Internet Appendix for
Shrinking the Term Structure

Damir Filipović* Markus Pelger† Ye Ye‡

May 9, 2024

Abstract

The Internet Appendix collects additional empirical results supporting the main text.

Keywords: Term structure of interest rates, bond returns, factor space, U.S. Treasury securities, non-parametric method, principal components, machine learning in finance, reproducing kernel Hilbert space

JEL classification: C14, C38, C55, G12

*École Polytechnique Fédérale de Lausanne and Swiss Finance Institute, Email: damir.filipovic@epfl.ch

†Stanford University, Department of Management Science & Engineering, and NBER, Email: mpelger@stanford.edu.

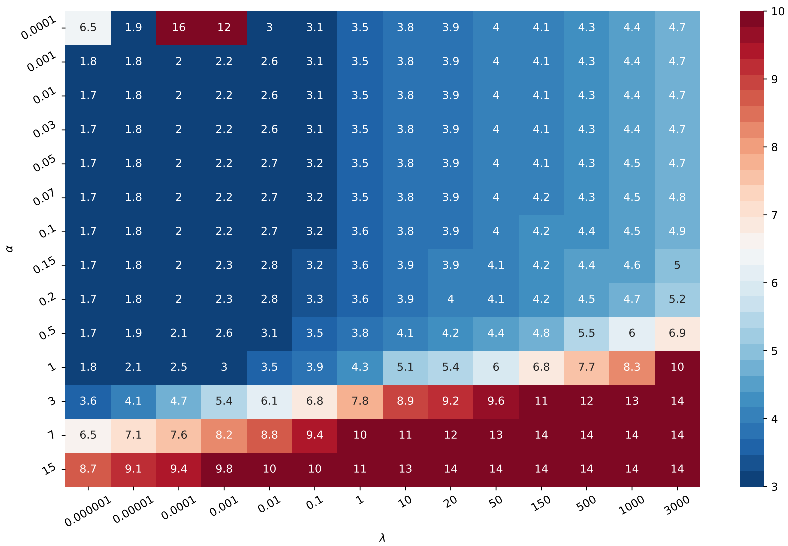
‡Stanford University, Department of Management Science & Engineering, Email: yeye@stanford.edu.

Contents

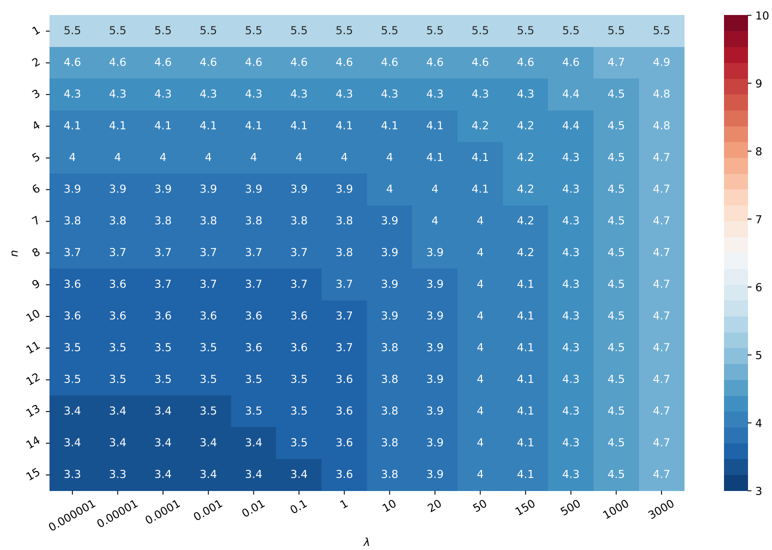
1 Term Structure Estimation

Figure IA.1 shows the in-sample results for the RMSE of the KR models for different choices of tuning parameters. It confirms the overall robustness of the parameter choices. As the in-sample results are prone to overfitting, a more regularized estimation with either larger smoothness penalty λ or fewer factors n has obviously higher in-sample RMSE.

Figure IA.1: Average in-sample excess return RMSE



(a) RMSE for λ and α



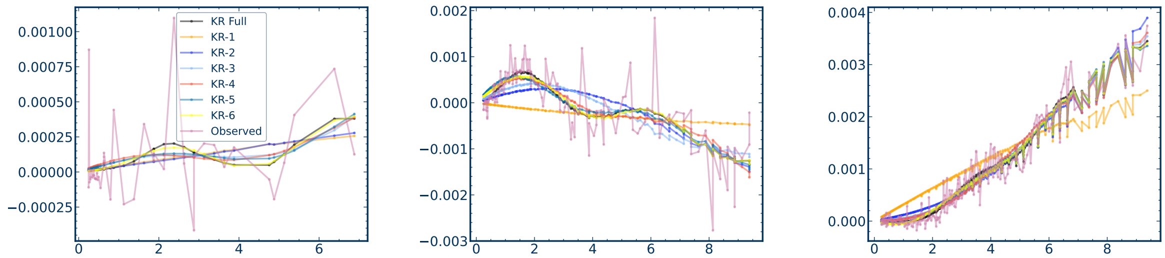
(b) RMSE for λ and n

This figure shows the average in-sample excess return RMSE of the full KR model in basis points (BPS) as a function of the smoothness parameter λ, the maturity weight α and the number of factors n. Subplot (a) displays the RMSE for λ and α, while subplot (b) reports the RMSE for λ and n. Factors are added by the order implied by the kernel decomposition, that is, in decreasing order of the eigenvalues of the kernel matrix $\boldsymbol{K}$. Results are calculated using quarterly data from June 1961 to December 2020.

Figure IA.2 illustrates the estimated discount bond excess return curve on three representative example days. Including more factors allows the estimated curves to capture more complex patterns and move closer to the full KR curves. A KR factor model with four to six factors approximates the full KR curve relatively well. The first KR factor seems to capture a linear slope pattern. The lower panels of the figures apply the sparse basis functions to yield estimation instead of return estimation. They illustrate that the yield estimates of Filipović, Pelger, and Ye (2022) can also be approximated well by a small number of yield factors.

