

# ONLINE APPENDIX

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Section A of this Online Appendix includes extended analytical derivations. Section B describes the three formal extensions discussed in Section 5 in the body of the paper, along with the associated proofs. Section C presents additional results, including the introduction of a deposit insurance fund, a comparison between traditional and generalized social welfare weights, and a more general directional test. Finally, Section D includes additional material supporting the quantitative application in Section 4 of the paper.

### A Extended Analytical Results

In this section, to facilitate the understanding of the results, we provide detailed analytical characterizations of several outcomes of the model.

#### A.1 Thresholds $\hat{s}(R_1)$ and $s^*(\delta, R_1)$

The threshold  $\hat{s}(R_1)$  is given by the minimum among the value of  $s$  that satisfies

$$\frac{R_1 - \rho_1(s)}{1 - \frac{1}{\rho_2(s)}} = (1 - \lambda) R_1, \quad (\text{A1})$$

and  $\bar{s}$ . Note that this threshold is not a function of  $\delta$ . Similarly, the value of  $s^*(\delta, R_1)$  is given by the minimum among the value of  $s$  that satisfies

$$\frac{R_1 - \rho_1(s)}{1 - \frac{1}{\rho_2(s)}} = (1 - \lambda) R_1 \zeta(\delta, R_1), \quad (\text{A2})$$

and  $\bar{s}$ , where  $\zeta(\delta, R_1) \equiv \frac{\int_{i \in I} \min\{D_0(i)R_1, \delta\} dG(i)}{D_0 R_1}$  denotes the share of insured deposits.<sup>34</sup> Note that  $\zeta(\delta, R_1) \in [0, 1]$ ,  $\frac{\partial \zeta}{\partial \delta} \geq 0$ , and  $\frac{\partial \zeta}{\partial R_1} \leq 0$ .

The left-hand side of both equations,  $z(s, R_1) \equiv \frac{R_1 - \rho_1(s)}{1 - \frac{1}{\rho_2(s)}}$ , is a decreasing function of  $s$ , since both  $\rho_1(s)$  and  $\rho_2(s)$  are monotonically increasing in  $s$  and  $\rho_2(s)$  strictly so. Since we have assumed that  $\rho_2(\underline{s}) < 1$ , it is always guaranteed that  $\hat{s}(R_1) > \underline{s}$ . Note that

$$\lim_{\rho_2(s) \rightarrow 1^+} z(s, R_1) = \infty \quad \text{and} \quad \lim_{\rho_1(s), \rho_2(s) \rightarrow \infty} z(s, R_1) < 0,$$

which is sufficient to establish that both Equation (A1) and Equation (A2) have a unique solution strictly

<sup>34</sup>When  $\rho_1(s) = 1$  and  $\rho_2(s) = s$ , the thresholds  $\hat{s}(R_1)$  and  $s^*(\delta, R_1)$  can be explicitly computed as

$$\hat{s}(R_1) = \frac{(1 - \lambda) R_1}{1 - \lambda R_1} \quad \text{and} \quad s^*(\delta, R_1) = \min \left\{ \rho_2^{-1} \left( \frac{(1 - \lambda) R_1}{(1 - \lambda) R_1 - \frac{R_1 - 1}{\zeta}} \right), \bar{s} \right\}.$$

higher than  $\hat{s}(R_1)$ . Since  $\zeta(\delta, R_1) \in [0, 1]$ , we can also conclude that  $\hat{s}(R_1) \leq s^*(\delta, R_1)$ , with equality only when all deposits are insured,  $\delta \rightarrow \overline{\overline{D}}R_1$ , since  $\lim_{\delta \rightarrow \overline{\overline{D}}R_1} \zeta(\delta, R_1) = 1$ .

In order for  $s^*(\delta, R_1) < \bar{s}$ , as in Figure 3 in the text, it must be that  $\rho_1(\bar{s}) > R_1$ . In that case, there are three regions (unique failure equilibrium, multiple equilibria, and unique no-failure equilibrium) for any value of  $\delta$ , including  $\delta = 0$ . If  $\rho_1(\bar{s}) < R_1$ , then there are only two regions (unique failure equilibrium and multiple equilibria) for small values of  $\delta$ .

The relevant comparative statics for  $\hat{s}(R_1)$  and  $s^*(\delta, R_1)$  are the following. First, it follows directly from Equation (A2) that

$$\frac{\partial s^*}{\partial \delta} \leq 0,$$

since its right-hand side is increasing in  $\delta$ . The effect of  $\delta$  on  $s^*(\delta, R_1)$  is modulated by the behavior of  $\zeta(\delta, R_1)$ . Second, similar arguments imply that

$$\frac{\partial \hat{s}}{\partial R_1} \geq 0 \quad \text{and} \quad \frac{\partial s^*}{\partial R_1} \geq 0. \quad (\text{A3})$$

Finally, it also follows immediately from Equations (A1) and (A2) that

$$\frac{\partial \hat{s}}{\partial \lambda} \geq 0 \quad \text{and} \quad \frac{\partial s^*}{\partial \lambda} \geq 0,$$

since the right-hand side of both equations is decreasing in  $\lambda$ . Intuitively, all else constant, an increase in the mass of early depositors, who withdraw their deposits inelastically, increases the probability of failure.

### A.1.1 Parametric assumptions

Under the following parametric assumptions:  $\rho_2(s) = s$  and  $\rho_1(s) = 1 + \varphi(s - 1)$ , we can express the thresholds  $\hat{s}(R_1)$  and  $s^*(\delta, R_1)$  implied by Equations (A1) and (A2) as follows:

$$\begin{aligned} \hat{s}(R_1) &= \left\{ s \mid 0 = \varphi s^2 - (\lambda R_1 + \varphi - 1)s - (1 - \lambda)R_1 \right\} \\ s^*(\delta, R_1) &= \left\{ s \mid 0 = \varphi s^2 - ((1 - (1 - \lambda)\zeta)R_1 + \varphi - 1)s - (1 - \lambda)R_1\zeta \right\}, \end{aligned}$$

where

$$\hat{s}(R_1) = \frac{\lambda R_1 + \varphi - 1 \pm \sqrt{(\lambda R_1 + \varphi - 1)^2 + 4\varphi(1 - \lambda)R_1}}{2\varphi} \quad (\text{A4})$$

$$s^*(\delta, R_1) = \frac{(1 - (1 - \lambda)\zeta)R_1 + \varphi - 1 \pm \sqrt{((1 - (1 - \lambda)\zeta)R_1 + \varphi - 1)^2 + 4\varphi(1 - \lambda)R_1\zeta}}{2\varphi}. \quad (\text{A5})$$

Both quadratic equations have a unique positive root. Note that by setting  $\zeta = 1$ , Equation (A5) collapses to Equation (A4). Note also that  $\hat{s}(R_1)$  is exclusively a function of  $\varphi$ ,  $\lambda$ , and  $R_1$ . The threshold  $s^*(\delta, R_1)$  is a function of  $\varphi$ ,  $\lambda$ ,  $R_1$ , and  $\zeta$ , which in turn depends on  $R_1$ ,  $\delta$ , and the distribution of deposits  $G(i)$ . Explicitly characterizing  $\hat{s}(R_1)$  and  $s^*(\delta, R_1)$  simplifies the numerical solution of the

model.

## A.2 Probability of bank failure

Starting from Equation (12), we can express  $\frac{\partial q^F}{\partial \delta}$  and  $\frac{\partial q^F}{\partial R_1}$  as follows:

$$\begin{aligned}\frac{\partial q^F}{\partial \delta} &= \pi f(s^*(\delta, R_1)) \frac{\partial s^*}{\partial \delta} \leq 0 \\ \frac{\partial q^F}{\partial R_1} &= (1 - \pi) f(\hat{s}(R_1)) \frac{\partial \hat{s}}{\partial R_1} + \pi f(s^*(\delta, R_1)) \frac{\partial s^*}{\partial R_1} \geq 0,\end{aligned}$$

where the sign results follow from Equation (A3). As explained in the text, higher levels of coverage reduce the probability of failure, holding  $R_1$  constant, while higher deposit rates increase the probability of failure, holding  $\delta$  constant.

## A.3 Insured/uninsured deposits

A fraction of a depositor's claims at date 1 is insured, while the remaining is potentially uninsured. Formally,

$$D_0(i) R_1 = \underbrace{\min\{D_0(i) R_1, \delta\}}_{\text{Insured}} + \underbrace{\max\{D_0(i) R_1 - \delta, 0\}}_{\text{Uninsured}}. \quad (\text{A6})$$

We can express aggregate insured and uninsured deposits at date 1 as follows:

$$\begin{aligned}\int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i) &= \int_{i \in \mathcal{FI}} D_0(i) R_1 dG(i) + \delta \int_{i \in \mathcal{PI}} dG(i) \quad (\text{Insured Deposits}) \\ \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i) &= \int_{i \in \mathcal{PI}} (D_0(i) R_1 - \delta) dG(i), \quad (\text{Uninsured Deposits}),\end{aligned}$$

where we formally define the sets of fully insured ( $\mathcal{FI}$ ) and partially insured depositors ( $\mathcal{PI}$ ) as

$$\begin{aligned}\mathcal{FI} &= \{i \mid D_0(i) R_1 \leq \delta\} \\ \mathcal{PI} &= \{i \mid D_0(i) R_1 > \delta\}.\end{aligned}$$

The counterpart of Equation (A6) in the aggregate is given by

$$\bar{D}_0 R_1 = \underbrace{\int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i)}_{\text{Insured Deposits}} + \underbrace{\int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i)}_{\text{Uninsured Deposits}}.$$

We repeatedly use the fact that

$$\begin{aligned}\frac{d\left(\int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i)\right)}{dR_1} &= \int_{i \in \mathcal{FI}} D_0(i) dG(i) \\ \frac{d\left(\int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i)\right)}{dR_1} &= \int_{i \in \mathcal{PI}} D_0(i) dG(i),\end{aligned}$$

as well as

$$\begin{aligned}\frac{d\left(\int_{i \in I} \min\{D_0(i)R_1, \delta\} dG(i)\right)}{d\delta} &= \int_{i \in \mathcal{PI}} dG(i) \\ \frac{d\left(\int_{i \in I} \max\{D_0(i)R_1 - \delta, 0\} dG(i)\right)}{d\delta} &= - \int_{i \in \mathcal{PI}} dG(i).\end{aligned}$$

Note that while  $\hat{s}(R_1)$  only depends on  $R_1$  and  $\lambda$ ,  $s^*(\delta, R_1)$  also depends on those two objects in addition to the whole distribution of deposits, through its impact on the share of insured deposits.

## A.4 Properties of depositors' and taxpayers' consumption

### A.4.1 Individual consumption levels

For reference, we reproduce here the expressions for depositors' equilibrium consumption in the cases of failure and no-failure:

$$\begin{aligned}C_t^F(i, \delta, R_1) &= \min\{D_0(i)R_1, \delta\} + \alpha_F(s) \max\{D_0(i)R_1 - \delta, 0\} + Y_t(i, s) \\ C_1^N(i, R_1) &= D_0(i)R_1 + Y_1(i, s) \\ C_2^N(i, R_1) &= \alpha_N(s) D_0(i)R_1 + Y_2(i, s).\end{aligned}$$

The equilibrium objects  $\alpha_F(s)$  and  $\alpha_N(s)$  are given by

$$\begin{aligned}\alpha_F(s) &= \frac{\max\{\chi(s)\rho_1(s)\bar{D}_0 - \int_{i \in I} \min\{D_0(i)R_1, \delta\} dG(i), 0\}}{\int_{i \in I} \max\{D_0(i)R_1 - \delta, 0\} dG(i)} = \max\left\{1 - \frac{(R_1 - \chi(s)\rho_1(s))\bar{D}_0}{\int_{i \in I} \max\{D_0(i)R_1 - \delta, 0\} dG(i)}, 0\right\} \\ \alpha_N(s) &= \rho_2(s) \frac{\rho_1(s) - \lambda R_1}{(1 - \lambda)R_1}.\end{aligned}$$

The rate  $\alpha_N(s)$  captures the additional gross return obtained by late depositors at date 2 when there is no bank failure. The rate  $\alpha_F(s)$  corresponds to the individual recovery rate on uninsured deposits in the case of bank failure. Note that  $\alpha_F(s)$  is a function of  $s$  only through  $\chi(s)\rho_1(s)$  and that  $\alpha_N(s)$  is a function of  $s$  through  $\rho_2(s)$  and  $\rho_1(s)$ . Note that  $\alpha_N(s) D_0(i)R_1 = \rho_2(s) \frac{\rho_1(s) - \lambda R_1}{1 - \lambda} D_0(i)$ . The fact  $\rho_1(s) > R_1$  is incompatible with the existence of a failure equilibrium, which implies that  $\alpha_F(s) < 1$ . Given our assumptions on  $\chi(s)$ ,  $\rho_1(s)$ , and  $\rho_2(s)$ , it follows that  $\frac{\partial \alpha_F(s)}{\partial s} \geq 0$  and  $\frac{\partial \alpha_N(s)}{\partial s} > 0$ . It also follows that  $\frac{\partial T(s)}{\partial s} \leq 0$ .

