Online Appendix for: The Impact of Bretton Woods International Capital Controls on the Global Economy and the Value of Geopolitical Stability: A General Equilibrium Analysis

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Contents

A Model Solution and Computation

We begin by describing the pseudo social planner's problem used to compute equilibria, and prove its equivalence with the competitive equilibrium problem. We next show how we transform both problems into intensive form problems that are stationary. Finally, we discuss the balanced growth path of the deterministic version of our model or, equivalently, the steady state of the deterministic intensive form model.

The Pseudo Social Planner's Problem

Consider a social planner whose problem is to choose state-, date-, and country-contingent sequences of consumption, capital, and hours worked to maximize:

$$E_0\left[\sum_j \chi_{jt}^C \sum_{t=0}^{\infty} \beta^t \left\{\ln\left(\frac{C_{jt}}{N_{jt}}\right) - \chi_{jt}^I \chi_{jt}^H \frac{\varphi}{1+\gamma}\left(\frac{h_{jt}N_{jt}}{N_{jt}}\right)^{1+\gamma}\right\} N_{jt}\right],$$

subject to a world resource constraint for each state and date

$$\begin{aligned} & \sum_j \left\{C_{jt} + \chi_{jt}^I X_{jt} + G_{jt}\right\} \\ = & \sum_j \chi_{jt}^I Y_{jt} + T_t^{PSPP} \\ = & \sum_j \chi_{jt}^I A_{jt} K_{jt}^{\alpha} \left(h_{jt}N_{jt}\right)^{1-\alpha} + T_t^{PSPP}, \end{aligned}$$

capital evolution equations for each country j of the form

$$K_{jt+1} = (1-\delta) K_{jt} + X_{jt} - \phi\left(\frac{X_{jt}}{K_{jt}}\right) K_{jt},$$

an exogenous path for the series of additive shocks to the resource constraint T^{PSPP} (which the social planner takes as given, but in equilibrium satisfy $T_t^{PSPP} = \sum_j \chi_{jt}^I \left(X_{jt} - Y_{jt}\right)$), and exogenous paths of population, productivity, and the social planner's "wedges"χ_{jt}^I, χ_{jt}^H, and χ_{jt}^C to be described next.

For χ_{jt}^H we assume the process is given by

$$\ln \chi_{jt+1}^H = \left(1 - \rho_j^h\right) \ln \chi_{jSS}^H + \rho_j^H \ln \chi_{jt}^H + \sigma_j^H \varepsilon_{jt+1}^H, \tag{1}$$

and link the process for this wedge to the processes for the competitive equilibrium wedge through parameter restrictions

$$\begin{aligned}\chi^H_{jSS} &= 1/\left(1-\tau^h_{jSS}\right),\\ \rho^H_j &= \rho^h_j,\\ \sigma^H_j &= \sigma^h_j.\end{aligned}$$

For the social planner's consumption wedge, we normalize $\chi^C_{Rt} = \chi^C_{RSS} = 1$, while for $j = U, E$ we require

$$\ln \chi^C_{jt+1} = \left(1-\rho^C_j\right)\ln \chi^C_{jSS} + \rho^C_j \ln \chi^C_{jt} + \varepsilon^C_{jt+1},$$

with the process for ε^C_{jt} assumed to be autoregressive and of the form

$$\varepsilon^C_{jt+1} = \rho^{\varepsilon^C}_j \varepsilon^C_{jt} + \sigma^{\varepsilon^C}_j \epsilon^{\varepsilon^C}_{jt+1},$$

with $\epsilon^{\varepsilon^C}_{jt+1}$ assumed to be standard normal. To ensure consistency with our competitive equilibrium problem we impose the parameter restrictions

$$\begin{aligned}1-\rho^C_j &= \frac{\psi_{j1}}{1+\psi_{j1}},\\ \chi^C_{jSS} &= \psi_{j0},\\ \rho^{\varepsilon^C}_j &= \frac{\rho^B_j}{1+\psi_{j1}},\\ \sigma^{\varepsilon^C}_j &= \frac{\sigma^B_j}{1+\psi_{j1}}.\end{aligned}$$

For the investment wedge, we assume that it's growth rate is related to past growth rates of itself, and to contemporaneous and lagged growth rates of the consumption wedge

$$\ln\left(\frac{\chi^I_{jt+1}}{\chi^I_{jt}}\right) = \left(1-\rho^I_j\right)\ln\left(1+g^{\chi^I}_{jSS}\right) - \ln\left(\frac{\chi^C_{jt+1}}{\chi^C_{jt}}\right) + \rho^I_j \ln\left(\frac{\chi^I_{jt}}{\chi^I_{jt-1}}\frac{\chi^C_{jt}}{\chi^C_{jt-1}}\right) + \sigma^{\chi^I}_j \varepsilon^I_{jt+1},$$

and impose parameter restrictions linking it to the evolution of the capital wedge in the competitive equilibrium problem.

$$\begin{aligned}\rho^I_j &= \rho^K_j,\\ 1+g^{\chi^I}_{jSS} &= 1-\tau^K_{jSS}\\ \sigma^{\chi^I}_j &= \sigma^K_j.\end{aligned} \tag{2}$$

Note that, compared to the competitive equilibrium problem, the formulation of this problem and the specification of the wedges are non-standard. As just one example, the investment wedge χ^I now appears in the objective function *and* multiplies *both* the production function *and* investment in the resource constraint. This specification is necessary to recover the competitive equilibrium allocations. This is quite intuitive: the investment wedge χ^I must multiply both output and investment in the resource constraint in order to replicate the capital wedge, which is modeled as a tax on the gross return to capital inclusive of the value of capital, but this in turn causes it to enter the planner's optimality condition for labor. The addition of the investment wedge as a multiplier on leisure ensures that the investment wedge cancels when determining optimal labor supply. As another example, the error term in the social planner's consumption wedge is autoregressive. As yet another example, we impose a specific relationship between the investment wedge and the consumption wedge. As a result of the unusual nature of this mapping to the planner's problem, we discuss the competitive equilibrium model in the paper.

