Online Appendix

Outsourcing, Inequality and Aggregate Output

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E Data description

Firm-level balance sheet data. We use the FICUS data (*"Fichier Complet Unifié de Suse"*) which covers the near universe of nonfarm French businesses. The unit of observation is a firm-year, and firms are identified by their tax identifier (*"siren"*). It details balance sheet information. We construct value added by substracting purchases of intermediate goods and other intermediate purchases from firm sales.

Firm-level survey data. We use the EAE data (*"Enquête Annuelle d'Entreprise*). It covers a random sample of firms and tracks them across years. We link it to other sources using the common tax identifier (*"siren"*). The unit of observation is a firm-year. Among others, the dataset breaks down intermediate purchases of goods and services. In particular, we use expenditures on external workers (*"Dépenses de personnel extérieur"*) as our main measure of outsourcing expenditures.

DADS panel. We use the 4% sample of the DADS panel, between 1996 and 2007. Once a worker enters the dataset in any year after 1976, all her subsequent employment spells are recorded. Individuals' employment history is recorded in the dataset if (a) they have at least one employment spell, and (b) they are born in October in even years. The dataset provides start and end days of each employment spell, the job's wage, four-digit occupation and industry, as well as establishment and firm tax identifiers that can be linked to other datasets. We follow Bilal (2021) to set sample restrictions and define unemployment.

DADS cross-section. The DADS *Postes*, are used by the French statistical institute to construct the DADS *Panel*. They cover the universe of French workers, but in the version available to researchers, worker identifiers are reshuffled every two years. The DADS Postes allow to compute employment, wages, occupational mix for the near universe of French establishments.

Firm-level customs data. We use customs data for the universe of French importers and exporters. The unit of observation is at the firm-product-year-country-export/import level. We aggregate French exports for every firm, year and destination country at the 4-digit industry level to construct our firm-level instrument.

F Additional proofs

F.1 Dynamic firm problem

We first show that the size constraint in (3) is consistent with the firm-level decision. Omit s indices whenever unambiguous. Denote by q the vacancy contact rate. Without loss of generality, we use a continuous offer distribution $F(w)$ to lighten notation. Start from the firm-level Kolmogorov Forward Equation:

$$\frac{dn(w,t)}{dt} = q[\phi + (1-\phi)G(w)] - [\delta + \lambda^E(1-F(w))]n,$$

where $\phi = \frac{u}{u+\frac{\lambda^E}{\lambda^U}(1-u)} = \frac{1}{1+k}$ is the probability of meeting an unemployed worker. In steady-state $dn/dt = 0$. Hence, from (1), $\phi + (1-\phi)G(w) = \frac{1}{1+k(1-F(w))}$, and so $n(w) = \frac{q}{\delta}\frac{1}{[1+k(1-F(w))]^2}$. Then, from a constant returns matching function, $\lambda^U = \theta q(\theta) = \frac{M}{m[u+(1-u)\lambda^E/\lambda^U]}q(\theta)$ where θ is labor market tightness. Re-arranging leads to $q = \frac{e\delta(1+k)}{M}$. Therefore,

$$n(w) = \frac{1}{M}\frac{(1+k)e}{[1+k(1-F(w))]^2}.$$

We now turn to showing that the decisions from the dynamic profit-maximization problem of the firm coincides with those from the static firm profit maximization problem (3) when the discount rate is low enough.

Consider the dynamic problem of a firm which may be out of its long-run size, while the rest of the economy is in steady-state. Assume that firms may freely adjust their wage each instant, but face an equal-pay constraint within worker type. Without loss of generality, we consider a single worker type to make notation lighter. Firms solve

$$rJ(z,n) = \max_w R(z,n) - wn + [q(\phi + (1-\phi)G(w)) - n(\delta + \lambda^E(1-F(w)))J_n(z,n).$$

Using $\phi = \frac{1}{1+k}$,

$$rJ(z,n) = \max_w R(z,n) - wn + \delta(1+k(1-F(w))(n(w)-n)J_n(z,n).$$

The first-order condition implies $-n + \delta(1+k(1-F))n'(w)J_n + kF'(n(w)-n)J_n = 0$. Evaluated at long-run size $n = n(w)$,

$$n(w) = \delta(1+k(1-F))n'(w)J_n(z,n(w)).$$

The envelope condition then yields $rJ_n = R_n - w + \delta(1+k(1-F))[-J_n + (n(w)-n)J_{nn}]$ which again evaluated at long-run size $n = n(w)$ leads to

$$rJ_n(z,n(w)) = R_n(z,n(w)) - w - \delta(1+k(1-F(w)))J_n(z,n(w)).$$

When the discount rate goes to zero $r \to 0$,

$$J_n(z, n(w)) = \frac{R_n(z, n(w)) - w}{\delta(1 + k(1 - F(w)))}.$$

Substituting into the first-order condition, we obtain

$$n(w) = n'(w)(R_n - w),$$

which coincides with the static first-order condition.

F.2 Welfare and rent-sharing implications of compensating differentials models of wage inequality

Models of wage inequality based on compensating differentials have become popular due to their tractability (see e.g. Card et al., 2018, Sorkin, 2018, Berger et al., 2022, Lamadon et al., 2022). These models are well-suited to study the effect of inequality on welfare at the market level. By contrast, they require strategic interactions to generate markdown variation across firms within markets. They also struggle to translate observed wage differences into welfare difference. We make that point by considering the simplest of such models.

There are M firms j. Each firm has some productivity z_j. Each firm choose the wage w_j it posts. For simplicity, suppose that each firm is small enough that it does not consider its impact on any aggregate. Workers choose from the continuum of wage offers. Worker draw random non-wage amenities a_j specific to each firm. Workers value wages and non-work amenities a_j to maximize $V = \max_j w_j a_j$.

Tractability is achieved by assuming that a_j are independent across firms j and have a Frechet distribution with shifter T_j and shape parameter ε: $F_j(a) = e^{-T_j a^{-\varepsilon}}$.[37] Classical results in extreme value theory ensure several results (see e.g. Eaton and Kortum, 2002) that we derive here for completeness.

[37]More involved correlation structures can be introduced with a nested Frechet distribution, but this additional complexity is not relevant for our purposes.

Welfare implications. First, the probability of choosing firm i is $\pi_i = \frac{T_i w_i^\varepsilon}{\sum_j T_j w_j^\varepsilon}$. To see this, consider the calculation

$$\begin{aligned}
\mathbb{P}[w_i a_i \geq w_j a_j \ \forall j \neq i] &= \int_0^\infty \prod_{j \neq i} F_j(w_i a_i / w_j) dF_i(a_i) \\
&= \int_0^\infty e^{-\Phi_{-i} w_i^{-\varepsilon} a_i^{-\varepsilon}} dF_i(a_i) \\
&= \int_0^\infty \big(F_i(a_i)\big)^{\Phi_{-i} w_i^{-\varepsilon} T_i^{-1}} dF_i(a_i) \\
&= \int_0^1 F^{\Phi_{-i} w_i^{-\varepsilon} T_i^{-1}} dF \\
&= \frac{1}{1 + \Phi_{-i} w_i^{-\varepsilon} T_i^{-1}} \\
&= \frac{T_i w_i^\varepsilon}{\sum_j T_j w_j^\varepsilon}
\end{aligned}$$

where we denote $\Phi_{-i} = \sum_{j \neq i} T_j w_j^\varepsilon$, and changed variables $F = F_i(a_i)$ from the third to fourth row.