Note that we can also express  $\alpha_F(s)$  as

$$\alpha_F(s) = \frac{\max\left\{\chi(s)\rho_1(s)\bar{D}_0 - \int_{i \in I} \min\{D_0(i)R_1, \delta\} dG(i), 0\right\}}{\int_{i \in I} \max\{D_0(i)R_1 - \delta, 0\} dG(i)},$$

which implies that when  $\alpha_F(s) > 0$ ,  $T(s) = 0$ , and when  $\alpha_F(s) = 0$ ,  $T(s) > 0$  (the funding shortfall is

introduced in Equation (18)). Note also that, whenever  $\alpha_F(s) > 0$ , we can express  $1 - \alpha_F(s)$  as

$$1 - \alpha_F(s) = \frac{(R_1 - \chi(s)\rho_1(s))\bar{D}_0}{\int_{i \in I} \max\{D_0(i)R_1 - \delta, 0\} dG(i)}.$$

Finally, note that we can express consumption ratios of the form  $\frac{C^F(j,s)}{C^F(j,s^*)}$  as

$$\frac{C^F(i,s)}{C^F(i,s^*)} = \begin{cases} \frac{D_0(i)R_1 + Y_t(i,s)}{D_0(i)R_1 + Y_t(i,s^*)}, & \text{if } D_0(i)R_1 < \delta \\ \frac{\delta + \alpha_F(s) \max\{D_0(i)R_1 - \delta, 0\} + Y_t(i,s)}{\delta + \alpha_F(s^*) \max\{D_0(i)R_1 - \delta, 0\} + Y_t(i,s^*)}, & \text{if } D_0(i)R_1 \geq \delta \end{cases}$$

for depositors and

$$\frac{C^F(\tau,s)}{C^F(\tau,s^*)} = \frac{Y(\tau,s) - T(s) - \kappa(T(s))}{Y(\tau,s^*) - T(s^*) - \kappa(T(s^*))}$$

for taxpayers. In order to understand whether  $\frac{C^F(j,s)}{C^F(j,s^*)}$  takes values above or below 1 across failure states, it is necessary to understand how  $\alpha_F(s)$ ,  $T(s)$ ,  $Y_t(i,s)$ , and  $Y(\tau,s)$  vary with  $s$ . Under the assumption that  $Y_t(i,s)$  and  $Y(\tau,s)$  are increasing in  $s$ , and, given that  $\alpha_F(s)$  is increasing in  $s$  and  $T(s)$  is decreasing in  $s$ , it follows that  $\frac{C^F(j,s)}{C^F(j,s^*)} \leq 1$  for both depositors and taxpayers. Note that if  $Y_t(i,s)$  is independent of  $s$ ,  $\frac{C^F(i,s)}{C^F(i,s^*)} = 1$  for fully insured depositors.

#### A.4.2 Comparative statics

We can show that  $\alpha_F(s)$  is decreasing in both  $R_1$  and  $\delta$ , as follows:

$$\frac{\partial \alpha_F(s)}{\partial R_1} = \begin{cases} -\frac{\int_{i \in \mathcal{FI}} D_0(i) dG(i) (\int_{i \in I} \max\{D_0(i)R_1 - \delta, 0\} dG(i)) + (\chi(s)\rho_1(s)\bar{D}_0 - \int_{i \in I} \min\{D_0(i)R_1, \delta\} dG(i)) \int_{i \in \mathcal{PI}} D_0(i) dG(i)}{(\int_{i \in I} \max\{D_0(i)R_1 - \delta, 0\} dG(i))^2} \leq 0, & \text{if } T(s) = 0 \\ 0, & \text{if } T(s) > 0 \end{cases}$$

$$\frac{\partial \alpha_F(s)}{\partial \delta} = \begin{cases} -\frac{(R_1 - \chi(s)\rho_1(s))\bar{D}_0 \int_{i \in \mathcal{PI}} dG(i)}{(\int_{i \in I} \max\{D_0(i)R_1 - \delta, 0\} dG(i))^2} = -(1 - \alpha_F(s)) \frac{\int_{i \in \mathcal{PI}} dG(i)}{\int_{i \in I} \max\{D_0(i)R_1 - \delta, 0\} dG(i)} \leq 0, & \text{if } T(s) = 0 \\ 0, & \text{if } T(s) > 0, \end{cases}$$

since  $R_1 - \chi(s)\rho_1(s) \geq 0$  in any failure equilibrium. Note that  $\frac{\int_{i \in \mathcal{PI}} dG(i)}{\int_{i \in I} \max\{D_0(i)R_1 - \delta, 0\} dG(i)}$  is the ratio of partially insured accounts to uninsured deposits.

In no-failure states, depositors' consumption levels vary with  $R_1$  as follows:

$$\frac{\partial C_1^N(i,s)}{\partial R_1} = D_0(i) \geq 0$$

$$\frac{\partial C_2^N(i,s)}{\partial R_1} = -\rho_2(s) \frac{\lambda}{1 - \lambda} D_0(i) \leq 0.$$

In no-failure states, depositors' consumption is not directly affected by  $\delta$ , so  $\frac{\partial C_t^N(i,s)}{\partial \delta} = 0$ .

In failure states, we can derive the following comparative statics, which are relevant inputs for the

characterization of the optimal deposit insurance policy:

$$\frac{\partial C_t^F}{\partial R_1}(i, s) = \begin{cases} D_0(i) \geq 0, & \text{if } D_0(i) R_1 < \delta \\ \alpha_F(s) D_0(i) + \frac{\partial \alpha_F(s)}{\partial R_1} (D_0(i) R_1 - \delta) \geq 0, & \text{if } D_0(i) R_1 \geq \delta \end{cases}$$

$$\frac{\partial C_t^F}{\partial \delta}(i, s) = \begin{cases} 0, & \text{if } D_0(i) R_1 < \delta \\ 1 - \alpha_F(s) + \frac{\partial \alpha_F(s)}{\partial \delta} (D_0(i) R_1 - \delta) \\ = (1 - \alpha_F(s)) \left( 1 - \frac{\int_{i \in \mathcal{P}\mathcal{I}} dG(i)}{\int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i)} (D_0(i) R_1 - \delta) \right) \geq 0, & \text{if } D_0(i) R_1 \geq \delta. \end{cases}$$

Hence, when  $T(s) > 0$ ,  $\alpha_F(s) = 0$  and  $\frac{\partial \alpha_F(s)}{\partial \delta} = 0$ , so  $\frac{\partial C_t^F}{\partial \delta}(i, s) = 1$  for all uninsured depositors — those for which  $D_0(i) R_1 \geq \delta$ . Note that when aggregated among depositors

$$\int_{i \in I} \frac{\partial C_t^F}{\partial \delta}(i, s) dG(i) = \begin{cases} 0, & \text{if } T(s) = 0 \\ \int_{i \in \mathcal{P}\mathcal{I}} dG(i), & \text{if } T(s) > 0. \end{cases}$$

Therefore, we can express  $\int \int_{i \in I} \frac{\partial C_t^F(i, s)}{\partial \delta} dG(i) dF(s) = \int_{i \in \mathcal{P}\mathcal{I}} dG(i) \int \mathbb{I}[T(s) > 0] dF(s)$ , where  $\mathbb{I}[\cdot]$  denotes the indicator function. Although for some individual depositors  $\frac{\partial C_t^F(i, s)}{\partial R_1}$  and  $\frac{\partial C_t^F(i, s)}{\partial \delta}$  can take negative values (this is more likely to occur to depositors with large uninsured balances) since, as shown above,  $\frac{\partial \alpha_F(s)}{\partial \delta} \leq 0$ , we show below that the aggregate consumption response among depositors to  $\delta$  and  $R_1$  is positive.

We can derive similar comparative statics for taxpayers' consumption as follows:

$$\frac{\partial C^F(\tau, s)}{\partial R_1} = - (1 + \kappa'(\cdot)) \frac{\partial T(s)}{\partial R_1} = \begin{cases} 0, & \text{if } T(s) = 0 \\ - (1 + \kappa'(\cdot)) \int_{i \in \mathcal{F}\mathcal{I}} D_0(i) dG(i) \leq 0, & \text{if } T(s) > 0 \end{cases} \quad (\text{A7})$$

$$\frac{\partial C^F(\tau, s)}{\partial \delta} = - (1 + \kappa'(\cdot)) \frac{\partial T(s)}{\partial \delta} = \begin{cases} 0, & \text{if } T(s) = 0 \\ - (1 + \kappa'(\cdot)) \int_{i \in \mathcal{P}\mathcal{I}} dG(i) \leq 0, & \text{if } T(s) > 0. \end{cases}$$

#### A.4.3 Individual consumption differences

We can express  $C_1^N(i, s) - C_1^F(i, s)$  for early depositors as

$$\begin{aligned} C_1^N(i, s) - C_1^F(i, s) &= D_0(i) R_1 - \min\{D_0(i) R_1, \delta\} - \alpha_F(s) \max\{D_0(i) R_1 - \delta, 0\} \\ &= \underbrace{(1 - \alpha_F(s)) \max\{D_0(i) R_1 - \delta, 0\}}_{\text{Partially Recovered Uninsured Deposits}}, \end{aligned}$$

and similarly  $C_2^N(i, s) - C_2^F(i, s)$  for late depositors as

$$\begin{aligned} C_2^N(i, s) - C_2^F(i, s) &= \alpha_N(s) D_0(i) R_1 - \min\{D_0(i) R_1, \delta\} - \alpha_F(s) \max\{D_0(i) R_1 - \delta, 0\} \\ &= \underbrace{(\alpha_N(s) - 1) D_0(i) R_1}_{\text{Net Return}} + \underbrace{(1 - \alpha_F(s)) \max\{D_0(i) R_1 - \delta, 0\}}_{\text{Partially Recovered Uninsured Deposits}}. \end{aligned}$$

## A.5 Properties of aggregate consumption

### A.5.1 Aggregate consumption differences

The aggregate change in consumption among early depositors is given by

$$\begin{aligned} \int_{i \in I} (C_1^N(i, s) - C_1^F(i, s)) dG(i) &= (1 - \alpha_F(s)) \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i) \\ &= \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i) - \max\left\{\chi(s) \rho_1(s) \bar{D}_0 - \int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i), 0\right\} \\ &= \begin{cases} (R_1 - \chi(s) \rho_1(s)) \bar{D}_0, & \text{if } T(s) = 0 \\ \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i), & \text{if } T(s) > 0. \end{cases} \end{aligned}$$

The aggregate change in consumption among late depositors is given by

$$\begin{aligned} \int_{i \in I} (C_2^N(i, s) - C_2^F(i, s)) dG(i) &= (\alpha_N(s) - 1) \bar{D}_0 R_1 + (1 - \alpha_F(s)) \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i) \\ &= (\alpha_N(s) - 1) \bar{D}_0 R_1 + \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i) \\ &\quad - \max\left\{\chi(s) \rho_1(s) \bar{D}_0 - \int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i), 0\right\} \\ &= \begin{cases} (\alpha_N(s) R_1 - \chi(s) \rho_1(s)) \bar{D}_0, & \text{if } T(s) = 0 \\ (\alpha_N(s) - 1) \bar{D}_0 R_1 + \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i), & \text{if } T(s) > 0. \end{cases} \end{aligned}$$

The aggregate change in consumption among depositors and taxpayers is given by

$$\begin{aligned} \int (C^N(j, s) - C^F(j, s)) dj &= \lambda \int_{i \in I} (C_1^N(i, s) - C_1^F(i, s)) dG(i) + (1 - \lambda) \int_{i \in I} (C_2^N(i, s) - C_2^F(i, s)) dG(i) + C^N(\tau, s) - C^F(\tau, s) \\ &= \begin{cases} \lambda(R_1 - \chi(s) \rho_1(s)) \bar{D}_0 + (1 - \lambda)(\alpha_N(s) R_1 - \chi(s) \rho_1(s)) \bar{D}_0, & \text{if } T(s) = 0 \\ (1 - \lambda)(\alpha_N(s) - 1) \bar{D}_0 R_1 + \int_{i \in I} \max\{D_0(i) R_1 - \delta, 0\} dG(i) + T(s) + \kappa(T(s)), & \text{if } T(s) > 0 \end{cases} \\ &= [(\rho_2(s) - 1)(\rho_1(s) - \lambda R_1) + (1 - \chi(s)) \rho_1(s)] \bar{D}_0 + \kappa(T(s)). \end{aligned}$$

Therefore, we can express  $\int (C^N(j, s) - C^F(j, s)) dj$  in Equation (23) as follows:

$$\int (C^N(j, s) - C^F(j, s)) dj = [(\rho_2(s) - 1)(\rho_1(s) - \lambda R_1) + (1 - \chi(s)) \rho_1(s)] \bar{D}_0 + \kappa(T(s)),$$

where  $T(s) = \max\left\{\int_{i \in I} \min\{D_0(i) R_1, \delta\} dG(i) - \chi(s) \rho_1(s) \bar{D}_0, 0\right\}$ .

### A.5.2 Aggregate consumption levels

Aggregate consumption among depositors in the case of bank failure is given by

$$\begin{aligned} \int_{i \in I} C_t^F(i, \delta, R_1) dG(i) &= \int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i) + \max \left\{ \chi(s) \rho_1(s) \bar{D}_0 - \int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i), 0 \right\} + \bar{Y}(s) \\ &= \begin{cases} \chi(s) \rho_1(s) \bar{D}_0 + \bar{Y}(s), & \text{if } T(s) = 0 \\ \int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i) + \bar{Y}(s), & \text{if } T(s) > 0 \end{cases} \\ &= \max \left\{ \chi(s) \rho_1(s) \bar{D}_0, \int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i) \right\} + \bar{Y}(s), \end{aligned}$$

where we define  $\bar{Y}(s) = \int_{i \in I} Y_t(i, s) dG(i)$ .

Aggregate consumption among depositors and taxpayers in the case of bank failure is given by

$$\begin{aligned} \int_{i \in I} C_t^F(i, \delta, R_1) dG(i) + C^F(\tau) &= \max \left\{ \chi(s) \rho_1(s) \bar{D}_0, \int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i) \right\} \\ &\quad - \max \left\{ \int_{i \in I} \min \{D_0(i) R_1, \delta\} dG(i) - \chi(s) \rho_1(s) \bar{D}_0, 0 \right\} - \kappa(T(s)) + \bar{Y}_j(s) \\ &= \chi(s) \rho_1(s) \bar{D}_0 - \kappa(T(s)) + \bar{Y}_j(s), \end{aligned}$$

where we define  $\bar{Y}_j(s) = \int_{i \in I} Y_t(i, s) dG(i) + Y(\tau, s)$ .