Under a restriction on the growth of the world economy (so that the expected summation in the objective function is finite), this problem is well defined. It is also concave. Hence, the necessary and sufficient conditions for an optimum include

$$C_{jt} \quad : \quad \beta^t \chi^C_{jt} \frac{N_{jt}}{C_{jt}} = \lambda^{PSPP}_t, \tag{3}$$

$$h_{jt} \quad : \quad \beta^t \chi^C_{jt} \chi^H_{jt} \psi h^{\gamma}_{jt} = \lambda^{PSPP}_t (1-\alpha) \frac{Y_{jt}}{h_{jt} N_{jt}} \tag{4}$$

$$\begin{aligned} K_{jt+1} \quad : \quad \mu^{PSPP}_{jt} = E \Bigg[& \lambda^{PSPP}_{t+1} \chi^I_{jt+1} \alpha \frac{Y_{jt+1}}{K_{jt+1}} \\ & + \mu^{PSPP}_{jt+1} \left(1 - \delta - \phi \left(\frac{X_{jt+1}}{K_{jt+1}} \right) + \phi' \left(\frac{X_{jt+1}}{K_{jt+1}} \right) \frac{X_{jt+1}}{K_{jt+1}} \right) \Bigg] \end{aligned} \tag{5}$$

$$X_{jt} \quad : \quad \lambda^{PSPP}_t \chi^I_{jt} = \mu^{PSPP}_{jt} \left(1 - \phi' \left(\frac{X_{jt}}{K_{jt}} \right) \right) \tag{6}$$

where λ^{PSPP}_t is the multiplier on the resource constraint at time t and μ^{PSPP}_{jt} is the one on the capital evolution equation in country j at time t.

To establish the mapping between the two problems, it is sufficient to show that a solution to these necessary and sufficient conditions is also a solution to the necessary conditions for the competitive equilibrium problem. We do this next.

Equivalence Between the Solution of the Pseudo Social Planner's Problem and the Competitive Equilibrium

To construct the mapping, we first find a solution to the competitive equilibrium problem (CEP), and show that the solution to the necessary and sufficient conditions for an optimum of the PSPP is also a solution to the necessary conditions for the CEP. For this, it is sufficient to show that both the prices and the Lagrange multipliers that ensure that the optimality conditions from the CEP are satisfied.

Consider the first-order condition (FOC) of the PSPP with respect to consumption (3). The corresponding FOC of the household's problem from the CEP is

$$\beta^t \frac{N_{jt}}{C_{jt}} = \lambda_{jt}^{HH},$$

and so the two conditions are equivalent iff

$$\lambda_{jt}^{HH} = \frac{\lambda_t^{PSPP}}{\chi_{jt}^C}. \tag{7}$$

Likewise, the FOC of the PSPP with respect to hours (4) can be compared with the corresponding FOC of the household's problem from the CEP

$$\beta^t \psi h_{jt}^{\gamma} = \lambda_{jt}^{HH} \left(1 - \tau_{jt}^h\right) W_{jt}.$$

Hence, the two conditions are equivalent iff

$$\lambda_{jt}^{HH} \left(1 - \tau_{jt}^h\right) W_{jt} = \frac{\lambda_t^{PSPP}}{\chi_{jt}^C} \frac{1}{\chi_{jt}^H} (1 - \alpha) \frac{Y_{jt}}{h_{jt} N_{jt}}.$$

But imposing (7), we can see that the conditions will be equivalent if

$$W_{jt} = (1 - \alpha) \frac{Y_{jt}}{h_{jt} N_{jt}}, \tag{8}$$

$$1 - \tau_{jt}^h = \frac{1}{\chi_{jt}^H}. \tag{9}$$

Note that (8) implies that the FOC for labor for the firm producing the consumption good in the CEP is now satisfied. Moreover, given assumption (1), the derived process for $1 - \tau_{jt}^h$ satisfied the law of motion (**??**) from the CEP because

$$\ln \chi_{jt+1}^H = \left(1 - \rho_j^h\right) \ln \chi_{jSS}^H + \rho_j^H \ln \chi_{jt}^H + \sigma_j^H \varepsilon_{jt+1}^H,$$

becomes

$$\ln\left(1-\tau_{jt+1}^{h}\right)=\left(1-\rho_{j}^{h}\right)\ln\left(1-\tau_{jSS}^{h}\right)+\rho_{j}^{h}\ln\left(1-\tau_{jt}^{h}\right)+\sigma_{j}^{h}\varepsilon_{jt+1}^{h},$$

under our assumptions on parameters above with $\varepsilon_{jt+1}^{H}=-\varepsilon_{jt+1}^{h}$.

The FOCs of the PSPP in consumption for country j and the rest of the world can be combined to yield

$$\frac{C_{jt}/N_{jt}}{C_{Rt}/N_{Rt}}=\frac{\chi_{jt}^{C}}{\chi_{Rt}^{C}}.$$

Under our normalization and parameter restrictions, this implies

$$\ln\frac{C_{jt+1}/N_{jt+1}}{C_{Rt+1}/N_{Rt+1}}=\frac{\psi_{j1}}{1+\psi_{j1}}\ln\psi_{j0}+\frac{1}{1+\psi_{j1}}\ln\frac{C_{jt}/N_{jt}}{C_{Rt}/N_{Rt}}+\varepsilon_{jt+1}^{C},$$

which is exactly equation (**??**) from the CEP problem with $\varepsilon_{jt+1}^{C}=\ln\left(1-\tau_{jt+1}^{*B}\right)$.

The FOC with respect to capital from the PSPP (5) combined with the FOC with respect to investment (6) can be rearranged to yield

$$\frac{\lambda_{t}^{PSPP}\chi_{jt}^{I}}{1-\phi'\left(\frac{X_{jt}}{K_{jt}}\right)}=E_{t}\left[\lambda_{t+1}^{PSPP}\chi_{jt+1}^{I}\left(\alpha\frac{Y_{jt+1}}{K_{jt+1}}+\frac{1-\delta-\phi\left(\frac{X_{jt+1}}{K_{jt+1}}\right)+\phi'\left(\frac{X_{jt+1}}{K_{jt+1}}\right)\frac{X_{jt+1}}{K_{jt+1}}}{1-\phi'\left(\frac{X_{jt+1}}{K_{jt+1}}\right)}\right)\right].$$

Comparing this with the FOC in capital from the household's problem

$$\lambda_{jt}^{HH}P_{jt}^{K}=E_{t}\left[\lambda_{jt+1}^{HH}\left(1-\tau_{jt+1}^{K}\right)\left(r_{jt+1}^{K}+P_{jt+1}^{*K}\right)\right],$$

we can see that the two will be equivalent if

$$\begin{aligned}
r_{jt+1}^{K} &= \alpha\frac{Y_{jt+1}}{K_{jt+1}},\\
P_{jt}^{K} &= \frac{1}{1-\phi'\left(\frac{X_{jt}}{K_{jt}}\right)},\\
P_{jt+1}^{*K} &= \frac{1-\delta-\phi\left(\frac{X_{jt+1}}{K_{jt+1}}\right)+\phi'\left(\frac{X_{jt+1}}{K_{jt+1}}\right)\frac{X_{jt+1}}{K_{jt+1}}}{1-\phi'\left(\frac{X_{jt+1}}{K_{jt+1}}\right)},\\
1-\tau_{jt+1}^{K} &= \frac{\chi_{jt+1}^{C}}{\chi_{jt}^{C}}\frac{\chi_{jt+1}^{I}}{\chi_{jt}^{I}},
\end{aligned}\tag{10}$$

where in the last line we substituted from (7). The first of these conditions is simply the FOC in choosing capital for the firm producing the consumption good in the CEP, while the second and third are the optimality conditions for the firm producing the capital good.