This property has made this class of models particularly attractive to model an upward-sloping labor supply curve. When every firm is small enough that it negelects its effect on the denominator, the labor supply curve faced by every firm j is of the form $n_j(w) = n_0 T_j w^\varepsilon$.

This tractability comes, however, at a cost. The cost is that welfare is equalized across firms, regardless of the wage they decide to post. To see this second result (also well-established, but less well-known), compute as above

$$\begin{aligned}
\mathbb{P}[w_i a_i \leq \omega \ \& \ w_i a_i > w_j a_i \ \forall j \neq i] &= \int_0^{\omega / w_i} \prod_{j \neq i} F_j(w_i a_i / w_j) dF_i(a_i) \\
&= \int_0^{\omega / w_i} e^{-\Phi_{-i} w_i^{-\varepsilon} a_i^{-\varepsilon}} dF_i(a_i) \\
&= \int_0^{\omega / w_i} \big(F_i(a_i)\big)^{\Phi_{-i} w_i^{-\varepsilon} T_i^{-1}} dF_i(a_i) \\
&= \int_0^{F_i(\omega / w_i)} F^{\Phi_{-i} w_i^{-\varepsilon} T_i^{-1}} dF \\
&= \frac{F_i(\omega / w_i)^{1 + \Phi_{-i} w_i^{-\varepsilon}}}{1 + \Phi_{-i} w_i^{-\varepsilon} T_i^{-1}} \\
&= \frac{T_i w_i^\varepsilon}{\sum_j T_j w_j^\varepsilon} e^{-\Phi \omega^{-\varepsilon}}
\end{aligned}$$

where $\Phi = \sum_j T_j w_j^\varepsilon$. Therefore, the conditional distribution of values within every chosen firm i is

$$\begin{aligned}
\mathbb{P}[w_i a_i \leq \omega \mid w_i a_i > w_j a_i \ \forall j \neq i] &= \frac{\mathbb{P}[w_i a_i \leq \omega \ \& \ w_i a_i > w_j a_i \ \forall j \neq i]}{\mathbb{P}[w_i a_i \geq w_j a_j \ \forall j \neq i]} \\
&= e^{-\Phi \omega^{-\varepsilon}}
\end{aligned}$$

The distribution of indirect utility of employed workers at any firm is Frechet with shifter Φ and shape parameter ε. It is independent of the wage offered by the firm. Thus, welfare is equalized across firms. In particular, expected utility of employed workers at any firm is $\Gamma(1-1/\varepsilon)\Phi^{1/\varepsilon}$, where Γ denotes Euler's Gamma function. Non-work amenities introduce mixing, but they do not change the core implications relative to a standard free-mobility condition without amenities, $\max_j w_j$.

Markdowns and labor shares. To study rent-sharing in this economy, we specify a revenue function $R(z,n)$ that may depend on productivity z and the number of workers that a firm hires n. Any given firm j then solves

$$\max_{w,n} R(z_j,n) - wn \text{ s.t } n \leq \pi_j(w) \equiv \frac{T_j w^\varepsilon}{\sum_{i\neq j} T_i w_i^\varepsilon}$$

Assuming a large enough number of firms and thus no strategic interactions, $\sum_{i\neq j} T_i w_i^\varepsilon \equiv W_{0j}^\varepsilon T_j$ may be assumed to be constant from the perspective of firm j. The labor supply curve becomes $\pi_j(w) = W_{0j}^{-\varepsilon} w^\varepsilon$. The wage it pays is $w_j(n) = W_{0j} n^{1/\varepsilon}$. The problem of the firm becomes

$$\max_n R(z_j,n) - W_{0j} n^{1+1/\varepsilon}.$$

Thus, the firm chooses n_j such that $R_n(z_j,n) = (1+1/\varepsilon)W_{0j} n_j^{1/\varepsilon}$. Under isoelastic revenue $R(z,n) = zn^\rho$, $R_n(z_j,n_j) = \rho R(z_j,n_j)/n_j = \rho z_j n_j^{\rho-1} = (1+1/\varepsilon)W_{0j} n_j^{1/\varepsilon} = (1+1/\varepsilon)w_j(n_j)$.

Hence, the markdown is constant across firms and given by

$$\text{markdown} = \frac{\rho\varepsilon}{1+\varepsilon}.$$

The labor share is also constant across firms and given by

$$\text{LS} = \frac{\varepsilon}{1+\varepsilon}$$

Taking stock. Thus, it is difficult to connect models of wage inequality based on compensating differentials to welfare inequality across firms.[38] These results are specific to the Frechet distribution however. Potentially, more complex distribution of compensating differentials could lead to meaningful welfare effects. Yet, more complex distributions would also break the tractability provided by the Frechet distribution. Similarly, models of wage inequality based on compensating differentials deliver constant markdowns and labor shares under standard isoelastic revenue functions absent market power.

[38] Of course, the shifter Φ depends on the distribution of wages inside the market and so cross-market welfare inequality is still meaningful.

G Simulation and estimation

G.1 Model

Good producers Good producers solve the following problem

$$\pi(z,\varepsilon) = \max_{\boldsymbol{n},\boldsymbol{w},\boldsymbol{v},o} R(z,\boldsymbol{n}) - ((1-o)w_1 - op\varepsilon)\, n_1 - (1-o)c(v_1) - \sum_{s>1}(w_s n_s + c(v_s)) - \eta, \tag{46}$$

subject to

$$n_s \leq \mathrm{n}_s(w)\left(\frac{v_s}{V_s}\right) \quad \text{if } o_s = 0,$$

where $\boldsymbol{x} \equiv \{x_s\}_{s=1}^S$ and n_s is the firm-specific labor supply curve,

$$\mathrm{n}_s(w) \equiv \frac{(1+k_s)e_s}{(1+k_s(1-F_s(w_s)))^2}.$$

In the formulation of the problem above, we have already assumed that only the workers of skill type one (service workers) can be outsourced. We use the following parametric assumptions. The revenue function is Cobb-Douglas nested in a decreasing returns upper tier,

$$R(z,\boldsymbol{n}) = \left(z\prod_{s=1}^{S} n_s^{a_s}\right)^{\rho}$$

with $\sum_s a_s = 1$. The vacancy cost function is isoelastic with elasticity γ,

$$c(v) = \frac{v^{1+\gamma}}{1+\gamma},$$

where we have normalized the vacancy cost of a vacancy for good producers to one, $c_0 = 1$. Finally, the joint distribution of (z,ε), denoted by Γ, is log-normal with zero means and variance-covariance matrix $\boldsymbol{\Sigma}$.

