B Online Appendix – not for publication

B.1 Discussion of the Poaching Mechanism of Section 1.3

To fix ideas, consider an ascending auction for the worker's services. Denote the firms' reservation surpluses by $S \equiv S(x, y)$ and $S' \equiv S(x, y')$. As the types are unobserved, reservation surpluses are also private information. Intuitively, an ascending auction should deliver the efficient outcome, and truthfulness should be a dominant strategy for firms.

The ascending auction argument for truthfulness is as follows. First, firms should not make offers above their reservation values S and S', respectively. Second, if the incumbent bids B, then the poacher should bid $B' > B - \xi$, as it knows that the worker will receive an additional ξ when moving. If the poacher bids B', the incumbent should bid $B \geq B' + \xi$, as the worker will need to be compensated for the forgone ξ. Firms continue to make alternating bids until one of them drops out. Therefore, the incumbent retains the worker if $S \geq S' + \xi$ and pays the second price $S' + \xi$. The poacher hires the worker if $S' + \xi > S$ and pays the second price $S - \xi$. When moving to the poacher, the worker receives an additional ξ and thus receives S in total. In the event that $S' = 0$ and $\xi > S$, the worker can choose to leave the incumbent for unemployment and collect ξ. It isconvenient to represent the outcome of this ascending auction as a direct mechanism that resembles the usual second-price sealed bid auction, augmented to account for the mobility shock ξ.

B.2 Additional details for Section 2

The present value for the employed worker The value to a type-x worker employed by a type-y firm at wage w is given by

$$
\begin{aligned}
W_1(x, y, w) = u(w) - c(x, y) + \frac{\delta(x, y)}{1 + r} W_0(x) + \frac{\overline{\delta}(x, y)\overline{\lambda}_1}{1 + r} \left(W_0(x) + R_0\right) \\
+ \frac{\overline{\delta}(x, y)\lambda_1}{1 + r} \iint \Big[\mathbf{1}_{\{B(\xi) \geq B'^{+} + \xi\}} \left(W_0(x) + R_1(B', \xi)\right) \\
+ \mathbf{1}_{\{B(\xi) < B'^{+} + \xi\}} \left(W_0(x) + \max\{B(\xi), \xi\}\right) \Big] \, \mathrm{d}F_0(B'|x, \xi) \, \mathrm{d}G_1(\xi),
\end{aligned}
$$

with $S = S(x,y)$ and $S' = S(x,y')$. Substituting in the equilibrium R_1, R_0, the outcomes of the auction mechanism, and rearranging we obtain:

$$
\begin{aligned}
rW_1(x,y,w) &= u(w) - c(x,y) + \frac{\delta(x,y)}{1+r}\left(W_0(x) - W_1(x,y,w)\right) \\
&+ \frac{\lambda_1 \overline{\delta}(x,y)}{1+r} \int \Bigg[\int_{W_1(x,y,w)-W_0(x)-S(x,y')}^{S(x,y)-S(x,y')} \left[W_0(x) + S(x,y') + \xi - W_1(x,y,w)\right] g(\xi)\,\mathrm{d}\xi \\
&\qquad + \int_{S(x,y)-S(x,y')}^{S(x,y)} \left[S(x,y) + W_0(x) - W_1(x,y,w)\right] g(\xi)\,\mathrm{d}\xi \\
&\qquad\qquad + \int_{S(x,y)}^{\infty} \left[W_0(x) + \xi - W_1(x,y,w)\right] g(\xi)\,\mathrm{d}\xi \Bigg] \frac{v(y')}{V}\,\mathrm{d}y'.
\end{aligned}
$$

Let $w(x,y,R)$ be the wage that provides surplus R to the worker. Multiplying by $(1+r)$, subtracting $W_1(x,y,w)$ and $rW_0(x)$ from both sides, and evaluating at $w(x,y,R)$, we obtain the utility flow of the wage equation (7).

And further integrating by part the integrals with respect to ξ, we finally obtain,

$$
\begin{aligned}
u(w(x,y,R)) = \frac{r+\delta(x,y)}{1+r} R &+ \frac{r}{1+r} W_0(x) + c(x,y) \\
&- \frac{\overline{\delta}(x,y)\lambda_1}{1+r} \int_{S(x,y)}^{\infty} \overline{G}_1(\xi)\,\mathrm{d}\xi \\
&- \frac{\overline{\delta}(x,y)\lambda_1}{1+r} \int \left[\int_R^{S(x,y)} \overline{G}_1(R' - S(x,y')^+)\,\mathrm{d}R' \right] \frac{v(y')}{V}\,\mathrm{d}y'. \quad (24)
\end{aligned}
$$

The minimum wage is $\underline{w}(x,y) = w(x,y,0)$ yielding zero surplus. The maximal wage is $\overline{w}(x,y) = w(x,y,S(x,y))$ yielding maximal surplus $S(x,y)$.