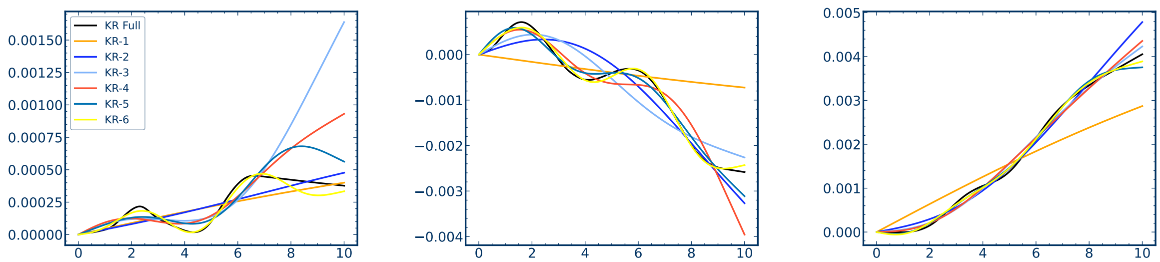
Figure IA.2: Illustration of KR estimator

Panel A: Observed and Fitted Excess Returns of Securities



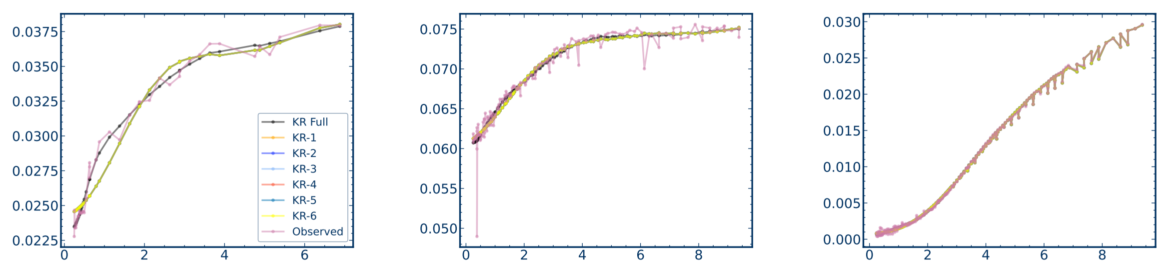
(a) 1961-06-30 **(b)** 1986-06-30 **(c)** 2013-12-30

Panel B: Fitted Excess Returns of Discount Bonds



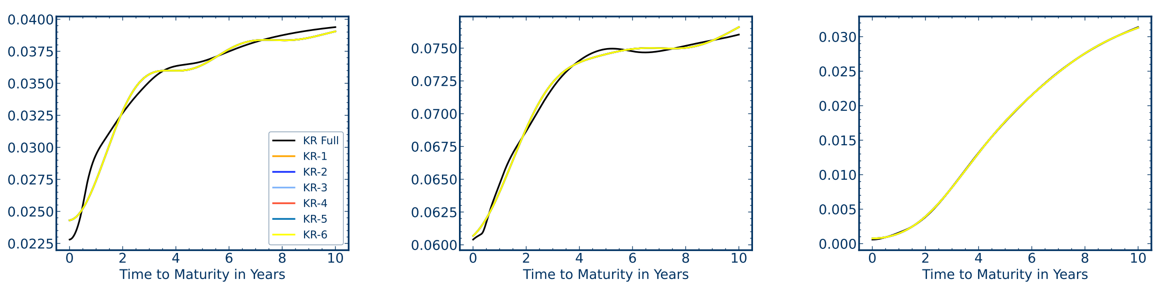
(d) 1961-06-30 **(e)** 1986-06-30 **(f)** 2013-12-30

Panel C: Observed and Fitted YTM of Securities



(g) 1961-06-30 **(h)** 1986-06-30 **(i)** 2013-12-30

Panel D: Fitted Yield Curves



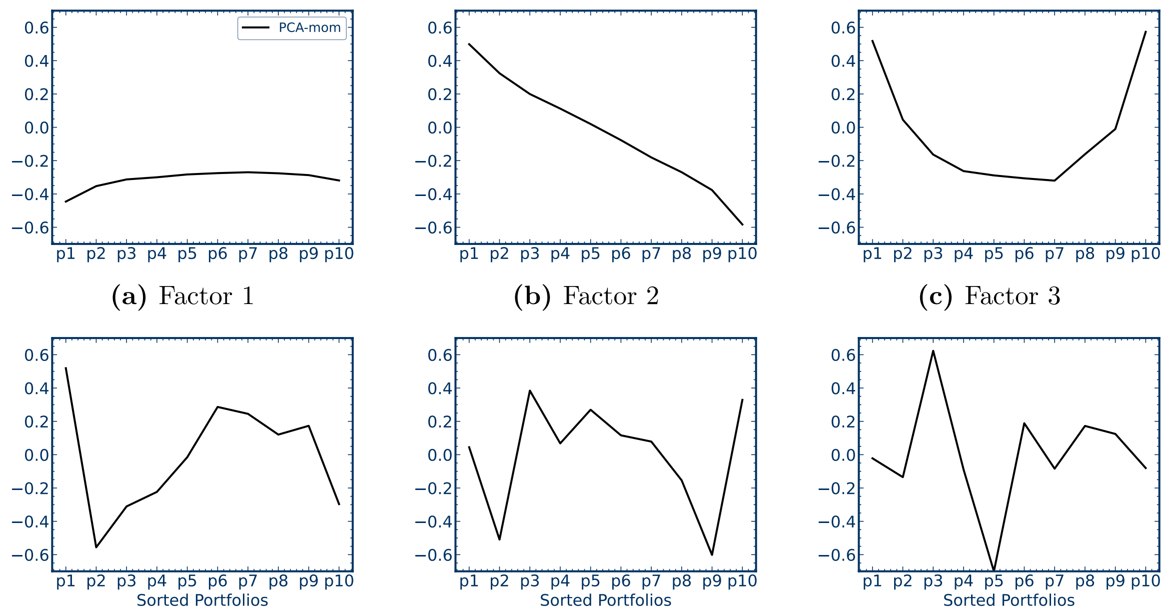
(j) 1961-06-30 **(k)** 1986-06-30 **(l)** 2013-12-30

This figure illustrates the KR estimates on three example dates. The panel A plots observed and fitted excess returns of securities. The panel B displays fitted excess returns of discount bonds. Panel C plots observed and fitted YTM of securities, and Panel D shows fitted yield curves.

