Appendix

For
"Monetary stabilization of sectoral tariffs"

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Appendix A. Equations from the benchmark model not listed in the main text

1. Demand equations

The composition of expenditure on adjustment costs, both for prices and bond holding, follows the same preferences as for consumption, and the associated demands mirror Eqs. (4)-(9). Adjustment costs for bond holding are as follows:

$$AC_{B,D,t} = \theta P_t AC_{B,t} / P_{D,t}$$

$$AC_{B,N,t} = (1-\theta) P_t AC_{B,t} / P_{N,t}$$

$$d_{AC,B,t}(h) = \left(p_t(h) / P_{D,t}\right)^{-\phi} AC_{B,D,t}$$

$$d_{AC,B,t}(f) = \left(p_t(f) T_{D,t} / P_{D,t}\right)^{-\phi} AC_{B,D,t}$$

$$AC_{B,H,t} = \nu\left(P_{H,t} / P_{N,t}\right)^{-\eta} AC_{B,N,t}$$

$$AC_{B,F,t} = (1-\nu)\left(P_{F,t} T_{N,t} / P_{N,t}\right)^{-\eta} AC_{B,N,t} .$$

The economy-wide demand for goods arising from price adjustment costs sums across the demand arising among *n* home firms: $AC_{P,t} = n_t AC_{P,t}(h)$. This is allocated as follows:

$$AC_{P,D,t} = \theta P_t AC_{P,t} / P_{D,t}$$

$$AC_{P,N,t} = (1-\theta) P_t AC_{P,t} / P_{N,t}$$

$$d_{AC,P,t}(h) = \left(p_t(h) / P_{D,t}\right)^{-\phi} AC_{P,D,t}$$

$$d_{AC,P,t}(f) = \left(p_t(f) T_{D,t} / P_{D,t}\right)^{-\phi} AC_{P,D,t}$$

$$AC_{P,H,t} = \nu\left(P_{H,t} / P_{N,t}\right)^{-\eta} AC_{P,N,t}$$

$$AC_{P,F,t} = (1-\nu)\left(P_{F,t} T_{N,t} / P_{N,t}\right)^{-\eta} AC_{P,N,t} .$$

The demand for differentiated goods for use as intermediates in production mirrors Eqs. (6)-(7), as follows:

$$d_{G,t}(h) = \left(p_t(h) / P_{D,t}\right)^{-\phi} G_t$$

$$d_{G,t}(f) = \left(p_t(f) T_{D,t} / P_{D,t}\right)^{-\phi} G_t .$$

The demand for differentiated goods for use in the sunk entry investment of new firms mirrors Eqs. (6)-(7), as follows:

$$d_{K,t}(h) = \left(p_t(h) / P_{D,t}\right)^{-\phi} ne_t K_t$$

$$d_{K,t}(f) = \left(p_t(f) T_{D,t} / P_{D,t}\right)^{-\phi} ne_t K_t .$$

2. Market clearing conditions

Market clearing for the non-differentiated goods market requires:

$$y_{H,t} = C_{H,t} + AC_{P,H,t} + AC_{B,H,t} + \left(1+\tau_N\right)\left(C^*_{H,t} + AC^*_{P,H,t} + AC^*_{B,H,t}\right)$$

$$y_{F,t} = \left(1+\tau^*_N\right)\left(C_{F,t} + AC_{P,F,t} + AC_{B,F,t}\right) + C^*_{F,t} + AC^*_{P,F,t} + AC^*_{B,F,t} .$$

Labor market clearing requires:

$$\int_0^{n_t} l_t(h)dh + l_{H,t} = l_t .$$

Bond market clearing requires:

$$B_{Ht} + B^*_{Ht} = 0$$

$$B_{Ft} + B^*_{Ft} = 0 .$$

Balance of payments requires:

$$\int_0^{n_t} p^*_t(h)\left(d^*_t(h)\right)dh - \int_0^{n^*_t} p_t(f)\left(d_t(f)\right)df + P^*_{Ht}\left(C^*_{H,t} + AC^*_{P,H,t} + AC^*_{B,H,t}\right)$$
$$-P_{F,t}\left(C_{F,t} + AC_{P,F,t} + AC_{B,F,t}\right) - i_{t-1}B^*_{H,t-1} + e_t i^*_{t-1}B_{F,t-1} = \left(B^*_{H,t} - B^*_{H,t-1}\right) + e_t\left(B_{F,t} - B_{F,t-1}\right).$$

Appendix B: Derivation of Analytical Results, Producer Currency Pricing

1. Demands

The modified consumption index implies the following demands:

$$C_{DHt} = \frac{1}{2}\frac{P_{Dt}C_{Dt}}{P_{DHt}} = \frac{\theta}{2}\frac{P_tC_t}{P_{DHt}}$$

$$C_{DFt} = \frac{1}{2}\frac{P_{Dt}C_{Dt}}{T_{Dt}e_tP^*_{DFt}} = \frac{\theta}{2}\frac{P_tC_t}{T_{Dt}e_tP^*_{DFt}}$$

$$C^*_{DHt} = \frac{\theta}{2}\frac{e_tP^*_tC^*_t}{T^*_{Dt}P_{DHt}} = \frac{\theta}{2}\frac{P_t\,C_t}{T^*_{Dt}P_{DHt}}$$

$$C_{Dt} = \theta\frac{P_tC_t}{P_{Dt}}$$

$$C_{Nt} = \left(1-\theta\right)\frac{P_tC_t}{P_{Nt}}$$

$$c_t(h) = \left(p_t\left(h\right)/P_{DH,t}\right)^{-\phi}C_{DH,t}$$

2. Optimal price setting for differentiated good

The home firm maximizes

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left[\left(p_t\left(h\right)-\frac{W_t}{\alpha_D}\right)\left(p_t\left(h\right)/P_{DH,t}\right)^{-\phi}C_{DH,t}+\left(p_t\left(h\right)-\frac{W_t}{\alpha_D}\right)\left(\left(\frac{T_t^*p_t\left(h\right)}{e_t}\right)/\left(\frac{T_t^*P_{DH,t}}{e_t}\right)\right)^{-\phi}C^*_{DH,t}\right]\right]$$

or $E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(p_t\left(h\right)-\frac{W_t}{\alpha_D}\right)\left(p_t\left(h\right)/P_{DH,t}\right)^{-\phi}\left(C_{DH,t}+C^*_{DH,t}\right)\right]$,

implying the price setting rule

$$P_{Ht} = \frac{\phi}{\phi-1}\frac{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}+C^*_{DH,t}\right)\frac{W_t}{\alpha_D}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}+C^*_{DH,t}\right)\right]}.$$

Substitute in demands from above

$$P_{Ht} = \frac{\phi}{\phi-1}\frac{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(\frac{\theta}{2}\frac{P_tC_t}{P_{DHt}}+\frac{\theta}{2}\frac{e_tP^*_tC^*_t}{T^*_{Dt}P_{DHt}}\right)\frac{W_t}{\alpha_D}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(\frac{\theta}{2}\frac{P_tC_t}{P_{DHt}}+\frac{\theta}{2}\frac{e_tP^*_tC^*_t}{T^*_{Dt}P_{DHt}}\right)\right]},$$

and substitute in for μ_t and exchange rate

$$P_{DHt}=\frac{\phi}{\phi-1}\frac{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(\frac{\theta}{2}\frac{\mu_t}{P_{DHt}}+\frac{\theta}{2}\frac{e_t\mu_t^*}{T_{Dt}^*P_{DHt}}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(\frac{\theta}{2}\frac{\mu_t}{P_{DHt}}+\frac{\theta}{2}\frac{e_t\mu_t^*}{T_{Dt}^*P_{DHt}}\right)\right]}.$$

Use $e_t=\frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}$ from the main text:

$$P_{DHt}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}.$$

The foreign firm counterpart is:

$$P_{DFt}^*=\frac{\phi}{\phi-1}\frac{E_{t-1}\beta\left[\frac{\mu_{t-1}^*}{\mu_t^*}\left(C^*{}_{DFt}+C_{DF,t}\right)\frac{W_t^*}{\alpha_D^*}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}^*}{\mu_t^*}\left(C^*{}_{DFt}+C_{DF,t}\right)\right]}.$$

Substitute in for in μ_t^* mu and exchange rate:

$$P_{DFt}^*=\frac{\phi}{\phi-1}\frac{E_{t-1}\beta\left[\frac{\mu_{t-1}^*}{\mu_t^*}\left(\frac{\mu_t^*}{P_{DFt}^*}+\frac{\mu_t}{T_{Dt}e_tP_{DFt}^*}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\beta\left[\frac{\mu_{t-1}^*}{\mu_t^*}\left(\frac{\mu_t^*}{P_{DFt}^*}+\frac{\mu_t}{T_{Dt}e_tP_{DFt}^*}\right)\right]}$$

$$P_{DFt}^*=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}$$

So the home price index can be written:

$$P_t=2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]P_{DHt}{}^{\theta/2}\left(T_{Dt}e_tP_{DFt}^*\right)^{\theta/2}P_{NHt}{}^{1-\theta}.$$

Use $P_{NHt}=\frac{W_t}{\alpha_N}=\frac{\kappa\mu_t}{\alpha_N}$ to write the price index in terms of exogenous variables:

$$P_t=2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]P_{DHt}{}^{\theta/2}\left(T_{Dt}e_tp_t^*(f)\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}$$

$$P_t = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\right)^{\theta/2}\left(T_{Dt}e_t\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_{Dt}^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}.$$

3. Labor

Given the homogenous second sector, the easiest way to derive equilibrium labor is from the household budget constraint, which under balanced trade, implies labor income equals total nominal expenditure minus profits from the home differentiated sector.