Therefore we can easily calculate  $\int \frac{\partial C^F(j, s)}{\partial \delta} dj$ , which is a relevant input to set the optimal level of coverage, as well as  $\int \frac{\partial C^F(j, s)}{\partial R_1} dj$ , as follows:

$$\begin{aligned} \int \frac{\partial C^F(j, s)}{\partial \delta} dj &= \begin{cases} 0, & \text{if } T(s) = 0 \\ -\kappa'(T(s)) \int_{i \in \mathcal{P}\mathcal{I}} dG(i), & \text{if } T(s) > 0 \end{cases} \\ \int \frac{\partial C^F(j, s)}{\partial R_1} dj &= \begin{cases} 0, & \text{if } T(s) = 0 \\ -\kappa'(T(s)) \int_{i \in \mathcal{F}\mathcal{I}} D_0(i) dG(i), & \text{if } T(s) > 0. \end{cases} \end{aligned}$$

Finally, aggregate consumption among depositors if there is no bank failure is given by

$$\begin{aligned} \int_{i \in I} C_t^N(i, \delta, R_1) dG(i) &= \lambda \bar{D}_0 R_1 + (1 - \lambda) \alpha_N(s) \bar{D}_0 R_1 + \bar{Y}(s) \\ &= \lambda \bar{D}_0 R_1 + \rho_2(s) (\rho_1(s) - \lambda R_1) \bar{D}_0 + \bar{Y}(s). \end{aligned}$$

### A.6 Regularity conditions and limits

Continuity and differentiability of the problems faced by banks and regulators are guaranteed whenever distributions and parameters that vary with the realization of the state  $s$  are sufficiently smooth. The one potential source of non-differentiability that emerges in the model is related to the form of the fiscal costs. To guarantee that social welfare is differentiable, it must be that  $\frac{dW}{d\delta}$  is continuous. For this to be the case, it must be that either  $\min_s \{\chi(s) \rho_1(s)\} = 0$  or  $\lim_{T \rightarrow 0} \kappa'(T) = 0$ . Otherwise, for sufficiently low values of  $\delta$  it is the case that there is no need to raise fiscal resources for any realization of  $s$ , so

the second term in Equation (22) changes from 0 to a positive value at a point, making the planner's objective non-differentiable.

As usual in normative exercises, it is hard to guarantee the convexity of the planning problem (quasi-concavity of the planner's objective) in general: there are no simple conditions on primitives that guarantee that the planner's problem is well-behaved. In practice, for natural parametrizations of the model,  $W(\delta)$  is well-behaved and features a single interior optimum. Similarly, it is not easy to establish the convexity of the problem solved by competitive banks (quasi-concavity of the bank's objective) to choose  $R_1$ , although the problem solved by banks is also well-behaved in practice for standard parametrizations, utility, and distributional choices.

In Remark 3 on page 24, we make a statement about the behavior of  $\frac{dW}{d\delta}$  in the limit when  $\delta \rightarrow 0$ . Formally, we can write  $\lim_{\delta \rightarrow 0^+} \frac{dW}{d\delta}$  as

$$\lim_{\delta \rightarrow 0^+} -\frac{\partial q^F}{\partial \delta} \int [U(C^N(j, s^*)) - U(C^F(j, s^*))] dj + \lim_{\delta \rightarrow 0^+} q^F \mathbb{E}_s^F \left[ \int U'(C^F(j, s)) \frac{\partial C^F(j, s)}{\partial \delta} dj \right].$$

Since  $U(C^N(j, s^*)) - U(C^F(j, s^*))$  is non-negative for both depositors and taxpayers, the sign of the first element is given by  $\lim_{\delta \rightarrow 0^+} \left(-\frac{\partial q^F}{\partial \delta}\right)$ . Similarly, since  $q^F$  is strictly positive for any  $\delta$ , including  $\delta = 0$ , the sign of the second term depends on whether the average marginal cost of funds across states is zero or positive when  $\delta \rightarrow 0$ . In a previous version of this paper, we explored a scenario in which  $\lim_{\delta \rightarrow 0^+} \left(-\frac{\partial q^F}{\partial \delta}\right) = 0$  and  $q^F \mathbb{E}_s^F \left[ \int \frac{\partial C^F(j, s)}{\partial \delta} dj \right] \Big|_{\delta=0} < 0$ . In that case, as long as there is a fiscal cost of paying for deposit insurance, increasing coverage around  $\delta = 0$  may decrease welfare locally, since very small coverage levels are costly in equilibrium but are not enough to reduce the probability of bank failure. This is likely to be a very local result.

## B Formal Extensions

For simplicity, we study every extension separately, and focus on the characterization of marginal changes in the level of deposit insurance under perfect regulation of the deposit rate, although the analysis can be extended to other scenarios along the lines of Section 3. When appropriate, we discuss the implications for the optimal design of ex-ante regulation.<sup>35</sup>

### B.1 Banks' Moral Hazard: General Portfolio and Investment Decisions

In our baseline formulation, neither depositors nor banks make portfolio decisions. Allowing for both sets of decisions is important to allow banks or depositors to adjust their risk-taking behavior in response to changes in the level of coverage — these effects are also often referred to as moral hazard. Depositors now have a consumption-savings decision at date 0 and a portfolio decision among  $K$  securities. In particular, depositors have access to  $k = 1, 2, \dots, K$  assets, with returns  $\theta_{1k}(s)$  at date 1 in state  $s$  for early depositors and returns  $\theta_{2k}(s)$  at date 2 in state  $s$  for late depositors. Hence, the resources of early and late depositors are respectively given by  $Y_1(i, s) = \sum_k \theta_{1k}(s) y_k(i)$  and  $Y_2(i, s) = \sum_k \theta_{2k}(s) y_k(i)$ . We preserve the structure of the distribution of deposits. Therefore, the budget constraint of depositors at date 0 is given by

$$\sum_k y_k(i) + D_0(i) + C_0(i) = Y_0(i), \quad (\text{A8})$$

where  $Y_0(i)$ , which denotes the initial endowment of depositor  $i$ , and  $D_0(i)$  are primitives of the model. Subject to Equation (A8), the ex-ante utility of depositors now corresponds to

$$U(C_0(i)) + \mathbb{E}_s [\lambda U(C_1(i, s)) + (1 - \lambda) U(C_2(i, s))], \quad (\text{A9})$$

where  $C_1(i, s)$  and  $C_2(i, s)$  respectively denote the consumption of early and late depositors with initial deposits  $D_0(i)$  for a given realization of the state  $s$ . Depositors optimally choose their holdings of the different assets  $y_k(i)$  to maximize their expected utility.

Additionally, banks have access to  $h = 1, 2, \dots, H$  investment opportunities, which offer a gross return  $\rho_{1h}(s)$  at date 1 and a return  $\rho_{2h}(s)$  between dates 1 and 2 in state  $s$ . Hence, at date 0, banks must choose shares  $\psi_h$  for every investment opportunity such that  $\sum_h \psi_h = 1$ . We assume that banks liquidate an equal share of every type of investment at date 1. This is a particularly tractable formulation to introduce multiple investment opportunities. Our results could be extended to the case in which different investments have different liquidation rates at date 1 and banks have the choice of liquidating different investments in different proportions.

Given our assumptions, we can show that the counterpart to the failure threshold  $\tilde{D}_1(s)$  in Equation (7) is given by

$$\tilde{D}_1(s) = \frac{(R_1 - \sum_h \rho_{1h}(s) \psi_h) \bar{D}_0}{1 - \frac{1}{\sum_h \rho_{1h}(s) \psi_h} \sum_h \rho_{2h}(s) \rho_{1h}(s) \psi_h}, \quad (\text{A10})$$

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<sup>35</sup>In this section, for simplicity, we do not make explicit some of the additional arguments of  $V(i)$  and  $V(\tau)$ .

allowing us to characterize the equilibrium thresholds  $\hat{s}$  and  $s^*$  as in the baseline model. It is equally straightforward to generalize the values taken by  $\alpha_F(s)$ ,  $\alpha_N(s)$ , and  $T(s)$ . We characterize below the optimal choices of  $y_k(i)$  and  $\psi_h$  by depositors and banks and focus again on the directional test for how welfare varies with the level of coverage.

**Proposition 6. (Directional test for  $\delta$  under general investment opportunities)** *The change in welfare induced by a marginal change in the level of deposit insurance  $\frac{dW}{d\delta}$  under perfect regulation is given by*

$$\frac{dW}{d\delta} = \int \omega(j) \left( -\frac{\partial q^F}{\partial \delta} \left( \frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \right) dj + q^F \mathbb{E}_s^F \left[ \int \frac{U'(C^F(j, s))}{U'(C^F(j, s^*))} \frac{\partial C^F(j, s)}{\partial \delta} \right] \right) dj,$$

where  $\mathbb{E}_s^F[\cdot]$  stands for a conditional expectation over bank failure states and  $q^F$  denotes the unconditional probability of bank failure. If  $\frac{dW}{d\delta} > (<) 0$ , it is optimal to locally increase (decrease) the level of coverage.

Proposition 6 extends the results of the baseline model by showing that introducing a consumption-savings and portfolio choices for depositors does not modify the set of sufficient statistics already identified under perfect regulation. However, allowing unregulated banks to make investment choices requires accounting for a new set of fiscal externality terms. The new set of fiscal externalities, which capture the direct effects of banks' changes in behavior on taxpayers welfare, is now given by

$$\underbrace{\frac{\partial V_m(\tau)}{\partial R_1} \frac{dR_1}{d\delta}}_{\text{Liability-side regulation}} + \underbrace{\sum_h \frac{\partial V_m(\tau)}{\partial \psi_h} \frac{d\psi_h}{d\delta}}_{\text{Asset-side regulation}}. \quad (\text{A11})$$

As in Section 3.2, we expect more generous levels of coverage to increase the deposit rate, so  $\frac{\partial V_m(\tau)}{\partial R_1} \frac{dR_1}{d\delta} < 0$ , making it socially costlier to increase  $\delta$ , since banks do not internalize the fiscal consequences of offering higher deposit rates. In principle, it is impossible to individually sign each of the  $H$  terms  $\frac{\partial V_m(\tau)}{\partial \psi_h} \frac{d\psi_h}{d\delta}$  that determine the regulation of banks' asset allocations. However, in most cases, it is reasonable to expect that the sum of all these terms takes negative values, since competitive banks have incentives to increase their risk-taking when the level of coverage is higher. Previous research has nonetheless shown that the risk-taking behavior of banks is sensitive to the details of the market environment; see, for instance, Boyd and De Nicolo (2005) and Martinez-Miera and Repullo (2010). In imperfectly competitive environments, it should not be surprising for the asset-side regulation term in Equation (A11) to take positive values.<sup>36</sup>

However, regardless of their sign, our results robustly point out that both liability-side regulations, controlling the deposit rate offered by banks, and asset-side regulations, controlling the investment portfolio of banks, are in general needed to maximize social welfare when ex-ante policies are feasible.<sup>37</sup>

<sup>36</sup>See also Corbae and D'Erasmus (2019) and Corbae and Levine (2018) for recent work that explores the role of imperfectly competitive intermediaries.

<sup>37</sup>In practice, capital requirements and net stable funding ratios are forms of liability-side regulations, while liquidity coverage ratios are an example of asset-side regulations. See Diamond and Kashyap (2016) for a recent assessment of these policy measures in a model of runs and Hachem and Song (2017) for a study of its ex-ante consequences in an environment with strategic banks.

The optimal corrective policy introduces wedges on banks' choices that can be approximated as follows:

$$\tau_{R_1} = -\frac{\partial V_m(\tau)}{\partial R_1} \approx \frac{\partial \mathbb{E}_s [T(s) + \kappa(T(s))]}{\partial R_1} \quad \text{and} \quad \tau_{\psi_h} = -\frac{\partial V_m(\tau)}{\partial \psi_h} \approx \frac{\partial \mathbb{E}_s [T(s) + \kappa(T(s))]}{\partial \psi_h}. \quad (\text{A12})$$

As discussed above, restrictions on the set of ex-ante instruments available to the planner deliver intermediate outcomes between the two extremes analyzed here. Equation (A12) provides direct guidance on how to set ex-ante policies to correct the ex-ante distortions on banks' behavior caused by deposit insurance.

## B.2 Alternative Equilibrium Selection Mechanisms

In the baseline model, depositors coordinate following an exogenous sunspot. We now show that varying the information structure and the equilibrium selection procedure does not change the sufficient statistics we identify. We consider a global game structure in which late depositors observe at date 1 an arbitrarily precise private signal about the date 2 return on banks' investments before deciding  $D_1(i, s)$ . With that information structure, Goldstein and Pauzner (2005) show, in a model which can be mapped to our baseline model with no deposit insurance, that there exists a unique equilibrium in threshold strategies in which depositors withdraw their deposits when they receive a sufficiently low signal but leave their deposits in the bank otherwise.

Since our goal in this paper is to show the robustness of our optimal policy characterization and to directly use the set of sufficient statistics that we identify, we take the outcome of a global game as a primitive. In particular, we take as a prediction of the global game that there exists a threshold  $s^G(\delta, R_1)$  such that when  $s \leq s^G(\delta, R_1)$  there is a bank failure with certainty but when  $s > s^G(\delta, R_1)$  no-failure occurs, with the following properties:

$$\frac{\partial s^G}{\partial R_1} \geq 0 \quad \text{and} \quad \frac{\partial s^G}{\partial \delta} \leq 0.$$

Goldstein and Pauzner (2005) formally show that  $\frac{\partial s^G}{\partial R_1} \geq 0$ , while Allen et al. (2018) formally show that  $\frac{\partial s^G}{\partial \delta} \leq 0$  in a special case of our framework. In fact, any model of behavior which generates a threshold with these properties, not necessarily a global game, is consistent with our results.

Therefore, given the behavior of depositors at date 1, the ex-ante welfare of depositors is now given by  $\int_{i \in I} V(i, \delta, R_1) dG(i)$ , where

$$V(i, \delta, R_1) = \lambda \mathbb{E}_s [U(C_1(i, s))] + (1 - \lambda) \mathbb{E}_s [U(C_2(i, s))], \quad (\text{A13})$$

and we define

$$\mathbb{E}_s [U(C_t(i, s))] = \int_{\underline{s}}^{s^G(\delta, R_1)} U(C_t^F(i, s)) dF(s) + \int_{s^G(\delta, R_1)}^{\bar{s}} U(C_t^N(i, s)) dF(s),$$

where early and late depositors' consumption is exactly defined by Equations (4) and (5). We can then

show that the characterization of  $\frac{dW}{d\delta}$  remains valid in this context.

**Proposition 7. (Directional test for  $\delta$  under an alternative equilibrium selection)** *The change in welfare induced by a marginal change in the level of deposit insurance  $\frac{dW}{d\delta}$  under perfect regulation is given by*

$$\frac{dW}{d\delta} = \int \omega(j) \left( -\frac{\partial q^F}{\partial \delta} \left( \frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \right) + q^F \mathbb{E}_s^F \left[ \int \frac{U'(C^F(j, s))}{U'(C^F(j, s^*))} \frac{\partial C^F(j, s)}{\partial \delta} \right] \right) dj, \quad (\text{A14})$$

where  $\mathbb{E}_s^F[\cdot]$  stands for a conditional expectation over bank failure states and  $q^F$  denotes the unconditional probability of bank failure. If  $\frac{dW}{d\delta} > (<) 0$ , it is optimal to locally increase (decrease) the level of coverage.

