Online Appendix for "The Cost of Intermediary Market Power for Distressed Borrowers"

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Abstract

This is the supplemental material to the paper titled "The Cost of Intermediary Market Power for Distressed Borrowers" (Dou et al., 2023). It includes the detailed description of the estimation of risk-adjusted yield spreads and comprehensive explanations on the steps of deriving the model solutions and performing estimation, as well as additional estimation results and quantitative analyses.

Keywords: Loan Pricing, Imperfect Competition, Collusion in Syndication, Bankruptcy and Distress, Bayesian MCMC, Antitrust Policy. (**JEL:** G12, G23, G30, L13)

Contents

OA.1 Estimation of Credit-Risk Components Based on CDS Prices

Under a CDS contract, the protection seller promises to buy the reference bond at its par value when a predetermined default event occurs. In return, the protection buyer makes periodic payments to the seller until the maturity date of the contract or until a credit event occurs. This periodic payment, which is usually expressed as a percentage (in basis points) of the notional value, is called the CDS spread. The reference bond is typically a senior unsecured bond. A CDS contract written on a particular reference bond normally provides coverage for all obligations of the reference entity that have equal or higher seniority.

Owing to the specification of CDS contracts and the liquid market for CDS transactions among sophisticated institutional investors, the CDS spread provides a pure measure of the credit risk component in the credit spread of the reference bond.

We observe T-year CDS premium, which is paid every period of Δ. The frequency $\Delta = 0.5$ is semiannual. We also have reliable estimation on the expected recovery rates δ and δ_L for the corporate bond and the loan, respectively. Recovery rate is the extent to which principal and accrued interest on defaulted debt can be recovered, expressed as a percentage of face value (i.e., par value). Moreover, we have the information on zero-coupon risk-free bond prices, denoted by $Z(0, i\Delta)$ with $i = 1, 2, \cdots, T/\Delta$. Let $n = T/\Delta$ and $t_i = i\Delta$ for $i = 1, \cdots, n$.

Back out constant hazard rate p^*. If CDS only reflects credit risk, the non-arbitrage CDS premium formula is

$$s = \frac{1}{\Delta} \frac{(1-\delta) \sum_{i=1}^{n} \left[P^*(0, t_{i-1}) - P^*(0, t_i)\right] Z(0, t_i)}{\sum_{i=1}^{n} P^*(0, t_i) Z(0, t_i)}, \tag{OA.1}$$

where $P^*(0, t)$ is the risk-neutral probability of survival up to time t, modeled as

$$P^*(0, t) \equiv \exp(-p^* \times t). \tag{OA.2}$$

We can estimate p^* using the equations (OA.1) and (OA.2) based on the data of s, δ, and $Z(0, t_i)$.

Estimate loan credit spreads. If a loan has maturity $T_L = t_m$, it holds that

$$1 = P^*(0, t_m) Z(0, t_m) + \sum_{i=1}^{m} \left[P^*(0, t_{i-1}) - P^*(0, t_i)\right] Z(0, t_i) \delta_L + y\Delta \sum_{i=1}^{m} P^*(0, t_i) Z(0, t_i),$$

where y is the annualized loan yield. The equality above recognizes that convention that loans are sold at par, and that the loan yield is exactly equal to the coupon rate.

Then, the credit-risk component of the annualized 6-month loan yield is

$$y = \frac{1}{\Delta} \frac{1 - P^*(0,t_m)Z(0,t_m) - \sum_{i=1}^{m} \left[P^*(0,t_{i-1}) - P^*(0,t_i)\right] Z(0,t_i)\delta_L}{\sum_{i=1}^{m} P^*(0,t_i)Z(0,t_i)}. \tag{OA.3}$$

Thus, the credit spread is $y - r_f$, where y is the annualized 6-month loan yield that is derived according to (OA.3), and $r_f\Delta = 1/Z(0,1) - 1$ is the annualized 6-month (simple) risk-free rate.

Risk-neutral expected loan maturity and annualized default fee. By utilizing the CDS-implied risk-neutral default probability, we derive the risk-neutral expected loan maturity and annualized default fee, both of which are utilized to compute the TCB. Given the estimated risk-neutral hazard rate p^*, the risk-neutral expected maturity is

$$\begin{aligned}
&\text{Risk Neutral Expected Maturity} \\
&= \sum_{i=1}^{\infty} \left[P^*(0,t_{i-1}) - P^*(0,t_i)\right] \min\{t_i, t_n\} && \text{(OA.4)} \\
&= \sum_{i=1}^{n} \left[P^*(0,t_{i-1}) - P^*(0,t_i)\right] t_i + \sum_{i=n+1}^{\infty} \left[P^*(0,t_{i-1}) - P^*(0,t_i)\right] t_n && \text{(OA.5)} \\
&= \sum_{i=1}^{n} \left[P^*(0,t_{i-1}) - P^*(0,t_i)\right] t_i + P^*(0,t_n)t_n. && \text{(OA.6)}
\end{aligned}$$

Suppose the default interest is D and the default borrower on average pays back in one year after the occurrence of default. The risk-neutral expected annualized default fee is

$$\begin{aligned}
&\text{Risk Neutral Expected Annualized Default Fee} \\
&= \sum_{i=1}^{n} \left[P^*(0,t_{i-1}) - P^*(0,t_i)\right] Z(0,t_i)D/t_i. && \text{(OA.7)}
\end{aligned}$$

OA.2 Estimating Liquidity Risk Components from Bid-Ask Prices in the Secondary Loan Market

To estimate the liquidity risk components within the yield spread, we employ bid-ask spreads observed in the secondary market by following a structured approach. Initially, we capture the bid (P_b) and ask (P_a) prices of term loans at their trading inception. We then proceed to convert the spread between these bid and ask prices into a yield spread. Loan pricing equation implies the following relations:

$$P_i = \sum_{n=1}^{N} \frac{C_n}{(1 + y_i\Delta)^{n\Delta}}, \quad \text{with } i \in \{a, b\}, \tag{OA.8}$$

where P_i represents the bid ($i = b$) or ask ($i = a$) price, y_i is the yield to maturity associated with the bid ($i = b$) or ask ($i = a$) price, N is the total number of payments made by the loan, C_n is the amount of payment made in the nth period that contains interest payment and principal amortization if any, Δ is the length of each payment period (e.g., $\Delta = 0.5$ means that payment is made every 6 months). To compute the payment schedule C_n, we first use the LIBOR curve observed on loan issuance to compute its component of the expected interest payment. Then, for amortized loans, we use the amortization schedule specified in the loan contracts to compute its component of amortized principal. For unamortized loans, the principal is only paid in full on maturity. We use $(y_a - y_b)/2$ as an approximation for the liquidity premium.

