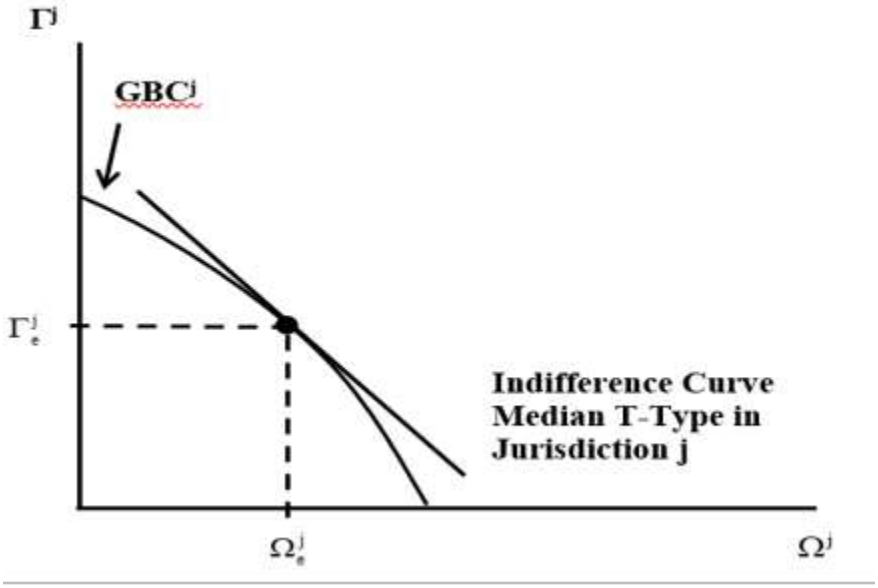
On-Line Appendix for Calabrese, S., Epple, D., and Romano, R., "The Political Economy of School Finance Systems with Endogenous State and Local Tax Policies"

A. Proof of Proposition 2: The proof applies a standard single-crossing argument, but we present it for readers less familiar with such arguments and because we will examine variants of this type of argument. Refer to Figure A.1. Indifference curves for a T-type, $\Gamma^j + T\Omega^j = \text{const.}$, drawn in the the (Ω^j,Γ^j)-plane, are downward sloping and linear, and steepen through any point as T increases. Figure 1 depicts the preferred (Ω^j,Γ^j) choice of a median type.[1] The policy that achieves this (Γ_e^j,Ω_e^j) pair is implicit, i.e., that which implies these composite public good values. To confirm this is the majority choice equilibrium, first consider policies implying any points NE of the indifference curve through T_{med}^j's preferred point. These are infeasible, or the preference of the median type would be contradicted. Consider any other policies not in the latter (infeasible) set with $\Gamma^j \geq \Gamma_e^j$. All types with $T > (=) T_{med}^j$ have (weakly) steeper indifference curves through (Γ_e^j,Ω_e^j) and do not (strictly) prefer a policy implying such a point. As this is half the population, such a point and corresponding policy does not majority defeat (Γ_e^j,Ω_e^j). Analogously, any remaining points are not majority preferred due to the preferences of lower T types, implying the policy yielding (Γ_e^j,Ω_e^j) is a majority choice equilibrium. A majority choice equilibrium as a preferred choice of a median type always exists (for any GBC^j). A similar argument implies only a preferred choice of a median preferred type is a majority choice equilibrium. Since Ω^j increases with G^j, the stated relative voting preferences of non-median types in j follow immediately. ■

Figure A.1:



[1] The preferred choice of a household is generically unique, though the Proposition applies if the median type has multiple preferred choices, any of which would then be a majority choice equilibrium.

B. Analysis of G_s^j for Proposition 3: When voting over m^s, households take as given residences, local tax rates, and the federal policy pair, and they anticipate the continuation equilibrium values in their district in Stage 3. A voter's preferred policy is that which would maximize indirect utility in (11). Consider a voter residing in district j. Using equations (3) – (4) and (6) – (9), we find the values that determine the continuation equilibrium (Γ,Ω) pair in j for any m^s. We refer to the frontier of this choice set as the state government budget set for j, denoted GBC_s^j. This frontier is *not* the same as the GBC^j, because m^s varies along GBC_s^j with (t^j,m^j) held constant at their equilibrium values.

Figure A,2 depicts the frontier of a choice set in district j, GBC_s^j which we write as: $\Gamma^j = \tilde{\Gamma}_s^j(\Omega^j)$. A voter living in j has preferred m^s that satisfies problem (12) with modified constraint: $\Gamma^j = \tilde{\Gamma}_s^j(\Omega^j)$. Except in anomalous cases, m^s increases moving down a GBC_s^j in the (Ω^j,Γ^j) – plane. To see this, using (3) – (4) and (6) – 98) and holding constant the above-noted variables, the slope is:

$$\text{(B.1)} \qquad \tilde{\Gamma}_s^{j\prime}(\Omega^j) = \frac{d\Gamma^j / dm^s}{d\Omega^j / dm^s} = \frac{-\kappa[(\gamma+\lambda)(1-M^j)^{\gamma+\lambda-1} - \lambda(1-M^j)^{\gamma+\lambda}(p_h^j)^{-1}\frac{dp_h^j}{dm^s}](p_h^j)^{-\lambda}}{(1+t^j)^{\lambda}\beta(Q^j)^{\beta-1}(\bar{T}^j)^{\eta/\beta}[Y + t^j\frac{d(p_h^j\bar{H}^j)}{dm^s}]}$$

Ignoring the effects of m^s on the housing market (the derivative terms on the RHS), one can readily see that Γ^j decreases with m^s (the numerator of (B.1) is then negative), Ω^j increases with m^s (the denominator of (B.1) is then positive), and thus GBC_s^j slopes downward. However, this is complicated by the effects in the housing market (the derivative terms) of increasing the state income tax. For example, a higher state income tax will reduce demand for housing and local taxes collected to spend on g^j, tending to lower G^j and thus Ω^j, this effect captured by the derivative term in the denominator of (B.1). Proposition 3 provides sufficient conditions for existence. We show that these properties hold for realistic parameterizations in our computational analysis.

Remark 2 to Proposition 3 states sufficient conditions for the requirements of Proposition 3. To confirm these, let $k^j = \kappa(1-t^j)^{-\lambda}$, write:

$$\text{(B.2)} \qquad \Gamma^j = k^j[1-(m^f + m^s + m^j)]^{\gamma+\lambda}(p_h^j)^{-\lambda} \rightarrow \frac{d\Gamma^j}{dm^s} = -k^j[1-M^j]^{\gamma+\lambda-1}(p_h^j)^{-\lambda} < 0,$$

where we use in the derivative in (B.2) that infinitely elastic housing supply implies p_h^j is constant. Now write:

$$\text{(B.3)} \qquad \Omega^j = (g^f + g^s + g^j)^{\beta}(\bar{T}^j)^{\eta} = (g^f + m^s Y + m^j\bar{y}^j + t^j p_h^j\bar{H}^j)^{\beta}(\bar{T}^j)^{\eta},$$

where we have written out the components of G^j and substituted the balanced budget constraints.

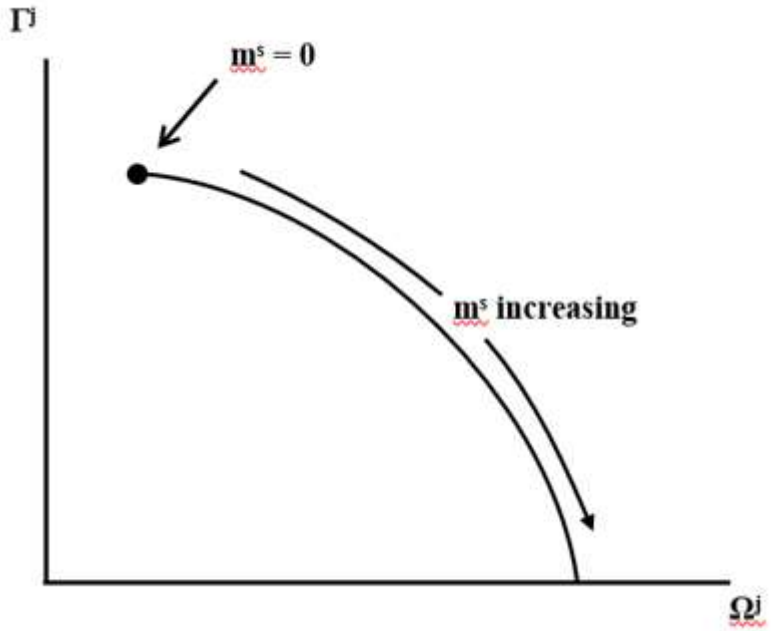


Figure A.2: A Concave District Budget Constraint for State Policy

Recall that $\bar{H}^j$ is average housing consumption, which, using demand for housing, is given by:

$$\text{(B.4)} \qquad \bar{H}^j = \frac{\lambda}{\gamma+\lambda}\frac{[1-(m^f+m^s+m^j)]\bar{y}^j}{(1+t^j)p_h^j} \rightarrow \frac{d\bar{H}^j}{dm^s} = -\frac{\lambda}{\gamma+\lambda}\frac{\bar{y}^j}{(1+t^j)p_h^j},$$

again using that the supplier price of housing is constant. Using the derivative in (B.4), differentiating (B.3) one gets:

$$\text{(B.5)} \qquad \frac{d\Omega^j}{dm^s} = \beta(G^j)^{\beta-1}[Y - \frac{\lambda}{\gamma+\lambda}\frac{t^j}{1+t^j}\bar{y}^j](\bar{T}^j)^{\eta/\beta} > 0,$$

the inequality using the assumption of the result. Positivity of the bracketed term in (B.5) is what is needed for assumption (ii) in Remark 3 to Proposition 3. Note that t^j not too high is sufficient for this. These imply that m^s is increasing as one moves down GBC_s^j, i.e., as Γ^j increases and Ω^j declines. Using the results, we have:

$$\text{(B.6)} \qquad \tilde{\Gamma}_s^{j\prime}(\Omega^j) = \frac{d\Gamma^j/dm^s}{d\Omega^j/dm^s} = -\frac{k^j(\gamma+\lambda)(1-M^j)^{\gamma+\lambda-1}(p_h^j)^{-\lambda}}{\beta(G^j)^{\beta-1}(\bar{T}^j)^{\eta/\beta}[Y - \frac{\lambda}{\gamma+\lambda}\frac{t^j}{1+t^j}\bar{y}^j]}.$$

For the next expression, define the latter expression as –N/D (i.e., the negative of the numerator over the denominator). To confirm the concavity of GBC_s^j, compute:

(B.7)

$$\frac{d}{dm^s}\left[\tilde{\Gamma}_s^{j'}\right] = -\left[\frac{D[1-(\gamma+\lambda)]k^j(\gamma+\lambda)(1-M^j)(p_h^j)^{-\lambda} + N(1-\beta)\beta(G^j)^{\beta-2}(\bar{T}^j)^{2\eta/\beta}[Y - \frac{\lambda}{\gamma+\lambda}\frac{t^j}{1+t^j}\bar{y}^j]^2}{D^2}\right].$$

Using $\gamma+\lambda<1$ and $\beta<1$ one can see the expression is negative, implying concavity.

