Online Appendix

A. Key Components for Generating the Inverse S-shaped Probability Weighting Function

In this section, we argue that both efficient coding and a U-shaped prior are fundamentally important for generating an inverse S-shaped weighting function. To illustrate how the weighting function depends on both components, we examine two scenarios. The first scenario keeps the U-shaped prior but replaces the efficient coding rule in equation (3) of the main text by an inefficient one

$$\theta(p) = \left(\sin\left(\frac{\pi}{2}p\right)\right)^2. \tag{A.1}$$

An important observation about this alternative coding rule is that it does not connect the prior belief $f(p)$ with the likelihood function $f(R_p|p)$.[22] Figure A.1 Panel A plots the weighting function $v(p)$ when the coding rule takes the form of equation (A.1). It shows that, in the absence of *efficient* coding, the U-shaped prior alone is insufficient to generate an inverse S-shaped probability weighting function.

[Place Figure A.1 about here]

The second scenario keeps the efficient coding rule but replaces the U-shaped prior by a uniform prior. That is, we replace the U-shaped prior in equation (8) of the main text by a uniform prior $f(p) = 1$. Figure A.1 Panel B shows that, with the uniform prior, efficient coding alone is also insufficient to generate an inverse S-shaped weighting function. The results in Figure A.1 therefore highlight that it is the particular set of likelihood functions that are optimized under a U-shaped prior that are crucial for generating the inverse S-shaped weighting function.

[22] There are an infinite number of inefficient coding rules; equation (A.1) is just one of them that we use to explain why a noisy coding model with a U-shaped prior is insufficient to generate an inverse S-shaped weighting function.

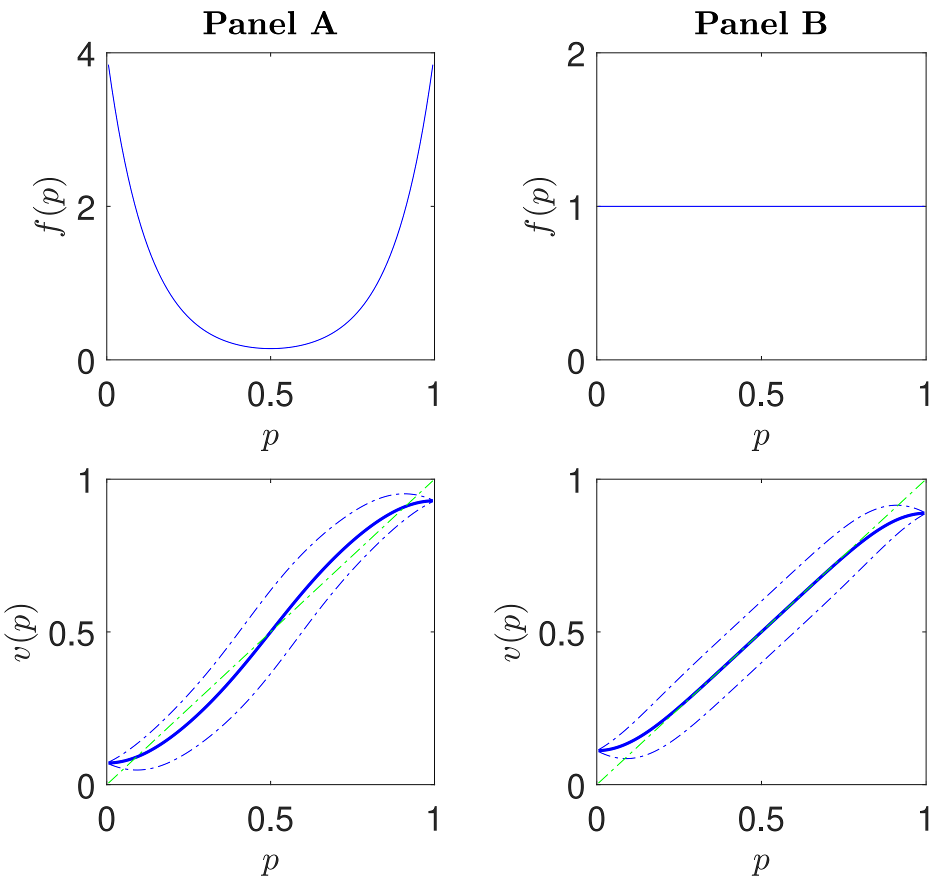


Figure A.1

Model implications when relaxing the efficient coding or U-shaped prior assumption

