Supplemental Appendix to "Optimal Tariffs When Labor Income Taxes Are Distortionary"

This supplemental appendix first derives an implementability constraint in a T period deterministic version of the model. (The deterministic model can easily be made stochastic if financial markets are assumed to be complete.) It is then straightforward to prove a generalization of Proposition 3 for this dynamic framework.

Consider an environment in which Agents in Home live for T periods and have utility function:

$$\sum_{t=1}^{T} \delta^{t-1}\{u(c_{1t}, c_{2t}) - v(n_t)\}, 0 < \delta < 1$$

over consumptions of good 1 and good 2, and over labor. Home agents can borrow and lend good 1 in a world market at a fixed interest rate r. The government needs to finance a sequence of purchases $\{G_t\}_{t=1}^{T}$ of good 1. It is able to use tariffs $(\tau_t)_{t=1}^{T}$, labor income taxes $(\Phi_t)_{t=1}^{T}$, and interest income taxes $(\alpha_t)_{t=2}^{T}$. Agents begin life with no assets, and so there is no interest income tax in period 1.

Home agents have linear technologies to turn labor into good 1 at each date, with labor productivity A_t at date t. Home agents take prices as given and so face budget constraints of the form:

$$c_{1t} + P_t c_{2t}(1+\tau_t) + B_{t+1} + b_{t+1}$$
$$\leq n_t A_t(1-\Phi_t) + B_t(1+r(1-\alpha_t)) + b_t(1+r(1-\alpha_t)) + M_t, T \geq t \geq 1$$
$$B_{T+1}, b_{T+1} \geq 0$$

where $\{P_t\}_{t=1}^T$ represent the world prices of imports in terms of non-imports at each date. Here, B_t represents holdings of international debt (which pays off in terms of good 1) and b_t represents holdings of Home government debt that also pay off in terms of good 1. Both B_t and b_t are allowed to be negative prior to period $(T+1)$. In that case, the tax on interest income is actually a subsidy to borrowers. Finally, $\{M_t\}_{t=1}^T$ is a sequence of non-negative lump-sum transfers.

The world price P_t of good 2 is a fixed increasing function P of c_{2t} and there is an exogenous sequence $\{G_t\}_{t=1}^T$ of government purchases of good 1 that needs to be financed. It follows that the period-by-period flow resource constraints for Home is given by:

$$c_{1t} + P(c_{2t})c_{2t} + G_t + B_{t+1} = n_t A_t + B_t(1+r), T \geq t \geq 1$$
$$B_1 = 0, B_{T+1} = 0$$

These take account of the international inflows and outflows associated with asset purchases and payoffs. Note that if $A_t n_t < c_{1t} + G_t$ in period t, then Home is importing good 1 as well as good 2.

The government can borrow and lend from Home agents, but not from abroad. As a result, its flow budget constraint is given by:

$$\tau_t P(c_{2t})c_{2t} + n_t A_t \Phi_t + b_{t+1} + r\alpha_t b_t$$
$$= G_t + b_t(1+r) + M_t, T \geq t \geq 1$$
$$b_1 = 0, b_{T+1} = 0$$

To summarize, given $(\tau_t, A_t, \Phi_t, M_t)_{t=1}^{\infty}$, an equilibrium $(c_{1t}, c_{2t}, n_t, b_t, B_t)_{t=1}^{T}$ in Home is defined by the first order conditions:

$$u_1(c_{1t}, c_{2t})(1-\Phi_t)A_t = v'(n_t), T \geq t \geq 1$$
$$u_1(c_{1t}, c_{2t})P(c_{2t})(1+\tau_t) = u_2(c_{1t}, c_{2t}), T \geq t \geq 1$$
$$\delta(1 + r(1-\alpha_{t+1}))u_1(c_{1,t+1}, c_{2,t+1}) = u_1(c_{1t}, c_{2t}), (T-1) \geq t \geq 1$$

the resource constraints

$$c_{1t} + P(c_{2t})c_{2t} + G_t + B_{t+1} = B_t(1+r) + n_t A_t, T \geq t \geq 1$$
$$B_1, B_{T+1} = 0$$

and the government's sequence of budget constraints:

$$c_{2t}P(c_{2t})\tau_t + n_t\Phi_t + \alpha_t r(b_t + B_t) = b_{t+1} + G_t + M_t, T \geq t \geq 1$$
$$b_1, b_{T+1} = 0$$

We can then characterize the set of budget-feasible and resource-feasible allocations much as we did in the static case.