Write the household budget constraint:

$$W_t l_t + \pi_t = P_t C_t,$$

where π is profits of home differentiated goods firms, used in the firm maximization problem above to determine price setting. Use labor supply condition to substitute out wage: $\kappa\mu_t l_t + \pi_t = \mu_t$, and use this to compute the term in welfare including labor, $\kappa E_{t-1}\left[l_t\right]$:

$$\kappa E_{t-1}\left[l_t\right] = 1 - E_{t-1}\left[\frac{\pi_t}{\mu_t}\right].$$

Compute profit for the home differentiated good producer:

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(P_{DHt}C_{DHt} + P_{DHt}C^*_{DHt}\right)\right].$$

Use $C_{DHt} + C^*_{DHt} = \frac{\theta}{2}\frac{\mu_t}{P_{DHt}} + \frac{\theta}{2}\frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}\frac{\mu_t^*}{T_{Dt}^* P_{DHt}}$ from price setting derivation

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(P_{DHt}\frac{\theta}{2}\frac{\mu_t}{P_{DHt}} + P_{DHt}\frac{\theta}{2}\frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}\frac{\mu_t^*}{T_{Dt}^* P_{DHt}}\right)\right]$$

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = \frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]$$

So $\kappa E_{t-1}\left[l_t\right] = 1 - \frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right].$

Note that there is no μ_t left in this term, so the labor term in the welfare condition will have no bearing on the optimal monetary policy under our specifcation.

4. Home optimal policy, Nash

Write home welfare, and express as a function of exogenous variables.

$$W_t = E_{t-1} \ln C_t - E_{t-1} \kappa l_t$$

$$W_t = E_{t-1}\left[\ln \mu_t\right] - E_{t-1}\left[\ln P_t\right] - E_{t-1}\kappa l_t$$

$$W_t = E_{t-1}\left[\ln \mu_t\right] - E_{t-1}\left[\ln\left(2\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\right)^{\theta/2}\left(T_{Dt}\frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_{Dt}^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}\right)\right] - \left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]\right)$$

$$W_t = E_{t-1}\left[\ln \mu_t\right] - \ln\left(2\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\right) - \theta\ln\left(\frac{\phi}{\phi-1}\right) - \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]\right) + \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\right)$$
$$-\frac{\theta}{2}E_{t-1}\left[\ln\left(T_{Dt}\right) + \ln\mu_t - \ln\mu_t^* + \ln\alpha_N^* - \ln\alpha_N - \ln T_{N,t}\right] - \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]\right) + \frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]\right) - (1-\theta)E_{t-1}\left(\ln\kappa + \ln\mu_t - \ln\alpha_N\right)$$
$$-1 + \frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]$$

Differentiate welfare with respect to the home monetary policy variable μ:

$$\frac{1}{\mu} - \frac{\theta}{2}\frac{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]} - \left(\frac{\theta}{2}+1-\theta\right)\frac{1}{\mu} = 0$$

$$\mu = \frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu\right]}{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)}$$

Conjecture the solution: $\mu = a\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)^{-1}$,

which is easily verified by substituting this in the equation immediately above.

e) Foreign optimal policy, Nash

The foreign price index is:

$$P_t^* = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(T_{Dt}^*\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}/e_t\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}$$

$$P_t^* = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(T_{Dt}^*\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\frac{\mu_t^*\alpha_N T_{N,t}}{\mu_t\alpha_N^*}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}$$

Foreign labor:

$$\kappa E_{t-1}\left[l_t^*\right] = 1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t}T_{Nt}^*}\right]$$

Welfare:

$$W_t^* = E_{t-1}\ln C_{t\,t}^* - E_{t-1}\kappa l_t^*$$

$$W_t^* = E_{t-1}\left[\ln\mu_t\right]-E_{t-1}\left[\ln P_t^*\right]-E_{t-1}\kappa l_t^*$$

$$W_t^* = E_{t-1}\left[\ln\mu_t^*\right]-E_{t-1}\left[\ln\left(2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]}\right)^{\theta/2}\left(T_{Dt}^*\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\frac{\mu_t^*\alpha_N T_{N,t}}{\mu_t\alpha_N^*}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}\right)\right]-\left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t}T_{Nt}^*}\right]\right)$$

$$W_t^* = E_{t-1}\left[\ln\mu_t^*\right]-\ln\left(2\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\right)-\theta\ln\left(\frac{\phi}{\phi-1}\right)-\frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]\right)+\frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\right]\right)$$

$$-\frac{\theta}{2}E_{t-1}\left[\ln\left(T_{Dt}^*\right)+\ln\mu_t^*-\ln\mu_t-\ln\alpha_N^*+\ln\alpha_N+\ln T_{Nt}\right]-\frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]\right)+\frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\right)-(1-\theta)E_{t-1}\left(\ln\kappa+\ln\mu_t^*-\ln\alpha_N^*\right)$$

$$1+\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t}T_{Nt}^*}\right]$$

Which is directly analogous to home, with solution $\mu_t^* = a\left(1+\frac{\alpha_N}{\alpha_N^*}\frac{T_{Nt}}{T_{Dt}}\right)^{-1}$

5. Cooperative policy

Take derivative of sum of home and foreing welfare with respect to μ_t

$$\frac{1}{\mu}-\frac{\theta}{2}\frac{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}-\left(\frac{\theta}{2}+1-\theta\right)\frac{1}{\mu}+\frac{\theta}{2}\frac{1}{\mu}-\frac{\theta}{2}\frac{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}=0$$

$$\mu=\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}}.$$

Conjecture solution: $\mu = a\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)^{-1}$, which is easily verified

We note that the cooperative solution is same as Nash in this case.

Now take the derivative with respect to μ_t^*:

$$\frac{1}{\mu_t^*}-\frac{\theta}{2}\frac{\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{1}{\alpha_D^*}}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}-\left(\frac{\theta}{2}+1-\theta\right)\frac{1}{\mu}+\frac{\theta}{2}\frac{1}{\mu}-\frac{\theta}{2}\frac{\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{1}{\alpha_D^*}}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}=0$$

$$\theta\frac{1}{\mu_t^*}-\theta\frac{\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{1}{\alpha_D^*}}{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\frac{\mu_t^*}{\alpha_D^*}\right]}=0$$

$$\mu_t^*=\frac{E_{t-1}\left[\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)\mu_t^*\right]}{\left(1+\frac{\alpha_N T_{N,t}}{\alpha_N^* T_{Dt}}\right)}$$

Conjecture same solution as before: $\mu_t^* = a\left(1+\frac{\alpha_N}{\alpha_N^*}\frac{T_{Nt}}{T_{Dt}}\right)^{-1}$, verified.

This is also same as Nash solution above.

6. Defining conditions under which home is both producer and net importer of homogeneous good

We can easily compute home consumption of the non-differentiated good:

$$C_{Nt}=\left(1-\theta\right)\frac{P_t C_t}{P_{Nt}},$$

where $P_{Nt}=e_t T_{N,t}P_{Nt}^*$ and $P_{Nt}=\frac{W_t}{\alpha_N}=\frac{\kappa\mu_t}{\alpha_N}$, $P_{Nt}^*=\frac{W_t^*}{\alpha_N^*}=\frac{\kappa\mu_t^*}{\alpha_N^*}$.

So $C_{Nt} = (1-\theta)\frac{P_t C_t}{\frac{\kappa\mu_t}{\alpha_N}} = (1-\theta)\frac{\mu_t}{\frac{\kappa\mu_t}{\alpha_N}} = (1-\theta)\frac{\alpha_N}{\kappa}$.

Note this is consant, unaffected by tariffs.

Now compute the level of home production in this sector based on labor allocation. Recall total labor allocation above:

$$l_t = \frac{1}{\kappa} - \frac{1}{\mu_t}\frac{1}{\phi\kappa}\left(P_{DHt}C_{DHt} + P_{DHt}C^*_{DHt}\right).$$

We next subtract labor for the differentiated goods sector: $\frac{C_{DHt} + C^*_{DHt}}{\alpha_D}$.