The particular information structure considered and the equilibrium selection procedure only enter in the expression of  $\frac{dW}{d\delta}$  through the sufficient statistics identified in this paper. In particular, this is true even though the sensitivity of the probability of bank failure to changes in the level of coverage  $\frac{\partial q^F}{\partial \delta}$  will depend on the assumptions on the informational structure of the economy. Studying a global game model, as in Allen et al. (2018), is appealing because it makes it possible to understand how the probability of failure is endogenously determined. However, Proposition 7 shows that it is enough to measure the sufficient statistics identified in this paper.

### B.3 General Equilibrium Spillovers/Macroprudential Considerations

In our baseline formulation, as in Diamond and Dybvig (1983), bank decisions do not affect aggregate variables, so our analysis so far can be defined as microprudential. When the decisions made by banks affect aggregate variables, for instance, asset prices, further exacerbating the possibility of a bank failure, the optimal deposit insurance formula may incorporate a macroprudential correction. These general equilibrium effects arise in models in which economy-wide outcomes determined by decentralized choices directly interact with coordination failures. Our extension captures in a simple form the macro implications of banks' choices, which may operate through pecuniary externalities or aggregate demand externalities (Dávila and Korinek, 2018; Farhi and Werning, 2016).

Formally, we now assume that, given a level of aggregate withdrawals  $\bar{\Omega}(s) = \bar{D}_0 R_1 - \bar{D}_1(s)$ , banks must liquidate  $\theta(\bar{\Omega}(s))$  of their investments, where  $\theta(\cdot) \geq 1$  is a well-behaved increasing function. By assuming that banks have to liquidate more than one-for-one their investments at a rate that increases with the aggregate level of liquidations, we capture the possibility of illiquidity in financial markets when many banks unwind existing investments. This is a parsimonious way of incorporating aggregate linkages, but there is scope for richer modeling of interbank markets as in, for instance, Freixas, Martin and Skeie (2011). Under this assumption, the level of resources available to banks with withdrawals  $\Omega(s)$ , when the level of total withdrawals is  $\bar{\Omega}(s)$ , is given by

$$\rho_2(s) \left( \rho_1(s) \bar{D}_0 - \theta(\bar{\Omega}(s)) \Omega(s) \right). \quad (\text{A15})$$

Equation (A15) generalizes the left-hand side of Equation (3). When  $\theta(\cdot) > 1$ , it captures that the

unit price of liquidating investments is increasing in the aggregate level of withdrawals. Following the same logic used to solve the baseline model, we can define thresholds  $\hat{s}$  and  $s^*$ , which now have  $\bar{\Omega}(s)$  as a new argument. When the regulator sets  $\delta$  optimally, she takes into account the effects of individual banks' choices on the aggregate level of withdrawals  $\bar{\Omega}(s)$ . Under these assumptions, we show that  $\frac{dW}{d\delta}$  satisfies the same equation as in our baseline model when ex-ante regulation is available, although it must incorporate a macroprudential correction when ex-ante regulation is not available.

**Proposition 8. (Directional test for  $\delta$  incorporating aggregate spillovers)** *The change in welfare induced by a marginal change in the level of deposit insurance  $\frac{dW}{d\delta}$  under perfect regulation is given by*

$$\frac{dW}{d\delta} = \int \omega(j) \left( -\frac{\partial q^F}{\partial \delta} \left( \frac{U(C^N(j, s^G)) - U(C^F(j, s^G))}{U'(C^F(j, s^G))} \right) dj + q^F \mathbb{E}_s^F \left[ \frac{U'(C^F(j, s))}{U'(C^F(j, s^G))} \frac{\partial C^F(j, s)}{\partial \delta} \right] \right) dj, \quad (\text{A16})$$

where  $\mathbb{E}_s^F[\cdot]$  stands for a conditional expectation over bank failure states and  $q^F$  denotes the unconditional probability of bank failure. If  $\frac{dW}{d\delta} > (<) 0$ , it is optimal to locally increase (decrease) the level of coverage.

In this case, ex-ante regulation can directly target the wedges caused by aggregate spillovers. In this case, the ex-ante regulation faced by banks partly addresses both the fiscal externality that emerges from the presence of deposit insurance and the externality induced by the aggregate spillovers caused by competitive deposit rate setting. Similar formulas would apply when banks have general portfolio decisions, as in our analysis earlier in this section.

As in the case of moral hazard, it is possible to correct the welfare impact of aggregate spillovers with ex-ante regulation. The optimal corrective policy can be expressed in this case as

$$\tau_{R_1} = - \underbrace{\frac{\partial V_m^P(\tau)}{\partial R_1}}_{\text{Fiscal. Ext.}} - \underbrace{\int_{i \in I} \left( \frac{\partial V_m^P(i)}{\partial R_1} - \frac{\partial V_m(i)}{\partial R_1} \right) dG(i)}_{\text{Spillovers}},$$

where the superscript  $P$  corresponds to the welfare assessment from the planner's perspective, as described below. The first term accounts for banks' fiscal externalities, as studied above. The second term, which accounts for the general equilibrium spillovers of banks decisions, is a function of the terms  $\frac{\partial s_P^*}{\partial R_1} - \frac{\partial s^*}{\partial R_1}$  and  $\frac{\partial \hat{s}_P}{\partial R_1} - \frac{\partial \hat{s}}{\partial R_1}$ , which account for the fact that the planner acknowledges that when banks offer higher rates, withdrawals are higher and bank failures more likely.

## B.4 Proofs and Derivations

### Proposition 6. (Directional test for $\delta$ under general investment opportunities)

First, we establish the new failure threshold, which corresponds to Equation (A10) in the text. For a given common liquidation rate  $\varphi$ , the resources at date 2 for a bank are now given by

$$\begin{aligned} \sum_h \rho_{2h}(s) \left( \rho_{1h}(s) \psi_h \bar{D}_0 - \varphi \rho_{1h}(s) \psi_h \bar{D}_0 \right) &= \sum_h \rho_{2h}(s) \left( \rho_{1h}(s) \psi_h \bar{D}_0 - \frac{\rho_{1h}(s) \psi_h \bar{D}_0}{\sum_h \rho_{1h}(s) \psi_h \bar{D}_0} \Omega(s) \right) \\ &= \sum_h \rho_{2h}(s) \left( \rho_{1h}(s) \psi_h \bar{D}_0 - \frac{\rho_{1h}(s) \psi_h}{\sum_h \rho_{1h}(s) \psi_h} \left( \bar{D}_0 R_1 - \bar{D}_1(s) \right) \right), \end{aligned}$$

where we use the fact that the level of withdrawals  $\Omega(s)$  pins down the liquidation rate  $\varphi = \frac{\Omega(s)}{\sum_h \rho_{1h}(s) \psi_h \bar{D}_0}$ . It is therefore easy to show that the threshold for the level of deposits that delimits the probability of failure is

$$\tilde{D}_1(s) = \frac{(R_1 - \sum_h \rho_{1h}(s) \psi_h) \bar{D}_0}{1 - \frac{\sum_h \rho_{2h}(s) \rho_{1h}(s) \psi_h}{\sum_h \rho_{1h}(s) \psi_h}},$$

which depends on  $R_1$  and  $\psi_h$ . It is straightforward to compute consumption for early and late depositors, as in Equations (4) and (5). In this case

$$T(s) = \max \left\{ \int_{i \in I} \min \{ D_0(i) R_1, \delta \} dG(i) - \sum_h \chi_h(s) \rho_{1h}(s) \psi_h \bar{D}_0, 0 \right\}.$$

We can express  $\frac{dW}{d\delta} = \int_{i \in I} \frac{\frac{dV(i)}{d\delta}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{dV(\tau)}{d\delta}}{U'(C^F(\tau, s^*))}$  as follows:

$$\begin{aligned} \frac{dW}{d\delta} &= \int_{i \in I} \frac{\frac{\partial V(i)}{\partial \delta}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{\partial V(\tau)}{\partial \delta}}{U'(C^F(\tau, s^*))} \\ &+ \left( \int_{i \in I} \frac{\frac{\partial V(i)}{\partial R_1}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{\partial V(\tau)}{\partial R_1}}{U'(C^F(\tau, s^*))} \right) \frac{dR_1}{d\delta} \\ &+ \sum_h \left( \int_{i \in I} \frac{\frac{\partial V(i)}{\partial \psi_h}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{\partial V(\tau)}{\partial \psi_h}}{U'(C^F(\tau, s^*))} \right) \frac{d\psi_h}{d\delta} \\ &+ \left( \int_{i \in I} \sum_k \frac{\frac{\partial V(i)}{\partial y_k(i)}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{\partial V(\tau)}{\partial y_k}}{U'(C^F(\tau, s^*))} \right) \frac{dy_k}{d\delta}, \end{aligned}$$

where

$$\begin{aligned} \int_{i \in I} \frac{\frac{\partial V(i)}{\partial \delta}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{\partial V(\tau)}{\partial \delta}}{U'(C^F(\tau, s^*))} &= - \int \frac{\partial q^F}{\partial \delta} \left( \frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(i, s^*))} \right) dj \\ &+ \int q^F \mathbb{E}_s^F \left[ \frac{U'(C^F(j, s))}{U'(C^F(\tau, s^*))} \frac{\partial C^F(j, s)}{\partial \delta} \right] dj \end{aligned}$$

which corresponds to  $\frac{dW}{d\delta}$  under perfect regulation. The definition of  $V(i)$  now corresponds to the updated utility specification (A9), and it is subject to Equations (A8) and  $\sum_h \psi_h = 1$ .

Unregulated banks optimally set  $\int_{i \in I} \frac{\partial V_m(i)}{\partial R_1} dG(i) = 0$ ,  $\int_{i \in I} \frac{\partial V_m(i)}{\partial \psi_h} dG(i) = 0$ ,  $\forall h$ , and  $\frac{\partial V_m(i)}{\partial y_k(i)} = 0$ ,  $\forall i, \forall k$ . Since  $T(s)$  is independent of  $y_k(i)$ , it is always the case that  $\frac{\partial V_m(\tau)}{\partial y_k(i)} = 0$ . Therefore, in that case,  $\frac{dW}{d\delta}$  corresponds to

$$\frac{dW}{d\delta} = \int_{i \in I} \frac{\frac{dV(i)}{d\delta}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{dV(\tau)}{d\delta}}{U'(C^F(\tau, s^*))} + \frac{\frac{\partial V(\tau)}{\partial R_1}}{U'(C^F(\tau, s^*))} \frac{dR_1}{d\delta} + \sum_h \frac{\frac{\partial V(\tau)}{\partial \psi_h}}{U'(C^F(\tau, s^*))} \frac{d\psi_h}{d\delta}.$$

We can express  $\frac{\partial V(\tau)}{\partial R_1}$  exactly as in Equation (39), and  $\frac{\partial V(\tau)}{\partial \psi_h}$  as follows:

$$\begin{aligned} \frac{\partial V(\tau)}{\partial \psi_h} &= q^F \mathbb{E}_s^F \left[ U'(C^F(\tau, s)) \frac{\partial C^F(\tau, s)}{\partial \psi_h} \right] + \left[ U(C^F(\tau, s^*)) - U(C^N(\tau, s^*)) \right] \pi f(s^*) \frac{\partial s^*}{\partial \psi_h} \\ &\quad + \left( U(C^F(\tau, \hat{s})) - U(C^N(\tau, \hat{s})) \right) (1 - \pi) \frac{\partial \hat{s}}{\partial \psi_h} f(\hat{s}), \end{aligned}$$

where

$$q^F \mathbb{E}_s^F \left[ U'(C^F(\tau, s)) \frac{\partial C^F(\tau, s)}{\partial \psi_h} \right] = \int_{\underline{s}}^{\hat{s}} U'(C^F(\tau, s)) \frac{\partial C^F(\tau, s)}{\partial \psi_h} dF(s) + \pi \int_{\hat{s}}^{s^*} U'(C^F(\tau, s)) \frac{\partial C^F(\tau, s)}{\partial \psi_h} dF(s).$$

Using an approximation as in Proposition 2, we can write

$$\frac{\partial V_m(\tau)}{\partial R_1} \approx - \frac{\partial \mathbb{E}_s [T(s) + \kappa(T(s))]}{\partial R_1} \quad \text{and} \quad \frac{\partial V_m(\tau)}{\partial \psi_h} \approx - \frac{\partial \mathbb{E}_s [T(s) + \kappa(T(s))]}{\partial \psi_h}.$$

**Proposition 7. (Directional test for  $\delta$  under an alternative equilibrium selection)**

Under the new equilibrium selection assumption,  $V(i, \delta, R_1)$  is defined in Equation (A13) and  $V(\tau, \delta, R_1)$  is defined as

$$V(\tau, \delta, R_1) = \int_{\underline{s}}^{s^G(\delta, R_1)} U(C^F(\tau, s)) dF(s) + \int_{s^G(\delta, R_1)}^{\bar{s}} U(C^N(\tau, s)) dF(s).$$

Therefore, under perfect regulation, we can express  $\frac{dW}{d\delta}$  as follows:

$$\frac{dW}{d\delta} = \int_{i \in I} \frac{\frac{\partial V(i, \delta, R_1)}{\partial \delta}}{U'(C^F(i, s^G))} dG(i) + \frac{\frac{\partial V(\tau, \delta, R_1)}{\partial \delta}}{U'(C^F(i, s^G))},$$

where the marginal impact of a change in  $\delta$  on depositors' welfare corresponds to

$$\int_{i \in I} \frac{\frac{\partial V(i, \delta, R_1)}{\partial \delta}}{U'(C^F(i, s^G))} dG(i) = \mathbb{E}_\lambda \left[ \int \frac{\frac{\partial \mathbb{E}_s [U(C_t(i, s))]}{\partial \delta}}{U'(C^F(i, s^G))} dG(i) \right],$$

where

$$\frac{\partial \mathbb{E}_s [U(C_t(i, s))]}{\partial \delta} = \left( U(C_t^F(i, s^G)) - U(C_t^N(i, s^G)) \right) \underbrace{f(s^G) \frac{\partial s^G}{\partial \delta}}_{=\frac{\partial q^F}{\partial \delta}} + \underbrace{\int_s^{s^G} U'(C_t^F(i, s)) \frac{\partial C_t^F(i, s)}{\partial \delta} dF(s)}_{=-q^F \mathbb{E}_s^F \left[ U'(C_t^F(i)) \frac{\partial C_t^F(i)}{\partial \delta} \right]}$$

since  $\frac{\partial C_t^N(i, s)}{\partial \delta} = 0$ , and where we can write

$$\frac{\partial V(\tau, \delta, R_1)}{\partial \delta} = \left( U(C^F(\tau, s^G)) - U(C^N(\tau, s^G)) \right) \underbrace{f(s^G) \frac{\partial s^G}{\partial \delta}}_{=\frac{\partial q^F}{\partial \delta}} + \underbrace{\int_s^{s^G} U'(C^F(\tau, s)) \frac{\partial C^F(\tau, s)}{\partial \delta} dF(s)}_{=-q^F \mathbb{E}_s^F \left[ U'(C^F(\tau, s)) \frac{\partial C^F(\tau, s)}{\partial \delta} \right]}$$

Equation (A14) follows immediately.

