac{c_i''(e_i)}{c_i'(e_i)}$ *for all* i *and* $\frac{a_1'(N_1)}{a_1(N_1)}>\frac{a_2'(N_1)}{a_2(N_1)}$*. Then*

$$
\frac{\partial e_1^*}{\partial N_1}<0,\quad \frac{\partial e_1^*}{\partial N_2}>0,\quad \frac{\partial e_2^*}{\partial N_1}<0,\quad \textit{and}\quad \frac{\partial e_2^*}{\partial N_2}<0.
$$

In addition, the signs of $\frac{df_1(e_1^*,N_1)}{dN_1}$ *and* $\frac{df_2(e_2^*,N_1)}{dN_1}$ *are ambiguous.* □

Proof. Let $\boldsymbol{e^*}=(e_1^*,e_2^*)$ and $\boldsymbol{N}=(N_1,N_2)$. Appealing to the implicit differentiation theorem yields,

$$
\begin{aligned}
D_{\boldsymbol{N}}\boldsymbol{e^*}&=\begin{bmatrix}\frac{\partial e_1^*}{\partial N_1} & \frac{\partial e_1^*}{\partial N_2}\\ \frac{\partial e_2^*}{\partial N_1} & \frac{\partial e_2^*}{\partial N_2}\end{bmatrix}=-\left[D_{\boldsymbol{e^*}}F(e_1^*,e_2^*,N_1,N_2)\right]^{-1}\left[D_{\boldsymbol{N}}F(e_1^*,e_2^*,N_1,N_2)\right]\\
&=\frac{-1}{\text{DET}}\begin{bmatrix}\left(\frac{\partial\widetilde{P}_2}{\partial e_2^*}-c_2''\right)\left(\frac{\partial\widetilde{P}_1}{\partial N_1}\right)-\left(\frac{\partial\widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial\widetilde{P}_2}{\partial N_1}\right) & \left(\frac{\partial\widetilde{P}_2}{\partial e_2^*}-c_2''\right)\left(\frac{\partial\widetilde{P}_1}{\partial N_2}\right)-\left(\frac{\partial\widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial\widetilde{P}_2}{\partial N_2}\right)\\ \left(\frac{\partial\widetilde{P}_1}{\partial e_1^*}-c_1''\right)\left(\frac{\partial\widetilde{P}_2}{\partial N_1}\right)-\left(\frac{\partial\widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial\widetilde{P}_1}{\partial N_1}\right) & \left(\frac{\partial\widetilde{P}_1}{\partial e_1^*}-c_1''\right)\left(\frac{\partial\widetilde{P}_2}{\partial N_2}\right)-\left(\frac{\partial\widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial\widetilde{P}_1}{\partial N_2}\right)\end{bmatrix},
\end{aligned}
$$

where

$$
\text{DET}=\underbrace{\left(\frac{\partial\widetilde{P}_1}{\partial e_1^*}-c_1''\right)}_{(-)}\underbrace{\left(\frac{\partial\widetilde{P}_2}{\partial e_2^*}-c_2''\right)}_{(-)}-\underbrace{\left(\frac{\partial\widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial\widetilde{P}_2}{\partial e_1^*}\right)}_{(\leq 0)}>0 \quad \text{by Lemma 7.}
$$

Combining the above result with Lemma 7, we obtain

$$
\frac{\partial e_1^*}{\partial N_1}=\underbrace{\frac{-1}{DET}}_{(-)}\left(\underbrace{\left(\frac{\partial\widetilde{P}_2}{\partial e_2^*}-c_2''\right)}_{(-)}\underbrace{\left(\frac{\partial\widetilde{P}_1}{\partial N_1}\right)}_{(-)}-\underbrace{\left(\frac{\partial\widetilde{P}_1}{\partial e_2}\right)}_{(+)}\underbrace{\left(\frac{\partial\widetilde{P}_2}{\partial N_1}\right)}_{(-)}\right)<0.
$$

$$\frac{\partial e_2^*}{\partial N_2} = \underbrace{\frac{-1}{DET}}_{(-)}\left(\underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)}_{(-)}\underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right)}_{(-)} - \underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial e_1}\right)}_{(-)}\underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right)}_{(+)}\right) < 0.$$