Panel A: the upper graph plots a U-shaped prior distribution in the form of (8) described in the main text; the parameter values are: $\lambda_1 = \lambda_2 = 8$ and $w = 0.5$. The lower graph plots the subjective valuation implied by inefficient coding, $v(p)$, and its one-standard-deviation bounds $v(p) \pm \sigma(p)$; when computing $v(p)$ and $\sigma(p)$, we use the coding rule from equation (A.1). Panel B: the upper graph plots a uniform prior distribution $f(p) = 1$. The lower graph plots the subjective valuation implied by efficient coding, $v(p)$, and its one-standard-deviation bounds $v(p) \pm \sigma(p)$; when computing $v(p)$ and $\sigma(p)$, we use the coding rule from equation (3) in the main text. For both panels, we set the parameter n to 10. In the lower graph of each panel, the green dash-dot line is the forty-five degree line.

B. Multi-Dimensional Efficient Coding

Consider a binary risky lottery: $(\$X, p; \$0, 1-p)$. In this section, we analyze an efficient coding model with two dimensions: the first dimension is X, the lottery upside; the second dimension is p, the probability that the risky lottery delivers X. Suppose that the DM holds prior beliefs about X and p, denoted by $f(X, p)$. When the lottery is revealed to the DM, she draws a noisy signal, R_x, of X, from the likelihood function $f(R_x|X, p)$ and a noisy signal, R_p, of p, from the likelihood function $f(R_p|X, p)$. We then assume that the DM encodes X and p through a total number of n "neurons" and that she chooses $f(R_x|X, p)$ and $f(R_p|X, p)$ to maximize

$$I((X, p); (R_x, R_p)), \tag{B.1}$$

which is the mutual information between the payoff-probability pair, (X, p), and its noisy representation (R_x, R_p).

Here, we analyze a special case of the above model in which the DM has a prior that X and p are drawn independently. In this case, p contains no information about X and therefore $f(R_x|X, p) = f(R_x|X)$; similarly, X contains no information about p and therefore $f(R_p|X, p) = f(R_p|p)$. Moreover, it is easy to show that, when X and p are drawn independently,

$$I((X, p); (R_x, R_p)) = I(X; R_x) + I(p; R_p). \tag{B.2}$$

That is, the overall mutual information in (B.1) is equal to the sum of $I(X; R_x)$, the mutual information between X and R_x, and $I(p; R_p)$, the mutual information between p and R_p.

We then follow Heng et al. (2020) and obtain

$$\begin{aligned} f(R_x|X) &= \binom{n_x}{R_x}(\theta(X))^{R_x}(1-\theta(X))^{n_x-R_x}, \\ f(R_p|p) &= \binom{n_p}{R_p}(\theta(p))^{R_p}(1-\theta(p))^{n_p-R_p}, \end{aligned} \tag{B.3}$$

where

$$\theta(X) = \left(\sin\left(\frac{\pi}{2}F(X)\right)\right)^2, \quad \theta(p) = \left(\sin\left(\frac{\pi}{2}F(p)\right)\right)^2. \tag{B.4}$$

That is, when the DM maximizes mutual information within each dimension, she arrives at the coding functions, $\theta(X)$ and $\theta(p)$, derived in Heng et al. (2020). As such, the constrained optimization problem faced by the DM reduces to

$$\max_{n_x>0, n_p>0}\{I(X; R_x) + I(p; R_p)\}, \tag{B.5}$$

subject to equations (B.3) and (B.4) and $n_x + n_p = n$. We solve this constrained optimization problem numerically.

[Place Figure B.1 about here]

Figure B.1 provides a numerical example of the model's solution. Suppose the DM holds a prior that X is drawn uniformly from $[23, 27]$ and that X and p are drawn independently. Further suppose that the DM has a total of $n = 15$ neurons to encode X and p. We then consider the two mixed priors, which take the form of (9) described in Section IV. For both mixed priors, the stable component takes the form of (10); the parameter values are: $\lambda_1 = \lambda_2 = 8$ and $w = 0.5$. In the intermediate condition, the fast-moving component takes the form of (11). In the extreme condition, the fast-moving component takes the form of (12). The parameter values are: $p_l = 0.38$, $p_h = 0.62$, $p_{l,1} = 0.1$, $p_{l,2} = 0.21$, $p_{h,1} = 0.79$, and $p_{h,2} = 0.9$. The weight ξ the DM assigns to the stable component is 0.5.

