A Additional Results

A.1 Derivations for Analytical Model

To derive equation (10), start by taking a total derivative of utility (1) with respect to the tax rate (recognizing that G is fixed and thus $dG = 0$), substitute in the household first-order conditions (3), and divide through by the marginal utility of income (λ). Then substitute in the equations for output prices (8) and total derivatives of the production equations (4), of the labor market clearing condition (5), and of covered emissions (7) (in each case with respect to the tax rate τ_{Z_J}) and rearrange to get

$$\frac{1}{\lambda}\frac{dU}{d\tau_{Z_J}} = \tau_{Z_J}\frac{dZ^C}{d\tau_{Z_J}} + \tau_L\frac{dL}{d\tau_{Z_J}}. \tag{18}$$

The first term in (18) is the direct effect, but to get to (10), we need to separate the second term into the tax-interaction and revenue-recycling effects. To do so, first take a total derivative of the government budget constraint (9) with respect to the tax rate τ_{Z_J} to get

$$Z^C + \tau_{Z_J}\frac{dZ^C}{d\tau_{Z_J}} + L\frac{d\tau_L}{d\tau_{Z_J}} + \tau_L\frac{dL}{d\tau_{Z_J}} = \frac{dT}{d\tau_{Z_J}}. \tag{19}$$

First consider the case in which emissions tax revenue is used to reduce the tax rate on labor. In this case, dT will equal zero, and the total derivative of labor supply with respect to the emissions tax is

$$\frac{dL}{d\tau_{Z_J}} = \frac{\partial L}{\partial\tau_{Z_J}} + \frac{\partial L}{\partial\tau_L}\frac{d\tau_L}{d\tau_{Z_J}}. \tag{20}$$

Rearranging (19), substituting in $dT = 0$, and substituting the result into (20) gives

$$\frac{dL}{d\tau_{Z_J}} = \frac{\partial L}{\partial\tau_{Z_J}} - \frac{\partial L}{\partial\tau_L}\frac{1}{L}\left[Z^C + \tau_{Z_J}\frac{dZ^C}{d\tau_{Z_J}} + \tau_L\frac{dL}{d\tau_{Z_J}}\right]. \tag{21}$$

One can then rearrange (20) to get

$$\tau_L \frac{dL}{d\tau_{Z_J}} = \tau_L \frac{dL}{d\tau_{Z_J}} \eta_{\tau_L} + (\eta_{\tau_L} - 1) \left[Z^C + \tau_{Z_J} \frac{dZ^C}{d\tau_{Z_J}} \right] \tag{22}$$

where

$$\eta_{\tau_L} = \frac{L}{L + \tau_L \partial L / \partial \tau_{Z_J}}, \tag{23}$$

which is the marginal cost of public funds (MCPF) for the labor tax (a partial-equilibrium version that ignores effects on markets other than the labor market): the numerator is the marginal cost to the household of an increase in the labor tax rate, while the denominator is the marginal revenue.

Now consider the case in which emissions tax revenue is used to increase the lump-sum transfer to households. In this case, $d\tau_{Z_J}$ will equal zero and the total derivative of labor supply with respect to the emissions tax is

$$\frac{dL}{d\tau_{Z_J}} = \frac{\partial L}{\partial \tau_{Z_J}} + \frac{\partial L}{\partial T} \frac{dT}{d\tau_{Z_J}}. \tag{24}$$

Rearranging (19), substituting in $d\tau_{Z_J} = 0$, and substituting the result into (24), gives

$$\frac{dL}{d\tau_{Z_J}} = \frac{\partial L}{\partial \tau_{Z_J}} + \frac{\partial L}{\partial T} \left[Z^C + \tau_{Z_J} \frac{dZ^C}{d\tau_{Z_J}} + \tau_L \frac{dL}{d\tau_{Z_J}} \right]. \tag{25}$$

One can then rearrange (25) to get

$$\tau_L \frac{dL}{d\tau_{Z_J}} = \tau_L \frac{\partial L}{\partial \tau_{Z_J}} \eta_T + (\eta_T - 1) \left[Z^C + \tau_{Z_J} \frac{dZ^C}{d\tau_{Z_J}} \right], \tag{26}$$

where

$$\eta_T = \frac{1}{1 - \tau_L \partial L / \partial T}, \tag{27}$$

which is the MCPF for the lump-sum transfer (again, a partial-equilibrium version): the numerator is the marginal cost to the household of a reduction in the lump-sum transfer, while the denominator is the marginal effect on the government budget.