OA.3 Perfect Cartel Benchmark and Non-Collusive Nash Equilibrium

(i) Perfect Cartel Benchmark. We begin with the perfect cartel benchmark, in which all specialized lenders coordinate to maximize the joint profit of the lending group, effectively behaving as a monopoly. Setting $L_{other} \equiv L$ in equation (4), since all specialized lenders choose the same loan size simultaneously and monopolistically, and then taking the first-order condition with respect to L, we obtain the optimal loan spread and the individual loan size provided by each lender:

$$R_2^M(k,x,m) = \frac{\varepsilon_k}{\varepsilon_k - 1} e^{\phi_2 + \varsigma u} \quad \text{and} \quad L_2^M(A,k,x,m) = \frac{1}{m}\left(1 - \frac{1}{\varepsilon_k}\right)^{\varepsilon_k} h_2(k,x)A, \tag{OA.9}$$

where $h_2(k,x)$ is defined in (9) with $l = 2$. The optimal individual loan supply, $L_2^M(A,k,x,m)$, equals $\frac{1}{m}$ of the total optimal monopolistic loan size. Thus, the optimal expected profit for each specialized lender in the syndication is

$$\Pi_2^M(A,k,m) = \frac{1}{m\varepsilon_k}\left(1 - \frac{1}{\varepsilon_k}\right)^{\varepsilon_k - 1} \mathcal{H}_2(k)A, \tag{OA.10}$$

where $\mathcal{H}_2(k)$ is defined in (11) with $l = 2$.

(ii) Non-Collusive Nash Equilibrium. Although the perfect cartel maximizes total syndicate profits, it is generally difficult to sustain. When other lenders restrict their loan supply, each specialized lender has a strong incentive to deviate by unilaterally expanding its lending to capture additional short-term gains, which undermines coordination and ultimately leads to a non-collusive Nash equilibrium. This equilibrium corresponds to the case in which specialized lenders supply loans competitively within the syndication, though not under conditions of perfect competition.

The loan spread and individual loan size in the non-collusive equilibrium are obtained by differentiating equation (4) with respect to L, while treating the loan supply of the other specialized lenders, L_{other}, as fixed at the equilibrium amount $L_2^N(A,k,x,m)$, which depends on the variables $\{A,k,x,m\}$ characterizing the syndication case. This yields the following closed-form expressions for the equilibrium risk-adjusted yield spread and the optimal loan size:

$$R_2^N(k,x,m) = \frac{\varepsilon_k}{\varepsilon_k - 1/m} e^{\phi_2 + \varsigma u} \quad \text{and} \quad L_2^N(A,k,x,m) = \frac{1}{m}\left(1 - \frac{1}{m\varepsilon_k}\right)^{\varepsilon_k} h_2(k,x)A, \tag{OA.11}$$

where $h_2(k,x)$ is defined in (9). When $m > 1$, the loan spread decreases and the loan size increases in the non-collusive equilibrium compared to the perfect cartel benchmark. Consequently, each specialized lender's expected profit is lower than in the perfect cartel benchmark, given by:

$$\Pi_2^N(A,k,m) = \frac{1}{m^2\varepsilon_k}\left(1 - \frac{1}{m\varepsilon_k}\right)^{\varepsilon_k - 1} \mathcal{H}_2(k)A, \tag{OA.12}$$

where $\mathcal{H}_2(k)$ is defined in (11).

OA.4 Likelihood Functions and MCMC

This section provides technical details on the estimation of the structural model. We first derive the likelihood function and then describe the Bayesian MCMC estimator used to recover the posterior distribution of model parameters and the latent cluster assignments.

OA.4.1 Likelihood Function

Likelihood. Given a classification of all observations into different demand curves and size groups (i.e., give a realization of $\{k_i\}_{i=1}^N$), the observable variables include $y_i \equiv (m_i, l_i, \ln(L_i/A_i), \ln(R_i))$ for deals indexed by $i = 1, \cdots, N$. Here, l_i is the type of lenders in deal i, m_i is the number of specialized lenders, L_i/A_i is the loan amount normalized by the borrower's asset size, and R_i is the risk-adjusted loan yield spread. We denote by θ the parameter vector that contains all the model parameters $\{\alpha_k, \varepsilon_k, \lambda_k\}_{k=1}^K$, $\{\gamma_k, \beta_k\}_{k=2}^K$, σ, ς, ξ, μ, ϕ_1, ϕ_2, and ϕ_3.

Conditioning on a latent demand curve classification $\{k_i\}_{i=1}^N$, the model-implied log-likelihood of these observable variables can be factorized as follows:

$$P\left(\{y_i\}_{i=1}^N | \theta, \{k_i\}_{i=1}^N\right) = \prod_{i=1}^N P\left(y_i|\theta, k_i\right), \tag{OA.13}$$

$$\text{where} \quad P\left(y_i|\theta,k_i\right) = P\left(l_i|\theta,k_i\right) \times P\left(m_i|\theta,l_i,k_i\right) \times P\left(\ln(L_i/A_i)|\theta,l_i,m_i,k_i\right) \times P\left(\ln(R_i)|\theta,\ln(L_i/A_i),l_i,m_i,k_i\right). \tag{OA.14}$$

We summarize below the individual component of the likelihood function in Equation (OA.14) for each observation. First, the likelihood of $\ln(R_i)$, conditioning on the parameters θ and the variables $\ln(L_i/A_i), l_i, m_i, k_i$, is a normal distribution with mean $\alpha_{k_i} - \varepsilon_{k_i}\ln(R_i)$ and variance σ^2. Thus, $P\left(\ln(R_i)|\theta, \ln(L_i/A_i), l_i, m_i, k_i\right)$ can be rewritten as $P\left(\ln(R_i)|\alpha_{k_i}, \varepsilon_{k_i}, \sigma, \ln(L_i/A_i), l_i, m_i, k_i\right)$, which identifies α_k and ε_k for $k \in \{1, \cdots, K\}$, as well as σ.