C. Analysis of the Foundation Policy. Propositions 1 (indirect utility) and 4 (stratification) continue to apply as stated in the text. Proposition 2, regarding local majority choice of (t^j,m^j), applies replacing "for any state income tax" with "for any state foundation property tax."

Showing majority choice equilibrium for the state foundation policy is also analogous to the baseline model, though the GBC relevant to state policy is a bit more complex due to housing market effects of changing t^s. Refer to Figure A.3, which depicts the GBC in district j *relevant to voting over state policy*, which we denote for the foundation case GBC_f^j. Similarly to the flat-state grant policy that relies on a state income tax, we need to show that as t^s increases one moves down this budget constraint and that it is concave. The relevant equations are:

(C.1) $\sum_{k\in J^+}(g^s - t^s p_h^k \bar{H}^k)n^k = 0,$

(C.2) $g^j = t^j p_h^j \bar{H}^j + m^j \bar{y}^j,$

(C.3) $p^j = (1+t^s+t^j)p_h^j,$

(C.4) $G^j = g^f + g^s + g^j,$

(C.5) $\bar{H}^j = \frac{\lambda}{\lambda+\gamma}\cdot\frac{\bar{y}^j(1-m^f-m^j)}{(1+t^s+t^j)p_h^j},$

and

(C.6) $n^j\bar{H}^j = H_s^j(p_h^j).$

Equations (C.1), (C.2), (C.4), and (C.5) imply:

(C.7) $G^j = t^j\frac{\lambda}{\gamma+\lambda}\frac{(1-m^f-m^j)\bar{y}^j}{1+t^j+t^s} + \sum_{k\in J^+} t^s\frac{\lambda}{\gamma+\lambda}\frac{(1-m^f-m^k)\bar{y}^k n^k}{1+t^k+t^s} + g^f.$

Using $\Omega^j = \left(G^j\right)^\beta = \left((\bar{T}^j)^{\eta/\beta}G^j\right)^\beta$ and (C.7), by differentiation one obtains:

(C.8) $\frac{d\Omega^j}{dt^s} = \beta(Q^j)^{\beta-1}(\theta^j)^{\eta/\beta}\frac{\lambda}{\gamma+\lambda}\left[-\frac{t^j\bar{y}^j(1-m^f-m^j)}{(1+t^j+t^s)^2} + \sum_{k\in J^+}\frac{(1+t^k)\bar{y}^k n^k(1-m^f-m^k)}{(1+t^k+t^s)^2}\right].$

Using $\Gamma^j = \kappa[1-(m^f+m^j)]^{\gamma+\lambda}[(1+t^j+t^s)p_h^j]^{-\lambda}$, $K \equiv \left(\frac{\gamma}{\gamma+\lambda}\right)^\gamma\left(\frac{\lambda}{\gamma+\lambda}\right)^\lambda$, and housing market clearance (i.e., (C.5) and (C.6), by differentiation one obtains:

(C.9) $$\frac{d\Gamma^j}{dt^s} = -Z^j \cdot \frac{\varepsilon_H^{s\,j}}{\varepsilon_H^{s\,j} - \varepsilon_H^{d\,j}} = -Z^j \cdot \frac{\varepsilon_H^{s\,j}}{\varepsilon_H^{s\,j} + 1} < 0 \quad \text{where} \quad Z^j \equiv \lambda\kappa \frac{(1 - m^j - m^f)^{\gamma+\lambda}}{p_h^{\lambda}(1 + t^s + t^j)^{1+\lambda}},$$

where ε_H^s and ε_H^d denote the housing supply and demand elasticities, and the last equality in the first expression uses that $\varepsilon_H^d = -1$ in our model. One can confirm the inequality by inspection.

If (and only if) increasing the foundation tax t^s implies an increase in G^j, then the expression in (C.8) will be positive. While increasing t^s necessarily increases the foundation grant g^s, this corresponding to the second term in the brackets being positive, it has the effect of reducing g^j by lowering the value of housing in j and thus leeway funding (if t^j is positive). The latter effect is captured by the first term in the brackets of (C.8). However, various sufficient conditions imply $d\Omega^j / dt^s$ is positive, and we find this to hold in our computational model. Then the slope of GBC_f^j, given by $\frac{d\Gamma^j / dt^s}{d\Omega^j / dt^s}$, is negative with t^s increasing as one moves down it, as depicted in Figure A.3. If it is also concave in all districts, then the direct analogue of Proposition 3 holds, replacing "m^s" with "t^s" and "state income tax" with "foundation property tax." We confirm concavity in our computational analysis for realistic parameterizations. Note, too, that if $d\Omega^j / dt^s$ were negative over a range of t^s, then GBC_f^j would slope upward and all residents of the district would vote against any values of t^s in this range.

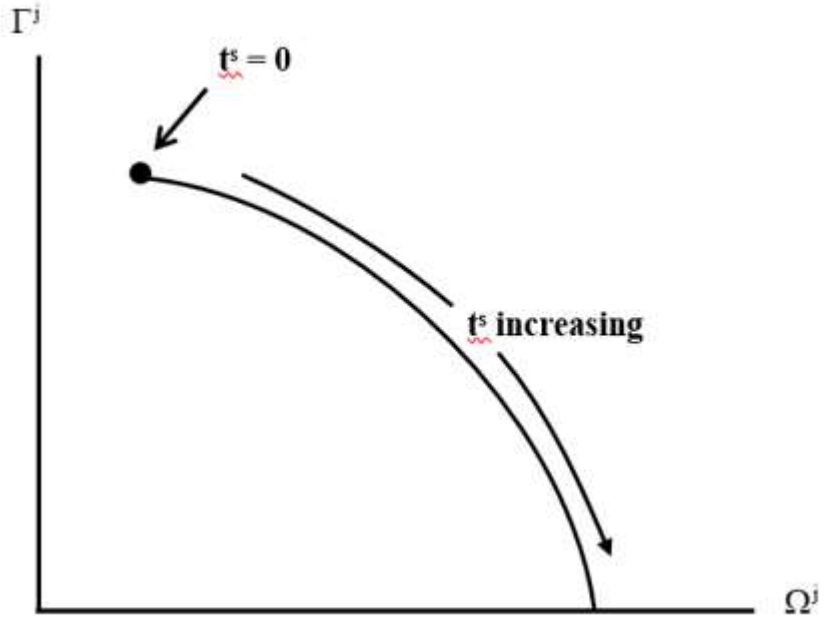


Figure A.3: A Concave District Budget Constraint for Foundation State Policy

D. District Power Equalization. District Power Equalizing (DPE) programs were common for a period of time and remain the primary school finance regime in Vermont and Wisconsin (Verstegan 2014). The central idea of such a program is to guarantee equal expenditure for equal local taxation, but to allow local choice of taxation. As described by Skinner (2019), "… this program type focuses specifically on equalizing the ability of different LEAs [local educational authorities] in a state to raise revenues from their available taxable property … this program type provides for a minimum guaranteed tax base …." DPE programs vary (historically) with whether there is "recapture" or not, with the recapture provision essentially precluding any local add-ons beyond that implied by the local property tax and the guaranteed

tax base. We present District Power Equalizing (DPE) plans with and without recapture, though we have only develop fully the more empirically relevant case with recapture to investigate the extension of the propositions.

DPE with Recapture Model: The state policy parameters are $(m^s, \bar{V}^s)$, where m^s is the state income tax and $\bar{V}^s$ is the legislated property tax base, defined *per capita*. Thus, local expenditure in district j is given by:

$$(D.1) \quad G^j = t^j \bar{V}^s + g^f,$$

with t^j the majority determined property tax rate in district j. No local income taxation is allowed. The state balance budget condition is:

$$(D.2) \quad m^s Y + \sum_{k \in J^+} t^k p_h^k \bar{H}^k n^k = \sum_k n^k t^k \bar{V}^s.$$

Note that the LHS of (D.2) equals state tax revenues, as the local property taxes actually collected are transferred to the state, while the RHS is the total expenditure on education. At the state level, everyone votes over m^s, this determining $\bar{V}^s$ given the local policies (i.e., the t^i's). At the local level, majority choice of t^j occurs given the state policy.[2] Everything else is the same in the model as in the other models (e.g., timing of choices).