2 Explaining Slope and Curvature by Smoothness for Equities

The relevance of our findings extends beyond term structure modeling. Slope and curvature patterns in PCAs of panels of asset returns have been demonstrated for a variety of asset classes, including equities and foreign exchange markets. In equities it is common to use portfolio sorts to reflect the conditional impact of firm characteristics. For example, for book-to-market ratios or momentum, one can obtain a panel of decile sorts for the different quantiles of the characteristics. These basis assets are the analogue to our discount bonds as they have close to constant exposure to the sorting variable in contrast to individual stocks. As shown in Figures IA.3 and IA.4, a PCA on the decile sorts leads to the same slope and curvature patterns as for "maturity sorted" discount bonds. Our findings suggest that these type of factors simply arise because the functional dependency of stock returns on momentum or book-to-market is inherently smooth. This non-parametric perspective is novel, and we are able to make it because our approach unifies non-parametric curve estimation with cross-sectional factor modeling.

Figure IA.3: PCA factor loadings of momentum sorted decile portfolios

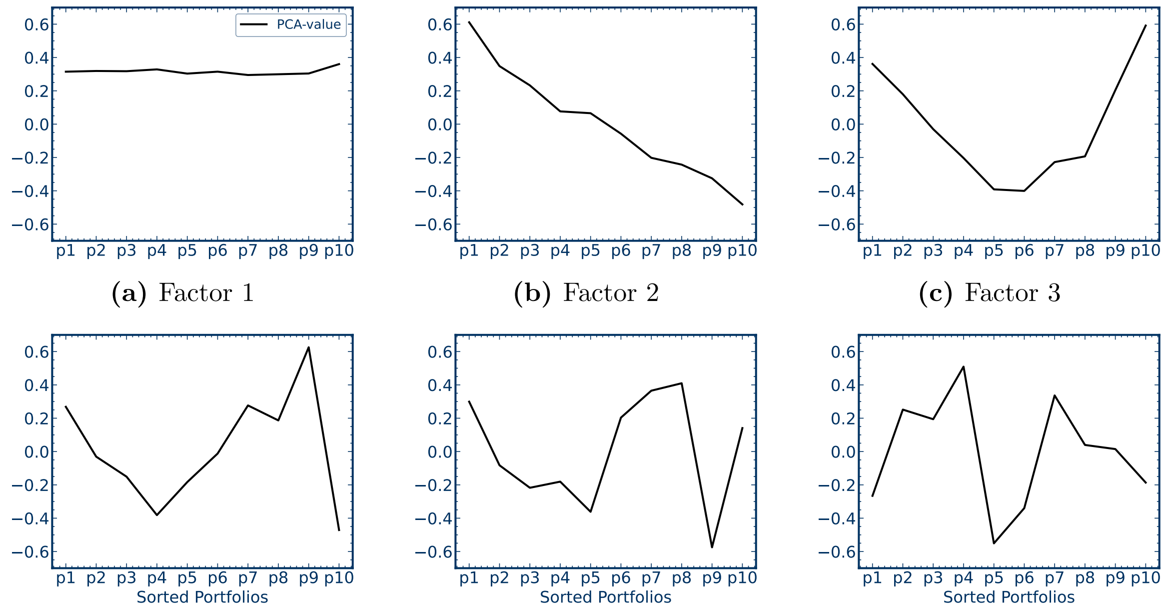


(a) Factor 1 **(b)** Factor 2 **(c)** Factor 3

(d) Factor 4 **(e)** Factor 5 **(f)** Factor 6

This figure shows the normalized loadings of the first 6 PCA factors from a panel of monthly excess returns of decile sorted portfolios based on momentum. The data is from Lettau and Pelger (2020). The panel of ten portfolios starts in November 1963 and ends in December 2017 (T=650). Note that PCA factors are not identified up to a sign, and without loss of generality the sign of the loadings can be changed.

Figure IA.4: PCA factor loadings of value sorted decile portfolios



(a) Factor 1 **(b)** Factor 2 **(c)** Factor 3

(d) Factor 4 **(e)** Factor 5 **(f)** Factor 6

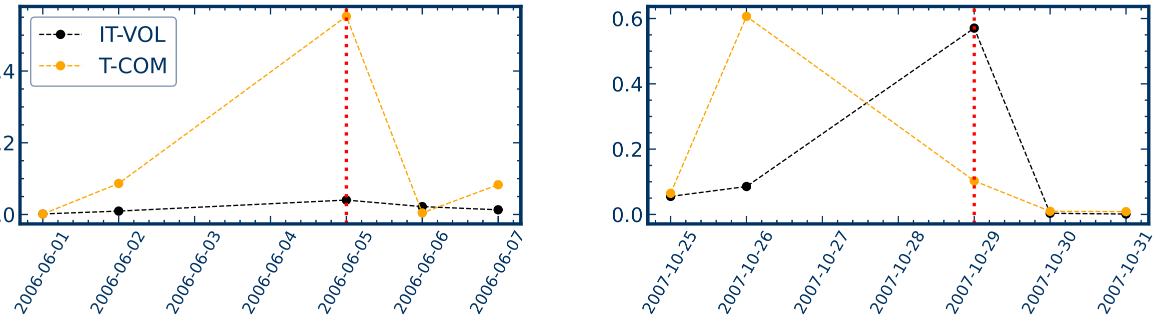
This figure shows the normalized loadings of the first 6 PCA factors from a panel of monthly excess returns of decile sorted portfolios based on book-to-market ratios. The data is from Lettau and Pelger (2020). The panel of ten portfolios starts in November 1963 and ends in December 2017 (T=650). Note that PCA factors are not identified up to a sign, and without loss of generality the sign of the loadings can be changed.