So the labor allocation for the non-differentiated sector becomes:

$$l_{Nt} = \frac{1}{\kappa} - \frac{1}{\mu_t}\frac{1}{\phi\kappa}\left(P_{DHt}C_{DHt} + P_{DHt}C^*_{DHt}\right) - \frac{C_{DHt} + C^*_{DHt}}{\alpha_D}$$

$$l_t = \frac{1}{\kappa} - \frac{1}{\phi\kappa}\frac{\theta}{2}\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] - \frac{1}{\alpha_D}\left(\frac{\theta}{2}\frac{\mu_t}{P_{DHt}} + \frac{\theta}{2}\frac{\mu_t \alpha_N^*}{\mu_t^* \alpha_N T_{N,t}}\frac{\mu_t^*}{T_{Dt}^* P_{DHt}}\right)$$

$$l_t = \frac{1}{\kappa} - \left(\frac{1}{\phi\kappa}\frac{\theta}{2} + \frac{1}{\alpha_D}\frac{\theta}{2}\frac{\mu_t}{P_{DHt}}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right]$$

$$l_{Nt} = \frac{1}{\kappa} - \left(\frac{1}{\phi\kappa}\frac{\theta}{2} + \frac{1}{\alpha_D}\frac{\theta}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right].$$

So output in the sector may be written:

$$y_{Nt} = \frac{\alpha_N}{\kappa} - \left(\frac{\alpha_N}{\phi\kappa}\frac{\theta}{2} + \frac{\alpha_N}{\alpha_D}\frac{\theta}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right].$$

We conclude that the condition for where $C_{Nt} > y_{Nt}$ may be written:

$$(1-\theta)\frac{\alpha_N}{\kappa} - \frac{\alpha_N}{\kappa} + \left(\frac{\alpha_N}{\phi\kappa}\frac{\theta}{2} + \frac{\alpha_N}{\alpha_D}\frac{\theta}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1 + \frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}\right)\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] > 0$$

$$(-1)+\left(\frac{1}{\phi}\frac{1}{2}+\frac{1}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu_t\right]}\right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]>0$$

$$\left(\frac{1}{\phi}\frac{1}{2}+\frac{1}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu_t\right]}\right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]>1.$$

Evaluate this condition under perfect foresight, abstracting from risk premium in pricing of the sticky price good:

$$\left(\frac{1}{\phi}\frac{1}{2}+\frac{1}{2}\frac{\phi-1}{\phi}\right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]>1$$

$$1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}>2$$

$$\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}>1.$$

Derive the condition under which we also can guarantee positive home production of the non-differentiated good:

$$y_{Nt}=\frac{\alpha_N}{\kappa}-\left(\frac{\alpha_N}{\phi\kappa}\frac{\theta}{2}+\frac{\alpha_N}{\alpha_D}\frac{\theta}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}\right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]>0$$

$$y_{Nt}=1-\left(\frac{1}{\phi}\frac{\theta}{2}+\frac{\theta}{2}\frac{\phi-1}{\phi}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\mu_t}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu_t\right]}\right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]>0.$$

Again, evaluate in perfect foresight, abstracting from the risk premium in pricing of sticky-price goods:

$$y_{Nt}=1-\left(\frac{1}{\phi}\frac{\theta}{2}+\frac{\theta}{2}\frac{\phi-1}{\phi}\right)\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]>0$$

$$y_{Nt} = 1 - \frac{\theta}{2}\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right] > 0$$

$$\frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*} < \frac{2}{\theta} - 1\,.$$

Appendix C: Derivation of Analytical Results, Local Currency Pricing

1. Price setting:

Home good now has distinct prices in home and foreign market, P_{DHt} and P^*_{DHt}, with foreign price in foreign currency.

The home firm maximizes :

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left[\left(p_t(h)-\frac{W_t}{\alpha_t}\right)\left(p_t(h)/P_{DH,t}\right)^{-\phi}C_{DH,t}+\left(e_t p_t^*(h)-\frac{W_t}{\alpha_t}\right)\left(\frac{T_t^* p_t^*(h)}{T_t^* P_{DH,t}^*}\right)^{-\phi}C^*{}_{DH,t}\right]\right]$$

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left[\left(p_t(h)-\frac{W_t}{\alpha_t}\right)\left(p_t(h)/P_{DH,t}\right)^{-\phi}C_{DH,t}+\left(e_t p_t^*(h)-\frac{W_t}{\alpha_t}\right)\left(p_t^*(h)/P_{DH,t}^*\right)^{-\phi}C^*{}_{DH,t}\right]\right]$$

With respect to $p_t(h)$:

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\right]=E_{t-1}\beta\left[\phi\frac{1}{P_{DHt}}\frac{\mu_{t-1}}{\mu_t}\left(P_{DHt}-\frac{W_t}{\alpha_D}\right)\left(C_{DH,t}\right)\right]$$

$$P_{dHt}E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\right]=P_{DHt}E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\right]-E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\frac{W_t}{\alpha_D}\right]$$

$$(\phi-1)P_{DHt}E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\right]=E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}\left(C_{DH,t}\right)\frac{W_t}{\alpha_D}\right]$$

$$P_{DHt}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}C_{DH,t}\frac{W_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}C_{DH,t}\right]}$$

$$P_{DHt}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\theta}{2}\frac{\mu_t}{P_{DH,t}}\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\theta}{2}\frac{\mu_t}{P_{DH,t}}\right]}$$

$$P_{DHt}=\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D}E_{t-1}\left[\mu_t\right]$$

Maximizing with respect to $p_t^*(h)$:

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]=E_{t-1}\beta\left[\phi\frac{1}{P_{DH,t}^*}\frac{\mu_{t-1}}{\mu_t}\left(e_t P_{DH,t}^*-\frac{W_t}{\alpha_D}\right)C^*{}_{DH,t}\right]$$

$$E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]=E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]-\frac{1}{P_{DH,t}^*}E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}C^*{}_{DH,t}\frac{W_t}{\alpha_D}\right]$$

$$(\phi-1)P^*_{DH,t}E_{t-1}\beta\left[\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]=E_{t-1}\beta\left[\phi\frac{\mu_{t-1}}{\mu_t}C^*{}_{DH,t}\frac{W_t}{\alpha_D}\right]$$

$$P^*_{DH,t}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}C^*{}_{DH,t}\frac{W_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}e_t C^*{}_{DH,t}\right]}$$

$$P^*_{DH,t}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\theta}{2}\frac{P^*_t}{T^*_{Dt}P^*_{DHt}}C^*_t\frac{W_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}e_t\frac{\theta}{2}\frac{P^*_t}{T^*_{Dt}P^*_{DHt}}C^*_t\right]}.$$

Use the property of a homogeneous sector above: $e_t=\frac{\mu_t\alpha^*_N}{\mu^*_t\alpha_N T_{N,t}}$:

$$P^*_{DH,t}=\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\theta}{2}\frac{\mu^*_t}{T^*_{Dt}P^*_{DHt}}\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\frac{\mu_{t-1}}{\mu_t}\frac{\mu_t\alpha^*_N}{\mu^*_t\alpha_N T_{N,t}}\frac{\theta}{2}\frac{\mu^*_t}{T^*_{Dt}P^*_{DHt}}\right]}$$

and cancel terms:

$$P^*_{DH,t}=\frac{\phi}{\phi-1}\frac{\frac{\kappa}{\alpha_D}E_{t-1}\left[\frac{\mu^*_t}{T^*_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha^*_N}{\alpha_N T_{Nt}T^*_{Dt}}\right]}.$$

Analogously for foreign differentiated good:

$$P^*_{Ft}=\frac{\phi}{\phi-1}\frac{\kappa}{\alpha^*_D}E_{t-1}\left[\mu^*_t\right]$$

$$P_{Ft}=\frac{\phi}{\phi-1}\frac{\kappa}{\alpha^*_D}\frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha^*_N T_{Dt}}\right]}.$$

Non-differentiated prices are the same as in PCP case above. Substitute into the home price index:

$$P_t = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D}E_{t-1}\left[\mu_t\right]\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}\frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}.$$

Analogously for foreign price index:

$$P_t^* = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}E_{t-1}\left[\mu_t^*\right]\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\frac{\kappa}{\alpha_D}E_{t-1}\left[\frac{\mu_t^*}{T_{Dt}^*}\right]}{E_{t-1}\left[\frac{\alpha_N^*}{\alpha_N T_{Nt} T_{Dt}^*}\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}$$

2. Home equilibrium labor:

Use labor supply condition to substitute out wage:

$$\kappa\mu_t l_t + \pi_t = \mu_t .$$

Use this to compute term for labor required in the welfare fucntion: $\kappa E_{t-1}\left[l_t\right]$:

$$\kappa E_{t-1}\left[l_t\right] = 1 - E_{t-1}\left[\frac{\pi_t}{\mu_t}\right].$$

Compute profit for the home D good producer:

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(P_{DHt}C_{DHt} + e_t P_{DHt}^* C_{DHt}^*\right)\right].$$

Use demands from above: $C_{DH,t} = \frac{\theta}{2}\frac{P_t C_t}{P_{DH,t}}$, $C_{DF,t} = \frac{\theta}{2}\frac{P_t C_t}{T_{Dt} P_{DF,t}}$, $C_{DHt}^* = \frac{\theta}{2}\frac{P_t^* C_t^*}{T_{Dt}^* P_{DHt}^*}$

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(P_{DHt}\frac{\theta}{2}\frac{P_t C_t}{P_{DH,t}} + e_t P_{DHt}^*\frac{\theta}{2}\frac{P_t^* C_t^*}{T_{Dt}^* P_{DHt}^*}\right)\right]$$

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(\frac{\theta}{2}\mu_t + e_t\frac{\theta}{2}\frac{\mu_t^*}{T_{Dt}^*}\right)\right].$$

Sub in for exchange rate:

$$E_{t-1}\left[\frac{\pi_t}{\mu_t}\right] = E_{t-1}\left[\frac{1}{\mu_t}\frac{1}{\phi}\left(\frac{\theta}{2}\mu_t + \frac{\mu_t\alpha_N^*}{\mu_t^*\alpha_N T_{N,t}}\frac{\theta}{2}\frac{\mu_t^*}{T_{Dt}^*}\right)\right]$$

So $\kappa E_{t-1}\left[l_t\right] = 1 - \frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1 + \frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right].$

Note this is the same as under PCP pricing.