Existence of majority choice of local policy is assured by the same kind of argument using single crossing and the composite public goods (Propositions 1 and 2 apply[3]). At the state level, one sufficient condition for existence of majority choice of policy is that the relevant state GBC is concave (a version of Proposition 3). Using (D.1) and (D.2) one obtains:

$$(D.3) \quad \begin{aligned} \Omega^j &= \left(t^j \cdot \frac{m^s Y + \sum_{k \in J^+} t^k p_h^k \bar{H}^k n^k}{\sum_{k \in J^+} t^k n^k} + g^f \right)^{\beta} (T^j)^{\eta/\beta} \\ \Gamma^j &= \kappa \cdot \left(\frac{[1-(m^s + m^f)]^{\gamma+\lambda}}{[p_h^j (1+t^j)]^{\lambda}} \right) \end{aligned}$$

Holding constant the district tax and using housing demand, it is straightforward to find:

$$(D.4) \quad \frac{d\Omega^j}{dm^s} = \frac{\beta (Q^j)^{\beta-1} (\bar{T}^j)^{\eta/\beta} t^j}{\sum_{k \in J^+} t^k n^k} \cdot \left[Y - \frac{\lambda}{\gamma + \lambda} (1 - m^f - m^s) \sum_{k \in J^+} \frac{t^k}{1+t^k} n^k \bar{y}^k \right] > 0.$$

The bracketed term and thus the whole expression is positive because $Y = \sum_{k \in J^+} n^k \bar{y}^k$, while one can see by inspection that the second term in brackets (without sign) is less than $Y = \sum_{k \in J^+} n^k \bar{y}^k$. It is also straightforward to confirm:

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[2] The state policy variable that *local* voters hold constant could be m^s or $\bar{V}^s$. To be clear, these are different. It is arguably more natural to assume local voters hold constant $\bar{V}^s$.

[3] Regarding the extension of Proposition 2, here no local income tax is allowed. Corollary 1 then comes into play.

(D.5) $$\frac{d\Gamma^j}{dm^s} = \kappa \cdot [p_h^j(1+t^j)]^{-\lambda} \cdot [\lambda(1+\varepsilon_H^{s,j})^{-1} - \lambda - \gamma] < 0.$$

The expression is negative, though the first term in brackets is positive. The positive term is because increasing the state income tax decreases housing demand in any district j, this (weakly) lowering the price of housing. However, one can see by inspection that the net effect on the composite public good Γ^j is a decline, this because the direct undesirable effect of a higher state income tax dominates.

Given these results, the GBC relevant to voting on the state policy is downward sloping in the $(\Omega^j, \Gamma^j)-$plane, with m^s increasing as one moves down it. If it is concave, then voting preferences are single peaked, etc. The analogue of Proposition 3 applies.

Proposition 4 applies. Propositions 5 and 6 are irrelevant since there is no local income taxation.

DPE without Recapture. Here (D.1) and (D.2) are replaced with:

(D.6)
$$G^j = t^j \bar{V}^s + g^f \text{ if } \bar{V}^s > p_h^j \bar{H}^j \text{ (poor districts)}$$
$$= t^j p_h^j \bar{H}^j + g^f \text{ if } \bar{V}^s < p_h^j \bar{H}^j \text{ (rich districts)}$$

(D.7) $$m^s Y + \sum_{k \in poor} t^k n^k p_h^k \bar{H}^k = \sum_{i \in poor} t^k n^k \bar{V}^k.$$

Districts that are "poor," as defined by their housing tax base being below the policy threshold, have effectively equalized tax bases. Rich districts are exempted, but all households face an income tax that permits an effectively increased tax base in poorer districts. All residents of rich districts prefer 0 state income tax.

E. Derivation of Housing Supply Function.

Assume the production function for housing is:

(E.1) $$H = L^{\mu} q^{1-\mu},$$

where q is an elastically supplied input at factor price p_w, and L is inelastically supplied at equilibrium price p_L. Since L is fixed, the employment of q will be given by:

(E.2) $$q(H) = H^{\frac{1}{1-\mu}} L^{-\frac{\mu}{1-\mu}}.$$

Total cost of producing H is equal to:

(E.3) $$C(H) = p_L L + p_w q(H) = p_L L + p_w H^{\frac{1}{1-\mu}} L^{-\frac{\mu}{1-\mu}},$$

the second equality using (E.2). Competition in provision of housing implies the supplier price of housing, denoted p_h, equals the marginal cost of housing (with input price taking). Thus, differentiating (E.3):

$$\text{(E.4)} \qquad p_h = C'(H) = \frac{p_w}{1-\mu} H^{\frac{\mu}{1-\mu}} L^{\frac{\mu}{1-\mu}},$$

which is inverse supply. Solving (E.4) for H gives housing supply, (19) in the text.

F. Equilibrium with No Local Income Taxation. Table A1 reports equilibrium properties in the various regimes with no local income taxation allowed. The hybrid case that is included, with $\phi = .13$, is the most preferred among other values of ϕ (over a grid with .01 increments). Other cases of ϕ are available from the authors. The case with state finance is not included in Table A1, since it is the same as in Table 2 in the text.

Table A1: Equilibrium Properties Schooling Regimes: No Local Income Tax					
	Foundation	**Pure Local**	**Flat State Grant**	**Pure State**	**FCG (ϕ = .13)**
Positive Properties					
Sorting:					
n^1=	30.05%	22.60%	32.43%		35.27%
n^2=	19.14%	18.09%	17.97%	71.43%	14.94%
n^3=	22.24%	30.74%	21.03%		21.22%
n^R=	28.57%	28.57%	28.57%	28.57%	28.57%
Gross Housing Price					
p^1=	$10.42	$8.44	$8.85		$10.64
p^2=	$12.27	$11.76	$10.62	$10.64	$13.27
p^3=	$19.52	$17.88	$16.10		$19.53
p^R=	$13.59	$13.08	$11.23	$10.31	$13.36
Avg. Housing Value					
D1	$40,751	$33,276	$49,076		$39,792
D2	$65,591	$68,470	$90,525	$116,512	$77,541
D3	$145,348	$140,341	$175,552		$143,782
R	$76,685	$80,771	$93,742	$102,660	$78,462
Per Student Exp.					
g^f=	$1,333	$1,333	$1,333	$1,333	$1,333
g^s=	$4,529	$0	$5,209	$7,342	$4,776 (D1&R) $3,256 (D2&D3)
Per Student Exp. All Sources					
G^1=	$5,863	$2,447	$6,542		$6,109
G^2=	$7,503	$6,168	$8,782		$9,239
G^3=	$20,844	$15,887	$21,108	$8,675	$20,724
G^R=	$8,616	$7,572	$8,924		$8,640
G^{avg}=	$10,295	$8,715	$10,689	$8,675	$10,402
Peer Measure					
θ^1=	0.3715	0.3636	0.3738		0.3765
θ^2=	0.4165	0.4037	0.4194	.4125	0.4215
θ^3=	0.4645	0.4536	0.4663		0.4660
θ^R=	0.4098	0.4098	0.4098	0.4098	0.4098
Quality Index					
Q^1=	216	85	245		233
Q^2=	372	282	444	420	473
Q^3=	1,378	987	1,409		1,381
Q^R=	410	360	425	413	411

(Table A1 cont. on next page)

Table A1 Cont.: Equilibrium Properties Schooling Regimes: No Local Income Tax					
	Foundation	**Pure Local**	**Flat State Grant**	**Pure State**	**FCG (ϕ=.13)**
Positive Properties Cont.					
Property Taxation (Poterba Adjusted):					
t^s=	2.83%	n/a	n/a	n/a	2.66%
t^1=	0.00%	1.65%	0.00%	n/a	0.00%
t^2=	1.24%	3.49%	1.22%	n/a	2.96%
t^3=	5.09%	5.12%	4.10%	n/a	5.54%
t^R=	1.77%	3.81%	1.26%	n/a	1.59%
Income Taxation:					
m^f=	1.01%	1.01%	1.01%	1.01%	1.01%
m^s=	n/a	n/a	3.95%	5.57%	n/a
$m^{1=}$	n/a	n/a	n/a	n/a	n/a
m^2=	n/a	n/a	n/a	n/a	n/a
m^3=	n/a	n/a	n/a	n/a	n/a
m^R=	n/a	n/a	n/a	n/a	n/a
Normative Properties					
EV^{avg}=	\$168	baseline	\$174	-\$308	\$190
Δ House Rents =	-\$176	baseline	\$207	\$624	-\$197

G. Computational Program to solve for equilibrium: Here we sketch the structure of the computational program used to compute equilibrium. We take the example of the baseline case policy regime with flat state grants to local jurisdictions.

Stage 1: The first stage of these computations is to derive the equations for performing numerical integration to calculate aggregate income in each of the three urban communities in equilibrium. In equilibrium, the boundary loci in the $(\ln(\alpha), \ln(y))$ plane for communities j and j+1 are determined by first calculating the numerical value of T^J, the type that is indifferent between living in community j and j+1:

(G.1) $$\Gamma^j + T^j\Omega^j = \Gamma^{j+1} + T^j\Omega^{j+1}$$

(G.2) $$T^j = \frac{\Gamma^j - \Gamma^{j+1}}{\Omega^{j+1} - \Omega^j}$$

By definition, $T^J = \alpha y^{1-(\gamma+\lambda)}$, where T^J is a numerical value. Therefore, the boundary loci in equilibrium in the $(\ln(y), \ln(\alpha))$ plane is defined by:

(G.3) $$\ln(\alpha) = \ln(T^j) - (1-(\gamma+\lambda))\ln(y)$$

Therefore, aggregate income in community j is computed by the numerical integration of the following function:

$$\text{(G.4)} \qquad Y^j = \int_{-\infty}^{\infty} \int_{\ln\left(T^{j-1}\right)-[1-(\gamma+\lambda)]\ln y}^{\ln\left(T^{j}\right)-[1-(\gamma+\lambda)]\ln y} y f\left(\ln(\alpha), \ln(y)\right) d\ln(\alpha) d\ln(y)$$

To simplify the numerical integration algorithm, we exploit the following properties of the bivariate normal distribution of (ln(y), ln(α)):

$$\text{(G.5)} \qquad \mu_{\ln(\alpha)|\ln(y)} \equiv \mu_{\ln(\alpha)} + \rho\sigma_{\ln(\alpha)} \frac{\ln(y) - \mu_{\ln(y)}}{\sigma_{\ln(y)}}$$

$$\text{(G.6)} \qquad \sigma_{\ln(\alpha)|\ln(y)} = \sqrt{1-\rho^2}\,\sigma_{\ln(\alpha)}$$