Proposition. *An allocation* $(c_{1t}, c_{2t}, n_t, B_t)_{t=1}^{T}$ *is an equilibrium for some* $(\alpha_t, \tau_t, \Phi_t)_{t=1}^{T}$ *if and* *only if the allocation satisfies the intertemporal resource constraint:*

$$\sum_{t=1}^{T}(1+r)^{-t}[A_t n_t - c_{1t} + P(c_{2t})c_{2t} + G_t] = 0$$

and an implementability constraint:

$$\sum_{t=1}^{T}\delta^{t-1}(u_1(c_{1t},c_{2t})c_{1t} + u_{2t}(c_{1t},c_{2t})c_{2t} - n_t v'(n_t)) \geq 0.$$

Proof. Suppose $(c_{1t}, c_{2t}, n_t, B_t)_{t=1}^{T}$ is an equilibrium given $(\tau_t, \Phi_t, M_t)_{t=1}^{T}$ and $(\alpha_t)_{t=2}^{T}$. These satisfy the sequence of resource constraints:

$$c_{1t} + P(c_{2t})c_{2t} + G_t + B_{t+1} = B_t(1+r) + n_t A_t, T \geq t \geq 1$$
$$B_1, B_{T+1} = 0$$

These imply the intertemporal resource constraint in the usual way. The government's period t budget constraint implies:

$$P(c_{2t})c_{2t}\tau_t + n_t\Phi_t A_t + \alpha_t r(B_t + b_t) + b_{t+1} = G_t + b_t(1+r) + M_t$$

Add this together with the period t resource constraint, and we get:

$$c_{1t} + P(c_{2t})c_{2t}(1+\tau_t) - n_t(1-\Phi_t)A_t + (B_{t+1} + b_{t+1}) = (b_t + B_t)(1 + (1-\alpha_t)r) + M_t$$

Multiply by $\delta^{t-1}u_1(c_{1t}, c_{2t})$ and add up to obtain the implementability constraint:

$$\sum_{t=1}^{T}\delta^{t-1}(u_1(c_{1t},c_{2t})c_{1t} + u_2(c_{1t},c_{2t})c_{2t} - v'(n_t)n_t) \geq 0.$$

This adding up makes use of the non-negativity of transfers and of equilibrium first order conditions:

$$\delta^{t-1} u_1(c_{1t}, c_{2t}) = \delta^t u_1(c_{1,t+1}, c_{2,t+1})(1 + (1 - \alpha_{t+1})r)$$

$$u_1(c_{1t}, c_{2t}) P(c_{2t})(1 + \tau_t) = u_2(c_{1t}, c_{2t})$$

$$u_1(c_{1t}, c_{2t})(1 - \Phi_t) A_t = v'(n_t)$$

$$B_{T+1} = b_{T+1} = 0$$

Now consider $(c_{1t}, c_{2t}, n_t)_{t=1}^T$ that satisfies the intertemporal resource constraint and the implementability constraints. Define $\{B_t\}_{t=2}^{T+1}$ by:

$$B_{t+1} = A_t n_t + B_t(1 + r) - c_{1t} - P(c_{2t}) c_{2t} - G_t$$

$$B_1 = 0.$$

The intertemporal resource constraint implies that $B_{T+1} = 0$.

Define:

$$\tau_t = \frac{u_2(c_{1t}, c_{2t})}{u_1(c_{1t}, c_{2t}) P(c_t)} - 1$$

$$\Phi_t = 1 - \frac{v'(n_t)}{u_1(c_{1t}, c_{2t})}$$

$$\alpha_{t+1} = 1 - \frac{\frac{u_1(c_{1t}, c_{2t})}{\delta u_1(c_{1,t+1}, c_{2,t+1})} - 1}{r}$$

to be taxes/tariff that satisfy the agents' first order conditions. Define a sequence of government bondholdings and a final-period government lump-sum transfer by:

$$b_{t+1} = G_t + b_t(1 + r) - P(c_{2t}) c_{2t} \tau_t - \Phi_t A_t n_t - \alpha_t r(b_t + B_t), T - 1 \geq t \geq 1.$$

$$-M_T = (G_T + b_T(1 + r) - P(c_{2T}) c_{2T} \tau_T - \Phi_T A_T n_T - \alpha_T r(b_T + B_T))$$