This wage equation is the value of the wage chosen by the firm given types (x,y) and a given worker surplus R (or value $W_0(x) + R$). First, we note that a greater promised surplus R implies a greater wage:

$$
\begin{aligned}
&(1+r)u'(w(x,y,R))\frac{\partial w(x,y,R)}{\partial R} = \\
&\qquad r + \delta(x,y) + \overline{\delta}(x,y)\lambda_1 \int \overline{G}_1\left[R - S(x,y')^+\right] \frac{v(y')}{V}\,\mathrm{d}y' > 0. \quad (25)
\end{aligned}
$$

Second, for a fixed R, a greater amenity $-c(x,y)$ requires a lower wage, as in Rosen's (1986) compensating wage differential story. Third, the worker may get an outside offer as a result of holding this job. The wage function depends on y through $c(x,y)$,

$\delta(x,y)$ and $S(x,y)$, and we have

$$(1+r)u'(w(x,y,R))\frac{\partial w(x,y,R)}{\partial S(x,y)} = \\ -\overline{\delta}(x,y)\lambda_1 \int \left(G_1[S(x,y)] - G_1[S(x,y)-S(x,y')^+]\right)\frac{v(y')}{V}\,\mathrm{d}y' < 0. \quad (26)$$

The effect of the current reservation surplus is another compensating differential. A greater surplus decreases the current wage required to achieve the promised value. However, there are two effects that go in opposite directions. By increasing $S(x,y)$ we reduce job-to-job mobility as fewer vacancies can beat the current job. This reduces future payoffs and increases the wage for a given R. But by increasing $S(x,y)$ we also increase the likelihood of future wage raises (in the current job). It happens that the former effect dominates the latter. The intuitions of the basic sequential auction model of Postel-Vinay and Robin (2002) thus carry through to the more general setup, perhaps in a more transparent way.

Surplus equation Substituting expression (8) for the maximum wage along with $R = S(x,y)$ into the wage equation (7) we have

$$u(f(x,y) - r(1+r)^{-1}\Pi_0(y)) = c(x,y) + \frac{r+\delta(x,y)}{1+r}S(x,y) + \frac{r}{1+r}W_0(x) \\ - \frac{\lambda_1\overline{\delta}(x,y)}{1+r}\int \phi(x,y')\Bigg[\int_{S(x,y)-S(x,y')}^{S(x,y)-S(x,y')} [S(x,y') + \xi - S(x,y)]\,g(\xi)\,\mathrm{d}\xi \\ + \int_{S(x,y)-S(x,y')}^{\infty} [\max\{S(x,y)-\xi, 0\} + \xi - S(x,y)]\,g(\xi)\,\mathrm{d}\xi\Bigg]\frac{v(y')}{V}\,\mathrm{d}y'.$$

The first inner integral evaluates to zero. The second inner integral simplifies by noting that when $\xi < S(x,y)$, $\max\{S(x,y)-\xi,0\} + \xi - S(x,y) = 0$ and when $\xi \geq S(x,y)$, $\max\{S(x,y)-\xi,0\} + \xi - S(x,y) = \xi - S(x,y)$, so we only need to consider $\xi \geq S(x,y)$, which implies that the inner integral does not depend on y'. The equation defining the surplus then simplifies to equation (9).

Firm profit Substitution in the outcomes of the auction mechanism we can write the firm profit as

$$
\begin{aligned}
[r+\delta(x,y)]\,\Pi_1(x,y,R) &= (1+r)\,[f(x,y)-w(x,y,R)] + \delta(x,y)\Pi_0(y) \\
&+ \overline{\delta}(x,y)\lambda_1 \int \Big[\int_{R-S(x,y')^+}^{S(x,y)-S(x,y')^+} \big(\Pi_1(x,y,S(x,y')^+ + \xi) - \Pi_1(x,y,R)\big)\, \mathrm{d}G_1(\xi) \\
&\qquad + \int_{S(x,y)-S(x,y')^+}^{S(x,y)} (\Pi_0(y) - \Pi_1(x,y,R)) \Big]\, \mathrm{d}G_1(\xi) \frac{v(y')}{V}\, \mathrm{d}y' \Big\}. \qquad (27)
\end{aligned}
$$

Moreover, Theorem 1 establishes that

$$\frac{\partial \Pi_1(x,y,R)}{\partial R} = -\frac{1}{u'(w(x,y,R))}.$$

This is a simple consequence of the Envelope Theorem, which can also be obtained by differentiating equation (27) and using equation (25).

If we define the match surplus as the maximal value of R such that $\Pi_1(x,y,R) \geq \Pi_0(y)$, then the profit function being decreasing, $S(x,y)$ is defined by the equality

$$\Pi_1(x,y,S(x,y)) = \Pi_0(y). \qquad (28)$$

We can therefore also deduce firm profits from worker wages as

$$\Pi_1(x,y,R) = \Pi_0(y) + \int_R^{S(x,y)} \frac{\mathrm{d}R'}{u'(w(x,y,R'))}. \qquad (29)$$

Value of a vacancy Let $J(x,y,R) = \Pi_1(x,y,R) - \Pi_0(y)$ be the gain for the firm of offering a surplus R to the worker. At the equilibrium offers and counter offers the value of a vacancy (3) becomes:

$$
\begin{aligned}
r\Pi_0(y) &= \lambda_0 \frac{L_0}{V} \iint_{-S(x,y)}^{\infty} J(x,y,\max\{-\xi,0\})\, \mathrm{d}G_0(\xi) \frac{\ell_0(x)}{L_0}\, \mathrm{d}x \\
&+ \lambda_1 \frac{L_1}{V} \iiint_{S(x,y')-S(x,y)}^{\infty} J(x,y,\max\{S(x,y')-\xi,0\})\, \mathrm{d}G_1(\xi)(1-\delta(x,y')) \frac{\ell_1(x,y')}{L_1}\, \mathrm{d}x\, \mathrm{d}y'.
\end{aligned}
$$