3. Compute welfare:

Home:

$$W_t = E_{t-1} \ln C_t - E_{t-1} \kappa l_t$$
$$W_t = E_{t-1}\left[\ln \mu_t\right] - E_{t-1}\left[\ln P_t\right] - E_{t-1}\kappa l_t$$

$$W_t = E_{t-1}\left[\ln \mu_t\right] - E_{t-1}\left[\ln\left(\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D}E_{t-1}\left[\mu_t\right]\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}\frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}\right)\right]$$
$$-\left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right]\right)$$

$$W_t = E_{t-1}\left[\ln \mu_t\right] - \frac{\theta}{2}E_{t-1}\ln\left(E_{t-1}\left[\mu_t\right]\right) - \frac{\theta}{2}E_{t-1}\ln\left(E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]\right) - (1-\theta)E_{t-1}\left[\ln \mu_t\right]$$
$$-E_{t-1}\left[\ln\left(\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D}\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}\frac{1}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]}\right)^{\theta/2}\left(\frac{\kappa}{\alpha_N}\right)^{(1-\theta)}\right)\right] - \left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t} T_{Dt}^*}\right]\right)$$

Compute derivative of home welfare with respect to μ_t:

$$\frac{1}{\mu_t} - \frac{\theta}{2}\frac{1}{E_{t-1}\left[\mu_t\right]} - \frac{\theta}{2}\frac{\frac{1}{T_{Dt}}}{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]} - (1-\theta)\frac{1}{\mu_t} = 0$$

$$\theta\frac{1}{\mu_t} - \frac{\theta}{2}\frac{1}{E_{t-1}\left[\mu_t\right]} - \frac{\theta}{2}\frac{\frac{1}{T_{Dt}}}{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]} = 0$$

$$\frac{1}{\mu_t} = \frac{1}{2}\frac{1}{E_{t-1}\left[\mu_t\right]} + \frac{1}{2}\frac{\frac{1}{T_{Dt}}}{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}$$

Foreign:

$$W_t^* = E_{t-1}\ln C_{t\,t}^* - E_{t-1}\kappa l_t^*$$

$$W_t^* = E_{t-1}\left[\ln \mu_t\right] - E_{t-1}\left[\ln P_t^*\right] - E_{t-1}\kappa l_t^*$$

$$W_t^* = E_{t-1}\left[\ln \mu_t^*\right] - E_{t-1}\left[\ln\left(2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1} + \left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}E_{t-1}\left[\mu_t^*\right]\right)^{\theta/2}\left(\frac{\phi}{\phi-1}\frac{\frac{\kappa}{\alpha_D}E_{t-1}\left[\frac{\mu_t^*}{T_{Dt}^*}\right]}{E_{t-1}\left[\frac{\alpha_N^*}{\alpha_N T_{Nt} T_{Dt}^*}\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}\right)\right]$$

$$-\left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t} T_{Nt}^*}\right]\right)$$

This is directly analogous to home. Now W* include only μ_t^* and no μ_t .

So directly analogous optimality condition:

$$\frac{1}{\mu_t^*} = \frac{1}{2}\frac{1}{E_{t-1}\left[\mu_t^*\right]} + \frac{1}{2}\frac{\frac{1}{T_{Dt}^*}}{E_{t-1}\left[\frac{\mu_t^*}{T_{Dt}^*}\right]}.$$

Appendix D: Derivation of Analytical Results, Home Dominant Currency Pricing

Home exporters face the PCP problem from above, with the implied price setting rule:

$$P_{DHt} = \frac{\phi}{\phi-1} \frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]},$$

which implies the export price:

$$P_{DHt}^* = \frac{1}{e_t}\frac{\phi}{\phi-1} \frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}$$

where the exchange rate still follows: $e_t = \frac{\mu_t \alpha_N^*}{\mu_t^* \alpha_N T_{N,t}}$.

Foreign exporters follow the price setting rules derived for the LCP case above:

$$P_{Ft}^* = \frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}E_{t-1}\left[\mu_t^*\right]$$

$$P_{Ft} = \frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}\frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]}.$$

These prices imply the home price index:

$$P_t = 2^{\theta}\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\right)^{\theta/2}\left(T_{Dt}\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}\frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]}\right)^{\theta/2}\left(\frac{\kappa\mu_t}{\alpha_N}\right)^{(1-\theta)}$$

Note that this is a hybrid of cases above, since the home good prices are PCP, while home import prices are LCP.

The foreign price index is:

$$P_t^* = 2^\theta\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}E_{t-1}\left[\mu_t^*\right]\right)^{\theta/2}\left(T_{Dt}^*\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\frac{\mu_t^*\alpha_N T_{N,t}}{\mu_t\alpha_N^*}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}$$

This too is a hybrid case, since the foreign good prices are LCP, while foreign imports of home goods are PCP.

Home labor supply is the same as the PCP case:

$$\kappa E_{t-1}\left[l_t\right] = 1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right],$$

and foreign the same as the LCP case:

$$\kappa E_{t-1}\left[l_t\right] = 1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right].$$

Home welfare may be computed:

$$W_t = E_{t-1}\left[\ln\mu_t\right]-\ln\left(2\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\right)-\theta\ln\left(\frac{\phi}{\phi-1}\right)-\frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa\mu_t}{\alpha_D}\right]\right)$$
$$+\frac{\theta}{2}\ln\left(E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]\right)-\frac{\theta}{2}E_{t-1}\left[\ln\left(T_{Dt}\right)-\ln\alpha_D^*\right]-\frac{\theta}{2}\ln\left(E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]\right)+\frac{\theta}{2}\ln\left(E_{t-1}\left[\frac{\alpha_N T_{Nt}}{\alpha_N^* T_{Dt}}\right]\right)$$
$$-\left(1-\theta\right)E_{t-1}\left(\ln\kappa+\ln\mu_t-\ln\alpha_N\right)-1+\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N^*}{\alpha_N T_{N,t}T_{Dt}^*}\right]$$

Note that the home policy variable, μ, interacts with both home and foreign tariffs, T_{Dt} and T_{Dt}^*, inside the expectation operator of price setting. This will imply that, when taking a derivative with respect to the policy variable, the optimality condition will involve both tariffs. Note also that the foreign policy variable, μ_t^*, does not appear in this welfare computation for home.

Foreign welfare is

$$W_t^* = E_{t-1}\left[\ln\mu_t^*\right]-E_{t-1}\left[\ln\left(\begin{array}{c}2^\theta\left[\left(\frac{1-\theta}{\theta}\right)^{\theta-1}+\left(\frac{1-\theta}{\theta}\right)^{\theta}\right]\left(\frac{\phi}{\phi-1}\frac{\kappa}{\alpha_D^*}E_{t-1}\left[\mu_t^*\right]\right)^{\theta/2}\\ \left(T_{Dt}^*\frac{\phi}{\phi-1}\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\frac{\kappa}{\alpha_D}\right]}{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\right]}\frac{\mu_t^*\alpha_N T_{N,t}}{\alpha_N^*}\right)^{\theta/2}\left(\frac{\kappa\mu_t^*}{\alpha_N^*}\right)^{(1-\theta)}\end{array}\right)\right]-\left(1-\frac{1}{\phi}\frac{\theta}{2}E_{t-1}\left[1+\frac{\alpha_N}{\alpha_N^* T_{D,t}T_{Nt}^*}\right]\right)$$

Note that the foreign policy variable, μ_t^*, does not interact with either home or foreign tariffs, T_{Dt} and T_{Dt}^*, inside the expectation operator of price setting. This will imply that, when taking a derivative with respect to the policy variable, the optimality condition will not involve either tariff.