Second, the likelihood of $\ln(L_i/A_i)$, conditioning on the parameters θ and the variables l_i, m_i, k_i, is a normal distribution with mean $\ln\widehat{L}(k_i, m_i) + \alpha_{k_i} - \varepsilon_{k_i}\phi_{l_i}$ and variance ς^2. Thus, given that $\{\alpha_k, \varepsilon_k\}_{k=1}^K$ and σ, the conditional likelihood $P\left(\ln(L_i/A_i)|\theta, l_i, m_i, k_i\right)$ and its associated data identify ξ and $\{\phi_l\}_{l=1}^3$.

Third, the likelihood of m_i, conditioning on the parameters θ and the variables l_i and k_i, is a multinomial distribution. Specifically, when $l_i = 1$ or $l_i = 3$, it holds that $P\left(m_i = 0|\theta, l_i, k_i\right) = 1$; when $l_i = 2$, it holds that $P\left(m_i|\theta, l_i, k_i\right) = \begin{pmatrix} M \\ m_i \end{pmatrix} F(\omega_C^*)^{m_i}\left[1 - F(\omega_C^*)\right]^{M-m_i}$. Conditioning on the parameters $\{\alpha_k, \varepsilon_k\}_{k=1}^K$, $\{\phi_l\}_{l=1}^3$, and ξ, which have already been identified using other conditional likelihoods, and the variables l_i and k_i, the equilibrium cutoff point w_C^* only depends on the parameter μ. Therefore, $P\left(m_i|\theta, l_i, k_i\right)$ identifies μ.

Lastly, we know that

$$\begin{aligned}
P\left(l_i = 1|\theta, k_i\right) &= \lambda_{k_i}, \\
P\left(l_i = 2|\theta, k_i\right) &= \left(1 - \lambda_{k_i}\right)\left\{1 - \left[1 - F(\omega_C^*)\right]^M\right\}, \\
P\left(l_i = 3|\theta, k_i\right) &= \left(1 - \lambda_{k_i}\right)\left[1 - F(\omega_C^*)\right]^M.
\end{aligned}$$

Thus, it is clear that $P\left(l_i|\theta, k_i\right)$ identifies $\{\lambda_k\}_{k=1}^K$.

Bayesian estimation. Using the Bayesian approach, we estimate the posterior distribution for the variables of interest, $P\left(\{k_i\}_{i=1}^N, \theta|\{y_i\}_{i=1}^N\right)$. This posterior distribution describes the estimate of model parameters and the augmented latent cluster variable of each deal based on the observed variables. Based on the Hammersley-Clifford Theorem (Besag, 1974), this posterior distribution is fully characterized by two conditional distributions $P\left(\theta|\{y_i\}_{i=1}^N, \{k_i\}_{i=1}^N\right)$ and $P\left(\{k_i\}_{i=1}^N|\theta, \{y_i\}_{i=1}^N\right)$, which in turn can be broken down into more lower dimensional conditional distributions.

Conditioning on $\{k_i\}_{i=1}^N$, the distribution $P\left(\theta|\{y_i\}_{i=1}^N, \{k_i\}_{i=1}^N\right)$ is determined by the distribution $P\left(\theta|\{k_i\}_{i=1}^N\right)$ and the likelihood function, $P\left(\{y_i\}_{i=1}^N|\theta, \{k_i\}_{i=1}^N\right)$, implied by the model solution. Specifically,

$$P\left(\theta|\{y_i\}_{i=1}^N, \{k_i\}_{i=1}^N\right) \propto P\left(\theta|\{k_i\}_{i=1}^N\right) \times P\left(\{y_i\}_{i=1}^N|\theta, \{k_i\}_{i=1}^N\right) \tag{OA.15}$$

Meanwhile, conditioning on θ, the distribution $P\left(\{k_i\}_{i=1}^N|\theta, \{y_i\}_{i=1}^N\right)$ is determined by $P\left(\{k_i\}_{i=1}^N|\theta\right)$

and the likelihood function $P\left(\{y_i\}_{i=1}^N|\theta,\{k_i\}_{i=1}^N\right)$:

$$P\left(\{k_i\}_{i=1}^N|\theta,\{y_i\}_{i=1}^N\right) \propto P\left(\{k_i\}_{i=1}^N|\theta\right) \times P\left(\{y_i\}_{i=1}^N|\theta,\{k_i\}_{i=1}^N\right) \tag{OA.16}$$

OA.4.2 MCMC Estimator

To implement the MCMC estimator, for each loan i in the data, we define a vector z_i of dimension K (i.e., the number of possible demand curves) so that the kth element in the vector describes the probability of this deal belonging to demand curve k, such that $z_{ik} \geq 0$ and $\sum_{k=1}^K z_{ik} = 1$. We then apply the Metropolis-Hastings algorithm with the following procedure.

Step 1: In each iteration g, we first update $z_i^{(g)}$ for each deal i using the Bayes rule for classification based on outputs from the last iteration round $g-1$:

$$z_{ik}^{(g)} = \frac{z_{ik}^{(g-1)} P\left(y_i|\theta^{(g-1)}, k_i = k\right)}{\sum_{j=1}^K z_{ij}^{(g-1)} P\left(y_i|\theta^{(g-1)}, k_i = j\right)} \tag{OA.17}$$

where $z_{ik}^{(g-1)}$ is the (posterior) probability of $k_i = k$, determined in the previous iteration (i.e., iteration $g-1$), and $P\left(y_i|\theta, k_i = k\right)$ is the likelihood function of observables evaluated at the parameter values simulated from the previous iteration (i.e., iteration $g-1$). We then follow the SEM algorithm (Celeux, 1985; Celeux and Diebolt, 1992) and draw the realization of $k_i^{(g)}$ from the multinomial distribution with weights in Equation (OA.17). We discard the draw and repeat this step if for any cluster k, the total number of deals assigned to that cluster falls below a minimum threshold (e.g., 3%).