**Proposition 8. (Directional test for  $\delta$  incorporating aggregate spillovers)**

First, we establish the new failure threshold, which corresponds to Equation (A15) in the text. That is, the total resources available to a given bank at date 2, given aggregate withdrawals  $\bar{\Omega}(s)$ , corresponds to  $\rho_2(s) \left( \rho_1(s) \bar{D}_0 - \theta(\bar{\Omega}(s)) \Omega(s) \right)$ , which can be expressed as

$$\rho_2(s) \left( \theta(\bar{\Omega}(s)) \bar{D}_1(s) + \left( \rho_1(s) - \theta(\bar{\Omega}(s)) R_1 \right) \bar{D}_0 \right).$$

As in the baseline model, we can implicitly define a threshold level of deposits, denoted by  $\tilde{D}_1(s)$  and given by

$$\tilde{D}_1(s) = \frac{\theta(\bar{\Omega}(s)) R_1 - \rho_1(s)}{\theta(\bar{\Omega}(s)) - \frac{1}{\rho_2(s)}} \bar{D}_0,$$

which delimits the failure regions. When banks choose  $R_1$  unregulated, they do not internalize that deposit rates affect  $\theta(\cdot)$ . In that case, we can define two types of thresholds. We denote the thresholds used by banks ex-ante to choose  $R_1$  by  $\hat{s}(R_1)$  and  $s^*(\delta, R_1)$ . Those perceived by the deposit insurance authority, incorporating the effects on aggregate withdrawals  $\bar{\Omega}(s) = \bar{D}_0 R_1 - \bar{D}_1(s)$ , are denoted by  $\hat{s}_P(R_1)$  and  $s_P^*(\delta, R_1)$ . In equilibrium,  $\hat{s}(R_1) = \hat{s}_P(R_1)$  and  $s^*(\delta, R_1) = s_P^*(\delta, R_1)$ , even though, crucially, the partial derivatives of each set of thresholds with respect to  $R_1$  are different.

As above, we can express  $\frac{dW}{d\delta}$  as follows:

$$\begin{aligned} \frac{dW}{d\delta} &= \int_{i \in I} \frac{\frac{dV^P(i)}{d\delta}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{dV^P(\tau)}{d\delta}}{U'(C^F(\tau, s^*))} \\ &= \int_{i \in I} \frac{\frac{\partial V^P(i)}{\partial \delta}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{\partial V^P(\tau)}{\partial \delta}}{U'(C^F(\tau, s^*))} + \left( \int_{i \in I} \frac{\frac{\partial V^P(i)}{\partial R_1}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{\partial V^P(\tau)}{\partial R_1}}{U'(C^F(\tau, s^*))} \right) \frac{dR_1}{d\delta}, \end{aligned}$$

where we use the  $P$  notation to emphasize that  $V^P(i)$  and  $V^P(\tau)$  are calculated from the perspective

of a planner who uses thresholds  $\hat{s}_P(R_1)$  and  $s_P^*(\delta, R_1)$ , which account for equilibrium spillovers. Note that the first two terms are given by

$$\int_{i \in I} \frac{\frac{\partial V^P(i)}{\partial \delta}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{\partial V^P(\tau)}{\partial \delta}}{U'(C^F(\tau, s^*))} = \int q^F \mathbb{E}_s^F \left[ \frac{U'(C^F(j, s))}{U'(C^F(j, s^*))} \frac{\partial C^F(j, s)}{\partial \delta} \right] dj \\ + \underbrace{\pi f(s^*)}_{= \frac{\partial q^F}{\partial \delta}} \int \left( \frac{U(C^F(j, s^*)) - U(C^N(j, s^*))}{U'(C^F(j, s^*))} \right) dj.$$

We can express  $\int_{i \in I} \frac{\frac{\partial V^P(i)}{\partial R_1}}{U'(C^F(i, s^*))} dG(i) = \int_{i \in I} \left( \frac{\frac{\partial V^P(i)}{\partial R_1}}{U'(C^F(i, s^*))} - \frac{\frac{\partial V(i)}{\partial R_1}}{U'(C^F(i, s^*))} \right) dG(i)$  as follows:

$$\int_{i \in I} \frac{\frac{\partial V^P(i)}{\partial R_1}}{U'(C^F(i, s^*))} dG(i) = \mathbb{E}_\lambda \left[ \int_{i \in I} \frac{\frac{\partial \mathbb{E}_s^P[U(C_t(i, s))]}{\partial R_1}}{U'(C^F(i, s^*))} dG(i) \right], \\ = \mathbb{E}_\lambda \left[ \int_{i \in I} \left( \frac{U(C_t^F(i, s^*)) - U(C_t^N(i, s^*))}{U'(C^F(i, s^*))} \right) \pi f(s^*) \left( \frac{\partial s_P^*}{\partial R_1} - \frac{\partial s^*}{\partial R_1} \right) dG(i) \right] \\ + \mathbb{E}_\lambda \left[ \int_{i \in I} \left( \frac{U(C_t^F(i, \hat{s})) - U(C_t^N(i, \hat{s}))}{U'(C^F(i, s^*))} \right) (1 - \pi) f(\hat{s}_P) \left( \frac{\partial \hat{s}_P}{\partial R_1} - \frac{\partial \hat{s}}{\partial R_1} \right) dG(i) \right],$$

where we use the fact that for depositors  $\mathbb{E}_\lambda \left[ \int_{i \in I} \frac{\frac{\partial \mathbb{E}_s[U(C_t(i, s))]}{\partial R_1}}{U'(C^F(i, s^*))} dG(i) \right] = 0$ . Similarly, we can express  $\frac{\partial V^P(\tau)}{\partial R_1}$  as follows:

$$\frac{\partial V^P(\tau)}{\partial R_1} = q^F \mathbb{E}_s^F \left[ U'(C^F(\tau, s)) \frac{\partial C^F(\tau, s)}{\partial R_1} \right] + \left( U(C^F(\tau, s^*)) - U(C^N(\tau, s^*)) \right) \pi f(s^*) \frac{\partial s_P^*}{\partial R_1} \\ + \left[ U(C^F(\tau, \hat{s})) - U(C^N(\tau, \hat{s})) \right] (1 - \pi) \frac{\partial \hat{s}_P}{\partial R_1} f(\hat{s}),$$

where

$$q^F \mathbb{E}_s^F \left[ U'(C^F(\tau, s)) \frac{\partial C^F(\tau, s)}{\partial R_1} \right] = \int_{\underline{s}}^{\hat{s}} U'(C^F(\tau, s)) \frac{\partial C^F(\tau, s)}{\partial R_1} dF(s) + \pi \int_{\hat{s}}^{s^*} U'(C^F(\tau, s)) \frac{\partial C^F(\tau, s)}{\partial R_1} dF(s).$$

Using an approximation as in Proposition 2, we can write  $\frac{\partial V_m(\tau, s)}{\partial R_1} \approx -\frac{\partial \mathbb{E}_s[T(s) + \kappa(T(s))]}{\partial R_1}$ .

Therefore, under perfect regulation,  $\frac{dW}{d\delta} = \int_{i \in I} \frac{\frac{\partial V^P(i)}{\partial \delta}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{\partial V^P(\tau)}{\partial \delta}}{U'(C^F(\tau, s^*))}$ , which corresponds to Equation (A16). The optimal regulation of banks, set so that  $\int_{i \in I} \frac{\frac{\partial V^P(i)}{\partial R_1}}{U'(C^F(i, s^*))} dG(i) + \frac{\frac{\partial V^P(\tau)}{\partial R_1}}{U'(C^F(\tau, s^*))} = 0$ , now incorporates a correction that accounts for aggregate spillovers. The optimal regulation is set so that banks internalize their fiscal externality and their aggregate spillovers.

## C Additional Results

### C.1 Deposit Insurance Fund

Here we describe a simple way to explicitly introduce a deposit insurance fund in our environment. We assume that banks must set aside a fraction  $\tau^f$  of their initial deposits to be invested in a fund. Therefore, the date 0 investment of a bank becomes  $(1 - \tau^f) \bar{D}_0$ . Consequently, bank failure is determined in this case by

$$\begin{aligned} \text{Bank Failure,} & \quad \text{if } \rho_2(s) \left( \rho_1(s) \left( 1 - \tau^f \right) \bar{D}_0 - \Omega(s) \right) < \bar{D}_1(s) \\ \text{No Bank Failure,} & \quad \text{if } \rho_2(s) \left( \rho_1(s) \left( 1 - \tau^f \right) \bar{D}_0 - \Omega(s) \right) \geq \bar{D}_1(s), \end{aligned}$$

which is the counterpart of Equation (3) in the text. Following the same steps as in the baseline model, the new deposit failure threshold is now given by

$$\tilde{D}_1(s) = \begin{cases} \frac{(R_1 - \rho_1(s)(1 - \tau^f))\bar{D}_0}{1 - \frac{1}{\rho_2(s)}}, & \text{if } \rho_2(s) > 1 \\ \infty, & \text{if } \rho_2(s) \leq 1, \end{cases}$$

which is the counterpart of Equation (7) in the text. It follows immediately that the thresholds  $\hat{s}$  and  $s^*$  are now increasing functions of  $\tau^f$ . That is, the higher the amount contributed to the deposit fund, the higher the probability of failure.

We assume that the return on the resources held in the fund is  $\rho_1^f(s)$ . In this case, the funding shortfall  $T(s)$  takes the form:

$$T(s) = \max \left\{ \int_{i \in I} \min \{ D_0(i) R_1, \delta \} dG(i) - \chi(s) \rho_1(s) \left( 1 - \tau^f \right) \bar{D}_0 - \rho_1^f(s) \tau^f \bar{D}_0, 0 \right\}. \quad (\text{A17})$$

Compared to its counterpart in the text in Equation (18), now the funds available after liquidation are only  $\chi(s) \rho_1(s) \left( 1 - \tau^f \right) \bar{D}_0$ , but there are  $\rho_1^f(s) \tau^f \bar{D}_0$  funds available through the fund.<sup>38</sup> A similar adjustment applies to  $\alpha_F(s)$  and  $\alpha_N(s)$ .

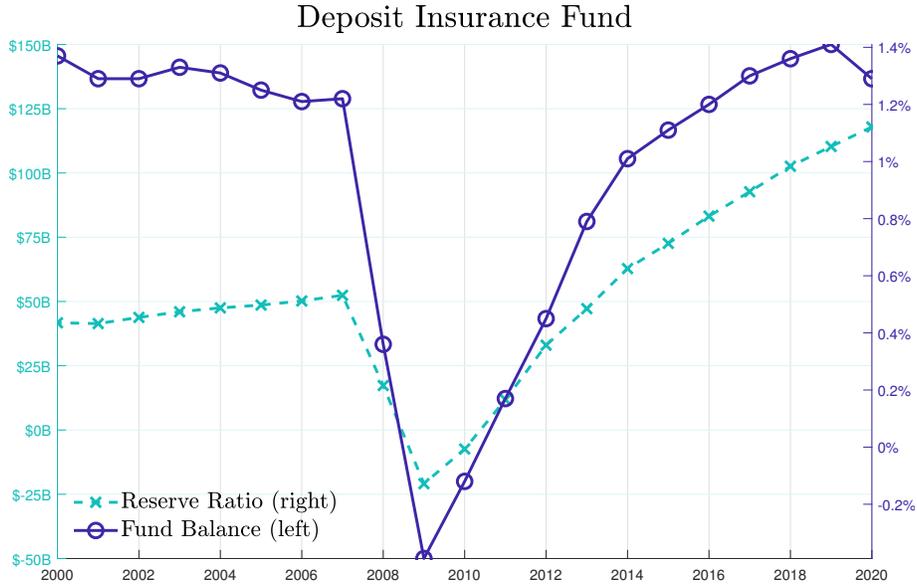
It is evident from Equation (A17) that whenever

$$\chi(s) \rho_1(s) = \rho_1^f(s),$$

the funding shortfall would be equivalent with or without a deposit insurance fund. In that case, our characterization of  $\frac{dW}{d\delta}$  in the paper would be almost unchanged, but for the supposedly small impact of  $\tau^f$  on  $\hat{s}$  and  $s^*$ . If the return on the deposit insurance fund is too low ( $\chi(s) \rho_1(s) > \rho_1^f(s)$ ), and supposing that an increase in level of coverage is associated with a higher  $\tau^f$ , there would be an additional cost to increasing the level of coverage. In any case, there is scope to explore further any issues related to the financing of deposit insurance obligations.

For reference, Figure OA-1 shows the evolution between 2000 and 2020 of i) the balance of the

<sup>38</sup>This formulation is valid for a representative bank. With multiple banks, it is necessary to model carefully the correlation of shocks across banks and understand whether bank failures are isolated events or systemic.



**Note:** Figure OA-1 shows the evolution between 2000 and 2020 of i) the balance of the deposit insurance fund (in nominal terms) in the US and ii) the reserve ratio, which corresponds to the ratio between the balance of the deposit insurance fund and the total estimated insured deposits of the industry. Data are from the FDIC.