Contractors To solve numerically the contractors' problem in a similar fashion as the problem of good producers, we introduce some minimal productivity heterogeneity across contractors. Specifically, let Ψ denote the distribution of contractors' productivity. We parametrize it to be log-normal with zero mean and variance σ^C and we set $\sigma^C \approx 0$. Then, contractors solve the problem

$$\pi^C(z) = \max_{n,w,v}\; p\tau z n - wn - \frac{(\bar{c}^C)^{\gamma} v^{1+\gamma}}{1+\gamma} \quad \text{s.t.} \quad n \leq \mathrm{n}_1(w)\left(\frac{v}{V_1}\right), \tag{47}$$

where z denote the productivity of the contractor. The effective amount of labor that a contractor with productivity z provides to the outsourcing market is $\tau z n$. Let $\underline{z}^C$ be the least productive contractor firms. In equilibrium, this firm offers the reservation wage and $\underline{z}^C = \underline{w}_1/(p\tau)$.[39]

G.2 A tractable reformulation of the model

Due to its large amount of heterogeneity, this model is *a priori* numerically non-tractable. In particular, two objects are complicated to compute. First, the wage offered to in-house workers by good producers that outsource their service workers. These wages depend indeed on (z, ε) rather than on a uni-dimensional variable. Second, the wage offer distributions is non-trivial to compute since different types of firms are now competing on the same skill-specific job ladder.[40] In this section, we derive a reformulation of the problem that simplifies these two problems. This derivation is feasible under three assumptions: a Cobb-Douglas revenue function, a single outsourceable worker type, and a log-normal distribution for (z, ε).

Outsourcing good producers Index good producers that outsource their service workers by the superscript o. Similarly, index good producers that hire their service workers in-house by i. The problem of a good producer of type o reads

$$\pi^o(z,\varepsilon) = \max_{\boldsymbol{n},\{w_s\}_{s>1},\{v_s\}_{s>1}} R(z,\boldsymbol{n}) - n_1 p\varepsilon - \sum_{s>1}(w_s n_s + c(v_s)) - \eta. \tag{48}$$

The optimal number of service workers hired from the contractor sector is obtained by taking the first order condition of this problem with respect to n_1,

$$n_1^o(z,\varepsilon) = \left(\frac{\rho a_1}{p\varepsilon}\right)^{\frac{1}{1-a_1\rho}} \left(z\prod_{s>1} n_s^{a_s}\right)^{\frac{\rho}{1-a_1\rho}}.$$

Plugged back into (48), the profit of the firm rewrites

$$\pi^o(z,\varepsilon) = \mathcal{G}\left(z\varepsilon^{-a_1}\prod_{s>1} n_s^{a_s}\right)^{\kappa} - \sum_{s>1} w_s n_s - c(v_s) - \eta \equiv \pi^o(\hat{z}),$$

for κ a parametric constant and $\mathcal{G}$ a general equilibrium constant.[41] In the above expression, payroll and vacancy costs are independent from z and ε. Meanwhile, revenues only depend on the TFP

[39] When contractor firms post their optimal number of vacancy, their profits are given by

$$\pi^C(z) \propto \max_w \left[(p\tau z - w)\mathrm{n}_1(w)\right]^{\frac{1+\gamma}{\gamma}}.$$

Hence, $\underline{z}^C = w(\underline{z}^C)/(p\tau)$. Furthermore, the least productive active contractor firms must offer the reservation wage for otherwise contractor firms with lower productivity would be able to make a profit by posting a wage in $[\underline{w}, w(\underline{z}^C))$.

[40] In a standard Burdett-Mortensen model, the wage offer distribution is directly recovered from two differential equations obtained from the wage and vacancy first-order conditions. This is not the case here as two firms with a similar revenue TFP z may offer different wages depending on their outsourcing choice.

[41] Specifically, we have

$$\kappa = \frac{\rho}{1-\rho a_1} \quad \text{and} \quad \mathcal{G} = \frac{\rho}{\kappa}\left(\frac{\rho a_1}{p}\right)^{a_1\kappa}.$$

aggregator $\hat{z} \equiv z\varepsilon^{a_1}$. As a result, the policy functions of type-o good producers are only a function of $\hat{z}$ and it is not needed to keep track of z and ε separately. Since (z, ε) is jointly log-normally distributed, so is $(z, \hat{z})$ and a closed-form expression exists for its variance-covariance matrix. Let Φ denote the log-normal distribution under the change of variable $(z, \varepsilon) \rightarrow (z, \hat{z})$. Finally, profits of in-house and outsourcing good producers are increasing in z and $\hat{z}$ respectively, and there exists two productivity lower bounds, $\underline{z}$ and $\underline{\hat{z}}$, so that $\pi^i(z) > 0$ iff $z > \underline{z}$ and $\pi^o(\hat{z}) > 0$ iff $\hat{z} > \underline{\hat{z}}$. With a slight abuse of notation, we refer to Φ as the truncated log-normal distribution, Φ_z as the z-marginal, and $\Phi_{\hat{z}|z}$ as the distribution of $\hat{z}$ condition on z.

Wage distributions To derive the wage offer distributions, it is required to know which firms outsource their service workers and which do not. Let $\varphi(z)$ denote the productivity level that renders an outsourcing firm indifferent between the two outsourcing choices, $\pi^i(z) = \pi^o[\varphi(z)]$ and $\pi^i(z) < \pi^o(\hat{z})$ for all $\hat{z} > \varphi(z)$. With this notation, the wage offer distribution of skill $s > 1$ is given by

$$F_s(w) = \frac{M^G}{V_s} \left(\int \mathbf{1}\{w_s^i(z) \leq w\} v_s^i(z) \Omega^i(z) \mathrm{d}\Phi_z(z) + \int \mathbf{1}\{w_s^o(\hat{z}) \leq w\} v_s^o(\hat{z}) \Omega^o(\hat{z}) \mathrm{d}\Phi_{\hat{z}}(\hat{z}) \right), \tag{49}$$

where

$$V_s = M^G \left(\int v_s^i(z) \Omega^i(z) \mathrm{d}\Phi_z(z) + \int v_s^o(\hat{z}) \Omega^o(\hat{z}) \mathrm{d}\Phi_{\hat{z}}(\hat{z}) \right) \tag{50}$$

is the mass of vacancy posted for skill s and $\Omega^i(x) \equiv \Phi_{\hat{z}|z}[\varphi(x) \mid x]$ is the probability that good producer z hires its service workers in-house. Similarly, $\Omega^o(x) \equiv \Phi_{z|\hat{z}}[\varphi^{-1}(x) \mid x]$. The first integral in (49) is the relative mass of vacancy attached to wages lower than w and offered by good producers hiring their service workers in-house. The second integral is the relative mass of vacancy attached to wages lower than w and offered by good producers outsourcing their service workers. Similarly, the wage offer distribution for service workers is

$$F_1(w) = \frac{M^G}{V_1} \int \mathbf{1}\{w_1^i(z) \leq w\} v_1^i(z) \Omega^i(z) \mathrm{d}\Phi_z(z) + \frac{M^C}{V_1} \int \mathbf{1}\{w_1^C(z) \leq z\} v_1^C(z) \mathrm{d}\Psi(z), \tag{51}$$

where

$$V_1 = M^G \int v_1^i(z) \Omega^i(z) \mathrm{d}\Phi_z(z) + M^C \int v_1^C(z) \mathrm{d}\Psi(z). \tag{52}$$

In equation (51), the first integral is the relative mass of vacancy attached to wages lower than w and offered by good producers hiring their service workers in-house. The second integral is the mass of vacancy attached to wages lower than w and offered by contractors.