Breaking the inner integrals into regions above and below the value of ξ at which $\max\{\cdot, 0\}$ becomes 0, we have

$$\begin{aligned}
r\Pi_0(y) = \lambda_0 \frac{L_0}{V} \int \phi(x,y) \left[\overline{G}_0(0) J(x,y,0) + \int_{-S(x,y)}^{0} J(x,y,0-\xi)\,\mathrm{d}G_0(\xi) \right] \frac{\ell_0(x)}{L_0}\,\mathrm{d}x \\
+ \lambda_1 \frac{L_1}{V} \iint \phi(x,y) \Big[\overline{G}_1(S(x,y')) J(x,y,0) \\
+ \int_{S(x,y')-S(x,y)}^{S(x,y')} J(x,y,S(x,y')-\xi)\,\mathrm{d}G_1(\xi) \Big] (1-\delta(x,y')) \frac{\ell_1(x,y')}{L_1}\,\mathrm{d}x\,\mathrm{d}y'. \quad (30)
\end{aligned}$$

Now, consider the two integrals involving ξ in turn. In the first case we have

$$\begin{aligned}
\int_{-S(x,y)}^{0} J(x,y,-\xi)\,\mathrm{d}G_0(\xi) &= -\left[J(x,y,-\xi)\overline{G}_0(\xi) \right]_{-S(x,y)}^{0} - \int_{-S(x,y)}^{0} \frac{\partial J}{\partial R}(x,y,-\xi)\overline{G}_0(\xi)\,\mathrm{d}\xi \\
&= -J(x,y,0)\overline{G}_0(0) + \int_{-S(x,y)}^{0} \frac{\overline{G}_0(\xi)}{u'(w(x,y,-\xi))}\,\mathrm{d}\xi,
\end{aligned}$$

where the first equality uses integration by parts and the second equality uses the fact that we defined $J(x,y,S(x,y)) = 0$ and that Lemma (A.7) implies that that $\frac{\partial J}{\partial R}(x,y,R) = -\frac{1}{u'(w(x,y,R))}$. Similarly, in the second case we have

$$\begin{aligned}
&\int_{S(x,y')-S(x,y)}^{S(x,y')} J(x,y,S(x,y')-\xi)\,\mathrm{d}G_1(\xi) \\
&\quad = -\left[J(x,y,S(x,y')-\xi)\overline{G}_1(\xi) \right]_{S(x,y')-S(x,y)}^{S(x,y')} \\
&\qquad - \int_{S(x,y')-S(x,y)}^{S(x,y')} \frac{\partial J}{\partial R}(x,y,S(x,y')-\xi)\overline{G}_1(\xi)\,\mathrm{d}\xi \\
&\quad = -J(x,y,0)\overline{G}_1(S(x,y')) + \int_{S(x,y')-S(x,y)}^{S(x,y')} \frac{1}{u'(w(x,y,S(x,y')-\xi))}\overline{G}_1(\xi)\,\mathrm{d}\xi \\
&\quad = -J(x,y,0)\overline{G}_1(S(x,y')) + \int_{-S(x,y)}^{0} \frac{\overline{G}_1(\xi+S(x,y'))}{u'(w(x,y,-\xi))}\,\mathrm{d}\xi.
\end{aligned}$$

Substituting these two expressions into equation (30) we obtain (11), where, given the bound of the integrals, we can remove $\phi(x,y)$.

B.3 Proof of Lemma 1: Trajectories are Markovian

We treat the different m_t states separately and show that the Markov property holds for each. When the worker is unemployed, we simply write $y_t = 0$ and $w_t = \varnothing$. Our goal is to show

$$\mathbb{P}[w_{t+1}, y_{t+1}, m_{t+1}|x, w_t, y_t, m_t, \Omega_{t-1}] = \mathbb{P}[w_{t+1}, y_{t+1}, m_{t+1}|x, w_t, y_t, m_t]$$

By applying successive conditioning,

$$\begin{aligned}\mathbb{P}[w_{t+1}, y_{t+1}, m_{t+1}|x, w_t, y_t, m_t, \Omega_{t-1}] &= \mathbb{P}[m_{t+1}|w_{t+1}, y_{t+1},\ x, w_t, y_t, m_t, \Omega_{t-1}] \\ &\times \mathbb{P}[w_{t+1}|y_{t+1},\ x, w_t, y_t, m_t, \Omega_{t-1}] \times \mathbb{P}[y_{t+1}|x, w_t, y_t, m_t, \Omega_{t-1}].\end{aligned}$$

We proceed by showing that each of these three probabilities are independent of Ω_{t-1}.

Mobility m_{t+1} It follows from the model that mobility is only a function of the surplus and not of the wage itself.

When an unemployed worker x meets a firm y, the match is formed if and only if $0 \leq -\xi \leq S(x,y)$ (remember that G_0 has negative support):

$$\mathbb{P}\left[m_{t+1} = \mathrm{UE}|x, y_{t+1} = 0, \Omega_t\right] = \lambda_0 \int G_0(-S(x,y))\phi(x,y)\frac{v(y)}{V}\,\mathrm{d}y,$$

and for workers employed in $y_t > 0$ we have (using the short-hand notations $S = S(x,y)$ and $S' - S(x,y')$):

$m_{t+1} = \mathrm{EU}$ if $\xi > \max\{S, S' + \xi\}$:

$$\begin{aligned}&\mathbb{P}\left[m_{t+1} = \mathrm{EU}|x, y_{t+1} = y, w_{t+1}, \Omega_t\right] \\ &\qquad = \delta(x,y) + \overline{\delta}(x,y)\lambda_1 \int \left[1 - \phi(x,y')\right]\frac{v(y')}{V}\,\mathrm{d}y' \times \overline{G}_1(S),\end{aligned}$$