Note that in each case, the result (equation (22) or (26)) equals

$$\tau_L \frac{dL}{d\tau_{Z_J}} = \tau_L \frac{\partial L}{\partial \tau_{Z_J}} \eta_R + (\eta_R - 1) \left[Z^C + \tau_{Z_J} \frac{dZ^C}{d\tau_{Z_J}} \right], \tag{28}$$

where η_R is the MCPF corresponding to how the emissions tax revenue is recycled. Now substitute (12) into (11) (from the main text) and rearrange to get

$$\mu_{IJ} = -\frac{\tau_L}{Z^C} \frac{\partial L}{\partial \tau_{Z_J}}. \tag{29}$$

Substituting (29) into the first term on the right-hand side of (28) gives

$$\tau_L \frac{dL}{d\tau_{Z_J}} = -\eta_R \mu_{IJ} Z^C + (\eta_R - 1) \left[Z^C + \tau_{Z_J} \frac{dZ^C}{d\tau_{Z_J}} \right]. \tag{30}$$

Finally, reversing the order of the two terms on the right-hand side of (30), substituting into (18), and dividing through by

$$-dZ^C / d\tau_{Z_J}$$

gives (10).

To derive equation (16), note that the only way that the derivation of (13) depends on the assumption that there is no leakage is in defining marginal cost as the cost per unit of reduction in covered emissions (Z^C): the rest of the derivation makes no assumptions at all about leakage. Allowing for leakage, marginal cost is the cost per unit of reduction in total emissions (Z). One can rearrange the equation defining leakage (17) to get

$$dZ/d\tau_{Z_J} = (1 - \theta)\, dZ^C / d\tau_{Z_J} \,. \tag{31}$$

Substituting (31) into (13) yields (16).

Table A-1: Calculating the Crossover Points between Economy-Wide and Narrower Policies

	EW Carbon Price	Narrow Carbon Price	Emissions Reductions (NPV)	Welfare Cost per Ton Reduced
Lump-Sum Recycling				
Power Sector Only	$17.49	$69.40	17.4%	$65.84
Motor Vehicle Fuel Exemption	$44.68	$50.80	29.6%	$98.25
EITE Industry Exemption	$7.58	$8.50	9.9%	$50.05
Individual Income Tax Cuts				
Power Sector Only	$7.25	$15.34	9.4%	$27.75
Motor Vehicle Fuel Exemption	n/a	n/a	n/a	n/a
EITE Industry Exemption	$7.61	$8.37	9.8%	$28.31
Corporate Income Tax Cuts				
Power Sector Only	n/a	n/a	n/a	n/a
Motor Vehicle Fuel Exemption	n/a	n/a	n/a	n/a
EITE Industry Exemption	$7.22	$7.85	9.3%	$7.72

B The Numerical Model

The numerical model is based off of the Goulder-Hafstead E3 model in Goulder and Hafstead (2017) with some changes to international trade, the aggregation of producer and consumer goods, and the nested consumption and production functions.