Following the constrained optimization described by (B.5), the DM optimally allocates $n_x = 8$ neurons to encode X and $n_p = 7$ neurons to encode p in the intermediate condition; she optimally allocates $n_x = 7$ neurons to encode X and $n_p = 8$ neurons to encode p in the extreme condition. Figure B.1 plots the probability weighting functions implied by the intermediate and the extreme conditions. Comparing Figure B.1 with Figure V, we observe that the theoretical predictions discussed in Section IV regarding the malleability of probability weighting remain robust to allowing for noisy coding of X.

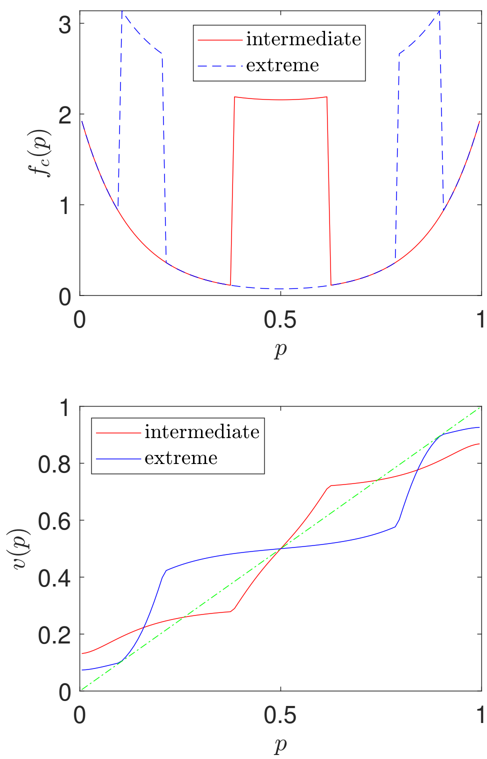


Figure B.1
Prior distribution and value function: Multi-dimensional efficient coding

The upper graph plots a mixed prior distribution $f_c(p)$ in the form of (9) described in the main text. The first, stable component takes the form of (10); the parameter values are: $\lambda_1 = \lambda_2 = 8$ and $w = 0.5$. The second, fast-moving component takes the form of (11) in the intermediate condition and takes the form of (12) in the extreme condition; the parameter values are: $p_l = 0.38$, $p_h = 0.62$, $p_{l,1} = 0.1$, $p_{l,2} = 0.21$, $p_{h,1} = 0.79$, and $p_{h,2} = 0.9$. The weight ξ the DM assigns to the stable component is 0.5. The lower graph plots, for both the intermediate condition and the extreme condition, the probability weighting function implied by the multi-dimensional efficient coding model described in Online Appendix B; here we set $n = 15$. For both the intermediate condition and the extreme condition, X is drawn uniformly from $[23, 27]$. In the intermediate condition, the DM optimally chooses $n_x = 8$ and $n_p = 7$; in the extreme condition, the DM optimally chooses $n_x = 7$ and $n_p = 8$. The green dash-dot line is the forty-five degree line.

C. Expected Payoff from Experiment 2

In this section, we derive the expected payoff a subject receives from Experiment 2. As described in Section V, each subject receives a fix amount of \$3.00 for participating in the experiment. As such, when computing the subject's expected payoff, we focus only on the bonus component.