Figure OA-1: Evolution of Deposit Insurance Fund

deposit insurance fund (in nominal terms) in the US and ii) the reserve ratio, which corresponds to the ratio between the balance of the deposit insurance fund and the total estimated insured deposits of the industry. In 2009 and 2010, the fund had a negative balance, although part of this fact was due to provisions to cover expected future losses. However, note that the ultimate backstop of the deposit insurance fund is a line of credit from the Treasury.<sup>39</sup>

The Dodd-Frank Act of 2010 sets a minimum reserve ratio of 1.35%. Whenever the reserve ratio falls (or is expected to fall) below this threshold, the FDIC must adopt a restoration plan to return to this level within 8 years.<sup>40</sup>

## C.2 Equivalence of Traditional and Generalized Social Welfare Weights

In this subsection, we show that there is a one-to-one mapping between traditional welfare weights and generalized social welfare weights. First, for completeness, we reproduce here Equation (21) in the text after substituting  $\frac{dV_m(j,\delta,R_1)}{d\delta} = \frac{dV(j,\delta,R_1)}{d\delta} \frac{1}{U'(C^F(j,s^*))}$ :

$$\frac{dW}{d\delta} = \int \omega(j) \frac{\frac{dV(j,\delta,R_1)}{d\delta}}{U'(C^F(j,s^*))} dj. \quad (\text{A18})$$

Alternatively, we can identify social welfare with a weighted sum of depositors' and taxpayers' ex-ante expected utility. We denote traditional welfare weights by  $\theta(j) = \{\theta(i), \theta(\tau)\}$ . In this case, social

<sup>39</sup>See <https://www.nytimes.com/2009/11/25/business/economy/25fdic.html> for a discussion of the 2009 episode.

<sup>40</sup>See <https://www.fdic.gov/deposit/insurance/fund.html> for more information on the management of the deposit insurance fund.

welfare  $W(\delta)$  formally corresponds to

$$W(\delta) = \int \theta(j) V(j, \delta, R_1) dj = \underbrace{\int_{i \in I} \theta(i) V(i, \delta, R_1) dG(i)}_{\text{Depositors}} + \underbrace{\theta(\tau) V(\tau, \delta, R_1)}_{\text{Taxpayers}}.$$

In this case, we can express  $\frac{dW}{d\delta}$  as follows:

$$\frac{dW}{d\delta} = \int \theta(j) \frac{dV(j, \delta, R_1)}{d\delta} dj. \quad (\text{A19})$$

Figure OA-10 on page OA-31 illustrates the magnitudes taken by the traditional welfare weights in the context of our quantitative model.

*Remark. (Equivalence of traditional and generalized social welfare weights)* A direct comparison of Equations (A18) and (A19) concludes that the value of  $\frac{dW}{d\delta}$  under both approaches is identical when

$$\theta(j) = \frac{\omega(j)}{U'(C^F(j, s^*))}. \quad (\text{A20})$$

Equation (A20) implies that using traditional utilitarian weights ( $\theta(j) = 1$ ) is equivalent to choosing generalized welfare weights  $\omega(j) = U'(C^F(j, s^*))$ . Similarly, it also implies that using uniform generalized weights ( $\omega(j) = 1$ ) is equivalent to using traditional weights of the form  $\theta(j) = \frac{1}{U'(C^F(j, s^*))}$ . In that sense, using generalized welfare weights is equivalent to working with a set of endogenous traditional welfare weights. Note that once we select a set of generalized weights and find the optimal level of coverage and the equilibrium allocation associated with those weights, it is possible to use Equation (A20) to find the set of  $\theta(j)$  that would deliver the same allocation.

Finally, note that we could have chosen a state other than  $s^*$  to compute the money-metric normalization. In that case, denoting the arbitrary state by  $\check{s}$ , the normalized welfare change is given by  $\frac{dV_m(j, \delta, R_1)}{d\delta} = \frac{dV(j, \delta, R_1)}{U'(C^F(j, \check{s}))}$ , and the counterpart of Equation (22) becomes

$$\frac{dW}{d\delta} = \int \omega(j) \left( \underbrace{-\frac{\partial q^F}{\partial \delta} m(j, s^*) \left( \frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \right)}_{\text{Marginal Benefit}} + \underbrace{q^F \mathbb{E}_s^F \left[ m(j, s) \frac{\partial C^F(j, s)}{\partial \delta} \right]}_{\text{Marginal Cost}} \right) dj,$$

where  $m(j, s) = \frac{U'(C^F(j, s))}{U'(C^F(j, \check{s}))}$ . In this case, we simply have to normalize  $m(j, s)$  by the reference state. It is evident that Proposition 2 remains valid in this case.

### C.3 Directional Test: General Case

In this section, we provide a directional test for the level of coverage under minimal assumptions. While using the Diamond and Dybvig (1983) framework allows us to completely characterize a fully specified model, here we show that our insights extend more generally. As in the text, we focus on characterizing

the welfare impact of a marginal change in the level of coverage under perfect regulation or when banks do not respond to the level of coverage. When banks are unregulated, our characterization needs to be augmented by the fiscal externality component(s).

Consider an economy in which  $V(j)$  denotes the utility of depositors or taxpayers. We can then write

$$V(j) = \mathbb{E}_s [U_j(C(j, \delta, s); s)] = \int_{\mathcal{F}} U_j(C^F(j, \delta, s); s) dF(s) + \int_{\mathcal{N}} U_j(C^N(j, \delta, s); s) dF(s),$$

where  $\mathcal{F}$  and  $\mathcal{N}$  denote the set of failure and no-failure states, respectively. The value of  $C(j, \delta, s)$  incorporates the final consumption by agent  $j$  in state  $s$ . By making utility state-dependent, we can implicitly account for early and late types.

Once we define the probability of bank failure as  $q(\delta) = \int_{\mathcal{F}} dF(s)$ , we can express the change in individual welfare as

$$\begin{aligned} \frac{dV(j)}{d\delta} &= -\frac{\partial q^F}{\partial \delta} \left( U_j(C^N(j, \delta, s^*); s^*) - U_j(C^F(j, \delta, s^*); s^*) \right) + q^F \mathbb{E}_{\mathcal{F}} \left[ U_j'(C^F(j, \delta, s); s) \frac{\partial C^F(j, \delta, s)}{\partial \delta} \right] \\ &+ (1 - q^F) \mathbb{E}_{\mathcal{N}} \left[ U_j'(C^N(j, \delta, s); s) \frac{\partial C^N(j, \delta, s)}{\partial \delta} \right]. \end{aligned}$$

In money-metric form,  $\frac{dV_m(j, \delta, R_1)}{d\delta} = \frac{\frac{dV(j)}{d\delta}}{U_j'(C^F(j, \delta, s^*); s^*)}$  is given by

$$\begin{aligned} \frac{dV_m(j, \delta, R_1)}{d\delta} &= -\frac{\partial q^F}{\partial \delta} \left( \frac{U_j(C^N(j, \delta, s^*); s^*) - U_j(C^F(j, \delta, s^*); s^*)}{U_j'(C^F(j, \delta, s^*); s^*)} \right) \\ &+ q^F \mathbb{E}_{\mathcal{F}} \left[ \frac{U_j'(C^F(j, \delta, s); s) \frac{\partial C^F(j, \delta, s)}{\partial \delta}}{U_j'(C^F(j, \delta, s^*); s^*)} \right] \\ &+ (1 - q^F) \mathbb{E}_{\mathcal{N}} \left[ \frac{U_j'(C^N(j, \delta, s); s) \frac{\partial C^N(j, \delta, s)}{\partial \delta}}{U_j'(C^F(j, \delta, s^*); s^*)} \right]. \end{aligned}$$

Since  $C^N(\cdot)$  does not depend on  $\delta$  directly, we can express  $\frac{dW}{d\delta}$  as follows:

$$\begin{aligned} \frac{dW}{d\delta} &= \int -\frac{\partial q^F}{\partial \delta} \left( \frac{U_j(C^N(j, \delta, s^*); s^*) - U_j(C^F(j, \delta, s^*); s^*)}{U_j'(C^F(j, \delta, s^*); s^*)} \right) dj \\ &+ \int q^F \mathbb{E}_{\mathcal{F}} \left[ \frac{U_j'(C^F(j, \delta, s); s) \frac{\partial C^F(j, \delta, s)}{\partial \delta}}{U_j'(C^F(j, \delta, s^*); s^*)} \right] dj. \end{aligned} \tag{A21}$$

Equation (A21) is a direct generalization of Equation (22) in the text.

## D Quantitative Application: Additional Material

### D.1 Measurement with CDS data

**Data Description** For our calculations, we use Markit CDS data, as distributed by Wharton Research Data Services (WRDS). Our full sample includes daily data from January 2006 until December 2014. We focus on five-year CDS spreads (these are the most liquid) on the following banks (ticker in parentheses): Bank of America Corp (BACORP), Bank of NY Mellon (BK), Citigroup Inc (C), Goldman Sachs (GS), JP Morgan Chase (JPM), Merrill Lynch & Co Inc (MER), Morgan Stanley (MWD), State Street Corp (STT), Wachovia Corp (WB), and Wells Fargo & Co (WFC). We exclusively consider CDS contracts with CR (Complete Restructuring) as a restructuring clause, so any restructuring event counts as a bank failure for our purposes. Similar results arise when using restructuring clauses MR (Modified Restructuring), MM (Modified Modified Restructuring), or XR (No Restructuring). We only consider CDS contracts on Senior Unsecured Debt and use the recovery rate provided by Markit.

**Measurement** The implied probability of failure can be read from spreads and recovery rates provided one is willing to make some assumptions. We use a simple constant hazard rate model (Hull, 2013), to calculate implied yearly default probabilities as follows:

$$\text{Implied Default Probability} = \frac{\text{5 Year Spread}}{1 - \text{Recovery Rate}}.$$

On October 3, 2008, President George W. Bush signed the Emergency Economic Stabilization Act of 2008, raising the limit on federal deposit insurance coverage from \$100,000 to \$250,000 per depositor. Initially, this change was temporary through the end of 2010, but it was made permanent by the Dodd-Frank Act in July 2010. Our discussion in the text is based on the jumps in the default probability caused by changes in the level of deposit insurance on October 3, 2008. We also gauge our measures by the impact of the Temporary Liquidity Guarantee Program (TLGP), announced on October 14, 2008, in which the FDIC guaranteed in full noninterest-bearing transaction accounts.<sup>41</sup>

### D.2 Model-Based Quantification: Matching Sufficient Statistics

Table OA-1 compares the sufficient statistics used in the direct measurement approach in Section 4.1 with the sufficient statistics implied by our calibration of the structural model in Section 4.2. The value of  $\frac{dW}{d\delta}$  implied by Table 1 using the direct measurement approach is  $\frac{dW}{d\delta} = \$4.5 \times 10^{-4}$ . The value of  $\frac{dW}{d\delta}$  implied by Table OA-1 using the model-based quantification is  $\frac{dW}{d\delta} = \$3.9 \times 10^{-4}$ . We have made sure that the calibration errs on the side of displaying somewhat lower marginal benefits than in the direct measurement approach to be able to robustly argue that the model implies a large level of coverage.

<sup>41</sup>See <https://www.fdic.gov/regulations/resources/TLGP/index.html> for a description of the TLGP.

Table OA-1: Sufficient Statistics as Calibration Targets

	Variable	Direct Measurement	Calibrated Model
	$q^F$	0.025	0.0249
Mg. Benefit	$\frac{\partial \log q^F}{\partial \delta}$	$-2 \times 10^{-6}$	$-1.96 \times 10^{-6}$
	$\int (C^N(j, s^*) - C^F(j, s^*)) dj / \bar{G}$	\$13,809	\$12,700
	$q^{T^+ F}$	1	1
Mg. Cost	$\mathbb{E}_s^F [\kappa'(\cdot)   T > 0]$	0.15	0.15
	$\int_{i \in \mathcal{PI}} dG(i) / \bar{G}$	0.064	0.064

**Note:** To compare with Table 1, note that  $-\frac{0.3}{150,000} = -2 \times 10^{-6}$ . The value of  $\frac{dW}{d\delta}$  implied by Table 1 using the direct measurement approach is  $\frac{dW}{d\delta} = \$4.5 \times 10^{-4}$ . The value of  $\frac{dW}{d\delta}$  implied by Table OA-1 using the model-based quantification is  $\frac{dW}{d\delta} = \$3.9 \times 10^{-4}$ .

### D.3 Model-Based Quantification: Additional Material

The left panel in Figure OA-2 shows the share of insured and uninsured deposits, respectively given by

$$\frac{\int \min \{D_0(i) R_1, \delta\} dG(i)}{\bar{D}_0 R_1} \quad \text{and} \quad \frac{\int \max \{D_0(i) R_1 - \delta, 0\} dG(i)}{\bar{D}_0 R_1}.$$

The right panel in Figure OA-2 shows the share of fully insured accounts/depositors and partially insured accounts/depositors, respectively given by

$$\int_{i \in \mathcal{FI}} dG(i) \quad \text{and} \quad \int_{i \in \mathcal{PI}} dG(i),$$

where  $\mathcal{FI} = \{i | D_0(i) R_1 > \delta\}$  and  $\mathcal{PI} = \{i | D_0(i) R_1 > \delta\}$ . The fact that the distribution of depositors is heavily right-skewed is necessary to be able to match at the same time (when  $\delta = 1$ ) a share of partially insured accounts of 6.4% and share of insured deposits of 62%.

Figures OA-3 and OA-4 are the counterparts in the calibrated model of Figures 4 and 5 in the text. Figure OA-3 shows how  $s^*(\delta, R_1)$  and  $\hat{s}(R_1)$  vary as a function of the level of deposit insurance coverage  $\delta$ . The left panel in Figure OA-4 shows the recovery rate on uninsured deposits in case of failure,  $\alpha_F(s)$ , as well as the funding shortfall,  $T(s)$ , for different values of the realizations of the state  $s$  when  $\delta = 1$ . The right panel in Figure OA-4 shows the additional gross return earned by the deposits that stay within the bank until date 2,  $\alpha_N(s)$ .