B.1 Goods and Trade

Let g_i denote the demand for a good produced by sector i. A good consumed could be produced by the domestic producer or it could be imported from an international producer. The model uses a simple CES Armington structure such that g_i is a

Table B-1: Industry and Consumer Goods

Industrial Producers	**Consumption Goods**
Oil Extraction	Motor Vehicles and Services
Gas Extraction	Motor Vehicle Fuels
Coal Mining	Motor Vehicle Electricity
Electricity Generation: Coal	Non-Personal Transportation
Electricity Generation: Gas/Petroleum	Housing
Electricity Generation: Nuclear	Electricity (Non-vehicle)
Electricity Generation: Hydro/Other	Natural Gas
Electricity Generation: Wind	Fuel Oils and Other Fuels
Electricity Generation: Solar	Other Goods and Services
Electric Transmission and Distribution	
Petroleum Refining	
Natural Gas Distribution	
Agriculture and Forestry	
Other Mining	
Mining Services	
Construction	
Water Utilities	
EITE Manufacturing	
Non-EITE Manufacturing	
Transportation	
Commercial (Finance, Communication, Services)	
Real Estate and Owner-Occupied Housing	

composite of the domestic good g_i^d and the foreign supplied good g_i^f. The prices of foreign supplied goods p_i^f are exogenous in DR-GEM, but there is an endogenous exchange rate $exch$ that balances international trade each period such that the real value of the trade balance is constant each period; the price of good g_i^f to a domestic consumer is $p_i^f/exch$. Let p^g denote the optimal unit price for the optimal basket of goods from different regions (domestic and international).

Supply of goods to different regions will be explained below in the producer section

B.2 Households

B.3 Goods and Trade

Let g_i denote the demand for a good produced by sector i. A good consumed could be produced by the domestic producer or it could be imported from an international producer. The model uses a simple CES Armington structure such that g_i is a composite of the domestic good g_i^d and the foreign supplied good g_i^f. The prices of foreign supplied goods p_i^f are exogenous in DR-GEM, but there is an endogenous exchange rate $exch$ that balances international trade each period such that the real value of the trade balance is constant each period; the price of good g_i^f to a domestic consumer is $p_i^f/exch$. Let p^g denote the optimal unit price for the optimal basket of goods from different regions (domestic and international).

Supply of goods to different regions will be explained below in the producer section.

B.4 Households

The numerical model uses an infinite-lived representative household framework. A single representative household chooses between work and leisure, savings and consumption, and consumption expenditure across various consumer goods and services to maximize their utility subject to a budget constraint. In year t, the household chooses a path of "full consumption" C to maximize

$$U_t = \sum_{s=t}^{\infty}(1+\beta)^{t-s}\frac{\sigma}{\sigma-1}C_s^{\frac{\sigma-1}{\sigma}}, \tag{32}$$

where β is the subjective rate of time preference and σ is the intertemporal elasticity of substitution in full consumption. C is a CES composite of consumption of goods and services, $\bar{C}$, and leisure, l:

$$C_s = \left[\bar{C}_s^{\frac{v-1}{v}} + \alpha_l^{\frac{1}{v}} l_s^{\frac{v-1}{v}}\right]^{\frac{v}{v-1}}, \tag{33}$$

where v is the elasticity of substitution between goods and leisure and α_l is a leisure intensity parameter.

The household's intertemporal budget constraint is

$$W_{t+1} - W_t = \bar{r}_t W_t + Y_t^l + GT_t - GL_t + BOP_t - \bar{p}_t \bar{C}_t, \tag{34}$$

where W_t is the household's financial wealth at time t, $\bar{r}$ is the nominal after-tax return on the household's financial wealth holdings, Y^l is the household's after-tax labor income, GT is transfer income from the government, GL is lump-sum taxes paid to the government, BOP_t is the international balance of payments, and $\bar{p}$ is the unit price of the consumption composite $\bar{C}$.