Step 2: In the same iteration g, we then update the remaining parameters in θ using the random walk Metropolis-Hastings algorithm based on the updated draws of $\{k_i^{(g)}\}_{i=1}^N$ from Step 1. We compute the acceptance/rejection threshold:

$$\alpha\left(\theta^{(g)}, \theta^{(g-1)}\right) = \min\left\{\frac{P\left(\theta^{(g)}|\{k_i^{(g)}\}_{i=1}^N\right)\prod_{i=1}^N P\left(y_i|\theta^{(g)}, k_i^{(g)} = k\right)}{P\left(\theta^{(g-1)}|\{k_i^{(g)}\}_{i=1}^N\right)\prod_{i=1}^N P\left(y_i|\theta^{(g)}, k_i^{(g)} = k\right)}, 1\right\}$$

where $\theta^{(g-1)}$ is the parameter vector from the last iteration, and $\theta^{(g)} = \theta^{(g-1)} + \Sigma\epsilon$ is the vector of proposed parameters.

Step 3: In the next iteration (i.e., iteration $g+1$), we repeat the procedure in Steps 1 and 2 by simulating the classification realization $k_i^{(g+1)}$ according to the updated posterior probabilities $\pi_{ik}^{(g+1)}$ for all i, k, and simulating the model parameters from the posterior distribution $P\left(\theta | \{y_i\}_{i=1}^N, \{k_i^{(g+1)}\}_{i=1}^N\right)$.

OA.5 Policy Analysis: Additional Results

OA.5.1 Government Intervention

In the main text, we presented the results for DIP loans with government intervention. In this section, we report the results for highly speculative loans. The findings are qualitatively similar: government intervention overall reduces the risk-adjusted spreads borrowers have to pay, increases the loan size they obtain (and thus the credit access), and improves borrower welfare.

Table OA1 reports the results for direct and indirect effects, and Figure OA1 illustrates the overall effect of government intervention on the risk-adjusted spreads and loan size and the extensive margin.

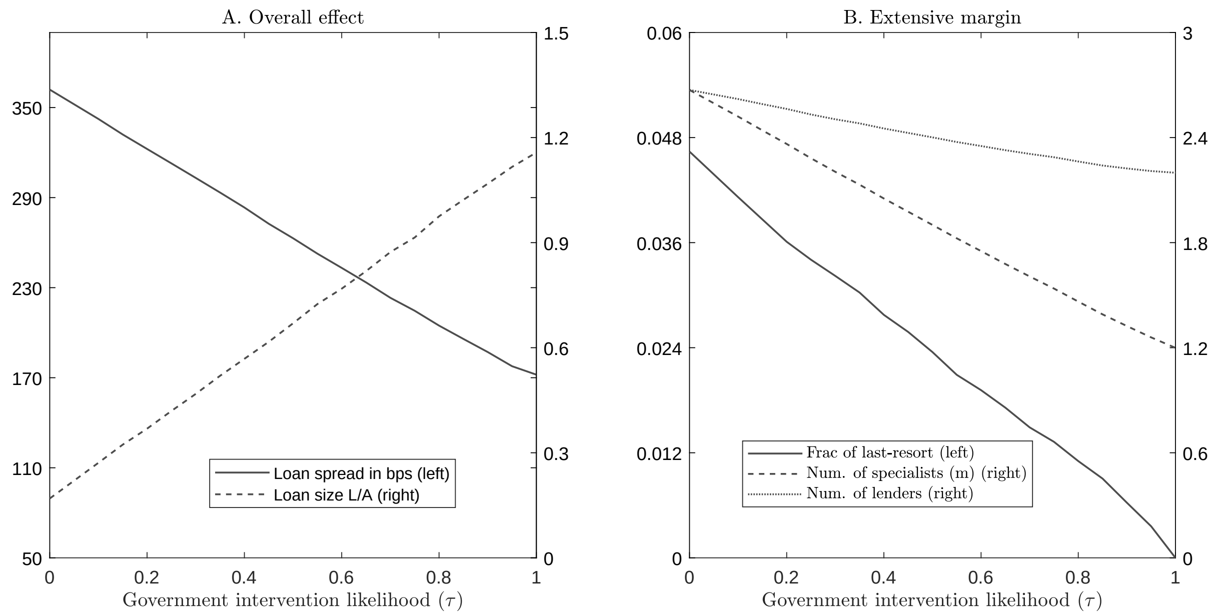
Table OA1: The Effect of Government Intervention: Highly-Speculative Loans.

		Direct effect	Indirect effect	
			$\tau = 0.8$	$\tau = 1.0$
R (bps)	Small	−249	0	−240
	Large	−162	−102	−156
L/A	Small	1.380	0.000	1.359
	Large	0.762	0.274	0.748
Borrower	Small	0.120	0.000	0.117
Welfare	Large	0.122	0.078	0.118

Note: This table examines the impact of government lending facilities on borrowers of varying sizes. It focuses on type-2 loans, i.e., syndicated loans made by specialized lenders. We report the changes from the baseline model caused by the direct and indirect effects of government lending facilities.

OA.5.2 Interest Rate Cap

Interest rate caps have historically and geographically been one of the most ubiquitous economic legislations, playing a vital role in economic activities across different economies (Blitz and Long, 1965). For example, in New York, corporations and limited liability companies (LLCs) cannot be charged more than 16% interest per annum for commercial loans whose amounts are above certain thresholds.



Note: This figure shows the overall effects and extensive margins of the government lending facility for highly speculative loans. Panel A depicts the overall effect on the risk-adjusted yield spread, R, and the loan-to-asset ratio, L/A. Panel B shows the extensive-margin effect of the government lending facility on reducing the number of participating specialized lenders and decreasing the likelihood of borrowers' turning to lenders of last resort. The total number of lenders, including the expected participation of the government, is also plotted.