The following variable ξ has a standard normal distribution:

$$\text{(G.7)} \qquad \xi = \frac{\ln(\alpha) - \mu_{\ln(\alpha)|\ln(y)}}{\sigma_{\ln(\alpha)|\ln(y)}}$$

Therefore, given (G.5), (G.6), and (G.7), equation (G.4) can be written as:

$$Y^j = \int_{-\infty}^{\infty} y f(\ln(y)) \left[\int_{Z_{j-1}(y)}^{Z_j(y)} \phi(\xi) d\xi \right] d\ln(y)$$

where

$$Z_j(y) = K_j - \omega \ln(y),$$

$$\omega = \omega_1 + \omega_2,$$

$$\omega_1 = \frac{\left[1-(\gamma+\lambda)\right]}{\sigma_{\ln(\alpha)}\sqrt{1-\rho^2}},$$

$$\omega_2 = \frac{\rho}{\sigma_{\ln(y)}\sqrt{1-\rho^2}}, \text{ and}$$

$$\text{(G.8)} \qquad K_j = \frac{\ln(T^j) - \mu_{\ln(\alpha)} + \left[\frac{\rho\sigma_{\ln(\alpha)}\mu_{\ln(y)}}{\sigma_{\ln(y)}}\right]}{\sqrt{1-\rho^2}\,\sigma_{\ln(\alpha)}},$$

Equation (G.8) means the numerical integration to compute equilibrium aggregate income in each community can be derived by one-dimensional Riemann numerical integration over y.

Stage 2: The second stage of the program is computing intracommunity and intercommunity equilibrium among the local communities given the state and federal tax policies.

To begin with, each of the four communities has an equation describing housing market equilibrium and local government budget constraint, implying 8 equations. From equation (G.2), the two

boundary T's in the urban area are derived, which is 2 equations. Given the boundary T's, 3 equations determine the three pivotal median T's in the three urban communities.

The derivatives of the housing market equilibrium and the local government budget constraint equations with respect m^j and g^j in each of the four communities are derived and set = 0. The p^j and t^j in each community are implicitly defined as functions of m^j and g^j through the housing market and local government budget constraint equations. There are 16 equations of these derivatives. These equations determine the values of the derivatives $\frac{\partial p^j}{\partial m^j}, \frac{\partial p^j}{\partial g^j}, \frac{\partial t^j}{\partial m^j}$, and $\frac{\partial t^j}{\partial g^j}$ in each of the four communities. These derivative values are used in computing the median pivotal T's most preferred $\left(p^j, t^j, m^j, g^j\right)$ in each community. These most preferred policies are determined through the F.O.C's with respect to m^j and g^j in each community, for 8 more equations. Hence, 37 total equations characterize intracommunity and intercommunity equilibrium among the local communities given the state and federal tax/expenditure policies.

Stage 3: The third stage of the program is deriving the equilibrium m^s. This derivation consists of an equation that needs to be solved simultaneously with the 37 equations that define local community equilibrium given the equilibrium value of m^s. Hence, a total of 38 equations needs to be solved simultaneously to compute system-wide equilibrium. This last equation to determine equilibrium m^s is derived as follows. Note the slope of type T's indifference curve in the (Ω, Γ) plane is -T. Set the slope of the GBC^s in community j, $\left.\frac{d\Gamma^j}{d\Omega^j}\right|_{\text{along } GBC^s}$, at the equilibrium value of m^s equal to $-T_j^s$. Assuming the GBC^s in community j is concave, all type T > (<) T_j^s in community j prefer an m^s greater (less) than the equilibrium value of m^s. Set the sum of the population measures of types in each community with T > T_j^s equal to NT^s. The equilibrium m^s is determined by the solution to the equation $NT^s = 0.5$.

Stage 4: Graph GBC^s in each community j, which is $\Gamma^j(\Omega(m^s))$, to verify it is concave.

These computations are written in Gauss using the subroutine "nlsys" to find the solution of the system of 38 nonlinear equations. The nlsys uses a quasi-Newton method for finding the solution of a system of nonlinear equations.

As stated, the sketch of the computations above is based on the baseline case policy regime with flat state grants to local jurisdictions. This program and the programs for the computations for all the school finance regimes analyzed in the paper are available upon request.

H. Computational Program for Computing Welfare and Distributional Results.

H.1. We calculated the average Equivalent Variation (ev) of going to each of the regimes from the local regime. Again, we take as an example going from the local regime to the baseline case policy regime with flat state grants.

For instance, the ev for a household represented by (y, α) combination who locates in community j in the urban area under the local regime and who locates in community k under the flat state grant regime, where k could = j, is derived by solving for ev in the following equation:

(H.1.1)
$$\left(y+ev\right)^{\lambda+\gamma}\kappa(1-M_L^j)^{\gamma+\lambda}[p_L^j]^{-\lambda}+\alpha\left(y+ev\right)\left(Q_L^j\right)^{\beta}=$$
$$y^{\lambda+\gamma}\kappa(1-M_{FG}^k)^{\gamma+\lambda}[p_{FG}^k]^{-\lambda}+\alpha y\left(Q_{FG}^k\right)^{\beta}$$

where the superscript on M. p, and Q represents the community and the subscript represents the school finance regime, L= local finance and FG = Flat Grant.

We use monte carlo simulation by randomly selecting 1,000,000 (y,α) combinations from the bivariate normal distribution of (ln(y), ln(α)) to represent 1,000,000 households. For each household we solved the ev from equation (H.1.1) by using the "nlsys" subroutine that solves nonlinear equations. We then take the average of these 1,000,000 values.

We then use the same procedure to calculate the average ev in the rural community which has a different income distribution. The average ev for the total metropolitan area is derived by taking the weighted average of the urban and rural average ev's = (average ev in the urban area + 0.4 x average ev in the rural area)/1.4.

The monte carlo simulations create two 1,000,000 (households) x 3 (y,α,ev), matrices; one for the urban area and one for the rural area. Each of these matrices is parsed into 100 matrices with each new matrix containing the households with a (y,α) pair that are within a certain decile combination of the income distribution and the preference distribution. For instance, all households within the i-decile $\left(i\in(1,10)\right)$ of the income distribution and within the j-decile $\left(j\in(1,10)\right)$ of the preference distribution are grouped together forming the 100 matrices for the urban area and 100 matrices for the rural area. The average ev in each decile combination is reported in Tables 5 -8.

H.2. The land rents in the metropolitan area are defined as the sum over the communities' producer surplus associated with housing production. Thus, average land rent under any regime is given by:

(H.2.1)
$$\text{Average land rent} = \sum_{j=1}^{4}\int_0^{p_h^j} L_j\left(p_h^j\right)^3 dp_h = \sum_{j=1}^{4}\frac{L_j\left(p_h^j\right)^4}{4}$$

H.3. We computed the percentage of the metropolitan population that prefers a row regime over a column regime in Table 4a. by one-dimensional Riemann numerical integration over T. Since $Ln(T)=\ln(\alpha)+\left(1-\left(\gamma+\lambda\right)\right)\ln(y)$, the distribution of T is:

(H.3.1) $\ln(T) \sim N(u_{\ln(\alpha)}+(1-(\gamma+\lambda))u_{\ln(y)}, \sigma^2_{\ln(\alpha)}+(1-(\gamma+\lambda))^2\sigma^2_{\ln(y)}+2\rho\sigma_{\ln(\alpha)}(1-(\gamma+\lambda))\sigma_{\ln(y)})$

The value of T was varied from 0 to 1 in increments of 0.001. The cumulative distribution = 100% at T = 1. In the urban area, we computed the utility each of these T's receives in each community for the row regime and took the maximum of these three values. We did the same for the column regime in the comparison. If the utility for a certain T was greater under the row regime, we computed the probability density function (PDF) value at that T and multiplied it by 0.001. If the utility for a certain T was lower under the row regime, we set the PDF value = 0. We then summed all these values from $T \in [0,1]$.

We performed the same process for the rural area, which has a different T distribution, although of course we only needed to compute one utility value for each T under a regime. The total percentage of the metropolitan population that prefers a row regime to a column regime is:

$$\text{(H.3.2)} \qquad \%\text{ of pop. who prefer row regime} = \frac{\%\text{ in urban area} + 0.4 \times \%\text{ in rural area}}{1.4}$$

I. Calibration Details

The proportion of after-tax expenditure on housing in the model is given by $\frac{\lambda}{\lambda+\gamma}$. We calibrate this so that the proportion of after-tax expenditure on housing is 20% of pre-tax income in the foundation equilibrium. Specifically, $\frac{\lambda}{\lambda+\gamma} = .2036$. In 2010, average household expenditure on "shelter" was 15.7% of pre-tax income, but average household expenditure on "housing" was 26.5%. Housing expenditure includes utilities (second to shelter in proportion), household operations, housekeeping supplies, and furnishings. Thus, we count roughly half of non-shelter housing-related expenditures in the 20% calibration. These data are from: Table A of "Consumer Expenditures in 2010: Lingering Effects of the Great Recession," U.S. Department of Labor, Bureau of Labor Statistics, Report #1037, August 2011. https://www.bls.gov/cex/csxann10.pdf

The ratio η/β measures the relative importance of peer students and expenditure in demanded school quality. We set this equal to 2.623 using the estimates of the Tiebout equilibrium in Calabrese, Epple, Romer, and Sieg (2006).

One can think of the two remaining parameters of the utility function to calibrate as γ and β, measuring the relative values of numeraire consumption and school quality in utility. To calibrate these and the federal income tax rate, we target the empirical values of educational spending per student and the empirical proportions of local, state, and federal education finance, also taking account of the empirical number of school-aged children per household.