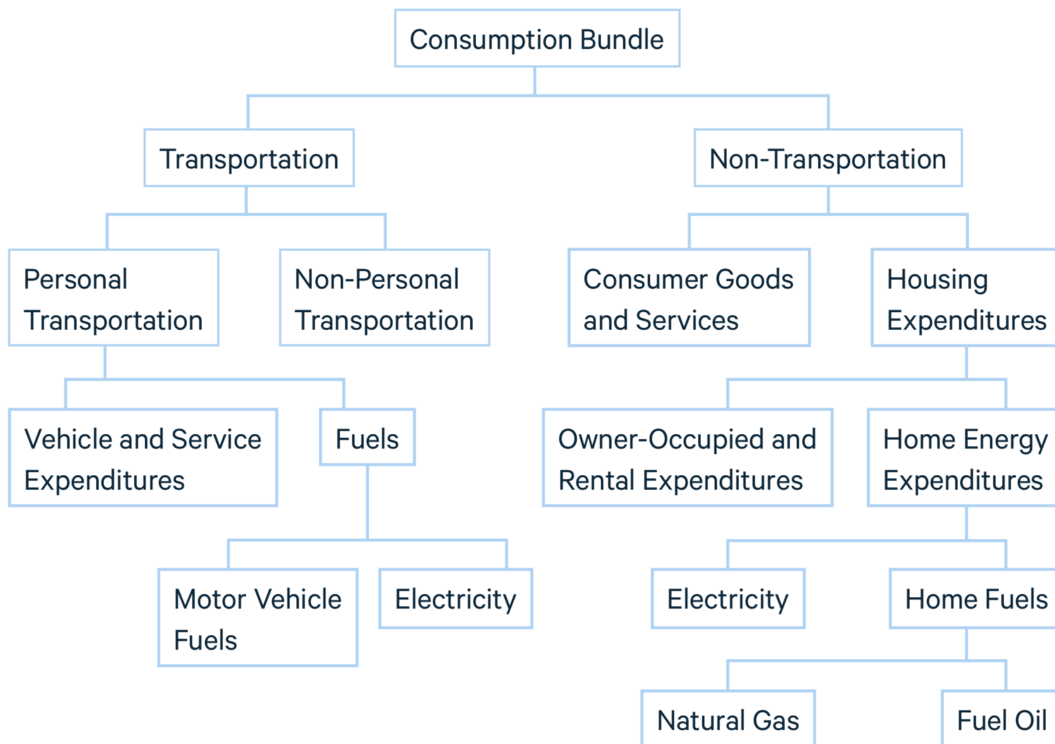
Household labor income, Y^l, is equal to total time worked (total potential labor time, $\bar{l}$, minus leisure), times the net-of-tax wage rate, $\bar{w} = (1 - \tau_l)w$. The household maximizes utility by choosing $\bar{C}_s$, l_s, and W_{s+1}.

The aggregate consumption good $\bar{C}$ is a composite of n_c consumption goods, $c_1, ..c_j$. Unlike Goulder and Hafstead (2017), which used a Cobb-Douglas specification for demand across goods, this numerical mode uses a nested CES utility structure, as specified in Figure B-1. At each level of the nest, households choose consumption intensities to achieve the least cost combination of goods.

Elasticities of substitution are common values taken from the literature.

Taxes on labor and capital income are sources of the tax interaction effect and the elasticities of substitution across consumption goods (along with baseline levels of expenditures on goods) determine the semi-elasticity for household motor vehicle fuel emissions and residential sector emissions, both of which determine the intercept for these two sources of emissions when they are subject to small carbon taxes.

Figure B-1: Consumption Bundle Nest



Each consumer good is itself a composite of producer goods. We use a fixed "G" matrix to map spending on consumer goods into spending on producer goods such that $cp_j = \sum_i G_{ij} p_i^g$, where cp_j is the price of consumer good j, G_{ij} is the amount of spending on consumer good j that flows to producer i and p_i^g is the price of consumption spending on producer good i. Finally, the amount of spending on producer good i is $pce_i = \sum_j G_{ij} cg_j$.

B.5 Firms

A representative firm in each sector produces a distinct output y_i using capital $\bar{K}$, labor l, and intermediate inputs IO (energy and materials). Producers choose variable inputs to minimize costs and choose investment to maximize its payment to the shareholders - the households.