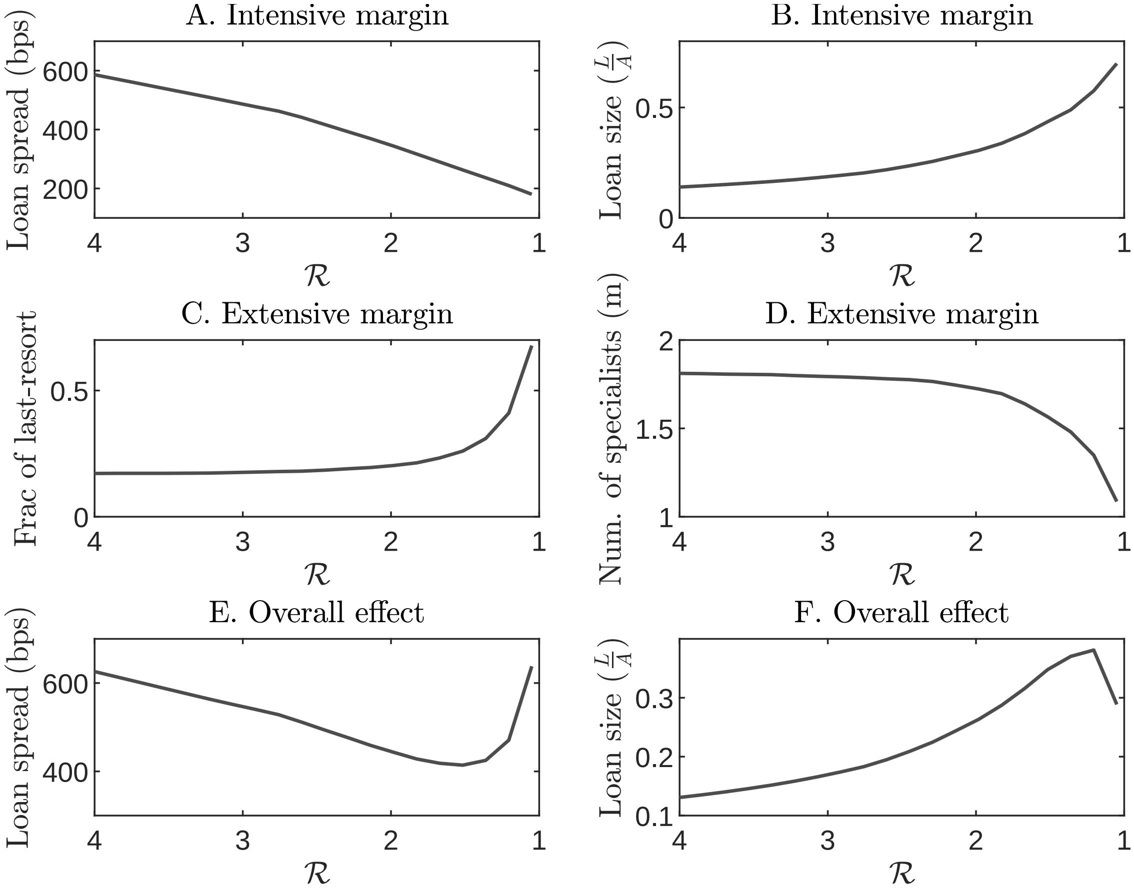
Figure OA1: The Intensive and Extensive Effects of the Government Lending Facility

Formulation of Polices in the Model. The interest rate cap regulation can be implemented after conditioning on the characteristics of a borrower and adjusting the risk premium to better account for borrower risk heterogeneity and risk pricing. This can substantially improve the effectiveness of the interest rate cap policy because unsophisticated constant interest rate caps have severe limitations on balancing the tradeoff between borrower production and credit access (e.g., Cuesta and Sepúlveda, 2021). Assuming that the regulator can observe or estimate the marginal cost of providing distressed loans $e^{\phi+\varsigma u}$ and the risk premium of the loan, we can directly consider the interest rate cap that is imposed on the risk-adjusted spread R in the following form:

$$R_{max}(x) \equiv \mathcal{R}_{max} e^{\phi+\varsigma u}, \tag{OA.18}$$

where $\mathcal{R}_{max}$ is a positive constant. A lower $\mathcal{R}_{max}$ reflects a stricter government intervention.

Similar to government lending facilities, the total effect of an interest rate cap is the result of both its direct and indirect effects. The direct effect measures how the cap alters the loan spread and loan size in the current syndication. Meanwhile, an interest rate cap imposed on future deals can influence specialized lenders' incentives for tacit collusion in the current syndication, which constitutes the indirect effect. Unlike the case of government lending facilities, both direct and indirect effects manifest simultaneously for each deal, and given the direct effect, the



Note: This figure shows the effects of the interest rate cap policy. Panels A and B depict the intensive-margin effect of the interest rate cap policy on reducing the risk-adjusted yield spread charged by specialized lenders and increasing the corresponding loan size, respectively; panels C and D show the extensive-margin effect of the interest rate cap policy on reducing the number of participating specialized lenders and increasing the likelihood of borrowers' turning to lenders of last resort, respectively; and panels E and F illustrate the overall effect of the interest rate cap policy on the average risk-adjusted yield spread and loan size, respectively.

Figure OA2: The Effects of Interest Rate Cap Policy

indirect effect of the interest rate cap is usually quite small and often negligible.

The Effects of Intensive and Extensive Margins. We start with demonstrating the intended consequences of the interest rate cap policy. In panels A and B of Figure OA2, we plot the risk-adjusted yield spread (Panel A) and loan amount (Panel B) as a function of the tightness of interest rate caps characterized by $\mathcal{R}_{max}$. As $\mathcal{R}_{max}$ decreases, the tightness of interest rate caps increases; as a result, the equilibrium risk-adjusted yield spread drops (see panel A) and the loan size increases (see panel B), for those borrowers who are able to receive loans from existing and specialized lenders. As expected, a tighter interest rate cap directly reduces the risk-adjusted yield spread. Moreover, because the loan spread for future deals is capped, the present value of future tacit coordination becomes lower, and thus the collusion capacity of specialized lenders is weakened. A lower loan price faced by borrowers would boost the demand for loans, and meanwhile, a lower collusion capacity would incentivize specialized lenders to offer loans of larger size. Consequently, in panel B, we observe a sharp increase in the average equilibrium loan size for those borrowers who are able to receive loans from existing and specialized lenders as interest rate cap tightens. Both the decline in risk-adjusted yield spread and increase in loan size by specialized lenders benefit the borrowers if they can receive the loans from existing and specialized lenders.