The optimality conditions for both Nash and cooperative problems are

$$\mu_t = \frac{1}{2}\left(\frac{E_{t-1}\left[\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)\mu_t\right]}{\left(1+\frac{\alpha_N^*}{\alpha_N}\frac{1}{T_{N,t}T_{Dt}^*}\right)} + \frac{E_{t-1}\left[\frac{\mu_t}{T_{Dt}}\right]}{\frac{1}{T_{Dt}}} \right)$$

for home policy, and

$$\mu_t^* = E_{t-1}\left[\mu_t^*\right]$$

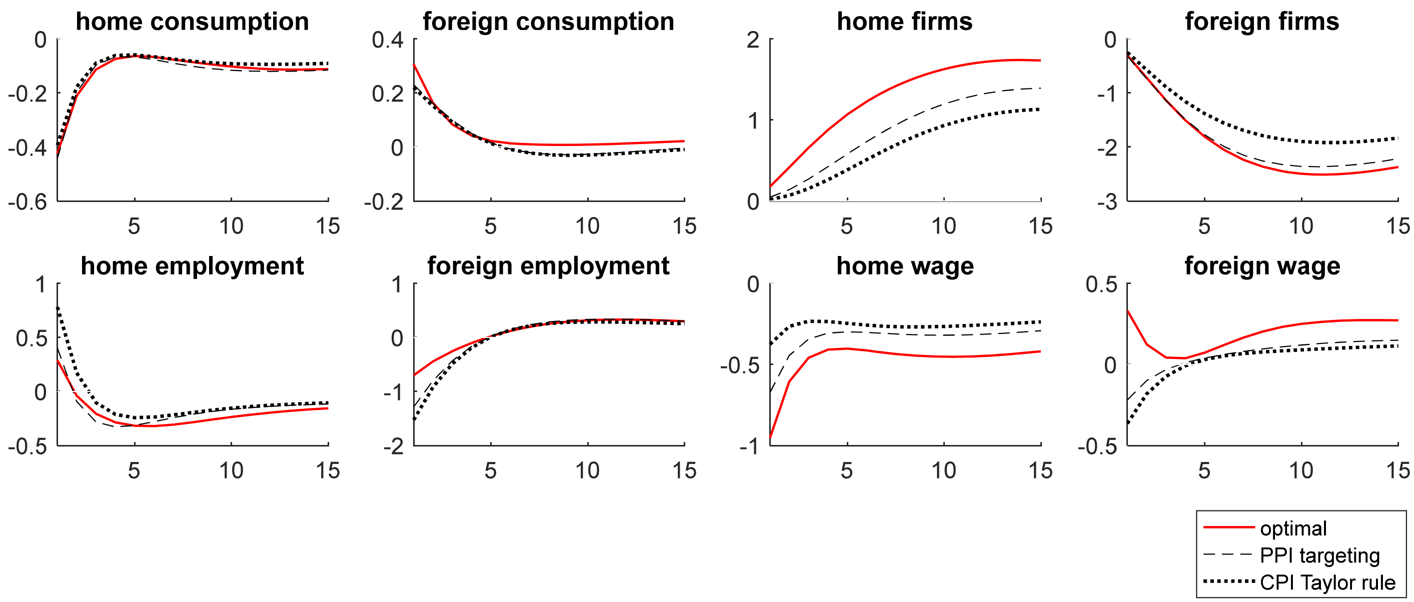
for foreign.

Appendix E. Discussion of Simulation Results for LCP and Foreign Dominant Cases

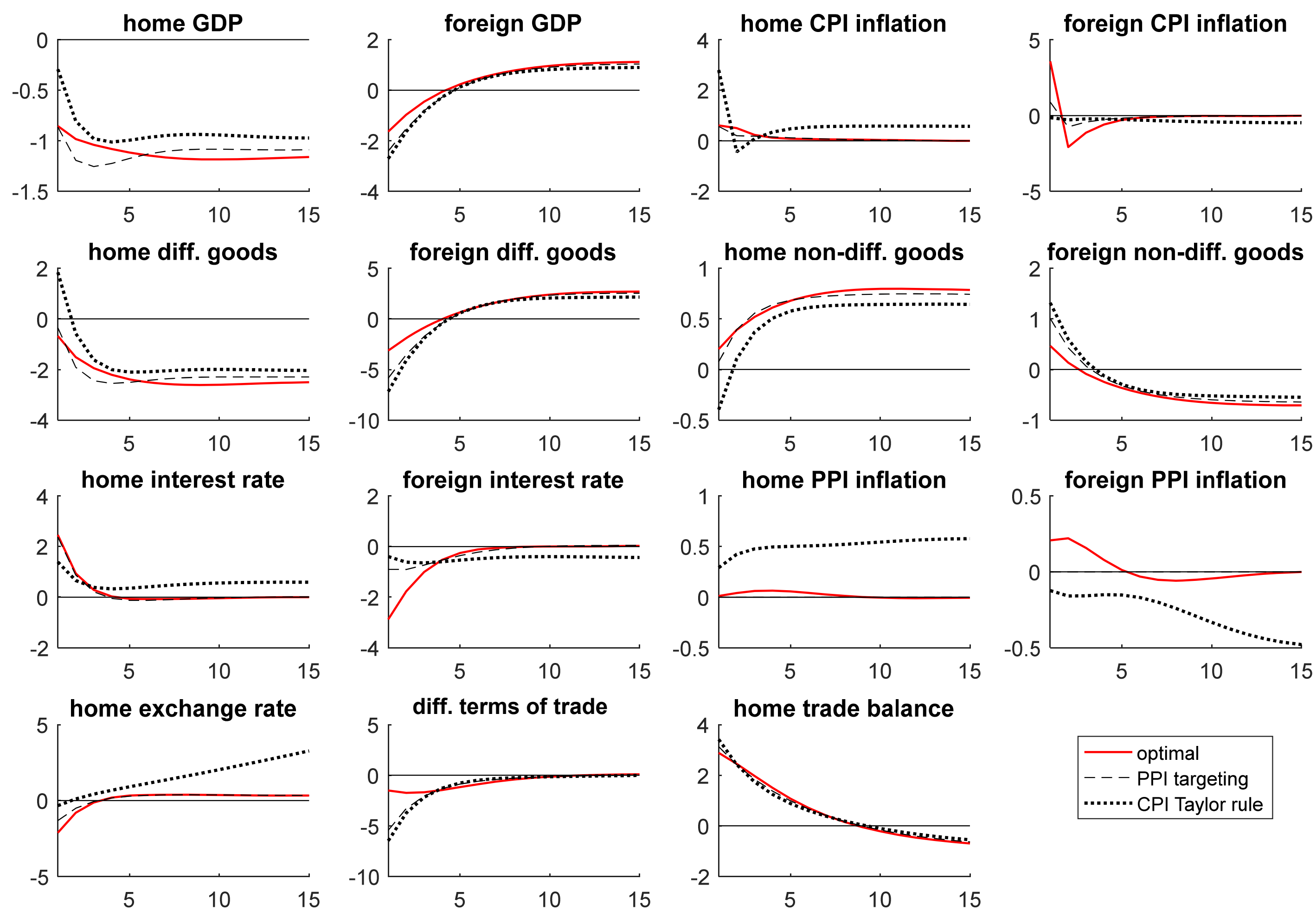
Appendix Figure 3 shows impulse responses for an environment with local currency pricing (LCP) in both countries. In this case, consistent with our analytical section, the home interest is lower than that implied by the PPI targeting rule, implying a relative expansion, and home PPI inflation as sharply positive. While the analytical result does not indicate any response in foreign monetary policy stance, in the richer environment of the simulation model, the optimal foreign response to the home tariff is mildly contractionary—the interest rate is above that implied by the PPI target, and there is a moderate fall in PPI inflation.

We also analyze the case in which the dominant currency is issued by the country targeted by tariff, i.e., the foreign country. The dynamic responses are shown in Appendix Figure 4. The analytical result in the simplified model implied that optimal policy in the country with the dominant currency in this case should resemble that under PCP. In the case of the present simulation, this prescription implies the foreign (dominant) country should expand in response to the home (non-dominant) tariff. Indeed, Figure 4 shows a sharp foreign expansion, with interest rate falling more than that implied by PPI targeting, and with a substantial PPI inflation. While the analytical result did not call for any policy response from the non-dominant country (home country in this case), the simulation in Figure 4 shows a mild contraction, with interest rate slightly above that of the PPI targeting rule, and a small reduction in PPI inflation.