B.5.1 Production

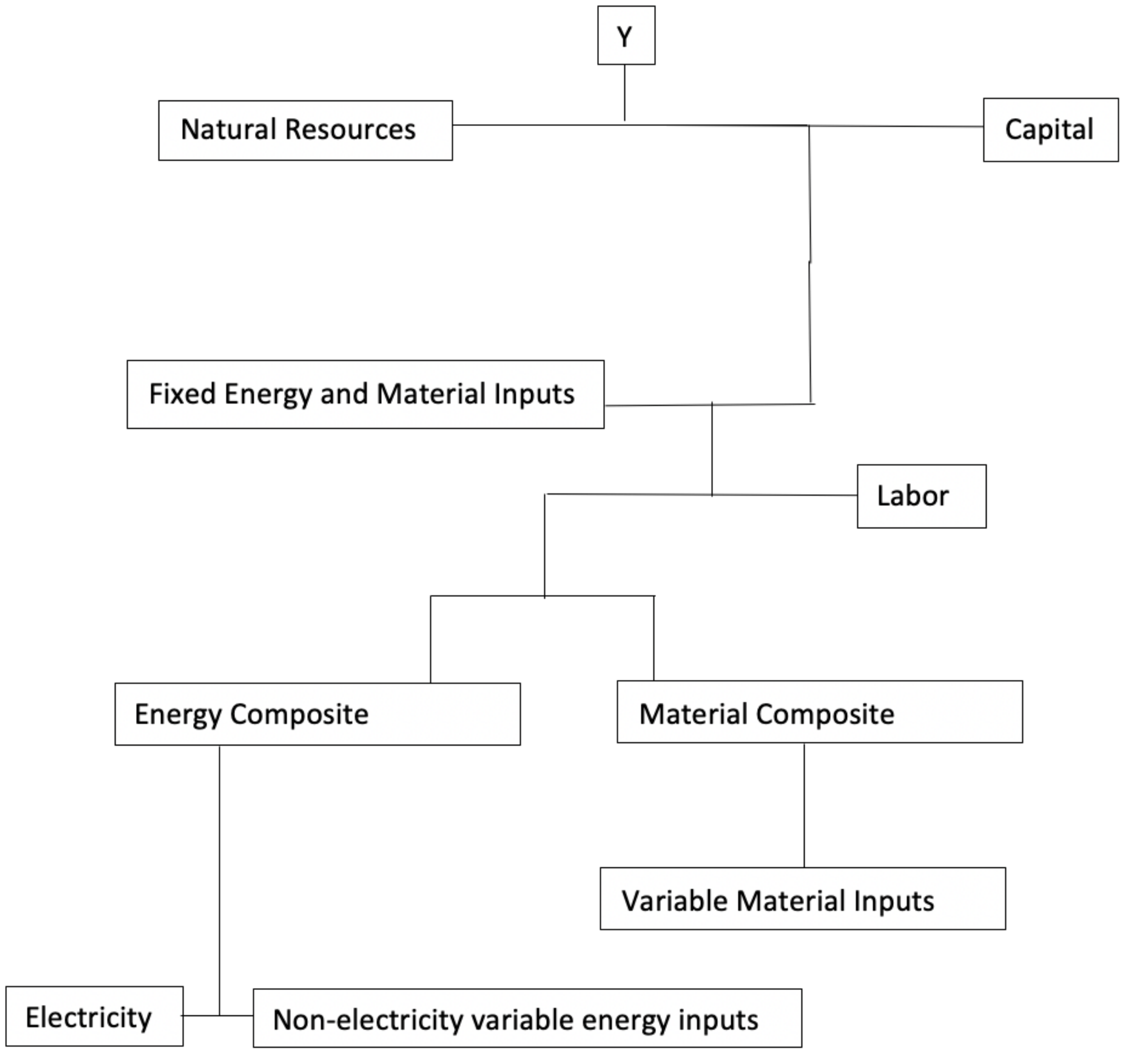
Output from each sector stems from a nested structure of constant-elasticity-of substitution (CES) production functions. Figure 2 displays this structure. For each of these CES nests, elasticities of substitution for each industry are taken from common estimates from the economic literature. These elasticities (along with the benchmark levels of expenditures on each input) determine the semi-elasticities for emissions from the non-household transportation sector, commercial sector, industrial sector, and the electric power sector.