3 Interpretation of Bond Market Conditions

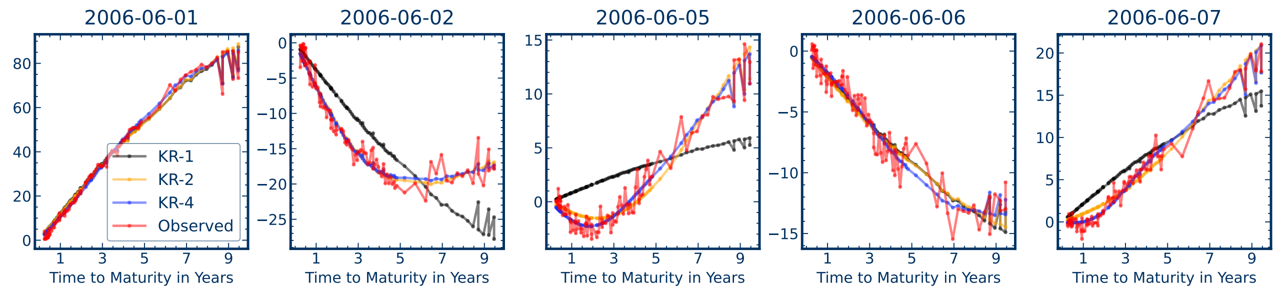
3.1 High-Frequency Changes in Bond Markets

Our novel measures also detect high-frequency changes in bond markets. Figure IA.5 illustrates the two measures and the fitted coupon bond excess returns for a time window around example days. In Panel (a), T-COM peaks on 2006-06-05 while IT-VOL remains low. In Panel (b), IT-VOL peaks on 2007-10-25, while T-COM stays low. The start and end date for the windows around the example days indicate a “simple” term structure, that is a simple linear model provides a reasonable fit, while the noise level is comparatively low. These examples indicate that bond market conditions can change quickly. In summary, we provide two refined measures of the state of the Treasury bond market that can be used for high frequencies.

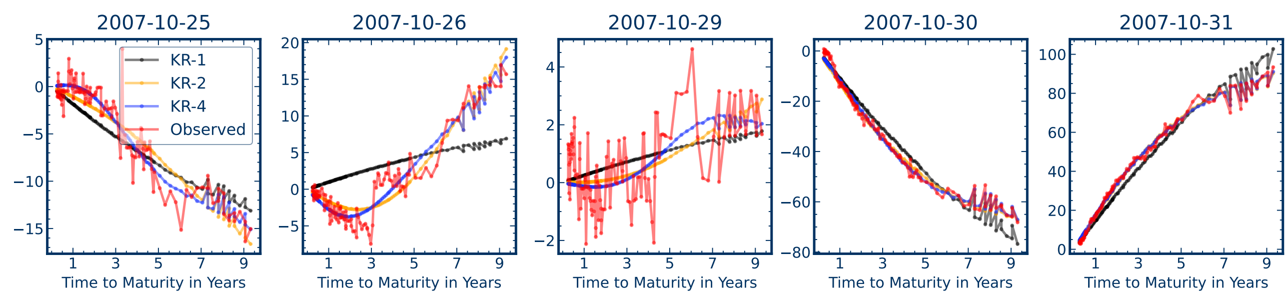
Figure IA.5: The Idiosyncratic Treasury Volatility (IT-VOL) measure and the Treasury Market Complexity (T-COM) measure on example dates over time



(a) IT-VOL and T-COM centered on 2006-06-05 **(b)** IT-VOL and T-COM centered on 2007-10-29



(c) Fits of coupon bond excess returns centered on 2006-06-05



(d) Fits of coupon bond excess returns centered on 2007-10-29

Panels (a) and (b) show the Idiosyncratic Treasury Volatility (IT-VOL) measure and the Treasury Market Complexity measure (T-COM) measure centered on two example dates: 2006-06-05 for (a) and 2007-10-29 for (b). Example dates are marked with red vertical lines. Panels (c) and (d) display fits of coupon bond excess returns on dates adjacent to the example dates. Numbers are in basis points. In Panel (a), T-COM peaks on 2006-06-05 while IT-VOL remains low. Panel (b), IT-VOL peaks on 2007-10-25, while T-COM stays low.

3.2 T-COM and Unemployment Rates

The measures of bond market conditions directly relate to real economic conditions. We show that the bond market complexity is correlated with future unemployment rates.

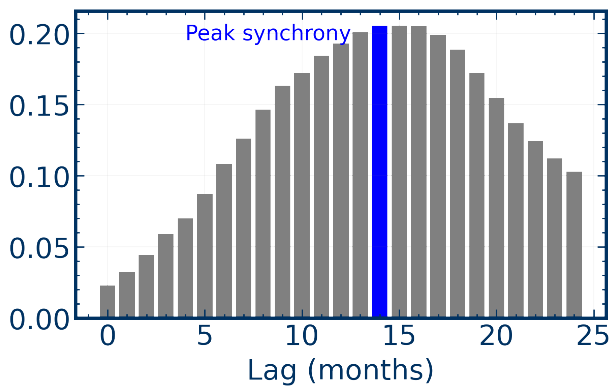
Changes in T-COM predict changes in unemployment rate one year ahead. Figure IA.6 shows the correlation of changes in T-COM and the unemployment rate relative to last year for different lags. The correlation is computed by shifting the time series of changes in the unemployment rate one month forward at a time. The largest correlation of 0.21 occurs at the 14-month lag. This means that that changes in T-COM have the highest correlations with changes in unemployment rates roughly one year in the future.

This prediction is statistically significant. First, we confirm that the two time-series of changes do not have unit-roots. Second, we run a Granger causality test. We find that changes in T-COM Granger cause changes in unemployment rates with a p-value of 0.01. Lastly, we estimate a simple regression model using the 14-month lagged change in T-COM to predict changes in unemployment rates:

$$\text{unemployment}_t = 0.01 + 0.14 \cdot \text{T-COM}_{t-14} + \epsilon_t \tag{1}$$

The regression coefficient on T-COM is highly significant with a t-statistic of 4.7 and an $R^2 = 0.05$. We conclude that more complex shapes of the term structure are predictive for future bad economic conditions.