We calculate .494 school-aged children per household as follows. The number of children under the age of 18 in the U.S. is estimated to be 74.1 million, this from *Number of Children,"* Child Trends September 13, 2019, https://www.childtrends.org/indicators/number-of-children. We take this times 14/18 to estimate the number of school-aged children, and then divide by the estimated 116.7 million households in 2010 ("*Households and Families: 2010"* 2010 Census Briefs, https://www.census.gov/prod/cen2010/briefs/c2010br-14.pdf). This implies .494 school-aged children per household. The following provides some additional explanation regarding this choice of school-aged children per household.

Our decision maker unit is a household, and most theoretical models of education assume one child per household. However, to match empirical values for households and student variables, it is crucial to take into account the actual number of students per household. To take an example, per household expenditure from all sources on education is $5,268, this corresponding to per student expenditure of ($10,655) = ($5,268)/.494, these values predicted in equilibrium of the foundation regime. Education expenditure per student in 2010-11 was $10,665 (rounded to nearest $5), this including expenditure from all sources, federal, state, and local, with respective proportions of 12.5%, 44.1%, and 43.4%. The total is for 2010-11 from the National Center for Education Statistics, Table 236.15 "Current

Expenditure and Current Expenditure per pupil in public elementary and secondary schools: 1989-1990 through 2018-2019." The percentages are also from National Center for Education Statistics, "Revenues and Expenditures for Public Elementary and Secondary Education: School Year 2010-11 (Fiscal Year 2011)," Figure 1, p. 6.

The federal per student expenditure is \$1,333, implying a per household federal income tax rate of $m^f = 1.01\%$ that we then assume. Then γ and β are calibrated so the foundation equilibrium predicts state and local per household and student expenditures corresponding to the empirical proportions with a per student expenditure from all sources of \$10,665. Thus, the model can reproduce in majority choice equilibrium these fundamental values.

J. Normalcy of Demand for Quality.

While schooling is provided through a political process, to interpret (1), suppose a household purchased Q at price p_Q and assume no income taxation for simplicity. Straightforward calculation implies demand for Q is defined implicitly in:

$p_Q^{1/[1-(\gamma+\lambda)]} = Q^{-\frac{1-\beta}{1-(\gamma+\lambda)}} \cdot [y - p_Q Q] \cdot q(\alpha y, p);\ q(\alpha y, p) \equiv (\alpha y)^{1/[1-(\gamma+\lambda)]} \left[\beta \frac{1}{\kappa(\gamma+\lambda)} p^{\lambda} \right]^{1/[1-(\gamma+\lambda)]}$, where p is the price of a unit of housing. From this one can verify that $\partial Q / \partial(\alpha y) > 0$ if $\gamma + \lambda < 1$.

K. Equivalent Variation of Change from Pure Local to the Categorical and the Pure State Regimes

Tables A6 and A7 in Section M below shows the distributions of the within cell average EV's of a change from the pure local regime to, respectively, the flat grant and foundation regimes. To be complete, we provide Tables A2 and A3, constructed analogously, that show the within cell average EV's of a change from the pure local regime to, respectively, the pure state and categorical regime. We have chosen the pure local regime as the benchmark for most of these tables though, as discussed in the text, one can compare the EV changes across regime switches and obtain a good approximation of changing between the "changed to" regimes.

In the panels of Table A2, one can see a strong preference for the pure local over the pure state regime consistent with the 95% preference shown in Table 3a of the text. This is due to a combination of Tiebout gains in the local equilibrium in the urban area and the high state tax in the pure state regime. The exception are the wealthiest households in the rural area. The pure state regime has higher schooling expenditure and school quality in the rural area and the wealthiest households are willing to pay higher taxes for this quality gain.

The panels of Table A3 show that all rural households and middle and the wealthiest urbanites prefer the categorical regime over the local regime, though the local regime is overall majority preferred (see Table 4a in the text). The strong preference of rural residents for the categorical regime is because of the large subsidy they obtain. While the poor urban residents also obtain a large tax subsidy in the categorical regime, housing prices rise substantially in the city as households are drawn in, implying net welfare losses to them. The Tiebout resorting in the urban area under the categorical regime has net benefit to the richest and lower-middle income households. The former value a large gain in school quality in spite of higher taxes and housing prices; the latter also gain in school quality without too high a cost. The upper middle class of urbanites does not gain enough in school quality to offset the high state tax.

We asserted in the text that urbanites have strongest preference for the foundation regime, while rural residents have the strongest preference for the categorical regime. Regarding the urbanites, comparing the average EV's in the cells in the upper panels of Tables A2, A3, A6, and A7 (the latter two further below) one can see the largest EV gains (in every cell!) relative to the local regime of a switch to the foundation regime. Regarding the rural residents, the analogue for the lower cells holds for a change to the categorical regime.

Table A2: Average Equivalent Variation from Pure Local Regime to Pure State Regime
Urban Area

	Income Deciles									
Preference Deciles	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	-$149.81	-$190.96	-$206.79	-$188.12	-$156.23	-$222.50	-$314.97	-$455.55	-$705.84	-$1,731.91
2	-$145.97	-$184.49	-$198.11	-$161.35	-$162.08	-$230.72	-$325.53	-$467.61	-$724.03	-$1,763.93
3	-$144.01	-$181.11	-$193.56	-$149.32	-$166.13	-$235.42	-$330.48	-$474.00	-$733.76	-$1,779.56
4	-$142.44	-$178.52	-$190.09	-$140.93	-$168.93	-$238.53	-$334.89	-$480.32	-$744.17	-$1,797.40
5	-$141.00	-$176.07	-$186.89	-$134.30	-$171.36	-$242.00	-$338.02	-$482.68	-$745.99	-$1,812.03
6	-$139.29	-$173.72	-183.84	-$128.41	-$174.07	-$244.99	-$342.20	-$489.37	-$753.36	-$1,823.96
7	-$138.10	-$171.40	-$180.43	-$123.20	-$176.51	-$247.95	-$346.06	-$492.84	-$758.10	-$1,816.27
8	-$137.04	-$168.69	-$171.42	-$123.51	-$179.19	-$251.53	-$350.87	-$499.69	-$768.85	-$1,841.10
9	-$134.57	-$165.29	-$158.87	-$126.37	-$183.07	-$255.73	-$355.10	-$506.16	-$776.19	-$1,830.75
10	-$130.99	-$158.85	-$138.65	-$132.01	-$189.64	-$263.99	-$366.12	-$518.58	-$793.06	-$1,877.60

Table A3: Average Equivalent Variation from Pure Local Regime to Pure State Regime
Rural Area

	Income Deciles									
Preference Deciles	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	-$13.39	-$16.05	-$16.95	-$17.17	-$16.81	-$15.76	-$13.71	-$9.84	-$1.44	$38.78
2	-$12.98	-$15.46	-$16.19	-$16.26	-$15.74	-$14.50	-$12.22	-$8.00	$0.81	$42.88
3	-$12.79	-$15.15	-$15.81	-$15.79	-$15.18	-$13.86	-$11.44	-$7.09	$2.05	$43.99
4	-$12.63	-$14.90	-$15.50	-$15.42	-$14.75	-$13.34	-$10.84	-$6.34	$2.90	$45.79
5	-$12.50	-$14.69	-$15.23	-$15.10	-$14.36	-$12.87	-$10.28	-$5.64	$3.74	$46.86
6	-$12.35	-$14.48	-$14.96	-$14.77	-$13.97	-$12.43	-$9.75	-$4.98	$4.63	$48.88
7	-$12.21	-$14.26	-$14.69	-$14.44	-$13.58	-$11.96	-$9.20	-$4.40	$5.41	$50.20
8	-$12.07	-$14.02	-$14.38	-$14.07	-$13.15	-$11.45	-$8.61	-$3.64	$6.34	$50.59
9	-$11.89	-$13.72	-$14.00	-$13.61	-$12.60	-$10.81	-$7.82	-$2.69	$7.50	$52.77
10	-$11.49	$13.14	-$13.21	-$12.70	-$11.52	-$9.55	-$6.36	-$0.88	$9.79	$57.23

L. Equilibrium Properties with Fixed Populations

Table A4 reports the positive equilibrium properties for the regimes examined with populations fixed at the baseline foundation values, this corresponding to the analysis in Section VII-B of the text. Note that the first column has the same values as in the baseline model, corresponding to Table 2 in the text.

Table A4: Equilibrium Properties Schooling Regimes with Fixed Populations			
	Foundation	**Pure Local**	**Flat State Grant**
Positive Properties			
Sorting:			
n^1=	30.36%	30.36%	30.36%
n^2=	19.07%	19.07%	19.07%
n^3=	22.00%	22.00%	22.00%
n^R=	28.57%	28.57%	28.57%
Gross Housing Price			
p^1=	$10.48	$10.18	$8.71
p^2=	$12.33	$12.07	$9.93
p^3=	$18.41	$15.99	$15.89
p^R=	$13.56	$12.97	$11.03
Avg. Housing Value			
D1	$40,831	$42,434	$49,181
D2	$66,775	$68,690	$83,890
D3	$153,492	$183,209	$178,631
R	$76,915	$81,407	$95,344
Per Student Exp.			
g^f=	$1,333	$1,333	$1,333
g^s=	$4,703	$0	$5,858
Per Student Exp. All Sources			
$G^{1=}$	$6,037	$3,351	$7,192
G^2=	$7,683	$6,452	$7,994
G^3=	$22,245	$21,649	$21,546
G^R=	$8,660	$7,638	$9,097
G^{avg}=	$10,665	$9,192	$11,046
Peer Measure			
θ^1=	0.3718	0.3718	0.3718
θ^2=	0.4169	0.4169	0.4169
θ^3=	0.4648	0.4648	0.4648
θ^R=	0.4098	0.4098	0.4098
Quality Index			
Q^1=	223	124	265
Q^2=	382	321	398
Q^3=	1,473	1,434	1,427
Q^R=	412	363	433

(Table A4 cont. on next page)