Suppose, for a given trial in Experiment 2, the objective probability is p. Given p, the subject's perceptual system generates a noisy signal R_p from $f(R_p|p)$. Then, given R_p, the subject reports a certainty equivalent of $25 \cdot \mathbb{E}[\tilde{p}|R_p]$. With the Becker-Degroot-Marschak (Becker et al., 1964) incentive scheme, the expected bonus payoff the subject receives, conditional on p and R_p, is

$$\begin{aligned}
& \frac{1}{25}\int_{25\cdot\mathbb{E}[\tilde{p}|R_p]}^{25} q dq + \frac{1}{25}\int_{0}^{25\cdot\mathbb{E}[\tilde{p}|R_p]} (25 \times p) dq \\
= \quad & \frac{1}{25}\int_{25p}^{25} q dq + \frac{1}{25}\int_{0}^{25p} (25 \times p) dq - \frac{1}{25}\int_{25p}^{25\cdot\mathbb{E}[\tilde{p}|R_p]} (q - 25p) dq \\
= \quad & \frac{25}{2}(1+p^2) - \frac{25}{2}(\mathbb{E}[\tilde{p}|R_p] - p)^2. \qquad \text{(C.1)}
\end{aligned}$$

Note that equation (C.1) corresponds to the limiting case of the 2,500-row payoff scheme we implemented in Experiment 2; the details of this 2,500-row payoff scheme are provided in Online Appendix E.2; as the number of rows increases from 2,500 to infinity, the expected bonus payoff approaches the expression given by equation (C.1).

Averaging across different R_p for a given p, and further averaging across different values of p drawn from the subject's prior belief $f(p)$, the expected bonus payment is

$$\mathbb{E}[\text{payoff}] = \frac{25}{2}\int_0^1 (1+p^2) f(p) dp - \frac{25}{2}\int_0^1 \left(\sum\nolimits_{R_p=0}^{n} (\mathbb{E}[\tilde{p}|R_p] - p)^2 f(R_p|p)\right) f(p) dp, \qquad \text{(C.2)}$$

which is equation (15) in the main text.

[Place Figure C.1 about here]

Given the DM's performance objective in equation (C.2) and the U-shaped prior from equation (8) of the main text, Figure C.1 plots the optimal coding rule $\hat{\theta}(p)$, which we solve numerically, and the subjective valuation $\hat{v}(p)$ implied by $\hat{\theta}(p)$.

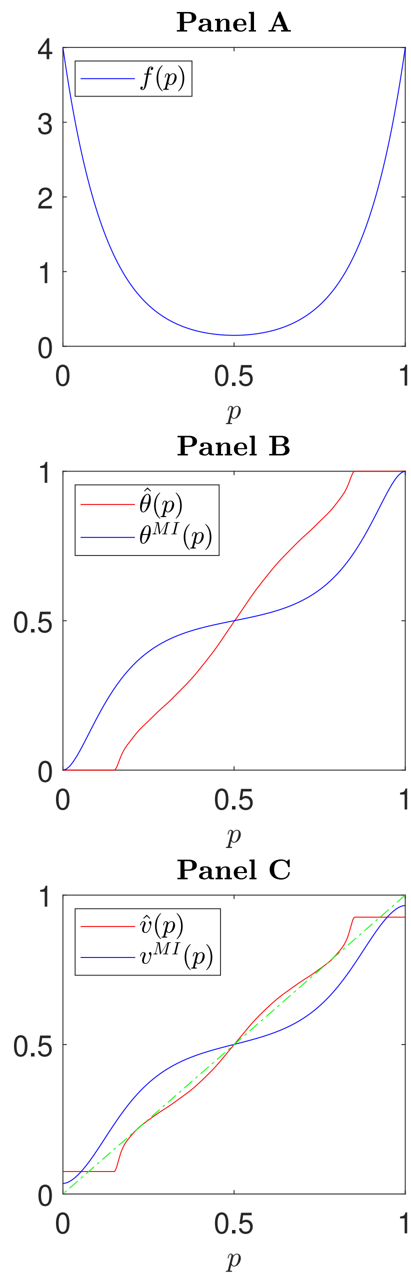


Figure C.1

Comparing performance objectives: Maximizing mutual information vs. maximizing expected payoff

Panel A plots a U-shaped prior distribution in the form of (8) described in the main text; the parameter values are: $\lambda_1 = \lambda_2 = 8$ and $w = 0.5$. Panel B plots the coding rule $\hat{\theta}(p)$, numerically solved for maximizing the expected payoff given by equation (C.2); it also plots the coding rule $\theta^{MI}(p)$, defined in equation (3), that maximizes the mutual information between p and R_p. Panel C plots the subjective valuation $\hat{v}(p)$ implied by $\hat{\theta}(p)$; it also plots the subjective valuation $v^{MI}(p)$ implied by $\theta^{MI}(p)$; the green dash-dot line is the forty-five degree line. For both Panel B and Panel C, we set the parameter n to 10.