The fourth line gives us the relationship between the consumption and investment wedges in the PSPP and the capital wedge from the CEP. This is straightforward to impose in our analysis; for any process for the growth of the PSPP consumption wedge, we simply implicitly assume whatever process for the growth of the PSPP investment wedge necessary to generate a first-order autoregressive process for the product of its growth rate with that of the consumption wedge. To see that the conditions presented above are sufficient to ensure that this is true, note that under this restriction we have

$$\ln\left(1-\tau^K_{jt+1}\right) = \ln\left(\chi^I_{jt+1}/\chi^I_{jt}\right) + \ln\left(\chi^C_{jt+1}/\chi^C_{jt}\right),$$

so that after substituting for (10) and imposing the restrictions in (2) we obtain the evolution equation for the capital wedge in the CEP

$$\ln\left(1-\tau^K_{jt+1}\right) = \left(1-\rho^K_j\right)\ln\left(1-\tau^K_{jSS}\right) + \rho^K_j \ln\left(1-\tau^K_{jt}\right) + \sigma^K_j \varepsilon^K_{jt+1}.$$

Lastly, note that the resource constraint of the PSPP is equal to the sum of the budget constraints of the CEP after imposing market clearing in bonds. Or, conversely, substituting for the allocations, prices and transfers in the CEP budget constraints from the PSPP problem, we can deduce the implied sequences of foreign bond holdings.

The Intensive Form Problem

As discussed in Section 2.1 of the paper, the world economy follows a stochastic trend identified with the rest of the world's level of effective labor $Z_t = A_{Rt}^{1/(1-\alpha)} N_{Rt}$. As the trend possesses a unit root, to make the model stationary we will work with first differences of this trend $z_{t+1} = Z_{t+1}/Z_t$ and scale all variables by the level of effective labor in the previous period Z_{t-1}. We also define

$$\begin{aligned} \pi_{t+1} &= \frac{A_{Rt+1}}{A_{Rt}}, \\ \eta_{t+1} &= \frac{N_{Rt+1}}{N_{Rt}}, \end{aligned}$$

so that

$$z_{t+1} = \frac{Z_{t+1}}{Z_t} = \frac{A_{Rt+1}^{1/(1-\alpha)} N_{Rt+1}}{A_{Rt}^{1/(1-\alpha)} N_{Rt}} = \pi_{t+1}^{1/(1-\alpha)} \eta_{t+1}.$$

For notational simplicity it helps to define $a_{Rt} = n_{Rt} = 1$ for all t in all states.

This section outlines this process and derives the resulting intensive form competitive equilibrium. We also derive the intensive form social planning problem that forms the basis for our numerical algorithm and parameter estimation. In the next section, we use the intensive form versions of both problems to establish that solutions to the pseudo social planner's problem are also competitive equilibria.

Competitive Equilibrium Problem

Recall that the problem of country j is to maximize

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left\{\ln\left(\frac{C_{jt}}{N_{jt}}\right)-\frac{\psi}{1+\gamma}h_{jt}^{1+\gamma}\right\}N_{jt}\right],$$

subject to a flow budget constraint for each state and date

$$\begin{aligned}C_{jt}+P_{jt}^{K}K_{jt+1}+E_t\left[q_{t+1}B_{jt+1}\right] \;\leq\; & \left(1-\tau_{jt}^{h}\right)W_{jt}h_{jt}N_{jt}+\left(1-\tau_{jt}^{B}+\Psi_{jt}\right)B_{jt}+T_{jt}\\ & +\left(1-\tau_{jt}^{K}\right)\left(r_{jt}^{K}+P_{jt}^{*K}\right)K_{jt},\end{aligned}$$

where, from the perspective of the country, Ψ_{jt} is a fixed sequence of interest penalties (analogous to a debt elastic interest rate that is not internalized) and where P_{jt}^{K} is the price of new capital goods, and P_{jt}^{*K} is the price of old capital goods.

Substituting for the evolution of the exogenous states and scaling by Z_{t-1}, and denoting all scaled variables by lower case, yields the household's objective function

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\prod_{s=0}^{t}\eta_s\right)\left\{\ln\left(\frac{C_{jt}}{N_{jt}}\right)-\frac{\psi}{1+\gamma}h_{jt}^{1+\gamma}\right\}n_{jt}N_{R0}\right],$$

which is an affine transformation of

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\prod_{s=0}^{t}\eta_s\right)\left\{\ln\left(c_{jt}\right)-\frac{\psi}{1+\gamma}h_{jt}^{1+\gamma}\right\}n_{jt}\right].$$

For the household budget constraint we get

$$\begin{aligned}c_{jt}+P_{jt}^{K}z_tk_{jt+1}+z_tE_t\left[q_{t+1}b_{jt+1}\right] \;\leq\; & \left(1-\tau_{jt}^{h}\right)\frac{W_{jt}h_{jt}N_{jt}}{A_{Rt-1}^{1/(1-\alpha)}N_{Rt-1}}+\left(1-\tau_{jt}^{B}+\Psi_{jt}\right)b_{jt}+t_{jt}\\ & +\left(1-\tau_{jt}^{K}\right)\left(r_{jt}^{K}+P_{jt}^{*K}\right)k_{jt}.\end{aligned}$$

Recall that there are two types of (representative) firms in this economy. The first produces the final consumption good. Optimization for these firms implies that

$$\begin{aligned} W_{jt} &= (1-\alpha)\, A_{jt} \left(\frac{K_{jt}}{h_{jt} N_{jt}} \right)^{\alpha}, \\ r_{jt}^{K} &= \alpha A_{jt} \left(\frac{K_{jt}}{h_{jt} N_{jt}} \right)^{-(1-\alpha)}. \end{aligned}$$

