Online Appendix

The Macroeconomic Dynamics for Labor Market Policies

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A Model Without Labor Market Policies

We start by studying the model without any labor market policies. This analysis underlies the results in Sections 1 and 2 of the main text. In Appendix B, we introduce labor market policies and use this BGP as the initial condition.

A.1 Households

From the main text, the household's utility maximization problem is

$$\max_{c_{it}, s_{ijt}, n_{ijt+1}} \sum_{t=0}^{\infty} \beta^t U_t(c_{it}, n_{it}, s_{it})$$

$$\text{s.t. } n_{ijt+1} = (1-\sigma) n_{ijt} + \lambda_w(\theta_{ijt}) s_{ijt} \quad (\times \beta^{t+1} \widehat{V}_{ijt+1})$$

$$\sum_{t=0}^{\infty} Q_{0,t} c_{it} = \zeta_i \mathbb{P} + \mathbb{I}_i + \sum_{t=1}^{\infty} Q_{0,t} \sum_j \lambda_w(\theta_{ijt-1}) s_{ijt-1} W_{ijt}. \quad (\times \Gamma) \tag{A1}$$

Here, the variables in parentheses denote the (often rescaled) Lagrange multiplier associated with the constraint. The first-order condition for consumption c_{it} is $\beta^t U_{cit} = \Gamma Q_{0,t}$. Taking ratios of this equation across adjacent time periods gives $Q_{t,t+1} = \beta \frac{U_{cit+1}}{U_{cit}}$. The first-order condition for employment n_{ijt+1} is

$$\beta^{t+1} U_{nit+1} \left(\frac{n_{ijt+1}}{n_{it+1}} \right)^{\frac{1}{\omega}} - \beta^{t+1} \widehat{V}_{ijt+1} + \beta^{t+2} (1-\sigma) \widehat{V}_{ijt+2} = 0,$$

which implies

$$\widehat{V}_{ijt+1} = U_{nit+1} \left(\frac{n_{ijt+1}}{n_{it+1}} \right)^{\frac{1}{\omega}} + \beta (1-\sigma) \widehat{V}_{ijt+2}.$$

which uses the fact that $\frac{\partial n_{it+1}}{\partial n_{ijt+1}} = \left(\frac{n_{ijt+1}}{n_{it+1}} \right)^{\frac{1}{\omega}}$. Note that $\widehat{V}_{ijt+1}$ is in units of utility at $t+1$. Going forward, it will be useful to put this object in consumption units by dividing by the marginal utility of consumption in period $t+1$:

$$\begin{aligned} V_{ijt+1} \equiv \frac{\widehat{V}_{ijt+1}}{U_{cit+1}} &= \frac{U_{nit+1}}{U_{cit+1}} \left(\frac{n_{ijt+1}}{n_{it+1}} \right)^{\frac{1}{\omega}} + \beta \frac{U_{cit+2}}{U_{cit+2}} (1-\sigma) \frac{\widehat{V}_{ijt+2}}{U_{cit+1}} \\ &= \frac{U_{nit+1}}{U_{cit+1}} \left(\frac{n_{ijt+1}}{n_{it+1}} \right)^{\frac{1}{\omega}} + Q_{t+1,t+2} (1-\sigma) V_{ijt+2}, \end{aligned} \tag{A2}$$

where the second line uses the fact that $Q_{t+1,t+2} = \beta \frac{U_{cit+2}}{U_{cit+1}}$. This equation defines V_{ijt+1} as the present value of marginal disutilities of work for workers that are hired in period t and begin working in period $t+1$, in terms of their consumption units in period $t+1$.

The first-order condition for search effort s_{it} is

$$\begin{aligned}
&\beta^t U_{sit} + \beta^{t+1}\widehat{V}_{ijt+1}\lambda_w(\theta_{ijt}) + \Gamma Q_{0,t+1}\lambda_w(\theta_{ijt})W_{ijt+1} = 0 \\
\implies & -U_{sit} = \lambda_w(\theta_{ijt})\beta U_{cit+1}\left(\frac{\widehat{V}_{ijt+1}}{U_{cit+1}} + W_{ijt+1}\right) \\
\implies & -\frac{U_{sit}}{U_{cit}} = \lambda_w(\theta_{ijt})Q_{t,t+1}\left[V_{it+1} + W_{ijt+1}\right],
\end{aligned} \tag{A3}$$

where the second line uses the fact that $\Gamma Q_{0,t+1} = U_{cit+1}$ and the third line uses the fact that $Q_{t,t+1} = \beta\frac{U_{cit+1}}{U_{cit}}$. Recall from the main text that we will write the participation constraint as

$$\lambda_w(\theta_{ijt})Q_{t,t+1}(W_{ijt+1} + V_{ijt+1}) \geq \mathcal{W}_{it} \equiv \lambda_w(\theta_{it})Q_{t,t+1}(W_{it+1} + V_{it+1}). \tag{A4}$$

A.2 Firms

We now turn to the firm's profit maximization problem, which is the main challenge of solving the model. We abstract from initial conditions because we use these results to derive the limiting BGP. We also ignore the non-negativity constraint on vacancy posting $a_{ijt} \geq 0$ because that constraint is not binding along the BGP. We begin by showing how to group collect the Lagrange multipliers on the participation constraints in terms of the auxiliary variable M_{ijt} defined in the main text. This allows us to specify the full profit maximization problem of the firm. We then derive the first-order conditions of the profit maximization problem. Finally, we summarize the resulting conditions which characterize the solution to the firm's problem.

Grouping Multipliers on the Participation Constraint. We collect across time the corresponding terms of the participation constraints of each period, in a way that has become standard in the dynamic contracting literature (Marcet and Marimon 2019), so as to isolate the impact of additional hires of a type-i family by firm j in t on the disutility of work of all members of the family hired by the firm in future periods. To do this, we attach the (scaled) Lagrange multiplier $Q_{0,t+1}\mu_i\gamma_{ijt+1}$ to the time-t participation constraint (5) from the main text. It is instructive to write out how the first few period's participation constraints enter firm j's expected profit maximization problem:

$$\begin{aligned}
&Q_{0,1}\gamma_{ij1}\Big[\frac{U_{ni1}}{U_{ci1}}\left(\frac{n_{ij1}}{n_{i1}}\right)^{\frac{1}{\omega}} + Q_{1,2}(1-\sigma)\frac{U_{ni2}}{U_{ci2}}\left(\frac{n_{ij2}}{n_{i2}}\right)^{\frac{1}{\omega}} + Q_{1,3}(1-\sigma)^2\frac{U_{ni3}}{U_{ci3}}\left(\frac{n_{ij3}}{n_{i3}}\right)^{\frac{1}{\omega}} + \ldots + W_{ij1} - \frac{\mathcal{W}_{i0}}{Q_{0,1}\lambda_w(\theta_{ij0})}\Big] \\
&Q_{0,2}\gamma_{ij2}\Big[\frac{U_{ni2}}{U_{ci2}}\left(\frac{n_{ij2}}{n_{i2}}\right)^{\frac{1}{\omega}} + Q_{2,3}(1-\sigma)\frac{U_{ni3}}{U_{ci3}}\left(\frac{n_{ij3}}{n_{i3}}\right)^{\frac{1}{\omega}} + \ldots + W_{ij2} - \frac{\mathcal{W}_{i1}}{Q_{1,2}\lambda_w(\theta_{ij1})}\Big] \\
&Q_{0,3}\gamma_{ij3}\Big[\frac{U_{ni3}}{U_{ci3}}\left(\frac{n_{ij3}}{n_{i3}}\right)^{\frac{1}{\omega}} + \ldots + W_{ij3} - \frac{\mathcal{W}_{i2}}{Q_{2,3}\lambda_w(\theta_{ij2})}\Big].
\end{aligned}$$

By collecting the multipliers associated with the terms $\frac{U_{nit}}{U_{cit}}\left(\frac{n_{ijt}}{n_{it}}\right)^{\frac{1}{\omega}}$ for some t and noting that $Q_{0,\tau}Q_{\tau,t} = Q_{0,t}$ for $\tau < t$, it is easy to see that all such terms are summarized by the auxiliary variable $M_{ijt+1} = (1-\sigma)M_{ijt} + \gamma_{ijt+1}$ as in Marcet and Marimon (2019). Hence, the contributions of the participation constraint to the Lagrangian can be reduced to

$$\sum_{t=0}^{\infty} Q_{0,t+1}\mu_i M_{ijt+1}\frac{U_{nit+1}}{U_{cit+1}}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}} + \sum_{t=0}^{\infty} Q_{0,t+1}\mu_i\gamma_{ijt+1}\left[W_{ijt+1} - \frac{\mathcal{W}_{it}}{Q_{t,t+1}\lambda_w(\theta_{ijt})}\right].$$

Profit-Maximization Problem. Using these results, we can write the firm's problem as choosing utilization $u_{jt}(v,\varepsilon,A_{t-\tau})$, the labor allocation $N_{ijt}(v,A_{t-\tau},\varepsilon)$, total employment N_{ijt}, vacancy posting a_{ijt}, market tightness θ_{ijt}, present value of wage offers W_{ijt+1}, investment $X_{jt}(v)$, and capital $K_{jt+\tau+1}(v,A_t)$, in order to maximize the expected present value of profits:

$$\sum_t Q_{0,t}\left(\sum_\tau \int_{v,\varepsilon} u_{jt}(v,A_{t-\tau},\varepsilon)A_{t-\tau}\varepsilon f(v)K_{jt}(v,A_{t-\tau})\pi(\varepsilon)d\varepsilon dv - \sum_i \mu_i(\lambda_f(\theta_{ijt-1})a_{ijt-1}W_{ijt} + \kappa_{it}a_{ijt})\right.$$
$$\left. - \int X_{jt}(v)dv\right) + \sum_{t=0}^{\infty} Q_{0,t+1}\mu_i M_{ijt+1}\frac{U_{nit+1}}{U_{cit+1}}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}} + \sum_{t=0}^{\infty} Q_{0,t+1}\mu_i\gamma_{ijt+1}\left[W_{ijt+1} - \frac{\mathcal{W}_{it}}{Q_{t,t+1}\lambda_w(\theta_{ijt})}\right]$$

such that $\quad u_{jt}(v,A_{t-\tau},\varepsilon) \geq 0 \quad (\times Q_{0,t}\lambda^L_{jt}(v,A_{t-\tau},\varepsilon))$

$u_{jt}(v,A_{t-\tau},\varepsilon) \leq 1 \quad (\times Q_{0,t}\lambda^U_{jt}(v,\varepsilon,A_{t-\tau}))$

$u_{jt}(v,A_{t-\tau},\varepsilon)v_i K_{jt}(v,A_{t-\tau})\pi(\varepsilon) \leq N_{ijt}(v,A_{t-\tau},\varepsilon)$ for all $i \quad (\times Q_{0,t}\lambda_{ijt}(v,A_{t-\tau},\varepsilon))$

$\sum_\tau \int_{v,\varepsilon} N_{ijt}(v,A_{t-\tau},\varepsilon)d\varepsilon dv \leq \mu_i n_{ijt}$ for all $i \quad (\times Q_{0,t}\chi_{ijt})$

$\mu_i n_{ijt+1} \leq (1-\sigma)\mu_i n_{ijt} + \lambda_f(\theta_{ijt})\mu_i a_{ijt}$ for all $i \quad (\times Q_{0,t+1}\nu_{ijt+1})$

$K_{jt+\tau+1}(v,A_t) = (1-\delta)^\tau X_{jt}(v) \quad (\times Q_{0,t+\tau+1}q_{jt,t+\tau+1}(v))$

$X_{jt}(v) \geq 0 \quad (\times Q_{0,t}\mu_{jt}(v)),$

with the side conditions that $M_{ijt+1} = (1-\sigma)M_{ijt} + \gamma_{ijt+1}$ and $\frac{\mathcal{W}_{it}}{Q_{t,t+1}\lambda_w(\theta_{it})} = W_{it+1} + V_{it+1}$. As before, variables in parenthesis denote scaled Lagrange multipliers on the associated constraint. In this problem, we have explicitly written the measure of workers as $N_{ijt} = \mu_i n_{ijt}$, where n_{ijt} is the share of family i working at firm j. We make this substitution because the participation constraint naturally depends on per-capital n_{ijt} rather than the total measure N_{ijt}.

We now proceed to take the first-order conditions of this problem. We group these conditions into three blocks: the utilization block, the hiring block, and the investment block.

Utilization Block. The first-order condition for labor assignment $N_{ijt}(v, A_{t-\tau}, \varepsilon)$ is simply $\lambda_{ijt}(v, A_{t-\tau}, \varepsilon) = \chi_{ijt}$. The first-order condition for utilization $u_{jt}(v, A_{t-\tau}, \varepsilon)$ is is given by

$$A_{t-\tau}\varepsilon f(v)K_{jt}(v, A_{t-\tau})\pi(\varepsilon) - \sum_i \lambda_{ijt}(v, A_{t-\tau}, \varepsilon)v_i K_{jt}(v, A_{t-\tau})\pi(\varepsilon) = \lambda^U_{ijt}(v, A_{t-\tau}, \varepsilon) - \lambda^L_{ijt}(v, A_{t-\tau}, \varepsilon).$$

Substituting the first-order condition for labor assignment from above, namely $\lambda_{ijt}(v, A_{t-\tau}, \varepsilon) = \chi_{ijt}$, we get

$$A_{t-\tau}\varepsilon f(v) - \sum_i \chi_{ijt}v_i = \frac{\lambda^U_{ijt}(v, A_{t-\tau}, \varepsilon) - \lambda^L_{ijt}(v, A_{t-\tau}, \varepsilon)}{K_{jt}(v, A_{t-\tau})\pi(\varepsilon)}. \tag{A5}$$

If $A_{t-\tau}\varepsilon f(v) - \sum_i \chi_{ijt}v_i > 0$ or, equivalently, $\varepsilon > \frac{\sum_i \chi_{ijt}v_i}{A_{t-\tau}f(v)} \equiv \underline{\varepsilon}(v, A_{t-\tau}; \chi_{jt})$ for $\chi_{jt} = (\chi_{1jt}, \ldots, \chi_{Ijt})$, then (A5) implies that $\lambda^U_{ijt}(v, A_{t-\tau}, \varepsilon) - \lambda^L_{ijt}(v, A_{t-\tau}, \varepsilon) > 0$ and so $u_{jt}(v, A_{t-\tau}, \varepsilon) = 1$ by complementary slackness. If $A_{t-\tau}\varepsilon f(v) - \sum_i \chi_{ijt}v_i < 0$ or, equivalently, $\varepsilon < \frac{\sum_i \chi_{ijt}v_i}{A_{t-\tau}f(v)} = \underline{\varepsilon}(v, A_{t-\tau}; \chi_{jt})$, then $\lambda^U_{ijt}(v, A_{t-\tau}, \varepsilon) - \lambda^L_{ijt}(v, A_{t-\tau}, \varepsilon) < 0$ by (A5), which implies that $u_{jt}(v, A_{t-\tau}, \varepsilon) = 0$ by complementary slackness. So, the utilization decision has the form: fully utilize if $\varepsilon > \underline{\varepsilon}(v, A_{t-\tau}; \chi_{jt})$ and do not utilize at all if $\varepsilon < \underline{\varepsilon}(v, A_{t-\tau}; \chi_{jt})$.[19]

Note that the solution to the static utilization problem (12) from the main text coincides with this solution from the dynamic problem if we set the static multipliers $\widehat{\chi}_{ijt} = \chi_{ijt}$.

Hiring Block. The first-order condition for employment n_{ijt+1} is

$$\nu_{ijt+1} = \chi_{ijt+1} + M_{ijt+1}\frac{U_{nit+1}}{U_{cit+1}}\frac{1}{\omega}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}-1}\frac{1}{n_{it+1}} + Q_{t+1,t+2}(1-\sigma)\nu_{ijt+2}, \tag{A6}$$

which uses the fact that $\frac{\partial}{\partial n_{ijt+1}}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}} = \frac{1}{\omega}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}-1}\frac{1}{n_{it+1}}$. This equation identifies the multiplier ν_{ijt+1} as the present value of a marginal worker to the firm, taking into account both their marginal product χ_{ijt+1} and the monopsony distortion $M_{ijt+1}\frac{u_{nit+1}}{u_{cit+1}}\frac{1}{\omega}\left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}-1}\frac{1}{n_{it+1}}$.