Table A4 Continued			
	Foundation	**Pure Local**	**Flat State Grant**
Positive Properties Cont.			
Property Taxation (Poterba Adjusted):			
t^s=	2.87%	n/a	n/a
t^1=	0.00%	2.35%	0.00%
t^2=	1.22%	3.67%	0.47%
t^3=	3.78%	3.67%	3.67%
t^R=	1.68%	3.67%	0.99%
Income Taxation:			
m^f=	1.01%	1.01%	1.01%
m^s=	n/a	n/a	4.44%
$m^{1=}$	0.00%	0.00%	0,00%
m^2=	0.00%	0.02%	0.00%
m^3=	1.61%	2.43%	0.40%
m^R=	0.00%	0.22%	0.00%

M. Local Equilibrium Schooling Expenditures with 1970 Income Distribution

As in the rest of the analysis, both the urban and rural income distributions are assumed to be lognormal. The urban distribution is calibrated to match the 1970 U.S. Census values for mean and median household income ($10,001 and $8,734 respectively[4]) adjusted to 2010 dollars based on Bureau of Labor Statistics CPI inflation Calculator[5] ($55,906 and $48,826 respectively), implying ln y ~ N(10.796, 0.520) there. Also as in the rest of the analysis, the rural area income distribution is adjusted so that the relative median incomes in the urban and rural areas and their relative poverty rates are as reported in Bishaw and Posey (2016). This implies the rural household income distribution is given by ln y ~ N(10.760, 0.433). Thus, the rural area is slightly poorer, e.g., with household mean and median income of $51,618 and $47,106. Table A5 reports the aggregate district per student district schooling expenditures, where we continue to assume even federal finance in the same proportion.[6] Here the ratio of G^3 to G^1 is given by 3.63.

Table A5: Pure Local Regime Equilibrium Per Student Expenditures by District Based on 1970 US Income Distribution in 2010 Dollars	
$G^{1=}$	$3,370
$G^{2=}$	$6,202
$G^{3=}$	$12,241
$G^{R=}$	$6,572
$G^{avg=}$	$3,612

[4] https://www.census.gov/data/tables/time-series/demo/income-poverty/historical-income-households.html -Table H-5

[5] https://data.bls.gov/cgi-bin/cpicalc.pl

[6] All the positive results reported in Table 2 for this case are available from the authors

N. Tables of Equivalent Variation from Regime Changes

The tables that follow show the within cell average equivalent variation of regime changes, these corresponding to the graphic presentation in Figures 5-8 of the paper.

Table A6-a: Average Equivalent Variation from Pure Local Regime to Flat State Grant Regime Urban Area

	Income Deciles									
Preference Deciles	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	$3.48	$40.15	$80.31	$120.86	$146.05	$222.95	$194.26	$227.66	$334.96	$751.89
2	$5.63	$43.86	$85.21	$120.03	$153.04	$224.21	$174.31	$231.26	$340.08	$765.08
3	$6.70	$45.62	$87.79	$119.36	$160.58	$213.60	$171.13	$233.56	$343.26	$760.72
4	$7.61	$46.94	$89.79	$118.86	$166.64	$202.18	$172.60	$235.15	$344.83	$765.40
5	$8.50	$48.42	$91.33	$118.27	$170.55	$192.84	$173.77	$236.30	$347.71	$773.56
6	$9.15	$49.42	$92.95	$117.43	$173.97	$184.74	$174.91	$237.71	$349.02	$776.32
7	$10.10	$51.12	$94.78	$116.73	$177.50	$175.98	$175.88	$239.25	$350.17	$773.63
8	$11.19	$52.52	$95.42	$117.39	$180.45	$167.41	$177.13	$240.62	$352.65	$782.79
9	$11.87	$54.54	$95.49	$118.61	$183.45	$156.84	$178.61	$242.55	$354.66	$780.44
10	$14.01	$58.02	$95.34	$124.95	$179.92	$144.02	$182.05	$246.51	$360.67	$796.92

Table A6-b: Average Equivalent Variation from Pure Local Regime to Flat Grant Regime Rural Area

	Income Deciles									
Preference Deciles	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	-$4.24	-$1.33	$2.16	$6.31	$11.48	$18.18	$27.13	$40.47	$63.93	$149.72
2	-$3.81	-$0.69	$2.99	$7.34	$12.69	$19.54	$28.77	$42.44	$66.58	$156.74
3	-$3.60	-$0.35	$3.41	$7.87	$13.30	$20.24	$29.66	$43.45	$67.79	$157.81
4	-$3.42	-$0.08	$3.74	$8.26	$13.79	$20.83	$30.27	$44.26	$68.81	$159.29
5	-$3.28	$0.14	$4.04	$8.61	$14.20	$21.29	$30.95	$45.02	$69.98	$160.30
6	-$3.14	$0.40	$4.34	$8.94	$14.59	$21.77	$31.50	$45.73	$70.56	$162.24
7	-$2.97	$0.62	$4.66	$9.35	$15.03	$22.27	$32.06	$46.36	$71.47	$163.79
8	-$2.80	$0.90	$4.99	$9.74	$15.52	$22.89	$32.73	$47.16	$72.52	$164.36
9	-$2.58	$1.22	$5.40	$10.25	$16.15	$23.58	$33.58	$48.17	$73.71	$167.35
10	-$2.18	$1.86	$6.25	$11.24	$17.34	$24.96	$35.23	$50.20	$76.31	$171.43

Table A7-a: Average Equivalent Variation from Pure Local Regime to Foundation Regime
Urban Area

	Income Deciles									
Preference Deciles	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	$12.09	$51.12	$92.18	$134.70	$172.28	$247.74	$207.50	$243.02	$358.85	$789.43
2	$14.09	$54.32	$96.44	$134.50	$186.79	$238.23	$186.63	$248.01	$365.03	$806.13
3	$14.87	$55.94	$98.71	$134.12	$193.24	$226.85	$183.30	$249.90	$366.71	$815.57
4	$15.56	$57.21	$100.50	$133.64	$197.14	$215.48	$185.22	$251.42	$369.43	$823.09
5	$16.62	$58.68	$102.21	$133.20	$200.80	$204.65	$185.80	$253.18	$370.40	$824.80
6	$17.19	$59.66	$103.55	$132.63	$203.62	$196.18	$187.28	$254.26	$372.22	$829.90
7	$18.08	$60.83	$105.19	$132.12	$206.19	$187.28	$188.39	$255.83	$374.65	$821.33
8	$18.75	$62.26	$105.87	$132.89	$207.89	$178.01	$189.97	$257.50	$376.99	$844.69
9	$19.58	$63.70	$106.07	$139.62	$203.86	$167.34	$191.36	$259.95	$379.96	$837.41
10	$21.56	$66.83	$106.43	$151.65	$191.92	$153.42	$194.79	$264.06	$385.48	$854.44

Table A7-b: Average Equivalent Variation from Pure Local Regime to Foundation Regime
Rural Area

	Income Deciles									
Preference Deciles	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	-$0.33	$3.59	$7.56	$12.05	$17.44	$24.16	$33.04	$46.08	$68.30	$150.53
2	$.001	$4.10	$8.22	$12.85	$18.40	$25.27	$34.36	$47.58	$70.34	$152.33
3	$0.17	$4.36	$8.54	$13.25	$18.82	$25.77	$35.00	$48.39	$71.43	$155.46
4	$0.30	$4.54	$8.80	$13.55	$19.21	$26.25	$35.50	$49.01	$72.03	$154.47
5	$0.43	$4.74	$9.03	$13.83	$19.53	$26.66	$36.00	$49.46	$72.67	$157.68
6	$0.53	$4.89	$9.27	$14.15	$19.85	$27.00	$36.39	$50.18	$73.53	$157.14
7	$0.64	$5.10	$9.47	$14.39	$20.17	$27.41	$36.94	$50.70	$74.09	$158.28
8	$0.75	$5.30	$9.73	$14.72	$20.57	$27.85	$37.45	$51.33	$75.13	$160.58
9	$0.93	$5.54	$10.08	$15.12	$21.04	$28.38	$38.09	$52.12	$75.99	$161.23
10	$1.28	$6.06	$10.71	$15.91	$21.96	$29.47	$39.39	$53.58	$78.11	$164.72

Table A8-a: Average Equivalent Variation from Pure State Regime to Flat Grant Regime-Urban Area

	Income Deciles									
Preference Deciles	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	$154.47	$232.43	$287.56	$340.66	$394.41	$374.42	$486.06	$689.19	$1,055.54	$2,520.15
2	$152.94	$229.14	$283.95	$335.82	$379.41	$361.80	$499.37	$704.08	$1,076.00	$2,525.76
3	$151.99	$227.65	$281.92	$332.91	$363.06	$367.71	$507.11	$714.28	$1,085.15	$2,596.20
4	$151.12	$226.20	$280.47	$331.00	$350.51	$372.61	$512.10	$721.33	$1098.62	$2,584.48
5	$149.99	$225.52	$278.89	$329.28	$340.57	$376.37	$517.94	$728.70	$1,108.19	$2,291.41
6	$149.22	$223.98	$277.39	$327.77	$331.31	$380.57	$522.48	$734.83	$1,116.71	$2,613.09
7	$149.03	$223.38	$276.28	$325.62	$322.81	$384.63	$525.85	$740.29	$1,123.76	$2,649.59
8	$148.36	$221.99	$274.44	$323.46	$314.38	$388.90	$532.91	$748.91	$1,131.00	$2,632.55
9	$147.32	$220.30	$272.63	$320.91	$304.71	$394.62	$539.66	$757.26	$1,142.16	$2,661.34
10	$146.34	$217.53	$268.33	$307.53	$298.61	$406.25	$553.34	$772.94	$1,167.33	$2,680.31

Table A8-b: Average Equivalent Variation from Pure State Regime to Flat Grant Regime-Rural Area