Appendix Figure 1. Additional Impulse responses for benchmark case: home tariff on differentiated imports

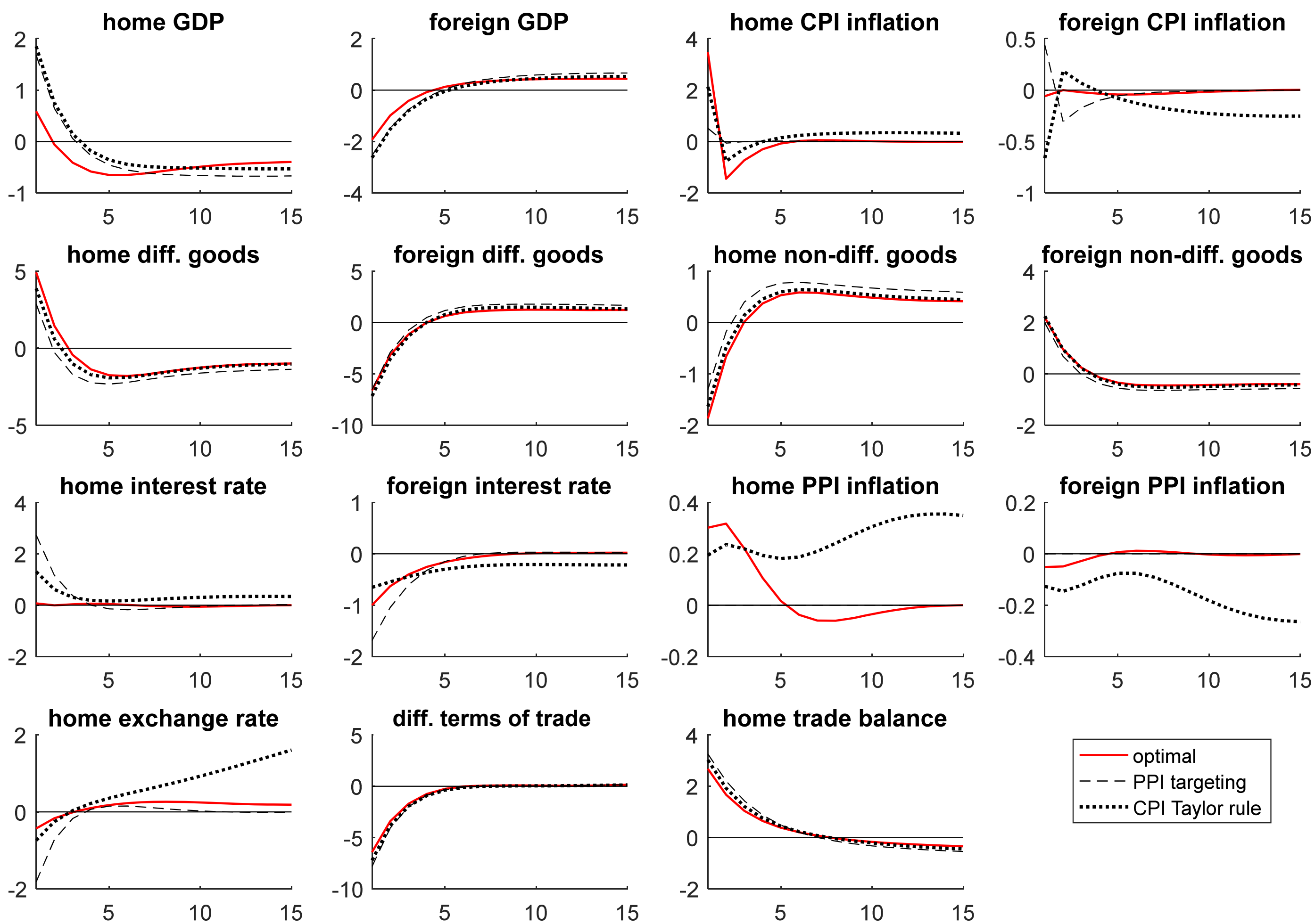


Appendix Figure 2. Impulse responses to a rise in home tariff on differentiated imports, lower trade elasticity (3.8)



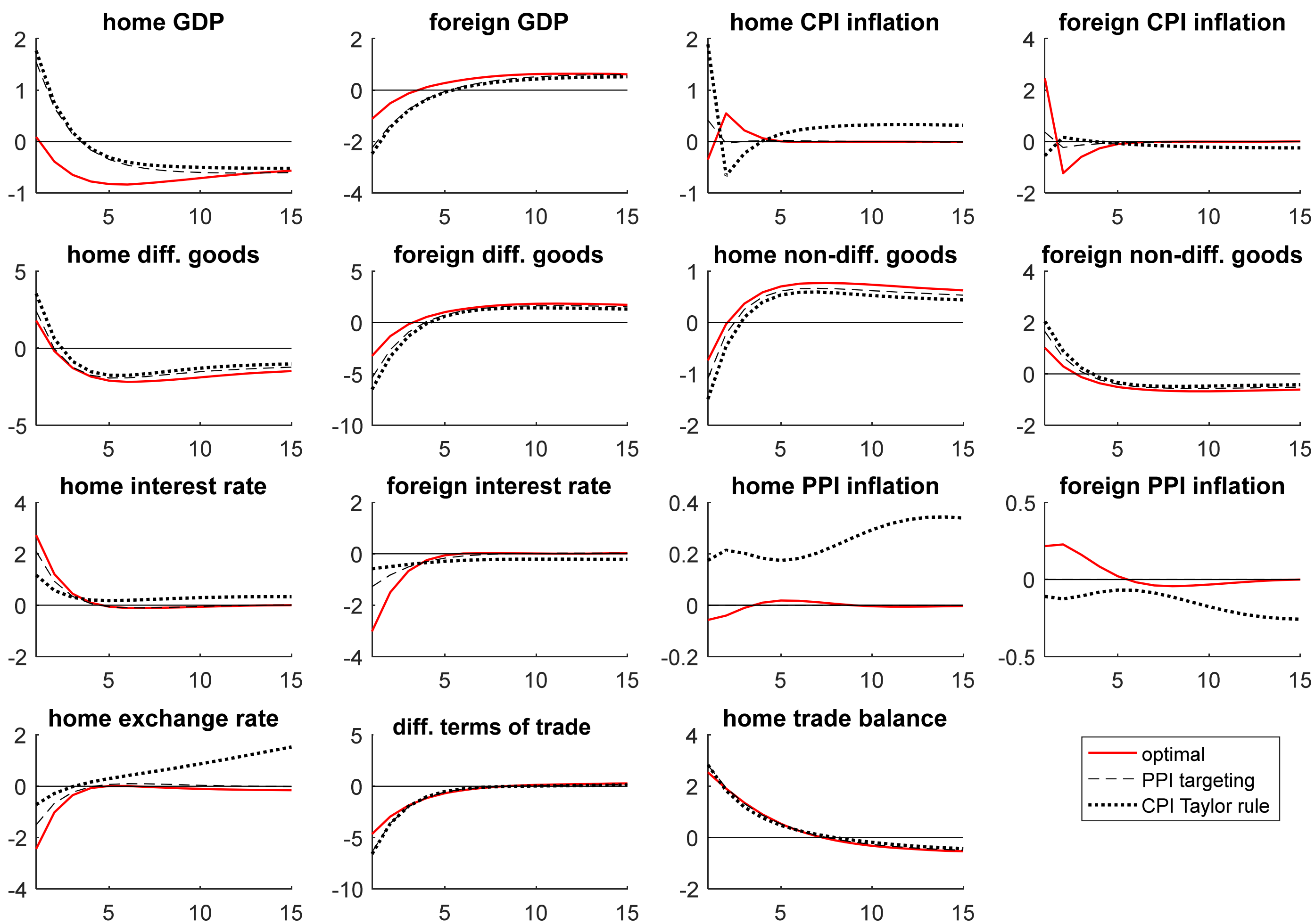
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 3. Impulse responses to a rise in home tariff on differentiated imports;
LCP price stickiness in both countries