Figure IA.6: Correlation of changes in T-COM and unemployment rate

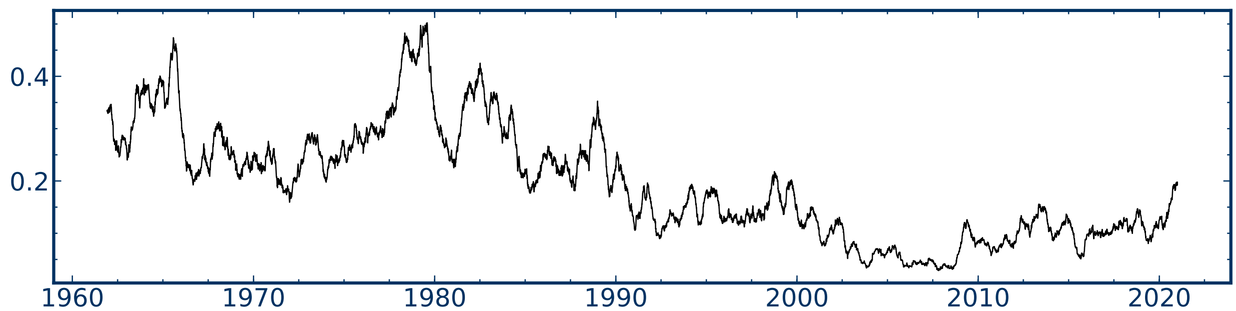


The figure shows the correlation of changes in T-COM and the unemployment rate relative to last year for different lags. The correlation is computed by shifting the time series of changes in unemployment rate one month forward at a time. Changes of T-COM are calculated using the moving average of the prior 120 business day observations of T-COM. The largest correlation of 0.21 occurs at the 14-month lag.

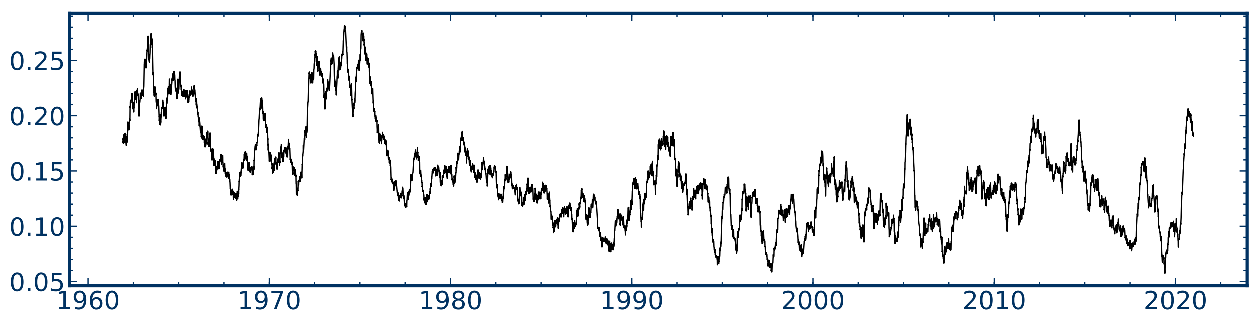
4 Alternative Bond Market Condition Measures

The results in the main text are robust to alternative formulations of T-COM and IT-VOL. Table IA.1 and Figures IA.7, IA.8, and IA.9 consider an alternative formulation based on the full KR model. There, the IT-VOL and T-COM measures are constructed with the full KR model, that is, the errors for the unexplained variation are based on the full KR model. Similarly, the alternative complexity measure is based on the normalized difference in cross-sectionally explained variation between the first KR factor and the full KR model instead of only the first four KR factors. The results are virtually identical, showing that our findings do not depend on the choice of the number of factors.

Figure IA.7: Time-series of market condition measures based on full KR model



(a) Idiosyncratic Treasury Volatility measure (IT-VOL)



(b) Treasury Market Complexity measure (T-COM)

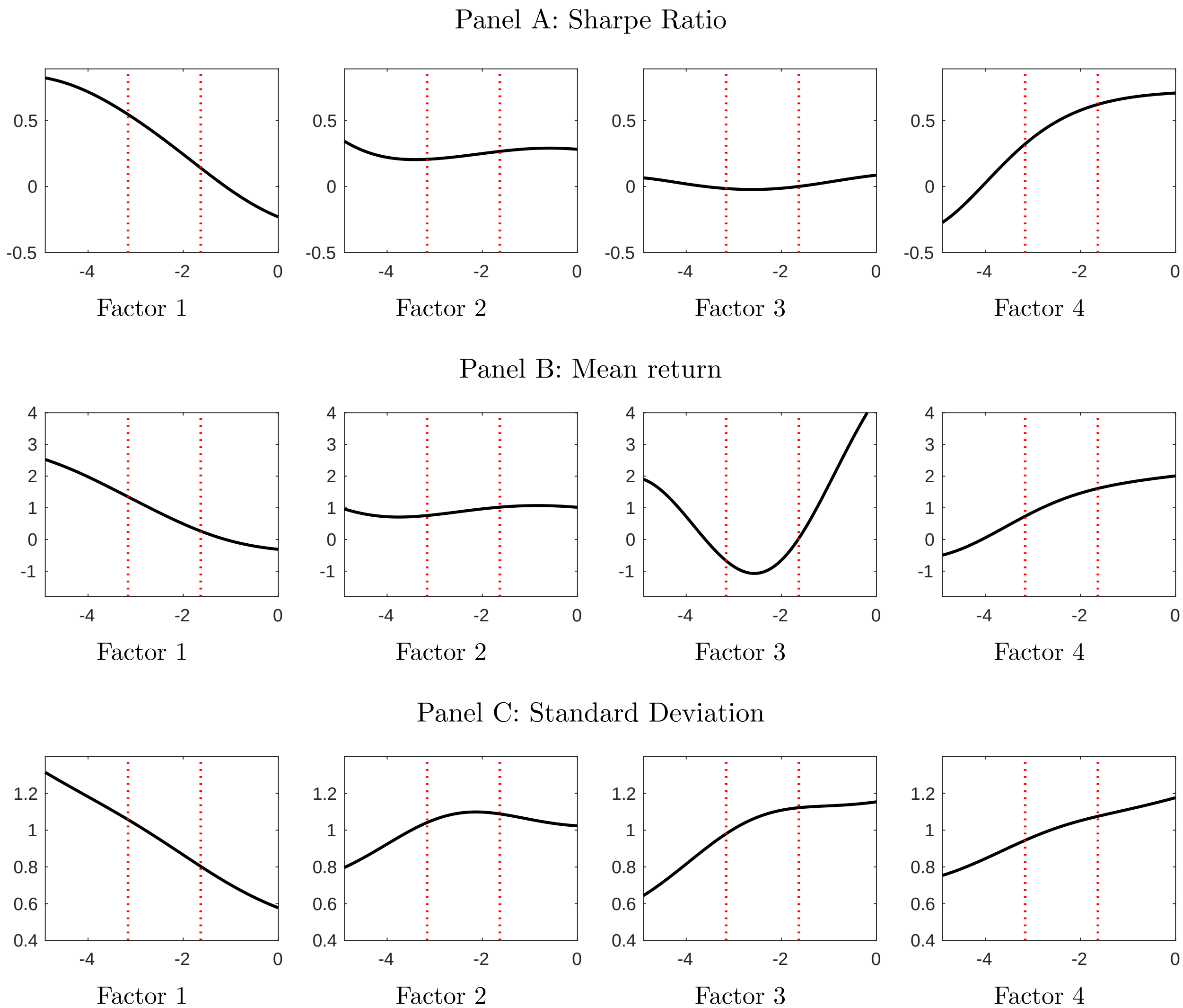
This figure shows the 120-day moving average of the two market condition measures. Subfigure (a) displays the Idiosyncratic Treasury Volatility measure (IT-VOL) as the unexplained variation by the full KR model. Subfigure (b) presents the Treasury Market Complexity measure (T-COM) as the difference between unexplained variation by the 1-factor and the full KR model. The unexplained cross-sectional variation of a factor model is normalized by the overall cross-sectional variation on that day.