	Income Deciles									
Preference Deciles	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	$9.34	$15.08	$19.57	$24.07	$28.95	$34.64	$41.69	$51.32	$66.61	$115.56
2	$9.41	$15.13	$19.62	$24.12	$29.04	$34.75	$41.83	$51.37	$66.94	$115.27
3	$9.45	$15.16	$19.66	$24.18	$29.09	$34.78	$41.86	$51.56	$66.96	$115.48
4	$9.45	$15.20	$19.67	$24.20	$29.14	$34.86	$41.94	$51.53	$67.07	$115.92
5	$9.49	$15.19	$19.73	$24.24	$29.17	$34.92	$42.01	$51.67	$67.20	$116.02
6	$9.46	$15.19	$19.73	$24.28	$29.24	$34.94	$42.04	$51.72	$67.30	$115.94
7	$9.49	$15.24	$19.77	$24.32	$29.26	$34.96	$42.09	$51.78	$67.27	$116.17
8	$9.50	$15.25	$19.80	$24.32	$29.29	$35.05	$42.18	$51.81	$67.50	$116.48
9	$9.52	$15.25	$19.84	$24.38	$29.37	$35.09	$42.26	$51.92	$67.56	$116.49
10	$9.53	$15.36	$19.89	$24.47	$29.48	$35.23	$42.38	$52.12	$67.69	$117.26

Table A9-a: Average Equivalent Variation from Pure State Regime to Foundation Regime-Urban Area

Income Deciles										
Preference Deciles	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	$163.11	$243.08	$299.60	$352.42	$388.76	$364.37	$498.63	$705.49	$1,077.33	$2,587.20
2	$161.08	$240.01	$295.17	$347.08	$354.58	$371.41	$512.27	$723.20	$1,100.62	$2,606.00
3	$159.84	$238.01	$293.00	$344.06	$338.88	$376.70	$518.73	$731.11	$1,109.84	$2,613.29
4	$158.73	$236.80	$291.12	$341.59	$325.74	$382.17	$524.47	$738.75	$1,121.40	$2,616.19
5	$159.51	$235.41	$289.45	$339.61	$316.62	$385.91	$530.04	$744.60	$1,130.40	$2,670.67
6	$158.60	$234.37	$288.03	$337.96	$308.24	$389.61	$534.64	$751.18	$1,137.34	$2,666.93
7	$157.15	$233.31	$286.37	$335.81	$300.08	$394.04	$539.64	$758.46	$1,146.77	$2,689.00
8	$159.27	$231.65	$284.54	$332.21	$292.20	$398.32	$545.01	$765.03	$1,159.28	$2,695.85
9	$155.18	$230.15	$282.32	$317.61	$295.46	$404.79	$551.27	$771.52	$1,169.15	$2,719.18
10	$153.76	$226.70	$277.69	$289.45	$305.18	$415.20	$566.31	$791.23	$1,191.22	$2,764.44

Table A9-b: Average Equivalent Variation from Pure State Regime to Foundation Regime-Rural Area

Income Deciles										
Preference Deciles	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	$13.31	$19.97	$24.96	$29.78	$34.89	$40.67	$47.61	$56.80	$71.09	$112.50
2	$13.21	$19.88	$24.82	$29.63	$34.71	$40.47	$47.42	$56.52	$70.70	$112.68
3	$13.21	$19.86	$24.80	$29.56	$34.66	$40.37	$47.27	$56.40	$70.59	$112.28
4	$13.15	$19.83	$24.73	$29.51	$34.60	$40.27	$47.19	$56.28	$70.29	$111.43
5	$13.13	$19.78	$24.69	$29.48	$34.52	$40.23	$47.07	$56.18	$70.30	$111.15
6	$13.09	$19.76	$24.67	$29.41	$34.47	$40.16	$47.02	$56.13	$70.16	$111.47
7	$13.07	$19.69	$24.61	$29.36	$34.41	$40.08	$46.92	$55.93	$70.08	$111.35
8	$13.09	$19.65	$24.56	$29.35	$34.34	$39.99	$46.86	$55.90	$69.85	$110.61
9	$13.09	$19.62	$24.50	$29.24	$34.26	$39.92	$46.76	$55.73	$69.78	$110.46
10	$12.95	$19.55	$24.39	$29.11	$34.09	$39.72	$46.53	$55.51	$69.40	$110.26

O. Analysis of Generalized Tax Prices, Proposition 5, and Proposition 6.

We focus first on the flat state grant case (i.e., with the state income tax). We derive expressions for the generalized tax prices associated with local taxation of income and property. We drop the j superscript indicating district values. Relevant equations are:

(O.1) $\Gamma = \kappa[1-M]^{\gamma+\lambda}[(1+t)p_h]^{-\lambda}$

(O.2) $G = g^f + g^s + g$

(O.3) $M = m^f + m^s + m$

(O.4) $g^s = m^s Y$

(O.5) $\bar{H} \equiv \frac{\lambda}{\gamma+\lambda}\frac{(1-M)\bar{y}}{(1+t)p_h}$

(O.6) $g = tp_h\bar{H} + m\bar{y} = \frac{t}{1+t}\frac{\lambda}{\lambda+\gamma}(1-M)\bar{y} + m\bar{y}$

(O.7) $n\bar{H} = H_s(p_h) \rightarrow n\cdot\frac{\lambda}{\lambda+\gamma}\cdot\frac{[1-(m^f+m^s+m)]}{(1+t)\bar{y}p_h} = H_s(p_h)$

All these come from the text, but we provide a quick review. (O.1) is the definition of the composite public good Γ. (O.2) is that local expenditure sums that from all sources and (O.3) likewise for the income tax rate in a district. (O.4) and (O.6) indicate the determination of the state and local levels of per student expenditure, with the second equality in (O.6) using (O.5). (O.5) is average housing demand in the district. Finally, (O.7) is housing market clearance, with substitution of (O.3) and (O.5) in the second statement of it.

Substitute (O.3) into (O.6). Then substitute (O.6) and (O.4) into (O.2) to get:

(O.8) $G = g^f + m^s Y + \frac{t}{1+t}\frac{\lambda}{\lambda+\gamma}[1-(m^f+m^s+m)\bar{y}] + m\bar{y}$

The equations used are then (O.1), (O.7), and (O.8). Keep in mind that m^s and g^f are constant. Differentiate (B.1):

(O.9) $\frac{d\Gamma}{dm} = -\kappa(\gamma+\lambda)[1-M]^{\gamma+\lambda-1}(1+t)^{-\lambda}p_h^{-\lambda} - K[1-M]^{\gamma+\lambda}(1+t)^{-\lambda}p_h^{-(\lambda+1)}\frac{dp_h}{dm}$

From (O.7), one obtains:

(O.10) $\frac{dp_h}{dm} = -\frac{p_h}{(1-M)(1+\varepsilon_H^s)}$

Substituting (O.10) into (O.9) yields:

$$(O.11)\quad \frac{d\Gamma}{dm} = -\kappa\{[1-M]^{\gamma+\lambda-1}(1+t)^{-\lambda}p_h^{-\lambda}\}\cdot\{(\gamma+\lambda)-\frac{\lambda}{1+\varepsilon_H^s}\}$$

Now differentiating (O.8) yields:

$$(O.12)\quad \frac{dG}{dm} = \bar{y}[1-\frac{t}{1+t}\cdot\frac{\lambda}{\lambda+\gamma}]$$

Using (O.11) and (O.12), one obtains (after multiplying the numerator and denominator by $(1+t)(\gamma+\lambda)$:

$$(O.13)\quad -\frac{d\Gamma/dm}{dG/dm} = \kappa\frac{(\gamma+\lambda)(1+t)^{1-\lambda}\left[(\gamma+\lambda)-\frac{\lambda}{(1+\varepsilon_H^s)}\right]}{\bar{y}(1-M)^{1-(\gamma+\lambda)}(p_h)^{\lambda}[\gamma(1+t)+\lambda]},$$

which is the expression for the generalized local income-tax price.

To find the analogue for the local property tax, one needs from (O.7) the result that:

$$(O.14)\quad \frac{dp_h}{dt} = -\frac{p_h}{(1+t)(1+\varepsilon_H^s)}$$

Using the latter, (O.1), and (O.8) to compute $\frac{d\Gamma}{dt}$ and $\frac{dG}{dt}$, it is straightforward to confirm that:

$$(O.15)\quad -\frac{d\Gamma/dt}{dG/dt} = \frac{\kappa(\gamma+\lambda)(1+t)^{1-\lambda}}{\bar{y}(1-M)^{1-(\gamma+\lambda)}(p_h)^{\lambda}}\frac{\varepsilon_s^H}{1+\varepsilon_s^H}.$$

Using (O.13) and (O.15), the results in Proposition 5 are easily obtained.

Now we verify Proposition 6. We find the tax price of varying the state income tax: $\frac{d\Gamma/dm^s}{dG/dm^s}$ and compare it to (O.13). The derivation of the tax price of the state income tax holds constant the local taxes. From (O.1) and that (O.7), $\frac{dp_h}{dm^s} = \frac{dp_h}{dm}$, and one can see that $\frac{d\Gamma}{dm^s} = \frac{d\Gamma}{dm}$, given by the RHS of (O.11). Differentiating (O.8):

$$(O.16)\quad \frac{dG}{dm^s} = Y - \frac{t}{1+t}\frac{\lambda}{\gamma+\lambda}\bar{y}.$$

Using these results, Proposition 6 is implied.