Next, let $g_i = \frac{v_i f_i'' N_j f_j}{(N_1 f_1 + N_2 f_2)^2}$. Appealing to Lemma 3 yields,

$$g_i - c_i'' = \frac{v_i f_i'' N_j f_j}{(N_1 f_1 + N_2 f_2)^2} - c_i'' = \frac{f_i'' c_i'}{f_i'} - c_i'' < 0 \quad \text{since } \tfrac{f_i''}{f_i'} < \tfrac{c_i''}{c_i'} \text{ by assumption.}$$

In addition, we have

$$\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - g_2\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) = \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right).$$

To see this, note that the derivatives found in the proof of Lemma 7 implies that the above equation is equivalent to:

$$\begin{aligned}
&\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - g_2\right)\left(\frac{v_1 f_1' f_2 \left(N_1 f_1 - N_2 f_2\right)}{\left(N_1 f_1 + N_2 f_2\right)^3}\right) = \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(-\frac{2 v_2 f_2' N_1 f_1 f_2}{\left(N_1 f_1 + N_2 f_2\right)^3}\right) \\
\iff & -\frac{2 v_2 f_2' N_1 f_1 N_2 f_2'}{\left(N_1 f_1 + N_2 f_2\right)^3}\left(\frac{v_1 f_1' f_2 \left(N_1 f_1 - N_2 f_2\right)}{\left(N_1 f_1 + N_2 f_2\right)^3}\right) = \frac{v_1 \left(N_1 f_1 - N_2 f_2\right) f_1' N_2 f_2'}{\left(N_1 f_1 + N_2 f_2\right)^3}\left(-\frac{2 v_2 f_2' N_1 f_1 f_2}{\left(N_1 f_1 + N_2 f_2\right)^3}\right) \\
\iff & f_2' N_1 f_1 N_2 f_2' \left(f_1' f_2 \left(N_1 f_1 - N_2 f_2\right)\right) = \left(N_1 f_1 - N_2 f_2\right) f_1' N_2 f_2' \left(f_2' N_1 f_1 f_2\right),
\end{aligned}$$

which is clearly true. Thus,

$$\begin{aligned}
\frac{\partial e_1^*}{\partial N_2} &= \frac{-1}{DET}\left(\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right)\right) \\
&= \frac{-1}{DET}\left(\left(\frac{\partial \widetilde{P}_2}{\partial e_2^*} - g_2 + g_2 - c_2''\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right) - \left(\frac{\partial \widetilde{P}_1}{\partial e_2^*}\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_2}\right)\right) = \underbrace{\frac{-1}{DET}}_{(-)}\underbrace{\left(g_2 - c_2''\right)}_{(-)}\underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial N_2}\right)}_{(+)} > 0.
\end{aligned}$$

Next, we have

$$\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1\right)\left(\frac{v_2 f_2' f_1 \left(N_2 f_2 - N_1 f_1\right)}{\left(N_1 f_1 + N_2 f_2\right)^3}\right) = \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{-2 v_1 N_2 f_1' f_2 f_1}{\left(N_1 f_1 + N_2 f_2\right)^3}\right) \tag{3}$$

This holds because the equation is equivalent to:

$$\begin{aligned}
&-\frac{2 v_1 f_1' N_2 f_2 N_1 f_1'}{\left(N_1 f_1 + N_2 f_2\right)^3}\left(\frac{v_2 f_2' f_1 \left(N_2 f_2 - N_1 f_1\right)}{\left(N_1 f_1 + N_2 f_2\right)^3}\right) = \frac{v_2 \left(N_2 f_2 - N_1 f_1\right) f_2' N_1 f_1'}{\left(N_1 f_1 + N_2 f_2\right)^3}\left(\frac{-2 v_1 N_2 f_1' f_2 f_1}{\left(N_1 f_1 + N_2 f_2\right)^3}\right) \\
&\iff \left(f_1' N_2 f_2 N_1 f_1'\right)\left(f_2' f_1\right)\left(N_2 f_2 - N_1 f_1\right) = \left(N_2 f_2 - N_1 f_1\right) f_2' N_1 f_1' \left(N_2 f_1' f_2 f_1\right),
\end{aligned}$$

which clears holds. Thus,

$$\begin{aligned}
\frac{\partial e_2^*}{\partial N_1} &= \frac{-1}{DET}\left(\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - c_1''\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right)\right) \\
&= \frac{-1}{DET}\left(\underbrace{\left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1\right)\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{\partial \widetilde{P}_1}{\partial N_1}\right)}_{(*)} + \underbrace{\left(g_1 - c_1''\right)}_{(-)}\underbrace{\left(\frac{\partial \widetilde{P}_2}{\partial N_1}\right)}_{(-)}\right),
\end{aligned}$$