Table IA.1: Sharpe ratios, means, and standard deviation of KR factors conditioned on IT-VOL and T-COM based on full KR model

	SR				Mean (BPS)				Standard Deviation (BPS)			
	F_1	F_2	F_3	F_4	F_1	F_2	F_3	F_4	F_1	F_2	F_3	F_4
IT-VOL												
Tercile 1	0.64	0.26	0.02	-0.22	22.22	17.52	1.70	-25.83	34.55	66.39	83.73	117.04
Tercile 2	0.49	0.13	-0.12	0.41	12.43	14.64	-22.64	72.25	25.16	114.45	188.51	177.83
Tercile 3	-0.58	0.38	0.18	0.65	-7.21	32.00	29.44	123.34	12.53	84.88	167.24	188.53
T-COM												
Tercile 1	1.13	-0.25	-0.14	-0.48	2.47	-0.75	-0.77	-3.57	34.70	47.07	86.71	117.32
Tercile 2	-0.25	0.37	0.09	0.77	-0.37	1.80	0.75	7.41	23.06	77.17	127.74	152.86
Tercile 3	-0.38	0.37	0.04	0.52	-0.37	2.99	0.56	6.85	15.63	128.58	216.06	209.06

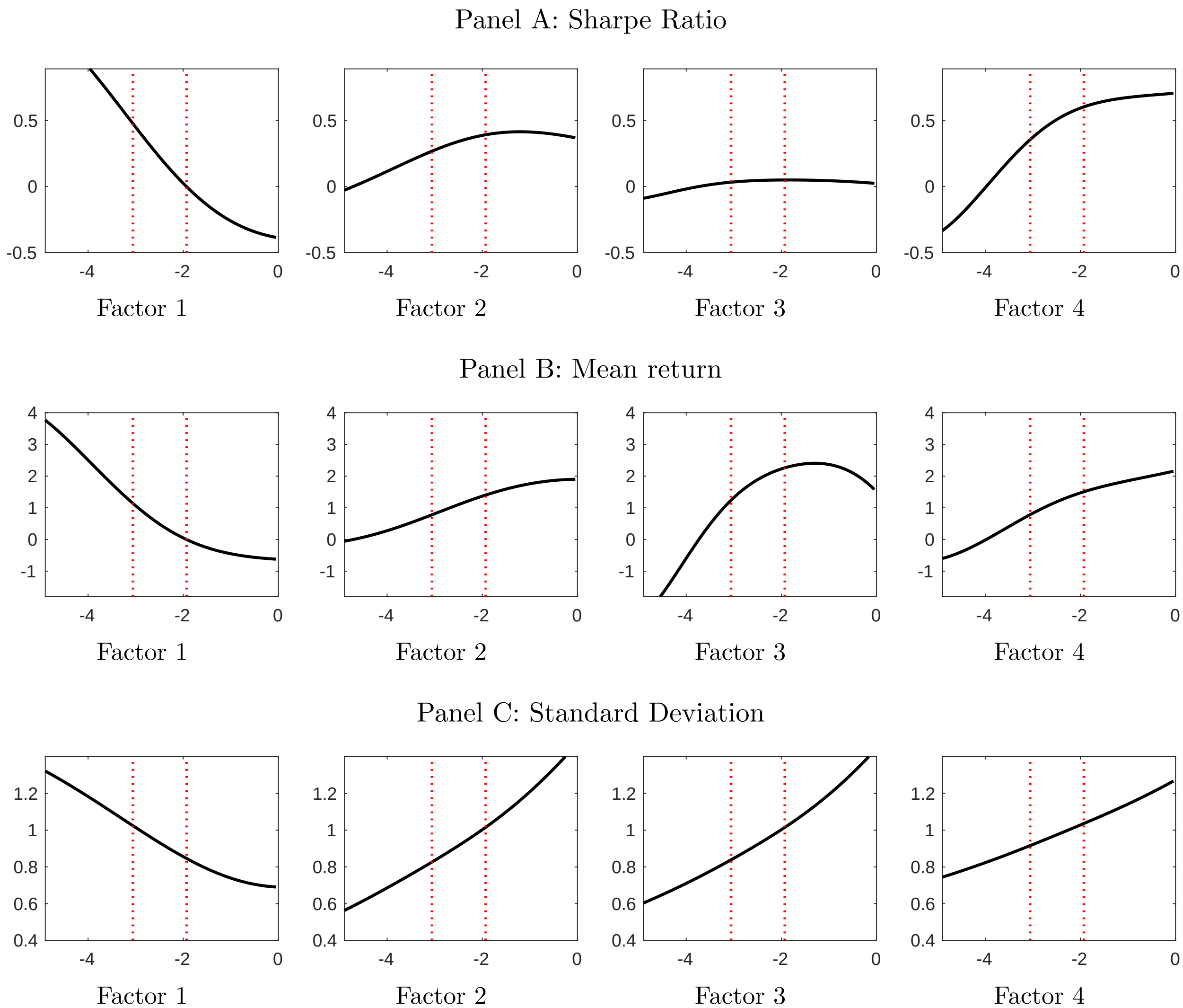
This table shows the annualized Sharpe ratio, mean, and standard deviation of the four KR factors conditioned on terciles of the Idiosyncratic Treasury Volatility (IT-VOL) measure and the Treasury Market Complexity (T-COM) measure. The IT-VOL and T-COM measures are constructed with the full KR model, that is, the cross-sectional unexplained variation with respect to the full KR model, respectively, the difference in cross-sectionally explained variation between the first KR factor and the full KR model (instead of only the first four KR factors). Mean and standard deviation are annualized and in basis points. The sample is from June 1961 to December 2020.