$m_{t+1} = \mathrm{JJ}$ if $S' + \xi > \max\{S, \xi\}$ or $S' + \xi > S$ and $S' > 0$:

$$\mathbb{P}\left[m_{t+1} = \mathrm{JJ}|x, y_{t+1} = y, w_{t+1}, \Omega_t\right] = \overline{\delta}(x,y)\lambda_1 \int \overline{G}_1(S - S')\phi(x,y')\frac{v(y')}{V}\,\mathrm{d}y',$$

$m_{t+1} = \mathrm{EE}$ if $S \geq S'^{+} + \xi$ or there was no offer:

$$\mathbb{P}\left[m_{t+1} = \mathrm{EE}|x, y_{t+1} = y, w_{t+1}, \Omega_t\right] = \overline{\delta}(x,y)\overline{\lambda}_1 + \overline{\delta}(x,y)\lambda_1 \int G_1(S - S'^{+})\frac{v(y')}{V}\,\mathrm{d}y'.$$

Therefore $\mathbb{P}[m_{t+1}|x, w_t, y_t, m_t, w_{t+1}, y_{t+1}, \Omega_{t-1}] = \mathbb{P}[m_{t+1}|x, w_t, y_t, m_t, w_{t+1}, y_{t+1}]$.

Wage w_{t+1} The next step is to examine the law of motion of wages. Whenever unemployed the wage is missing ($w_t = \varnothing$), so we do not need to consider cases $m_t \in \{\mathrm{UU}, \mathrm{EU}\}$ for the law of motion of the wage. We then need to look at UE, EE, and JJ. In each case, we seek an expression for $\mathbb{P}\left[w_{t+1}|x, y_t, w_t, y_{t+1}, m_t, \Omega_{t-1}\right]$.

When $m_t = \mathrm{UE}$, we know that, conditional on moving, the offer is set to deliver a surplus $-\xi$ to the worker, where ξ is a draw from G_0, truncated below by $-S(x, y_{t+1})$. The wage is set through the injective function $w(x, y, R)$ and so:

$$\begin{aligned}\mathbb{P}\left[w_{t+1} \leq w'|x, w_t = w, y_t = 0, y_{t+1} = y, m_t = \mathrm{UE}, \Omega_{t-1}\right] \\ = \mathbb{P}\left[w(x, y, -\xi) \leq w'|\xi \geq -S(x, y)\right] := F_{\mathrm{UE}}(w'|x, y).\end{aligned}$$

Similarly when $m_t = \mathrm{JJ}$, we know that conditional on moving from $y_t = y$ to $y_{t+1} = y'$ the offer is set to deliver a surplus $(S(x, y) - \xi)^+$ to the worker, where ξ is a draw from G_1, truncated below by $S(x, y) - S(x, y')$:

$$\begin{aligned}&\mathbb{P}\left[w_{t+1} \leq w'|x, w_t = w, y_t = y, y_{t+1} = y', m_t = \mathrm{JJ}, \Omega_{t-1}\right] \\ &\quad = \mathbb{P}\left[w(x, y', (S(x, y) - \xi)^+) \leq w' \,|\, \xi \geq S(x, y) - S(x, y')\right] := F_{\mathrm{JJ}}(w'|x, y, y').\end{aligned}$$

Here, we note that in addition it is independent of the previous wage.

We then consider our final case of $m_t = \mathrm{EE}$. In this case, the wage only changes if an outside offer comes in and is above the surplus the worker is getting from their current wage. We know that the surplus the worker receives at wage w from a firm y is equal to $0 \leq R(x, y, w) \leq S(x, y)$. The joint probability of $m_t = \mathrm{EE}$ and $w_{t+1} \leq w' \in [w, w(x, y, S(x, y))]$ for a worker currently employed at a firm $y_t = y$ with a wage $w_t = w$ is the probability of drawing an offer $y_{t+1} = y'$ and a ξ such that $S(x, y')^+ + \xi \leq R(x, y, w')$:

$$\begin{aligned}&\mathbb{P}\left[m_t = \mathrm{EE}, w_{t+1} \leq w'|x, w_t = w, y_t = y, \Omega_{t-1}\right] \\ &\quad = \overline{\delta}(x, y)(1 - \lambda_1) + \overline{\delta}(x, y)\lambda_1 \int G_1\left[R(x, y, w') - S(x, y')^+\right] \frac{v(y')}{V}\, \mathrm{d}y'.\end{aligned}$$

Hence

$$\begin{aligned}&\mathbb{P}\left[w_{t+1} \leq w'|x, w_t, y_t, y_{t+1}, m_t = \mathrm{EE}, \Omega_{t-1}\right] \\ &\quad = \mathbf{1}\left\{w' \geq w\right\} \frac{\mathbb{P}\left[m_t = \mathrm{EE}, w_{t+1} \leq w'|x, w_t = w, y_t = y, \Omega_{t-1}\right]}{\mathbb{P}\left[m_t = \mathrm{EE}|x, y_t\right]} \\ &\quad = \mathbf{1}\left\{w' \geq w\right\} \frac{1 - \lambda_1 \int \overline{G}_1\left[R(x, y, w') - S(x, y')^+\right] \frac{v(y')}{V}\, \mathrm{d}y'}{1 - \lambda_1 \int \overline{G}_1\left[S(x, y) - S(x, y')^+\right] \frac{v(y')}{V}\, \mathrm{d}y'} := F_{\mathrm{EE}}(w'|x, y, w).\end{aligned}$$

This shows that $\mathbb{P}[w_{t+1}|x, w_t, y_t, y_{t+1}, m_t{=}\mathrm{EE}, \Omega_{t-1}] = \mathbb{P}[w_{t+1}|x, w_t, y_t, y_{t+1}, m_t{=}\mathrm{EE}]$.