Finally, two further conditions close the equilibrium. First, the reservation wages which constitute the lower bound of the wage offer distributions,

$$\underline{w}_s = b_s + \left(k_s^U - k_s\right)\left(\int_{\underline{w}_s} \frac{1 - F_s(w)}{1 + k_s(1 - F_s(w))}\mathrm{d}w\right). \tag{53}$$

Second, the market clearing condition of the outsourcing market is

$$\begin{aligned} M^C \int \tau z n^c(z)\mathrm{d}\Psi(z) &= M^G \int \varepsilon n_1^o(z,\varepsilon)\mathbf{1}\{\pi^o(z,\varepsilon) > \pi^i(z)\}\mathrm{d}\Phi(z,\varepsilon) \\ &= M^G \left(\frac{a_1\rho}{p}\right)^{\frac{\kappa}{\rho}} \int \left(\hat{z}\prod_{s>1} n_s^o(\hat{z})^{a_s}\right)^{\kappa} \Omega^o(\hat{z})\mathrm{d}\Phi_{\hat{z}}(\hat{z}), \end{aligned} \tag{54}$$

where, as explained in the main text, ε is interpreted as an idiosyncratic iceberg shock and as such appears in the aggregate demand for outsourced service workers. The second line is obtained from plugging the expression for n^o and performing the change of variable $z\varepsilon^{-a_1} \to \hat{z}$.

Definition 1 (Equilibrium)**.** *An equilibrium is a collection of wage and vacancy functions for good producers, $\{w_s^\theta, v_s^\theta\}_{s\in\{1,\dots S\},\theta\in\{i,o\}}$, wage and vacancy functions for contractor firms, w_1^C and v_1^C, an indifference function, φ, wage distributions $\{F_s\}_{s=1}^S$, productivity cutoffs $(\underline{z}, \underline{\hat{z}}, \underline{z}^C)$, and aggregate quantities, $\{\underline{w}_s\}_{s=1}^S$ and p, such that*

1. *Given $\{F_s\}_{s=1}^S$, p, and an outsourcing decisions $\theta \in \{i, o\}$, the functions $\{w_s^\theta, v_s^\theta\}_{s=1}^S$ solve (46);*
2. *Given F_1 and p, the functions w_1^C and v_1^C solve (47);*
3. *Given the policy functions $\{w_s^\theta, v_s^\theta\}_{s=1}^S$, the indifference function is such that $\pi^i(z) = \pi^o[\varphi(z)]$;*
4. *Given the policy functions, the wage distributions satisfy (49) and (51);*
5. *Given $\{F_s\}_{s=1}^S$, the reservation wages are given by (53);*
6. *Given the firms' profits, the productivity cutoffs solve $\underline{z} = \inf\{z : \pi^i(z) > 0\}$, $\underline{\hat{z}} = \inf\{\hat{z} : \pi^o(\hat{z}) > 0\}$ and $\underline{z}^C = \underline{w}_1/(p\tau)$;*
7. *Given the policy functions and φ, the price p solves the market clearing condition (54).*

G.3 Expressing the model as a system of differential equations

To compute numerically the equilibrium defined in Definition 1, we take the first order conditions of the firms' problem (46) and (47). In doing so, we show that it is never required to solve for the solution of the contractors' problem as long as their wage overlap with the wages offered by good producers. We then rewrite the wage offer distributions (49) and (51) in differentials so as to obtain them jointly with the wage and vacancy policy functions as the solution to a system of differential equations.

Optimality conditions Let $\Upsilon_s^\theta(z) \equiv F_s[w_s^\theta(z)]$ denote the rank on the skill-s job ladder of a good producer with productivity z and outsourcing choice $\theta \in \{i, o\}$. For instance, for $s > 1$ and $\theta = i$, we have

$$\Upsilon_s^i(z) = \frac{M^G}{V_s}\left(\int^z v_s^i(x)\Omega^i(z)\mathrm{d}\Phi_z(x) + \int \mathbf{1}\{w_s^o(\hat{z}) \leq w_s^i(z)\} v_s^o(\hat{z})\Omega^o(\hat{z})\mathrm{d}\Phi_{\hat{z}}(\hat{z})\right). \tag{55}$$

Taking the first-order condition of (46) with respect to w and using the function Υ_s^θ, the wage optimality condition can be expressed as the differential equation

$$\frac{\partial w_s^\theta(z)}{\partial z} = \left(\frac{2k_s}{1 + k_s(1 - \Upsilon_s^\theta(z))}\right)\left(\frac{\partial R(z, \boldsymbol{n}^\theta(z))}{\partial n_s} - w_s^\theta(z)\right)\frac{\partial \Upsilon_s^\theta(z)}{\partial z}, \tag{56}$$

while the vacancy optimality condition is

$$v_s^\theta(z)^\gamma = \mathrm{n}_s^\theta(z)\left(\frac{\partial R(z, \boldsymbol{n}^\theta(z))}{\partial n_s} - w_s^\theta(z)\right). \tag{57}$$

Similarly, the wage optimal condition for a contractor firm is

$$\frac{\partial w_1^C(z)}{\partial z} = \left(\frac{2k_1}{1 + k_1(1 - \Upsilon_1^C(z))}\right)\left(p\tau z - w_1^C(z)\right)\frac{\partial \Upsilon_1^C(z)}{\partial z}, \tag{58}$$

while the vacancy optimality condition is

$$\left(\bar{c}^C v_1^C(z)\right)^\gamma = \mathrm{n}_1^C(z)(p\tau z - w_1^C(z)). \tag{59}$$

Solving for the wage in (56) and (58) requires to know $\mathrm{d}\Upsilon_s^\theta$. The goal of the algorithm developed in Section G.4 is to solve jointly for w_s^θ and Υ_s^θ. This requires to know how the second integral in (55) moves with z. To deal with this problem, we define two wage equivalence functions. First, let $\zeta^{G\to C}$ be the TFP of contractors firms posting wage $w_1^i(z)$; that is, $w_1^i(z) = w_1^C[\zeta^{G\to C}(z)]$. Similarly, let $\zeta_s^{i\to o}$ be such that $w_s^i(z) = w_s^o[\zeta_s^{i\to o}(z)]$. We now derive expressions for these two functions.