Last here, consider the tax prices for local policy in the foundation case. Here the relevant equations are (C.1) – (C.7) above. Straightforward derivation yields:

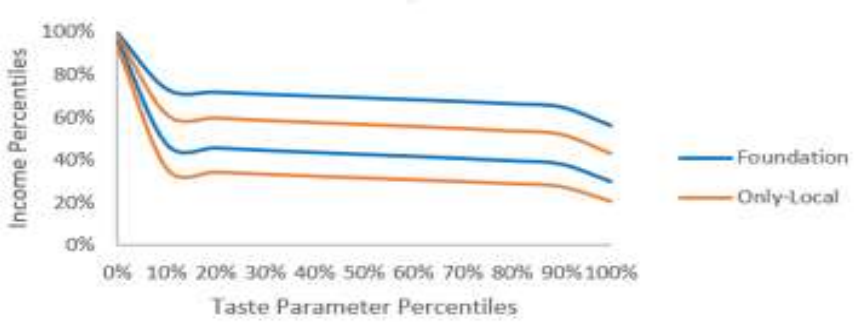
$$(O.17)\quad -\frac{d\Gamma / dm}{dG / dm} = \kappa \frac{(1+t+t^s)^{-\lambda}\left[(\gamma+\lambda)-\dfrac{\lambda}{(1+\varepsilon_H^s)}\right]}{\bar{y}(1-M)^{1-(\gamma+\lambda)}(p_h)^{\lambda}\left[1-\dfrac{\lambda}{\lambda+\gamma}\dfrac{1}{1+t+t^s}(t+nt^s)\right]};$$

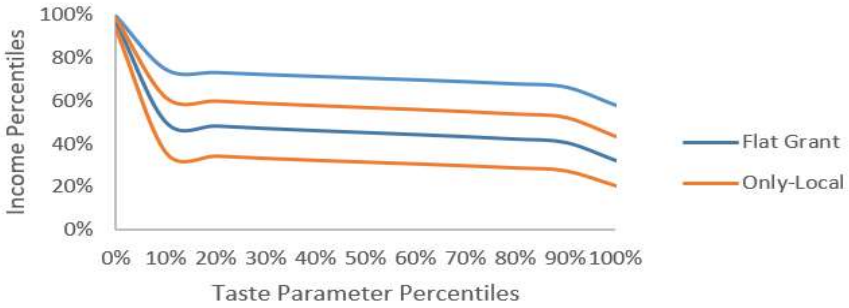
and

$$(O.18)\quad -\frac{d\Gamma / dt}{dG / dt} = \kappa \frac{(1+t+t^s)^{1-\lambda}(\lambda+\gamma)}{\bar{y}(1-M)^{1-(\gamma+\lambda)}(p_h)^{\lambda}[1+(1-n)t^s]}\frac{\varepsilon_s^H}{1+\varepsilon_s^H}.$$

As a partial check, observe that these would be the same as in the flat-grant case if t_s were to equal 0 here and if $m^s = 0$ in the flat grant case. From (O.17) and (O.18), one obtains (16) in the text and then the analogue of the results in Proposition 5 applied to the foundation case (see the text).

P. Household Sorting in the Urban Area under Mixed Finance Regimes. It was indicated in the text that household sorting in the urban area was similar in the foundation and flat-grant school finance systems, but the figure for the flat-grant case was omitted to conserve space. The second figure below depicts this for the flat-grant case (including the only-local case), while, for comparison, the upper figure is Figure 2 from the text.





Q. Applying the Model to Intergovernmental Finance of Other Local Public Goods. The structure has the potential to be applied to other cases of multiple-tier governmental funding of local public goods. Let $Q^P = Q^P(\theta^j, G^P(g^f, g^s, g^j))$ denote the local public good in local jurisdiction j, with G^P a production function aggregating provision at the three government expenditures, and arguments of Q^P and G^P weakly increasing. Consider policing. Federal, state, and local monies might be provided, which may or may not be perfect substitutes. A peer effect might arise from resident propensity to commit crimes. Other

applications might not have a peer effect, $\partial Q^p / \partial \theta^j = 0$, an example being road construction. Any particular application would likely require further modification, especially with regard to jurisdictional spill overs. The basic structure has potential to facilitate determination of majority choice equilibrium.

R. Equilibrium Properties for Various Values of ϕ in the Foundation Regime with Categorical Funding

Table A10 reports equilibrium values (as in Table 2 of the paper) for several values of ϕ in the FCG regime. The first column has $\phi = 0$, which corresponds to the (pure) foundation regime. The column with $\phi = .11$ is that majority preferred. The fourth column has all state funding allocated to the poorer districts.

Table A10: Equilibrium Properties FCG Regimes				
	Foundation	$\phi = .11$	$\phi = .20$	$\phi = \max$
Positive Properties				
Sorting:				
$n^1=$	30.36%	34.98%	38.58%	44.86%
$n^2=$	19.07%	15.24%	13.27%	9.51%
$n^3=$	22.00%	21.21%	19.58%	17.06%
$n^R=$	28.57%	28.57%	28.57%	28.57%
Avg. Housing Value				
D1	$40,831	$39,779	$42,180	$47,161
D2	$66,775	$77,595	$81,462	$79,616
D3	$153,492	$153,816	$145,775	$144,867
R	$76,915	$78,563	$78,987	$80,587
Per Student Exp.				
$g^f=$	$1,333	$1,333	$1,333	$1,333
$g^s=$	$4,703	$4,838 (D1&R) $3,523 (D2&D3)	$6,343 (D1&R) $3,125 (D2&D3)	$8,114 (D1&R) $0 (D2&D3)
Per Student Exp. All Sources				
$G^{1=}$	$6,037	$6,171	$7,677	$9,448
$G^2=$	$7,683	$9,220	$10,521	$11,229
$G^3=$	$22,245	$22,308	$22,594	$24,541
$G^R=$	$8,660	$8,664	$8,973	$9,448
$G^{avg}=$	$10,665	$10,770	$11,346	$12,191
Peer Measure				
$\theta^1=$	0.3718	0.3762	0.3795	0.3852
$\theta^2=$	0.4169	0.4213	0.4256	0.4335
$\theta^3=$	0.4648	0.4660	0.4685	0.4727
$\theta^R=$	0.4098	0.4098	0.4098	0.4098
Quality Index				
$Q^1=$	223	235	299	382
$Q^2=$	382	472	552	619
$Q^3=$	1,473	1,487	1,469	1,698
$Q^R=$	412	412	427	449
Property Taxation (Poterba Adjusted):				
$t^s=$	2.87%	2.66%	3.34%	3.85%
$t^1=$	0.00%	0.00%	0.00%	0.00%
$t^2=$	1.22%	2.78%	3.68%	4.95%
$t^3=$	3.78%	3.95%	4.26%	4.95%
$t^R=$	1.68%	1.57%	0.81%	0.00%
Income Taxation				
$m^f=$	1.01%	1.01%	1.01%	1.01%
$m^s=$	n/a	n/a	n/a	n/a
$m^{1=}$	0.00%	0.00%	0.00%	0.00%
$m^2=$	0.00%	0.00%	0.00%	1.20%
$m^3=$	1.61%	1.86%	2.00%	2.93%
$m^R=$	0.00%	0.00%	0.00%	0.00%
Normative Properties				
$EV^{avg}=$	$179	$203	$161	$121
Δ House Rents =	-$219	-$228	-$302	-$357

S. Some Equilibrium Properties in State Analysis of FCG Regimes

Table A11 contains equilibrium properties corresponding to the equilibria in the state FCG regimes in Table 7 of the paper. Note that all the property taxes that are reported are on rental values, not property values. To obtain the corresponding tax rates on property values, simply divide by 11 (see the text of the paper). The t_i's are the district property taxes, with i = 1 the city, i = 2 the non-elite suburb, i = 3 the elite suburb, and the R subscript indicating the rural district. The t_s is the foundation (state) tax. The m_i's are the local income taxes.

The N_i's are the population proportion of households in district i that would prefer a higher foundation tax. Note that these sum to 50% in every state to have a majority choice equilibrium. In every state: (i) all residents of the elite suburb would prefer a lower foundation tax; and (ii) there are multiple pivotal voting households (usually three). Property (i) holds because of the high property values in the elite suburb, where households are cross subsidizing the rest of households with a foundation tax. Property (ii) means that there are households in multiple districts with the same preference for a foundation tax, in particular the equilibrium one.

Table A11: Some Equilibrium Elements in State Level Analysis

State	t_s	N_1	N_2	N_3	N_R	m_1	m_2	m_3	m_R	t_1	t_2	t_3	t_R
CA	33.40%	29.48%	18.08%	0%	2.44%	0%	0%	2.16%	0%	0%	8.88%	31.70%	8.24%
AL	41.22%	23.77%	0%	0%	26.23%	0%	0%	4.14%	0%	0%	40.18%	40.86%	7.64%
NY	54.44%	24.32%	13.65%	0%	12.03%	0%	0%	9.75%	0%	0%	20.79%	17.78%	30.47%
PA	26.52%	16.83%	12.20%	0%	20.96%	0%	0%	8.42%	3.00%	0%	31.96%	25.45%	33.82%
AZ	27.42%	37.34%	8.67%	0%	3.99%	0%	0%	1.57%	0%	0%	12.88%	32.68%	11.31%
AR	49.48%	19.71%	0%	0%	30.29%	0%	0%	6.04%	0%	0%	40.07%	38.48%	11.76%
CO	24.39%	35.95%	2.96%	0%	11.09%	0%	0%	3.03%	0%	0%	23.99%	33.21%	22.74%
CT	36.03%	37.02%	0%	0%	12.98%	0%	0.41%	6.09%	0%	0%	41.14%	33.73%	28.17%
IN	39.54%	31.87%	0%	0%	18.13%	0%	0%	3.41%	0%	0%	42.39%	40.39%	7.52%
KS	41.93%	31.43%	0%	0%	18.57%	0%	0%	3.19%	0%	0%	42.94%	40.27%	6.83%
MD	6.72%	19.70%	19.60%	0%	11.70%	0%	0.83%	4.66%	2.25%	18.54%	33.91%	31.33%	34.28%
MA	24.93%	24.65%	20.55%	0%	4.80%	0%	0%	5.92%	0.1%	0%	17.30%	26.32%	38.75%
NJ	26.52%	24.27%	19.53%	0%	6.20%	0%	0%	6.64%	0%	0%	21.34%	24.50%	38.46%
WA	11.31%	17.92%	15.49%	0%	16.59%	0%	0.99%	6.24%	2.24%	8.95%	34.78%	29.61%	34.57%