Figure IA.8: Sharpe ratio, mean, and standard deviation of KR factors conditioned on IT-VOL based on full KR model



This figure shows the annualized Sharpe ratio (Panel A), normalized mean (Panel B), and normalized standard deviation (Panel C) of the four KR factors conditioned on the Idiosyncratic Treasury Volatility (IT-VOL) measure. The IT-VOL measure is constructed with the full KR model, that is, the cross-sectional unexplained variation with respect to the full KR model (instead of only the first four KR factors). The mean returns and standard deviation are normalized by their unconditional values. We condition on the logarithm of IT-VOL. The vertical dotted lines represent the tercile values of IT-VOL. The conditional first and second moment are estimated with kernel smoothing regressions. The sample is from June 1961 to December 2020.

Figure IA.9: Sharpe ratio, mean, and standard deviation of KR factors conditioned on T-COM based on full KR model

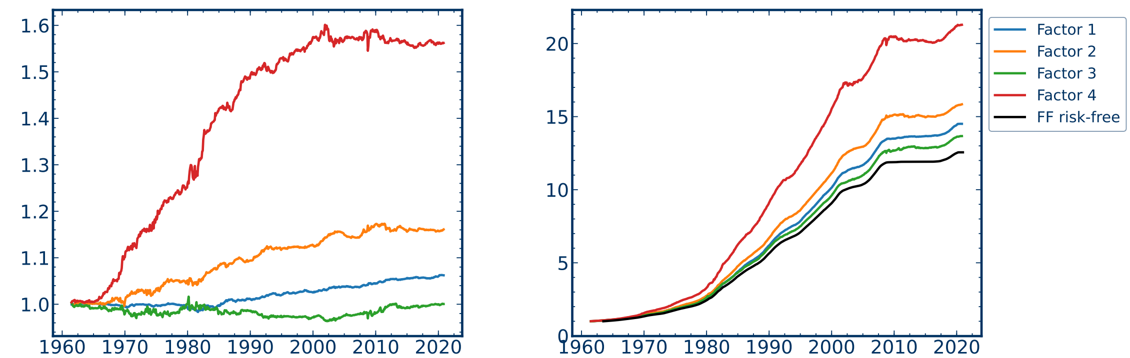


This figure shows the annualized Sharpe ratio (Panel A), normalized mean (Panel B), and normalized standard deviation (Panel C) of the four KR factors conditioned on the Treasury Market Complexity (T-COM) measure. The T-COM measure is constructed with the full KR model, that is, the difference in cross-sectionally explained variation between the first KR factor and the full KR model (instead of only the first four KR factors). The mean returns and standard deviation are normalized by their unconditional values. We condition on the logarithm of T-COM. The vertical dotted lines represent the tercile values of T-COM. The conditional first and second moment are estimated with kernel smoothing regressions. The sample is from June 1961 to December 2020.

5 Alternative One-Business Day Risk-Free Rate

Our results are robust to the choice of the one-business day risk-free rate. We study the excess returns of the traded bonds, that is, returns in excess of the one-business day risk-free rate. In the main text we use the risk-free one-business day return estimated in Filipović, Pelger, and Ye (2022). As robustness test we show that our results are not affected when we use the risk-free one-business day return implied by the 1-month risk-free rate of Fama and French (1993). The 1-month risk-free rate of Fama and French (1993) is based on the discount bond prices of Fama and Bliss (1987). We specify this alternative risk-free one-business day return such that it compounds to the 1-month return corresponding to the 1-month risk-free rate. If the next calendar day is a weekend or bank holiday, then we compound that risk-free one-calendar day return to obtain the desired risk-free one-business day return.