Variable energy inputs are a combination of (retail) electricity and non-electricity energy inputs —crude oil, natural gas (raw or distributed), coal, and petroleum refining. Variable intermediate inputs V are an aggregate of the energy and material composite (which is a simple aggregate of non-energy goods).

Let p^e, p^m, and p^v denote the unit prices of the optimal composites of energy, material, and total intermediate inputs, respectively. The intermediate input composite V is combined with labor l, which has costs $w(1+\tau^p)$ (τ^p represents federal payroll taxes), to create a variable input composite $\bar{V}$ with unit cost $p^{\bar{v}}$.

For each industry, specific intermediate inputs are considered fixed proportion inputs: the firm cannot utilize more or less of these inputs relative to other inputs (for example, crude oil input into petroleum refining). A fixed Leontief function

Figure B-2: Nested Production Function



combines these inputs with the other inputs $\bar{V}$ into a total variable input composite Z with unit price p^z.[43]

Fixed natural resources NR are required to produce outputs for certain industries (nuclear and hydro electricity generation, for example). Output is a function of total capital $\bar{K}$, variable inputs Z, and natural resources NR: $Y = F(\bar{K}, Z, NR)$. Natural resources are fixed and exogenous; the elasticity of substitution between the

[43] If α^z_{ij} is the share for the fixed proportion input of good i to sector j, then $p^z_j == \sum_i \alpha^z_{ij} p^g_i + (1 - \sum_i \alpha^z_{ij}) p^{\bar{v}_j}$.

three inputs determines the supply elasticity for these industries. Payments for natural resources are made to the households at price p_i^{nr}. For industries without natural resources inputs, output is $Y = F(\bar{K}, Z)$.

B.5.2 Investment

Capital adjustment costs are modeled as the sacrifice of output associated with the process of investing in capital. Specifically, net output is equal to gross output minus adjustment costs, $\phi(I/K) \times I$, represents the adjustment costs (in terms of lost output). Adjustment costs have the same functional form as in Goulder and Hafstead (2017), where net output $\bar{Y} = Y - \phi(I/K) \times I$ and adjustment costs are quadratic in deviations from the level of investment consistent with the steady state growth path, $\phi(I/K) = \frac{\upsilon(I/K-(\delta+gr))^2}{I/K}$, where υ is the primary adjustment cost parameter and gr represents exogenous steady state growth, and δ is the rate of capital depreciation.

Capital goods are a Leontief aggregate of goods from different sectors. There are three types of capital stocks in the model —structures, equipment/intellectual property, residential structures —with different depreciation rates δ^k. Each capital stock evolves as $K_{t+1}^k = I_t^k - \delta^k K_t^k$. Let α_i^k denote the input intensity for good i into the capital good k: $p^k = \sum_i \alpha_i^k p_i^g$ and let $\bar{K} = K(K^s, K^e, K^r)$ denote "total capital", the composite of the three types of capital stocks, and let $pfi_i = \sum_k \alpha_i^k \sum_j I_j^k$ denote the amount of investment expenditure received by producer i.

B.5.3 Profits and Behavior of Firms

Firms choose inputs and capital to maximize the value of the firm, VV. The value of the firm is the discounted flow of after-tax profits π net of new share issues. Capital income is (dropping time and sectoral subscripts)

$$\pi^b = (1-\tau_y)p^y y - p^{nr} nr - p^z Z, \tag{35}$$

where p^y is the unit price of output (see below) and τ_y is a potential tax on output. Firms face capital taxes on their profits, but are allowed to deduct depreciation, debt payments, and property taxes. After-tax profits are equal to after-tax capital income, with adjustments for deductions and payments on outstanding debt and property tax payments,

$$\pi = (1-\tau_a)\pi^b - (1-\tau_a)(rDEBT + TPROP) + \tau_a \sum_k \delta_D^k D^k, \tag{36}$$

where τ_a is the corporate income tax rate, which contributes to the tax interaction effect. Interest rate payments on existing debt are equal to the gross-of-tax interest rate r times the value of current debt $DEBT$. Debt is assumed to be a constant fraction of the current value of capital and property tax payments are also paid on the current value of the capital stock.[44] The term D^k represents the current depreciated capital stock base for capital stock k which can be depreciated for tax purposes at rate δ_D^k.