D. Experimental Designs

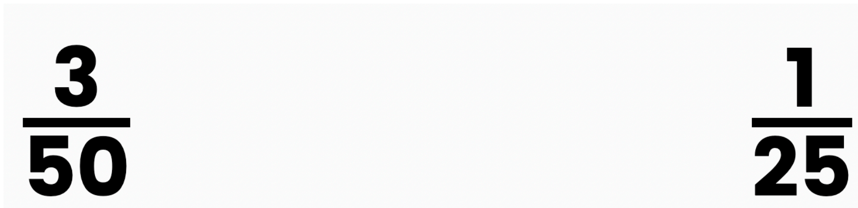


Figure D.1
Screenshot from Experiment 1

Subjects are incentivized to quickly and accurately select the larger of the two displayed fractions. Subjects input their response on the keyboard using either the “Q” key (for left) or the “P” key (for right).

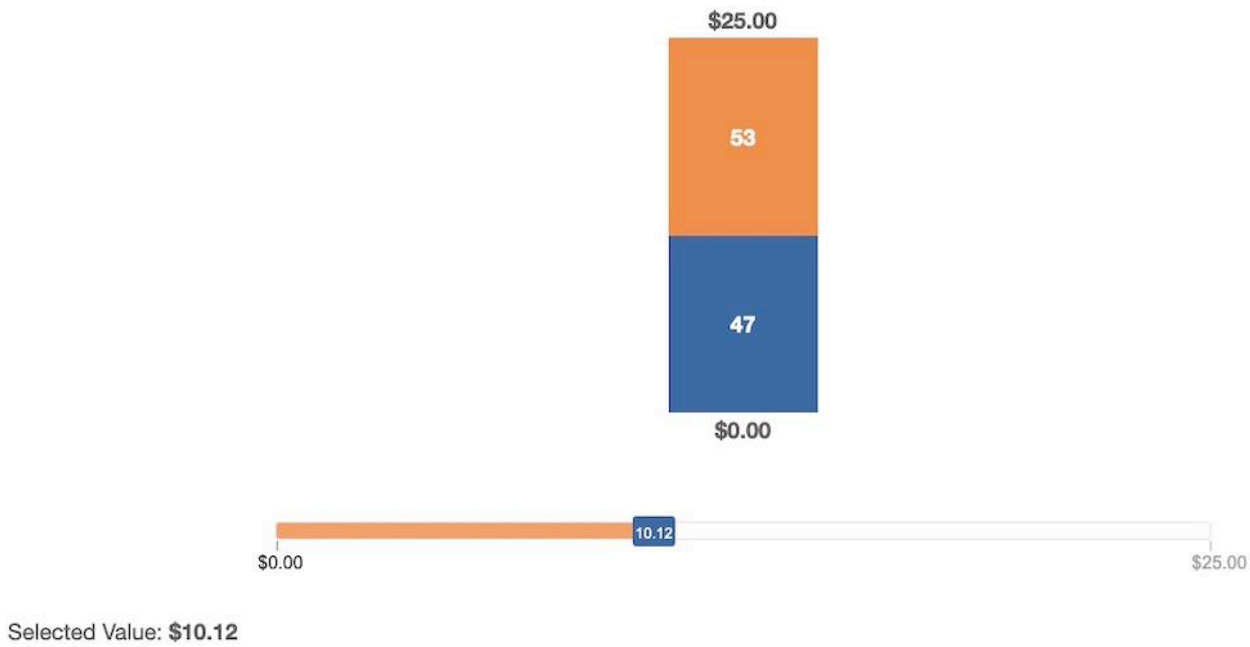


Figure D.2
Screenshot of trial from Experiment 2

In this example, the risky lottery pays $25 with 53% probability and $0 with 47% probability. The slider position indicates that the subject's reported certainty equivalent on this trial is $10.12.