Substitute for G_t using the resource constraint:

$$G_t = B_t(1+r) + n_t A_t - c_{1t} - P(c_{2t})c_{2t} - B_{t+1}, T \geq t \geq 1$$

and we get:

$$b_{t+1} = (B_t + b_t)(1 + r(1-\alpha_t)) - c_{1t} - P(c_{2t})c_{2t}(1+\tau_t) + n_t(1-\Phi_t)A_t - B_{t+1}, T-1 \geq t \geq 1$$
$$-M_T = (B_T + b_T)(1 + r(1-\alpha_T)) - c_{1T} - P(c_{2T})c_{2T}(1+\tau_T) + n_T(1-\Phi_T)A_T$$

since $B_{T+1} = 0$.

We need to verify that the lump-sum transfer in the final period is indeed non-negative. Multiply through by $\delta^{t-1}u_1(c_{1t}, c_{2t})$ and adding up. we obtain:

$$\delta^{t-1}u_1(c_{1t}, c_{2t})b_{t+1} = -\delta^{t-1}u_1(c_{1t}, c_{2t})B_{t+1} - \sum_{s=1}^{t} \delta^{s-1}(u_1(c_{1s}, c_{2s})c_{1s} + u_2(c_{1s}, c_{2s})c_{2s} - v'(n_s)n_s), T-1 \geq t \geq$$
$$-M_T = -\sum_{s=1}^{T} \delta^{s-1}(u_1(c_{1s}, c_{2s})c_{1s} + u_2(c_{1s}, c_{2s})c_{2s} - v'(n_s)n_s)$$

The implementability constraint then implies that the last period transfer is non-negative:

$$M_T \geq 0.$$

□

With this proposition in hand, we can define the second-best policy problem in this dynamic setting as:

$$\max_{\{c_{1t},c_{2t},n_t\}_{t=1}^T} \sum_{t=1}^{T} \delta^{t-1}(u(c_{1t},c_{2t}) - v(n_t)\}$$

$$\text{s.t.}\sum_{t=1}^{T}(1+r)^{1-t}(c_{1t} + P(c_{2t})c_{2t} + G_t - A_t n_t) = 0$$

$$\sum_{t=1}^{T}\delta^{t-1}(u_1(c_{1t},c_{2t})c_{1t} + u_2(c_{1t},c_{2t})c_{2t} - v'(n_t)n_t) \geq 0.$$

The following proposition is a generalization of Proposition 3. The notation η_{imp} and η_{dom} refers, as in that Proposition, to within-period income elasticities of imports and non-imports respectively.

Proposition. *Suppose u has decreasing marginal rates of substitution. Suppose too that $(c_{1t}^{SB}, c_{2t}^{SB}, n_t^{SB})_{t=1}^T$ solves the above second-best policy problem and that $(c_{1t}^{FB}, c_{2t}^{FB}, n_t^{FB})_{t=1}^T$ solves the analog first-best policy problem without the implementability constraint. Assume that the standard optimal tariff revenue does not raise sufficient revenue to fund the stream of government purchases:*

$$\sum_{t=1}^{T}(1+r)^{1-t}(\tau_{STD}(c_{2t}^{FB})P(c_{2t}^{FB})c_{2t}^{FB} - G_t) < 0$$

Then:

$$\eta_{imp}(c_{1t}^{SB}, c_{2t}^{SB}) = \eta_{dom}(c_{1t}^{SB}, c_{2t}^{SB}) \Rightarrow \tau_t^{SB} = \tau_{STD}(c_{2t}^{SB})$$

$$\eta_{imp}(c_{1t}^{SB}, c_{2t}^{SB}) > \eta_{dom}(c_{1t}^{SB}, c_{2t}^{SB}) \Rightarrow \tau_t^{SB} < \tau_{STD}(c_{2t}^{SB})$$

$$\eta_{imp}(c_{1t}^{SB}, c_{2t}^{SB}) < \eta_{dom}(c_{1}^{SB}, c_{2}^{SB}) \Rightarrow \tau^{SB} > \tau_{STD}(c_{2}^{SB}).$$