Noting that

$$\begin{aligned} W_{jt} &= (1-\alpha)\, A_{jt} \left(\frac{K_{jt}}{h_{jt} N_{jt}} \right)^{\alpha} \\ &= (1-\alpha)\, a_{jt} A_{Rt} \left(\frac{K_{jt}}{h_{tj} n_{jt} N_{Rt}} \right)^{\alpha}, \end{aligned}$$

we let

$$\begin{aligned} w_{jt} &= \frac{W_{jt}}{A_{Rt-1}^{1/(1-\alpha)}} = (1-\alpha)\, a_{jt} \left(\frac{K_{jt}}{h_{tj} n_{jt} A_{Rt-1}^{1/(1-\alpha)} N_{Rt}} \right)^{\alpha} \\ &= (1-\alpha)\, a_{jt} \pi_t \left(\frac{k_{jt}}{h_{jt} n_{jt} \eta_t} \right)^{\alpha}. \end{aligned}$$

But note that for the return to capital

$$\begin{aligned} r_{jt}^{K} &= \alpha A_{jt} \left(\frac{K_{jt}}{h_{jt} N_{jt}} \right)^{-(1-\alpha)} \\ &= \alpha a_{jt} A_{Rt} \left(\frac{K_{jt}}{h_{jt} n_{jt} N_{Rt}} \right)^{-(1-\alpha)} \\ &= \alpha a_{jt} A_{Rt} \left(\frac{K_{jt}}{h_{jt} n_{jt} A_{Rt-1}^{1/(1-\alpha)} N_{Rt-1}} \frac{A_{Rt-1}^{1/(1-\alpha)} N_{Rt-1}}{N_{Rt}} \right)^{-(1-\alpha)} \\ &= \alpha a_{jt} \pi_t \left(\frac{k_{jt}}{h_{jt} n_{jt} \eta_t} \right)^{-(1-\alpha)}, \end{aligned}$$

so that no scaling of capital returns is required.

The second type of firm produces new capital goods $z_t k_{jt+1}$ using x_{jt} units of deferred consumption and k_{jt} units of the old capital good. Their objective function is

$$P_{jt}^{K} z_t k_{jt+1} - x_{jt} - P_{jt}^{*K} k_{jt}.$$

Assuming a capital accumulation equation with adjustment costs of the form

$$z_t k_{jt+1} = (1-\delta)\, k_{jt} + x_{jt} - \phi \left(\frac{x_{jt}}{k_{jt}} \right) k_{jt},$$

we get that the firm's problem is to choose x_{jt} and k_{jt} to maximize

$$P_{jt}^{K}\left[\left(1-\delta\right)k_{jt}+x_{jt}-\phi\left(\frac{x_{jt}}{k_{jt}}\right)k_{jt}\right]-x_{jt}-P_{jt}^{*K}k_{jt}.$$

The FOC in x implies

$$P_{jt}^{K}=\frac{1}{1-\phi'\left(\frac{x_{jt}}{k_{jt}}\right)},$$

while the one in k yields

$$P_{jt}^{*K}=P_{jt}^{K}\left(1-\delta-\phi\left(\frac{x_{jt}}{k_{jt}}\right)+\phi'\left(\frac{x_{jt}}{k_{jt}}\right)\frac{x_{jt}}{k_{jt}}\right).$$

The first-order conditions of the household's intensive form problem are

$$\begin{aligned}
c_{jt} &: \beta^{t}\left(\prod_{s=0}^{t}\eta_{s}\right)n_{jt}\frac{1}{c_{jt}}=\lambda_{jt}^{CE},\\
h_{jt} &: \beta^{t}\left(\prod_{s=0}^{t}\eta_{s}\right)n_{jt}\psi h_{jt}^{\gamma}=\lambda_{jt}^{CE}\left(1-\tau_{jt}^{h}\right)w_{jt}n_{jt}\eta_{t},\\
k_{jt+1} &: 1=E\left[\frac{\lambda_{jt+1}^{CE}}{\lambda_{jt}^{CE}}\left(1-\tau_{jt+1}^{K}\right)\frac{r_{jt+1}^{K}+P_{jt+1}^{*K}}{P_{jt}^{K}z_{t}}\right],\\
b_{jt+1} &: z_{t}q_{t+1}\lambda_{jt}^{CE}=\lambda_{jt+1}^{CE}\left[\left(1-\tau_{jt+1}^{B}+\Psi_{jt+1}\right)\right],
\end{aligned}$$

where λ_{jt}^{CE} is the multiplier on the budget constraint.

With transfers rebating all "tax revenues" beyond that required to finance government expenditure, then in equilibrium we have

$$c_{jt}+z_{t}k_{jt+1}+z_{t}E_{t}\left[q_{t+1}b_{jt+1}\right]+g_{jt}=w_{jt}h_{jt}n_{jt}\eta_{t}+\left(r_{jt}^{K}+1-\delta\right)k_{jt}-\phi\left(\frac{x_{jt}}{k_{jt}}\right)k_{jt}+b_{jt}.$$

From the labor-leisure condition we get

$$\psi h_{jt}^{\gamma}=\frac{1}{c_{jt}}\left(1-\tau_{jt}^{h}\right)w_{jt}n_{jt}\eta_{t}.$$

From the Euler equation in physical capital we get

$$1=E\left[\frac{\lambda_{jt+1}^{CE}}{\lambda_{jt}^{CE}}\left(1-\tau_{jt+1}^{K}\right)\frac{r_{jt+1}^{K}+\left(1-\delta-\phi\left(\frac{x_{jt+1}}{k_{jt+1}}\right)+\phi'\left(\frac{x_{jt+1}}{k_{jt+1}}\right)\frac{x_{jt+1}}{k_{jt+1}}\right)/\left(1-\phi'\left(\frac{x_{jt+1}}{k_{jt+1}}\right)\right)}{z_{t}\left(1-\phi'\left(\frac{x_{jt}}{k_{jt}}\right)\right)^{-1}}\right].$$

After substituting for λ^{CE} we obtain

$$1=E\left[\beta\eta_{t+1}\frac{c_{jt}}{c_{jt+1}}\frac{n_{jt+1}}{n_{jt}}\left(1-\tau_{jt+1}^{K}\right)\frac{r_{jt+1}^{K}+\left(1-\delta-\phi\left(\frac{x_{jt+1}}{k_{jt+1}}\right)+\phi'\left(\frac{x_{jt+1}}{k_{jt+1}}\right)\frac{x_{jt+1}}{k_{jt+1}}\right)/\left(1-\phi'\left(\frac{x_{jt+1}}{k_{jt+1}}\right)\right)}{z_{t}\left(1-\phi'\left(\frac{x_{jt}}{k_{jt}}\right)\right)^{-1}}\right].$$

Lastly, from the Euler equation in foreign assets, we obtain

$$z_t q_{t+1} \frac{n_{jt}}{c_{jt}} = \beta \eta_t \frac{n_{jt+1}}{c_{jt+1}} \left(1 - \tau_{jt}^B + + \Psi_{jt}\right).$$