Service workers' wages Given that wages are striclty increasing in productivity, the function $\zeta^{G\to C}$ is well-defined on the joint support of the wages offered by good producers and contractor firms. Since z is unbounded above for both good producers and contractor firms, $w_1^i(z) \to \bar{w}_1$ and $w_1^C(z) \to \bar{w}_1$ as $z \to \infty$. In addition, we have already argued that the least productive contractor firm offers the reservation wage. It follows that the function $\zeta^{i\to o}$ is well-defined for $z \geq \underline{z}$. Furthermore, a closed-form expression exists for $\zeta^{i\to o}$. For all $z \geq \underline{z}$, $w_1^i(z) = w_1^C[\zeta^{G\to C}(z)]$ implies by definition $\Upsilon_1^i(z) = \Upsilon_1^C[\zeta^{G\to C}(z)]$ and therefore $\mathrm{n}_1^i(z) = \mathrm{n}_1^C[\zeta^{G\to C}(z)]$. Furthermore, differentiating the first two equations implies $\partial_z w_1^i(z)/\partial_z \Upsilon_1^i(z) = \partial_z w_1^C[\zeta^{G\to C}(z)]/\partial_z \Upsilon_1^C[\zeta^{G\to C}(z)]$, where $\partial_z x(z) \equiv \partial x/\partial z$. Comparing equations (56) and (58), it must then be that the marginal product of labor of good producer z and contractor firms $\zeta^{G\to C}(z)$ are equal. But since contractor firms are facing constant returns to scale, their MPL are independent from their size, and we can therefore invert this MPL equality condition and solve for $\zeta^{G\to C}(z)$ to obtain

$$\zeta^{G\to C}(z) = \left(\frac{\rho a_1}{p\tau}\right)\left(\frac{R^i(z)}{n_1^i(z)}\right). \tag{60}$$

Finally, since the right hand side of (57) and (59) are equal, it must also be that

$$v_1^i(z) = \bar{c}^C v_1^C[\zeta^{G\to C}(z)].$$

With the help of these results, we can differentiate $\Upsilon_1^i(z)$ for $z \geq \underline{z}$ to obtain

$$\frac{\partial \Upsilon_1^i(z)}{\partial z} = \frac{v_1^i(z)}{V_1}\left[M^G\Omega^i(z)\frac{\partial \Phi_z(z)}{\partial z} + \left(\frac{M^C}{\bar{c}^C}\right)\frac{\mathrm{d}\Psi[\zeta^{G\to C}(z)]}{\mathrm{d}z}\right]. \tag{61}$$

Ignoring for now the complementarity across skills, equations (56) and (61) constitute a system of differential equations, which, together with (57) and (60), allows us to solve for $(w_1^i, v_1^i, \Upsilon_1^i)$. These differential equations are subject to two boundary conditions, $\underline{w}_1^i \equiv w_1^i(\underline{z})$ and $\underline{\Upsilon}_1^i \equiv \Upsilon_1^i(\underline{z})$. If the least productive good producers decide to offer the reservation wage, then $\underline{w}_1^i = \underline{w}_1$ and $\Upsilon_1^i(\underline{z}) = 0$. However, since good producers compete on the job ladder of service workers with the contractor firms, the least productive good producers may decide optimally *not* to offer the reservation wage.[42] If that is the case, then all wages between $[\underline{w}_1, \underline{w}_1^i)$ are offered by contractor firms. These wages have to satisfy (58) together with

$$\frac{\partial \Upsilon_1^C(z)}{\partial z} = \left(\frac{M^C}{V_1}\right) v_1^C(z)\frac{\partial \Psi(z)}{\partial z}.$$

Combined with (59), this differential equation becomes

$$\frac{\partial \Upsilon_1^C(z)}{\partial z} = \frac{1}{\bar{c}^C}\left((p\tau z - w_1^C(z))\mathrm{n}_1^C(z)\right)^{1/\gamma}\left(\frac{N}{V_1}\right)\frac{\partial \Psi(z)}{\partial z}. \tag{62}$$

This differential equation, together with (58) and the boundary conditions $w_1^C(\underline{z}^C) = \underline{w}_1$ and $\Upsilon_1^C(\underline{z}^C) = 0$, pin down the wages and the wage offer distribution for contractors with TFP in $[\underline{z}^C, \tilde{z}^C)$, where $\tilde{z}^C$ is such that $w_1^C(\tilde{z}^C) = w_1^i(\underline{z})$. By a similar argument as before, this wage equality also implies that the MPL of the two firms have to be equal, or

$$\tilde{z}^C = \left(\frac{\rho a_1}{p\tau}\right)\left(\frac{R^i(\underline{z})}{n_1^i(\underline{z})}\right). \tag{63}$$

Hence, on $[\underline{z}^C, \tilde{\underline{z}}^C)$, the policy functions of contractor firms are obtained from solving a standard uni-skill constant returns to scale Burdett-Mortensen model. For $z \geq \tilde{\underline{z}}^C$, it is not needed to solve for the problem of contractor firms since we know that $w_1^C[\zeta^{G\to C}(z)] = w_1^i(z)$ and $v_1^C[\zeta^{G\to C}(z)] = v_1^i(z)/\bar{c}^C$.

[42]Precisely, the least productive good producers may not want to offer the reservation wage due to two reasons. First, due to the presence of the contractor firms on the job ladder. Second, due to the existence of decreasing returns to scale in the revenue function. If either feature were absent, then $\underline{w}_1^i = \underline{w}_1$ as in the standard Burdett-Mortensen model.

Other skills While only good producers are hiring workers with skills $s > 1$, there is effectively two type of goods producers in this model: those that hire service workers in-house, and those that outsource their service workers. As such, a similar derivation as in the previous paragraph is needed to solve efficiently for the wage distribution of skills $s > 1$. To avoid complicated combinatorial issues, we assume that the least productive good producers post the reservation wage regardless of their outsourcing decision; that is, $w_s^i(\underline{z}) = w_s^o(\underline{\hat{z}}) = \underline{w}_s$ for $s > 1$.[43] With this assumption, the wage equivalence function $\zeta^{i\to o}$ is globally well-defined. Then, using a similar argument as in the previous paragraph, $w_s^i(z) = w_s^o[\zeta^{i\to o}(z)]$ implies $v_s^i(z) = v_s^o[\zeta_s^{i\to o}(z)]$ as well as the equalization of the MPL of these two firms. Together, we therefore know that

$$\frac{R^i(z)}{n_s^i(z)} = \left(\frac{\kappa}{\rho}\right)\left(\frac{R^o[\zeta_s^{i\to o}(z)]}{n_s^o[\zeta_s^{i\to o}(z)]}\right) = \left(\frac{\kappa}{\rho}\right)\left(\frac{R^o[\zeta_s^{i\to o}(z)]}{n_s^i(z)}\right), \tag{64}$$

where the second equality follows from $\mathrm{n}_s^i(z) = \mathrm{n}_s^o[\zeta_s^{i\to o}(z)]$ and $v_s^i(z) = v_s^o[\zeta_s^{i\to o}(z)]$ so that $n_s^i(z) = n_s^o[\zeta_s^{i\to o}(z)]$. Hence, it must be that $R^o[\zeta_s^{i\to o}(z)] = \rho R^i(z)/\kappa$, which holds for each $s > 1$. This in turn implies $R^o[\zeta_s^{i\to o}(z)] = R^o[\zeta_{s'}^{i\to o}(z)]$ for any pair of skill (s, s'). But the function R^o is strictly increasing in z, and therefore it must be that $\zeta_s^{i\to o}(z) = \zeta_{s'}^{i\to o}(z) \equiv \zeta^{i\to o}(z)$. The skill independence of the function $\zeta^{i\to s}$ allows us to simplify the equation (64) to

$$\zeta^{i\to o}(z) = \left(\frac{\rho}{\mathcal{G}\kappa}\right)^{\frac{1}{\kappa}}\left(z n_1^i(z)^{a_1}\prod_{s>1} n_s^i(z)^{a_s\left(1-\frac{\kappa}{\rho}\right)}\right)^{\frac{\rho}{\kappa}}. \tag{65}$$