The first-order condition for vacancy posting a_{ijt} is

$$\begin{aligned} & -Q_{0,t}\kappa_{it} - Q_{0,t+1}\lambda_f(\theta_{ijt})W_{ijt+1} + Q_{0,t+1}\lambda_f(\theta_{ijt})\nu_{ijt+1} = 0 \\ \Longrightarrow & \frac{\kappa_i}{\lambda_f(\theta_{ijt})} = Q_{t,t+1}\left(\nu_{ijt+1} - W_{ijt+1}\right). \end{aligned} \tag{A7}$$

[19]In the knife-edge case where $A_{t-\tau}\varepsilon f(v) - \sum_i \chi_{ijt}v_i = 0$, the firm is indifferent over any $u_{jt}(v, A_{t-\tau}, \varepsilon) \in [0, 1]$.

The first-order condition for market tightness θ_{ijt} is

$$
\begin{aligned}
&- Q_{0,t+1}\lambda_f'(\theta_{ijt})a_{ijt}W_{ijt+1} + Q_{0,t+1}\nu_{ijt+1}\lambda_f'(\theta_{ijt})a_{ijt} + Q_{0,t+1}\gamma_{ijt+1}\frac{\mathcal{W}_{it}}{Q_{t,t+1}\lambda_w(\theta_{ijt})^2}\lambda_w'(\theta_{ijt}) = 0 \\
&\implies W_{ijt+1} = \nu_{ijt+1} + \frac{\gamma_{ijt+1}}{a_{ijt}}\frac{\mathcal{W}_{it}}{Q_{t,t+1}\lambda_w(\theta_{ijt})^2}\frac{\lambda_w'(\theta_{ijt})}{\lambda_f'(\theta_{ijt})}. \qquad \text{(A8)}
\end{aligned}
$$

The first-order condition for wages W_{ijt+1} is

$$
\gamma_{ijt+1} = \lambda_f(\theta_{ijt})a_{ijt}, \qquad \text{(A9)}
$$

Plugging in this expression for γ_{ijt+1} into the first-order condition for market tightness (A8) gives

$$
\begin{aligned}
W_{ijt+1} &= \nu_{ijt+1} + \frac{\lambda_f(\theta_{ijt})}{\lambda_w(\theta_{ijt})}\frac{\mathcal{W}_{it}}{Q_{t,t+1}\lambda_w(\theta_{ijt})}\frac{\lambda_w'(\theta_{ijt})}{\lambda_f'(\theta_{ijt})} \\
&= \nu_{ijt+1} + \frac{\lambda_f(\theta_{ijt})}{\lambda_w(\theta_{ijt})}\left(W_{ijt+1} + V_{ijt+1}\right)\frac{\lambda_w'(\theta_{ijt})}{\lambda_f'(\theta_{ijt})} \\
&= \nu_{ijt+1} - \frac{1-\eta}{\eta}(W_{ijt+1} + V_{ijt+1}),
\end{aligned}
$$

where in the second line we used $\frac{\mathcal{W}_{it}}{Q_{t,t+1}\lambda_w(\theta_{ijt})} = W_{ijt+1} + V_{ijt+1}$ and in the third line we used $\frac{\lambda_f(\theta_{ijt})}{\lambda_w(\theta_{ijt})}\frac{\lambda_w'(\theta_{ijt})}{\lambda_f'(\theta_{ijt})} = -\frac{1-\eta}{\eta}$. Solving for the present value of wages W_{ijt+1} yields

$$
W_{ijt+1} = \eta\nu_{ijt+1} - (1-\eta)V_{it+1}, \qquad \text{(A10)}
$$

which is the expression in the main text.

Investment Block. We now characterize the investment stage and, in the process, prove Proposition 2 from the main text. First, consider capital installed in period t — and therefore with vintage productivity A_t — in use in period $t+\tau$, $K_{jt+\tau}(v, A_t)$. The first-order condition for this variable is

$$
\begin{aligned}
&Q_{0,t+\tau}\int u_{jt+\tau}(v, A_t, \varepsilon)A_t\varepsilon f(v)\pi(\varepsilon)d\varepsilon \\
&\quad - Q_{0,t+\tau}\sum\nolimits_i v_i\int \lambda_{ijt+\tau}(v, A_t, \varepsilon)u_{jt+\tau}(v, \varepsilon, A)\pi(\varepsilon)d\varepsilon - Q_{0,t+\tau}q_{jt,t+\tau}(v) = 0 \\
&\implies q_{jt,t+\tau}(v) = \int_{\underline{\varepsilon}(v, A_t; \chi_{jt+\tau})}\left(A_t\varepsilon f(v) - \sum_i \chi_{ijt+\tau}v_i\right)\pi(\varepsilon)d\varepsilon, \qquad \text{(A11)}
\end{aligned}
$$

where the second line uses the facts that $\lambda_{ijt+\tau}(v, A_t, \varepsilon) = \chi_{ijt+\tau}$ and that $u_{jt}(v, A, \varepsilon) = 1$ for $\varepsilon \geq \underline{\varepsilon}(v, A_t; \chi_{jt+\tau})$ and 0 otherwise. The first-order condition for investment $X_{jt}(v)$ is

$$\mu_{jt}(v) = 1 - \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1} q_{jt,t+\tau}(v)$$
$$\implies \mu_{jt}(v) = 1 - \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1} \int_{\underline{\varepsilon}(v,A_t;\chi_{jt+\tau})} \left(A_t \varepsilon f(v) - \sum_i \chi_{ijt+\tau} v_i \right) \pi(\varepsilon) d\varepsilon, \tag{A12}$$

where the second line uses the expression for $q_{jt,t+\tau}(v)$ from (A11).

Optimal Capital Type. As in the main text, we use (A12) to show that there is unique type of capital in which firms invest in period t. To do so, first note that since $\mu_{jt}(v)$ is a Lagrange multiplier, it has a minimum value at zero. Furthermore, if the RHS of (A12) is single-peaked, then there is a unique value of v — call it v_{jt} — which achieves that minimum. Therefore, we have $\mu_{jt}(v) > 0$ for all $v \neq v_{jt}$, which by complementary slackness implies that $X_{jt}(v) = 0$ for all $v_{jt} \neq 0$. For the optimal type v_{jt}, we have that (A12) holds with $\mu_{jt}(v_{jt}) = 0$. Hence, under the optimal choice of capital type, the first-order condition for investment becomes

$$1 = \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1} \int_{\underline{\varepsilon}(v,A_t;\chi_{jt+\tau})} \left(A_t \varepsilon f(v_{jt}) - \sum_i \chi_{ijt+\tau} v_{ijt} \right) \pi(\varepsilon) d\varepsilon.$$

Since this optimal type v_{jt} is the minimizer of the RHS of (A12), it equivalently solves the maximization problem

$$v_{jt} = \operatorname*{argmax}_v \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1} \int_{\underline{\varepsilon}(v,A_t;\chi_{jt+\tau})} \left(A_t \varepsilon f(v_{jt}) - \sum_i \chi_{ijt+\tau} v_{ijt} \right) \pi(\varepsilon) d\varepsilon.$$

The first-order condition for v_{ijt} in this problem is

$$0 = \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1} \left(A_t \frac{\partial f(v)}{\partial v_i} \int_{\underline{\varepsilon}(v,A_t;\chi_{jt+\tau})}^{\infty} \varepsilon \pi(\varepsilon) d\varepsilon - \chi_{ijt+\tau} \int_{\underline{\varepsilon}(v,A_t;\chi_{jt+\tau})}^{\infty} \pi(\varepsilon) d\varepsilon \right)$$
$$- \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1} \frac{\partial \underline{\varepsilon}(v, A_t; \chi_{jt+\tau})}{\partial v_i} \pi(\underline{\varepsilon}(v, A_t; \chi_{jt+\tau})) \left(A_t \underline{\varepsilon}(v, A_t; \chi_{jt+\tau}) f(v) - \sum_i \chi_{ijt+\tau} v_i \right).$$

The top line is the derivatives holding fixed $\underline{\varepsilon}(v, A_t; \chi_{jt+\tau})$ and the second line is the derivatives with respect to $\underline{\varepsilon}(v, A_t; \chi_{jt+\tau})$, using the fundamental theorem of calculus.[20] However, each term in the summand in this second line is zero at the optimum. To see this, plug in $\underline{\varepsilon}(v, A_t; \chi_{jt+\tau}) = \frac{\sum_i \chi_{ijt+\tau} v_i}{A_t f(v)}$

[20] That is, $\int_a^b F'(x)dx = F(b) - F(a)$ so $\frac{\partial}{\partial a} \int_a^b F''(a)$.

to see that the term becomes 0 for each future period τ. In the language of Gilchrist and Williams (2000), the marginal unit of capital is earns zero quasi-rents in each period.

To summarize, the optimal investment policy is characterized by two conditions:

$$0 = \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1}\left(A_t f_i(v_t)\int_{\underline{\varepsilon}(v,A_t;\chi_{jt+\tau})}^{\infty}\varepsilon\pi(\varepsilon)d\varepsilon - \chi_{ijt+\tau}\int_{\underline{\varepsilon}(v,A_t;\chi_{jt+\tau})}^{\infty}\pi(\varepsilon)d\varepsilon\right)\ \forall i \tag{A13}$$

$$1 = \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1}\int_{\underline{\varepsilon}(v,A_t;\chi_{jt+\tau})}\left(A_t\varepsilon f(v_{jt}) - \sum_i \chi_{ijt+\tau}v_{ijt}\right)\pi(\varepsilon)d\varepsilon, \tag{A14}$$

where $f_i(v_t) = \frac{\partial f(v_{jt})}{\partial v_i}$. The first equation is the first-order condition for the optimal type of capital and the second equation is the first-order condition for investment in the optimal type.

Going forward, it will be useful to simplify notation. First, we let $\underline{\varepsilon}_{t,t+\tau} = \underline{\varepsilon}(v, A_t; \chi_{jt+\tau})$ denote the utilization cutoff for capital installed in period t to be used in period $t+\tau$. Second, following the main text, we define $\Pi^u(\underline{\varepsilon}_{t,t+\tau}) = \int_{\underline{\varepsilon}_{t,t+\tau}} \pi(\varepsilon)d\varepsilon$ and $\Pi^p(\underline{\varepsilon}_{t,t+\tau}) = \int_{\underline{\varepsilon}_{t,t+\tau}} \varepsilon\pi(\varepsilon)d\varepsilon$. With this notation, we can write (A13) and (A14) more compactly as

$$0 = \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1}\left(\Pi^p(\underline{\varepsilon}_{t,t+\tau})A_t f_i(v_t) - \Pi^u(\underline{\varepsilon}_{t,t+\tau})\chi_{ijt+\tau}\right)$$

$$1 = \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1}\left(\Pi^p(\underline{\varepsilon}_{t,t+\tau})A_t\varepsilon f(v_{jt}) - \Pi^u(\underline{\varepsilon}_{t,t+\tau})\sum_i \chi_{ijt+\tau}v_{ijt}\right).$$

Summary of Equilibrium Conditions. We now collect all of the equilibrium conditions of the model, including the optimality conditions from the households and the firms. Since we study a symmetric equilibrium, we drop the j notation for individual firms going forward.

$$Q_{t,t+1} = \beta\frac{U_{cit+1}}{U_{cit}} \tag{A15}$$

$$V_{it+1} = \frac{U_{nit+1}}{U_{cit+1}} + Q_{t+1,t+2}(1-\sigma)V_{it+2} \tag{A16}$$

$$-\frac{U_{sit}}{U_{cit}} = Q_{t,t+1}\lambda_w(\theta_{it})(W_{it+1} + V_{it+1}) \tag{A17}$$

$$\sum\nolimits_{t=0}^{\infty} Q_{0,t}c_{it} = \zeta_i\mathbb{P} + \mathbb{I}_i + \sum\nolimits_{t=1}^{\infty} Q_{0,t}\lambda_w(\theta_{it-1})s_{it-1}W_{it} \tag{A18}$$

$$\underline{\varepsilon}_{t,t+\tau} = \frac{\sum_i \chi_{it+\tau}v_{it}}{A_t f(v_t)} \tag{A19}$$

$$\nu_{it+1} = \chi_{it+1} + M_{it+1}\frac{U_{nit+1}}{U_{cit+1}}\frac{1}{\omega n_{it+1}} + Q_{t+1,t+2}(1-\sigma)\nu_{it+2} \tag{A20}$$

$$\frac{\kappa_{it}}{\lambda_f(\theta_{it})} = Q_{t,t+1}(\nu_{it+1} - W_{it+1}) \tag{A21}$$

$$W_{it+1} = \eta\nu_{it+1} - (1-\eta)V_{it+1} \text{ and } \gamma_{it+1} = \lambda_f(\theta_{it})a_{it} \tag{A22}$$

$$n_{it+1} = (1-\sigma)n_{it} + \lambda_f(\theta_{it})a_{it} \tag{A23}$$

$$M_{it+1} = \gamma_{it+1} + (1-\sigma)M_{it} \tag{A24}$$

$$\theta_{it} = a_{it}/s_{it} \tag{A25}$$

$$1 = \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1}\left(\Pi^p(\underline{\varepsilon}_{t,t+\tau})A_t\varepsilon f(v_t) - \Pi^u(\underline{\varepsilon}_{t,t+\tau})\sum_i \chi_{ijt+\tau}v_{it}\right) \tag{A26}$$

$$0 = \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1}\left(\Pi^p(\underline{\varepsilon}_{t,t+\tau})A_t f_i(v_t) - \Pi^u(\underline{\varepsilon}_{t,t+\tau})\chi_{ijt+\tau}\right) \tag{A27}$$

$$Y_t = \sum_{\tau=1}^{\infty} \Pi^p(\underline{\varepsilon}_{t-\tau,t})A_{t-\tau}f(v_{t-\tau})(1-\delta)^{\tau-1}X_{t-\tau} \tag{A28}$$

$$Y_t = \sum_i \mu_i c_{it} + X_t + \sum_i \mu_i \kappa_{it} a_{it}. \tag{A29}$$

$$\mu_i n_{it} = \sum_{\tau=1}^{\infty} \Pi^u(\underline{\varepsilon}_{t-\tau,t})v_{it-\tau}(1-\delta)^{\tau-1}X_{t-\tau}. \tag{A30}$$

Equation (A28) simplifies the expression for aggregate output from the main text using our results about optimal investment and the definition of $\Pi^p(\underline{\varepsilon}_{t-\tau,t})$. In particular, aggregate output equals output produced by each vintage of capital, $\Pi^p(\underline{\varepsilon}_{t-\tau,t})A_{t-\tau}f(v_{t-\tau})$, times the amount of capital of that vintage which is remaining, $(1-\delta)^{\tau-1}X_{t-\tau}$. Equation (A29) is market clearing for aggregate output. Finally, equation (A30) equates aggregate employment of type-i worker in period t to the amount of that type of labor assigned to each vintage.

A.3 Detrending

Due to capital-embodied technological progress in vintage productivity A_t, the equilibrium allocation is not stationary over time. In this subsection, we describe how to detrend the model into stationary form, which will be useful for numerically solving the model. As in the main text, we assume that $\kappa_{it} = (1+g)^t\kappa_i$ so that vacancy-posting costs grow along with the economy.

A balanced growth path will have the following properties:

(i) The following variables grow along with the economy: $c_{it}, W_{it+1}, V_{it+1}, \chi_{it}, \nu_{it}, Y_t, X_t$. Let tildes denote detrended variables, e.g. $\widetilde{c}_{it} = c_{it}/(1+g)^t$.

(ii) The following variables shrink over time: v_{it}. Let $\widetilde{v}_{it} = v_{it}(1+g)^t$.

(iii) The following variables are stationary: $s_{it}, n_{it+1}, a_{it}, \theta_{it}, \gamma_{it+1}, M_{it+1}, Q_{t,t+1}$.