Firm y_{t+1} Next we turn to y_{t+1}. When $m_t = \mathrm{EU}$, $y_{t+1} = 0$. When $m_t = \mathrm{EE}$, $y_{t+1} = y_t$. When $m_t = \mathrm{UE}$, $\mathbb{P}\left[y_{t+1} = y'|x, m_t = \mathrm{UE}, \Omega_{t-1}\right] \propto \frac{v(y')}{V}\overline{G}_0(-S(x, y'))$. Finally, when $m_t = \text{JJ}$, we get

$$\mathbb{P}\left[y_{t+1} = y'|x, m_t = \text{JJ}, y_t = y, w_t, \Omega_{t-1}\right] \propto \overline{\delta}(x,y)\lambda_1 \frac{v(y')}{V}\phi(x,y')\overline{G}_1(S(x,y)-S(x,y')).$$

This establishes $\mathbb{P}[y_{t+1}|x, w_t, y_t, m_t, \Omega_{t-1}] = \mathbb{P}[y_{t+1}|x, w_t, y_t, m_t]$ and concludes the proof for the Markov property of the model.

B.4 Identification Step 1

We adapt the proof of Bonhomme et al. (2019) to our context of Markovian wages on the job. For simplification, we consider the case with discrete wage outcomes but refer to the original paper for a proof with wages belonging to a continuum.

Throughout the proof, we assume that we have a discretization of the wage where the assumptions hold. This discretization is simply a list of support points w_p for $p \in 1, ..., n_w$. Let also $q \in \{1, ..., n_x\}$ denote the values for worker types x.

Lemma B.1. *We consider 2 firm types y, y' and one middle wage w_2. The distributions*

$$\mathbb{P}[w_1|x, y_1, m_1 = \text{JJ}, m_2 = \text{EE}], \mathbb{P}[w_3|x, w_2, y_2, m_1 = \text{JJ}, m_2 = \text{EE}],$$
$$\mathbb{P}[x, w_2|y_1, y_2, m_1 = \text{JJ}, m_2 = \text{EE}],$$

are identified from $\mathbb{P}[w_1, w_2, w_3|y_1, y_2, m_1 = \text{JJ}, m_2 = \text{EE}]$ for all $y_1, y_2 \in \{y, y'\}$ under the assumptions that for any $y_1, y_2 \in \{y, y'\}$,

1. *Wages are Markovian within job spells:*
$$\mathbb{P}[w_3|x, w_2, w_1, y_1, y_2, m_1 = \text{JJ}, m_2 = \text{EE}] = \mathbb{P}[w_3|x, w_2, y_1, y_2, m_1 = \text{JJ}, m_2 = \text{EE}].$$

2. *Wages after a move do not depend on wages before the move:*
$$\mathbb{P}[w_2|x, y_1, y_2, w_1, m_1 = \text{JJ}, m_2 = \text{EE}] = \mathbb{P}[w_2|x, y_1, y_2, m_1 = \text{JJ}, m_2 = \text{EE}].$$

3. *The distributions $\mathbb{P}[w_1|x, y_1, m_1 = \text{JJ}, m_2 = \text{EE}]$ and $\mathbb{P}[w_3|x, w_2, y_1, m_1 = \text{JJ}, m_2 = \text{EE}]$ are linearly independent with respect to x for all y_1, w_2 (that is, the CDFs given one x cannot be replicated by the linear combination of the CDFs of w_1 given the other x').*
4. *$d(x, y_1, y_2, w_2) = \mathbb{P}[x, w_2|y_1, y_2, m_1{=}\text{JJ}, m_2{=}\text{EE}] \neq 0$ for all x.*
5. *The following quantity is different for different x's:*
$$\frac{d(x, y, y, w_2)d(x, y', y', w_2)}{d(x, y', y, w_2)d(x, y, y', w_2)}.$$

Proof. We are going to show that given data around a move, we can identify the law of motion for each pair of worker and firm types that employ all types. Throughout y_1 and y_2 can be either y or y'. We can write the following joint density as

$$\begin{aligned}
&\mathbb{P}[w_1, w_2, w_3 | y_1, y_2, m_1 = \text{JJ}, m_2 = \text{EE}] \\
&= \sum_x \mathbb{P}[x | y_1, y_2, m_1 = \text{JJ}, m_2 = \text{EE}] \times \mathbb{P}[w_1, w_2, w_3 | x, y_1, y_2, m_1 = \text{JJ}, m_2 = \text{EE}] \\
&= \sum_x \mathbb{P}[x | y_1, y_2, m_1 = \text{JJ}, m_2 = \text{EE}] \times \mathbb{P}[w_1 | x, y_1, y_2, m_1 = \text{JJ}] \\
&\qquad \times \mathbb{P}[w_2 | x, y_1, y_2, m_1 = \text{JJ}, m_2 = \text{EE}] \times \mathbb{P}[w_3 | x, w_2, y_1, y_2, m_1 = \text{JJ}, m_2 = \text{EE}],
\end{aligned}$$

where we used assumptions i) and ii) to establish the last equality. We then denote the data matrix of the joint density of w_1, w_3 for a fixed value of w_2 and a given y_1, y_2 by $A(y_1, y_2, w_2) \in \mathbb{R}^{n_w \times n_w}$. Hence we have

$$A(y_1, y_2, w_2) = \Big[\mathbb{P}[w_1 \le w_p, w_2, w_3 \le w_q | y_1, y_2, m_1 = \text{JJ}, m_2 = \text{EE}]\Big]_{p,q}.$$