Pseudo Social Planner's Problem

Following an analogous process for the pseudo social planner's problem introduced above, the intensive form pseudo social planners objective function becomes

$$\begin{aligned} & E_0 \left[\sum_j \chi_{jt}^C \sum_{t=0}^{\infty} \beta^t \left\{ \ln\left(\frac{C_{jt}}{N_{jt}}\right) - \chi_{jt}^I \chi_{jt}^H \frac{\psi}{1+\gamma} h_{jt}^{1+\gamma} \right\} n_{jt} N_{Rt} \right] \\ = \quad & E_0 \left[\sum_j \chi_{jt}^C \sum_{t=0}^{\infty} \beta^t \left(\prod_{s=0}^{t} \eta_s \right) \left\{ \ln\left(\frac{C_{jt}}{N_{jt}}\right) - \chi_{jt}^I \chi_{jt}^H \frac{\psi}{1+\gamma} h_{jt}^{1+\gamma} \right\} n_{jt} N_{R0} \right], \end{aligned}$$

which is equivalent to maximizing

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\prod_{s=0}^{t} \eta_s \right) \sum_j \chi_{jt}^C \left\{ \ln\left(c_{jt}\right) - \chi_{jt}^I \chi_{jt}^H \frac{\psi}{1+\gamma} h_{jt}^{1+\gamma} \right\} n_{jt} \right].$$

The resource constraint becomes

$$\begin{aligned} & \sum_j \left\{ c_{jt} + \chi_{jt}^I x_{jt} + g_{jt} \right\} \\ = \quad & \sum_j \chi_{jt}^I y_{jt} + t_t^{SP} \\ = \quad & \sum_j \chi_{jt}^I a_{jt} \pi_t k_{jt}^{\alpha} \left(h_{jt} n_{jt} \eta_t\right)^{1-\alpha} + t_t^{SP}, \end{aligned}$$

and the capital evolution equation is

$$z_t k_{jt+1} = \left(1 - \delta\right) k_{jt} + x_{jt} - \phi\left(\frac{x_{jt}}{k_{jt}}\right) k_{jt}.$$

The first order-conditions of this problem are

$$\begin{aligned} c_{jt} \quad : \quad & \beta^t \left(\prod_{s=0}^{t} \eta_s \right) \chi_{jt}^C \frac{1}{c_{jt}} n_{jt} = \lambda_t^{SP}, \\ h_{jt} \quad : \quad & \beta^t \left(\prod_{s=0}^{t} \eta_s \right) \chi_{jt}^C \chi_{jt}^I \chi_{jt}^H \psi h_{jt}^{\gamma} n_{jt} = \lambda_t^{SP} \left(1 - \alpha\right) \chi_{jt}^I a_{jt} \pi_t n_{jt} \eta_t k_{jt}^{\alpha} \left(h_{jt} n_{jt} \eta_t\right)^{-\alpha} \\ k_{jt+1} \quad : \quad & \mu_{jt}^{SP} z_t = E\left[\lambda_{t+1}^{SP} \chi_{jt+1}^I \alpha a_{jt+1} \pi_{t+1} k_{jt+1}^{\alpha-1} \left(h_{jt+1} n_{jt+1} \eta_{t+1}\right)^{1-\alpha} + \right. \\ & \left. \mu_{jt+1}^{SP} \left(1 - \delta - \phi\left(\frac{x_{jt+1}}{k_{jt+1}}\right) + \phi'\left(\frac{x_{jt+1}}{k_{jt+1}}\right) \frac{x_{jt+1}}{k_{jt+1}} \right) \right], \\ x_{jt} \quad : \quad & \lambda_t^{SP} \chi_{jt}^I = \mu_{jt}^{SP} \left(1 - \phi'\left(\frac{x_{jt}}{k_{jt}}\right) \right), \end{aligned}$$

where λ_t^{SP} is the multiplier on the resource constraint at time t and μ_{jt}^{SP} is the multiplier on the capital evolution equation in country j at time t. We can rearrange these, after substituting for λ_t^{SP}, to get

$$1 = E\left[\beta\eta_{t+1}\frac{c_{jt}}{c_{jt+1}}\frac{n_{jt}}{n_{jt+1}}\frac{\chi_{t+1}^C}{\chi_t^C}\frac{\chi_{jt+1}^I}{\chi_{jt}^I}\times \right.$$
$$\left.\frac{\alpha a_{jt+1}\pi_{t+1}k_{jt+1}^{\alpha-1}\left(h_{jt+1}n_{jt+1}\eta_{t+1}\right)^{1-\alpha}+\left(1-\delta-\phi\left(\frac{x_{jt+1}}{k_{jt+1}}\right)+\phi'\left(\frac{x_{jt+1}}{k_{jt+1}}\right)\frac{x_{jt+1}}{k_{jt+1}}\right)/\left(1-\phi'\left(\frac{x_{jt+1}}{k_{jt+1}}\right)\right)}{z_t\left(1-\phi'\left(\frac{x_{jt}}{k_{jt}}\right)\right)}\right].$$

Imposing the "equilibrium" restriction on the wedges and additive shock yields

$$\sum_j\left\{c_{jt}+z_tk_{jt+1}-(1-\delta)k_t-\phi\left(\frac{x_{jt}}{k_{jt}}\right)k_{jt}+g_{jt}\right\}=\sum_j a_{jt}\pi_tk_{jt}^{\alpha}\left(h_{jt}n_{jt}\eta_t\right)^{1-\alpha}.$$

The Balanced Growth Path of the Deterministic Model

This section presents the balanced growth path of our model or, equivalently, the steady state of the intensive form version of our model. We then use this derivation to provide further detail about why the portfolio adjustment costs are needed to establish the existence of a non-degenerate balanced growth path for our model. Lastly, we use the derivation to show why the labor wedge has little role in the balanced growth path of the model, even though it matters a great deal along the transition to this balanced growth path, and hence why analyses based on steady-state relations will tend to understate the importance of the labor wedge in determining capital flows.

As noted in the text, which can be easily verified from the resource constraint of the economy, along the balanced growth path the growth rates of consumption, investment, capital, output, government spending, and net exports for all countries are all equal to the long-run growth rate of effective labor, or

$$z_{ss}=\eta_{ss}\pi_{ss}^{\frac{1}{1-\alpha}}.$$

From the household's optimality condition in the accumulation of international assets, we see that on the balanced growth path the price of these assets satisfies

$$\frac{1}{1+r_{ss}^W}\equiv q_{ss}=\beta\frac{\eta_{ss}}{z_{ss}}=\beta\pi_{ss}^{\frac{-1}{1-\alpha}},$$

where r_{ss}^W is the steady-state world interest rate. That is, as usual, the world interest rate increases in the discount rate (decreases in the discount factor) and increases in the rate of growth of productivity.