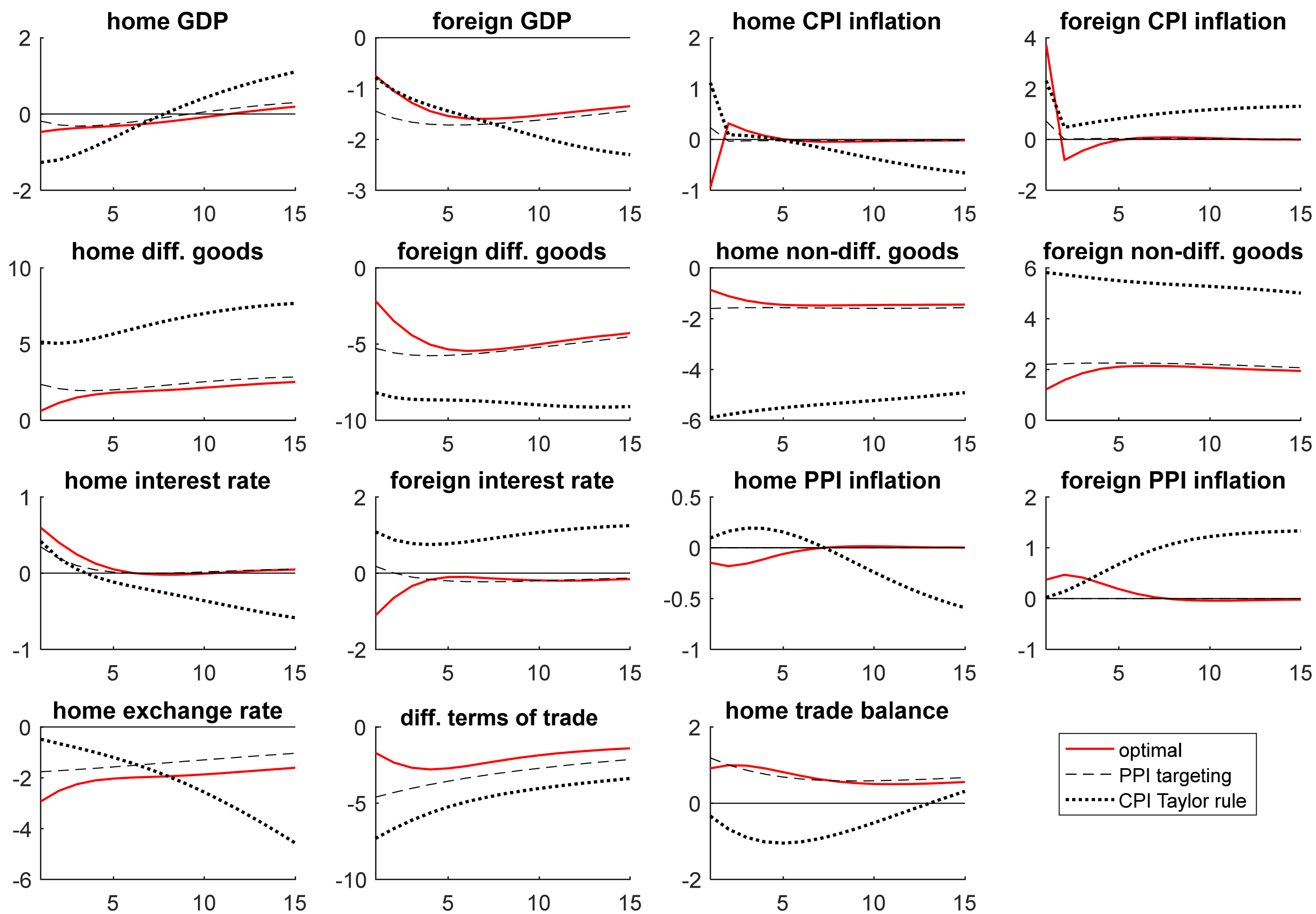
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 4. Impulse responses to a rise in home tariff on differentiated imports; foreign currency dominant



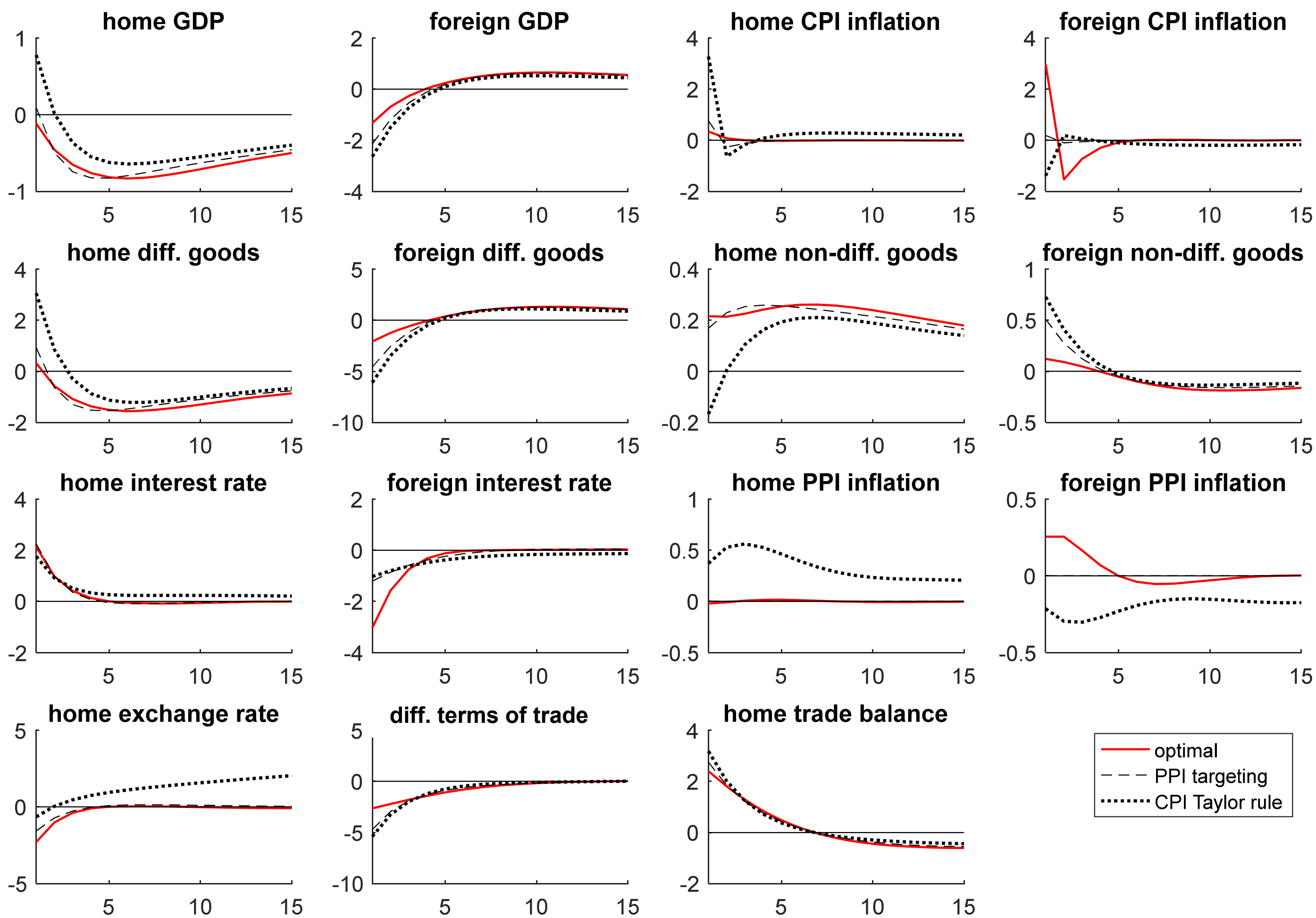
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 5. Impulse responses to a more persistent tariff on home differentiated imports



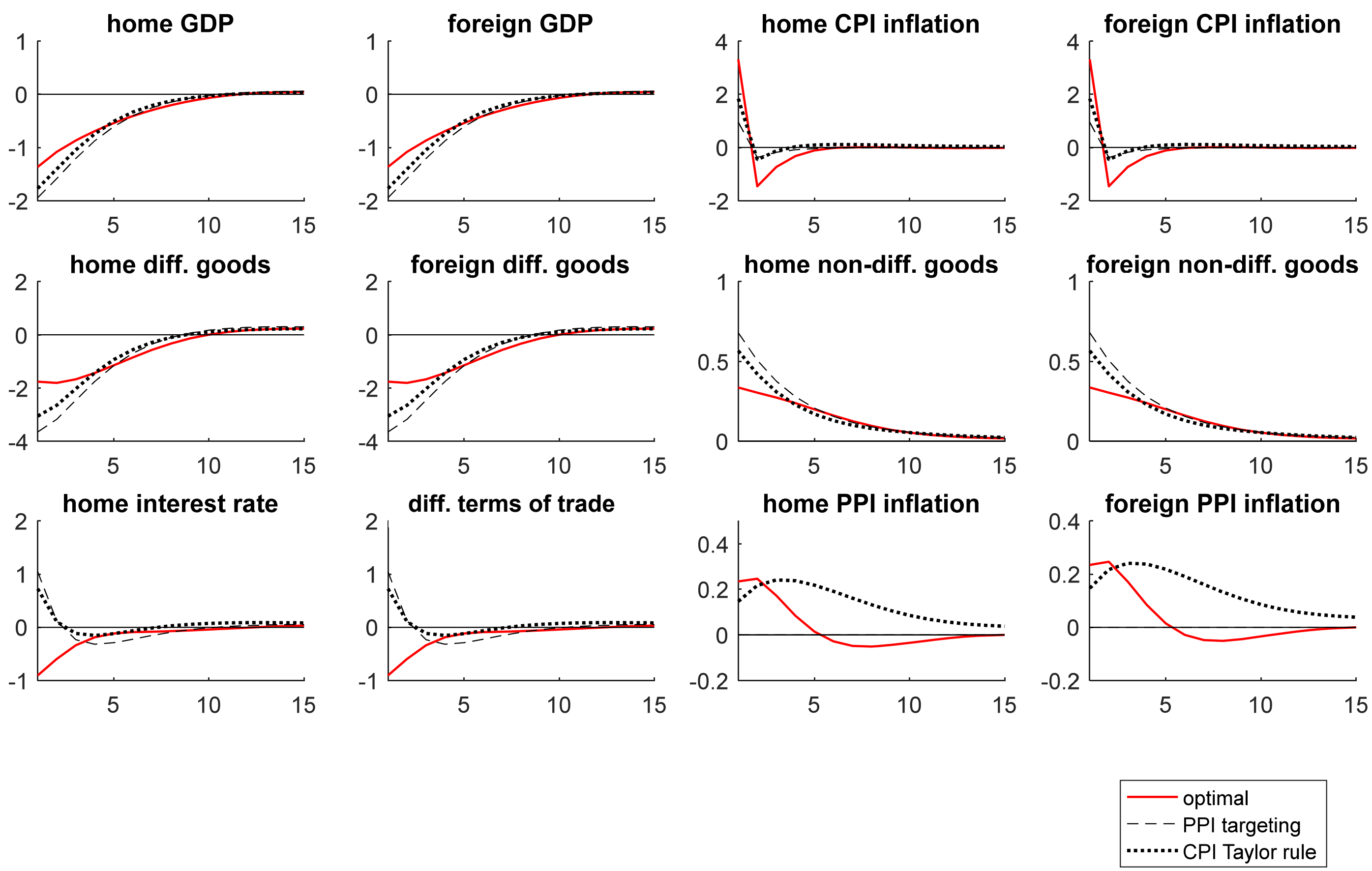
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 6. Impulse responses to home tariff on differentiated imports, nontraded non-diff. goods