Using the expressions for $\frac{\partial \widetilde{P}_2}{\partial N_1}$ and $\frac{\partial \widetilde{P}_1}{\partial N_1}$ derived in the proof of Lemma 7 and equation (3), we obtain:

$$
\begin{aligned}
(*) &= \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1\right)\left(v_2\left(\frac{b_2' N_1 b_1 \left(a_1 a_2' - a_1' a_2\right)\left(N_1 f_1 - N_2 f_2\right) + f_2' f_1 \left(N_2 f_2 - N_1 f_1\right)}{\left(N_1 f_1 + N_2 f_2\right)^3}\right)\right) \\
&\quad - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(v_1 N_2\left(\frac{b_1' b_2 \left(a_1' a_2 - a_1 a_2'\right)\left(N_2 f_2 - N_1 f_1\right) - 2 f_1' f_2 f_1}{\left(N_1 f_1 + N_2 f_2\right)^3}\right)\right) \\
&= \left(\frac{\partial \widetilde{P}_1}{\partial e_1^*} - g_1\right)\left(\frac{v_2 b_2' N_1 b_1 \left(a_1 a_2' - a_1' a_2\right)\left(N_1 f_1 - N_2 f_2\right)}{\left(N_1 f_1 + N_2 f_2\right)^3}\right) - \left(\frac{\partial \widetilde{P}_2}{\partial e_1^*}\right)\left(\frac{v_1 N_2 b_1' b_2 \left(a_1' a_2 - a_1 a_2'\right)\left(N_2 f_2 - N_1 f_1\right)}{\left(N_1 f_1 + N_2 f_2\right)^3}\right).
\end{aligned}
$$

In addition,

$$
\begin{aligned}
(*) > 0 \iff & \left(-\frac{2 v_1 f_1' N_2 f_2 N_1 f_1'}{\left(N_1 f_1 + N_2 f_2\right)^3}\right)\left(\frac{v_2 b_2' N_1 b_1 \left(a_1 a_2' - a_1' a_2\right)\left(N_1 f_1 - N_2 f_2\right)}{\left(N_1 f_1 + N_2 f_2\right)^3}\right) \\
& > \left(\frac{v_2 \left(N_2 f_2 - N_1 f_1\right) f_2' N_1 f_1'}{\left(N_1 f_1 + N_2 f_2\right)^3}\right)\left(\frac{v_1 N_2 b_1' b_2 \left(a_1' a_2 - a_1 a_2'\right)\left(N_2 f_2 - N_1 f_1\right)}{\left(N_1 f_1 + N_2 f_2\right)^3}\right) \\
\iff & -2 f_1' f_2 b_2' N_1 b_1 \left(a_1 a_2' - a_1' a_2\right) > -\left(N_2 f_2 - N_1 f_1\right) f_2' b_1' b_2 \left(a_1' a_2 - a_1 a_2'\right) \\
\iff & 2 a_1 b_1' a_2 b_2 b_2' N_1 b_1 \left(a_1' a_2 - a_1 a_2'\right) > \left(N_1 f_1 - N_2 f_2\right) a_2 b_2' b_1' b_2 \left(a_1' a_2 - a_1 a_2'\right) \\
\iff & 2 a_1 N_1 b_1 \left(a_1' a_2 - a_1 a_2'\right) > \left(N_1 f_1 - N_2 f_2\right)\left(a_1' a_2 - a_1 a_2'\right) \\
\iff & \left(N_1 f_1 + N_2 f_2\right)\left(a_1' a_2 - a_1 a_2'\right) > 0 \iff \frac{a_1'}{a} > \frac{a_2'}{a_2}.
\end{aligned}
$$

Thus when $\frac{a_1'}{a_1} > \frac{a_2'}{a_2}$, we have $\frac{\partial e_2^*}{\partial N_1} < 0$, as required. Finally, since

$$
\frac{df_1(e_1^*, N_1)}{dN_1} = \underbrace{\left(\frac{\partial f_1}{\partial e_1^*}\right)}_{(+)}\underbrace{\left(\frac{\partial e_1^*}{\partial N_1}\right)}_{(-)} + \underbrace{\frac{\partial f_1}{\partial N_1}}_{(+)} \quad \text{and} \quad \frac{df_2(e_2^*, N_1)}{dN_1} = \underbrace{\left(\frac{\partial f_2}{\partial e_2^*}\right)}_{(+)}\underbrace{\left(\frac{\partial e_2^*}{\partial N_1}\right)}_{(-)} + \underbrace{\frac{\partial f_2}{\partial N_1}}_{(+)},
$$

the signs of the two derivatives are ambiguous without further assumption on f_1 and f_2. □