Figure IA.10: Cumulative excess returns and returns of the first four KR factors



(a) Cumulative excess returns of factors **(b)** Cumulative returns of factors

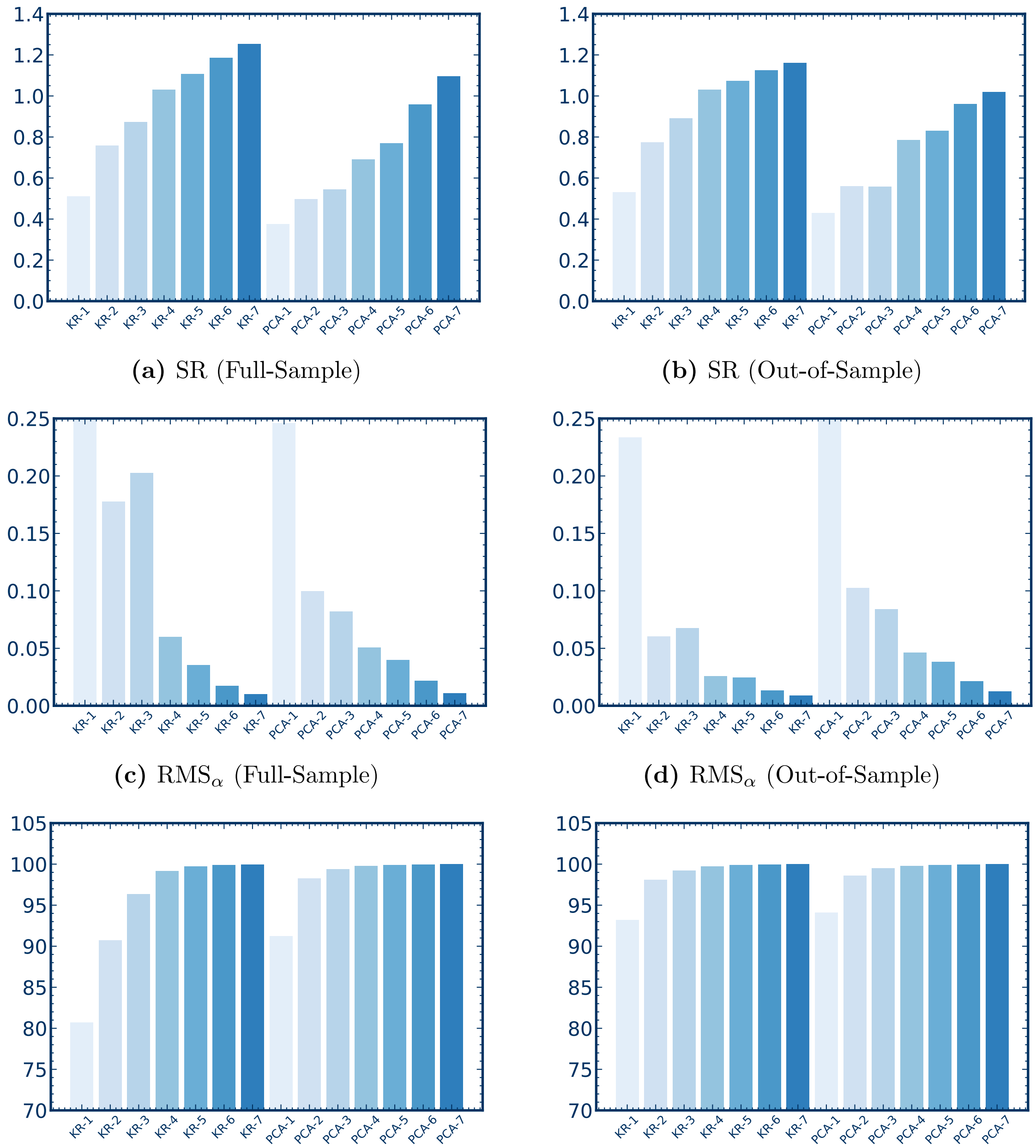
This figure shows cumulative excess returns (left) and cumulative returns (right) of the first four KR factors. The KR factors are portfolios of tradable Treasury securities. The risk-free one-business day return is based on the Fama–French monthly risk-free rate. The sampling period is from June 1961 to December 2017.

Table IA.2: Sharpe Ratios of Factors and Implied SDF with FF Risk-Free Rate

n	Factor	SDF	SDF (OOS)
1	0.51	0.51	0.54
2	0.53	0.77	0.78
3	0.37	0.89	0.88
4	0.46	1.03	1.02
5	0.34	1.10	1.05
6	0.32	1.18	1.10

This table shows the annualized Sharpe ratios for different KR factors and for their implied SDF. We report the Sharpe ratio of each factor and of the SDF, when forming the mean-variance efficient portfolio by successively including KR factors. The Sharpe ratios for the individual factors and SDF are in-sample, while the SDF (OOS) shows the out-of-sample Sharpe ratios for estimating the mean-variance efficient portfolio weights on a daily updated rolling window of the previous 10-years of daily excess returns. The risk-free one-business day return is based on the Fama–French monthly risk-free rate. The sample is daily data from June 1961 to December 2020.

Figure IA.11: Asset Pricing Results for FF one-day risk-free rate



(a) SR (Full-Sample) **(b)** SR (Out-of-Sample)

(c) RMS$_\alpha$ (Full-Sample) **(d)** RMS$_\alpha$ (Out-of-Sample)

(e) Explained Variation (%, Full-Sample) **(f)** Explained Variation (%, Out-of-Sample)

This figure summarizes the asset pricing results for different terms structure factors on a panel of discount bond excess returns. We compare the the first seven KR and PCA factors in- and out-of-sample in terms of their Sharpe ratios, root-mean-squared pricing errors and explained variation. The test assets are panel of 10 discount bond excess returns estimated with KR with yearly maturities ranging from 1 to 10 years. The annualized Sharpe ratios of the implied SDF are based on the mean-variance efficient combination of successively adding factors. The pricing errors are the intercepts in time-series regressions of the discount bond excess returns on factors. The explained variation is the percentage of variation spanned by the factors in the time-series regressions. Out-of-sample results are obtained from a regression and moments on a monthly updated 10-year rolling window. The out-of-sample pricing errors are estimated with a cross-sectional Fama Macbeth type regression with loadings from past data. The risk-free one-business day return is based on the Fama–French monthly risk-free rate. The sampling period is from July 1963 to December 2020.

References

Fama, E. F., and R. R. Bliss (1987): "The Information in Long-Maturity Forward Rates," *The American Economic Review*, 77(4), 680–692.

Fama, E. F., and K. R. French (1993): "Common risk factors in the returns on stocks and bonds," *Journal of Financial Economics*, 33(1), 3–56.

Filipović, D., M. Pelger, and Y. Ye (2022): "Stripping the Discount Curve – a Robust Machine Learning Approach," *Working paper*.

Lettau, M., and M. Pelger (2020): "Factors that Fit the Time-Series and Cross-Section of Stock Returns," *Review of Financial Studies*, 33(5), 2274–2325.