Even though interest rate cap improves the welfare of borrowers who can obtain loans from the existing and specialized lenders, which is referred to as the intensive-margin effect, an adverse consequence of this policy is to discourage the participation of specialized lenders, thereby hindering the credit supply of specialized lenders to distressed borrowers. Such adverse effect of interest rate caps on the credit accessibility of distressed borrowers is referred to the extensive-margin effect. Intuitively, specialized lenders choose to participate the loan syndication only when their expected profits are higher than the participation costs. Interest rate cap reduces the expected profits earned by specialized lenders and therefore excludes more specialist lenders from participating. If no specialized lenders are willing to participate in a specific deal, then the borrower is forced to borrow from the lenders of last-resort, such as private equity and hedge funds, which are not subject to the interest rate cap. Panels C and D confirm this model prediction. As $\mathcal{R}_{max}$ declines and the spread-cap tightens, the likelihood for the borrowers to borrow from lenders of the last-resort climbs sharply from about 15% to above 60%. Meanwhile, even if some borrowers can still borrow from specialized lenders, the average number of participating lenders becomes significantly smaller. Lack of competition among the last-resort lenders and the high variable costs they bear make the loan very expensive.

Combining the positive effect of interest rate cap on the intensive margin and its negative effect on the extensive margin, Panel E and F illustrate the net effect. The net effect captures the likelihood of the borrowers borrowing from different types of lenders and the risk-adjusted yield spread charged by these lenders. We observe a U-shaped relation between the average risk-adjusted yield spread paid by borrowers and the tightness of rate cap and a hump-shaped relation between the average loan amount obtained by these borrowers and the tightness of rate cap. These findings suggest that there exists an optimal level of rate cap that maximizes the borrowers' welfare. Specifically, the optimal interest rate cap maps to an average risk-adjusted yield spread of about 400 bps across all types of lenders in the DIP loan market, compared with the observed spread of 700 bps in the data.

OA.6 Solutions to Policy Analysis

In this section, we present the solutions to the policy analyses, including government intervention through lending facilities and imposing interest rate cap on distressed loans. These solutions are used to perform policy analyese presented in the section above.

OA.6.1 Model of Loan Syndication with Government Lending Facilities

Suppose government sets up a special purpose vehicle (SPV) to participate the loan syndicate for distressed borrowers. For each borrower, there is a probability τ with which government will join the loan syndicate. The probability τ captures the intervention intensity. We define a dummy variable $I_g = 1$ if and only if government intervenes, and we set $I_g = 0$ otherwise.

Compared with the baseline model, the continuation values $W^C(L^C)$ and W^N are different with government intervention. The participation threshold with government intervention, denoted by w^*_G, is also different from w^*_N. First, given the threshold w^*_G, the continuation value in a collusive Nash equilibrium is

$$W^C(L^C) = \mathbb{E}^{A',k'}\left\{\lambda_{k'}\frac{\Pi_1(A',k')}{M_0}\right\} \tag{OA.19}$$

$$+\,\mathbb{E}^{A',k'}\left\{(1-\lambda_{k'})\,[1-\tau]\,\mathbb{E}^{w',m',x'}\left[\left(\Pi_2(A',k',x',m';L^C)-w'\right)\mathbf{1}_{\{w'\le w^*_C\}}|I_g=0\right]\right\} \tag{OA.20}$$

$$+\,\mathbb{E}^{A',k'}\left\{(1-\lambda_{k'})\,\tau\mathbb{E}^{w',m',x'}\left[\left(\Pi_2(A',k',x',m'+1;L^N)-w'\right)\mathbf{1}_{\{w'\le w^*_G\}}|I_g=1\right]\right\} \tag{OA.21}$$

and the continuation value in a non-collusive equilibrium is

$$W^N = \mathbb{E}^{A',k'}\left\{\lambda_{k'}\frac{\Pi_1(A',k')}{M_0}\right\} \tag{OA.22}$$

$$+\,\mathbb{E}^{A',k'}\left\{(1-\lambda_{k'})\,[1-\tau]\,\mathbb{E}^{w',m',x'}\left[\left(\Pi_2(A',k',x',m';L^N)-w'\right)\mathbf{1}_{\{w'\le w^*_N\}}|I_g=0\right]\right\} \tag{OA.23}$$

$$+\,\mathbb{E}^{A',k'}\left\{(1-\lambda_{k'})\,\tau\mathbb{E}^{w',m',x'}\left[\left(\Pi_2(A',k',x',m'+1;L^N)-w'\right)\mathbf{1}_{\{w'\le w^*_G\}}|I_g=1\right]\right\} \tag{OA.24}$$

Here, conditioning on the government intervention, the expected profit from participating the syndication in the next period is

$$\mathbb{E}^{w',m',x'}\left[\left(\Pi_2(A',k',x',m'+1;L^N)-w'\right)\mathbf{1}_{\{w'\le w^*_G\}}|I_g=1\right] \tag{OA.25}$$

$$=\sum_{m'=1}^{M} q(m'|w'\le w^*_G, w^*=w^*_G, I_g=1)\left[F(w^*_G)\mathbb{E}^{x'}\left[\Pi_2(A',k',x',m'+1;L^N)\right]-\int_{w'\le w^*_G} w'dF(w')\right],$$

the profit of the syndicated lending with non-collusive loan size plan L^N is

$$\Pi_2(A', k', x', m'+1; L^N) \equiv \max_L \left[\left(e^{\alpha_{k'}+\sigma z'} \frac{A'}{L + m' L^N(A', k', m'+1)} \right)^{1/\varepsilon_{k'}} - e^{\phi_2 + \varsigma u'} \right] L, \quad \text{(OA.26)}$$

and the conditional probability $q(m'|w' \leq w_G^*, I_g = 1)$ is

$$\begin{aligned} & q(m'|w' \leq w_G^*, w^* = w_G^*, I_g = 1) \\ & = \frac{\mathbb{P}\left\{\text{This specialist and other } m'-1 \text{ specialists participate the lending} | w^* = w_G^*, I_g = 1\right\}}{\mathbb{P}\left\{\text{This specialist participates the lending} | w^* = w_G^*, I_g = 1\right\}} \\ & = \binom{M-1}{m'-1} F(w_G^*)^{m'-1} \left[1 - F(w_G^*)\right]^{M-m'}. \end{aligned} \quad \text{(OA.27)}$$