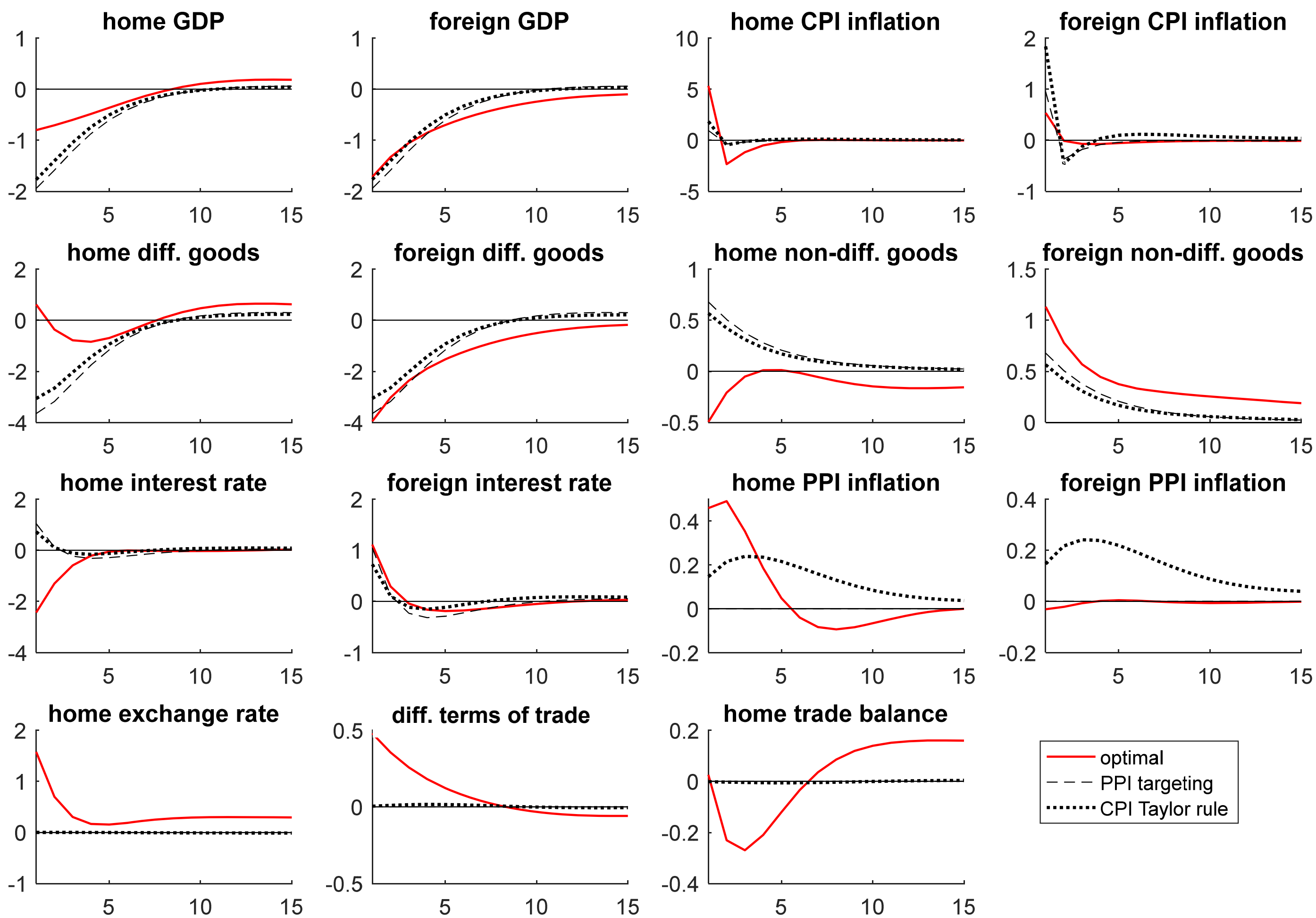
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 7. Impulse responses to a symmetric tariff to differentiated imports in both countries



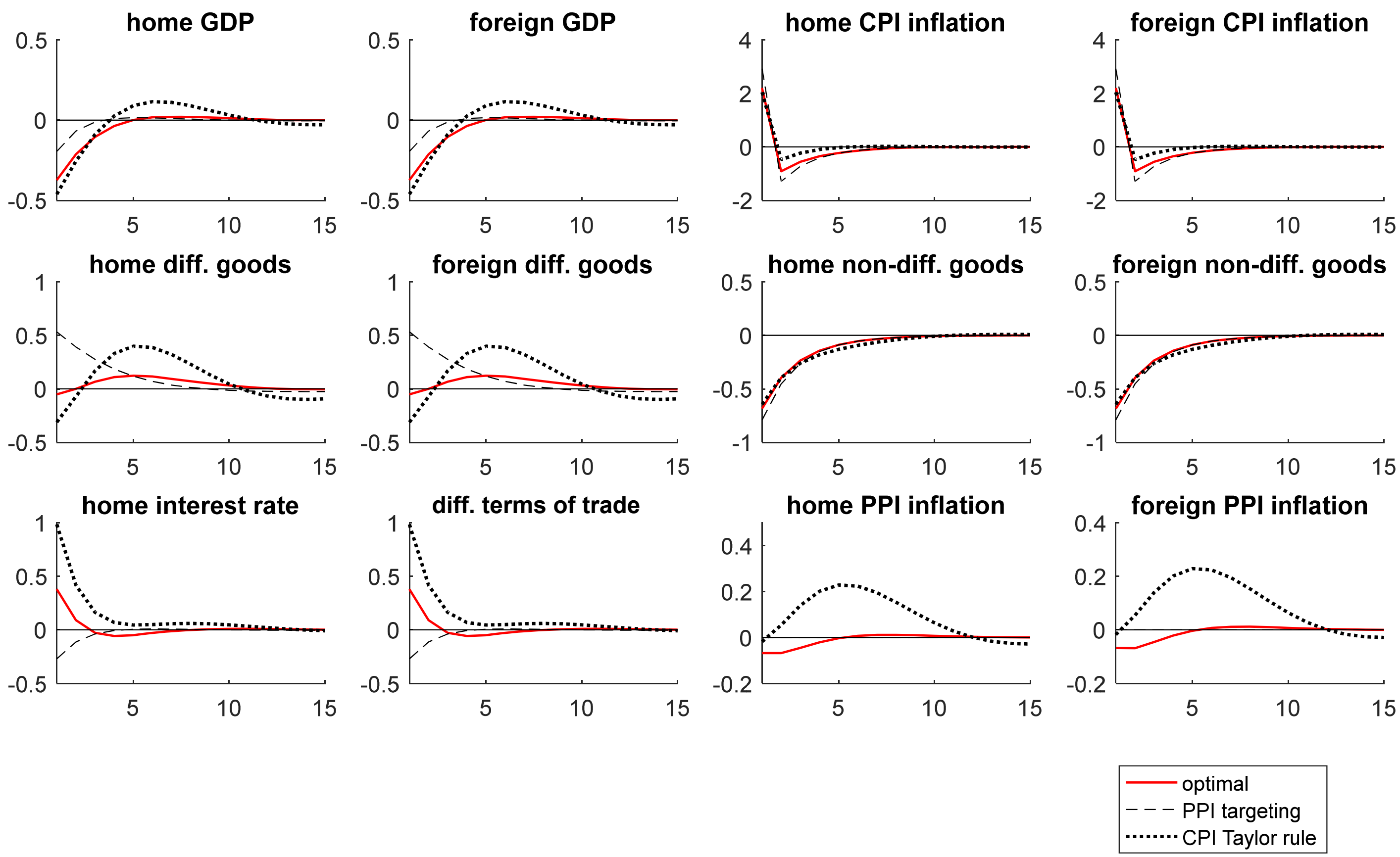
Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 8. Impulse responses to a symmetric tariff to differentiated imports in both countries; home currency dominant



Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.

Appendix Figure 9. Impulse responses to a symmetric tariff to non-differentiated imports in both countries



Vertical axis is percent deviation from steady state (1=1%); horizontal axis is time (in quarters). Trade balance reported as percent of GDP. Interest rates in percentage points. Inflation rates annualized.