The left panel in Figure OA-5 shows the average marginal cost of public funds in failure states,  $\mathbb{E}_s^F [\kappa'(T(s))]$ , and in failure states in which the funding shortfall is positive,  $\mathbb{E}_s^F [\kappa'(T(s)) | T(s) > 0]$ , as a function of the level of deposit insurance coverage  $\delta$ . The right panel in Figure OA-5 shows the unconditional probability of a positive funding shortfall,  $q^{T^+}$ , and the probability of a positive funding shortfall condition on a bank failure taking place, as a function of the level of deposit insurance coverage

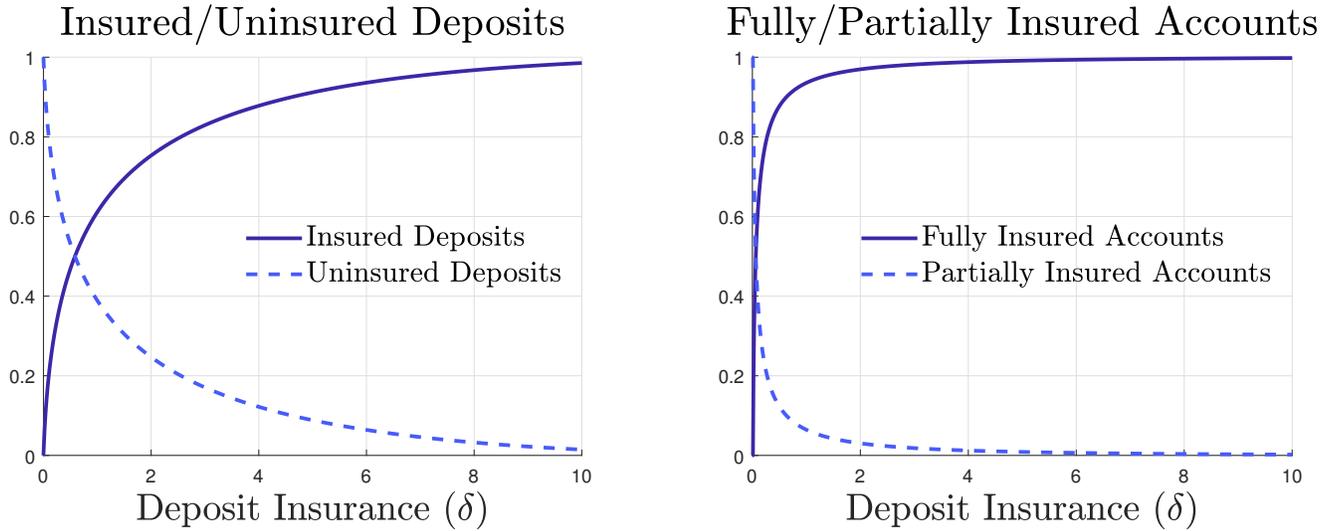


Figure OA-2: Share of Insured/Uninsured Deposits and Fully/Partially Insured Accounts

**Note:** The left panel in Figure OA-2 shows the share of insured deposits and its complement, the share of uninsured deposits, as a function of the level of deposit insurance coverage  $\delta$ . The right panel in Figure OA-2 shows the share of fully insured accounts/depositors and its complement, the share of partially insured accounts/depositors, as a function of the level of deposit insurance coverage  $\delta$ .

$\delta$ . Formally,

$$q^{T_+} = \int_{\underline{s}}^{\hat{s}} \mathbb{I}[T(s) > 0] dF(s) + \pi \int_{\hat{s}}^{s^*} \mathbb{I}[T(s) > 0] dF(s)$$

$$q^{T_+|F} = \frac{\int_{\underline{s}}^{\hat{s}} \mathbb{I}[T(s) > 0] dF(s) + \pi \int_{\hat{s}}^{s^*} \mathbb{I}[T(s) > 0] dF(s)}{\int_{\underline{s}}^{\hat{s}} dF(s) + \pi \int_{\hat{s}}^{s^*} dF(s)},$$

where  $\mathbb{I}[\cdot]$  denotes the indicator function. Figure OA-6 integrates over  $\frac{dW}{d\delta}$ , as defined in Equation (26), to build a measure of social welfare. Figure OA-6 uses the normalization  $W(0) = 0$ .

#### D.4 Sensitivity Analysis: Cost of Public Funds

Figure OA-7 illustrates how changes in the fiscal capacity of the economy, captured by the marginal cost of public funds  $\kappa_1$ , affect the desirability of changing the level of coverage. Changes in the marginal cost of public funds can be interpreted as a shock to the fiscal condition of the economy. Consistent with our analytical results, changes in the level of  $\kappa_1$  exclusively affect taxpayers' welfare, leaving unchanged depositors' welfare. The main effect of an increase in the cost of public funds is that the marginal cost of paying for coverage becomes higher. Given our calibration, as one would expect, a higher  $\kappa_1$  is associated with a lower optimal level of coverage  $\delta^*$ , since the deadweight loss associated with covering funding shortfalls becomes higher.

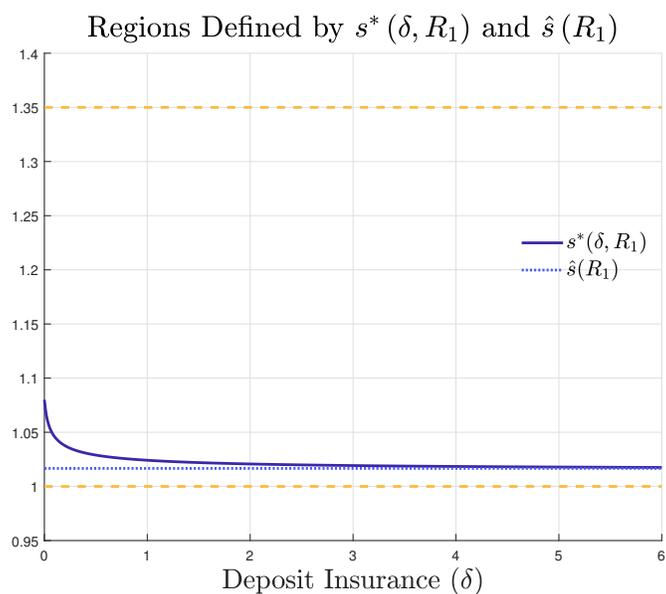


Figure OA-3: Regions Defined by  $s^*(\delta, R_1)$  and  $\hat{s}(R_1)$

**Note:** Figure OA-3 shows the thresholds  $s^*(\delta, R_1)$  and  $\hat{s}(R_1)$  as a function of the level of deposit insurance coverage  $\delta$ .

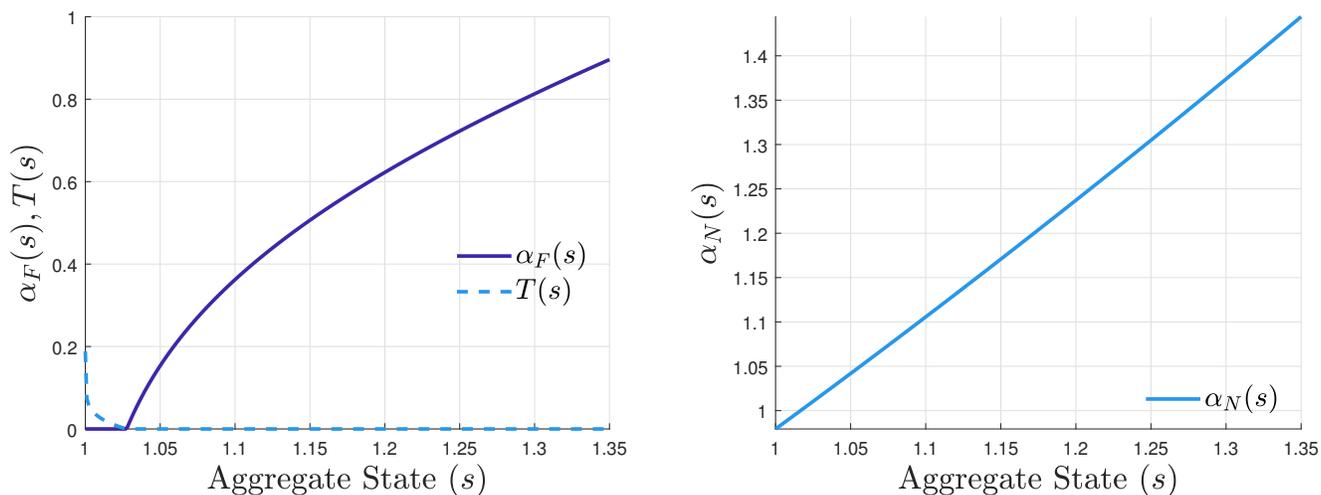


Figure OA-4: Depositors' Equilibrium Consumption Determinants and Funding Shortfall

**Note:** The left panel in Figure OA-4 shows the recovery rate on uninsured deposits in case of failure,  $\alpha_F(s)$ , as well as the funding shortfall,  $T(s)$ , as a function of the realizations of the state  $s$  when  $\delta = 1$ . The right panel in Figure OA-4 shows the additional gross return earned by the deposits that stay within the bank until date 2,  $\alpha_N(s)$ , as a function of the realizations of the state  $s$  when  $\delta = 1$ .

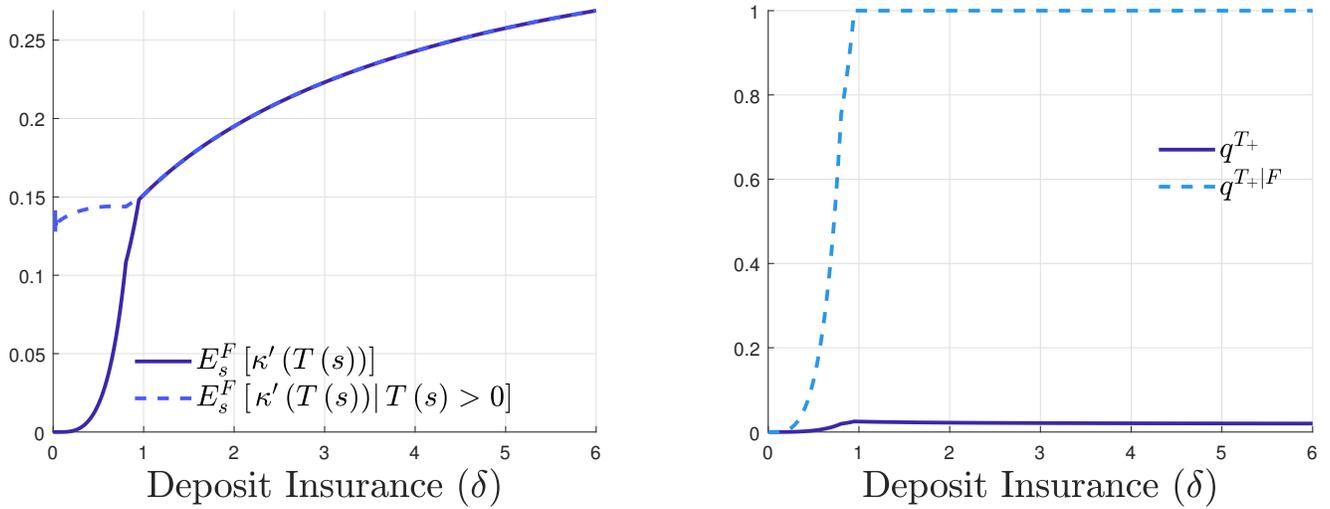


Figure OA-5: Marginal Cost of Public Funds/Probability of Funding Shortfall

**Note:** The left panel in Figure OA-5 shows the average marginal cost of public funds in failure states,  $\mathbb{E}_s^F[\kappa'(T(s))]$ , and in failure states in which the funding shortfall is positive,  $\mathbb{E}_s^F[\kappa'(T(s))|T(s) > 0]$ , as a function of the level of deposit insurance coverage  $\delta$ . The right panel in Figure OA-5 shows the unconditional probability of a positive funding shortfall,  $q^{T+}$ , and the probability of a positive funding shortfall condition on a bank failure taking place, as a function of the level of deposit insurance coverage  $\delta$ .

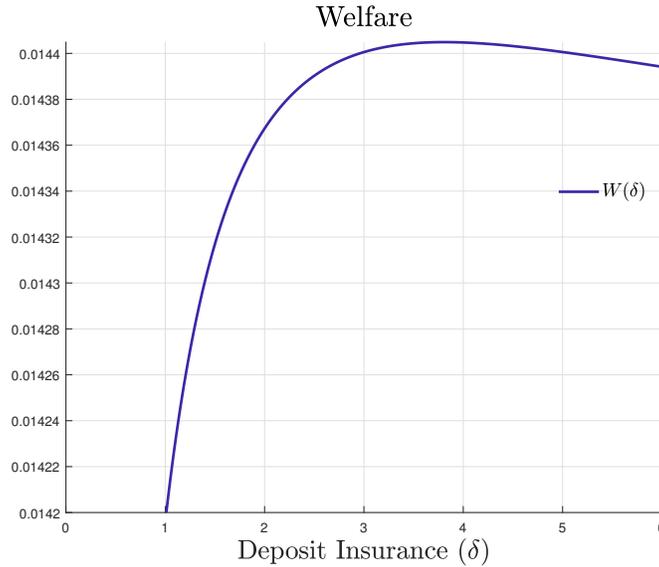


Figure OA-6: Social Welfare  $W(\delta)$

**Note:** Figure OA-6 integrates over  $\frac{dW}{d\delta}$ , as defined in Equation (26), to build a measure of social welfare. This figure uses the normalization:  $W(0) = 0$ .