As far as country-specific levels of variables, the steady-state level of government spending relative to output is given by assumption as g_{jss}. Steady-state investment is determined as

$$\left(\frac{X_j}{K_j}\right)_{ss} = \delta + z_{ss} - 1,$$

where we have imposed the fact that adjustment costs are zero on the balanced growth path (or steady state), and where we have written the subscript "ss" outside of the parentheses to denote the fact that the ratio of investment to capital is constant on the balanced growth path, but the levels of investment and capital themselves are not. Hence, investment relative to output is given by

$$\left(\frac{X_j}{Y_j}\right)_{ss} = (\delta + z_{ss} - 1)\left(\frac{K_j}{Y_j}\right)_{ss},$$

and so will be pinned down once we know the steady-state output to capital ratio.

From the Euler equation in capital, we have

$$1 + r^W_{ss} = \left(1 - \tau^K_{jss}\right)\left(\alpha\left(\frac{Y_j}{K_j}\right)_{ss} + 1 - \delta\right)$$

which pins down the capital to output ratio as

$$\frac{K_{jss}}{Y_{jss}} = \alpha \frac{1}{\frac{1+r^W_{ss}}{1-\tau^K_{jss}} - (1-\delta)}.$$

All that remains is to pin down consumption, hours, net exports and net foreign assets on the balanced growth path. It turns out that all of this can be done once we have the level of net foreign assets relative to output. Given $(B_j/Y_j)_{ss}$ we have that

$$\left(\frac{B_j}{Y_j}\right)_{ss}(1 - qz_{ss}) = -\left(\frac{NX_j}{Y_j}\right)_{ss}.$$

This simply states that the level of net exports in steady state is equal to the growth adjusted world interest rate on net foreign assets.

As an aside, it is worthwhile to note that, since net foreign assets are growing on the balanced growth path, the current account—in a deterministic model is equal to the change in the level of net foreign assets—and is not zero on the balanced growth path. Given our timing convention, the ratio of the current account CA to output is given by

$$\left(\frac{CA_j}{Y_j}\right)_{ss} = \left(\frac{B'_j - B_j}{Y_j}\right)_{ss} = (z_{ss} - 1)\left(\frac{B_j}{Y_j}\right)_{ss} = \frac{1 - z_{ss}}{1 - qz_{ss}}\left(\frac{NX_j}{Y_j}\right)_{ss}.$$

Given the ratio of net exports to output, we can back out the ratio of consumption to output from the resource constraint of a country

$$\left(\frac{C_j}{Y_j}\right)_{ss} = 1 - \left(\frac{X_j}{Y_j}\right)_{ss} - g_{jss} - \left(\frac{NX_j}{Y_j}\right)_{ss}.$$

The level of hours per person (which is constant on the balanced growth path) is then pinned down by the first-order condition in hours

$$h_{jss} = \left(\frac{1-\tau^h_{jss}}{\psi}\left(\frac{Y_j}{C_j}\right)_{ss}\right)^{1/(1+\gamma)}.$$

What determines the level of net foreign assets relative to output on the balanced growth path? In a complete markets model without wedges, this would be pinned down by initial conditions. In an incomplete markets model, in general, this level would not be pinned down at all, but would instead vary forever with the realized sequence of shocks that hit the economy. This is why the model does not possess a unique steady state: if the shocks are all set to zero after some date T, and the economy jumped immediately to the balanced growth path, the level of net foreign assets that had been accumulated up until that time period, scaled by output, would persist forever after. This is why we, and all of the literature up until this point, have adopted some mechanism for pinning down the long-run level of net foreign assets relative to output. Our specification of a tax on deviations of net foreign assets from a benchmark allows us to estimate the balanced growth path of assets from the data.

It is also worth pointing out that, as constructed above, the labor wedge had no impact on the balanced growth path except for determining the level of hours worked relative to consumption. This is a little misleading; in general, realizations of the labor wedge will affect the economy on the transition to steady state and hence will affect the accumulation of net foreign assets. However, a naive analysis of capital flows simply from the balanced growth will find that the labor wedge plays no role in impacting long-run capital flows.

B Data and Methods

As noted in the paper, recovering the wedges requires data on national accounts' expenditure aggregates—output Y_{jt}, consumption C_{jt}, investment X_{jt}, government spending G_{jt}, and net exports NX_{jt}—along with data on population N_{jt} and hours worked h_{jt}, for each of our three "countries" or regions. This section describe our data sources, data aggregation techniques, and sample definitions, and provide plots of the raw data used in our analysis. A data file will be made available after the paper has been accepted for publication. We then discuss our estimation method in greater detail than in the text.

Sample Definition

The rest of the world is defined to be the aggregate of Japan, Korea, Taiwan, Hong Kong, Singapore, Canada, Australia, New Zealand, Iceland, Argentina, Brazil, Chile, Colombia, Mexico, Peru, Venezuela, and Costa Rica.

Europe is the aggregate of Austria, Belgium, Denmark, Luxembourg, France, Germany, Greece, Italy, Netherlands, Norway, Portugal, Sweden, Switzerland, Turkey, and the Unted Kingdom.

General Data Sources

Data are obtained from a number of sources (this is also described in Ohanian and Wright (2008)). Briefly, where available, data from the Organization for Economic Cooperation and Development's *Annual National Accounts* are used for its member countries. For other countries, data from the World Bank's *World Development Indicators* (WDI) is our primary source. Data prior to 1960 are often scarce; our primary source is the World Bank's *World Tables of Economic and Social Indicators* (WTESI). The Groningen Growth and Development Center (GGDC) was a valuable source of data on hours worked. Taiwanese data came from the National Bureau of Statistics of China. More specifics are provided in the country-specific notes below. Data on exchange rates per dollar span the years 1950 to 2008. The country-level data were not available from a single source; thus, they were obtained from different sources, namely, the OECD, the World Development Indicators, and the Penn World Tables.

For the purpose of comparing our model-generated estimates of the level of TFP and cap-

ital stocks to the data, we use the estimate of capital stocks in 1950 from Nehru and Dhareshwar (1993) combined with the perpetual inventory method to construct a reference series for the capital stock and the level of TFP.

Data Aggregation and Cleaning

All national accounts data were transformed to constant 2000 U.S. dollar prices. Data were aggregated by summation for each region. Net exports for the rest of the world were constructed to ensure that the world trade balance with itself was zero, and any statistical discrepancy for a region was added to government spending.

Our measure of output is gross domestic product. Hence, net exports do not include net exports of factor services, and correspond to the trade balance (and not the current account balance). Where available, our measure of investment was gross capital expenditure. When this was not available, we used data on gross fixed capital expenditure.