$$D_{t+1}^k = p_t^k I_t^k + (1-\delta_D^K)D_t^k. \tag{37}$$

The model assumes dividends, DIV, are a constant fraction a of after-tax profits. A cash-flow identity links the sources and uses of revenues by the firms,

$$\pi_t + (DEBT_{t+1} - DEBT_t) = DIV_t + \sum_k p_t^k I_t^k + SR_t. \tag{38}$$

[44]The expressions for debt and property taxes are $DEBT = b\sum_k p_{t-1}^k K_t^k$, where b is fixed over time (but can vary across sectors) and $TPROP = \sum_k \tau_{pr}^k p_{t-1}^k K_t^k$, where τ_{pr}^k is the property tax rate levied on each type of capital.

The firm receives cash from after-tax profits and new debt issues and it uses that cash to finance dividend payments, investment expenditures, and share repurchases SR_t (or new share issues if dividend payments and investment expenditures exceed cash-on-hand).

Firms must offer a rate of return (in terms of dividends and capital gains) consistent with the rate of return from other owning public or private debt,

$$(1-\tau_d)DIV_t + (1-\tau_{cg})(VV_{t+1} - VV_t + SR_t) = (1-\tau_b)r_t VV_t, \tag{39}$$

where τ_d, τ_{cg}, and τ_b represent personal tax rates in dividend income, capital gains, and interest income. An expression for the value of the firm VV_t can be derived from equation (8) above.

Due to the nested production function, the optimal production and investment problem of the firm, which is described in Appendix B, requires firms to determine investment I and total variable inputs Z (natural resources are exogenous) to maximize the value of the firm. Given total variable inputs Z, the optimal composite functions are used to determine labor demand l^d and intermediate inputs (IO_{ij} represents total input of good i to firm j) for each sector.

B.5.4 Optimal Supply

This version of the Goulder-Hafstead E3 model uses a Constant Elasticity of Transformation (CET) function to determine the supply of goods between the domestic and international market: firms can choose to supply to either market based on relative prices. The unit output price p_i^y is given by

$$p_i^y = \left[\alpha_y^d (p_i^d)^{1+\sigma_s} + \alpha_y^f(r)(p_i^f/exch)^{1+\sigma_s}\right]^{\frac{1}{1+\sigma_s}}, \tag{40}$$

where α_y^x is a share parameter for market x and σ_s is the elasticity of transformation that determines the flexibility of producers to supply different markets. The optimal supply of good i to market x, y_i^x is

$$y_i^x = y_i \alpha_y^x (p_i^x / p_i^y)^{\sigma_s}. \tag{41}$$

B.6 Government

The model uses a representative government to represent federal, state, and local governments. The government levies taxes and uses that revenue to finance government spending on goods and services, labor, and transfers.

B.6.1 Government Expenditure

The production of government services is assumed to use a fixed amount of capital, intermediate inputs, and labor. The level of inputs on each good are assumed to grow at the real rate of economic growth each period. Let gov_i denote the real level of government expenditures on investment goods and intermediate inputs and let l_g^d denote government labor demand.[45] The government also sends transfers to households each period, which are assumed to be held fixed in real terms. The government also must pay the household interest payments on outstanding public nominal debt; the real level of debt is assumed to grow exogenously over time, and nominal debt is equal to real debt level times the price level.

[45] Because government expenditures are held fixed, the model does not explicitly distinguish between spending on new capital and intermediate inputs into the production of government services.

