

Appendix A. Estimating $Var(\tilde{\alpha}_i)$ if assignment to teachers depends on student growth.

Even at the true value of δ , we may underestimate $Var(\tilde{\alpha}_i)$ if assignment to teachers depends on student growth (since some of that variance will be absorbed into the teacher effects). In this appendix we derive an adjustment based on the within and between teacher variance in $W_i\gamma$ (the observable component of α_i) that yields a consistent estimate of $Var(\tilde{\alpha}_i)$ under the same assumption as that needed for the Oster bias correction (that the degree of selection on the observed ($W_i\gamma$) and unobserved ($\tilde{\alpha}_i$) components is equivalent).

Suppose the following two assumptions hold (same as those needed for Oster bias correction):

1. Let $\alpha_i = W_i\gamma + \tilde{\alpha}_i$ with $W_i\gamma$ independent of $\tilde{\alpha}_i$, $Var(W_i\gamma) = \sigma_{W_i\gamma}^2$ and $Var(\tilde{\alpha}_i) = \sigma_{\tilde{\alpha}}^2$.
2. Students are sorted to teachers based on $z_i = \alpha_i + u_i$, where u_i is independent of α_i and $Var(u_i) = \sigma_u^2$.

Note that assumption 2 implies that sorting on the unobservable component of student growth ($\tilde{\alpha}_i$) is as strong as sorting on the observable component ($W_i\gamma$). As Oster (2019) notes, this is likely to provide an upper bound on the amount of bias since sorting is, if anything, likely to be stronger on easily observable correlates of student growth in $W_i\gamma$.

Our goal is to estimate the unconditional variance of $\tilde{\alpha}_i$, $\sigma_{\tilde{\alpha}}^2$, based on our estimate of the variance in $\tilde{\alpha}_i$ that is conditional on the teacher assignment. If students are sorted to teachers based on z_i , then conditioning on teacher assignment is equivalent to conditioning on z_i , so that:

$$\text{Equation A1: } Var(\tilde{\alpha}_i|teacher) = Var(\tilde{\alpha}_i|z_i) = \sigma_{\tilde{\alpha}}^2 \cdot (1 - R_{\tilde{\alpha}_i|z_i}^2)$$

Where $R_{\tilde{\alpha}_i|z_i}^2$ is the R-squared from a regression of $\tilde{\alpha}_i$ on z_i , which under our assumptions is:

$$\text{Equation A2: } R_{\tilde{\alpha}_i|z_i}^2 = \frac{\sigma_{\tilde{\alpha}}^2}{\sigma_{W_i\gamma}^2 + \sigma_{\tilde{\alpha}}^2 + \sigma_u^2}$$

Similarly, the R-squared from a regression of $W_i\gamma$ on teacher effects is equivalent to the R-squared from a regression of $W_i\gamma$ on z_i , which under our assumptions is:

$$\text{Equation A3: } R_{W_i\gamma|z_i}^2 = \frac{\sigma_{W_i\gamma}^2}{\sigma_{W_i\gamma}^2 + \sigma_{\tilde{\alpha}}^2 + \sigma_u^2}$$

Taking the ratio of the R-squared values in Equations A2 and A3 yields:

$$\text{Equation A4: } \frac{R_{\tilde{\alpha}_i|z_i}^2}{R_{W_i\gamma|z_i}^2} = \frac{\sigma_{\tilde{\alpha}}^2}{\sigma_{W_i\gamma}^2}, \text{ or equivalently } R_{\tilde{\alpha}_i|z_i}^2 = \frac{\sigma_{\tilde{\alpha}}^2 \cdot R_{W_i\gamma|z_i}^2}{\sigma_{W_i\gamma}^2}$$

Plugging Equation A4 into Equation A1 yields:

$$\text{Equation A5: } Var(\tilde{\alpha}_i|z_i) = \sigma_{\tilde{\alpha}}^2 \cdot \left(1 - \frac{\sigma_{\tilde{\alpha}}^2 \cdot R_{W_i\gamma|z_i}^2}{\sigma_{W_i\gamma}^2}\right)$$

Or equivalently:

$$\text{Equation A6: } \text{Var}(\tilde{\alpha}_i|z_i) - \sigma_{\tilde{\alpha}}^2 + (\sigma_{\tilde{\alpha}}^2)^2 \left(\frac{R_{W_i\gamma|z_i}^2}{\sigma_{W_i\gamma}^2} \right) = 0$$

We use the quadratic formula to solve Equation A6 for $\sigma_{\tilde{\alpha}}^2$ as a function of our estimates of $\text{Var}(\tilde{\alpha}_i|z_i)$, $R_{W_i\gamma|z_i}^2$, and $\sigma_{W_i\gamma}^2$ (selecting the smaller root).

Appendix Table 1 reports the results of applying this correction. We use estimates of $\text{Var}(\tilde{\alpha}_i|z_i)$ and $W_i\gamma$ from the bias-corrected AC-VAM model (Table 3, columns 5 and 10), and an estimate of $R_{W_i\gamma|z_i}^2$ using the adjusted R-squared from a regression of $W_i\gamma$ on teacher fixed effects. For both math and reading, the estimates of $\sigma_{\tilde{\alpha}}^2$ are only slightly larger than the estimates of $\text{Var}(\tilde{\alpha}_i|z_i)$ that we report in the main tables.

Appendix Table 1. Estimates of $\text{Var}(\alpha_i)$ that correct for sorting to teachers on α_i

	Math	Reading
Estimates from bias-corrected ACVAM:		
$\text{Var}(\tilde{\alpha}_i z_i)$	0.000744	0.000257
$\sigma_{W_i\gamma}^2$	0.000722	0.001456
$R_{W_i\gamma z_i}^2$	0.1844	0.3303
Implied estimate of $\sigma_{\tilde{\alpha}}^2$	0.000999	0.000274