For some countries and variables, data were missing for a small number of years. More details on these cases are presented in the country-specific notes below; in general, missing data were filled in by assuming that data for the missing country evolved in the same way as the rest of the regional aggregate.

We proceed in several steps to create the regional exchange rates. First, we use data on each country's consumption across time, along with total yearly consumption by region. We then calculate the following weight for each country j: $w_{jt} = c_{jt} / \sum_{j \in R} c_{jt}$, where the denominator represents regional consumption, such that $R \in \{EUR, RoW\}$. Finally, we calculate the exchange rate for each region, as follows: $\sum_{j \in R} w_{jt} e_j$, where e_j is the real exchange rate between country j and the U.S.

Country-Specific Notes on Data

Below we provide a series of country-specific notes on data sources and construction. These notes focus on details about missing data that are specific to each country, and on any other issues with country-specific data.

Rest of the World

1. Hong Kong. NIPA and population data from 1960 to 2007 are from the WDI. NIPA and population data from 1950 to 1960 are from WTESI. Hours data are from GGDC. Inventory investment was not available prior to 1965 and so gross fixed capital expenditure was used instead.

2. Japan. NIPA and population data from 1960 to 2007 are from the OECD. NIPA and population data from 1950 to 1960 are from WTESI. Hours data are from GGDC. Inventory investment was not available prior to 1960 and so gross fixed capital expenditure was used instead. Hours data were missing for 1950 and were imputed using trends in the data for other Asian countries.

3. South Korea. NIPA and population data from 1960 to 2007 are from the WDI. NIPA and population data from 1950 to 1960 are from WTESI. Hours data from 1963 to 2007 are from GGDC; no hours data are available prior to 1963. Inventory investment was not available prior to 1960 and so gross fixed capital expenditure was used instead.

4. Singapore. Official NIPA data for Singapore first becomes available in 1960 and was taken from the WDI. Prior to 1960, NIPA estimates derived from colonial data were obtained from Sugimoto (2011). Hours worked data were taken from GGDC from 1960. Prior to 1960, we computed total hours worked from data on the employment and hours worked of laborers, shop assistants, shop clerks, and industrial clerks in both public- and private-sector establishments as tabulated in the *Annual Report of the Labour Department* for the Colony of Singapore (1950-1956) and State of Singapore (1957-1960).

5. Taiwan. NIPA data for Taiwan begin in 1951 and come from the National Bureau of Statistics of China. Hours worked data come from GGDC starting in 1960. Population, and hours worked data prior to 1960, come from the Penn World Tables v.9.0.

6. Argentina. NIPA and population data from 1960 to 2007 are from the WDI. NIPA and population data from 1950 to 1960 are from WTESI. Hours data are from GGDC. Inventory investment was not available prior to 1960, and for some years after 1979, and so gross fixed capital expenditure was used instead.

7. Brazil. NIPA and population data from 1960 to 2007 are from the WDI. NIPA and population data from 1950 to 1960 are from WTESI. Hours data are from GGDC. Inventory

investment was not available prior to 1960 and so gross fixed capital expenditure was used instead.

8. Chile. NIPA and population data from 1960 to 2007 are from the WDI. NIPA and population data from 1950 to 1960 are from WTESI. Hours data are from GGDC.

9. Colombia. NIPA and population data from 1960 to 2007 are from the WDI. NIPA and population data from 1950 to 1960 are from WTESI. Hours data are from GGDC. Inventory investment was not available prior to 1960 and so gross fixed capital expenditure was used instead.

10. Mexico. NIPA and population data from 1960 to 2007 are from the WDI. NIPA and population data from 1950 to 1960 are from WTESI. Hours data are from GGDC. Inventory investment was not available prior to 1960 and so gross fixed capital expenditure was used instead.

11. Peru. NIPA and population data from 1960 to 2007 are from the WDI. NIPA and population data from 1950 to 1960 are from WTESI. Hours data are from GGDC. Inventory investment was not available prior to 1960 and so gross fixed capital expenditure was used instead.

12. Australia. NIPA and population data are from the OECD. Hours worked were taken from GGDC back until 1953, and extended back to 1950 using the series in Butlin (1977).

13. Canada. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

14. New Zealand. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

15. Iceland. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

Europe

1. Austria. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

2. Belgium. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

3. Denmark. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

4. France. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

5. Germany. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

6. Greece. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

7. Italy. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

8. Luxembourg. NIPA and population data are from the OECD. Hours worked were taken from GGDC back until 1958.

9. Netherlands. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

10. Norway. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

11. Portugal. NIPA and population data are from the OECD. Hours worked were taken from GGDC back until 1956.

12. Spain. NIPA and population data are from the OECD. Hours worked were taken from GGDC back until 1954.

13. Sweden. NIPA and population data are from the OECD. Hours worked were taken from GGDC back until 1959.

14. Switzerland. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

15. United Kingdom. NIPA and population data are from the OECD. Hours worked were taken from GGDC.

United States of America

NIPA and population data are from the OECD. Hours worked were taken from GGDC.

C Conceptual Issues About Measuring Capital Flows

This paper uses net exports of goods and services as our measure of international capital flows. This is a common approach, although some researchers studying capital flows in more recent decades have focused on the current account as a measure of capital flows (which includes income from net exports of factor services, otherwise known as net factor income). Here, we discuss the reasons for our approach in more detail.

There are several reasons for our approach: (1) net factor income is poorly measured; (2) balance of payments data are limited by their focus on transactions data and their inconsistent treatment of transfers such as debt restructuring; (3) balance of payments data are not available for many countries prior to 1970 and sometimes have severe measurement issues; and (4) there is no unique mapping from model outcomes to implications for the balance of payments, although there is a unique mapping of net exports. We elaborate on these reasons in detail below.

Regarding data availability, it is important to note that data on net factor income (the difference between net exports and the current account balance) are often not available, particularly before 1970. For example, Alfaro, Kalemli-Ozcan, and Volosovych (2014), who conduct the most exhaustive study of data on international capital flows that we know of, focus most of their analysis on the period after 1980, for which the most data are available for 156 countries. Their "1970" sample covers only 46 countries and includes only a limited subset of the variables contained in their wider analysis. This means that these data do not speak to a key period of interest: the decades leading up to 1973.

Second, on the issue of data reliability, it is important to note that even when these data are available, they are subject to significant measurement error. As a number of people have pointed out, including the International Monetary Fund itself, according to their data the world often runs a large current account deficit with itself. Until recently, this deficit was almost entirely concentrated in the net factor income component of the current account. Moreover, the error has often been extremely large, peaking at around 5 percent of world imports in 1982 (see Marquez and Workman (2001)).