We similarly define the n_w-by-n_x matrices

$$\begin{aligned}
M_1(y_1) &= \Big[\mathbb{P}[w_1 \le w_p | x = q, y_1, m_1 = \text{JJ}, m_2 = \text{EE}]\Big]_{p,q}, \\
M_{\text{EE}}(y_2, w_2) &= \Big[\mathbb{P}[w_3 \le w_p | x = q, w_2, y_2, m_1 = \text{JJ}, m_2 = \text{EE}]\Big]_{p,q},
\end{aligned}$$

and the the following diagonal matrix

$$D(y_1, y_2, w_2) = \text{diag}\Big[\mathbb{P}[x = q, w_2 | y_1, y_2, m_1 = \text{JJ}, m_2 = \text{EE}]\Big]_q,$$

where $M_1(y), M_{\text{EE}}(y_2, w_2) \in \mathbb{R}^{n_w \times n_x}$ and $D(y_1, y_2, w_2) \in \mathbb{R}^{n_x \times n_x}$. We can then write the identifying restrictions as

$$A(y_1, y_2, w_2) = M_1(y_1) D(y_1, y_2, w_2) M_{\text{EE}}^\top(y_2, w_2).$$

The next step is to take the singular value decomposition of $A(y, y', w_2) = USV^\top$ where the matrix $S \in \mathbb{R}^{n_x \times n_x}$ by assumption iv) and iii) is diagonal, and $U, V \in \mathbb{R}^{n_w \times n_x}$ are such that $U^\top U = V^\top V = I_{n_x}$. For all $y_1, y_2 \in \{y, y'\}$, define $B(y_1, y_2, w_2) = S^{-\frac{1}{2}} U^\top A(y_1, y_2, w_2) V S^{-\frac{1}{2}}$ where $B(y_1, y_2, w_2) \in \mathbb{R}^{n_x \times n_x}$ is invertible again by assumption iii) and iv). Note that the different $B(y_1, y_2, w_2)$ matrices for each $y_1, y_2 \in \{y, y'\}$ use the same U, S, V matrices defined as the SVD for $A(y, y', w_2)$.

We first note a property we will use at the end. We have that $D(y, y', w_2) M_{\text{EE}}^\top(y', w_2)$ is full row rank (n_x) by assumption iii) and iv), hence there exist a matrix $\tilde{M} \in \mathbb{R}^{n_w \times n_x}$

such that

$$D(y, y', w_2) M_{\mathrm{EE}}^\top(y', w_2) \tilde{M} = I_{n_x}.$$

This implies that

$$\begin{aligned} UU^\top M_1(y) &= UU^\top M_1(y) D(y, y', w_2) M_{\mathrm{EE}}^\top(y', w_2) \tilde{M} \\ &= UU^\top USV^\top \tilde{M} \\ &= M_1(y) D(y, y', w_2) M_{\mathrm{EE}}^\top(y', w_2) \tilde{M} \\ &= M_1(y). \end{aligned}$$

We then construct

$$\begin{aligned} B(y, y, w_2) B(y', y, w_2)^{-1} = S^{-\frac{1}{2}} U^\top M_1(y) D(y, y, w_2) M_{\mathrm{EE}}^\top(y, w_2) V S^{-\frac{1}{2}} \\ \times \left(S^{-\frac{1}{2}} U^\top M_1(y') D(y', y, w_2) M_{\mathrm{EE}}^\top(y, w_2) V S^{-\frac{1}{2}} \right)^{-1} \\ = Q_1(y) D(y, y, w_2) D(y', y, w_2)^{-1} Q_1(y')^{-1}, \end{aligned}$$

where we used $Q_1(y) = S^{-\frac{1}{2}} U^\top M_1(y) \in \mathbb{R}^{n_x \times n_x}$. We note that $Q_1(y')$ is full rank since $A(y', y, w_2)$ has rank n_x from Assumption part iii) and iv). We have then established the following eigenvalue problem:

$$\begin{aligned} B(y, y, w_2) B(y', y, w_2)^{-1} B(y', y', w_2) B(y, y', w_2)^{-1} = \\ Q_1(y) D(y, y, w_2) D(y', y, w_2)^{-1} D(y', y', w_2) D(y, y', w_2)^{-1} Q_1(y)^{-1}. \end{aligned}$$

Provided that the eigenvalues are unique, as guaranteed by assumption v), this identifies $Q_1(y)$. We have established that $UU^\top M_1(y) = M_1(y)$ and hence we identified $M_1(y) = US^{\frac{1}{2}} Q_1(y)$ up to the scale of the eigenvalue. This scale is pinned down by the fact that the columns of $M_1(y)$ are each a c.d.f which allows using that they equal 1 at w_{n_w}.

With $Q_1(y)$ identified we can use the fact that

$$Q_1^{-1}(y) S^{-\frac{1}{2}} A(y, y', w_2) = D(y, y', w_2) M_{\mathrm{EE}}^\top(y', w_2).$$

All objects on the left hand side are known and hence $M_{\mathrm{EE}}(y', w_2)$ is identified up to scale. We can use again, that the columns are CDFs and hence equal 1 at the top.