The following equality characterizes the cutoff w_G^*:

$$\sum_{m=1}^{M} q(m|w = w_G^*, w^* = w_G^*, I_g = 1) \mathbb{E}^{A,k,x} \left[\Pi_2(A, k, x, m+1; L^N) \right] = w_G^*. \quad \text{(OA.28)}$$

OA.6.2 Model of Loan Syndication with Interest Rate Caps

According to the demand system of the borrowers, the loan amount per specialized lender corresponding to the ceiling on the risk-adjusted spread is

$$L_{min}(k, x, m) = \frac{1}{m} \mathcal{R}_{max}^{-\varepsilon(k)} e^{[\alpha(k) - \varepsilon(k)\phi] - \varepsilon(k)\varsigma u + \sigma z} A,$$

where m is the number of specialized lenders in the syndication, k indicates the type of the borrower, and x contains the characteristics of the deal. In fact, under the interest rate cap specified in (OA.18), the loan amount $L_{min}(k, x, m)$ is the minimum loan size each specialized lender will offer in equilibrium for the syndication characterized by (k, x, m).

Given that we focus on the risk-adjusted interest rate cap imposed on the spread, the optimal loan sizes for each specialized lender under non-collusive syndication, collusive syndication, and deviation have the following respective functional forms:

$$L^i(k, x, m; \mathcal{R}_{max}) \equiv \widehat{L}^i(k, m; \mathcal{R}_{max}) e^{[\alpha(k) - \varepsilon(k)\phi] - \varepsilon(k)\varsigma u + \sigma z} A, \quad \text{with } i \in \{N, C, D\}. \quad \text{(OA.29)}$$

Non-collusive equilibrium with interest rate cap $\mathcal{R}_{max}$. Under the interest rate cap regulation, the value function prior to paying the fixed cost w and observing the deal-specific characteristics $x = (z, u)$, denoted by $U^N(k, x, m; \mathcal{R}_{max})$, satisfies the following Bellman equation:

$$
\begin{aligned}
U^N(k, x, m; \mathcal{R}_{\max}) &= \Pi_2(k, x, m; L^N, \mathcal{R}_{\max}) + \frac{W^N(\mathcal{R}_{\max})}{1-\delta}, \quad \text{where} \\
W^N(\mathcal{R}_{\max}) &= \mathbb{E}^{k'}\left\{\lambda(k')\frac{\Pi_1(k'; \mathcal{R}_{\max})}{M_0}\right\} \\
&\quad + \mathbb{E}^{k'}\Bigg\{[1-\lambda(k')]\sum_{m'=1}^{M} q(m' \mid w' \leq w^*_{N,\mathcal{R}_{\max}}) \\
&\quad \times \left[F(w^*_{N,\mathcal{R}_{\max}})\Pi_2(k', m'; L^N, \mathcal{R}_{\max}) - \int_{w' \leq w^*_{N,\mathcal{R}_{\max}}} w'\, dF(w')\right]\Bigg\}. \quad \text{(OA.30)}
\end{aligned}
$$

where $\mathbb{E}^{k'}[\cdot]$ is the expectation over $k' \in \{1, \cdots, K\}$ with probability weight $\pi(k')$ for each k', and the cutoff $w^*_{N,\mathcal{R}_{max}}$ is determined in the same way as w^*_N but they can be different in the equilibrium.

The symmetric non-collusive Nash equilibrium can be characterized by the following condition:

$$
L^N(k, x, m; \mathcal{R}_{max}) = \underset{L \geq L_{min}(k,x,m)}{\operatorname{argmax}} \left[\left(e^{\alpha(k)+\sigma z}\frac{A}{L + (m-1)L^N(k, x, m; \mathcal{R}_{max})}\right)^{1/\varepsilon(k)} - e^{\phi + \varsigma u}\right] L. \quad \text{(OA.31)}
$$

Plugging (OA.29) into (OA.31) results in the following relation:

$$
\widehat{L}^N(k, m; \mathcal{R}_{max}) = \underset{\widehat{L} \geq \frac{1}{m}\mathcal{R}_{max}^{-\varepsilon(k)}}{\operatorname{argmax}} \left\{\left[\widehat{L} + (m-1)\widehat{L}^N(k, m; \mathcal{R}_{max})\right]^{-1/\varepsilon(k)} - 1\right\}\widehat{L}, \quad \text{(OA.32)}
$$

which leads to

$$
\widehat{L}^N(k, m; \mathcal{R}_{max}) = \max\left\{\frac{1}{m}\mathcal{R}_{max}^{-\varepsilon(k)}, \ \frac{1}{m}\left[\frac{m\varepsilon(k)}{m\varepsilon(k)-1}\right]^{-\varepsilon(k)}\right\}. \quad \text{(OA.33)}
$$

Collusive equilibrium with interest rate cap $\mathcal{R}_{max}$. Under the interest rate cap regulation, the value function of a specialist at the beginning of the "afternoon" when w, k, and x are already observed, denoted by $V^C(k, x, w, m; L^C, \mathcal{R}_{max})$, has the following functional form:

$$
V^C(k, x, w, m; L^C, \mathcal{R}_{max}) \equiv U^C(k, x, m; L^C, \mathcal{R}_{max}) - w. \quad \text{(OA.34)}
$$