We now go through each of the equilibrium conditions and replace the original non-stationary variables with their stationary version.

A.3.1 Household

Using our functional form for the utility function, namely

$$U_t(c_{it}, s_{it}, n_{it}) = \log\left(c_{it} - (1+g)^t v(n_{it}) - (1+g)^t h(s_{it})\right),$$

the ratio of date-0 output prices (A15) becomes

$$\begin{aligned}
Q_{t,t+1} &= \beta \frac{c_{it} - (1+g)^t v(n_{it}) - (1+g)^t h(s_{it})}{c_{it+1} - (1+g)^{t+1} v(n_{it+1}) - (1+g)^{t+1} h(s_{it+1})} \\
&= \beta \frac{(1+g)^t \widetilde{c}_{it} - (1+g)^t v(n_{it}) - (1+g)^t h(s_{it})}{(1+g)^{t+1} \widetilde{c}_{it+1} - (1+g)^{t+1} v(n_{it+1}) - (1+g)^{t+1} h(s_{it+1})} \\
&= \frac{\beta}{1+g} \frac{\widetilde{c}_{it} - v(n_{it}) - h(s_{it})}{\widetilde{c}_{it+1} - v(n_{it+1}) - h(s_{it+1})}.
\end{aligned}$$

The equation defining the disutility of labor (A16) becomes

$$\begin{aligned}
\widetilde{V}_{it+1}(1+g)^{t+1} &= -(1+g)^{t+1} v'(n_{it+1}) + Q_{t+1,t+2}(1-\sigma)\widetilde{V}_{it+2}(1+g)^{t+2} \\
\implies \widetilde{V}_{it+1} &= -v'(n_{it+1}) + Q_{t+1,t+2}(1+g)(1-\sigma)\widetilde{V}_{it+2}.
\end{aligned}$$

The first-order condition for optimal search effort (A17) becomes

$$\begin{aligned}
(1+g)^t h'(s_{it}) &= Q_{t,t+1} \lambda_w(\theta_{it})(1+g)^{t+1} \left(\widetilde{W}_{it+1} + \widetilde{V}_{it+1}\right) \\
\implies h'(s_{it}) &= Q_{t,t+1}(1+g)\lambda_w(\theta_{it}) \left(\widetilde{W}_{it+1} + \widetilde{V}_{it+1}\right).
\end{aligned}$$

The budget constraint (A18) becomes

$$\sum_{t=0}^{\infty} Q_{0,t}(1+g)^t \widetilde{c}_{it} = \zeta_i \mathbb{P} + \mathbb{I}_i + \sum_{t=0}^{\infty} Q_{0,t+1}(1+g)^{t+1} \lambda_w(\theta_{it}) s_{it} \widetilde{W}_{it+1}.$$

A.3.2 Firms

We go through the utilization block, the hiring block, and the investment block.

Utilization Block. The production stage is summarized by the expression for the productivity threshold $\underline{\varepsilon}_{t,t+\tau} = \sum_i \chi_{it+\tau} v_{it} / A_t f(v_t)$. Recalling that v_{it} *shrinks* are rate g so that $\widetilde{v}_{it} = (1+g)^t v_{it}$, in detrended terms, the numerator of the threshold is

$$\sum_i \chi_{it+\tau} v_{it} = \sum_i \left[(1+g)^{t+\tau} \frac{\chi_{it+\tau}}{(1+g)^{t+\tau}}\right] \left[\frac{1}{(1+g)^t} v_{it}(1+g)^t\right] = (1+g)^\tau \sum_i \widetilde{\chi}_{it+\tau} \widetilde{v}_{it}.$$

Recalling that $F(K, N_1, \ldots, N_I) = K^\alpha G(N_1, \ldots, N_I)^{1-\alpha}$ and both F and G are CRS gives

$$f(v_1, \ldots, v_I) = F(1, \frac{N_1}{K}, \ldots, \frac{N_I}{K}) = 1^\alpha G(\frac{N_1}{K}, \ldots, \frac{N_I}{K})^{1-\alpha} = G(v_1, \ldots, v_I)^{1-\alpha}.$$

Since $f(v_t) = G(v)^{1-\alpha}$, the denominator of the cutoff is

$$\begin{aligned} A_t f(v_t) &= ((1+g)^{1-\alpha})^t G(v_t)^{1-\alpha} = ((1+g)^{1-\alpha})^t G(\frac{\widetilde{v}_t}{(1+g)^t})^{1-\alpha} \\ &= ((1+g)^{1-\alpha})^t \left[\frac{1}{(1+g)^t} G(\widetilde{v}_t)\right]^{1-\alpha} = f(\widetilde{v}_t), \end{aligned}$$

where the first equality uses that $A_t = ((1+g)^{1-\alpha})^t$ and $f(v_t) = G(v_t)^{1-\alpha}$, and the third equality uses the fact that $G(v_t)$ is constant returns to scale. Putting these results about the numerator and denominator together, we get

$$\underline{\varepsilon}_{t,t+\tau} = (1+g)^\tau \frac{\sum_i \widetilde{\chi}_{it+\tau} \widetilde{v}_{it}}{f(\widetilde{v}_t)}.$$

Hiring Block. The expression for the present value of a worker (A20) becomes

$$\begin{aligned} &\widetilde{\nu}_{it+1}(1+g)^{t+1} = \widetilde{\chi}_{it+1}(1+g)^{t+1} + M_{it+1} v'(n_{it+1})(1+g)^{t+1} \frac{1}{\omega n_{it+1}} + Q_{t+1,t+2}(1-\sigma)\widetilde{\nu}_{it+2}(1+g)^{t+2} \\ &\implies \widetilde{\nu}_{it+1} = \widetilde{\chi}_{it+1} + M_{it+1} v'(n_{it+1}) \frac{1}{\omega n_{it+1}} + Q_{t+1,t+2}(1+g)(1-\sigma)\widetilde{\nu}_{it+2}. \end{aligned}$$

The first-order condition for optimal vacancy-posting (A21) becomes

$$\frac{\kappa_i (1+g)^t}{\lambda_f(\theta_{it})} = Q_{t,t+1}(1+g)^{t+1} \left(\widetilde{\nu}_{it+1} - \widetilde{W}_{it+1}\right) \implies \frac{\kappa_i}{\lambda_f(\theta_{it})} = Q_{t,t+1}(1+g) \left(\widetilde{\nu}_{it+1} - \widetilde{W}_{it+1}\right).$$

The-condition for wages (A22) becomes

$$\widetilde{W}_{it+1}(1+g)^{t+1} = \eta \widetilde{\nu}_{it+1}(1+g)^{t+1} - (1-\eta)\widetilde{V}_{it+1}(1+g)^{t+1} \implies \widetilde{W}_{it+1} = \eta \widetilde{\nu}_{it+1} - (1-\eta)\widetilde{V}_{it+1}$$

and $\gamma_{it+1} = \lambda_f(\theta_{it}) a_{it}$ is already stationary. The evolution of employment (A23), the definition of the quasi-multipliers (A24), and the definition of market tightness (A25) are already stationary.

Investment Block. First, consider the condition that equates marginal cost with marginal benefit of new capital, (A26). As argued with the productivity cutoff above, the terms with $A_t f(v_t) = f(\widetilde{v}_t)$, and the terms with $\sum_i \chi_{it+\tau} v_{it} = (1+g)^\tau \sum_i \widetilde{\chi}_{it+\tau} \widetilde{v}_{it}$. Thus

$$1 = \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1} \left(\Pi^p(\underline{\varepsilon}_{t,t+\tau}) f(\widetilde{v}_t) - \Pi^u(\underline{\varepsilon}_{t,t+\tau}) \sum_i \widetilde{\chi}_{it+\tau} \widetilde{v}_{it} \right).$$

Next consider the first-order condition for the optimal type of capital, (A27). Note that

$$\begin{aligned} f_i(v_t) &= \frac{\partial}{\partial v_i} f(v_t) = \frac{\partial}{\partial v_i} [G(v_t)]^{1-\alpha} \\ &= (1-\alpha) G(v_t)^{-\alpha} G_i(v_t) \\ &= (1-\alpha) G(\frac{\widetilde{v}_t}{(1+g)^t})^{-\alpha} G_i(\frac{\widetilde{v}_t}{(1+g)^t}) \\ &= ((1+g)^\alpha)^t (1-\alpha) G(\widetilde{v}_t)^{-\alpha} G_i(\widetilde{v}_t) = ((1+g)^\alpha)^t f_i(\widetilde{v}_t), \end{aligned}$$

where the fourth line uses the fact that $G(v_t)$ is homogenous of degree one (and therefore its derivatives are homogenous of degree zero). Therefore, we have that the terms $A_t f_i(v_t) = ((1+g)^{1-\alpha})^t ((1+g)^\alpha)^t f_i(\widetilde{v}_t) = (1+g)^t f_i(\widetilde{v}_t)$. Plugging these into the first-order condition for the optimal type of capital (A27) gives

$$\begin{aligned} 0 &= \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1} \left(\Pi^p(\underline{\varepsilon}_{t,t+\tau})(1+g)^t f_i(\widetilde{v}_t) - \Pi^u(\underline{\varepsilon}_{t,t+\tau})(1+g)^{t+\tau} \widetilde{\chi}_{it+\tau} \right) \\ \implies 0 &= \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1} \left(\Pi^p(\underline{\varepsilon}_{t,t+\tau}) f_i(\widetilde{v}_t) - \Pi^u(\underline{\varepsilon}_{t,t+\tau})(1+g)^\tau \widetilde{\chi}_{it+\tau} \right). \end{aligned}$$

A.3.3 Aggregate Conditions

We start with the definition of aggregate output (A28). As we argued above, the terms $A_{t-\tau} f(v_{t-\tau}) = f(\widetilde{v}_{t-\tau})$. Using this result, the equation becomes

$$\begin{aligned} \widetilde{Y}_t (1+g)^t &= \sum_{\tau=1}^{\infty} \Pi^p(\underline{\varepsilon}_{t-\tau,t}) f(\widetilde{v}_{t-\tau})(1-\delta)^{\tau-1}(1+g)^{t-\tau} \widetilde{X}_{t-\tau} \\ \implies \widetilde{Y}_t &= \sum_{\tau=1}^{\infty} \Pi^p(\underline{\varepsilon}_{t-\tau,t}) f(\widetilde{v}_{t-\tau})(1-\delta)^{\tau-1}(1+g)^{-\tau} \widetilde{X}_{t-\tau} \\ \implies \widetilde{Y}_t &= \sum_{\tau=1}^{\infty} \Pi^p(\underline{\varepsilon}_{t-\tau,t}) f(\widetilde{v}_{t-\tau}) \left(\frac{1-\delta}{1+g} \right)^{\tau-1} \frac{\widetilde{X}_{t-\tau}}{1+g}. \end{aligned}$$

For aggregate employment (A30) , note that $v_{it-\tau}X_{t-\tau} = \frac{\widetilde{v}_{it-\tau}}{(1+g)^{t-\tau}}\widetilde{X}_{t-\tau}(1+g)^{t-\tau} = \widetilde{v}_{it-\tau}\widetilde{x}_{t-\tau}$ is already stationary. So we have

$$\mu_i n_{it} = \sum_{\tau=1}^{\infty} \Pi^u(\underline{\varepsilon}_{t-\tau,t})\widetilde{v}_{it-\tau}(1-\delta)^{\tau-1}\widetilde{X}_{t-\tau}.$$

Finally, the output market clearing condition (A29) is

$$\widetilde{Y}_t(1+g)^t = \sum_i \mu_i \widetilde{c}_{it}(1+g)^t + \widetilde{X}_t(1+g)^t + \sum_i \mu_i\kappa_i(1+g)^t a_{it} \implies \widetilde{Y}_t = \sum_i \mu_i\widetilde{c}_{it} + \widetilde{X}_t + \sum_i \mu_i\kappa_i a_{it}.$$

A.3.4 Summary of Detrended Equilibrium Conditions

We now collect all of these detrended equilibrium conditions. In our quantitative work, we compute the transition paths by solving this large nonlinear system,

$$Q_{t,t+1} = \frac{\beta}{1+g}\frac{\widetilde{c}_{it} - v(n_{it}) - h(s_{it})}{\widetilde{c}_{it+1} - v(n_{it+1}) - h(s_{it+1})} \tag{A31}$$

$$\widetilde{V}_{it+1} = -v'(n_{it+1}) + Q_{t+1,t+2}(1+g)(1-\sigma)\widetilde{V}_{it+2} \tag{A32}$$

$$h'(s_{it}) = Q_{t,t+1}(1+g)\lambda_w(\theta_{it})\left(\widetilde{W}_{it+1} + \widetilde{V}_{it+1}\right) \tag{A33}$$

$$\sum_{t=0}^{\infty} Q_{0,t}(1+g)^t\widetilde{c}_{it} = \zeta_i\mathbb{P} + \mathbb{I}_i + \sum_{t=0}^{\infty} Q_{0,t+1}(1+g)^{t+1}\lambda_w(\theta_{it})s_{it}\widetilde{W}_{it+1} \tag{A34}$$

$$\underline{\varepsilon}_{t,t+\tau} = (1+g)^{\tau}\frac{\sum_i \widetilde{\chi}_{it+\tau}\widetilde{v}_{it}}{f(\widetilde{v}_t)} \tag{A35}$$

$$\widetilde{\nu}_{it+1} = \widetilde{\chi}_{it+1} + M_{it+1}v'(n_{it+1})\frac{1}{\omega n_{it+1}} + Q_{t+1,t+2}(1+g)(1-\sigma)\widetilde{\nu}_{it+2} \tag{A36}$$

$$\frac{\kappa_i}{\lambda_f(\theta_{it})} = Q_{t,t+1}(1+g)\left(\widetilde{\nu}_{it+1} - \widetilde{W}_{it+1}\right) \tag{A37}$$

$$\widetilde{W}_{it+1} = \eta\widetilde{\nu}_{it+1} - (1-\eta)\widetilde{V}_{it+1} \text{ and } \gamma_{it+1} = \lambda_f(\theta_{it})a_{it} \tag{A38}$$

$$\mathbb{P} = \sum_{t=0}^{\infty} Q_{0,t}(1+g)^t\big[\widetilde{Y}_t - \widetilde{X}_t - \sum_i \mu_i\left(\kappa_i a_{it} + \lambda_f(\theta_{it})a_{it}Q_{t,t+1}(1+g)\widetilde{W}_{it+1}\right)\big] - \sum_i \mu_i\mathbb{I}_i \tag{A39}$$

$$M_{it+1} = (1-\sigma)M_{it} + \gamma_{it+1} \tag{A40}$$

$$1 = \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1}\left(\Pi^p(\underline{\varepsilon}_{t,t+\tau})f(\widetilde{v}_t) - \Pi^u(\underline{\varepsilon}_{t,t+\tau})\sum_i \widetilde{\chi}_{it+\tau}\widetilde{v}_{it}\right) \tag{A41}$$

$$0 = \sum_{\tau=1}^{\infty} Q_{t,t+\tau}(1-\delta)^{\tau-1}\left(\Pi^p(\underline{\varepsilon}_{t,t+\tau})f_i(\widetilde{v}_t) - \Pi^u(\underline{\varepsilon}_{t,t+\tau})(1+g)^{\tau}\widetilde{\chi}_{it+\tau}\right) \tag{A42}$$

$$n_{it+1} = (1-\sigma)n_{it} + \lambda_w(\theta_{it})s_{it} \tag{A43}$$

$$\theta_{it} = \frac{a_{it}}{s_{it}} \tag{A44}$$

$$\widetilde{Y}_t = \sum_{\tau=1}^{\infty} \Pi^p(\underline{\varepsilon}_{t-\tau,t}) f(\widetilde{v}_{t-\tau}) \left(\frac{1-\delta}{1+g}\right)^{\tau-1} \frac{\widetilde{X}_{t-\tau}}{1+g} \tag{A45}$$

$$\mu_i n_{it} = \sum_{\tau=1}^{\infty} \Pi^u(\underline{\varepsilon}_{t-\tau,t}) \widetilde{v}_{it-\tau}(1-\delta)^{\tau-1}\widetilde{X}_{t-\tau} \tag{A46}$$

$$\widetilde{Y}_t = \sum_i \mu_i \widetilde{c}_{it} + \widetilde{X}_t + \sum_i \mu_i \kappa_i a_{it}. \tag{A47}$$

A.4 Balanced Growth Path

The balanced growth path is simply the steady state of the detrended system. In this subsection, we collect the conditions that define the BGP and then simplify them to prove Lemma 3 and Lemma 4 from the main text. We also derive the formula for the wage markdown (26) from the main text.