Once $M_{\mathrm{EE}}(y', w_2)$ and $M_1(y)$ are known we can get $\mathbb{P}[w_2, x | y_1 = y, y_2 = y', m_1 = \mathrm{JJ}, m_2 = \mathrm{EE}]$ from $A(y, y', w_2)$. This gives us the wage conditional on moving, as well as the destination firm for each of the worker types. $\square$

Lemma B.2. *We recover $M_1(y'')$ and $M_{EE}(y'', w)$ for all other y'' and w using the identified $M_{\text{EE}}(y, w_2)$ and $M_1(y)$ from Lemma B.1.*

Proof. We have identified $M_1(y)$ and $M_{EE}(y, w_2)$ for a specific y and w_2. We then note that

$$A(y'', y, w_2)M_{EE}(y, w_2) = M_1(y'')D(y'', y, w_2)M_{\text{EE}}^{\top}(y, w_2)M_{EE}(y, w_2),$$

where $M_{\text{EE}}^{\top}(y, w_2)M_{EE}(y, w_2)$ is know and invertible from Lemma B.1. Hence, the matrix $M_1(y'')D(y'', y, w_2)$ is identified. We finally use the fact that $M_1(y'')$ is a CDF to separate $M_1(y'')$ from $D(y'', y, w_2)$. This identifies $M_1(y'')$ and $D(y'', y, w_2)$ for all y'' with the same labeling as in Lemma B.1. Whenever $D(y'', y, w_2)$ we can't get the corresponding wage density since there are no movers. Though there are no movers only when that particular type never works in the firm.

Next we use the same reasoning for a different w_2. We note that for the y and y' of Lemma B.1 and a w_2' we have:

$$M_1(y)^{\top}A(y, y', w_2') = M_1(y)^{\top}M_1(y)D(y, y', w_2')M_{\text{EE}}^{\top}(y, w_2'),$$

where $M_1(y)^{\top}M_1(y)$ is known and invertible. Hence $D(y, y', w_2')M_{\text{EE}}^{\top}(y, w_2')$ is identified. We finally use the fact that $M_{\text{EE}}^{\top}(y, w_2')$ is a CDF to separate $M_{\text{EE}}^{\top}(y, w_2')$ from $D(y, y', w_2')$. This identifies both. □

Lemma B.3. $\mathbb{P}[y_4, w_4, m_3, y_3, w_3, m_2|m_1 = \text{EE}, y_1, w_1, w_2, x]$ *is identified from* $\mathbb{P}[y_4, w_4, m_3, y_3, w_3, m_2, w_2|m_1 = \text{EE}, y_1, w_1]$ *provided that*

1. *$P[w_2, x|m_1 = \text{EE}, y_1, w_1]$ are linearly independent.*
2. *$\mathbb{P}[w_2|x, w_1, y_1, m_1{=}\text{EE}]$ is known (from Lemma B.1)*

Proof. We first consider the following marginal distribution:

$$\mathbb{P}[w_2|m_1 = \text{EE}, y_1, w_1] = \sum_x \mathbb{P}[w_2|x, m_1 = \text{EE}, y_1, w_1]\mathbb{P}[x|m_1 = \text{EE}, y_1, w_1], \tag{31}$$

where $\mathbb{P}[w_2|x, m_1 = \text{EE}, y_1, w_1]$ is known and the column rank assumption of Lemma B.1 gives that $\mathbb{P}[x|m_1 = \text{EE}, y_1, w_1]$ is identified.

We then note that

$$\begin{aligned}\mathbb{P}[y_4,&w_4,m_3,y_3,w_3,m_2,w_2|m_1=\mathrm{EE},y_1,w_1]\\&=\sum_x \mathbb{P}[y_4,w_4,m_3,y_3,w_3,m_2|m_1=\mathrm{EE},y_1,w_1,w_2,x]\\&\qquad\times P[w_2,x|m_1=\mathrm{EE},y_1,w_1]\end{aligned}$$

where the left hand side is data and $P[w_2,x|m_1 = \mathrm{EE},y_1,w_1]$ is known from the previous step and Lemma B.1. The linear independence assumption concludes the proof. □

Corollary 1. $\mathbb{P}[y_4,w_4,m_3,y_3,w_3,m_2|m_1 = \mathrm{EE},y_1,w_1,w_2,x]$ *identifies the following quantities:*

- $\mathbb{P}[m_t = \mathrm{EU}|x,y_t]$
- $\mathbb{P}[m_t = \mathrm{UE}|x,y_t = 0]$
- $\mathbb{P}[y_{t+1},w_{t+1}|m_t = \mathrm{UE},x]$
- $\mathbb{P}[m_t = \mathrm{JJ}|x,y_t]$
- $\mathbb{P}[y_{t+1},w_{t+1}|m_t = \mathrm{JJ},x,y_t]$

Proof. The result follows from the Markovian properties of the model. For example:

$$\begin{aligned}\mathbb{P}[y_{t+1},&w_{t+1}|m_t=\mathrm{UE},x]\\&=\mathbb{P}[y_{t+1},w_{t+1}|m_t=\mathrm{UE},x,m_{t-2}=\mathrm{EE},m_{t-1}=\mathrm{EU},w_{t-2}]\\&=\mathbb{P}[y_4=y_{t+1},w_4=w_{t+1},m_3=\mathrm{UE},y_3,w_3,m_2|m_1=EE,y_1,w_1,w_2,x]\end{aligned}$$

□

Corollary 2. *Cross-sectional distributions are identified from using transition probabilities from Corollary 1.*

B.5 Data, sample, and variable construction

We used the raw matched employer-employee data set constructed in Friedrich, Laun, Meghir, and Pistaferri (2019). This links information from three data sources made available by The Institute for Evaluation of Labour Market and Education Policy (IFAU).