B.6.2 Government Revenue

The federal government collects taxes through labor income taxes τ_l, payroll taxes τ_p, capital income taxes τ_a, τ_d, τ_{cg}, τ_b, property taxes τ_{pr}, and output taxes τ_y. The government also issues new debt to finance its expenditures through changes in the nominal debt level each period.

Generally, lump-sum taxes adjust each period to close the government budget constraint. See Appendix B for complete description of the government budget constraint. In policy simulations, we allow lump-sum taxes to change annually but find levels of rebates and/or changes in pre-existing tax rates such that the intertemporal value of lump-sum taxes remains unchanged between the policy and non-policy simulations.

B.7 Market Clearing

Total demand for each good g_i must be consistent with final good demands,[46]

$$g_i = pce_i + pfi_i + gov_i + \sum_j IO_{ij}(r). \tag{42}$$

As described in section 1.1, total demand for each good is split into demand for domestic goods g_i^d and foreign goods g_i^f. Given these demands and the optimal supply of domestically-produced goods to the domestic and foreign region, the model finds prices p_i^d in each period such that

$$y_i^d \geq g_i^d. \tag{43}$$

[46]Time subscripts ignored for clarity.

Because foreign prices p_i^f are exogenous, the model finds the real exchange rate that holds fixed the trade balance each period,

$$\sum_i (p_i^f/exch)(g_{i0}^f - y_{i0}^f) - \sum_i (p_i^f/exch)(g_i^f - y_i^f) \tag{44}$$

where $(g_{i0}^f - y_{i0}^f)$ represents the benchmark real balance of payments. In equilibrium, BOP in the household budget constraint is $BOP = \sum_i (p_i^f/exch)(g_{i0}^f - y_{i0}^f)$.

The labor market clears when total labor demand equals total labor supply (potential time less leisure),[47]

$$(\bar{l} - l) \geq \sum_i l_i^d + l_g^d. \tag{45}$$

B.8 Emissions Accounting and Emissions Pricing

B.8.1 Emissions Accounting

The model includes coefficients μ that convert demand for different fuels by sectors, households, and the government into energy-related carbon dioxide emissions. These coefficients are applied to the downstream purchase of coal, natural gas, and refined petroleum products.

Coefficients are calibrated to match energy-related CO_2 emissions from the residential, commercial, industrial, transportation, and electric power sectors by fuel (coal, natural gas, and refined petroleum products). Using fuel-sector level emissions data and fuel-sector level energy expenditure data from the Energy Information Administration, fuel-sector level coefficients $\mu_{EIA}(f, s)$ are derived by dividing fuel-sector emissions by fuel-sector expenditures.

[47] The pre-tax wage is the numeraire in the model, so the labor market clearing constraint is dropped from the model.

A mapping from EIA sectors (residential, commercial, industrial, transportation, and electric power) to energy consumers in the model (households, different sectors, and government) are used to determine the model specific coefficients μ, μ^c, and μ^g, which represent coefficients for firms, households, and the government, respectively.

B.8.2 Emissions Pricing

Emissions pricing is introduced as a tax on the direct purchase of coal, natural gas, and refined petroleum products.[48] Let p^{CO2} denote the price on energy-related carbon dioxide emissions.[49] The new effective price for good j is $cp_j = \sum_i G_{ij} p_i^g + \mu_j^c p^{CO2}(r)$. For firms, the price of good i for sector j in region r becomes $p_i^g + \mu_{ij} p^{CO2}$.

Revenue from emissions pricing flows to the government is an added source of government revenue. These revenues can be rebated back to households through lump-sum dividends or through changes in preexisting tax rates. In all cases, as explained in section 1.4.2, all policies are inter-temporally revenue-neutral, such that the intertemporal value of lump-sum taxes remains unchanged between policy and non-policy simulations.

[48] Crude oil is purchased only by refiners and no emissions factor is applied to those purchases.
[49] The model allows for fuel-sectors to be omitted from carbon pricing policy through the use of fuel-sector dummy variables. The model also specifies the real price of carbon dioxide, and the price is scaled by the CPI each period. Both are excluded here for clarity.