Summary of BGP Conditions. Collecting the summary of detrended equilibrium conditions from above and imposing a steady state, we get the system

$$Q_{t,t+1} \equiv \widetilde{\beta} = \frac{\beta}{1+g} \tag{A48}$$

$$h'(s_i) = \frac{\beta}{1-\beta(1-\sigma)} \lambda_w(\theta_i) \left[\widetilde{w}_i - v'(n_i)\right] \tag{A49}$$

$$\sum_{t=0}^{\infty} \beta^t \widetilde{c}_i = \zeta_i \mathbb{P} + \mathbb{I}_i + \sum_{t=0}^{\infty} \beta^{t+1} \lambda_w(\theta_i) s_i \widetilde{W}_i \tag{A50}$$

$$\widetilde{\nu}_i = \frac{1}{1-\beta(1-\sigma)} \left[\widetilde{\chi}_i - \frac{1}{\omega} v'(n_i)\right] \tag{A51}$$

$$\widetilde{W}_i = \eta \widetilde{\nu}_i + (1-\eta) \frac{v'(n_i)}{1-\beta(1-\sigma)} \tag{A52}$$

$$\frac{\kappa_i}{\lambda_f(\theta_i)} = \beta(\widetilde{\nu}_i - \widetilde{W}_i) \tag{A53}$$

$$\underline{\varepsilon}_\tau = (1+g)^\tau \frac{\sum_i \widetilde{\chi}_i \widetilde{v}_i}{f(\widetilde{v})} \tag{A54}$$

$$0 = f_i(v) \sum_{\tau=1}^{\infty} \widetilde{\beta}^\tau (1-\delta)^{\tau-1} \Pi^p(\underline{\varepsilon}_\tau) - \widetilde{\chi}_i \sum_{\tau=1}^{\infty} \widetilde{\beta}^\tau (1-\delta)^{\tau-1} (1+g)^\tau \Pi^u(\underline{\varepsilon}_\tau) \tag{A55}$$

$$1 = f(\widetilde{v}) \sum_{\tau=1}^{\infty} \widetilde{\beta}^\tau (1-\delta)^{\tau-1} \Pi^p(\underline{\varepsilon}_\tau) - \sum_{\tau=1}^{\infty} \widetilde{\beta}^\tau (1-\delta)^{\tau-1} \Pi^u(\underline{\varepsilon}_\tau) \sum_i \widetilde{\chi}_i \widetilde{v}_i \tag{A56}$$

$$\widetilde{Y} = \sum_{\tau=1}^{\infty} \Pi^p(\underline{\varepsilon}_\tau) f(\widetilde{v}) \left(\frac{1-\delta}{1+g}\right)^{\tau-1} \frac{\widetilde{X}}{1+g} \tag{A57}$$

$$\mu_i n_i = \sum_{\tau=1}^{\infty} \Pi^u(\underline{\varepsilon}_\tau) \widetilde{v}_i (1-\delta)^{\tau-1} \widetilde{X} \tag{A58}$$

$$\widetilde{Y} = \sum_i \mu_i \widetilde{c}_i + \widetilde{X} + \sum_i \mu_i \kappa_i a_i \tag{A59}$$

$$\theta_i = a_i / s_i \tag{A60}$$

$$\mathbb{P} = \frac{\widetilde{Y} - \widetilde{X} - \sum_i \mu_i \left(\kappa_i a_i + \beta \sigma n_i W_i\right)}{1-\beta} - \sum_i \mu_i \mathbb{I}_i \tag{A61}$$

$$\sigma n_i = \lambda_w(\theta_i) s_i. \tag{A62}$$

Note that the BGP of the putty-clay model is not the same as the model with standard capital. The reason is that the labor intensities v in the putty-clay model are chosen before the realization of the capital quality shock ε in the putty-clay model, but after the realization of ε in the model with standard capital. Therefore, firms in the standard model will implicitly assign more workers to high-ε machines, which is not possible in the putty-clay model. If we allowed firms to choose the labor intensities v after the realization of capital quality shocks ε in the putty-clay model, then we would not have an active utilization margin, which is a key feature of our analysis.

Reduced System Characterizing the BGP. Under our preference specification, the labor market equilibrium and investment decisions are separable from the consumption allocation. This property allows us to significantly reduce the number of equations which characterize the BGP in Lemma 8 below (a version of Lemma 3 from the main text). In Appendix B, we use this Lemma to show that a combination of type-specific minimum wages and vacancy-posting subsidies can achieve the competitive allocation (Proposition 6 from the main text).

Lemma 8. *Along the balanced growth path, the labor allocations and wages are determined by the following equations:*

(i) optimal cut-off for idiosyncratic productivity of capital

$$\underline{\varepsilon}_1 = (1+g)(1-\alpha) m(\underline{\varepsilon}_1), \tag{A63}$$

where $m(\underline{\varepsilon}_1)$ *is defined by*

$$m(\underline{\varepsilon}_1) = \frac{\sum_{\tau=1}^{\infty} \widetilde{\beta}^{\tau}(1-\delta)^{\tau-1}\Pi^p((1+g)^{\tau-1}\underline{\varepsilon}_1)}{\sum_{\tau=1}^{\infty} \widetilde{\beta}^{\tau}(1-\delta)^{\tau-1}(1+g)^{\tau}\Pi^u((1+g)^{\tau-1}\underline{\varepsilon}_1)}; \tag{A64}$$

(ii) the marginal unit of capital earns zero profit

$$1 = \alpha\left[\sum_{\tau=1}^{\infty} \widetilde{\beta}^{\tau}(1-\delta)^{\tau-1}\Pi^p((1+g)^{\tau-1}\underline{\varepsilon}_1)\right] f(\widetilde{v}); \tag{A65}$$

(iii) flow wages

$$\widetilde{w}_i = \eta[f_i(\widetilde{v})m(\underline{\varepsilon}_1) - v'(n_i)/\omega] + (1-\eta)v'(n_i); \tag{A66}$$

(iv) optimal vacancy posting

$$\kappa_i = \beta\lambda_f(\theta_i)\frac{f_i(\widetilde{v})m(\underline{\varepsilon}_1) - \widetilde{w}_i - v'(n_i)/\omega}{1-\beta(1-\sigma)}; \tag{A67}$$

(v) optimal household search

$$h'(s_i) = \beta\lambda_w(\theta_i)\frac{\widetilde{w}_i - v'(n_i)}{1-\beta(1-\sigma)}; \tag{A68}$$

(vi) the steady-state law of motion for employment

$$\sigma n_i = \lambda_w(\theta_i)s_i; \tag{A69}$$

(vii) labor market clearing

$$\frac{\mu_i n_i}{\widetilde{v}_i} = \frac{\mu_1 n_1}{\widetilde{v}_1}. \tag{A70}$$

Proof. The system of equations consists of variables $1+5N$ variables $\underline{\varepsilon}_1$, $\widetilde{v}_i$, n_i, θ_i, s_i, $\widetilde{w}_i$, with $1+5N$ equations (A63)–(A70).

(i) On a BGP, the optimal investment equation (A55) equates capital's marginal product to the marginal cost of operation. Hence, the shadow value of a worker $\widetilde{\chi}_i$ is simply that worker's marginal product, given by

$$\widetilde{\chi}_i = f_i(\widetilde{v})m(\underline{\varepsilon}_1), \tag{A71}$$

where m is defined in equation (A64) as a weighted mean idiosyncratic productivity of capital that is utilized. Substituting $\widetilde{\chi}_i$ into the equation for $\underline{\varepsilon}_\tau$ in (A54) and evaluating at $\tau = 1$ gives

$$\underline{\varepsilon}_1 = (1+g)\frac{\sum_{i=1}^{N} f_i(\widetilde{v})\widetilde{v}_i}{f(\widetilde{v})} m(\underline{\varepsilon}_1).$$

From the definition of $f(v) = F(1, v) = v^{1-\alpha}$, we know that $f(v)$ is homogeneous of degree $1-\alpha$, which is the labor share of the production function. Applying Euler's theorem for homogeneous equations gives $\sum_{i=1}^{I} f_i(\widetilde{v})v_i = (1-\alpha)f(\widetilde{v})$, so we have

$$\underline{\varepsilon}_1 = (1+g)(1-\alpha)m(\underline{\varepsilon}_1),$$

which is equation (A63). This is independent of any other labor market condition and only depends on the parameters g, α, and the dispersion of idiosyncratic shocks σ_ε.

(ii) Substituting our expression for $\sum_{i=1}^{I} f_i(\widetilde{v})\widetilde{v}_i$ into the optimal investment condition (A56) obtains

$$\alpha f(\widetilde{v}) = \left[\sum_{\tau=1}^{\overline{\tau}} \widetilde{\beta}^\tau (1-\delta)^{\tau-1} \Pi^p((1+g)^{\tau-1}\underline{\varepsilon}_1)\right]^{-1}$$

which rearranges to the expression in equation (A65).

(iii) Substituting the value of $\widetilde{\chi}_i$ from (A71) into the definition of $\widetilde{\nu}_i$ (A51), the present value of a type i worker to the firm $\widetilde{\nu}_i$ is

$$\widetilde{\nu}_i = \frac{f_i(\widetilde{v})m(\underline{\varepsilon}_1) - v'(n_i)/\omega}{1-\beta(1-\sigma)}.$$

Substituting $\widetilde{\nu}_i$ into the vacancy-posting condition (A53) gives the simplified vacancy-posting condition in equation (A67).

(iv) The household optimal search condition (A68) is a restatement of equation (A49).

(v) Substituting $\widetilde{\nu}_i$ into the BGP wage equation (A52) gives (A66).

(vi) The transition law for labor (A69) is a restatement of equation (A62).

(vii) The BGP labor market clearing condition (A58) rearranges to

$$\frac{\mu_i n_i}{\widetilde{v}_i} = \widetilde{X} \sum_{\tau=1}^{\overline{\tau}} \Pi^u(\underline{\varepsilon}_\tau)(1-\delta)^{\tau-1}$$

Observe that the right-hand side of this equation is independent of i, so the left-hand side must be the same for all i, which gives equation (A70). □

Wage Markdowns. We now describe how we arrive at the expression for the BGP wage markdown (26) from the main text. We will use equations (A51), (A53), and (A52) from the balanced growth path, reproduced here in rearranged form:

$$\widehat{\nu}_i = \widetilde{\chi}_i - \frac{1}{\omega}v'(n_i) \tag{A72}$$

$$\frac{1-\beta(1-\sigma)}{\beta}\frac{\kappa_i}{\lambda_f(\theta_{it})} = (\rho+\sigma)\frac{\kappa_i}{\lambda_f(\theta_{it})} = \widehat{\nu}_i - \widetilde{w}_i \tag{A73}$$

$$\widetilde{w}_i = \eta\widehat{\nu}_i + (1-\eta)v'(n_i) \tag{A74}$$

where $\widehat{\nu}_i = [1-\beta(1-\sigma)]\widetilde{\nu}_i$ is the flow value of the worker to the firm and $\rho = \frac{1}{\beta} - 1$ is the rate of time preference such that $\frac{1-\beta(1-\sigma)}{\beta} = \frac{1}{\beta} - (1-\sigma) = \rho + \sigma$ in (A73).

The expression for $\widehat{\nu}_i$, (A72), can be rewritten as $\widetilde{\chi}_i = \widehat{\nu}_i + \frac{1}{\omega}v'(n_i)$. The expression for the annuitized vacancy posting costs, (A73), can be written as $\widehat{\nu}_i = (\rho+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)} + \widetilde{w}_i$. Substituting this expression for $\widehat{\nu}_i$ into $\widetilde{\chi}_i$ implies that the ratio of $\widetilde{w}_i$ to $\widetilde{\chi}_i$ is given by

$$\frac{\widetilde{w}_i}{\widetilde{\chi}_i} = \frac{\widetilde{w}_i}{\widetilde{w}_i + (\rho+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)} + \frac{1}{\omega}v'(n_i)} = \frac{1}{1 + (\rho+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)}\frac{1}{\widetilde{w}_i} + \frac{1}{\omega}\frac{v'(n_i)}{\widetilde{w}_i}}, \tag{A75}$$

where the second equation divides the numerator and denominator by $\widetilde{w}_i$. We now eliminate the wage from the RHS of (A75). Equation (A73) can be written $\widetilde{w}_i = \widehat{\nu}_i - (\rho+\sigma)\frac{\kappa_i}{\lambda_f(\theta_{it})}$. Plug this into the wage equation (A74) and rearrange to get $\widehat{\nu}_i = v'(n_i) + (\rho+\sigma)\frac{\kappa_i}{\lambda_f(\theta_{it})}\frac{1}{1-\eta}$. Then plug this back into (A73) to get $\widetilde{w}_i = v'(n_i) + \frac{\eta}{1-\eta}(\rho+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)}$. Finally, plug this into (A75) to get

$$\frac{\widetilde{w}_i}{\widetilde{\chi}_i} = \left[1 + \frac{1}{\omega} \times \frac{v'(n_i)}{v'(n_i) + \frac{\eta}{1-\eta}(\rho+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)}} + \frac{(r+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)}}{v'(n_i) + \frac{\eta}{1-\eta}(\rho+\sigma)\frac{\kappa_i}{\lambda_f(\theta_i)}}\right]^{-1} \tag{A76}$$

as in the main text.

Firm-Specific Labor Supply Elasticity. We can also use this algebra to derive equation the firm-specific labor supply elasticity along the BGP from the main text. As in the main text, first consider the participation constraint for firm j along the BGP:

$$\frac{\beta}{1-\beta(1-\sigma)}\lambda_w(\theta_{ij})\left[\widetilde{w}_{ij} - v'(n_i)\left(\frac{n_{ij}}{n_i}\right)^{\frac{1}{\omega}}\right] \geq \widetilde{\mathcal{W}}_i.$$

Differentiating with respect to the wage $\widetilde{w}_{ij}$ and n_{ij} holding θ_{ij} and $\widetilde{\mathcal{W}}_i$ fixed, we get

$$d\widetilde{w}_{ij} - v'(n_i)\frac{1}{\omega}\left(\frac{n_{ij}}{n_i}\right)^{\frac{1}{\omega}-1}\frac{dn_{ij}}{n_i} = 0 \implies d\log\widetilde{w}_{ij}\cdot\widetilde{w}_i = \frac{v'(n_i)}{\omega}d\log n_{ij},$$

where the second line uses the fact that $n_{ij} = n_i$ and $\widetilde{w}_{ij} = \widetilde{w}_i$ in a symmetric equilibrium. From the derivation of the markdown equation above, we know that $\widetilde{w}_i = v'(n_i) + \frac{\eta}{1-\eta}\frac{1-\beta(1-\sigma)}{\beta}\frac{\kappa_i}{\lambda_f(\theta_i)}$. Furthermore, from our calibration results, we also know that the annuitized portion of vacancy-posting costs are small, implying that $\widetilde{w}_i \approx v'(n_i)$. Plugging this in gives

$$d\log\widetilde{w}_{ij}\cdot v'(n_i) \approx \frac{v'(n_i)}{\omega}d\log n_{ij} \implies \frac{d\log n_{ij}}{d\log\widetilde{w}_{ij}} \approx \omega. \tag{A77}$$

B Labor Market Policies

In this appendix, we show how to add to the model the two labor market policies that we study in the main text: the minimum wage and transfer programs (like the EITC).