The primary data sources for this study are three datasets. The is the Longitudinal Database on Education, Income, and Employment (LOUISE), which provides comprehensive information on demographic and socioeconomic variables for the entire working-age population in Sweden, spanning from 1990 to the present.

The second dataset is the Register-Based Labor Market Statistics (RAMS), which covers employment spells in Sweden starting from 1985 and continuing up to the present time. RAMS includes essential details such as gross annual earnings, the initial and final remunerated month for each employee-firm spell, and unique firm identifiers at the Corporate Registration Number level.

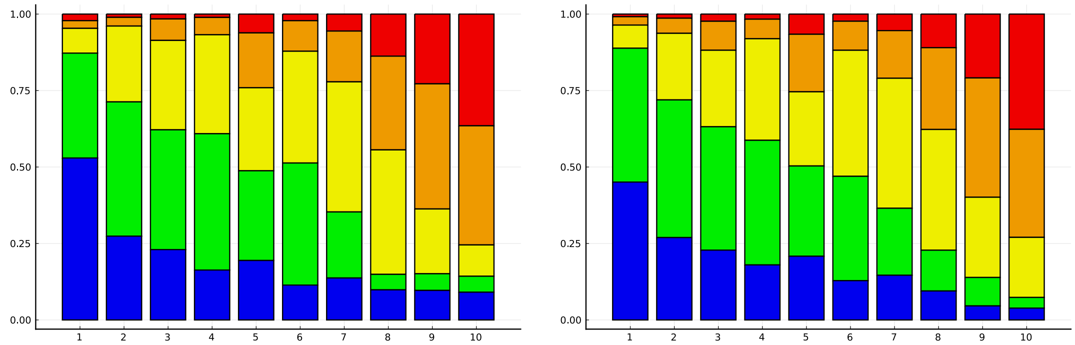
On the firm-related side, RAMS also records information about the industry and the type of legal entity for all firms that employ workers. Finally, we draw from the third data source, the Structural Business Statistics (SBS), which encompasses accounting and balance sheet information for all nonfinancial corporations in Sweden, spanning from 1997 to the present. Of particular interest within SBS is the variable called FORBRUKNINGSVARDE, which provides a measure of value added at both the firm and year level. All monetary variables are adjusted for inflation (detrended with the CPI).

Our analysis is centered on the years 2000 to 2004. The sample we examine comprises all firms classified as either a limited partnership or a limited company, excluding banking and insurance companies. There are two specific restrictions inherited from the original data construction: spells with monthly earnings below 3,416 in 2008 Swedish krona are excluded from the sample, spells that span less than two months of employment (i.e. instances where the start month is the same as the end month) are also excluded from our analysis.

In addition to CPI detrending, we remove year means in the data and we limit it to workers under the age of 50.

B.6 Likelihood for transitions

We estimate the following set of parameters $S(x,y)$ and $\tilde{v}(y) = v(y)/V$, λ_0, λ_1, ρ_0 and ρ_1, that we denote θ by maximizing the following log-likelihood subject to the



(a) Model Stationary

(b) Data (Step 1)

Figure 6: Model stationary distribution and distribution in first step.

Notes: comparing distribution implied by the surplus to distribution estimated with empirical model in Step 1.

moment constraints in the text:

$$
\begin{aligned}
\max_{\theta} \sum_{x} \ell_0(x) \Big[& \\
& \sum_{y'} \mathbb{P}[y_{t+1}{=}y', m_t{=}\text{UE}|x] \times \log \lambda_0 \tilde{v}(y') G_0(\rho_0 S(x,y')) \\
& + \mathbb{P}[m_t{=}\text{UU}|x] \times \log \Big(1 - \sum_{y'} \lambda_0 \tilde{v}(y') G_0(\rho_0 S(x,y'))\Big)\Big] \\
+ \sum_{x,y} \ell_1(x,y) \overline{\delta}(x,y) \Big[& \\
& \sum_{y'} \mathbb{P}[y_{t+1}{=}y', m_t{=}\text{JJ}|y_t{=}y, x] \times \log \lambda_1 \tilde{v}(y') G_1(\rho_1 S(x,y') - \rho_1 S(x,y)) \\
& + \mathbb{P}[m_t{=}\text{EE}|y_t{=}y, x] \times \log \Big(1 - \sum_{y'} \lambda_1 \tilde{v}(y') G_1(\rho_1 S(x,y') - \rho_1 S(x,y))\Big)\Big] \\
s.t. \quad m_1(\theta) = m_1, m_2(\theta) = m_2 &
\end{aligned}
$$

where $\mathbb{P}[y_{t+1}{=}y', m_t{=}\text{UE}|x]$, $\mathbb{P}[m_t{=}\text{UU}|x]$, $\mathbb{P}[y_{t+1}{=}y', m_t{=}\text{JJ}|y_t{=}y, x]$, $\mathbb{P}[m_t{=}\text{EE}|y_t{=}y, x]$ and $\ell_1(x,y)$ and $\ell_0(x)$ are known from step 1. The moments $m_1(\theta), m_2(\theta)$ are constructed by simulation, since given θ we can simulate wages and transitions.

References Online Appendix

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