Using the function $\zeta^{i\to o}$ in the expression for the wage offer distribution and differentiating the later, we obtain

$$\frac{\partial \Upsilon_s^i(z)}{\partial z} = M^G\left(\frac{v_s^i(z)}{V_s}\right)\left(\Omega^i(z)\frac{\partial \Phi_z(z)}{\partial z} + \Omega^o[\zeta^{i\to o}(z)]\frac{\mathrm{d}\Phi_{\hat{z}}[\zeta^{i\to o}(z)]}{\mathrm{d}z}\right). \tag{66}$$

Here as well, ignoring for now the skill complementarity, (56) and (66) together with (57) form a system of differential equations which, subject to the boundary conditions $\underline{w}_s^i(\underline{z}) = \underline{w}_s$ and $\Upsilon_s^i(\underline{z}) = 0$, returns the policy functions (w_s^i, v_s^i) and the job ladder ranks Υ_s^θ.

G.4 Algorithm

The algorithm has four levels of iteration. The most inner level solves for the policy functions and wage offer distributions using the system of differential equations obtained in Section G.3 while taking into account the complementarity across skills. The second most inner levels iterate on the aggregate number of vacancy, $\{V_s\}_s$, and the production cutoffs, $\underline{z}$ and $\underline{\hat{z}}$. The intermediate levels iterate on the indifference function φ, the reservation wages $\{\underline{w}_s\}_s$, and the productivity cutoff of contractor firms, $\underline{z}^C$. Finally, the outer level iterates on the marker clearing condition to solve for the price of outsourcing p.

[43] While we cannot guarantee that this assumption is satisfied for all calibrations of the model, we always check numerically whether there exists profitable deviations for $\underline{z}$ and $\underline{\hat{z}}$ and find that these do not exist.

Differential equations Given an outsourcing price o, the function φ, the reservation wages $\{\underline{w}_s\}_s$, the aggregate vacancies $\{V_s\}_s$ and the productivity cutoffs $\underline{z}$, $\underline{\hat{z}}$ and $\underline{z}^C$, the most-inner loop iterates forward on the differential equations to solve for the policy functions and the wage offer distributions. In particular, we iterate twice on the grid of productivity: one time to solve for the policy functions of good producers that hire their service workers in-house, and a second time to solve for the policy functions of good producers that outsource their service workers.

For the first iteration, we need to first find the initial conditions $\underline{w}_1^i$ and Υ_1^i depending on whether the least productive good producers offer the reservation wage to service workers. To find these boundary conditions, we proceed as follows. For each $z \geq \underline{z}^C$ starting from $\underline{z}^C$ for which we know that $w_1^C(\underline{z}^C) = \underline{w}_1$ and $\Upsilon_1^C(\underline{z}^C) = 0$:

1. Compute the policy functions of the least productive good producer with productivity $\underline{z}$ as if this firm was offering wages $w_1^i(\underline{z}) = w_1^C(z)$ and $w_s^i(\underline{z}) = \underline{w}_s$ for $s > 1$:
   (a) Compute $\mathrm{n}_s^i(\underline{z})$ for each s. For $s > 1$, this is $\mathrm{n}_s^i(\underline{z}) = \mathrm{n}_s(\underline{w}_s)$. For $s = 1$, this is $\mathrm{n}_1^i(\underline{z}) = \mathrm{n}_s[w_1^C(z)]$.
   (b) Compute $\{v_s^i(\underline{z})\}_s$ jointly by solving (57) using a non-linear solver.
   (c) Compute firm output and marginal products of labor.
2. Check if condition (63) holds at z.
   (a) If it does, set $w_1^i(\underline{z}) = w_1^C(z)$ and start the following algorithm.
   (b) If not, continue.
3. Compute $\partial_z w_1^C(z)$ and $\partial_z \Upsilon_1^C(z)$ from (58) and (62) respectively and go back to step 1.

Once the initial conditions $\underline{w}_1^i = w_1^C(\tilde{z}^C)$ and $\Upsilon_1^i = \Upsilon_1^C(\tilde{z}^C)$ have been found, we can iterate once on the differential equations for the good producers that hire their service workers in-house. The starting point for the wage equivalence functions are $\zeta^{i\to o}(\underline{z}) = \underline{\hat{z}}$ and $\zeta^{G\to C}(\underline{z}) = \tilde{z}^C$. Then, for any $z \geq \underline{z}$, given that we know $\{w_s^i(z)\}_s$, $\{\Upsilon_s^i(z)\}_s$ and the wage equivalence functions:

1. Compute $\{\mathrm{n}_s^i(z)\}_s$ from the labor supplies.
2. Compute $\{v_s^{Fi}(z)\}_s$ jointly by solving (57) using a non-linear solver.
3. Compute $\zeta^{G\to C}(z')$ from (60), where $z' = z + \Delta_z$ is the next point on the grid of z, and $R^i(z')$ and $n_1^i(z')$ are obtained from linear extrapolation of $\{n_s^i(z - \Delta_z), n_s^i(z)\}_s$. Similarly, compute $\zeta^{i\to o}(z')$ from (65).
4. Use (61) to compute $\Upsilon_1^i(z')$. Similarly, for $s > 1$, use (66) to compute $\Upsilon_s^i(z')$
5. Compute wages $\{w_s^i(z')\}_s$ from the wage ODEs (56).

Once this iteration over the z's is finished, proceed to iterate over the $\hat{z}$ to compute the policy functions for good producers that outsource their service workers. Given that these firms do not hire service workers, the first step to find the lower bounds of $w_1^o(\underline{\hat{z}})$ and $\Upsilon_1^o(\underline{\hat{z}})$ is not necessary. The remaining steps are identical except for step 4: in this iteration, it is not required to compute $\zeta^{G\to C}$ but it is needed to find the numerical inverse of $\zeta^{i\to o}$. Once this iteration is finished, we have recovered all the policy functions for the good producers, $\{w_s^\theta(z), v_s^\theta(z)\}_{s\in\{1,\ldots,S\},\theta\in\{i,o\}}$, and the firms' rank on the job ladders, $\{\Upsilon_s^\theta(z)\}_{s\in\{1,\ldots,S\},\theta\in\{i,o\}}$. The fact that we only need two iterations allows for a fast computation of the equilibrium despite its complexity.

Inner iteration Given an outsourcing price o, the function φ, the reservation wages $\{\underline{w}_s\}_s$ and the productivity cutoff of contractor firms $\underline{z}^C$, the inner iteration solves for the aggregate vacancies $\{V_s\}_s$ and the productivity cutoffs. In particular, $\{V_s\}_s$ need to be consistent with equations (50) and (52), while $\underline{z}$ and $\hat{\underline{z}}$ are given in condition 6 of Definition 1.