B.1 Minimum Wage

As in Appendix A, we first focus on the firm's problem ignoring initial conditions in order to see how the minimum wage changes the key decisions of the firm. We use this analysis to characterize the long-run effects of the minimum wage along the BGP and prove Proposition 6 from the main text. Finally, we add back in the initial conditions and discuss why firms are reluctant to fire workers in our quantitative work. Throughout, we focus only on the equations that change relative to the baseline model from Appendix A.

B.1.1 Introducing the Minimum Wage

The firm's problem is the same as in Appendix A except that we add a minimum wage constraint

$$W_{ijt+1} \geq \underline{W}_{t+1} \quad \text{for all } t \geq 0 \ (\times Q_{0,t}\rho_{ijt+1}) \tag{A1}$$

and a nonnegativity condition on vacancies

$$a_{ijt} \geq 0 \text{ for all } t \geq 0 \quad (\times Q_{0,t}\xi^a_{ijt}), \tag{A2}$$

where $Q_{0,t}\rho_{ijt+1}$ and $Q_{0,t}\xi^a_{ijt}$ are the scaled multipliers. We assume that the firm fulfills the present value by a constant wage per period that grows with time and satisfies the legislated minimum wage constraint on the flow minimum wage. That is, if the wage offered to workers in period t who begin working in period $t+1$ is $w_{ijt+1} \geq \bar{w}_{t+1}$, then in the net period we have $w_{ijt+2} = (1+g)w_{ijt+1} \geq (1+g)\bar{w}_{t+1}$ and so on. This leads to the constraint (A1) in terms of the present value $W_{ijt+1} = d_{t+1}w_{ijt+1}$ where d_{t+1} is a discounter defined by

$$d_{t+1} = 1 + Q_{t+1,t+2}(1-\sigma)(1+g) + Q_{t+1,t+3}(1-\sigma)^2(1+g)^2 + \dots,$$

which accounts for discounting, separations, and growth. The reason that we specify this constraint in terms of flow wages is that in practice that is how minimum wage legislation works. Our formulation restricts wages inwill minimal ways consistent with the constraint that in each period the flow wage is at least as high as its legislated minimum. Specifically, it prevents firms from offering present values of wages in which in some periods the associated flow wage falls below the legislated minimum.

First-Order Conditions. The only part of the firm's problem that is affected by the minimum wage are the equations in the hiring stage. Within the hiring stage, the first-order conditions for employment n_{ijt+1}, (A6), and market tightness θ_{ijt}, (A8), are not affected.

The first-order condition for vacancy posting a_{ijt} is now

$$\begin{aligned} & - Q_{0,t}\kappa_{it} - Q_{0,t+1}\lambda_f(\theta_{ijt})W_{ijt+1} + Q_{0,t+1}\lambda_f(\theta_{ijt})\nu_{ijt+1} + Q_{0,t}\xi^a_{ijt} = 0 \\ \implies & \frac{\kappa_i}{\lambda_f(\theta_{ijt})} + Q_{t,t+1}W_{ijt+1} \geq Q_{t,t+1}\nu_{ijt+1}, \text{ with equality if } a_{ijt} > 0. \end{aligned} \tag{A3}$$

Here, we explicitly keep track of the multiplier on the nonnegativity constraint on vacancies, since it will never bind without a minimum wage policy but it could bind with one. The only condition that is directly affected is the first-order condition for wages W_{ijt+1}, which now is

$$- \lambda_f(\theta_{ijt})a_{ijt} + \gamma_{ijt+1} + \rho_{ijt+1} = 0, \tag{A4}$$

where ρ_{ijt+1} is the multiplier on the minimum wage constraint. There are two cases. First, if the minimum wage is not binding, then this equation is reduces to $\gamma_{ijt+1} = \lambda_f(\theta_{ijt})a_{ijt}$. In this case, plugging this expression back into the first-order condition for market tightness (A8) we get the same equation as when there is no minimum wage, that is, (A10), which repeat here for convenience

$$W_{ijt+1} = \eta\nu_{ijt+1} - (1-\eta)V_{ijt+1}.$$

The interesting case is when the minimum wage is binding. Here we will simply use the first-order condition for market tightness (A8) as an equation that defines the multiplier γ_{ijt+1} given that $W_{ijt+1} = \underline{W}_t$. That is, we solve for γ_{ijt+1} using the following algebra

$$\begin{aligned}
W_{ijt+1} &= \nu_{ijt+1} + \frac{\gamma_{ijt+1}}{a_{ijt}} \frac{\mathcal{W}_{it}}{Q_{t,t+1}\lambda_w(\theta_{ijt})^2} \frac{\lambda'_w(\theta_{ijt})}{\lambda'_f(\theta_{ijt})} \\
\underline{W}_{t+1} &= \nu_{ijt+1} + \frac{\gamma_{ijt+1}}{a_{ijt}} \frac{\mathcal{W}_{it}}{Q_{t,t+1}\lambda_w(\theta_{ijt})} \frac{\lambda'_w(\theta_{ijt})}{\lambda'_f(\theta_{ijt})} \frac{1}{\lambda_w(\theta_{ijt})} \\
\underline{W}_{t+1} &= \nu_{ijt+1} + \frac{\gamma_{ijt+1}}{a_{ijt+1}} (\underline{W}_{t+1} + V_{ijt+1}) \frac{\lambda'_w(\theta_{ijt})}{\lambda'_f(\theta_{ijt})} \frac{1}{\lambda_w(\theta_{ijt})} \\
\implies \underline{W}_{t+1} &= \nu_{ijt+1} - \gamma_{ijt+1} \frac{1-\eta}{\eta} \frac{\underline{W}_{t+1} + V_{ijt+1}}{\lambda_f(\theta_{ijt}) a_{ijt}} \\
\implies \gamma_{ijt+1} &= \frac{\eta}{1-\eta} \frac{\nu_{ijt+1} - \underline{W}_{t+1}}{\underline{W}_{t+1} + V_{ijt+1}} \lambda_f(\theta_{ijt}) a_{ijt}.
\end{aligned}$$

In the second line, we plugged in $W_{ijt+1} = \underline{W}_{t+1}$. In the third line we used $\mathcal{W}_{it}/\left[Q_{t,t+1}\lambda_w(\theta_{ijt})\right] = \underline{W}_{t+1} + V_{ijt+1}$ and in the fourth line we used $\frac{\lambda_f(\theta_{ijt})}{\lambda_w(\theta_{ijt})} \frac{\lambda'_w(\theta_{ijt})}{\lambda'_f(\theta_{ijt})} = -\frac{1-\eta}{\eta}$. Summarizing

$$\gamma_{ijt+1} = \left\{ \begin{array}{c} \lambda_f(\theta_{ijt}) a_{ijt} \text{ if slack} \\ \frac{\eta}{1-\eta} \frac{\nu_{ijt+1} - \underline{W}_{t+1}}{\underline{W}_{t+1} + V_{ijt+1}} \lambda_f(\theta_{ijt}) a_{ijt} \text{ if bind} \end{array} \right\} \tag{A5}$$

$$W_{ijt+1} = \left\{ \begin{array}{c} \eta \nu_{ijt+1} - (1-\eta) V_{ijt+1} \text{ if slack} \\ \underline{W}_{t+1} \text{ if bind} \end{array} \right\} \tag{A6}$$

and the sequence of multipliers on the participation constraint, $\gamma_{ij1}, ..., \gamma_{it+1}$ show up in the value of a worker equation

$$\nu_{ijt+1} = \chi_{ijt+1} + M_{ijt+1} \frac{u_{nit+1}}{u_{cit+1}} \frac{1}{\omega} \left(\frac{n_{ijt+1}}{n_{it+1}} \right)^{\frac{1}{\omega}-1} \frac{1}{n_{it+1}} + Q_{t+1,t+2}(1-\sigma)\nu_{ijt+2}, \tag{A7}$$

since $M_{ijt+1} = \gamma_{ijt+1} + (1-\sigma)\gamma_{ijt} + ... + (1-\sigma)^t \gamma_{ij1}$. So, in general, the value of M_{ijt+1} depends on the entire binding pattern of the minimum wage.

Detrending. Here we state the conditions of the problem in stationary form to anticipate the balanced growth path and we impose symmetry. The only conditions that change are those for when the minimum wage is binding. When it is slack then, as before, $\gamma_{it+1} = \lambda_f(\theta_{it}) a_{it}$ is already stationary. When it binds then in detrended form the multiplier is

$$\gamma_{it+1} = \frac{\eta}{1-\eta}\frac{(1+g)^{t+1}(\widetilde{\nu}_{it+1} - \underline{\widetilde{W}}_{t+1})}{(1+g)^{t+1}(\underline{\widetilde{W}}_{t+1} + \widetilde{V}_{it+1})}\lambda_f(\theta_{it})a_{it} = \frac{\eta}{1-\eta}\frac{\widetilde{\nu}_{it+1} - \underline{\widetilde{W}}_{t+1}}{(\underline{\widetilde{W}}_{t+1} + \widetilde{V}_{it+1})}\lambda_f(\theta_{it})a_{it}.$$

So in detrended form (A5) becomes

$$\gamma_{it+1} = \left\{ \begin{array}{c} \lambda_f(\theta_{it})a_{it} \text{ if slack} \\ \frac{\eta}{1-\eta}\frac{\widetilde{\nu}_{it+1} - \underline{\widetilde{W}}_{t+1}}{\underline{\widetilde{W}}_{t+1} + \widetilde{V}_{it+1}}\lambda_f(\theta_{it})a_{it} \text{ if bind} \end{array} \right\} \tag{A8}$$

and in detrended form wages are

$$\widetilde{W}_{it+1} = \left\{ \begin{array}{c} \eta\widetilde{\nu}_{it+1} - (1-\eta)\widetilde{V}_{it+1} \text{ if slack} \\ \underline{\widetilde{W}}_{t+1} \text{ if bind} \end{array} \right\}. \tag{A9}$$

Of course, here also since the value of M_{ijt+1} depends on the entire binding pattern of the minimum wage, so does the value of a worker given by

$$\widetilde{\nu}_{it+1} = \widetilde{\chi}_{it+1} - M_{it+1}v'(n_{it+1})\frac{1}{\omega n_{it+1}} + Q_{t+1,t+2}(1+g)(1-\sigma)\widetilde{\nu}_{it+2}. \tag{A10}$$

B.1.2 BGP and Proof of Proposition 6

We summarize how the minimum wage impacts the BGP using the following Lemma, analogous to Lemma 8 from Appendix A:

Lemma 9. *Along the balanced growth path, the labor allocations and wages are determined by the following equations:*

(i) optimal cut-off for idiosyncratic productivity of capital

$$\underline{\varepsilon}_1 = (1+g)(1-\alpha)m(\underline{\varepsilon}_1), \tag{A11}$$

where $m(\underline{\varepsilon}_1)$ is defined by

$$m(\underline{\varepsilon}_1) = \frac{\sum_{\tau=1}^{\infty}\widetilde{\beta}^{\tau}(1-\delta)^{\tau-1}\Pi^p((1+g)^{\tau-1}\underline{\varepsilon}_1)}{\sum_{\tau=1}^{\infty}\widetilde{\beta}^{\tau}(1-\delta)^{\tau-1}(1+g)^{\tau}\Pi^u((1+g)^{\tau-1}\underline{\varepsilon}_1)}; \tag{A12}$$

(ii) the marginal unit of capital earns zero profit

$$1 = \alpha \left[\sum_{\tau=1}^{\infty} \widetilde{\beta}^{\tau} (1-\delta)^{\tau-1} \Pi^{p}((1+g)^{\tau-1} \underline{\varepsilon}_1) \right] f(\widetilde{v}); \tag{A13}$$

(iii) flow wages

$$\widetilde{w}_i = \left\{ \ \eta[f_i(\widetilde{v}) m(\underline{\varepsilon}_1) - v'(n_i)/\omega] + (1-\eta) v'(n_i) \ \textit{if slack}, \underline{w} \ \textit{if bind} \ \right\}; \tag{A14}$$

(iv) optimal vacancy posting

$$\kappa_i = \left\{ \begin{array}{c} \beta \lambda_f(\theta_i) \frac{f_i(\widetilde{v}) m(\underline{\varepsilon}_1) - \widetilde{w}_i - v'(n_i)/\omega}{1-\beta(1-\sigma)} \ \textit{if slack} \\ \beta \lambda_f(\theta_i) \frac{f_i(v) m(\underline{\varepsilon}_1) - \underline{w}}{1-\beta(1-\sigma)} \left[\frac{\underline{w} - v'(n_i)}{\underline{w} - v'(n_i)(1-1/\omega)} \right] \ \textit{if bind} \end{array} \right\}; \tag{A15}$$

(v) optimal household search

$$h'(s_i) = \beta \lambda_w(\theta_i) \frac{\widetilde{w}_i - v'(n_i)}{1-\beta(1-\sigma)}; \tag{A16}$$

(vi) the steady state law of motion for employment

$$\sigma n_i = \lambda_w(\theta_i) s_i; \tag{A17}$$

(vii) labor market clearing

$$\frac{\mu_i n_i}{\widetilde{v}_i} = \frac{\mu_1 n_1}{\widetilde{v}_1}. \tag{A18}$$

Proof. As described above, the only two equations change due to the presence of the minimum wage: (i) the expression for the multiplier on the participation constraint (equation (A8) in the detrended system) and (ii) the wage equation (equation (A9) in the detrended system). All the other equations characterizing the equilibrium from the summary of BGP conditions from the baseline model (A48)-(A62) continue to hold. Furthermore, the proof of conditions (i), (ii), (v), (vi), and (vii) from Lemma 8 relied only on those other conditions, so they apply equally here. Therefore, we only need to focus on the part of conditions (iii) and (iv) when the minimum wage binds. Clearly, the wage equation when the minimum wage binds is simply $\widetilde{w}_i = \underline{w}$, giving us the binding part of condition (iii).

The remaining challenge is to prove condition (iv). Recall, that in the case where the minimum wage is not binding, we substituted for the multiplier on the participation constraint γ_{it} from (A8) in the expression for the present value of a worker $\widetilde{\nu}_{it+1}$ from (A14) and simplified to arrive at the non-binding version of condition (iv) from Lemma 8. When the minimum wage is binding, we must follow a different strategy because the multiplier on the participation constraint γ_{it} in (A8) itself depends on the present value of a worker $\widetilde{\nu}_{it+1}$, and the present value of a worker $\widetilde{\nu}_{it+1}$ from (A10) implicitly depends on the value of γ_{it} through the auxiliary variable $M_{it+1} = (1-\sigma)M_{it} + \gamma_{it+1}$. Note that imposing balanced growth on (A10) gives

$$\widetilde{\nu}_i = \frac{\widetilde{\chi}_i - M_i v'(n_i)/(\omega n_i)}{1-\beta(1-\sigma)}. \tag{A19}$$

Next, note that the BGP version of the multiplier on the participation constraint (A8) becomes

$$\gamma_i = \frac{\eta}{1-\eta}\frac{\widetilde{\nu}_i - \underline{W}}{\underline{W} - \widetilde{V}_i}\lambda_f(\theta_i)a_i \implies \frac{\gamma_i}{\sigma}\frac{1}{n_i} = \frac{\widetilde{\nu}_i - \underline{W}}{\underline{W} + \widetilde{V}_i}, \tag{A20}$$

where the second line uses the BGP law of motion for employment $\lambda_f(\theta_i)a_i = \sigma n_i$. Next, define the numerator of (A19) as the *flow value of a worker* to the firm along the BGP by letting $\widehat{\nu}_i = \widetilde{\chi}_i - M_i v'(n_i)\frac{1}{\omega n_i}$. We then convert all the terms on the right side of (A20) by dividing both the numerator and denominator by $1-\beta(1-\sigma)$ to get

$$\frac{\gamma_i}{\sigma}\frac{1}{n_i} = \frac{\widehat{\nu}_i - \underline{w}}{\underline{w} - v'(n_i)}. \tag{A21}$$

Next, we will plug this expression into the flow value of a worker $\widehat{\nu}_i = \widetilde{\chi}_i - M_i v'(n_i)\frac{1}{\omega n_i}$. Note that the BGP version of the evolution of $M_{it+1} = (1-\sigma)M_{it} + \gamma_{it+1}$ is $M_i = \frac{\gamma_i}{\sigma}$, so we can write this flow value as

$$\widehat{\nu}_i = \widetilde{\chi}_i - \frac{\gamma_i}{\sigma}v'(n_i)\frac{1}{\omega n_i} \implies \widehat{\nu}_i = \widetilde{\chi}_i - \frac{\gamma_i}{\sigma n_i}\frac{v'(n_i)}{\omega}.$$

Now plug in the expression for $\frac{\gamma_i}{\sigma n_i}$ from (A21) into this equation to get

$$\widehat{\nu}_i = \widetilde{\chi}_i - \frac{v'(n_i)}{\omega}\frac{\widehat{\nu}_i - \underline{w}}{\underline{w} - v'(n_i)}. \tag{A22}$$

We will use this implicit expression for $\widehat{\nu}_i$ to obtain the expression for optimal vacancy posting (A15).