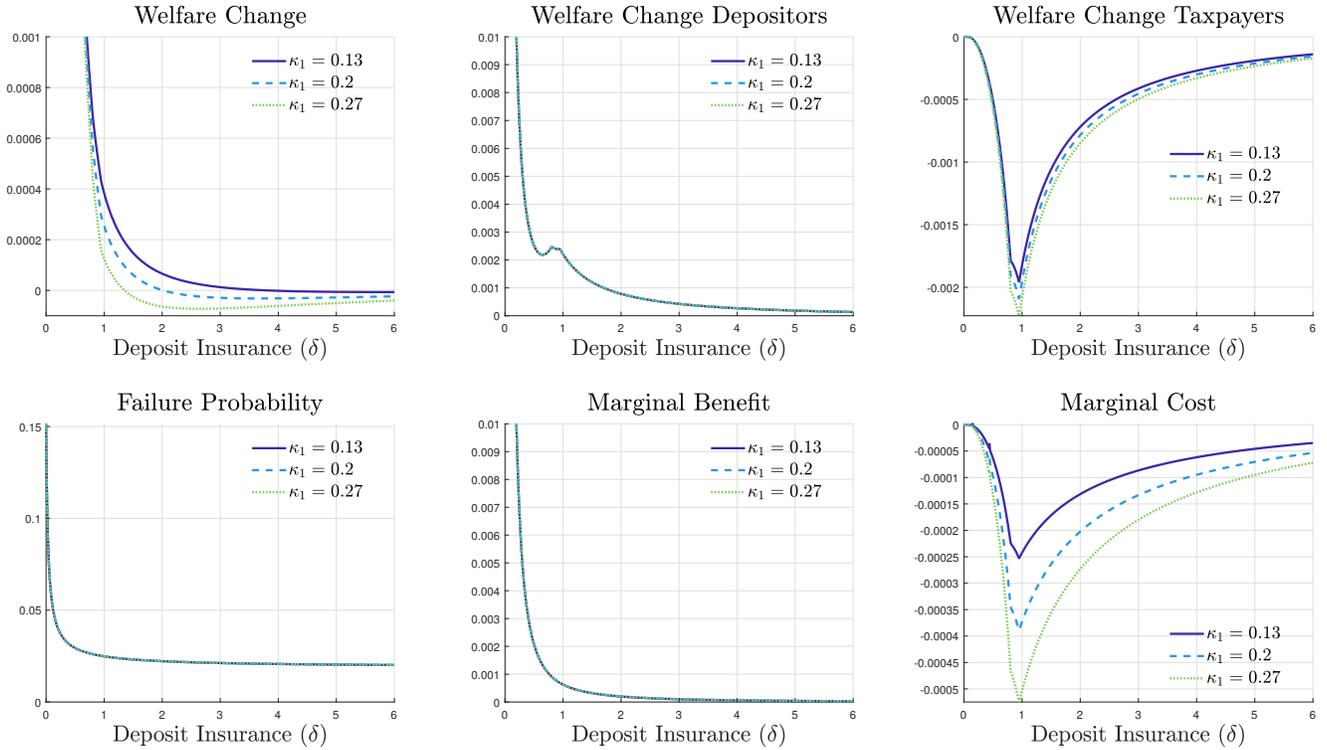


Figure OA-7: Sensitivity Analysis: Cost of Public Funds ( $\kappa_1$ )

**Note:** The top left panel in Figure OA-7 shows the change in social welfare induced by a marginal change in the level of deposit insurance coverage,  $\frac{dW}{d\delta}$ , as described in Equation (26), for  $\kappa_1 \in \{0.13, 0.2, 0.27\}$ . The top middle and right panels respectively show the welfare change for depositors,  $\int \frac{dV_m(i, \delta, R_1)}{d\delta} dG(i)$ , and taxpayers,  $\frac{dV_m(\tau, \delta, R_1)}{d\delta}$ . The bottom left panel shows the probability of failure,  $q^F(\delta, R_1)$ , and the probability of fundamental failure  $F(\hat{s}(R_1))$ . The bottom middle and right panels show the marginal benefit and marginal of increasing the level of coverage, as defined in Equation (26). The optimal levels of coverage are  $\delta^* = 3.81$ ,  $\delta^* = 2.02$ , and  $\delta^* = 1.35$  for  $\kappa_1 = 0.13$ ,  $\kappa_1 = 0.2$ , and  $\kappa_1 = 0.27$ , respectively.

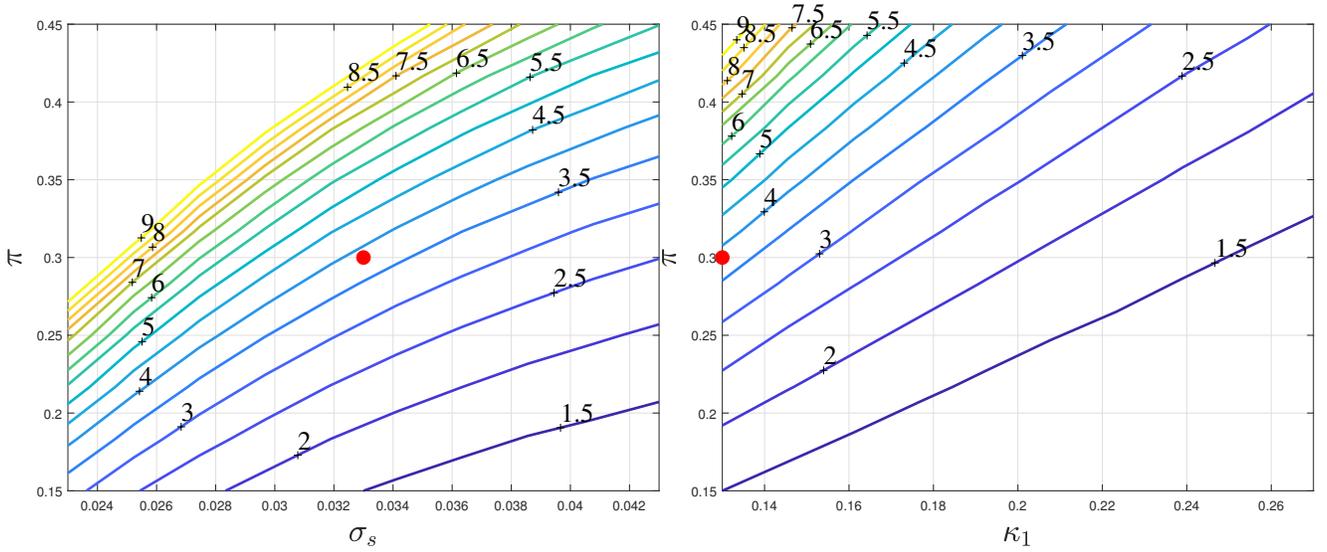


Figure OA-8: Optimal Deposit Insurance Coverage Isoquants

**Note:** The left panel in Figure OA-8 shows combinations of  $\pi$  and  $\sigma_s$  that yield the same optimal level of coverage  $\delta^*$ . The right panel in Figure OA-8 shows combinations of  $\pi$  and  $\kappa_1$  that yield the same optimal level of coverage  $\delta^*$ . The red solid bullet point in each of the figures represents the baseline combination used in Section 4.2:  $(\pi, \sigma_s) = (0.3, 0.033)$  and  $(\pi, \kappa_1) = (0.3, 0.13)$ , respectively.

## D.5 Model-Based Quantification: Optimal Coverage Isoquants

Figure OA-8 describes the set of parameters that yield the same optimal level of coverage  $\delta^*$  for combinations of  $\pi$  and  $\sigma_s$  and combinations  $\pi$  and  $\kappa_1$ , respectively. The left panel in Figure OA-8 shows that higher values of the sunspot probability  $\pi$  and lower values of the riskiness of bank's investments are associated with a higher optimal level of coverage. The right panel in Figure OA-8 shows that higher values of the sunspot probability  $\pi$  and lower values of the marginal cost of public funds  $\kappa_1$  are associated with a higher optimal level of coverage.

## D.6 Robustness of Approximation/Welfare Weights Choice

In our model-based quantitative analysis in Section 4.2 we have used the characterization of the marginal welfare change from Proposition 2. The main advantage of working with the result in Proposition 2 is that we can characterize  $\frac{dW}{d\delta}$  exclusively as a function of bank-level aggregates.

Here we explore the robustness of our results to the conditions necessary for Proposition 2 to hold. To simplify the exposition, we reproduce here Equation (22) in the text:

$$\frac{dW}{d\delta} = \int \omega(j) \left( -\frac{\partial q^F}{\partial \delta} \left( \frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \right) + q^F \mathbb{E}_s^F \left[ m(j, s) \frac{\partial C^F(j, s)}{\partial \delta} \right] \right) dj.$$

In Figure OA-9, we compute  $\frac{dW}{d\delta}$  under three different sets of assumptions. First, we use uniform generalized social welfare weights,  $\omega(j) = 1$ , and use the approximation assumptions required for Proposition 2 to hold, that is,  $\frac{U(C^N(j, s^*)) - U(C^F(j, s^*))}{U'(C^F(j, s^*))} \approx C^N(j, s^*) - C^F(j, s^*)$ , and  $m(j, s) = 1$ . This

is the approach we have followed in the body of the paper. Second, we use uniform generalized social welfare weights,  $\omega(j) = 1$ , but do not make any approximations. By comparing these two approaches, we can obtain a sense of the validity of the approximation. Finally, we compute  $\frac{dW}{d\delta}$  using utilitarian traditional social welfare weights. As shown in Section C.2 of this Online Appendix, this corresponds to setting  $\omega(j) = U'(C^F(j, s^*))$ . By comparing the second and third approaches, we can understand the differences between using generalized and traditional welfare weights.

There are two main takeaways from this exploration. First, Figure OA-9 clearly shows that the choice of uniform generalized welfare weights versus utilitarian traditional welfare weights does matter for the welfare calculations. A planner using utilitarian traditional welfare weights finds a lower optimal level of coverage because the gains from insuring the consumption depositors with medium to high levels of deposits have barely any weight in the planner's computations. This is illustrated in Figure OA-10 below, in which we compare the implied welfare weights  $\omega(j)$  for depositors with different deposit levels and taxpayers under different assumptions. The solid line shows the implied generalized social welfare weights of depositors when using a traditional utilitarian objective. The dashed line shows the implied generalize social welfare weight for taxpayers also when using a traditional utilitarian objective. The dotted line shows for reference uniform generalized social weights.

Second, we find that the planner who uses the approximate results tends to overestimate the welfare gains from increasing the level of coverage, but the approximate are results reasonably accurate for our baseline calibration. The optimal level of coverage changes from  $\delta^* = 3.81$  when using the approximation to  $\delta^* = 3.42$  in the exact case. Whether the approximation results are closer to the exact ones does depend on the value of  $\gamma$ . Intuitively, the quality of the approximate results is a function of the curvature of the utility functions of the agents in the economy, since the two approximations are exact when agents are risk-neutral. With isoelastic utility,  $m(j, s) = \frac{U'(C^F(j, s))}{U'(C^F(j, s^*))} = \left(\frac{C^F(j, s)}{C^F(j, s^*)}\right)^{-\gamma}$ , and we show in Section A of this Online Appendix that  $\frac{C^F(j, s)}{C^F(j, s^*)} \leq 1$  for both depositors and taxpayers. Therefore, increasing  $\gamma$  makes the value of  $m(j, s)$  move away from 1, worsening the approximation. When using the approximation, the welfare losses suffered by taxpayers in states with very low  $s$  carry a lower weight than when using the exact solution (since  $m(\tau, s) = 1$ ), which underestimates the marginal cost of increasing  $\delta$ , making a higher level of coverage more desirable. Finally, note that these effects are also modulated by the value of  $Y(\tau, s)$ , since the marginal utility of taxpayers becomes more sensitive to all these effects when  $Y(\tau, s)$  is lower.

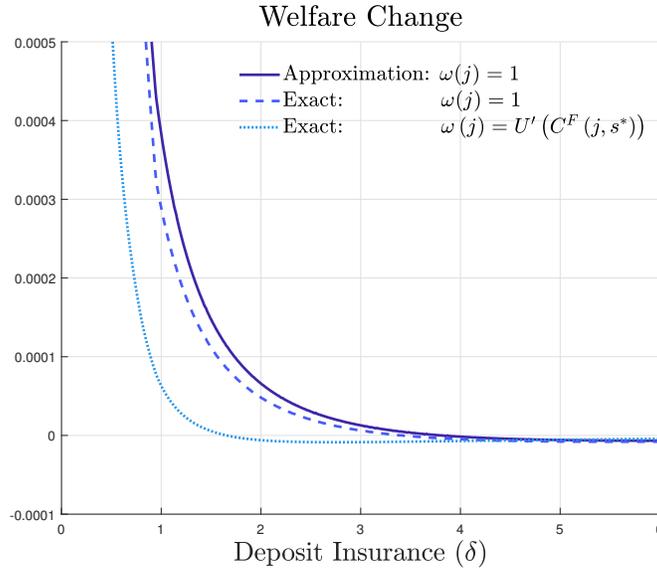


Figure OA-9: Robustness of Approximation

**Note:** Figure OA-9 shows the change in social welfare induced by a marginal change in the level of deposit insurance coverage,  $\frac{dW}{d\delta}$ , under different assumptions on how to aggregate and compute social welfare. First, the solid line computes  $\frac{dW}{d\delta}$  as in Equation (26), that is, using uniform generalized social welfare weights and under the approximation required for Proposition 2 to hold. Second, the dashed line computes  $\frac{dW}{d\delta}$  as in Equation (22) when setting  $\omega(j) = 1$ . Third, the dotted line computes  $\frac{dW}{d\delta}$  as in Equation (22) when using utilitarian traditional social welfare weights, so  $\omega(j) = U'(C^F(j, s^*))$ . The optimal levels of coverage in each case are  $\delta^* = 1.64$ ,  $\delta^* = 3.41$ , and  $\delta^* = 3.81$ , respectively.

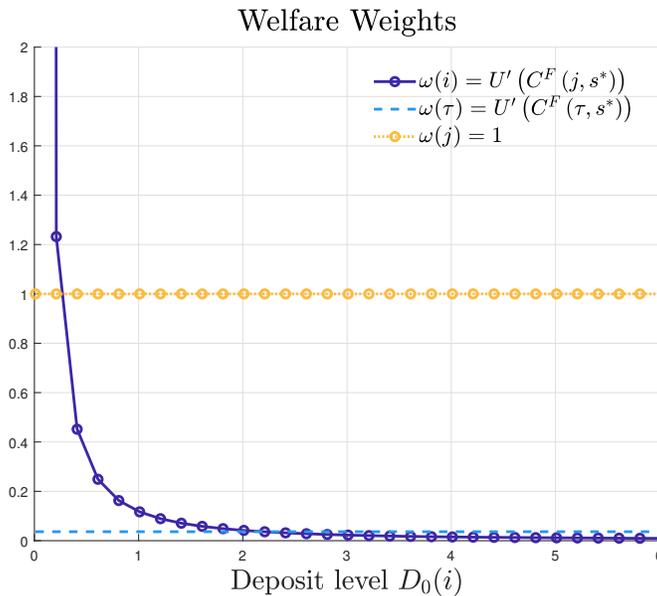


Figure OA-10: Generalized vs. Traditional Welfare Weighs

**Note:** Figure OA-10 shows the implied welfare weights  $\omega(j)$  for depositors with different deposit levels and taxpayers under different assumptions. The solid dark blue line shows the implied generalized social welfare weights of depositors when using a traditional utilitarian objective. The dashed light blue line shows the implied generalize social welfare weight for taxpayers also when using a traditional utilitarian objective. The dotted orange line shows for reference uniform generalized social weights.