Intermediate iteration Given an outsourcing price o, the intermediate iteration solves for the indifference function, φ, the reservation wages, $\{\underline{w}_s\}_s$ and the productivity cutoffs of contractor firms $\underline{z}^C$. Specifically, given the profit functions π^i and π^o, the function φ is found through numerical inversion of the condition $\pi^i(z) = \pi^o[\varphi(z)]$. Then, from the policy functions $\{w_s^\theta(z), v_s^\theta(z)\}_{s\in\{1,\ldots,S\},\theta\in\{i,o\}}$ and the updated indifference function φ, we compute the wage offer distributions $\{F_s\}_s$ that we then use to update the reservation wages according to (53). Finally, the productivity cutoff of contractor firms is computed as $\underline{z}^C = \underline{w}_1/(p\tau)$.

Outer iteration The outer iteration solves for the price of outsourcing through the market clearing condition (54). The supply of outsourcing services is computed directly from the contractors' policy functions on $[\underline{z}^C, \tilde{z}^C)$ and from the good producers' policy functions through $\zeta^{G\to C}$ for $z \geq \tilde{z}^C$.

G.5 Estimation of parameters

As described in the main text, the estimation of the parameters is broken down into three steps. The first step inverts some equations of the model to directly estimates $\{\delta_s\}_{s=1}^3$ and $\{\zeta_s\}_{s=1}^3$ from the data (see Section G.6 to derive a closed form expression for the EE rate). The second step sets $\xi = 0.5$. Finally, the third step estimates together the remaining parameters via a minimum distance estimator. The remaining parameters can be divided into two sets. The first set of parameters can be estimated by model inversion given the other parameters. This set consists of the matching efficiency, $\{\mu_s\}_s$, and the unemployment benefits, $\{b_s\}_s$. To recover the matching efficiency, note that, in equilibrium, unemployed workers always accept the job offers that they receive, so that $\lambda_s^U = \text{NE}_s$. Furthermore, in the model, $\lambda_s^U = \mathcal{M}(m_s[u_s + \zeta_s(1-u_s)], V_s)/m_s[u_s + \zeta_s(1-u_s)]$. Together, we obtain

$$\mu_s = \text{NE}_s \left(\frac{m_s[u_s + \zeta_s(1-u_s)]}{V_s}\right)^{1-\xi}.$$

Given V_s, the above equation identifies μ_s. To recover the unemployment benefits, rewrite the expression for the reservation wage (53) as

$$\underline{w}_s = \text{RE}_s\mathbb{E}[w_s] + \left(k_s^U - k_s\right)\int_{\underline{w}_s} \frac{1-F_s(w)}{1+k_s(1-F_s(w))}\text{d}w,$$

where $\text{RE}_s \equiv b_s/\mathbb{E}[w_s]$ is the replacement rate and is targeted in the estimation. Importantly, the above expression is the only equation in which b_s appears and $\mathbb{E}[w_s]$ is independent of b_s up to $\underline{w}_s$. Hence, by directly setting RE_s, it is possible to compute the reservation wage without knowing b_s and to recover b_s as a residual,

$$b_s = \underline{w}_s - (k_s^U - k_s) \int_{\underline{w}_s} \frac{1 - F_s(w)}{1 + k_s(1 - F_s(w))} \mathrm{d}w.$$

To estimate the second set of parameters, we define the loss function

$$\mathcal{L}(\boldsymbol{\theta}) = \sqrt{\frac{1}{N} \sum_{n=1}^{N} \left[h_n(\boldsymbol{\theta}) - \hat{h}_n \right]^2},$$

where $\boldsymbol{\theta}$ is the vector of parameter to be estimated, $\{\hat{h}_n\}_{n=1}^N$ is the set of empirical moments we are targeting, and $h : \mathbb{R}^N \mapsto \mathbb{R}^N$ maps parameters into simulated moments from our model. The simulated moments are computed as exact analogs of the empirical moments. To compute the simulated moments, we simulate a dataset in the (z, ε) space, projecting onto this space the policy functions found in the $(z, \hat{z})$ space using linear interpolations.

To find the minimum of $\mathcal{L}$, we use a gradient descent algorithm. That is, starting from $\boldsymbol{\theta}^0$, we obtain a sequence of parameters $\{\boldsymbol{\theta}^j\}_j$ by iterating on $\boldsymbol{\theta}^{j+1} = \boldsymbol{\theta} - \gamma_j \nabla \mathcal{L}(\boldsymbol{\theta})$, where the endogenous step size follows the Barzilai–Borwein method. Namely, for $j > 1$,

$$\gamma_j = \frac{\max\left\{ |\boldsymbol{\theta}^j - \boldsymbol{\theta}^{j+1}|^T \cdot |\nabla \mathcal{L}(\boldsymbol{\theta}^j) - \nabla \mathcal{L}(\boldsymbol{\theta}^{j-1})|, 10^{-3} \right\}}{\|\nabla \mathcal{L}(\boldsymbol{\theta}^j) - \nabla \mathcal{L}(\boldsymbol{\theta}^{j-1})\|^2}.$$

We impose a maximal step size as in Burdakov et al. (2019) to stabilize the descent. The gradient of the loss function is approximated with central finite difference to maximize accuracy. Given that we use $N = 13$ parameters, the loss function $\mathcal{L}(\boldsymbol{\theta})$ is high-dimensional and we cannot check for the existence of local minima. To avoid those, we first search manually to start the algorithm from a $\boldsymbol{\theta}^0$ with a relatively low loss function, in practice $\mathcal{L}(\boldsymbol{\theta}^0) \in [0.3, 0.5]$. The gradient descent attains its minimum at $\mathcal{L}(\boldsymbol{\theta}^*) = .085$. The gradient descent is implemented in Julia and parallelized over 6 CPUs. The descent is run on a standard laptop and takes about one hour to converge.

G.6 Estimation: expression for the employment-employment transition rate

Omit s indices for simplicity. Our argument requires only that the economy be stationary. Index firms by their wage offer w and thie vacancy decision v. Denote $H(v|w)$ the conditional c.d.f. of vacancies given the wage offer. Then

$$EE = \frac{\lambda^E \iint n(w, v)(1 - F(w)) dF(w) H(dv|w)}{\iint n(w, v) dF(w) H(dv|w)}.$$

The integral over $H(dv|w)$ produces the vacancy share of goods producers in the numerator and denominator, and hence drops out. Hence,

$$EE = \frac{\lambda^E \int \frac{(1+k)e}{(1+k(1-F(w)))^2}(1-F(w))dF(w)}{\int \frac{(1+k)e}{(1+k(1-F(w)))^2}dF(w)} = \frac{\lambda^E \int_0^1 \frac{(1-F)dF}{(1+k(1-F))^2}}{\int_0^1 \frac{dF}{(1+k(1-F))^2}},$$

after changing variables to $F = F(w)$. Both integrals admit closed-form expressions, and thus:

$$EE = \lambda^E \frac{\big((1+k)\log(1+k) - k\big)/\big(k^2(1+k)\big)}{1/(1+k)} = \delta\frac{(1+k)\log(1+k) - k}{k}.$$