Table D.1
Fraction pairs in Experiment 1

Test trials				Catch trials			
Fraction 1 num	Fraction 1 denom	Fraction 2 num	Fraction 2 denom	Fraction 1 num	Fraction 1 denom	Fraction 2 num	Fraction 2 denom
1	25	3	50	3	45	1	20
2	25	3	50	2	9	3	8
3	25	7	50	7	55	3	20
4	25	7	50	5	6	3	8
4	25	9	50	9	46	4	21
6	25	11	50	3	7	2	7
6	25	13	50	13	55	6	21
7	25	13	50	5	9	3	7
8	25	17	50	17	45	8	21
9	25	17	50	5	9	2	7
9	25	19	50	19	55	9	20
11	25	21	50	3	5	3	8
11	25	23	50	23	45	11	20
12	25	23	50	7	8	2	9
13	25	27	50	27	55	13	20
14	25	27	50	5	6	7	8
14	25	29	50	29	45	14	21
16	25	31	50	2	9	5	9
16	25	33	50	33	55	16	21
17	25	33	50	5	6	3	4
18	25	37	50	37	45	18	19
19	25	37	50	3	7	5	6
19	25	39	50	39	55	19	20
21	25	41	50	7	8	3	7
21	25	43	50	43	45	21	25
22	25	43	50	5	6	3	7
23	25	47	50	47	55	23	25
24	25	47	50	2	7	7	8

Notes. The table presents the fraction pairs used in the discrimination task from Experiment 1. The left panel provides the 28 fraction pairs used in the test trials; the right panel provides the 28 fraction pairs used in the catch trials. For each panel and each row, “Fraction 1 num” and “Fraction 1 denom” denote the numerator and denominator of the first fraction. “Fraction 2 num” and “Fraction 2 denom” denote the numerator and denominator of the second fraction. We counterbalance the left-right ordering of the two fractions so that each subject is presented with a total of 56 test trials.

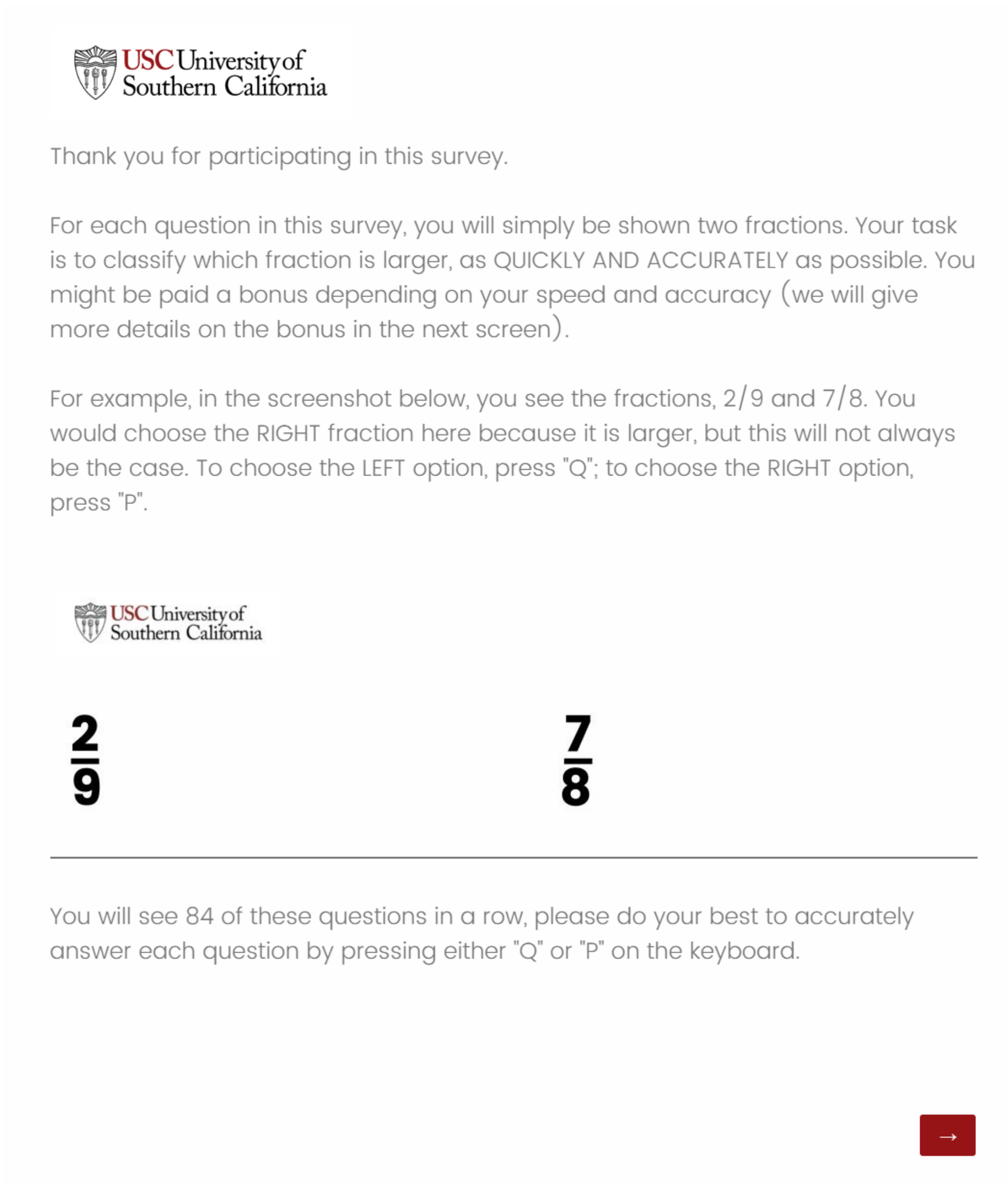
Table D.2
Design parameters for Experiment 2