Now subtract the minimum wage $\underline{w}$ and solve for $\widehat{\nu}_i - \underline{w}$ to get

$$\widehat{\nu}_i - \underline{w} = \widetilde{\chi}_i - \underline{w} - \frac{\widehat{\nu}_i - \underline{w}}{\underline{w} - v'(n_i)} \frac{v'(n_i)}{\omega} \implies (\widehat{\nu}_i - \underline{w}) \left[1 + \frac{1}{\underline{w} - v'(n_i)} \frac{v'(n_i)}{\omega} \right] = \widetilde{\chi}_i - \underline{w}$$
$$\implies (\widehat{\nu}_i - \underline{w}) \left[\frac{\underline{w} - v'(n_i) + v'(n_i)/\omega}{\underline{w} - v'(n_i)} \right] = \widetilde{\chi}_i - \underline{w}$$
$$\implies \widehat{\nu}_i - \underline{w} = \left[\frac{\underline{w} - v'(n_i)}{\underline{w} - v'(n_i)(1 - 1/\omega)} \right] (\widetilde{\chi}_i - \underline{w}) \,. \tag{A23}$$

Finally, plug this into the detrended vacancy-posting condition given by

$$\kappa_i = \frac{\beta}{1 - \beta(1 - \sigma)} \lambda_f(\theta_i) \left(\widehat{\nu}_i - \underline{w} \right) .$$

to get

$$\kappa_i = \frac{\beta}{1 - \beta(1 - \sigma)} \lambda_f(\theta_i) \left[\frac{\underline{w} - v'(n_i)}{\underline{w} - v'(n_i)(1 - 1/\omega)} \right] (\widetilde{\chi}_i - \underline{w}) \,. \tag{A24}$$

Finally, note that the expression $\widetilde{\chi}_i = f_i(v) m(\underline{\varepsilon}_1)$ continues to be true from the proof of Lemma 3. Using this we obtain

$$\kappa_i = \frac{\beta}{1 - \beta(1 - \sigma)} \lambda_f(\theta_i) \left[\frac{\underline{w} - v'(n_i)}{\underline{w} - v'(n_i)(1 - 1/\omega)} \right] [f_i(v) m(\underline{\varepsilon}_1) - \underline{w}] \,. \tag{A25}$$

which completes the proof. □

Proof of Proposition 6. We will now build on this characterization of the BGP under the minimum wage to prove Proposition 6 from the main text. To do so, we must first extend the space of policies to incorporate the two policies from Proposition 6. First, a type-specific minimum wage can be represented as the detrended flow minimum wage, $\underline{w}_i$, specific for a type i worker. Since the policy will set the minimum wage to its competitive level, which is strictly above the monopsonistically competitive equilibrium value, the minimum wage will be binding for each type of worker. Second, a subsidy to vacancy-posting can be represented by replacing the detrended vacancy-posting cost κ_i with its after-subsidy version $\kappa_i(1 - \tau_i)$. With these two changes, the system of equations characterizing the allocation under the policies is

$$\underline{\varepsilon}_1 = (1+g)(1-\alpha)m(\underline{\varepsilon}_1) \tag{A26}$$

$$1 = \alpha\left[\sum_{\tau=1}^{\infty}\widetilde{\beta}^{\tau}(1-\delta)^{\tau-1}\Pi^p((1+g)^{\tau-1}\underline{\varepsilon}_1)\right]f(\widetilde{v}) \tag{A27}$$

$$\kappa_i(1-\tau_i) = \lambda_f(\theta_i)\frac{f_i(v)m(\underline{\varepsilon}_1)-\underline{w}_i}{\rho+\sigma}\left[\frac{\underline{w}_i - v'(n_i)}{\underline{w}_i - v'(n_i)(1-1/\omega)}\right] \tag{A28}$$

$$h'(s_i) = \beta\lambda_w(\theta_i)\frac{\underline{w}_i - v'(n_i)}{1-\beta(1-\sigma)} \tag{A29}$$

$$\sigma n_i = \lambda_w(\theta_i)s_i \tag{A30}$$

$$\frac{\mu_i n_i}{\widetilde{v}_i} = \frac{\mu_1 n_1}{\widetilde{v}_1}. \tag{A31}$$

The allocation in the competitive model can be obtained by evaluating Lemma 3 at $\omega = \infty$:

$$\underline{\varepsilon}_1 = (1+g)(1-\alpha)m(\underline{\varepsilon}_1) \tag{A32}$$

$$1 = \alpha\left[\sum_{\tau=1}^{\infty}\widetilde{\beta}^{\tau}(1-\delta)^{\tau-1}\Pi^p((1+g)^{\tau-1}\underline{\varepsilon}_1)\right]f(\widetilde{v}^c) \tag{A33}$$

$$\kappa_i = \beta\lambda_f(\theta_i^c)\frac{f_i(\widetilde{v}^c)m(\underline{\varepsilon}_1)-\widetilde{w}_i^c}{1-\beta(1-\sigma)} \tag{A34}$$

$$h'(s_i^c) = \beta\lambda_w(\theta_i^c)\frac{\widetilde{w}_i^c - v'(n_i^c)}{1-\beta(1-\sigma)} \tag{A35}$$

$$\sigma n_i^c = \lambda_w(\theta_i^c)s_i^c \tag{A36}$$

$$\frac{\mu_i n_i^c}{\widetilde{v}_i^c} = \frac{\mu_1 n_1^c}{\widetilde{v}_1^c} \tag{A37}$$

$$\widetilde{w}_i^c = \eta f_i(\widetilde{v}^c)m(\underline{\varepsilon}_1) + (1-\eta)v'(n_i^c). \tag{A38}$$

We will show that the competitive allocation from equations also solves the equilibrium allocation under the policy choices from Proposition 6. To do so, we evaluate the system of equations characterizing the equilibrium under the policies, (A26)—(A31), at the competitive allocation (i.e. with $\widetilde{v} = \widetilde{v}^c$, $\widetilde{n}_i = \widetilde{n}_i^c$, and so on). We will show that doing so gives the same system of equations as the competitive system, (A32)—(A38).

Given this guessed allocation, the following equations are clearly the same because the wage does not enter them directly: (i) the productivity cutoffs (A26) and (A32), (ii) the zero profit condition for the marginal unit of capital (A27) and (A34), (iii) the law of of motion for employment (A30), and (iv) the labor ratios (A31) and (A37). The household's optimal search condition depends on the wage; since the policy sets the type-specific minimum wage $\underline{w}_i = \widetilde{w}_i^c$, these two conditions (A29) and (A35) also coincide. Finally, under the choice for τ_i from the main text, the optimal vacancy-posting conditions (A28) and (A34) also coincide.

Note that dividing the monopsony vacancy-posting condition (A28) evaluated at the competitive allocation with the minimum wage policy $\underline{w}_i = \widetilde{w}_i^c$ and the $1-\tau_i$ in the proposition gives that

$$1 - \tau_i = \left[\frac{\widetilde{w}_i c - v'(n_i^c)}{\widetilde{w}_i^c - v'(n_i^c)(1 - 1/\omega)} \right],$$

which is exactly how these subsidies are set. This establishes the result.

B.1.3 Initial Conditions

In all of our experiments, we assume that the economy is initially growing along the BGP without policies, characterized in Appendix A, and then the policy is unexpectedly introduced in the initial period $t = 0$. Up to this point, we have largely ignored the initial conditions faced by firms in this initial period $t = 0$ and focused on the behavior of the economy from period $t \geq 1$ onward. In this subsection, we specify the initial conditions and whether firms fire workers in the initial period $t = 0$. The firm takes as given four sets of initial conditions drawn from the initial BGP when solving its problem in period $t = 0$. First, the firm inherits a distribution of capital stocks $K_{j0}(v_{-\tau}, A_{-\tau})$, where $v_{-\tau}$ is the vector of labor intensities chosen along the initial BGP in periods $-1, -2, \ldots$ and $A_{-\tau}$ is the corresponding level of vintage productivity. Second, the firm inherits a measure of employed workers of each type i, $N_{ij0} = N_{i0}$, equal to the employment rate N_i of each group from the BGP. Third, the firm inherits the flow wage schedule initially promised to each of these worker types along the BGP. Given that flow wages grow at a constant rate within a match, this flow wage schedule is summarized by w_{i0}, the flow wage promised to workers of group i in period $t = 0$. Under the minimum wage, firms must now pay these workers $\widehat{w}_{i0} = \max\{w_{i0}, \underline{w}\}$ in period $t = 0$, $\widehat{w}_{i1} = \max\{(1+g)w_{i0}, (1+g)\underline{w}\}$ in the following period $t = 1$, and so on. Let $\widehat{W}_{i0} = \sum_{t=0}^{\infty} Q_{0,t}(1-\sigma)^t \widehat{w}_{it}$ denote the present value of wage payments promised to these workers going forward. Finally, the firm inherits the Marcet-Marimon cumulation of multipliers M_{i0} from the initial BGP to reflect promises made to workers hired before period $t = 0$.

We assume that when the minimum wage is unexpectedly introduced in period $t = 0$, a firm j can choose to fire a measure F_{ij0} of its initially employed workers. However, for all workers that it does not fire, the firm must pay them at least the flow minimum wage each period.

Initial Period Decisions. The majority of the firm's problem is identical to the what we have already studied except for decisions in the initial period $t = 0$. Furthermore, nearly all decisions about hiring and investment made in this period only impact the firm's objective starting in period $t \geq 1$ onward, so those are unchanged. The only exception is the option for the firm to fire F_{ij0} workers in the initial period. The option to fire workers affects the profit maximization problem in four ways. First, for each fired worker, the firm saves itself the present value of flow wages it would have been obliged to fire that worker had they remained employed. Hence, the term $\sum_i \widehat{W}_{i0} F_{ij0}$ is added to the firm's objective function. Second, the adding up constraint in the assignment of workers to machines must reflect the fact that the firm may fire some of the existing workers:

$$\sum_{\tau=1}^{\infty} v_{i,-\tau} u_{j0}(v_{-\tau}, \varepsilon, A_{-\tau}) K_{j0}(v_{-\tau}, A_{-\tau}) \pi(\varepsilon) d\varepsilon dv \leq N_{ij0} - F_{ij0} \quad (\times \chi_{ij0}),$$

where χ_{ij0} is the multiplier on this constraint. Note that, since this constraint holds with equality along the initial BGP, positive firing $F_{ij0} > 0$ requires lowering the utilization rates of existing capital. Third, we must modify the law of motion for employment to account for firings as well:

$$N_{ij1} \leq (1 - \sigma)(N_{ij0} - F_{ij0}) + \lambda_f(\theta_{ij0}) \mu_i a_{ij0} \quad (\times Q_{0,1} \nu_{ij1}),$$

where ν_{ij1} is the scaled multiplier on this constraint. Finally, firms must satisfy the non-negativity constraint $F_{ij0} \geq 0$ for $\quad (\times \xi^f_{ij0})$.

First-Order Condition. The first-order condition with respect to F_{ij0} is $\widehat{W}_{i0} - \chi_{ij0} - Q_{0,1}(1 - \sigma)\nu_{ij1} + \xi^f_{ij0} = 0$ or, equivalently,

$$\widehat{W}_{i0} - \chi_{ij0} - Q_{0,1}(1 - \sigma)\nu_{ij1} + \xi^f_{ij0} = 0 \implies \chi_{ij0} + Q_{0,1}(1 - \sigma)\nu_{ij1} \geq \widehat{W}_{i0}, \text{ with equality if } F_{ij0} > 0. \tag{A39}$$

That is, firms do not fire workers if the present value of the workers' benefits to the firm — their marginal product in period $t = 0$, χ_{ij0} plus their present value going forward, $Q_{0,1}(1 - \sigma)\nu_{ij1}$ on the LHS of (A39) — is strictly greater than the present value of wage payments to those workers — $\widehat{W}_{i0}$ on the RHS of (A39). This is the only new condition for the initial period $t = 0$; all other conditions are the same as in the baseline model.

B.2 Transfer Programs

We now turn to the transfer programs. Section B.2.1 shows how transfers impact the household's problem. Section B.2.2 shows how transfers impact the firms problem. Section B.2.3 shows how to detrend those conditions and arrives at the BGP.

B.2.1 Households

We first provide some additional notation related to the transfer system, and then show how it affects the solution to the household's problem.

Notation. As in the main text, we will represent the transfer system in terms of the after-transfers wages that households receive. In particular, if the firm pays the flow wage w_{ijt}, then households receive the flow payment $A_t(w_{ijt})$ which includes the transfers from the government. Also as in the main text, we assume that the transfer system satisfies the property $A_t(w_{ijt}) = (1+g)^t A(\widetilde{w}_{ijt})$ for some time-invariant function $A(\widetilde{w})$ where, as usual, tildes denote detrended variables. Note that this assumption also implies that $A'_t(w) = A'(\widetilde{w})$.[21] We use the discounter operator $d_{t+1} = 1 + Q_{t+1,t+2}(1+g)(1-\sigma) + Q_{t+1,t+3}(1+g)^2(1-\sigma)^2 + \dots$ to convert this stream of flow payments to the worker to the present value $W^H_{ijt+1} = d_{t+1}A_t(w_{ijt+1})$. From the firm's perspective, the present value of wage costs is the same as in the baseline model $W_{ijt+1} = d_{t+1}w_{ijt+1}$.

Household's Problem. The transfer program changes two parts of the household's problem relative to our baseline model. First, the wages received in the budget constraint are W^H_{ijt} from above, to reflect the present value of transfer payments. Second, the present value of profits need to reflect the corporate taxes to fund the program.

More formally, the household's utility maximization problem is now

$$\max_{c_{it}, s_{ijt}, n_{ijt+1}} \sum_{t=0}^{\infty} \beta^t U_t(c_{it}, n_{it}, s_{it}) \quad \text{such that}$$

$$n_{ijt+1} = (1-\sigma)n_{ijt} + \lambda_w(\theta_{ijt})s_{ijt} \quad (\times \beta^t \widehat{V}_{ijt+1})$$

$$\sum_{t=0}^{\infty} Q_{0,t}c_{it} = \psi_i(1-\tau_c)\mathbb{P} + \mathbb{I}_i + \sum_{t=1}^{\infty} Q_{0,t} \sum_j \lambda_w(\theta_{ijt-1})s_{ijt-1}W^H_{ijt} \quad (\times \Gamma),$$

[21] To see this, note that

$$A'_t(w) = \frac{d}{dw}A_t(w) = \frac{d}{dw}(1+g)^t A(\widetilde{w}_{ijt}) = (1+g)^t \frac{d}{dw} A(\frac{w}{(1+g)^t}) = \frac{(1+g)^t}{(1+g)^t} A'(\widetilde{w}) = A'(\widetilde{w}).$$

where τ_c is the profits tax rate. As usual, the variables in parentheses denote the (often rescaled) Lagrange multiplier associated with the constraint. Compared to the household problem in the baseline model from Appendix A, the only first-order condition which changes is the one for search effort to take into account that wage payments are now W^H_{ijt+1}:

$$-\frac{u_{sit}}{u_{cit}} = \lambda_w(\theta_{ijt})Q_{t,t+1}\left(V_{ijt+1} + W^H_{ijt+1}\right). \tag{A40}$$

The fact that the optimal search condition (A40) also changes the participation constraint which firms will take as given:

$$\lambda_w(\theta_{ijt})(W^H_{ijt+1} + V_{ijt+1}) \geq \mathcal{W}_{it}. \tag{A41}$$

B.2.2 Firms

We now turn to how the transfer program affects the solution to the firm's problem. We first restate the profit maximization problem and then derive the FOCs. As with our minimum wage analysis, we abstract from initial conditions in this section and focus on the conditions that change due to the presence of the transfer program.