Third, at a deeper level, our focus on net exports data (and not data on the current account or on the capital account) is driven by issues related to the way the balance of payments is constructed. Conceptually, a country's net foreign asset position can change for roughly

three reasons: first, because of a transaction in which assets change hands or income is paid; second, because of capital gains and valuation effects; and third, because of a gift or transfer, such as foreign aid, a nationalization, or an expropriation, or because of debt forgiveness and restructuring.

The way the balance of payments is constructed, it is designed to capture transactions. It is explicitly *not* designed to capture the effect of valuation changes on a country's net foreign asset position (this has, in and of itself, led to a significant debate about how to interpret data on the balance of payments and data on net foreign assets; see the issues raised by Lane and Milesi-Ferretti (2001, 2005, and 2007); Tille (2003); Higgins, Klitgaard, and Tille (2005) and Gourinchas and Rey (2007)). In addition, its ability to capture transfers such as sovereign default depends on whether the country has adopted accrual accounting standards (in which case, a debt restructuring is paired with an artificial accounting transaction) and whether it is believed that accrual accounting standards are adequate for this purpose (Sandleris and Wright (2013) and others have argued that, when a country defaults on its debts, it is better to use cash accounting concepts in evaluating its balance of payments). As a result of all these concerns, amplified by the fact that the asset structure of international finance has changed over time to emphasize more derivative securities and valuation effects have become more important in an era of floating exchange rates, confidence in the reliability and backwards comparability of balance of payments data is low, even in the absence of the measurement error noted above. The issues are well summarized by Alfaro, Kalemli-Ozcan, and Volosovych (2014), who write:

> There are substantial country differences in terms of time coverage, missing, unreported, or misreported data, in particular for developing countries. Some countries do not report data for all forms of capital flows. Outflows data tend to be misreported in most countries and, as the result, captured in the "errors and omissions" item.
>
> Unfortunately, it is hard to verify whether the data are really missing as opposed to simply being zero. Due to the debt crisis of the 1980s there are several measurement problems related to different methodologies of recording non-payments, rescheduling, debt forgiveness and reductions.

Fourth, on the issue of mapping models to the data, it has been known for a long time that a

given model of international capital markets can be mapped into data on the balance of payments in different ways depending on which of many alternative equivalent asset structures is used. For example, in a complete markets framework, it may be possible to decentralize the equilibrium allocations using Arrow securities, Arrow-Debreu securities, a portfolio of equities and debt, or a combination of debt and derivative securities and so on. Each will typically have different implications for the balance of payments. A model with only Arrow or Arrow-Debreu securities has many assets experiencing a 100 percent capital loss each period, with one asset experiencing a large capital gain. In principle, these capital gains would not be recorded in the balance of payments at all. With only Arrow-Debreu securities, no transactions occur after the first initial period. With Arrow securities, a portfolio of new securities is bought every period. Again, these can have very different implications for the balance of payments. Likewise, the equilibrium will look different if it is decentralized with a mixture of debt and equity or with financial derivatives.

As a consequence, it has become traditional in the literature (1) to work with models that have a very limited asset structure (such as with bonds only or a bond and one equity), which misses much of the richness of the international asset trade but can give precise predictions for the balance of payments, or (2) to work with complete market models to focus on allocations—such as net exports—which are invariant across different decentralizations. A particularly strong statement of this position is provided by Backus, Kehoe, and Kydland (1994). This is the approach we have adopted in this paper.

Moreover, even when a particular stand is taken on the asset structure in the model, it is not always obvious how best to map the model to the data. This might be more easily understood in the model of this paper, under the assumption that the asset structure is one in which the world trades Arrow securities each period (the assumption made in the text).

To begin, we can start by looking at the change in a country's net foreign asset position from one period to the next. If the current account in the data was constructed to include valuation effects, this would be the natural measure of the current account in the model. However, even with this simple concept, we can measure the change at different points within the period by looking at either start or end-of-period levels.

The start-of-period definition is

$$CA^1_{jt} = B_{jt+1} - B_{jt},$$

so that, recalling also that

$$B_{jt} = -NX_{jt} + E_t \left[q_{t+1} B_{jt+1} \right],$$

we can write the current account as

$$CA^1_{jt} = NX_{jt} + B_{jt+1} - E_t \left[q_{t+1} B_{jt+1} \right],$$

where the two terms after net exports correspond to net factor income (which can be thought of as earned between t and $t+1$),

$$NFI_{jt} = B_{jt+1} - E_t \left[q_{t+1} B_{jt+1} \right].$$

The end-of-period definition is

$$\begin{aligned} CA^2_{jt} &= E_t \left[q_{t+1} B_{jt+1} \right] - E_{t-1} \left[q_t B_{jt} \right] \\ &= NX_{jt} + B_{jt} - E_{t-1} \left[q_t B_{jt} \right]. \end{aligned}$$

This differs from the previous version in that it adds net factor income between periods $t-1$ and t to net exports in period t, as opposed to income earned between t and $t+1$.

As noted previously, current accounts are not measured this way in practice. Specifically, the current account does not include the capital gains or losses on foreign assets. One could try to compute a model analog of net factor income exclusive of capital gains and losses in the model. One way to do this, although far from the only way, would be to define the model in terms of the expected profits and losses from the country's net foreign asset position:

$$NII_{jt} = E_{t-1} \left[B_{jt} \left(1 - q_t \right) \right].$$

Intuitively, if we define the interest rate between $t-1$ and t as satisfying

$$q_t = \frac{1}{1 + r_t}$$

so that

$$1 - q_t = \frac{r_t}{1 + r_t},$$

we get

$$B_{jt} \left(1 - q_t \right) = r_t \frac{B_{jt}}{1 + r_t}.$$

This leads to an alternative measure of the current account, designed to more-closely mimic that available in the data, or

$$CA_{jt}^3 = NX_{jt} + E_{t-1}\left[\frac{r_t}{1+r_t}B_{jt}\right].$$

A fourth alternative would be to try to measure net foreign investment income using an average (or expected) interest rate. For example, we might define an average interest rate $\bar{r}_t$ from

$$\bar{q}_t = E_{t-1}\left[q_t\right]$$

as

$$1 + \bar{r}_t = 1/\bar{q}_t.$$

Then we have a fourth measure of the current account:

$$CA_{jt}^4 = NX_{jt} + \frac{\bar{r}_t}{1+\bar{r}_t}B_{jt}.$$

In summary, in the context of a complete markets model where multiple decentralizations are possible, even when attention is restricted to a decentralization using Arrow securities alone, there are multiple plausible ways of mapping model outputs into the analog of the current account measured in the data.