Proof. Let λ be a multiplier on the resource constraint and μ be a multiplier on the implementability constraint. Then at any date t, $(c_{1t}^{SB}, c_{2t}^{SB}, n_t^{SB})$ satisfies the same first order necessary conditions that arose in the proof of Proposition 3:

$$u_1(c_{1t}^{SB}, c_{2t}^{SB}) - \lambda \tag{1}$$

$$= -\mu u_{11}(c_{1t}^{SB}, c_{2t}^{SB})c_{1t}^{SB} - \mu u_{21}(c_{1t}^{SB}, c_{2t}^{SB})c_{2t}^{SB} - \mu u_1(c_{1t}^{SB}, c_{2t}^{SB}) \tag{2}$$

$$u_2(c_{1t}^{SB}, c_{2t}^{SB}) - \lambda P'(c_2^{SB})c_2^{SB} - \lambda P(c_2^{SB}) \tag{3}$$

$$= -\mu u_{12}(c_{1t}^{SB}, c_{2t}^{SB})c_{1t}^{SB} - \mu u_{22}(c_{1t}^{SB}, c_{2t}^{SB})c_{2t}^{SB} - \mu u_2(c_{1t}^{SB}, c_{2t}^{SB}) \tag{4}$$

As long as $\mu > 0$, we can follow the proof of Proposition 3 to use these first order conditions to prove the current Proposition.

All that is left to prove is that $\mu > 0$. Suppose $\mu = 0$, and $(c_{1t}^{SB}, c_{2t}^{SB}, n_t^{SB}) = (c_{1t}^{FB}, c_{2t}^{FB}, n_t^{FB})$ for all $t = 1, ..., T$. The first-best allocation satisfies the optimality conditions:

$$\begin{aligned}
u_1(c_{1t}^{FB}, c_{2t}^{FB})A_t &= v'(n_t^{FB}), t = 1, ..., T \\
\frac{u_1(c_{1t}^{FB}, c_{2t}^{FB})}{u_2(c_{1t}^{FB}, c_{2t}^{FB})} &= P(c_2^{FB})(1 + \tau_{STD}(c_2^{FB})), t = 1, ..., T \\
u_1(c_{11}^{FB}, c_{21}^{FB}) &= \delta^{t-1}(1+r)^{1-t}u_1(c_{1t}^{FB}, c_{2t}^{FB}), t = 2, .., T
\end{aligned}$$

Then, we can plug these into the implementability constraint:

$$\begin{aligned}
&\sum_{t=1}^{T} \delta^{t-1}(u_1(c_{1t}^{FB}, c_{2t}^{FB})c_{1t}^{FB} + u_2(c_{1t}^{FB}, c_{2t}^{FB})c_{2t}^{FB} - v'(n_t^{FB})n_t^{FB}) \\
&= \sum_{t=1}^{T} \delta^{t-1}u_1(c_{1t}^{FB}, c_{2t}^{FB})(c_{1t}^{FB} + \frac{u_2(c_{1t}^{FB}, c_{2t}^{FB})}{u_1(c_{1t}^{FB}, c_{2t}^{FB})}c_{2t}^{FB} - \frac{v'(n_t^{FB})}{u_1(c_{1t}^{FB}, c_{2t}^{FB})}n_t^{FB}) \\
&= u_1(c_{11}^{FB}, c_{21}^{FB}) \sum_{t=1}^{T}(1+r)^{1-t}(c_{1t}^{FB} + P(c_{2t}^{FB})(1 + \tau_{STD}(c_2^{FB}))c_{2t}^{FB} - A_t n_t^{FB}) \\
&= u_1(c_{11}^{FB}, c_{21}^{FB}) \sum_{t=1}^{T}(1+r)^{1-t}(c_{1t}^{FB} + P(c_{2t}^{FB})(1 + \tau_{STD}(c_2^{FB}))c_{2t}^{FB} - A_t n_t^{FB}) \\
&= u_1(c_{11}^{FB}, c_{21}^{FB}) \sum_{t=1}^{T}(1+r)^{1-t}(\tau_{STD}(c_2^{FB}))P(c_2^{FB})c_{2t}^{FB} - G_t) \\
&< 0.
\end{aligned}$$

The assumption that $\mu = 0$ leads to a contradiction and so $\mu > 0$. $\square$