Firm's Problem. The presence of the transfer system changes the firm's problem in three ways. First, the participation constraint (A41) now reflects the fact that households receive transfers, as derived above. Second, and related, we re-state the firm's wage choice in terms of the initial flow wage w_{ijt+1} instead of the present value. Using w_{ijt+1} as the choice variables allows us to capture how firms' wage-posting decisions affect both the present value of households' post-transfer income and firms' pre-transfer costs. Third, the profits tax multiplies flow profits each period by $1-\tau_c$. However, since the tax rate is constant, this change amounts to multiplying the objective function by $1-\tau_c$, which doesn't affect the profit-maximizing decisions. We therefore drop the $1-\tau_c$ from the exposition to keep the equations as close to our baseline model as possible. We abstract from the possibility that firms will want to fire initial workers given that it will not be relevant for this policy. With these changes, the profit maximization problem is

$$\sum_t Q_{0,t}\Bigg(\sum_\tau \int_{v,\varepsilon} u_{jt}(v, A_{t-\tau}, \varepsilon) A_{t-\tau}\varepsilon f(v) K_{jt}(v, A_{t-\tau})\pi(\varepsilon) d\varepsilon dv - \sum_i \mu_i(\lambda_f(\theta_{ijt-1}) a_{ijt-1} d_t w_{ijt} + \kappa_{it} a_{ijt})$$

$$- \int X_{jt}(v)dv\Bigg) + \sum_{t=0}^{\infty} Q_{0,t+1}\mu_i M_{ijt+1} \frac{U_{nit+1}}{U_{cit+1}} \left(\frac{n_{ijt+1}}{n_{it+1}}\right)^{\frac{1}{\omega}} + \sum_{t=0}^{\infty} Q_{0,t+1}\mu_i \gamma_{ijt+1} \left[d_{t+1} A_{t+1}(w_{ijt+1}) - \frac{\mathcal{W}_{it}}{Q_{t,t+1}\lambda_w(\theta_{ijt})}\right]$$

such that $\quad u_{jt}(v, A_{t-\tau}, \varepsilon) \geq 0 \quad (\times Q_{0,t}\lambda^L_{jt}(v, A_{t-\tau}, \varepsilon))$

$u_{jt}(v, A_{t-\tau}, \varepsilon) \leq 1 \quad (\times Q_{0,t}\lambda^U_{jt}(v, \varepsilon, A_{t-\tau}))$

$u_{jt}(v, A_{t-\tau}, \varepsilon) v_i K_{jt}(v, A_{t-\tau})\pi(\varepsilon) \leq N_{ijt}(v, A_{t-\tau}, \varepsilon)$ for all $i \quad (\times Q_{0,t}\lambda_{ijt}(v, A_{t-\tau}, \varepsilon))$

$\sum_\tau \int_{v,\varepsilon} N_{ijt}(v, A_{t-\tau}, \varepsilon) d\varepsilon dv \leq \mu_i n_{ijt}$ for all $i \quad (\times Q_{0,t}\chi_{ijt})$

$\mu_i n_{ijt+1} \leq (1-\sigma)\mu_i n_{ijt} + \lambda_f(\theta_{ijt})\mu_i a_{ijt}$ for all $i \quad (\times Q_{0,t+1}\nu_{ijt+1})$

$K_{jt+\tau+1}(v, A_t) = (1-\delta)^\tau X_{jt}(v) \quad (\times Q_{0,t+\tau+1} q_{jt,t+\tau+1}(v))$

$X_{jt}(v) \geq 0 \quad (\times Q_{0,t}\mu_{jt}(v))$.

First-Order Conditions. As with the minimum wage, the only part of the firm's problem that is affected are the equations in the hiring stage. Within the hiring stage, the first-order conditions for employment n_{ijt+1}, vacancies a_{ijt}, and market tightness θ_{ijt} are unaffected. However, the transfer system will change how we simplify the first-order condition for market tightness (A8), so we reproduce it here:

$$W_{ijt+1} = \nu_{ijt+1} + \frac{\gamma_{ijt+1}}{a_{ijt}} \frac{\mathcal{W}_{it}}{\lambda_w(\theta_{ijt})^2} \frac{\lambda'_w(\theta_{ijt})}{\lambda'_f(\theta_{ijt})}. \tag{A42}$$

The first-order condition which changes is the one for wages w_{ijt+1}:

$$-Q_{0,t+1}\mu_i\lambda_f(\theta_{ijt}) a_{ijt} d_{t+1} + Q_{0,t+1}\mu_i\gamma_{ijt+1} d_{t+1} A'_t(w_{ijt+1}) = 0 \implies \quad \gamma_{ijt+1} = \frac{\lambda_f(\theta_{ijt}) a_{ijt}}{A'_t(w_{ijt+1})}. \tag{A43}$$

Hence, as stated in the main text, the the transfer system changes the multiplier on the participation constraint. This multiplier then affects two things. First, it enters the law of motion for the auxiliary variable $M_{ijt+1} = (1-\sigma)M_{ijt} + \gamma_{ijt+1}$. Second, it affects how we simplify the FOC for market tightness (A42). Plugging the expression for the multiplier (A43) into the FOC for market tightness

(A42) gives

$$
\begin{aligned}
W_{ijt+1} &= \nu_{ijt+1} + \frac{\lambda_f(\theta_{ijt})}{\lambda_w(\theta_{ijt})} \frac{\mathcal{W}_{it}}{\lambda_w(\theta_{ijt})} \frac{\lambda_w'(\theta_{ijt})}{\lambda_f'(\theta_{ijt})} \frac{1}{A_t'(w_{ijt+1})} \\
&= \nu_{ijt+1} + \frac{\lambda_f(\theta_{ijt})}{\lambda_w(\theta_{ijt})} \left(W_{ijt+1}^H + V_{ijt+1}\right) \frac{\lambda_w'(\theta_{ijt})}{\lambda_f'(\theta_{ijt})} \frac{1}{A_t'(w_{ijt+1})} \\
&= \nu_{ijt+1} - \frac{1-\eta}{\eta} \frac{W_{ijt+1}^H + V_{ijt+1}}{A_t'(w_{ijt+1})}. \qquad \text{(A44)}
\end{aligned}
$$

where in the second line we used $\frac{\mathcal{W}_{it}}{\lambda_w(\theta_{ijt})} = W_{ijt+1}^H + V_{ijt+1}$ and in the third line we used $\frac{\lambda_f(\theta_{ijt})}{\lambda_w(\theta_{ijt})} \frac{\lambda_w'(\theta_{ijt})}{\lambda_f'(\theta_{ijt})} = -\frac{1-\eta}{\eta}$. In the baseline model, we were able to further simplify (A44) in order to get a closed-form expression for the present value of wage payments W_{ijt+1}. However, we're not able to do so in this case due to the transfer system.

B.2.3 Detrending and BGP

The only equilibrium conditions which have changed relative to the baseline are the search FOC (A40) and the wage equation (A44). We therefore focus our discussion of detrending on those conditions. For the search FOC (A40), first note that

$$
W_{it+1}^H = d_{t+1} A_t(w_{ijt+1}) = d_{t+1}(1+g)^t A(\widetilde{w}_{ijt+1}) \implies \widetilde{W}_{ijt+1}^H = d_{t+1} A(\widetilde{w}_{ijt+1}),
$$

where the second equation uses our assumption that $A_t(w_{ijt+1}) = (1+g)^t A(\widetilde{w}_{ijt+1})$. Plugging this into the search FOC gives

$$
\begin{aligned}
(1+g)^t h'(s_{it}) &= Q_{t,t+1} \lambda_w(\theta_{it})(1+g)^{t+1} \left(\widetilde{W}_{it+1}^H + \widetilde{V}_{it+1}\right) \\
\implies h'(s_{it}) &= Q_{t,t+1}(1+g)\lambda_w(\theta_{it}) \left(\widetilde{W}_{it+1}^H + \widetilde{V}_{it+1}\right). \qquad \text{(A45)}
\end{aligned}
$$

To detrend the wage equation (A44), recall the property that $A_t'(w_{ijt+1}) = A'(\widetilde{w}_{ijt+1})$. Therefore,

$$
\widetilde{W}_{ijt+1} = \widetilde{\nu}_{ijt+1} - \frac{1-\eta}{\eta} \frac{\widetilde{W}_{ijt+1}^H + \widetilde{V}_{ijt+1}}{A'(\widetilde{w}_{ijt+1})} \text{ and } \gamma_{ijt+1} = \frac{\lambda_f(\theta_{ijt}) a_{ijt}}{A'(\widetilde{w}_{ijt+1})}. \qquad \text{(A46)}
$$

C Data Appendix

This appendix contains details about our data sources and targeted moments. We use data from the pooled 2017-2019 American Community Survey (ACS).[22] Our sample includes all individuals

[22] We downloaded the data directly from https://usa.ipums.org/usa/.

aged 16 and over. All observations are weighted using the weights provided by the ACS.

Share of College Workers. We define *college* individuals as those individuals who report having a bachelor's degree or higher. During the 2017-2019 period, 31.3% of our sample had at least a bachelor's degree.

Employment Rates. We focus on full-time employment and on workers strongly attached to the labor force. We define individuals as being *full-time* employed if 1) they are currently working at least 30 hours per week; 2) they reported working at least 29 weeks during the prior year; and 3) they reported positive labor earnings during the prior 12 month period. For our 2017-2019 sample, 46.8% of non-college individuals and 62.4% of college individuals worked full-time.

Share of Income Earned by College Workers. For the 2017-2019 period, 37.8% of individuals working full-time were college educated. Conditional on being full-time employed, mean annual earnings for college individuals total $91,706, whereas mean annual earnings for non-college individuals total $44,871. Given these statistics, we calculate that 55.5% of all earnings of full-time workers accrued to workers with at least a bachelor's degree.

Table 1: Average Wages by Education Group in ACS Data

	Less than High School	High School	Some College	*College*
[0.5ex] Average wage	$16.6	$19.6	$21	$37.4

Notes: Average wages of full-time workers by education group in ACS data.

Wage Distributions. We compute hourly wages for our sample of full-time workers by dividing annual labor earning by annual hours worked. We calculate annual hours worked as the product of weeks worked last year and reported usual hours worked. We impose two additional sample restrictions when measuring the wage distribution. First, we restrict the sample to only those workers who report at least $5,000 of labor earnings during the prior year. Second, we truncate the resulting distribution of hourly wages of each education group at the top and bottom 1%. All wages are converted to 2019 dollars using the June CPI-U. From these data, we compute the median wage and standard deviation of wages for each education group as well as the ratios of wages between the 10th percentile and the median for each of the education groups. These moments are used as part of our parameterization strategy. We also show that even though only those moments are targeted for each education group, our model matches the full distribution of wages for each education group quite closely. The heterogeneity of wages within education groups swamps the heterogeneity across education groups, motivating our choice to primarily focus on within-group heterogeneity. Related to this choice, Table 1 shows that the average wage of *each* education group is higher than $15 per hour. Hence, modeling within-group heterogeneity is necessary for even a high minimum wage to be binding for any worker.

D Validating Use of Long-Run Elasticity Estimates

We now provide the details about how we replicate Card and Lemieux (2001)'s estimation strategy in our model. As explained in the main text, Card and Lemieux (2001) exploit within-education-group variation in employment rates by age, which they identify as a skill level, z_i. We replicate this variation through exogenous changes in the measure of families, μ_{it}. Section D.1 shows how we extend our model to incorporate time-varying measures of families. Section D.2 then explains how we choose the specific path of μ_{it} to mimic the empirical variation utilized by Card and Lemieux (2001). Finally, Section D.3 shows the results of this exercise and explains why this procedure recovers a value for the long-run elasticity of substitution among workers very close to that estimated by Card and Lemieux (2001).

D.1 Model Extension

We model changes in the measure of families, μ_{it}, as a one time unanticipated shock after which agents in the model have perfect foresight. Specifically, the economy starts at an initial BGP with measures μ_i, and at time $t = 0$ agents learn about a new path of measures $\{\mu_{it}\}$ over time that converge to a new constant level μ_i^* at some point T in the future.

To proceed, we must first specify the objective function of a type-i family now that the measure of the family changes over time. We assume the family maximizes

$$\sum_{t=0}^{\infty} \beta^t \mu_{it} U_t(c_{it}, n_{it}, s_{it}),$$

where c_{it}, n_{it}, and s_{it} are per-capita variables. This "utilitarian" utility function captures the average utility of each household member and weighs each member equally. In our numerical experiments, the path of μ_{it} is such that there are new members of the family available to work in each period. We assume that each of these new family members' initial labor market state is unemployment. Hence, they must search for a period before they can be hired by a firm. Therefore, the law of motion for employment of family i at firm j is

$$\mu_{it+1}n_{ijt+1} = (1-\sigma)\mu_{it}n_{ijt} + \lambda_w(\theta_{ijt})\mu_{it}s_{ijt},$$

where n_{ijt} and s_{ijt} correspond to per-capita variables as in the baseline model. In our baseline, $\mu_{it+1} = \mu_{it}$ so this equation reduces to $n_{ijt+1} = (1-\sigma)n_{ijt} + \lambda_w(\theta_{ijt})s_{ijt}$.

In per-capita terms, the budget constraint is now

$$\sum_{t=0}^{\infty} Q_{0,t}\mu_{it}c_{it} \leq \mu_{i0}\left(\zeta_i\mathbb{P}_0 + \mathbb{I}_i\right) + \sum_{t=1}^{\infty} Q_{0,t}\mu_{it-1}\sum_j \lambda_w(\theta_{ijt-1})s_{ijt-1}W_{ijt},$$

where, as before, $\zeta_i\mathbb{P}_0$ is the per-capita share of family i in the present value of firm's profits and $\mathbb{I}_i$ is the per-capita present value of wages promised to initial workers. Putting all this together, the household's utility maximization problem is

$$\begin{aligned}
&\max_{c_{it},s_{ijt},n_{ijt+1}} \sum_{t=0}^{\infty} \beta^t \mu_{it} U_t(c_{it}, n_{it}, s_{it}) \\
&\text{s.t. } \mu_{it+1}n_{ijt+1} = (1-\sigma)\mu_{it}n_{ijt} + \lambda_w(\theta_{ijt})\mu_{it}s_{ijt} \\
&\quad \sum_{t=0}^{\infty} Q_{0,t}\mu_{it}c_{it} \leq \mu_{i0}\left(\zeta_i\mathbb{P}_0 + \mathbb{I}_i\right) + \sum_{t=1}^{\infty} Q_{0,t}\mu_{it-1}\sum_j \lambda_w(\theta_{ijt-1})s_{ijt-1}W_{ijt}
\end{aligned}$$

It turns out that this utility maximization problem leads to equilibrium conditions that naturally extend those of our baseline model, with the constant measure of each family μ_i from the baseline replaced by the time-varying measure μ_{it}. Results are available upon request.

D.2 Mimicking the Variation in Card and Lemieux (2001)

Card and Lemieux (2001) estimate the elasticity of substitution ϕ using residual variation in employment rates across different groups of workers within a given education group. A key assumption is that this residual variation reflects changes in labor supply. In this spirit, we mimic their variation by assuming that the measure of each family i, μ_i, changes over time as denoted by μ_{it} in a way consistent with their data. Card and Lemieux (2001) identify z_i by assuming that, within an education group, all workers within a 5-year age group share the same z_i. In their published paper, Card and Lemieux (2001) report the time series of the ratios of college to non-college employment rates $\frac{N_{jHt}}{N_{jLt}}$ for each age group j, but not the employment rates of each group N_{jHt} and N_{jLt} separately. We therefore proceed in two steps. First, we use additional assumptions to infer the time-series variation in N_{jHt} and N_{jLt} from what Card and Lemieux (2001) report about $\frac{N_{jHt}}{N_{jLt}}$

in the data. Second, we then back out the variation in the measures of families μ_{it} which replicates this variation in employment rates within our model.