The value function $U^C(k, x, m; L^C, \mathcal{R}_{max})$ satisfies the following Bellman equation:

$$
\begin{aligned}
U^C(k, x, m; L^C, \mathcal{R}_{\max}) &= \Pi_2(k, x, m; L^C, \mathcal{R}_{\max}) + \frac{W^C(L^C, \mathcal{R}_{\max})}{1-\delta}, \quad \text{where} \\
W^C(L^C, \mathcal{R}_{\max}) &= \mathbb{E}^{k'}\left\{\lambda(k')\frac{\Pi_1(k', \mathcal{R}_{\max})}{M_0}\right\} \\
&\quad + \mathbb{E}^{k'}\Bigg\{[1-\lambda(k')]\sum_{m'=1}^{M} q(m' \mid w' \leq w^*_{C,\mathcal{R}_{\max}}) \\
&\qquad \times \left[F(w^*_{C,\mathcal{R}_{\max}})\Pi_2(k', m'; L^C, \mathcal{R}_{\max}) - \int_{w' \leq w^*_{C,\mathcal{R}_{\max}}} w' \, dF(w')\right]\Bigg\}.
\end{aligned}
\tag{OA.35}
$$

where $\mathbb{E}^{k'}[\cdot]$ is the expectation over $k' \in \{1, \cdots, K\}$ with probability weight $\pi(k')$ for each k', and the cutoff $w^*_{C,\mathcal{R}_{max}}$ is determined in the same way as w^*_C but they can be different in the equilibrium.

For a given scheme of collusive loan size captured by $\widehat{L}^C(k, m)$, the optimal deviation in terms of loan size is the one that maximizes the expected deviation profit, characterized as follows:

$$
\widehat{L}^D(k, m; \mathcal{R}_{max}) = \underset{\widehat{L} \geq \frac{1}{m}\mathcal{R}_{max}^{-\varepsilon(k)}}{\operatorname{argmax}} \left\{\left[\widehat{L} + (m-1)\widehat{L}^C(k, m; \mathcal{R}_{max})\right]^{-1/\varepsilon(k)} - 1\right\}\widehat{L}. \tag{OA.36}
$$

The benefit of deviation is the difference between the maximal expected deviation profit and the expected collusive profit without deviation, denoted by

$$
\Pi_2^D(k, m; \widehat{L}^C, \mathcal{R}_{max}) \equiv \mathbb{E}^x\left[\Pi_2^D(k, x, m; \widehat{L}^C, \mathcal{R}_{max})\right], \ \text{and} \tag{OA.37}
$$

$$
\Pi_2(k, m; \widehat{L}^C, \mathcal{R}_{max}) \equiv \mathbb{E}^x\left[\Pi_2(k, x, m; \widehat{L}^C, \mathcal{R}_{max})\right], \ \text{respectively.} \tag{OA.38}
$$

Given the collusive scheme $\widehat{L}^C(\cdot, \cdot; \mathcal{R}_{max})$ and the interest rate cap regulation captured by $\mathcal{R}_{max}$, the maximal expected deviation profit $\Pi_2^D(k, m; \widehat{L}^C, \mathcal{R}_{max})$ is achieved at the optimal deviation $\widehat{L}^D(k, m; \mathcal{R}_{max})$, that is,

$$
\begin{aligned}
&\Pi_2^D(k, m; \widehat{L}^C, \mathcal{R}_{max}) \\
&\quad = \left\{\left[\widehat{L}^D(k, m; \mathcal{R}_{max}) + (m-1)\widehat{L}^C(k, m; \mathcal{R}_{max})\right]^{-1/\varepsilon(k)} - 1\right\}\widehat{L}^D(k, m; \mathcal{R}_{max}) \\
&\qquad \times \exp\left\{\alpha(k) + \frac{1}{2}\sigma^2 + [1-\varepsilon(k)]\phi + \frac{1}{2}[1-\varepsilon(k)]^2\zeta^2\right\} A.
\end{aligned}
\tag{OA.39}
$$

Given the collusive scheme $\widehat{L}^C(\cdot, \cdot; \mathcal{R}_{max})$ and the interest rate cap regulation captured by $\mathcal{R}_{max}$, the expected collusive profit $\Pi_2(k,m;\widehat{L}^C,\mathcal{R}_{max})$ is

$$\begin{aligned}
&\Pi_2(k,m;\widehat{L}^C,\mathcal{R}_{max}) \\
&\quad = \left\{\left[m\widehat{L}^C(k,m;\mathcal{R}_{max})\right]^{-1/\varepsilon(k)} - 1\right\}\widehat{L}^C(k,m;\mathcal{R}_{max}) \\
&\qquad \times \exp\left\{\alpha(k) + \frac{1}{2}\sigma^2 + [1-\varepsilon(k)]\phi + \frac{1}{2}[1-\varepsilon(k)]^2\zeta^2\right\} A.
\end{aligned}$$

We define the IC compatible set of functionals $\widehat{L}^C(\cdot,\ ;\mathcal{R}_{max})$ as follows:

$$\begin{aligned}
\widehat{\mathcal{L}}^C(\mathcal{R}_{max}) \equiv \Bigg\{\widehat{L}^C:\ &\frac{\xi[W^C(\widehat{L}^C,\mathcal{R}_{max}) - W^N(\mathcal{R}_{max})]}{1-\delta} \geq \Pi_2^D(k,m;\widehat{L}^C,\mathcal{R}_{max}) - \Pi_2(k,m;\widehat{L}^C,\mathcal{R}_{max}), \\
&\widehat{L}^C(k,m) \geq \frac{1}{m}\mathcal{R}_{max}^{-\varepsilon(k)},\ \ \forall\ k,m\Bigg\}.
\end{aligned}$$

In the collusive Nash equilibrium we focus on, the loan size is

$$\widehat{L}^C(\cdot,\cdot) = \underset{\widehat{L}\in\widehat{\mathcal{L}}^C(\mathcal{R}_{max})}{\operatorname{argmax}}\ \mathbb{E}\left[U^C(k,x,m;\widehat{L},\mathcal{R}_{max})\right]. \tag{OA.40}$$

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