Adaptation conditions				Test conditions		
Extreme	Low	Intermediate	High	Low	Intermediate	High
10	10	38	67	11	38	69
11	11	39	68	15	42	73
12	12	40	69	19	47	77
13	13	41	70	23	53	81
14	14	42	71	27	58	85
15	15	43	72	31	62	89
16	16	44	73			
17	17	45	74			
18	18	46	75			
19	19	47	76			
20	20	48	77			
21	21	49	78			
79	22	51	79			
80	23	52	80			
81	24	53	81			
82	25	54	82			
83	26	55	83			
84	27	56	84			
85	28	57	85			
86	29	58	86			
87	30	59	87			
88	31	60	88			
89	32	61	89			
90	33	62	90			

Notes. The table provides the specific values of probability (in percentage) used in each of the adaptation and test conditions. Each of the four columns on the left corresponds to an adaptation condition, and it contains 24 trials. Each of the three columns on the right corresponds to a test condition, and it contains 6 trials. Each entry in the table denotes a probability p associated with the upside payoff of \$25. Subjects who are randomized into the low test condition are further randomized into either the low or intermediate adaptation condition; subjects who are randomized into the high test condition are further randomized into either the high or intermediate adaptation condition; subjects who are randomized into the intermediate test condition are further randomized into either the extreme or intermediate adaptation condition.

E. Experimental Instructions

E.1. Instructions for the discrimination task in Experiment 1



USC University of Southern California

Thank you for participating in this survey.

For each question in this survey, you will simply be shown two fractions. Your task is to classify which fraction is larger, as QUICKLY AND ACCURATELY as possible. You might be paid a bonus depending on your speed and accuracy (we will give more details on the bonus in the next screen).

For example, in the screenshot below, you see the fractions, 2/9 and 7/8. You would choose the RIGHT fraction here because it is larger, but this will not always be the case. To choose the LEFT option, press "Q"; to choose the RIGHT option, press "P".

USC University of Southern California

$\frac{2}{9}$ $\frac{7}{8}$

You will see 84 of these questions in a row, please do your best to accurately answer each question by pressing either "Q" or "P" on the keyboard.

→

Bonus Payment

1 out of every 10 survey respondents will be selected to receive a bonus payment. The bonus payment is $0.10 for every correct answer, but we will reduce this amount by $0.01 for every second it takes you to respond. Therefore, if you answered all 84 questions correctly, and it took you an average of 4 seconds to respond to each question, and you were randomly selected to receive the bonus payment, **you would receive a bonus of $8.40 - $3.36 = $5.04.** If instead, you only answered 50 out of the 84 questions correctly, and it took you an average of 4 seconds to respond, and you were randomly selected to receive a bonus, **you would receive a bonus of $5.00 - $3.36 = $1.64.** [Note that we impose a minimum bonus of $0.]

→

You are now ready to begin. Remember, you will see 84 of these questions, and your task is to CHOOSE THE LARGER FRACTION.

Press Q for LEFT and P for RIGHT.