Mimicking the Employment Rate Variation in Card and Lemieux (2001). Let $x_{jt} = \frac{N_{jHt}}{N_{jLt}}$ denote the ratio of college to non-college employment for age group j reported by Card and Lemieux (2001).[23] Card and Lemieux (2001) classify workers into $N = 6$ different age groups, which we assume correspond to different within-education group skill levels $z_{e,j}$ in our model so that the youngest group corresponds to $z_{e,1}$, the next youngest group corresponds to to $z_{e,2}$, and so on. The sample used by Card and Lemieux (2001) covers the period between 1960 and 1995. The spirit of our modeling exercise will be that the measures of families change over this 35-year sample, at which point they remain constant and the economy settles into a new BGP. We will further assume that this new BGP corresponds to the calibrated steady state of our model. Hence, we will choose the time path of the measures of families μ_{it} such that the percentage change in the model's employment series relative to the final BGP corresponds to Card and Lemieux (2001)'s variation relative to the end of their sample.[24]

We construct the percentage changes in the employment series N_{jLt} and N_{jHt} in the following way. First, we construct a time series of our targeted ratios $x_{jt} = N_{jHt}/N_{jLt}$ relative to their 1995 endpoint in Card and Lemieux (2001)'s sample. Second, in order to separately construct the levels of N_{jHt} and N_{jLt}, we will bring in additional information about the total employment of all workers in age group j, denoted $a_{jt} \equiv N_{jLt} + N_{jHt}$. Given a value of a_{jt} and the ratio x_{jt}, we can solve for $N_{jLt} = a_{jt}/(1 + x_{jt})$ and $N_{jHt} = x_{jt}a_{jt}/(1 + x_{jt})$. To compute the time series of a_{jt} for each age group j, we assume that the terminal value a_{jT} equals its value in our calibrated BGP, compute an initial value a_{j0} from the 1960 Census data described in Appendix C, and assume that a_{jt} grows at a constant rate over the sample.

Backing Out Variation in Measures of Families. Equipped with the resulting series of employment rates N_{it} for each worker type $i = (e, z)$, our second step consists of determining the measure of families μ_{it} in our model such that the model's equilibrium employment series $N_{it} = \mu_{it}n_{it}$ matches the data (recall that n_{it} is the per-capita employment rate of family of type

[23] In particular, Figure IV in Card and Lemieux (2001) reports cohort fixed effects, which capture 98% of the time-series variation in the employment ratios x_{jt}—intuitively, because college attainment decisions are made before workers enter the labor force. We back out the time-series variation in x_{jt} from these cohort effects. Since Card and Lemieux (2001) effectively report x_{jt} for 5-year intervals, we log-linearly interpolate between these points.

[24] Of course, the last year of the model's 35-year sample does not exactly equal the new BGP because it takes time for the economy to exactly converge to the new BGP once the measures stop changing. However, we find that the two points are close.

i in t). In principle, this step requires solving a complicated fixed-point problem. Namely, for any candidate path of measures μ_{it}, we must solve for the equilibrium of the model, derive the implied path of aggregate employment series N_{it}, check if it matches the empirical path, and if not, update the guess of each μ_{it}. Computationally, this procedure is prohibitively costly because it requires solving for the entire transition path for each candidate μ_{it}.

Instead, we compute the path of measures μ_{it} from a simpler approach that approximately matches the path of N_{it} from the data. In this exercise, we assume that the economy is initially along some BGP with measures μ_{i0}, where we choose μ_{i0} to match the initial values of N_{i0} from the data described above. At date $t = 0$, all agents unexpectedly learn about a new exogenous path of measures μ_{it} from $t = 0$ to a final point 35 years later, after which these measures remain constant. As described above, we assume that these final measures μ_{iT} correspond to the calibrated BGP from Section 4. In order to construct the time path of measures in between these two points, we compute the path of μ_{it} which solves the approximate relationship $N_{it}^{\text{data}} = \mu_{it} n_i^*$, where n_i^* is the per-capita employment rate for each family i in the final BGP. This relationship is an approximation because per-capital employment rates n_{it} may endogenously change over time in equilibrium in response to the changes in measures μ_{it}.

D.3 Results

Given the variation described, we construct an estimator for the long-run elasticity of substitution ϕ following an established approach in the literature. In particular, suppose — like the majority of the literature which estimates this elasticity — that we interpret the data through the lens of a static CES production function $G(N)$ under perfect labor market competition, and accordingly construct the natural estimator $\widehat{\phi}$ of ϕ. Our question now is: How biased would that estimator be if the true data-generating process were our model? We set to address this question next.

Estimator. To motivate the estimator we use, note first that under the assumption of a static and competitive labor market, the ratios of wages of workers with different skills equal the ratios of their marginal products,

$$\frac{w_{it}}{w_{jt}} = \frac{z_i}{z_j} \left(\frac{N_{it}}{N_{jt}}\right)^{-\frac{1}{\phi}}.$$

Under the CES form for $G(N)$, the ratio of marginal products depends on the ratio of employment rates as well as the ratio of skill levels z_i/z_j. Since these skill levels are constant over time, taking log-differences over time of the above condition gives

$$\Delta \log \left(\frac{w_{it}}{w_{jt}} \right) = -\frac{1}{\phi} \Delta \log \left(\frac{N_{it}}{N_{jt}} \right). \tag{D1}$$

Intuitively, we can use variation in labor supply to trace out firms' labor demand schedule, which depends on ϕ. We obtain an estimate of ϕ by estimating (D1) through a simple linear regression using our model-simulated data constructed above. To construct the ratios $\frac{w_{it}}{w_{jt}}$ and $\frac{N_{it}}{N_{jt}}$ in our model, we choose the middle skill level $j_0 = N/2$ to be in the denominator, and then compute $\frac{w_{e,j't}}{w_{e,j_0t}}$ for all $j' \neq j_0$ and $e \in \{L, H\}$. Following Card and Lemieux (2001), for each of these ratios, we compute 5-year time differences except for the first observation, which is a 10-year difference. We then run the unweighted regression across each group and time period.

Results. Table D.1, which reproduces Table 4 from the main text, illustrates the results of this exercise. The first column displays the true value of $\phi = 4$. The second column shows that our estimator leads to an estimate $\widehat{\phi} = 3.94$ of ϕ, very close to the true value. We conclude from this exercise that the estimates of the long-run elasticity of substitution in the literature provide appropriate discipline to pin down the value of ϕ in our model as well. The last column of the table shows that by estimating ϕ using a long difference between the final BGP and the initial BGP, we recover almost exactly the true value. The only slight difference stems from the presence of search frictions which, as noted in the main text, do not vary meaningfully across workers with different productivity. As noted in the main text, the estimator may not perform well in our model due to the existence of the putty-clay frictions. Because of these frictions, the 5-year time differences employed by Card and Lemieux (2001) reflects not only the true long-run substitution possibilities among workers assigned to new capital but also the employment ratios required to operate existing capital. Since these old employment ratios are rigid, taking them into account would bias the estimator $\widehat{\phi}$ downward relative to the true value of ϕ. From this perspective, it may be surprising that our estimator does so well, despite the importance of putty-clay frictions in determining the slow response of the economy to changes in policy.

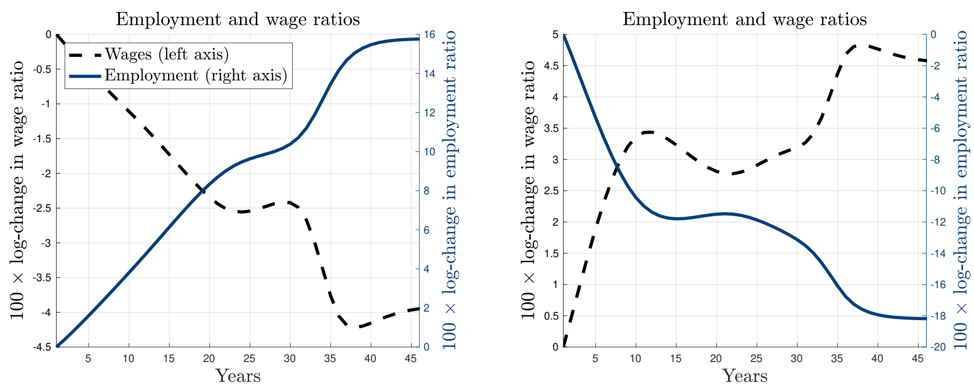
As argued in the main text, we interpret this finding as implying that putty-clay frictions are less important in shaping the value of ϕ that we recover than in shaping the response of the economy to, say, changes in the minimum wage. Figure D.1 further illustrates this logic. The two panels in the figure illustrate the wage and employment ratios for two non-college worker types j' relative to the base type j_0 over the entire transition path. For both types of workers, the log-change in

Table D.1: Estimation Strategy for Long-Run Elasticity ϕ in Literature

True value	Card and Lemieux (2001) Variation	Comparing BGPs Only
$\phi = 4$	$\widehat{\phi} = 3.94$	$\widehat{\phi} = 3.99$

Notes: Results from simulating the model for the time path of measures of families μ_{it} as described in text. *True value* reports the value of the long-run elasticity of substitution across workers $\phi = 4$ used to simulate the model. *Card and Lemieux (2001) Variation* reports the estimate $\widehat{\phi}$ from the regression in (D1) using 5-year time differences, with the exception of the first observation which is a 10-year difference. "Comparing BGPs Only" reports the estimator $\widehat{\phi}$ from (D1) using a long difference between the new BGP and the initial BGP.

Figure D.1: Variation in Employment and Wage Ratios for Estimating Elasticities



Notes: time path of employment ratios and wage ratios for individual non-college worker types for variation in measures of families μ_{it}. Employment ratios are $N_{L,j't}/N_{L,j_0t}$ where $N_{L,j't} = \mu_{L,j't} n_{L,j't}$ and $N_{L,j_0t} = \mu_{L,j_0t} n_{L,j_0t}$ are the total measures of workers of that type and $j_0 = N/3$ is the base type, as described in the text. Wage ratios are similarly defined as $w_{L,j't}/w_{L,j_0t}$, where $w_{L,j't}$ and w_{L,j_0t} are average wages among employed workers.

their employment ratios is approximately equal to $-\phi = -4$ times the log-change in their wage ratios by (D1). Firms' adjustment of these ratios is smooth because the underlying variation in the measures of families is itself smooth, and firms can predict it starting at $t = 0$.[25] In contrast, the policy experiments analyzed in the main text involve an immediate and unexpected jump in policy.

E Additional Quantitative Results

This Appendix contains two additional quantitative results about the minimum wage.

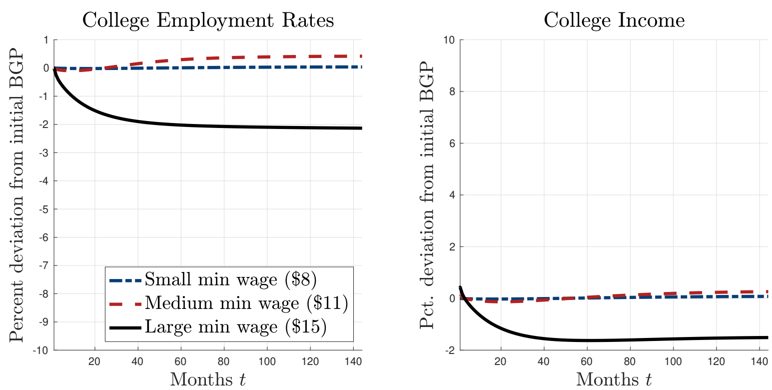
E.1 Impact of Minimum Wage Changes on College Workers

In the main text, we focused on how labor market policies affected employment and labor earnings of non-college workers. These workers are most likely to be effected by the labor market policies we studied. In this subsection, we show how minimum wage changes of various sizes would affect non-

[25] Consistent with this predictability, the adjustment of employment ratios vs. wage ratios is farthest from ϕ in the early stages of the transition.

college workers. Appendix Figure E.1 plots the transition paths of *aggregate college employment and labor income* following the introduction of our three illustrative minimum wages. The small minimum wage has almost no effect on college employment and labor income because it is not binding for nearly all college workers. The medium minium wage has a very small positive effect on college employment and income because it reduces the monopsony distortion of a few low productivity college workers. Finally, the large minimum wage has a slightly negative effect on college employment for two reasons. First, the large minimum wage now binds on some lower productivity college workers. Second, the large reduction in non-college employment also reduces the marginal product of college workers because they are complementary in production.

FIGURE E.1: Dynamic Effects of the Minimum Wage for College Workers

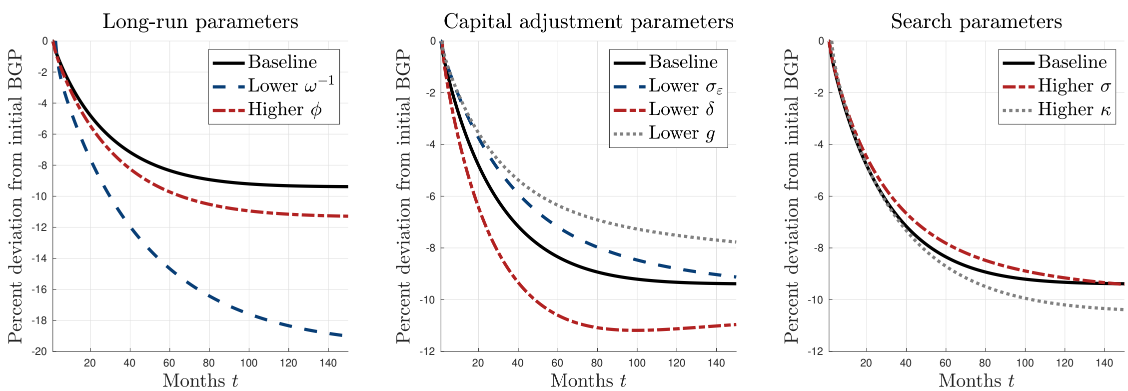


Notes: Transition paths of aggregate college employment (left panel) and labor income (right panel) following the introduction of the minimum wage. Employment is expressed in percentage deviation from the initial BGP. Labor income, net of trend growth $(1+g)^t$, is expressed relative to the initial BGP.

E.2 Additional Robustness

Figure 10 in the main paper showed the robustness of the speed of transition to the new BGP when we change various parameters. In this subsection of the appendix we show how those parameters affect long rung changes in aggregate employment for non-college workers in response to a $15 minimum wage. Specifically, Appendix Figure E.2 plots our sensitivity analysis from Figure 10 of the main paper in terms of percentage deviations from the initial BGP rather than percentage of the total long run change. In this space, we can better assess the long-run effects of these various parameterizations. For example, the parameterization with less monopsony power (i.e. higher ω) leads to a larger long-run decline in non-college employment.

FIGURE E.2: Sensitivity Analysis for \$15 Minimum Wage, Non-College Employment



Notes: Figure shows transition of non-college employment expresses as the percent deviation from the original BGP. "Baseline" corresponds to the model shown in Figure 6. "Higher ω^{-1}" corresponds to a degree of monopsony power of $\omega^{-1} = 1/6$ that produces an 85% markdown. "Higher ϕ" corresponds to a long-run elasticity of substitution within education groups of $\phi = 4.5$. "Lower σ_ε" corresponds to a standard deviation of idiosyncratic capital productivity of $\sigma_\varepsilon = 0.01$ that generates a steady-state capacity utilization rate of 97%. "Higher δ" sets the depreciation rate to $\delta = 20\%$ annually. "Lower g" corresponds to a trend growth rate of $g = 1\%$. "Higher σ" sets the job-destruction rate to $\sigma = 3.5\%$ monthly. "Higher κ" increases the baseline vacancy-posting cost κ_0 by 2.5 times, which approximately doubles the average hiring costs $\kappa_i/\lambda_f(\theta_i)$ to 125% of average monthly wage.