G.7 Estimation of counterfactual

The baseline parameters of our model are estimated as if the economy was in 2002. To quantify the effects of the rise of outsourcing between 1996 and 2007, we therefore estimate two outsourcing shocks: a negative outsourcing shock that pushes back the economy to 1996, and a positive outsourcing shock that brings the economy to 2007. To compute both shocks, we estimate the mixture of parameter changes that matches the (rescaled) elasticities presented in Table 2. Specifically, let $o(\boldsymbol{\theta})$ be the outsourcing share and $\boldsymbol{X}(\boldsymbol{\theta})$ the variables of interest (e.g. log value added, log employment, etc.) under the parameter vector $\boldsymbol{\theta}$. For each shock, the new vector of parameters $\boldsymbol{\theta}'$ as to be such that

$$\begin{aligned} \frac{\boldsymbol{X}(\boldsymbol{\theta}') - \boldsymbol{X}(\boldsymbol{\theta})}{o(\boldsymbol{\theta}') - o(\boldsymbol{\theta})} &= \boldsymbol{\gamma}, \\ o(\boldsymbol{\theta}') - o(\boldsymbol{\theta}) &= \Delta_o, \end{aligned}$$

where $\boldsymbol{\gamma}$ are the cross-industry scaled elasticity and Δ_o is the size of the outsourcing shock considered. To implement a fast estimation of the shocks, we solve locally for $\boldsymbol{\theta}'$. Taking a first order approximation around $\boldsymbol{\theta}$, the system above becomes

$$\begin{aligned} \left[\boldsymbol{D}\boldsymbol{X}(\boldsymbol{\theta}) - \boldsymbol{\gamma}\boldsymbol{D}o(\boldsymbol{\theta})\right](\boldsymbol{\theta}' - \boldsymbol{\theta}) &= \mathbf{0}, \\ \boldsymbol{D}o(\boldsymbol{\theta})(\boldsymbol{\theta}' - \boldsymbol{\theta}) &= \Delta_o, \end{aligned} \tag{67}$$

where $\boldsymbol{D}$ is the differential operator with respect to $\boldsymbol{\theta}$ and $\boldsymbol{D}\boldsymbol{X}$ and $\boldsymbol{D}o$ can be computed by perturbating the economy around its 2002 steady state. We can then invert this linear system to recover $\{\boldsymbol{\theta}_{1996}, \boldsymbol{\theta}_{2007}\}$. To construct the counterfactual reported in Section 5, we fit cubic polynomials on $\{\boldsymbol{\theta}_{1996}, \boldsymbol{\theta}_{2002}, \boldsymbol{\theta}_{2007}\}$ for each dimension of $\boldsymbol{\theta}$. We then use these polynomials to infer a vector of shock for each year in the 1996-2007 time frame. We also use the fitted cubic polynomials to extrapolate the consequences of outsourcing till 2016.

G.8 Accounting

This section details how the main micro and macro variables are computed in the model. For that, suppose that we have simulated a cross-sectional data set at the firm level from our model. Let i and

j describes the identity of a firm in this data set.

Goods producers profits and value added. Profits of goods producer i are

$$\text{Profits}_i^G = R_i^G - \sum_s w_{is}^G n_{is}^G - p\varepsilon_i n_i^G - \sum_s c(v_{is}^G) - \eta,$$

where the notation follows closely that of Section G.1. The value added of goods producer i is revenues net of spending on intermediaries, or

$$\text{VA}_i^G = R_i^G - p\varepsilon_i n_i^G.$$

The variable ε_i indeed represents iceberg costs faced by a given goods producer i. Goods producer thus i needs to purchase $\varepsilon_i n_i^G$ units of labor in the labor service market to obtain n_i^G units of effective labor in production.

Contractor profits and value added. Contractors j make profits

$$\text{Profits}_j^C = R_j^C - w_j^C n_j^C - c(v_j^C) = p\tau z_j n_j^C - w_j^C n_j^C - c(v_j^C),$$

and have value added

$$\text{VA}_j^C = R_j^C = p\tau z_j n_j^C.$$

Aggregate output. Aggregate output is the sum of value added of all sectors of the economy. Aggregate output coincides with the amount of goods available for consumption for workers, who receive wage payments, and capital owners, who receive vacancy costs and fixed costs. Thus,

$$\begin{aligned} \text{Ag. output} &= \sum_i \text{VA}_i^G + \sum_j \text{VA}_j^C \\ &= \sum_i \left(R_i^G - p\varepsilon_i n_i^G\right) + p\sum_j \tau z_j n_j^C \\ &= \sum_i \left(R_i^G - p\varepsilon_i n_i^G\right) + p\sum_i \varepsilon_i n_i^G \\ &= \sum_i R_i^G \end{aligned} \tag{68}$$

where the first equality uses the definitions of value added, and the second equality uses labor services market clearing (54).

Aggregate TFP We define TFP as

$$\text{TFP} = \frac{\text{Ag. output}}{\overline{N}}, \tag{69}$$

where the labor aggregator $\overline{N}$ is defined as

$$\overline{N} = \left(\prod_s \overline{N}_s^{a_s} \right)^{\rho},$$

and $\overline{N}_s = m_s(1 - u_s)$ is employment of skill s. To capture reallocation towards less productive contractors, we define the adjusted aggregator

$$\widetilde{N} = \widetilde{N}_1^{\rho a_1} \left(\prod_{s=2}^{3} \overline{N}_s^{a_s} \right)^{\rho} \quad , \quad \widetilde{N}_1 = \overline{N}_1^G + \tau_1 \overline{N}_1^C,$$

where $\overline{N}_1^G$ denotes aggregate employment of skill 1 by goods producers, and $\overline{N}_1^C$ aggregate employment by contractors. The effective measure $\widetilde{N}_1^G$ encodes the effective amount of labor used for task 1 in the economy. The ratio

$$\frac{\widetilde{N}}{\overline{\overline{N}}} = \left(\frac{\widetilde{N}_1}{\overline{\overline{N}}_1} \right)^{\rho a_1} = \left(\tau_1 x_1^C + (1 - x_1^C) \right)^{\rho a_1},$$

where $x_1^C = \overline{N}_1^C / \overline{N}_1$ is the employment share of contractors among low skill service workers, captures the TFP effect of reallocation towards more or less productive contractors. When $\tau_1 < 1$ and x_1^C rises as outsourcing increases, $\widetilde{N}/\overline{N}$ decreases: workers are reallocated towards less productive jobs as far as production of labor services is concerned. Aggregate TFP then writes

$$\text{TFP} = \frac{\text{Ag. output}}{\widetilde{N}} \times \frac{\widetilde{N}}{\overline{\overline{N}}},$$

and so changes in aggregate TFP are

$$\Delta \log \text{TFP} = \underbrace{\Delta \log \frac{\text{Ag. output}}{\widetilde{N}}}_{\substack{\text{Allocative efficiency} \\ \text{given effective labor} \\ \text{in the economy}}} + \underbrace{\Delta \log \frac{\widetilde{N}}{\overline{\overline{N}}}}_{\substack{\text{Productivity gains/losses} \\ \text{from contractor comparative} \\ \text{advantage/disadvantage:} \\ \text{change in effective labor}}} .$$