→

E.2. Instructions for the valuation task in Experiment 2

USC University of Southern California

Instructions (Page 1 of 3)

Please read these instructions carefully. There will be a short comprehension quiz at the end of the instructions, which you need to pass before continuing to the study.

The study consists of approximately 30 rounds. In each round, you will see a gamble which pays either $25 or $0, each with a different probability. For example, if you see the following gamble:

this means that the gamble pays $25 with 53% chance, and $0 with 47% chance. Importantly, there are no right or wrong answers in this study, please answer all questions depending on your personal opinions.

→

USC University of Southern California

Instructions (page 2 of 3)

On each round, you will be asked to tell us what amount of money -- in your opinion -- the gamble is worth to you. In order to help you form your opinions, think about the amount of money as the maximum price you'd be willing to pay to play the gamble. We will ask you to enter your decision on each round using a slider that goes from $0 (on the left) to $25 (on the right), like in the example below.

Selected Value: **$10.12**

In the above example, the person indicated that a 53% chance of winning $25 is worth $10.12 to them. In other words, the maximum amount they'd be willing to pay to play the gamble is $10.12. We emphasize that different people will give different answers to the same question, since answers depend on your own opinions. There are no right or wrong answers in this survey.

→

USC University of Southern California

Instructions (Page 3 of 3)

10% of all participants will be chosen to receive a bonus, in addition to the $3 for completing the survey.
We have developed the bonus payment method in a way that makes it optimal for you to report your true valuation of the gamble. If you don't care about the exact mechanism that we use to compute the bonus payment, you can feel free to skip the rest of the details on this screen (the comprehension quiz will not ask about how bonuses are computed). If you would like to know exactly how we determine the bonus, then please read below.

Details on how bonus is computed:

As you know, you will see a new gamble on each round and you will be asked to give a certain payment amount that is worth as much as the gamble to you. We will use your response to fill in the table below which actually contains 2,500 questions (for example, see the table below which corresponds to a round in which the gamble offers a 53% chance of winning $25 and 47% chance of winning $0). Question #1 asks if you'd rather have the gamble or $0.01 with certainty. Question #2 asks if you'd rather have the gamble or $0.02 for with certainty. Question #2,500 asks if you'd rather have the gamble or $25.00 with certainty.

Question #		Option A		Option B
1	Would you rather have:	With probability **53%: Get $ 25** With probability **47%: Get $ 0**	or	$0.01 with certainty
2	Would you rather have:		or	$0.02 with certainty
3	Would you rather have:		or	$0.03 with certainty
4	Would you rather have:		or	$0.04 with certainty
…	…		…	…
2,499	Would you rather have:		or	$24.99 with certainty
2,500	Would you rather have:		or	$25.00 with certainty

Once we get your response on the slider, we will fill in the answer to all 2,500 questions as follows. Suppose your response on the slider is denoted by "x". Then, we will assume you would have chosen Option A for all questions for which the certain amount in Option B is less than or equal to x. We will also assume you would have Chosen Option B for all questions for which the certain amount in Option B is greater than x.

If you are chosen to receive a bonus, then we will then randomly pick one of the 2,500 questions (from one randomly chosen round) and pay you according to what you chose on that one question. Each question and each round is equally likely to be chosen for payment. If you chose Option B on the randomly selected question, you would be paid the certain amount. If you chose Option A on the randomly selected question, then the gamble would be played out by the computer. Using the above table as an example, if you chose Option A then you would receive $25 with 53% chance and $0 with 47% chance.

Comprehension Quiz

The following two questions test your understanding of the instructions. Please answer them correctly in order to continue to the survey. Suppose that someone is presented with the gamble below and answers with the following slider location.

Selected Value: **$7.43**

1. Which one of the following statements is correct?

The person values the gamble at exactly $7.43.

The person values the gamble at more than $7.43

The person values the gamble at less than $7.43

2. Which one of of the following statements is correct?

The person would rather have $4.24 with certainty than play the gamble

The person would rather have $6.43 with certainty than play the gamble

The person would rather have $8.34 with certainty than play the gamble

→

You have successfully completed the comprehension quiz.

To begin the survey, please click the button below. Remember to answer each question carefully as you might be chosen to receive a bonus according to your answer on that question.

→