Figure A1: Distribution of Weekly Homework Hours for 10th-Grade Native-born Strong Students with Normal Density Plot

SOURCE: US Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988

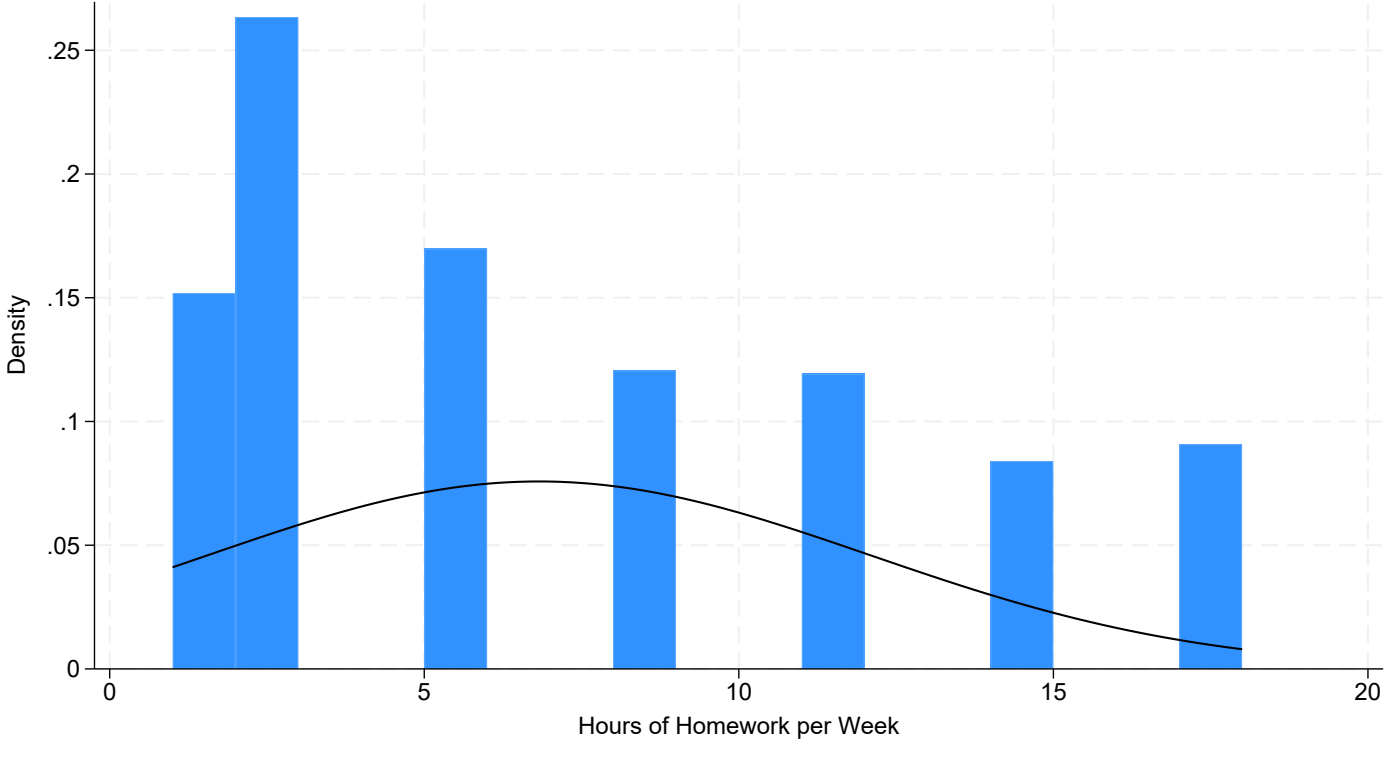

Figure A2: Distribution of Weekly Homework Hours for 12th-Grade Native-born Strong Students with Normal Density Plot

SOURCE: US Department of Education, National Center for Education Statistics, National Education Longitudinal Study of 1988